CHRIST (Deemed to be University), Bengaluru – 560 029
MID SEMESTER EXAMINATION - August 2018
(Semester-5)

PROGRAMME NAME: BSc   **MAX. MARKS: 50**
COURSE NAME: NUMBER THEORY   **TIME: 2 Hours**
COURSE CODE: MAT541C

INSTRUCTIONS

- **All rough work should be done in the answer script. Do not write or scribble in the question paper except your register number.**
- **Verify the Course code / Course title & number of pages of questions in the question paper.**
- **Make sure your mobile phone is switched off and placed at the designated place in the hall**
- **Malpractices will be viewed very seriously**.
- **Answers should be written on both sides of the paper in the answer booklet. No sheets should be detached from the answer booklet.**
- **Answers without the question numbers clearly indicated will not be valued. No page should be left blank in the middle of the answer booklet.**

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Part A – Answer any Four Questions. $4 \times 3 = 12$ Marks.

1. Show that $a|b$ and $b|a$ if and only if $a = \pm b$.
2. If d is the GCD of two integers a and b, then show that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.
3. Does the Diophantine equation $14x + 35y = 93$ has integer solutions? Justify your answer.
4. If p is a prime and $p|ab$, then show that $p|a$ or $p|b$.
5. For arbitrary integers a and b, show that $a \equiv b \pmod{n}$ if and only if a and b leave same remainder when divided by n.
6. Find the remainder when $1! + 2! + 3! + 4! + \cdots + 100!$ is divided by 5.

Part B – Answer any Four Questions. $4 \times 7 = 28$ Marks

7. If $a|c$, $b|c$, and $\gcd(a, b) = 1$, then show that $ab|c$.
8. Show that $\gcd(a, b). \operatorname{lcm}(a, b) = ab$, for any two positive integers a and b.
9. Find x and y where $\gcd(1769, 2378) = 1769\,x + 2378\,y$.
10. If p is a prime number, then show that $\sqrt{p}$ is irrational.
11. If $ca \equiv cb \pmod{n}$, then show that $a \equiv b \pmod{\frac{n}{d}}$, where $d = \gcd(c, n)$.

12. Find the remainders when 2^{50} and 41^{65} are divided by 7.

Part C – Answer any One Question. $1 \times 10 = 10$ Marks.

13. A customer bought a dozen pieces of fruits – apples and oranges – for Rs. 132/−. If an apple costs Rs. 3/− more than an orange and more apples than oranges were bought, how many pieces of each kind were bought?

14. State and prove the fundamental theorem of arithmetic.

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