**Theorem 15.** If S is a linearly independent subset of L and **P** is a point of L, not in span(S), then S ∪{**P**} is also linearly independent.

Proof:

Let and S is linearly independent. Also let and .

S is linearly independent, then:

(Theorem 12)

Now assume that

If we can show that , then S ∪{**P**} is linearly independent by Theorem 12.

Case 1:

(Adding both sides by )

So

(def. of additive inverse and axioms)

(Divide both sides by , since by assumption)

(Definition of span)

This contradicts our assumption that .

Therefore is false.

Case 2:

(our assumption of

(Theorem 12 as S is linearly independent)

is linearly independent

(Theorem 12)

Therefore, if S is a linearly independent subset of L and **P** is a point of L, not in span(S), then S ∪ {**P**} is also linearly independent. ֍

End of proof.