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Uncertainty Analysis for CFD Simulations of Flow over Plate and around Foil with OpenFOAM

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1 INTRODUCTION

The Computational Fluid Dynamics (CFD) is applied to a wide range of research and engineering problems, including aerodynamics, thermodynamics, ocean and naval architecture engineering and so on. It is necessary to asses and make clear the accuracy and reliability of the numerical results. The object of this paper is to do the solution verification exercises for OpenFOAM simulations and compare the uncertainty results with different estimators.

Many studies have been carried out to estimate numerical uncertainties in CFD simulations. A simple algorithm, Grid Convergence Index (GCI), derived by Roache [1] is widely used. In the work of Celik et al. [2] and Cosner et al. [3], GCI method was recommended for estimating the uncertainties due to grid-spacing and time-step. To improve the quality of uncertainty estimation, a variable factory of safety according to the distance of solution from the asymptotic range was introduced in Factor of Safety method (FS) by Xing and Stern [4]. It was applied successfully in the verification for the beam bending problem. For oscillatory converged results, the Least Squared Root method (LSR) was proposed by Eça and Hoekstra [5]. This method adopted weight to increase the influence of those solutions near the extrapolated values.

In this paper, the three methods were applied to estimate the uncertainties in 2D simulations of flow over plate and an foil with OpenFOAM due to grid spacing and turbulence modelling.

2 UNCERTAINTY ESTIMATION

2.1 Iteration Error Estimation

The iterative error is the difference between the value of flow quantity ϕ at n^{th} iteration and the exact value ϕ_{exact} . When the non-linear system meets the convergence criteria, the final value, ϕ_{final} , is usually considered as the exact solution. The change between consecutive iterations is considered as a proper measure for the iterative error:

$$e_i = \phi_i^n - \phi_{exact} \tag{1}$$

$$e_i = \phi_i^{final-1} - \phi_i^{final} \tag{2}$$

where e_i is the iterative error, ϕ_i^{final} is the flow quantity in the final iteration step and $\phi_i^{final-1}$ is that at the step before the final one.

However, these quantities are not reliable since the value at the last step is an estimate of the exact value [6]. Another estimator of the iterative error uses the solution converged to machine accuracy [7]. Based on this work, four levels of the convergence tolerance, e_t , were tested in the present studies: $10^{-4}, 10^{-6}, 10^{-8}$ and 10^{-14}. The last one corresponds to machine accuracy.

The L_∞ norm is also studied in the present work:

$$L_\infty(\Delta\phi) = max(\Delta\phi_i) \quad 1 \leq i \leq N_P \tag{3}$$

where N_P stands for the total number of nodes of a given grid and $\Delta\phi$ denotes the local change of the flow quantity ϕ, $\Delta\phi$ is the difference between the solution and machine accuracy. Furthermore, two special norms are also examined:

$$L_1(\Delta\phi) = \frac{\sum_{i=1}^{N_P} |\Delta\phi_i|}{N_P} \quad L_2(\Delta\phi) = \sqrt{\frac{\sum_{i=1}^{N_P} |\Delta\phi_i|^2}{N_P}} \tag{4}$$

Note that L_1 and L_2 are equal to the mean value of $\Delta\phi_i$ and to the root mean square of $\Delta\phi$, respectively.

2.2 Solution Verification

Solution verification is to estimate the numerical uncertainty, U_ϕ, of a solution, ϕ_i for which the exact solution, ϕ_{exact}, is unknown. The goal is to define an interval that contains the exact solution with a 95% confidence.

$$\phi_i - U_\phi \leq \phi_{exact} \leq \phi_i + U_\phi \tag{5}$$

Using the generalized Richardson extrapolation [8], the discretization error, ε can be estimated as:

$$\varepsilon \simeq \delta_{RE} = \phi_i - \phi_0 = \alpha h_i^p \tag{6}$$

where ϕ_i stands for any integral of local quantity, ϕ_0 is the estimate of the exact solution, α is a constant, h is the typical cell size, and p is the observed order of accuracy.

3 SIMULATION CASES

3.1 Flow over Plate

The computational domain of the flow over a flat plate is a rectangle of length $1.5L$ and width $0.25L$. The inlet is located $0.25L$ upstream of the leading edge of the plate and the outlet is placed $0.25L$ downstream of the trailing edge of the plate. The outer boundary is located $0.25L$ away from the surface of the plate. The Cartesian coordinate system has the origin at the leading edge of the plate and the x axis aligned with the plate. Figure 1 shows the grid with $r_i = 8$.

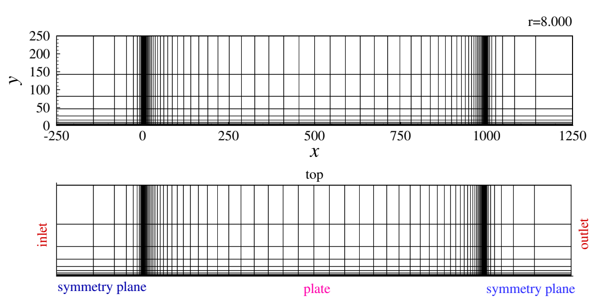

Figure 1: The grid of flow over a plate with $r_i = 8$

3.2 Flow around NACA0012

The grid of the flow around NACA0012 is shown in Figure 2, where c is the chord of the foil. The inlet is a semicircle with $R = 12c$ and the outlet is placed $13c$ downstream of the trailing edge of the foil. The external boundary is located approximately $12c$ away from the foil.

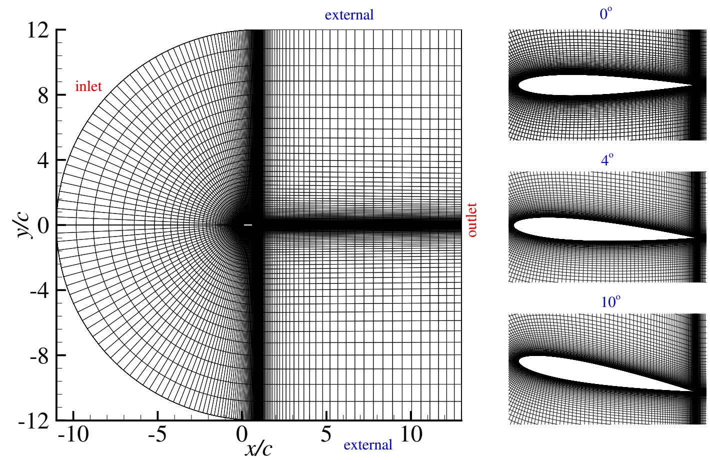

Figure 2: An example of the grid for flow around NACA0012

4 RESULTS

The iterative errors for flow over a plate with the Reynolds number of 10^8 are first presented, then the results of discretization errors are discussed.

4.1 Iteration Errors

For $e_t = 10^{-4}, 10^{-6}$ and 10^{-8}, L_∞, L_1 and L_2 norms of the iterative errors of the nondimensional horizontal velocity, U_x, were computed. The Iterative errors of the horizontal velocity with different turbulence models are presented in Figs. 3 and 4 as a function of the grid refinement ratio.

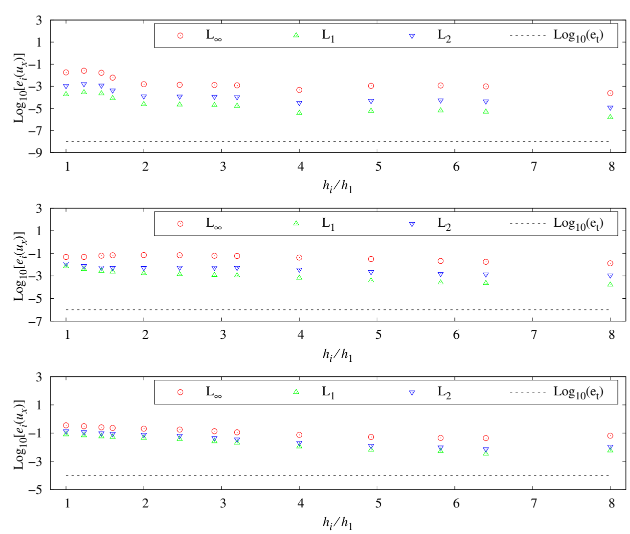

Figure 3: Iterative error of the horizontal velocity with SST $k-\omega$

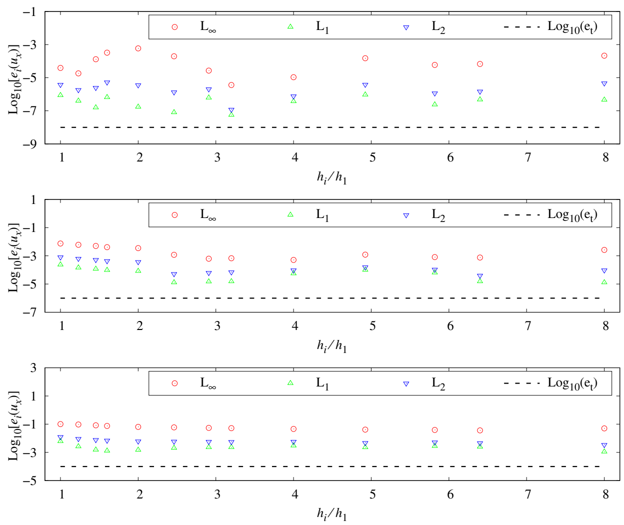

Figure 4: Iterative error of the horizontal velocity with SA

4.2 Discretization Errors

The discretization error can be calculated by:

$$e_d = \phi_i - \phi_0 \tag{7}$$

where the exact value, ϕ_0 , is evaluated by LSR. At the same time, the extrapolated curves can be calculated by using this uncertainty estimator and the error bars at grid ratios of r_i = 1, 2, 4 and 8 are included.

Figures 5 and 6 present the convergence of the friction drag coefficient, C_f, with the grid refinement for different levels of the convergence criteria by using two turbulence models.

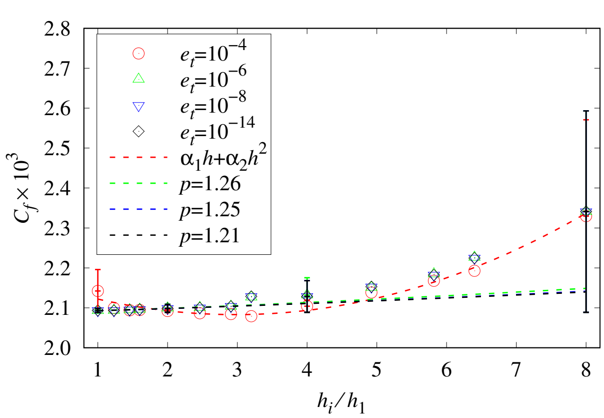

Figure 5: Convergence of the friction drag coefficient for different levels of the convergence criteria with the SA turbulence model

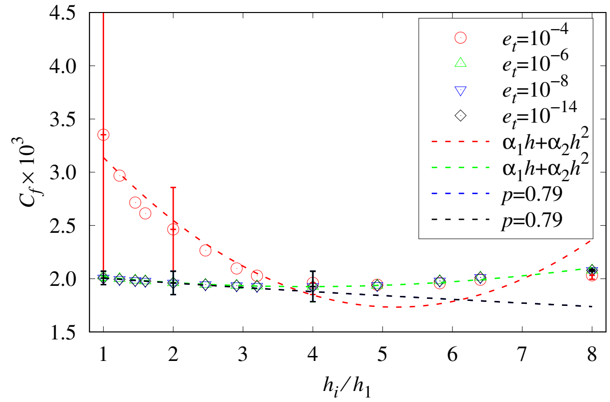

Figure 6: Convergence of the friction drag coefficient for different levels of the convergence criteria with SST $k-\omega$

4.3 Uncertainty results

As an example, the convergence of drag coefficient and corresponding uncertainty for flow around NACA0012 at angle of attack of 0° with SA are shown in Figs. 7 and 8, respectively.

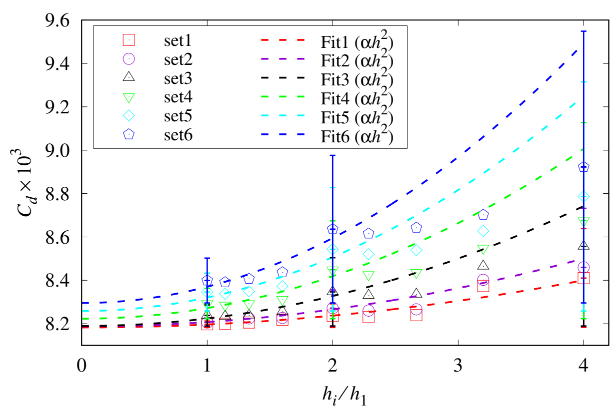

Figure 7: Convergence of the drag coefficient for flow around NACA0012 at angle of attack of 0° with SA

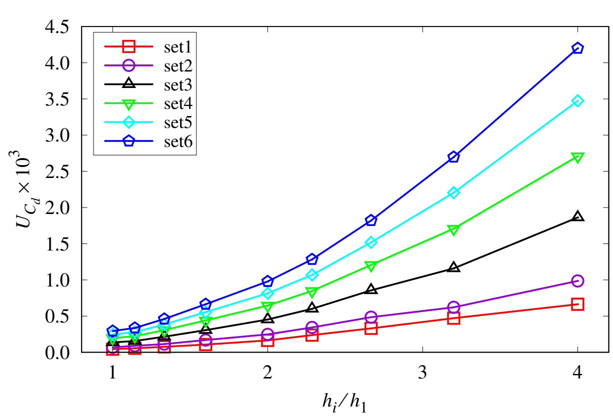

Figure 8: Uncertainty of the drag coefficient for flow around NACA0012 at angle of attack of 0° with SA

5 CONCLUSION

This paper presents solution verification exercises for the flow over a flat plate and flow around the NACA0012.

The results show that an iteration error study is required before the estimation of the discretization error. In terms of uncertainty estimation, LSR method has higher stability than GCI method and FS method, especially for the oscillatory converged results. When y^+ less than 1, SST $k-\omega$ model is more sensitive to grid than Spalart-Allmaras model in OpenFOAM.

REFERENCES

[1] P. J. Roache. Verification and validation in computational science and engineering. *Hermosa Albuquerque, NM*, **895**, 1998.

[2] I.B. Celik, U. Ghia, P.J. Roache, C.J. Freitas, H. Coleman, P.E. Raad, I. Celik, C. Freitas, H.P. Coleman. Procedure for estimation and reporting of uncertainty due to discretization in CFD applications. *Journal of Fluids Engineering*, **130**(7):078001, 2008.

[3] R. Cosner, B. Oberkampf, C. Rumsey, C. Rahaim, T. Shih. AIAA Committee on standards for computational fluid dynamics: status and plans. In *44th AIAA Aerospace Sciences Meeting and Exhibit*, **889**, 2006.

[4] T. Xing, F. Stern. Factors of safety for Richardson extrapolation. *Journal of Fluids Engineering*, **132**(6), 2010.

[5] L. Eça, M. Hoekstra. An uncertainty estimation exercise with the finite-difference and finite-volume versions of parnassos. In *Workshop on CFD Uncertainty Analysis*, **6**, 2004.

[6] L. Eça, M. Hoekstra. On the influence of the iterative error in the numerical uncertainty of ship viscous flow calculations. In *26th Symposium on Naval Hydrodynamics*, 17–22, 2006.

[7] L. Eça, G.N.V.B. Vaz, M. Hoekstra. A verification and validation exercise for the flow over a backward facing step. *Proceedings of the ECCOMAS CFD*, 2010.

[8] L.F. Richardson. IX. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, **210**(459-470):307–357, 1911.