

1. keys = {127, 372, 437, 452, 622, 647, 241}

use double hashing to resolve collision

hash function $h(k, i) = (h_1(k) + i \times h_2(k)) \pmod{10}$

$$= [k \pmod{10} + i \times (5 - k \pmod{5})] \pmod{10}$$

Initial table:

0	1	2	3	4	5	6	7	8	9

① hash 127

$$h_1(127) = 127 \% 10 = 7 \quad h_2(127) = 5 - 127 \% 5 = 5 - 2 = 3$$

$$h(127, 0) = (7 + 0 \times 3) \% 10 = 7 \quad \text{insert } 127 \text{ at index 7}$$

								127	
0	1	2	3	4	5	6	7	8	9

② hash 372

$$h_1(372) = 372 \% 10 = 2 \quad h_2(372) = 5 - 372 \% 5 = 5 - 2 = 3$$

$$h(372, 0) = (2 + 0 \times 3) \% 10 = 2 \quad \text{insert } 372 \text{ at index 2}$$

		372						127	
0	1	2	3	4	5	6	7	8	9

③ hash 437

$$h_1(437) = 437 \% 10 = 7 \quad h_2(437) = 5 - 437 \% 5 = 5 - 2 = 3$$

$$h(437, 0) = (7 + 0 \times 3) \% 10 = 7 \quad \text{collision } i++$$

$$h(437, 1) = (7 + 1 \times 3) \% 10 = 0 \quad \text{insert } 437 \text{ at index 0}$$

437	372						127		
0	1	2	3	4	5	6	7	8	9

④ hash 452

$$h_1(452) = 452 \% 10 = 2 \quad h_2(452) = 5 - 452 \% 5 = 5 - 2 = 3$$

$$h(452, 0) = (2 + 0 \times 3) \% 10 = 2 \quad \text{collision } i++$$

$$h(452, 1) = (2 + 1 \times 3) \% 10 = 5 \quad \text{insert } 452 \text{ at index 5}$$

437	372			452			127		
0	1	2	3	4	5	6	7	8	9

⑤ hash 622

$$h_1(622) = 622 \% 10 = 2 \quad h_2(622) = 5 - 622 \% 5 = 5 - 2 = 3$$

$$h(622, 0) = (2 + 0 \times 3) \% 10 = 2 \quad \text{collision } i++$$

$$h(622, 1) = (2 + 1 \times 3) \% 10 = 5 \quad \text{collision } i++$$

$$h(622, 2) = (2 + 2 \times 3) \% 10 = 8 \quad \text{insert } 622 \text{ at index 8}$$

437	372			452	127	622			
0	1	2	3	4	5	6	7	8	9

⑥ insert 647

$$h_1(647) = 647 \% 10 = 7 \quad h_2(647) = 5 - 647 \% 5 = 5 - 2 = 3$$

$$h(647, 0) = (7 + 0 \times 3) \% 10 = 7 \quad \text{collision } i++$$

$$h(647, 1) = (7 + 1 \times 3) \% 10 = 0 \quad \text{collision } i++$$

$$h(647, 2) = (7 + 2 \times 3) \% 10 = 3 \quad \text{insert } 647 \text{ at index 3}$$

437	372	647		452	127	622			
0	1	2	3	4	5	6	7	8	9

⑦ insert 241

$$h_1(241) = 241 \% 10 = 1 \quad h_2(241) = 5 - 241 \% 5 = 5 - 1 = 4$$

$$h(241, 0) = (1 + 0 \times 4) \% 10 = 1 \quad \text{insert } 241 \text{ at index 1}$$

437	241	372	647		452	127	622		
0	1	2	3	4	5	6	7	8	9

2a. key₄ = { 11, 34, 26, 78, 53, 95, 32, 44, 24, 166, 56 }

use linear probing to resolve collision

hash function $h(k) = (5k+2) \% 11$

initial table

0	1	2	3	4	5	6	7	8	9	10

① hash 11

$$h(11) = (5 \times 11 + 2) \% 11 = 2 \quad \text{insert 11 at index 2}$$

		11								
0	1	2	3	4	5	6	7	8	9	10

② hash 34

$$h(34) = (5 \times 34 + 2) \% 11 = 7 \quad \text{insert 34 at index 7}$$

		11					34			
0	1	2	3	4	5	6	7	8	9	10

③ hash 26

$$h(26) = (5 \times 26 + 2) \% 11 = 0 \quad \text{insert 26 at index 0}$$

26	11					34				
0	1	2	3	4	5	6	7	8	9	10

④ hash 78

$$h(78) = (5 \times 78 + 2) \% 11 = 7 \quad \text{collision}$$

insert 78 at index 8

26	11					34	78			
0	1	2	3	4	5	6	7	8	9	10

⑤ hash 53

$$h(53) = (5 \times 53 + 2) \% 11 = 3 \quad \text{insert 53 at index 3}$$

26	11	53				34	78			
0	1	2	3	4	5	6	7	8	9	10

⑥ hash 95

$$h(95) = (5 \times 95 + 2) \% 11 = 4 \quad \text{insert 95 at index 4}$$

26	11	53	95			34	78			
0	1	2	3	4	5	6	7	8	9	10

⑦ hash 32

$$h(32) = (5 \times 32 + 2) \% 11 = 8 \quad \text{collision}$$

insert 32 at index 9

26	11	53	95			34	78	32		
0	1	2	3	4	5	6	7	8	9	10

⑧ hash 44

$$h(44) = (5 \times 44 + 2) \% 11 = 2 \quad \text{collision}$$

insert 44 at index 5

26	11	53	95	44		34	78	32		
0	1	2	3	4	5	6	7	8	9	10

⑨ hash 24

$$h(24) = (5 \times 24 + 2) \% 11 = 1 \quad \text{insert 24 at index 1}$$

26	24	11	53	95	44		34	78	32	
0	1	2	3	4	5	6	7	8	9	10

⑩ hash 166

$$h(166) = (5 \times 166 + 2) \% 11 = 7 \quad \text{collision}$$

insert 166 at index 10

26	24	11	53	95	44		34	78	32	166
0	1	2	3	4	5	6	7	8	9	10

⑪ hash 56

$$h(56) = (5 \times 56 + 2) \% 11 = 7 \quad \text{collision}$$

insert 56 at index 6

26	24	11	53	95	44	56	34	78	32	166
0	1	2	3	4	5	6	7	8	9	10

2b. keys = { 11, 34, 26, 78, 53, 95, 32, 44, 24, 166, 56 }

use quadratic probing to resolve collision

hash function $h(k) = (3k+2) \% 11$

initial table

0	1	2	3	4	5	6	7	8	9	10

① hash 11

$$h(11) = (3 \times 11 + 2) \% 11 = 2 \quad \text{insert 11 at index 2}$$

		11								
0	1	2	3	4	5	6	7	8	9	10

② hash 34

$$h(34) = (3 \times 34 + 2) \% 11 = 7 \quad \text{insert 34 at index 7}$$

		11					34			
0	1	2	3	4	5	6	7	8	9	10

③ hash 26

$$h(26) = (3 \times 26 + 2) \% 11 = 0 \quad \text{insert 26 at index 0}$$

26	11					34				
0	1	2	3	4	5	6	7	8	9	10

④ hash 78

$$h(78) = (3 \times 78 + 2) \% 11 = 7 \quad \text{collision}$$

move to $7+1=8$ insert 78 at index 8

26	11					34	78			
0	1	2	3	4	5	6	7	8	9	10

⑤ hash 53

$$h(53) = (3 \times 53 + 2) \% 11 = 7 \quad \text{insert 53 at index 7}$$

26	11	53				34	78			
0	1	2	3	4	5	6	7	8	9	10

⑥ hash 95

$$h(95) = (3 \times 95 + 2) \% 11 = 4 \quad \text{insert 95 at index 4}$$

26	11	53	95			34	78			
0	1	2	3	4	5	6	7	8	9	10

⑦ hash 32

$$h(32) = (3 \times 32 + 2) \% 11 = 8 \quad \text{collision}$$

move to $8+1=9$ insert 32 at index 9

26	11	53	95			34	78	32		
0	1	2	3	4	5	6	7	8	9	10

⑧ hash 44

$$h(44) = (3 \times 44 + 2) \% 11 = 2 \quad \text{collision}$$

move to $2+1=3 \times 2+4=6$ insert 44 at index 6

26	11	53	95		44	34	78	32		
0	1	2	3	4	5	6	7	8	9	10

⑨ hash 24

$$h(24) = (3 \times 24 + 2) \% 11 = 1 \quad \text{insert 24 at index 1}$$

26	24	11	53	95		44	34	78	32	
0	1	2	3	4	5	6	7	8	9	10

⑩ hash 166

$$h(166) = (3 \times 166 + 2) \% 11 = 7 \quad \text{collision}$$

move to $7+1=8 \times 7+4=10+1=0 \times 7+9=5$ insert at index 5

26	24	11	53	95	166	44	34	78	32	
0	1	2	3	4	5	6	7	8	9	10

⑪ hash 56

$$h(56) = (3 \times 56 + 2) \% 11 = 7 \quad \text{collision}$$

$8 \times 0 \times 5 \times$, insert at index 10

26	24	11	53	95	166	44	34	78	32	56
0	1	2	3	4	5	6	7	8	9	10

$$3. \text{ keys} = \{56, 254, 24, 78, 15, 29, 45, 12, 9\}$$

use chaining to resolve collisions

$$\text{hash function } h(k) = k \% 9$$

Initial Table:

0	1	2	3	4	5	6	7	8

① hash 56

$$h(56) = 56 \% 9 = 2 \text{ insert 56 to index 2}$$

		56						
0	1	2	3	4	5	6	7	8

② hash 254

$$h(254) = 254 \% 9 = 2 \text{ insert 254 to index 2}$$

		56						
0	1	2	3	4	5	6	7	8

③ hash 24

$$h(24) = 24 \% 9 = 6 \text{ insert 24 to index 6}$$

		56				24		
0	1	2	3	4	5	6	7	8

④ hash 78

$$h(78) = 78 \% 9 = 6 \text{ insert 78 to index 6}$$

		56				24		
0	1	2	3	4	5	6	7	8

⑤ hash 15

$$h(15) = 15 \% 9 = 1 \text{ insert 15 to index 1}$$

		56				24		
0	1	2	3	4	5	6	7	8

⑥ hash 29

$$h(29) = 29 \% 9 = 2 \text{ insert 29 to index 2}$$

		56				24		
0	1	2	3	4	5	6	7	8

⑦ hash 45

$$h(45) = 45 \% 9 = 0 \text{ insert 45 to index 0}$$

45		56				24		
0	1	2	3	4	5	6	7	8

⑧ hash 12

$$h(12) = 12 \% 9 = 3 \text{ insert 12 to index 3}$$

45		56	12			24		
0	1	2	3	4	5	6	7	8

⑨ hash 9

$$h(9) = 9 \% 9 = 0 \text{ insert 9 to index 0}$$

45		56	12			24		
0	1	2	3	4	5	6	7	8