*Proposition 1: Let σ be an element of Sn. Then there exists a positive integer k such that σk = e.*

**Proof**:

There are n! possible permutations of a set of n elements. Therefore, there exists a positive integer k and n such that σk = σn and k < n.

We have σk (σ -1)k = σn (σ -1)k = σn-k = e

Therefore there exists a positive integer such that σ to that integer powered is equal to the identity element.

We are done 😊

*Proposition 2: Let k be the order of 𝜎, then iff k | m.*

**Proof**:

Let k be the order of 𝜎.

We have . Since k is the order of 𝜎, moves all elements around in a circle, which results in the same set. For , must behave the same way.

However, k is different from m that k is the smallest positive integer which satisfy so cycle all elements once. For m, it cycles the elements for some n times, and so m = kn.

Therefore, k | m.

We have k | m, therefore .

We are done 😊

*Proposition 3: Every cycle of length k has order k.*

**Proof**:

Suppose is a n-cycle which has an order of k.

Let

Assume that 0 < k < n

moves to . Since , must be equals to . However, the definition of cycle says that all are different. We see a contradiction here.

Therefore, .

Moreover, moves all in a circle one time. Since , then k = n.

We are done 😊

*Proposition 4: Disjoint cycles commute.*

**Proof**:

Let , where for all and .

Also,

The same goes for . Therefore, disjoint cycles commute.

We are done 😊

*Proposition 5: Let be a permutation of and let ~ be a relation of defined by:*

*for some*

*Prove that ~ is an equivalence relation.*

**Proof**:

Reflexive property:

Let k be the order of . That gives us

Symmetric property:

Let k be the order of . That gives us

Transitive property:

So

Let k be the order of . That gives us

Therefore

And so

Then ~ is an equivalence relation.

*Proposition 6: Every permutation can be written as the product of disjoint cycles.*

**Proof**:

Let be a permutation of S and let be the orbits of . For each , the permutation is a cycle . Since ~ is an equivalence relation, the orbits of are disjointed to each other. Also, . So , as well as any permutation, can be written as the product of disjoint cycles.

*Proposition 7: Every permutation is a product of unique transpositions.*

**Proof**:

For all cycles ,

We are done 😊

*Proposition 8: The Futurama Theorem*

**Proof**:

Let our set to be: {1, 2, 3, 4, 5, 6, … n}