Angle-based control of directed acyclic formations with three-leaders

Minh Hoang Trinh, Kwang-Kyo Oh and Hyo-Sung Ahn
School of Mechatronics
Gwangju Institute of Science and Technology (GIST)
Distributed Control and Autonomous System Laboratory (DCASL)
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Outline

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   - Control Strategy
   - System Dynamics
   - Stability Analysis
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Formation Control And Bearing Measurements

- Formation control [1]:
  - A research topic in the realm of cooperative control.
  - Goal: Achieve a prescribed geometric formation without a centralized sensing and processing unit.
  - Classification: based on measured and controlled variables. (position-, displacement-, and distance-based schemes)
  - Background: graph theory, matrix theory, control theory, etc.
- Bearing measurements:

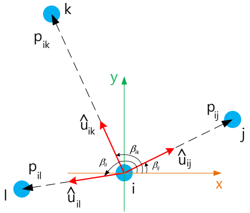


Figure 1: Agent i senses the bearing angles $\beta_{ij}, \beta_{ik}, \beta_{il}$ [2] and composes corresponding bearing vectors $\hat{u}_{ij}, \hat{u}_{ik}, \hat{u}_{il}$ w.r.t. the three agents j, k, l .

Graph Theory [4]

- Directed graph: a pair $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$: a set of nodes, $\mathcal{E}$: a set of ordered pairs of the nodes, called edges.
- A directed edge $(i, j) \in \mathcal{E}$ exists if agent i measures agent j's bearing angle. Agent j is a neighbor of agent i. $\mathcal{N}_i$: the neighbor set of i.
- Directed path: a sequence of edges $(i_1, i_2), (i_2, i_3), ..., (i_{k-1}, i_k)$, where $i_1, ..., i_k \in \mathcal{V}$. Directed cycle: a directed path with the same start vertex and end vertex.
- Acyclic directed graph: a directed graph with no directed cycle.

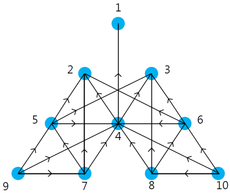


Figure 2: An acyclic directed graph representing the sensing topology of a system of ten agents

Input-to-State Stability for Cascaded System

Consider the system

$$\dot{x} = f(x, u), \tag{1}$$

where $f : D_x \times D_u \mapsto \mathbb{R}^n$ is locally Lipschitz in x and u, and $D_x \subseteq \mathbb{R}^n$ and $D_u \subseteq \mathbb{R}^m$ are domains containing $x = 0$ and $u = 0$, respectively.

Definition 1 ([5])

The system (1) is locally input-to-state stable (ISS) if there exist a class $\mathcal{KL}$ function γ, and positive constants k_x and k_u s.t. for any initial state $x(0)$ with $\|x(0)\| < k_x$ and any input $u(t)$ with $\sup_{0\leqslant\tau\leqslant t} \|u(\tau)\| < k_x$, the solution $x(t)$ exists and satisfies

$$\|x(t)\| \leqslant \beta(\|x(0)\|, t) + \gamma(\sup_{0\leqslant\tau\leqslant t} \|u(\tau)\|), \; t \geqslant 0. \tag{2}$$

Input-to-State Stability for Cascaded System

Lemma 2 ([5])

If there exists a neighborhood U of $(x = 0, u = 0)$ s.t. $f(x, u)$ in (1) is continuously differentiable and the unforced system $\dot{x} = f(x, 0)$ is asymptotically stable (a.s.) in U, then the system (1) is locally ISS.

Lemma 3 ([5])

For the cascade system

$$\dot{x}_1 = f_1(x_1, x_2), \tag{3a}$$

$$\dot{x}_2 = f_2(x_2), \tag{3b}$$

where $f_1 : D_1 \times D_2 \mapsto \mathbb{R}^{n_1}$ and $f_2 : D_2 \mapsto \mathbb{R}^{n_2}$ are locally Lipschitz in x_1 and x_2, if the system (3a), with x_2 as input, is locally ISS and the origin of the system (3b) is locally a.s., the origin of the cascade system (3) is locally a.s.

Problem Formulation

Consider the single-integrator modeled agents

$$\dot{p}_i = u_i,\ i = 1, \ldots, N. \tag{4}$$

Let $p_i^* \in \mathbb{R}^2$ be given for leader i ($i = 1, 2, 3$), and $\hat{u}_{ij}^*$, $j \in \mathcal{N}_i$ be given to follower i ($i = 4, \ldots, N$). The objective of the agents is to achieve $p_i \to p_i^*$ for $i = 1, \ldots, N$.

Assumption 1

- *Leader $i \in \{1, 2, 3\}$ measures p_i*
- *Follower $i \in \{4, 5, ..., N\}$ measures the bearing angle of its three neighbors.*
- *The bearing angle measuring topology is modeled by a directed acyclic graph $\mathcal{G}$*
- *Let agents j, k, and l be the neighbors of agent i. Then any three of p_i^*, p_j^*, p_k^*, and p_l^* are not collinear.*

Problem Formulation

Problem

Consider the single-integrator modeled agents (4). Let p_i^* be given to leader i and $\hat{u}_{ij}^*$, $j \in \mathcal{N}_i$ be given to follower i. Under the Assumption 1, design control laws for the leaders and followers such that $p^* = [{p_1^*}^T \cdots {p_N^*}^T]^T$ is asymptotically stable with respect to (4).

Proposed Control Strategy

- The control law
  - for leaders:

$$u_i = k_L(p_i^* - p_i) = k_L\tilde{p}_i, \tag{5}$$

    where $k_L > 0$.
  - for followers [3]:

$$u_i = -\sum_{j\in\mathcal{N}_i} (\hat{u}_{ij}^{*\,T}\hat{u}_{ij}^{\perp})\hat{u}_{ij}^{\perp}, \tag{6}$$

    where

$$\hat{u}_{ij} := \frac{p_{ij}}{\|p_{ij}\|} = \mathbf{1}\angle\beta_{ij}, \tag{7}$$

$$\hat{u}_{ij}^{\perp} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \hat{u}_{ij}.$$

note that in Eq. (7), **1** is the unit vector.

System Dynamics

- For $1 \leq i \leq j \leq N$, let

$$\mathcal{V}_{[i:j]} := \{i, ..., j\}, p_{[i:j]} = [p_i^T \dots p_j^T]^T.$$

- Let $\tilde{p}_i := p_i$ and consider $p_i^* = const$. The position error dynamics of the agents is

$$\dot{\tilde{p}}_i = -k_L \tilde{p}_i, i \in \mathcal{V}_{[1:3]} \tag{8a}$$

$$\dot{\tilde{p}}_i = f_i(\tilde{p}_i, \tilde{p}_{[1:i-1]}), i \in \mathcal{V}_{[4:N]}, \tag{8b}$$

where

$$f_i(\tilde{p}_i, \tilde{p}_{[1:i-1]}) := \sum_{j \in \mathcal{N}_i} (\hat{u}_{ij}^{*T} \hat{u}_{ij}^{\perp}) \hat{u}_{ij}^{\perp}.$$

Stability Analysis

Follower i can be described as

$$\dot{\tilde{p}}_i = f_i(\tilde{p}_i, \tilde{p}_{[1:i-1]}), \tag{9a}$$

$$\dot{\tilde{p}}_{[1:i-1]} = f_{[1:i-1]}(\tilde{p}_{[1:i-1]}), \tag{9b}$$

where

$$f_{[1:i-1]}(\tilde{p}_{[1:i-1]}) := \begin{bmatrix} -k_L \tilde{p}_{[1:3]} \\ f_4(\tilde{p}_4, \tilde{p}_{[1:3]}) \\ \vdots \\ f_{i-1}(\tilde{p}_{i-1}, \tilde{p}_{[1:i-2]}) \end{bmatrix} \tag{10}$$

Stability Analysis

- The stability analysis consists of 3 steps [6]:
  - *1st step*: Show that the origin $\tilde{p}_{[1:3]} = 0$ is exponentially stable wrt. $\dot{\tilde{p}}_{[1:3]} = -k_L \tilde{p}_{[1:3]}$.
  - *2nd Step*: Show that (9) is locally ISS with $\tilde{p}_{[1:i-1]}$ as input. Based on *Lemma 3*, $\tilde{p}_{[1:i]}$ is locally a.s. wrt. (9) if $\tilde{p}_{[1:i-1]} = 0$ is locally a.s. wrt. (9b).
  - *3rd Step*: Finally we show that the origin is locally a.s. wrt. $\dot{\tilde{p}}_{[1:N]} = f_{[1:N]}(\tilde{p}_i, \tilde{p}_{[1:N]})$ based on mathematical induction.
- The first step is obvious. For the second step, we show local stability of the following unforced dynamics of (9a)

$$\dot{\tilde{p}}_i = f_i(\tilde{p}_i, 0), \; i \in \mathcal{V}_{[4:N]}. \tag{11}$$

Main Results

Lemma 4

Let Assumption 1 hold. For $i \in \mathcal{V}_{[4:N]}$, the origin $\tilde{p}_i = 0$ is locally asymptotically stable with respect to (11).

Lemma 5

Let Assumption 1 hold. For $i \in \mathcal{V}_{[4:N]}$, (9a) is locally input-to-state stable with $\tilde{p}_{[1:i-1]}$ as input.

Theorem 6

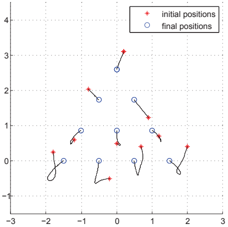
Let Assumption 1 hold. The origin $\tilde{p} = 0$ is locally asymptotically stable with respect to (8).

Proof.

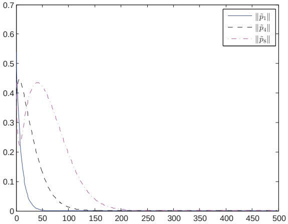
Please refer to the paper for detailed proofs. □

Simulation

- Ten agents with sensing topology as depicted in Fig. 2.
- The desired positions: $p_1^* = (0, \frac{3\sqrt{3}}{2}), p_2^* = (-\frac{1}{2}, \sqrt{3}), p_3^* = (\frac{1}{2}, \sqrt{3}), p_4^* = (0, \frac{\sqrt{3}}{2}), p_5^* = (-1, \frac{\sqrt{3}}{2}), p_6^* = (1, \frac{\sqrt{3}}{2}), p_7^* = (-\frac{1}{2}, 0), p_8^* = (\frac{1}{2}, 0), p_9^* = (-\frac{3}{2}, 0), p_{10}^* = (\frac{3}{2}, 0)$.



(a) Trajectories of the agents



(b) Position Error

Conclusion

Main contributions:

- A control strategy for N agents, with three leaders and the other followers. The leaders have position information. The followers only have three bearing measurements.
- Proving the locally asymptotically stable of the origin using mathematical induction.
- Simulation results of a system of ten agents.

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