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MATH-440

Midterm

1.

a) I found the interpolating polynomial using 3 (excluding x=40, 72), 4 (excluding x=72), and 5 points and here are the results respectively:

So,

b)

Upper estimate:

Lower estimate:

Central estimate:

I also plugged into the spreadsheet to find for the interpolating polynomial and evaluate these polynomials on a spreadsheet.

They are closely the same even with 5, 4, or 3 points interpolating polynomial.

So,

2. Convincing Evidence

Consider where , or

We know that this function has 2 roots based on its graph. Let be the roots of

We have: , so

Using the hybrid method on [-2, 0], we know that the 1st root of is between

-1.22675838491381 and -1.22675838491380 because:

-1.22675838491381 is too low since which is too high

-1.22675838491380 is too high since which is too high

So

Using the hybrid method on [0, 1], we know that the 2nd root of is between

0.872223101666470 and 0.872223101666471 because:

0.872223101666470 is too low since which is too low

0.872223101666471 is too high since which is too high

So

Therefore, has 2 roots:

3.

a) We know that .

Let the estimate polynomial . For :

By tweaking in the above formula, we figured out the polynomial estimation for sin(x) on :

With a maximum error of

b) This estimation is slightly worse than the cubic approximation from the Taylor series, unfortunately.

The Taylor approximation has an error of

c) By finding an interpolating polynomial where:

We find

With a maximum error of

This estimation is better than both estimations we found above.

4. Convincing Evidence

Consider

Using the hybrid method on [0, 2], we know that the 1st root of is between

2.404825557695773 and 2.404825557695774 because:

|  |  |  |
| --- | --- | --- |
| Terms |  |  |
| 2 | 0.0767854 | 0.0767854 |
| 4 | 4.215952 E-4 | 4.215952 E-4 |
| 8 | 2.066278 E-10 | 2.066272 E-10 |
| 16 | 2.220446 E-16 | -4.440892 E-16 |
| 32 | 2.2204460 E-16 | -4.440892 E-16 |
| 64 | 6.6613381 E-16 | -6.661338 E-16 |

As in the table above, is too high while is too low

So

Using the hybrid method on [2, 6], we know that the 2nd root of is between

5.5200781102863 and 5.5200781102864 because:

|  |  |  |
| --- | --- | --- |
| Terms |  |  |
| 2 | 7.8899630 | 7.8899630 |
| 4 | 1.4567772 | 1.4567772 |
| 8 | 6.0907603 E-4 | 6.0907603 E-4 |
| 16 | 5.32907052 E-15 | 3.8191672 E-14 |
| 32 | -4.44089210 E-15 | 3.0198066 E-14 |
| 64 | -4.44089210 E-15 | 3.0198066 E-14 |

As in the table above, is too low while is too high

So

Therefore, the first 2 roots of are:

And