CHRIST (Deemed to be University), Bengaluru – 560 029

MID SEMESTER EXAMINATION - August 2018

(Semester-5)

PROGRAMME NAME: BSc | **MAX. MARKS: 50**
COURSE NAME: NUMBER THEORY | **TIME: 2 Hours**
COURSE CODE: MAT541C

INSTRUCTIONS

- **All rough work should be done in the answer script. Do not write or scribble in the question paper except your register number.**
- **Verify the Course code / Course title & number of pages of questions in the question paper.**
- **Make sure your mobile phone is switched off and placed at the designated place in the hall**
- **Malpractices will be viewed very seriously**.
- **Answers should be written on both sides of the paper in the answer booklet. No sheets should be detached from the answer booklet.**
- **Answers without the question numbers clearly indicated will not be valued. No page should be left blank in the middle of the answer booklet.**

Part A – Answer any Four Questions. $4 \times 3 = 12$ Marks

1. If $a|bc$ and $\gcd(a, b) = 1$, then show that $a|c$.
2. Show that the sum of squares of two odd numbers is not a perfect square.
3. If $p, q_1, q_2, q_3, \dots, q_n$ are all primes and p divides $p|a_i$ for some $i, 1 \le i \le n$.
4. What is the remainder when $1^5 + 2^5 + 3^5 + \cdots + 100^5$ is divisible by 4?
5. If a is an odd integer, then show that $a^2 \equiv 1 \pmod 8$.
6. Let $P(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function of x with integer coefficients and let $a \equiv b \pmod n$. Then show that $P(a) \equiv P(b) \pmod n$.

Part B – Answer any Four Questions. $4 \times 7 = 28$ Marks.

7. For $n \ge 1$, verify that 43 divides $6^{n+2} + 7^{2n+1}$.
8. Use Euclidean algorithm to determine the GCD and LCM of 272 and 1479.
9. Determine all integer solutions of the Diophantine equation $56x + 72y = 40$.
10. Show that the number $\sqrt{2}$ is irrational.
11. Using the theory of congruences, verify that $89|2^{44} - 1$ and $97|2^{48} - 1$
12. Use Chinese Remainder Theorem, solve the following system of linear congruences.

$$x \equiv 2 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 2 \pmod{7}$$

Part C – Answer any One Question. $1 \times 10 = 10$ Marks

13. Given integers a and b, not both of which are zero, show that there exist integers x and y such that $\gcd(a, b) = ax + by$. Hence, show that the set $T = \{ax + by: x, y \text{ are integers}\}$ is the set of all multiples of $\gcd(a, b)$.

14. Given an integer $b > 1$, show that any positive integer N can uniquely be written in terms of the powers of b as $N = a_m b^m + a_{m-1} b^{m-1} + a_{m-2} b^{m-2} + \cdots + a_2 b^2 + a_1 b + a\_0$.

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