**Theorem 16.** S is a basis for L if and only if it is a maximal linearly independent subset of L. In other words, S is a basis for L if and only if S is linearly independent and not a proper subset of any other linearly independent set.

We can rewrite this, using the definition of basis, as:

Proof:

Assume **S** LI, and **S** spans **L**, or spanS = **L**.

Also assume . Let . We must show that **.**

(definition of span)

(our assumption of )

(Theorem 14)

**We see a contradiction here!** (We assumed **Q** is linearly independent)

Hence, if.

In other words, S = Q. ֍

Given .

Let

(Theorem 15)

**We see a contradiction here!** We assumed that S is the maximum linearly independent subset of L, but we showed that a bigger linearly independent subset exists.

Hence, .

In other words, **.** ֍

Therefore, S is a basis for L if and only if it is a maximal linearly independent subset of L. ֍

End of proof.