GNSS Introduction

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Part IX

GPS SIS generation

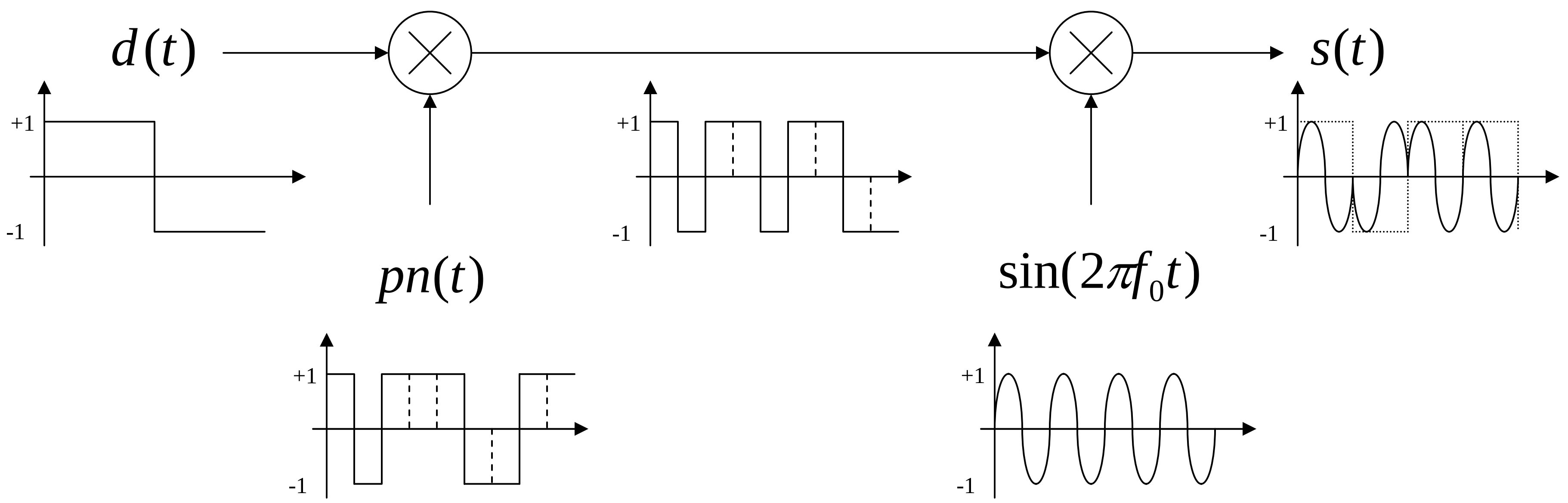
The navigation Signal-In-Space

- The signal broadcast by the navigation satellites is usually denoted as Signal-In-Space
- These signals must:
  - Allow the user to estimate the pseudodistance user-satellite
  - Carry some useful data
  - Be robust to the transmission through the atmosphere
  - Identify in a unique way the satellites

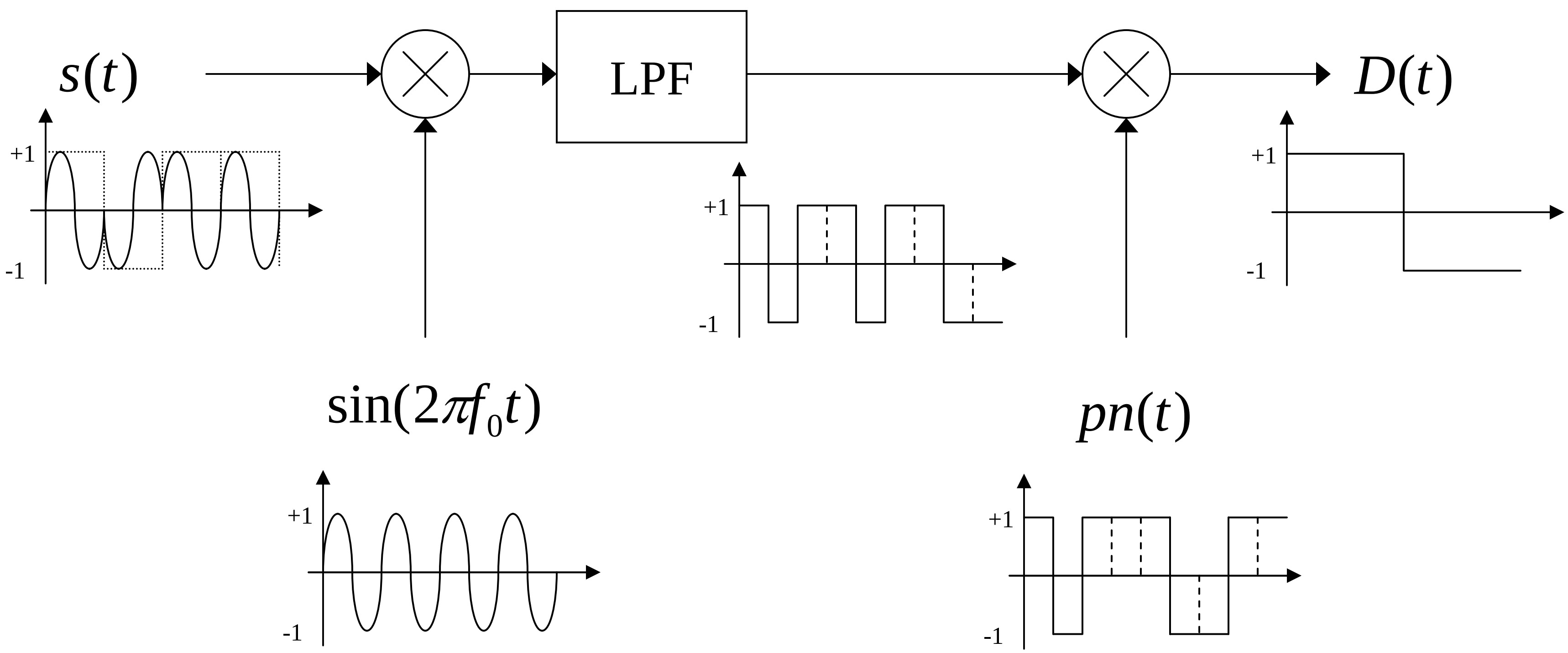
The multiple access technique

- Both GPS and Galileo use a Code-Division-Multiple-Access technique to identify the satellites without ambiguity
- Each satellite transmit a different code (ranging codes), using the same carrier frequencies, without time division
- Codes are mutually orthogonal in order to permit to the receiver to separate the signal of the satellite of interest from the others
- The data signals are modulated by the ranging codes

Direct Sequence Spread Spectrum: transmission scheme

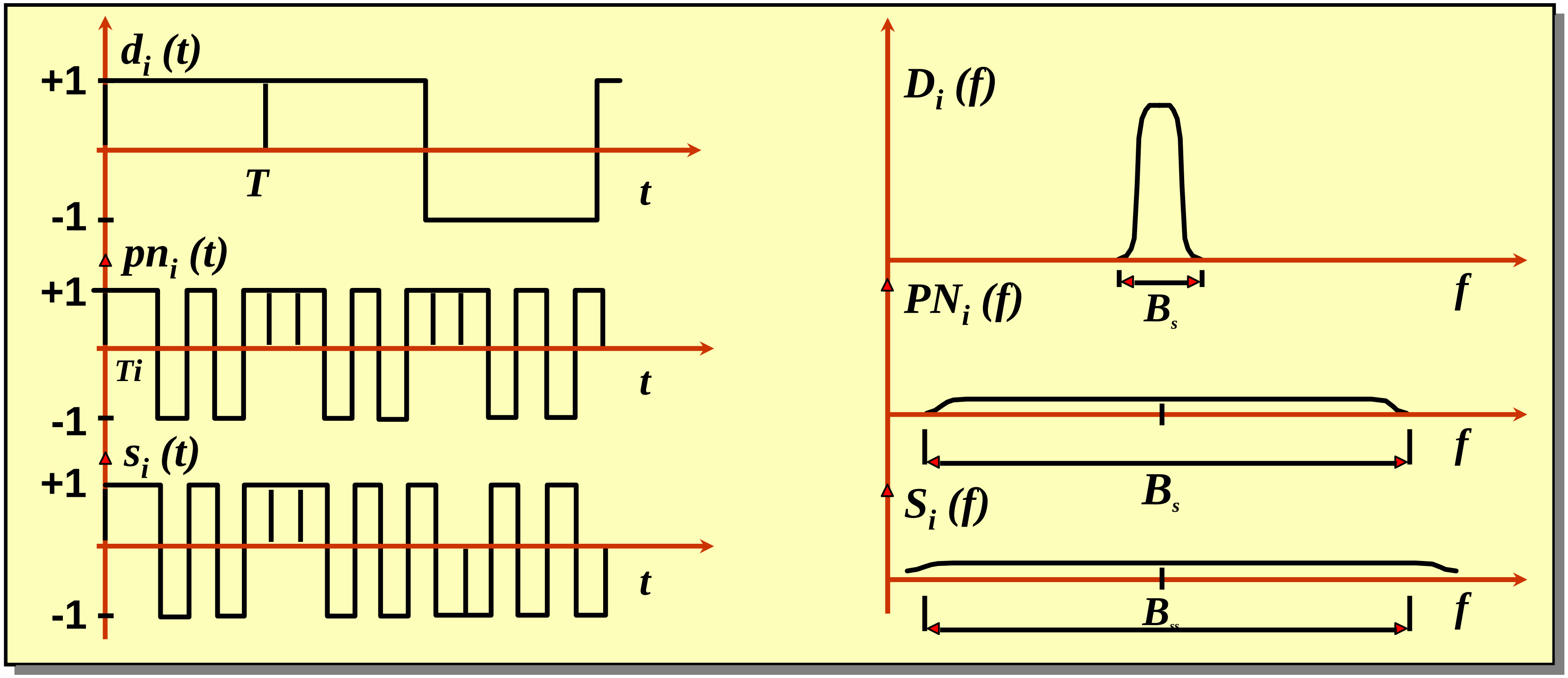


DSSS receiver



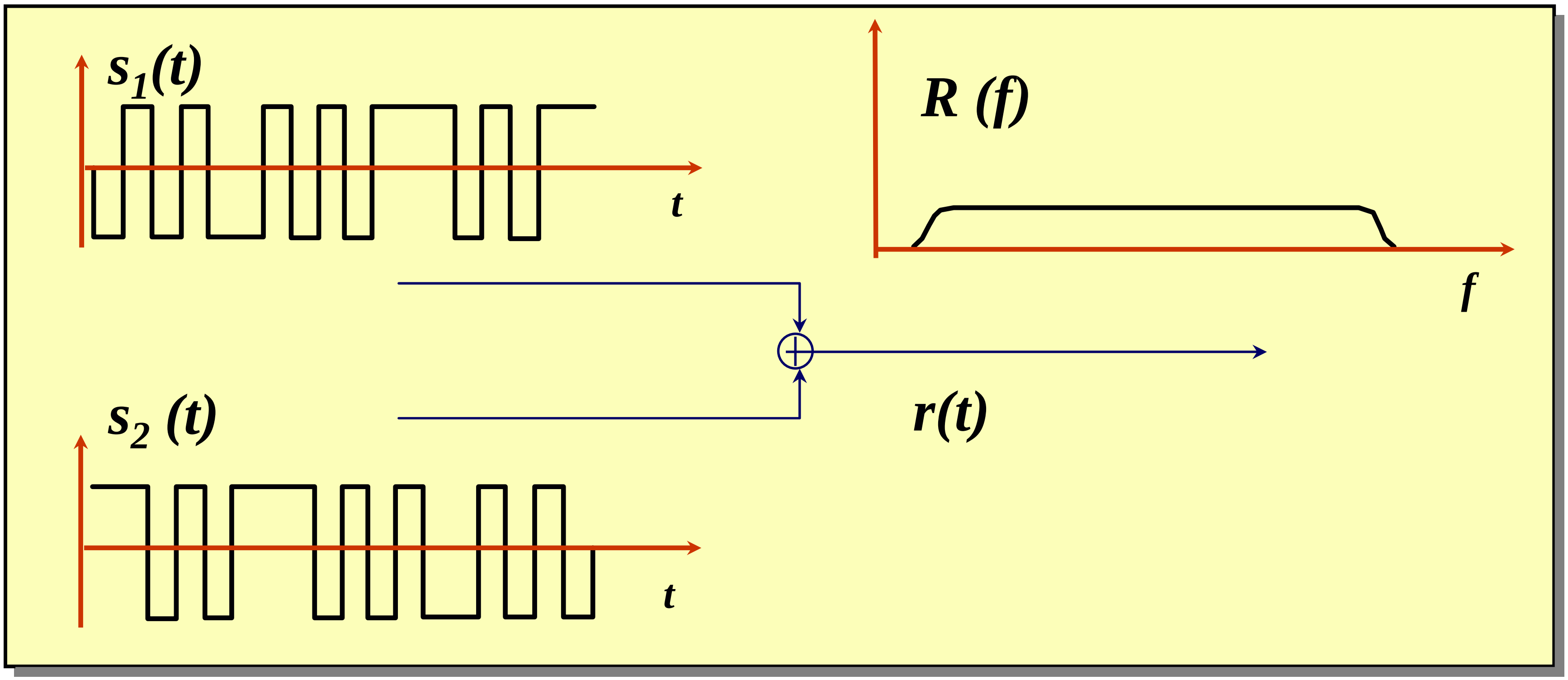
Code Division Multiple Access

- Using DSSS the signal power associated to a signal is spreaded over a larger bandwith



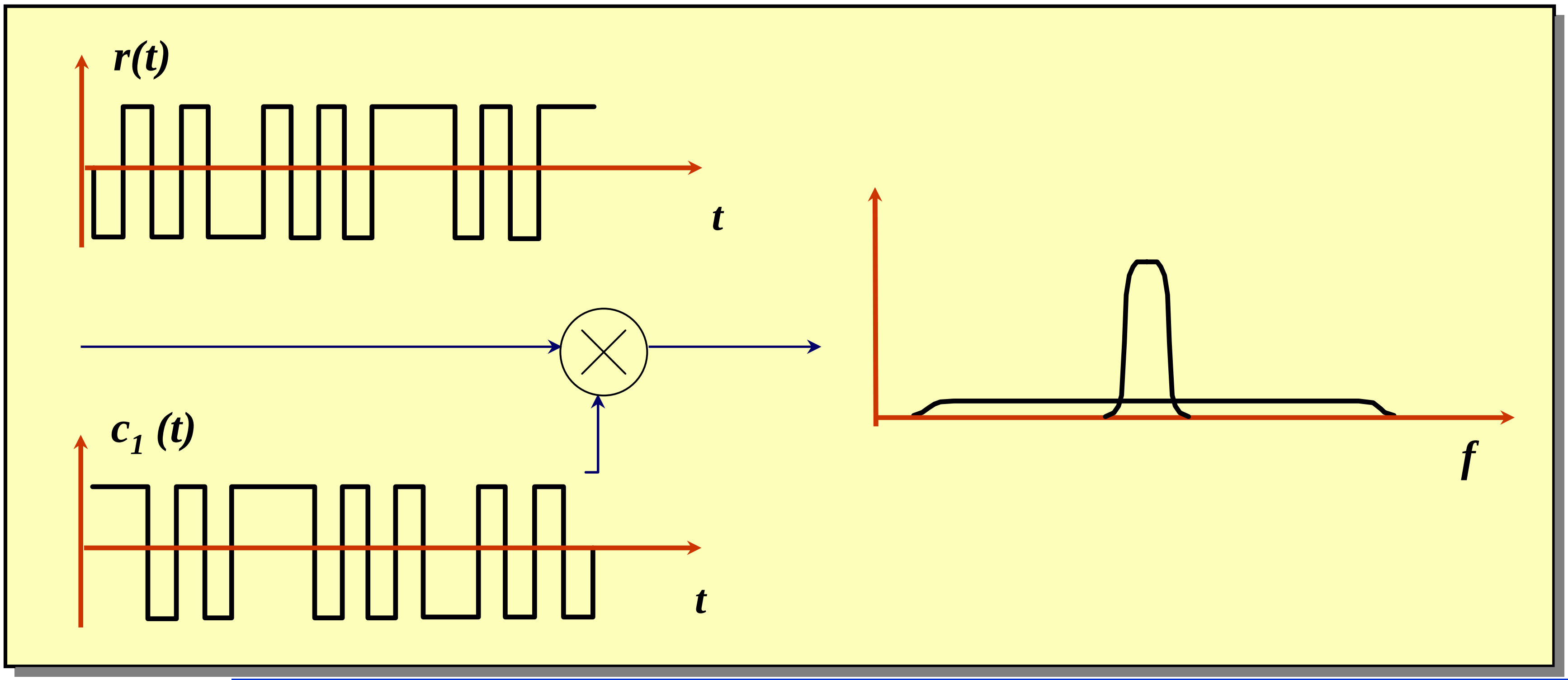
Code Division Multiple Access

- The use of ortogonal spreading sequences allows the transmission of different satellite SIS at the same time, sharing the same bandwidth



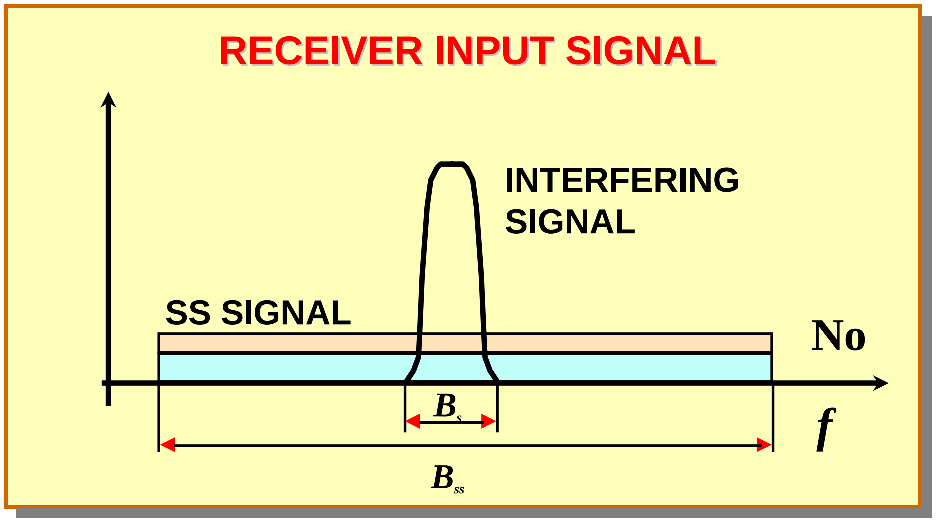
Code Division Multiple Access

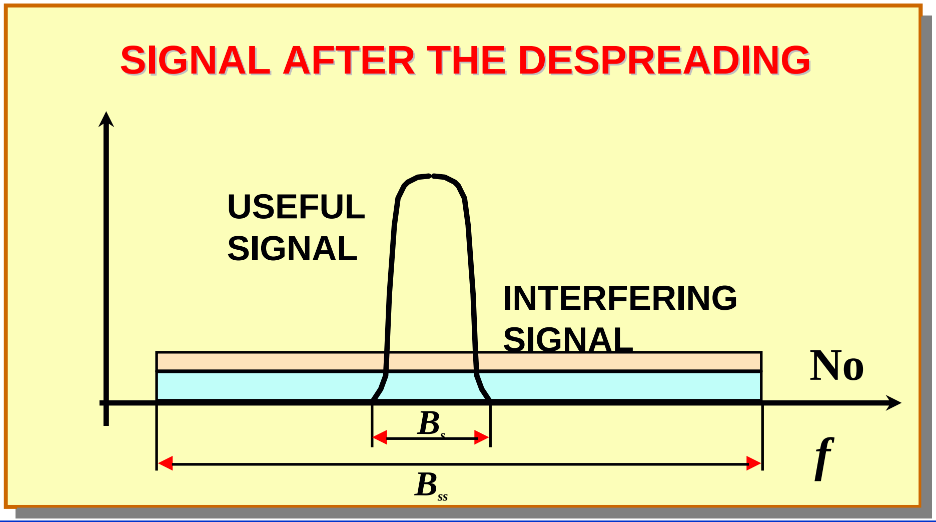
- the desired data signal can be recovered by multiplying the received signal by the spreading code of the *i*-th satellite



Code Division Multiple Access

- The CDMA is robust to the presence of interfering signals
- The despreading operation made at the receiver spreads the power of the interfering signal over a wide bandwidth





Code generation

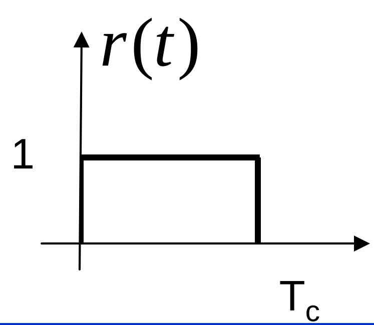
- The satellite SIS is a BPSK modulation of a periodic sequence of bits (code chips)

$$x_n \in \{\pm 1\}$$

- The code sequence of length l for satellite k is

$$x_{cod}^{(k)} = \sum_{n=0}^{l-1} x_n^{(k)} r\left(t - nT_c\right)$$

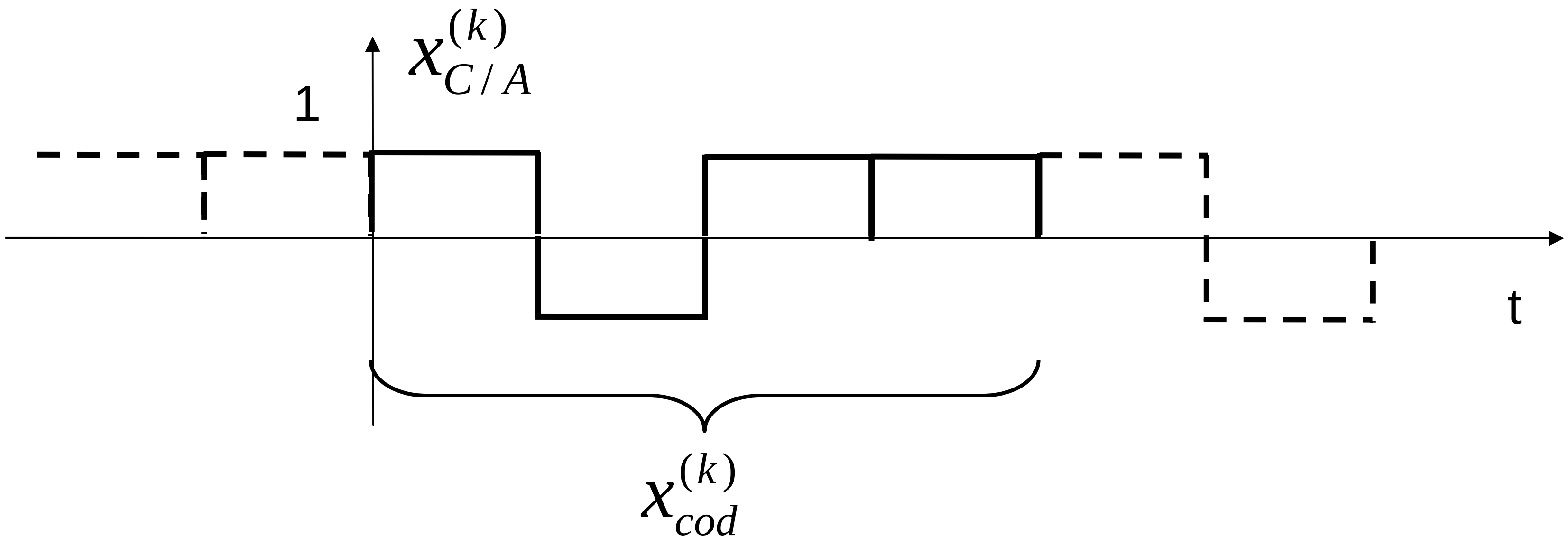
- where



Code generation

- The total transmitted code is then

$$x_{C/A}^{(k)} = \sum_{l=-\infty}^{+\infty} x_{cod}^{(k)} r(t - klT_c)$$

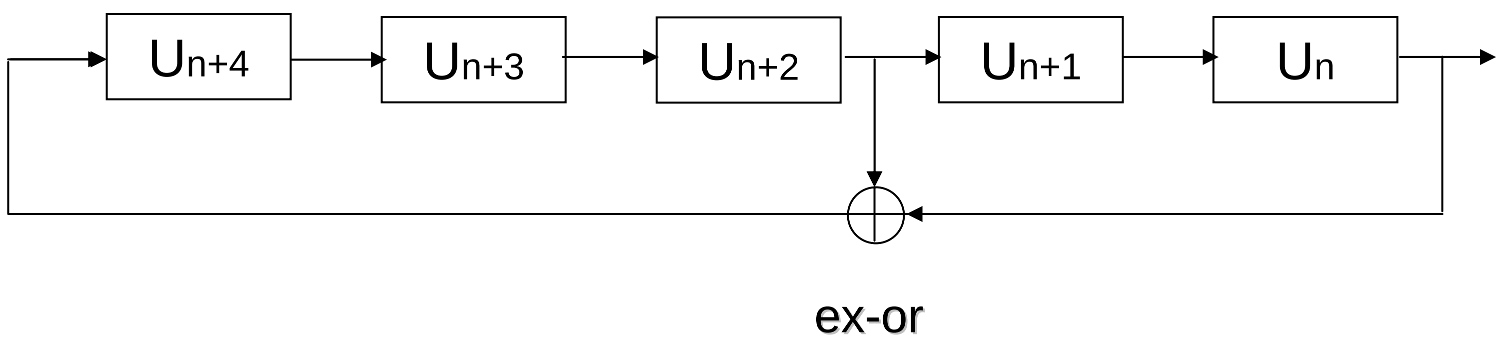


Generation of code sequences

- The code sequences can be generated through Maximal Length Linear Shift Registers (*m-sequences*)

$$u_n^{(k)} \in \{0,1\} \leftrightarrow x_n^{(k)} \in \{1,-1\}$$

- EXAMPLE 1

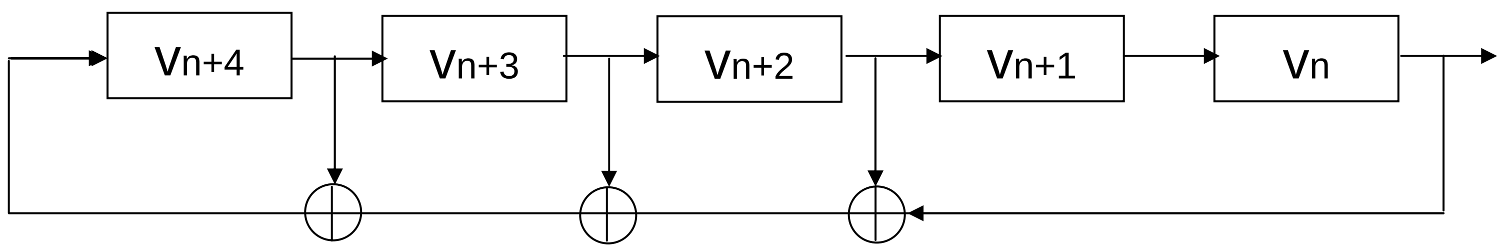


Generation of code sequences

step	u_{n+4}	u_{n+3}	u_{n+2}	u_{n+1}	u_n
1	0	0	0	0	1
2	1	0	0	0	0
3	0	1	0	0	0
4	0	0	1	0	0
5	1	0	0	1	0
6	0	1	0	0	1
7	1	0	1	0	0

Generation of m-sequences

- The shift register in the example can assume 2^5-1 states. (The all zeros state has to be avoided)
- If the register assumes all the possible states a *maximum length sequence* is obtained
- EXAMPLE II



- Different positions of the ex-ors give (or not) maximum length sequences

m-sequences: properties

- Shifted versions of the sequence are orthogonal

$$\sum_{n=1}^{p} x_n x_{(n+m)\bmod p} = 0$$

- Each sequence has length 2^m-1 bits
- Each sequence contains 2^{m-1} ones and 2^{m-1}-1 zeros
- The ex-or of a sequence and of a cyclically shifted version, is another shifted version of the same sequence

ML-sequences: properties

- Consider the notation:

$$\mathbf{u} = \{u_n\}_{n=0}^{p-1} \qquad \mathbf{v} = \{v_n\}_{n=0}^{p-1}$$

- where $S^m\mathbf{v}$ is a cyclical shift of *m* digits of the sequence **v**

Gold codes

- A family of codes with good correlation properties can be generated using two different sequences of the same length

$$\mathbf{u} \oplus S^{m}\mathbf{v} = \left\{ u_n \oplus v_{n-m} \right\}$$

- For a length p a family of p+2 Gold sequences can be generated
- In GPS p=1023; sequences for the satellites are chosen among the 1025 Gold sequences

GPS Signal in Space

It is common to describe the design of linear code generators by means of polynomials of the form

$$\boxed{1+\sum_i b^i}$$

where b^i means that the output of the i-th cell of the shift register is used at the input of the modulo-2 adder (ex-or)

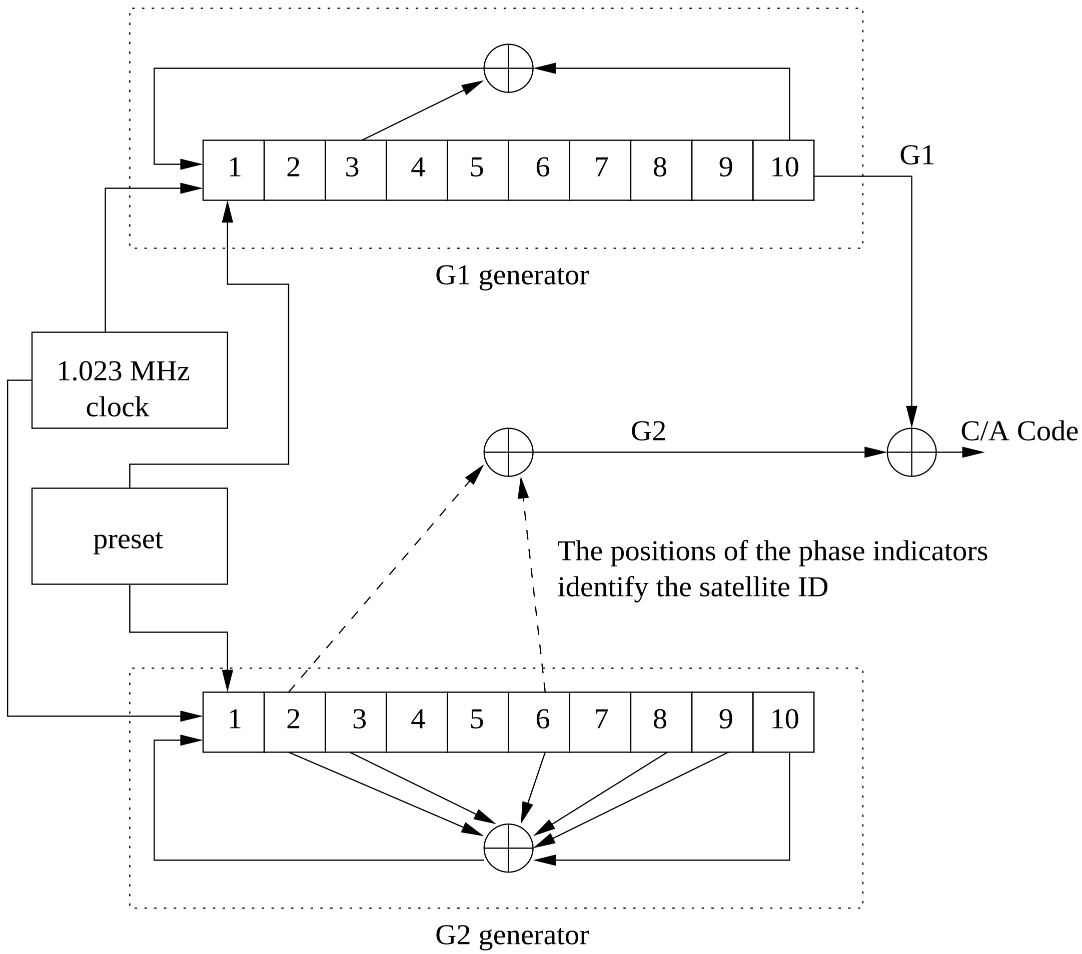
GPS C/A code generation

- Two sequences generate all the C/A codes for the satellites

$$G1:1+b^3+b^{10}$$
$$G2:1+b^2+b^3+b^6+b^8+b^9+b^{10}$$

- Each satellite is characterized by a different shifted version of G2
- The delay effect of the G2 output is obtained by ex-or of selected positions of two taps,
- This is possible thanks to the properties of the m-sequences that the sum of a sequence and of a shfted one is another shift of the same sequence

C/A Code generation



GPS Signal in Space

Satellite spreading codes

Satellite ID	PRN Code	Phase Selector
1	1	2 – 6
2	2	3 – 7
3	3	4 – 8
4	4	5 – 9
5	5	1 – 9
6	6	2 – 10
7	7	1 – 8
8	8	2 – 9
9	9	3 – 10
10	10	2 – 3
11	11	3 – 4
12	12	5 – 6

Satellite ID	PRN Code	Phase Selector
13	13	6 – 7
14	14	7 – 8
15	15	8 – 9
16	16	9 – 10
17	17	1 – 4
18	18	2 – 5
19	19	3 – 6
20	20	4 – 7
21	21	5 – 8
22	22	6 – 9
23	23	1 – 3
24	24	4 – 6

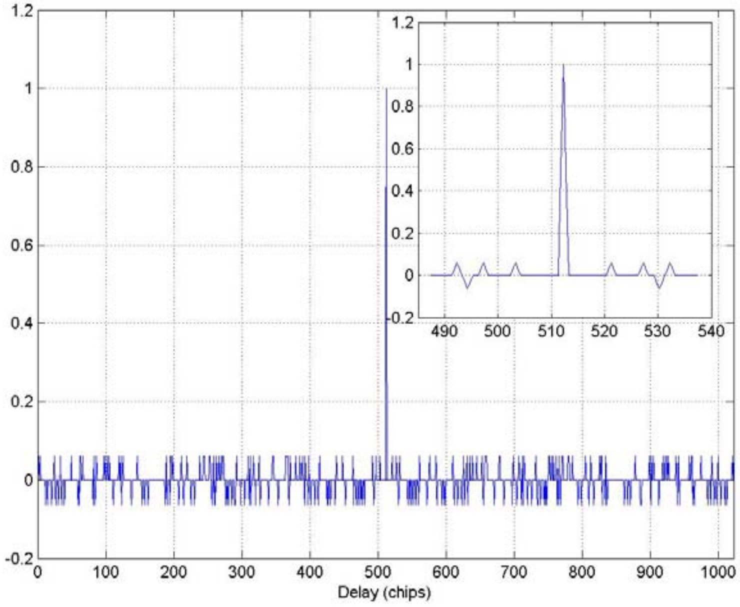
Satellite ID	PRN Code	Phase Selector
25	25	5 – 7
26	26	6 – 8
27	27	7 – 9
28	28	8 – 10
29	29	1 – 6
30	30	2 – 7
31	31	3 – 8
32	32	4 – 9
-	33	5 – 10
-	34	4 – 10
-	35	1 – 7
-	36	2 – 8
-	37	4 – 10

Gold codes properties

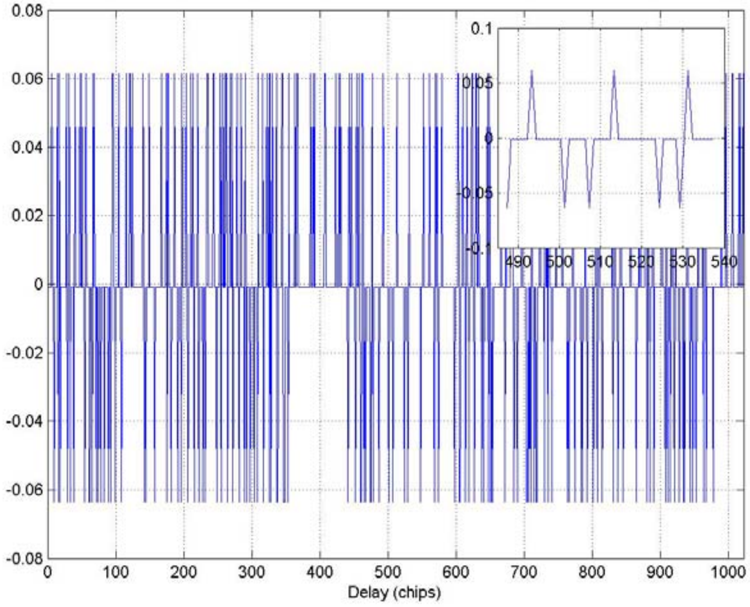
Code Period	*Shift Register Stage*	*Normalized level and statistic Frequency of Cross-Correlation*
p=2^n-1	**n=odd**	$\begin{cases} -\frac{2^{(n+1)/2}+1}{p} & 0.25 \\ -\frac{1}{p} & 0.5 \\ \frac{2^{(n+1)/2}-1}{p} & 0.24 \end{cases}$
p=2^n-1	**n=even**	$\begin{cases} -\frac{2^{(n+2)/2}-1}{p} & 0.125 \\ -\frac{1}{p} & 0.75 \\ \frac{2^{(n+2)/2}-1}{p} & 0.125 \end{cases}$

- C/A code: n=10 p=1023
- Cross-correlation values:
  - -65/1023 (12.5%), -1/1023 (75%), 63/1023 (12.5%)

C/A code properties



Autocorrelation of satellite 16



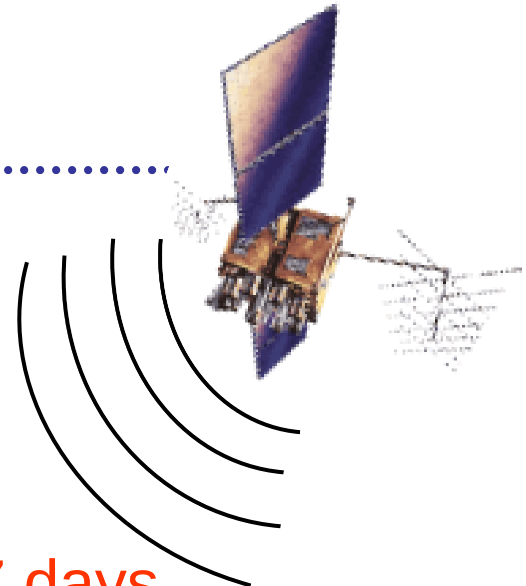
Cross-correlation between satellites 16 and 27

GPS C/A code

- Unique sequence of 1023 bits (*chips*)
- Repeated each millisecond
- Duration of each chip =1/1.023 MHz = 997.5 ns ~ 1µs
- *Chip width or wavelength* ~ 300 m

Chipping rate = 1.023 MHz or Mchips/s (Mcps)

GPS Signal in Space



Signal structure

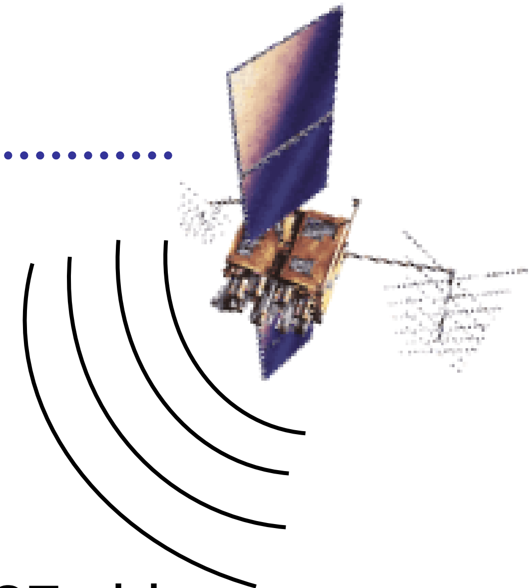
Ranging code

P(Y) code

- Unique segment of an extremely long PRN sequence (~ 10^{14} chips lasting 267 days, split into 37 sections of 7 days each)
- Repeated after one week
- *Chip width or wavelength* ~ 30 m
- Smaller wavelength = greater precision in range measurements w.r.t. C/A-codes

Chipping rate = 10.23 Mcps

GPS Signal in Space



Signal structure

Ranging code

P(Y) code

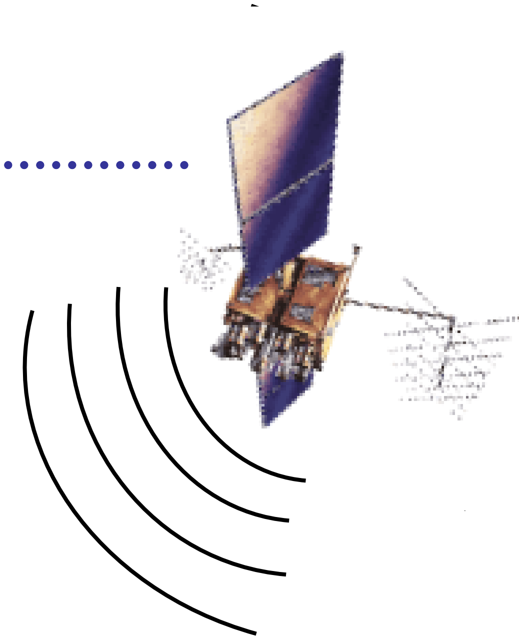
- The 7 days sequence is the modulo-2 sum of two sub-sequences (X1 and X2i)
- X1 = 15345000 chips and X2i =15345037 chips
- X2i is a X2 sequence selectively delayed by 1 to 37 chips
- This technique leads to a set of 37 mutually exclusive P Code sequences of 7 days in length

32 are designed for use by SVs, while the remaining 5 are reserved for other purpose (e.g. ground transmitter)

GPS Signal in Space

Signal structure

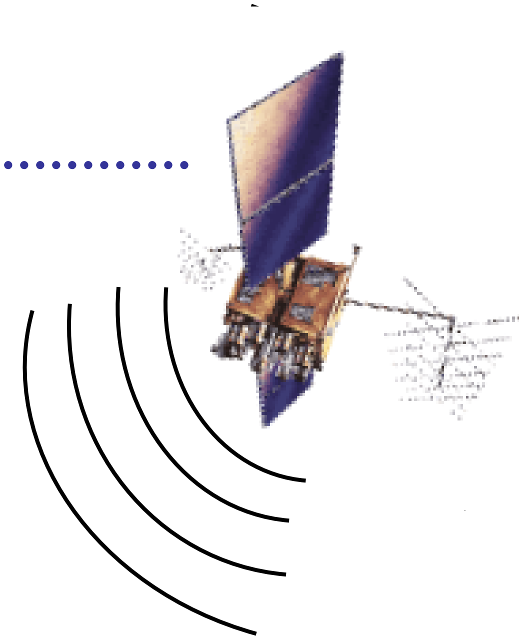
On L_1 2 signals
- 1 for civil user
- 1 for DoD-authorized users



On L_2 1 signal for DoD-authorized users only

- Other RF signals are transmitted at frequencies L_3 and L_4
- These signals are related to classified payloads on the satellites

GPS Signal in Space



Signal structure

Each signal consists of 3 components:

Carrier RF sinusoidal signal with frequency f_{L1} and f_{L2}

Ranging code Each satellites transmits 2 codes

- ✓ Coarse/Acquisition code (C/A)
- ✓ Precision [encrypted] code (P) [(Y)]

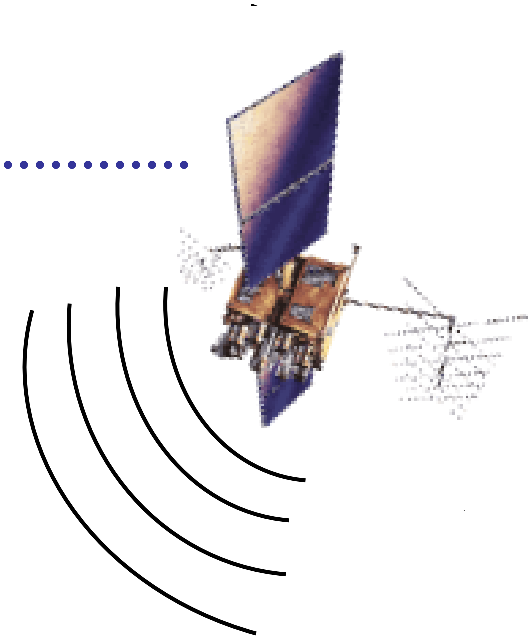
based on the unique PRN code assigned to each satellite

GPS Signal in Space

Signal structure

Ranging code

P(Y) code

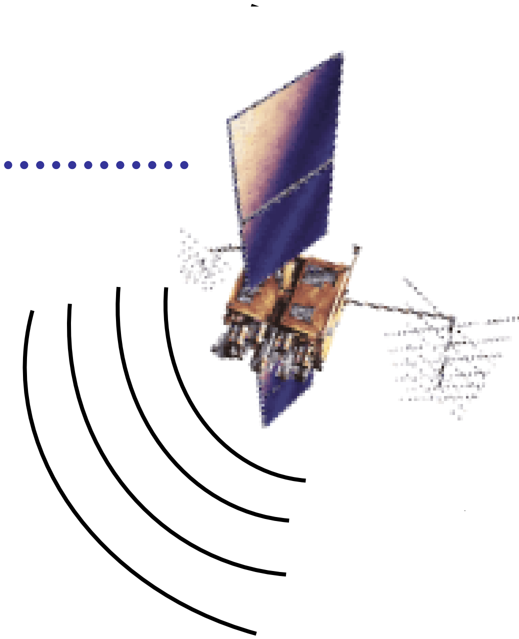


- GPS has the option to replace the P-Code with a secure Y-Code (for authorized U.S. Government users)
- Y-Code is employed when the AS (*AntiSpoof*) mode of operation is activated (since 1994)

The main purpose of the Y-Code is to assure that an opponent cannot spoof the Y-Code signal generating a Y-Code replica

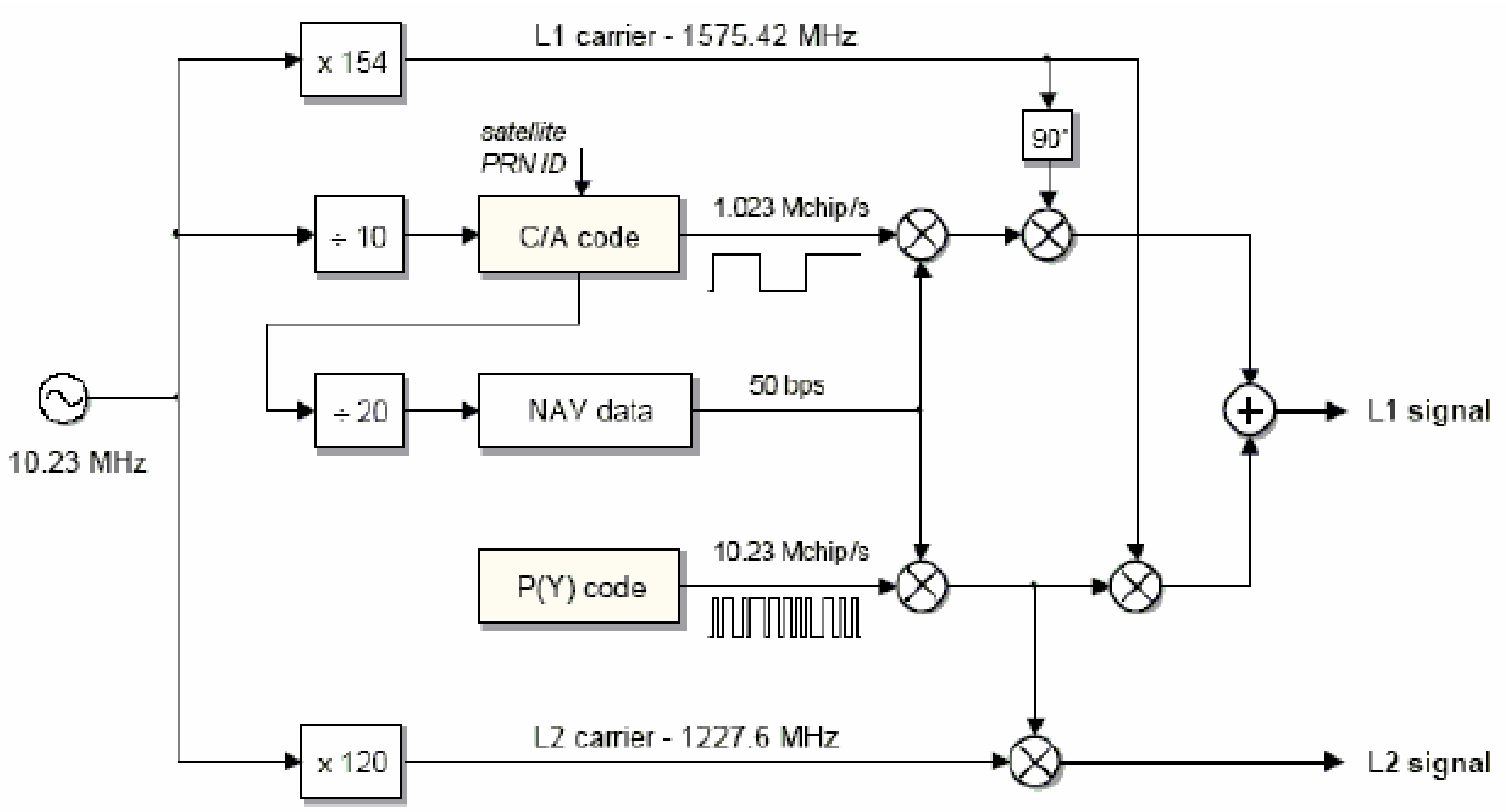
GPS Signal in Space

Signal assembly



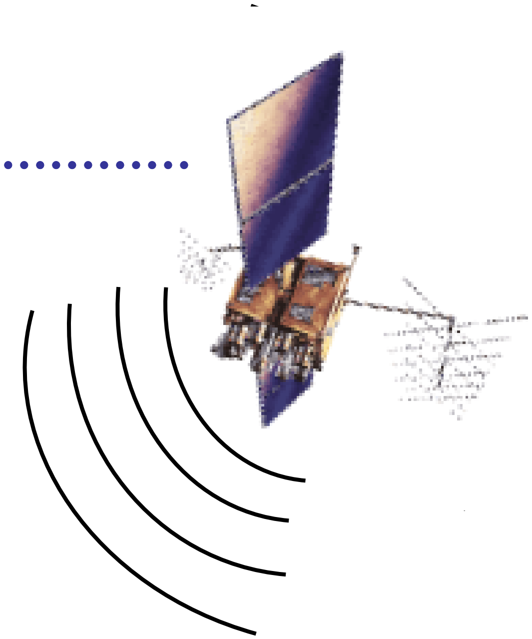
- C/A and P code are combined with binary navigation data with modulo-2 adders
- During a 20-ms data bit duration the C/A code repeats 20 times
- The resulting signal modulates the carrier with a BPSK scheme (Binary Phase Shift Keying)

On board signal generation



GPS Signal in Space

Signal structure

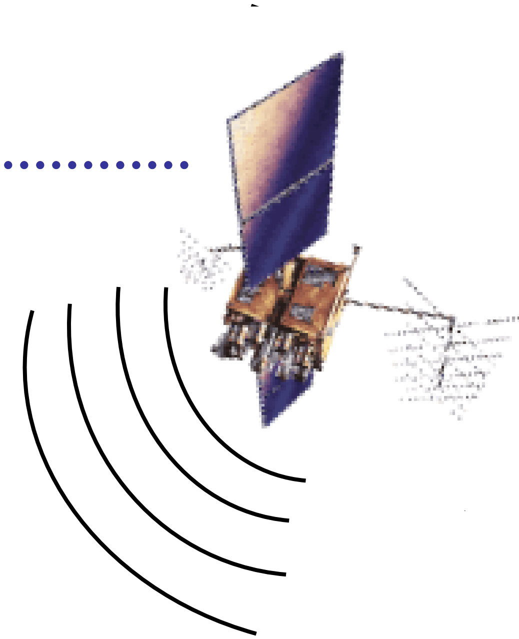


Navigation data Binary-coded message with data on the satellite's:

- ✓ health status
- ✓ ephemeris (position and velocity)
- ✓ clock bias parameters

and with an almanac giving reduced-precision ephemeris data on all satellites in the constellation

Navigation data



Signal structure

Navigation data

- Bit rate = 50 bits/s (bps)
- Bit duration = 20 ms
- Navigation message: 30 bits = 1 word

  10 words = 1subframe = 300 bits

  (6-second duration)

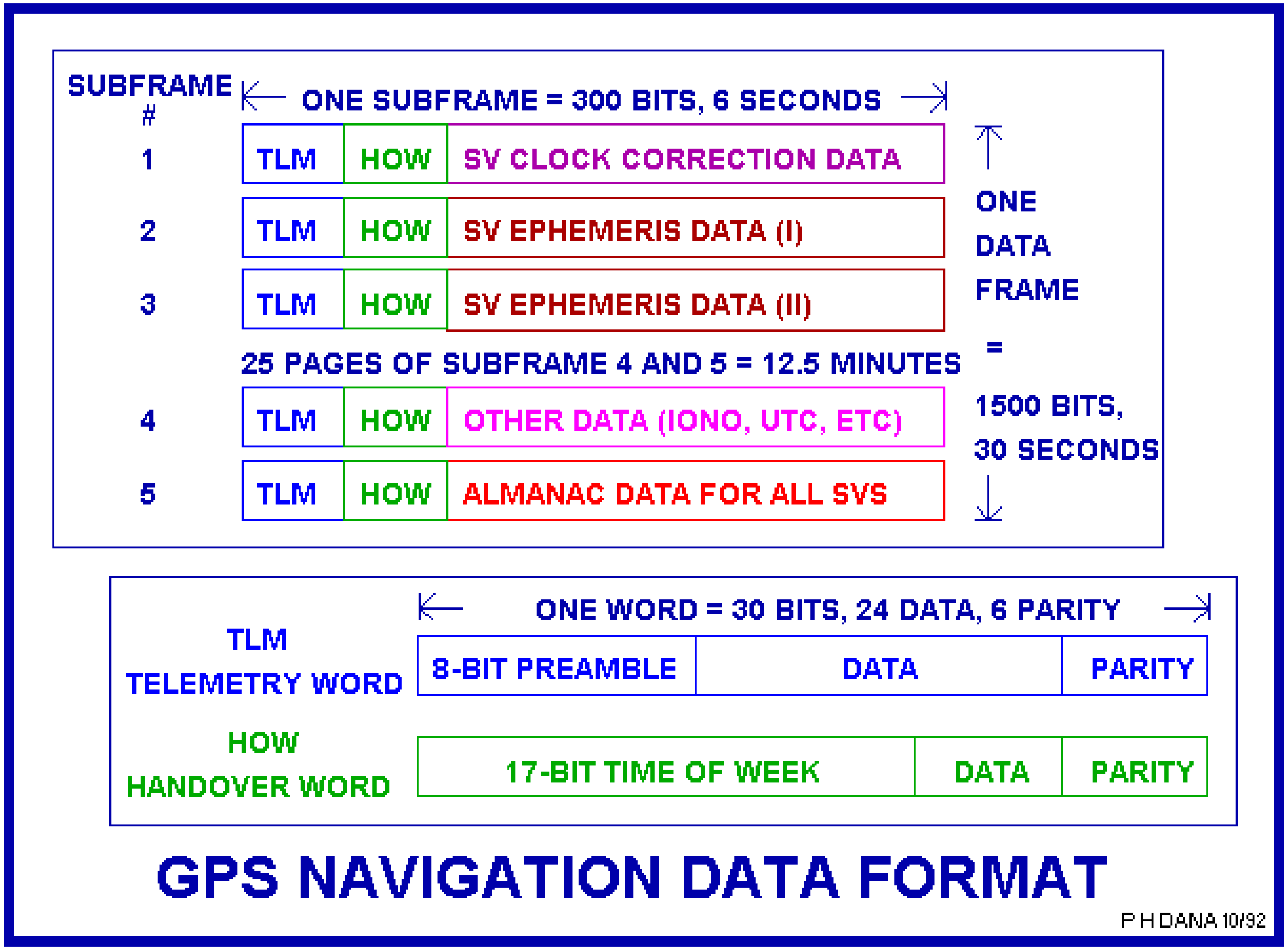
  1 Frame (or page) = 5 subframes = 1500 bits (30-second duration)

  1 superframe = 25 frames (12.5 m)

GPS Signal in Space

Signal structure

Navigation data



GPS Signal in Space

GPS signal in the time domain

The signal broadcasted by antenna of the *k*th satellite is:

$$s^{(k)}(t) = \sqrt{2P_C}\, x^{(k)}(t)\, D^{(k)}(t) \sin\left(2\pi f_{L1} t + \theta_{L1}\right) + \\ \sqrt{2P_{Y1}}\, y^{(k)}(t)\, D^{(k)}(t) \cos\left(2\pi f_{L1} t + \theta_{L1}\right) + \\ \sqrt{2P_{Y2}}\, y^{(k)}(t)\, D^{(k)}(t) \cos\left(2\pi f_{L2} t + \theta_{L2}\right)$$

P_C, P_{Y1}, P_{Y2} signal powers

$D(t)$ navigation data stream (±1)

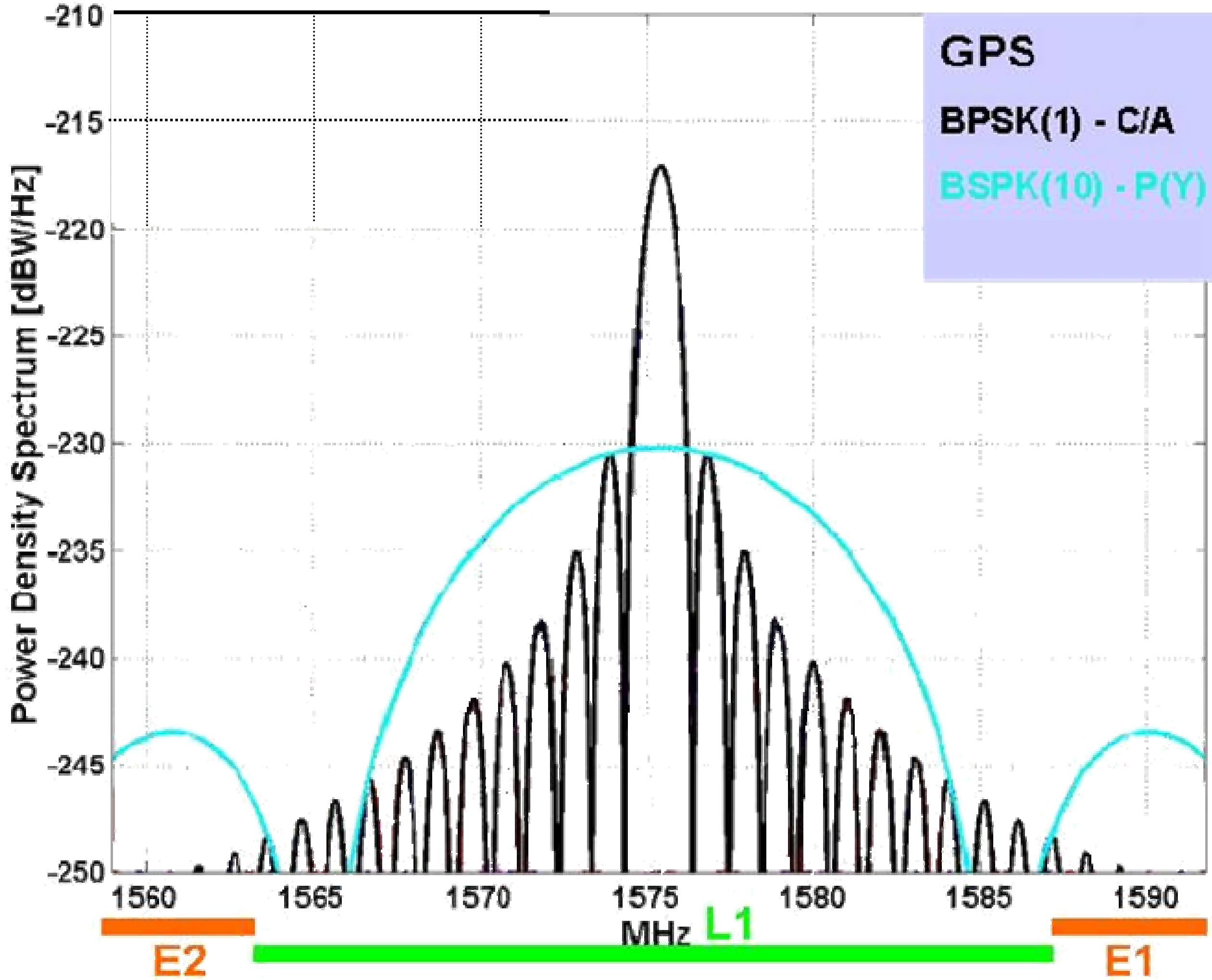
$x(t)$ C/A-code sequence (±1)

θ_{L1}, θ_{L2} phase offsets

$y(t)$ P-code sequence (±1)

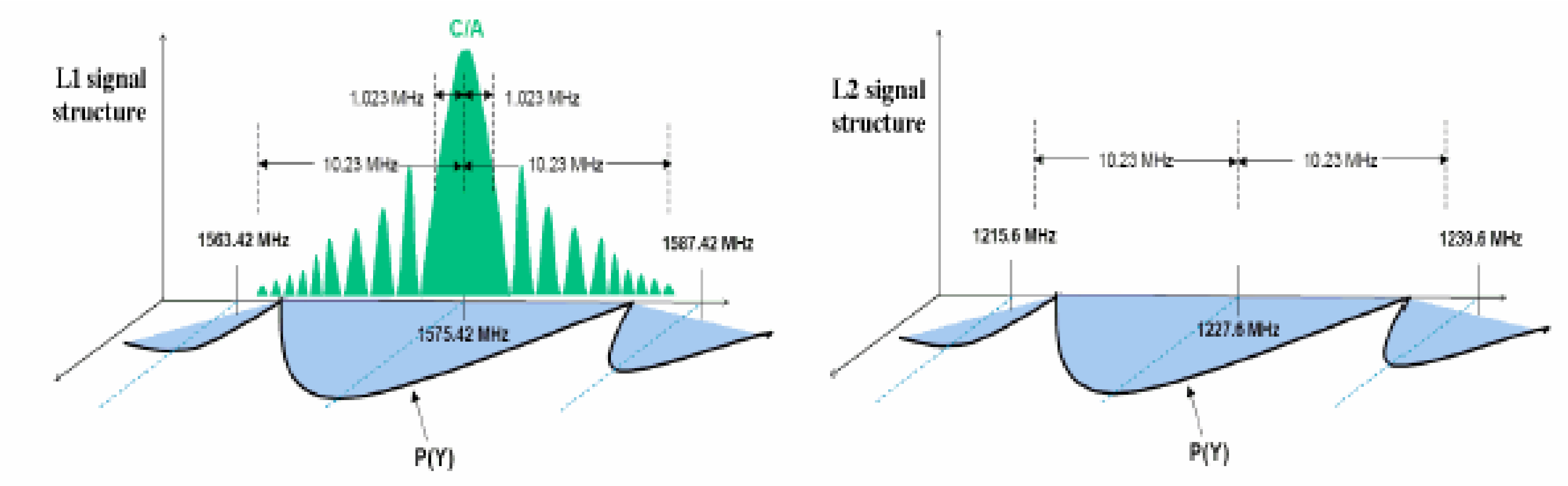
GPS Signal in Space

GPS signal in the frequency domain



GPS Signal in Space

GPS signals in the frequency domain



End of Part IX