P-VALUES, NULL HYPOTHESIS, TYPE I & TYPE II ERRORS

GROUP 4

INTRODUCTION

- Hypothesis is an assumption about a population that can be tested with a statistical method.
- It serves as the foundation for scientific research and statistical testing
- Formulated based on prior observations, or theories and are evaluated through experiments.
- **Key Characteristics of a Hypothesis**
    - **Testability**: Allows it to be tested using data or experiments.
    - **Falsifiability**: It must be possible to prove the hypothesis false.
    - **Specificity**: Defines the variables and the expected relationship or difference.

Why formulate a hypothesis?

- Data-driven decision making:
  - Provides a systematic approach to analyze data and make informed decisions based on statistical evidence rather than personal opinions or biases.
- Testing theories and assumptions:
  - Researchers can test their hypotheses against empirical data to determine whether their initial ideas are supported or need to be revised.
- Identifying significant relationships
  - Hypothesis testing reveal whether observed relationships between variables are statistically significant, allowing for meaningful interpretations of the results.

The Null and Alternative Hypotheses

- **Null Hypothesis (H_0)**
  - A statement of "no effect" : There is no relationship between A and B
- **Alternative Hypothesis (H_1)**:
  - A statement that challenges H_0. Posits that there is some difference between A and B
- **Key Points**
  - **H_0** and **H_1** are **mutually exclusive**: meaning that only one of them can be true at any given time.
  - Hypothesis testing assumes **H_0** is true until evidence suggests rejecting it.
  - Failing to reject **H_0** does not prove **H_0** is true; it means there’s insufficient evidence against it.

P-VALUE

- The p-value is the probability of obtaining the observed results or extreme ones if the null hypothesis is true.
- **Explanation**:
  - A small p-value (e.g.,): $\mathbf{p < 0.05}$: Evidence against H_0 : Reject H_0.
  - A large p-value : Insufficient evidence : Fail to reject H_0.
- **Misconceptions**:
  - The p-value **does not** measure the probability that H_0 is True or False.
    - Explain the likelihood of observing the data
  - It also **does not** indicate the strength or size of an effect.
    - Indicates a statistical difference but not the magnitude of this effect. Use Cohen’s d test

Setting The Significance Level (Alpha, α)

- **Significance Level (α)**
  - The threshold used to determine whether to reject the null hypothesis (H_0).
  - It is set before conducting the analysis.
  - Represents the probability of making a **Type I error** (rejecting H_0 when it is actually true).
- **Relationship to the P-Value**:
  - Compare the p-value to α:
    - **If p-value < α:** Reject H_0 (result is statistically significant).
    - **If p-value ≥ α:** Fail to reject H_0 (result is not statistically significant).

Confidence Level and Confidence Interval

Confidence level (C. L)

- The probability that the true population parameter lies within the confidence interval (e.g., 95%) in repeated studies
- Commonly expressed as a percentage, such as 90%, 95%, or 99%.
    - A 95% confidence level means we expect the true population parameter to fall within the range 95% of the time across repeated samples.
- Relationship with significant level
    - Confidence level = 1 − a
    - If you use an alpha value of $p < 0.05$ for statistical significance, then then the confidence level would be $1 - 0.05 = 0.95$, or 95%.

Confidence interval (C.I)

- Provides an estimated range, based on sample data, where the true population parameter is likely to fall.
- Typically written as a range: [lower bound,upper bound]
    - A 95% CI of [10,20] means the true parameter is likely to fall between 10 and 20, with 95% confidence.

Confidence Level and Significance Level: Inversely Proportional

- Inversely Proportional:
  - As you increase the **confidence level** (e.g., from 95% to 99%), you decrease the **significance level** (α), which reduces the likelihood of making a **Type I error** (rejecting H_0 when it is true).
  - However, increasing the confidence level can increase the likelihood of making a **Type II error** (failing to reject H_0 when it is false).

What Are Errors in Hypothesis Testing?

- Hypothesis testing involves making decisions based on sample data to draw conclusions about a population.
- These decisions are not always correct, and errors can occur when conclusions about the null hypothesis (H_0) are drawn incorrectly.
- There are two types of errors in hypothesis testing: Type I and Type II.

Type I Error (False Positive)

- Rejecting the null hypothesis when it is actually true.
- We conclude that there is an effect or difference when, in reality, there isn't.
- **Example**: A drug test shows effectiveness when the drug has no effect.
- **Significance Level (α)**: Type I error is directly related to the significance level; α represents the probability of making a Type I error.

Type II Error (False Negative)

- Failing to reject the null hypothesis when it is actually false.
- We conclude that there is no effect or difference when there actually is.
- **Example**: A drug test shows no effect when the drug actually works.
- **Power ($1-\beta$)**: The probability of correctly rejecting the null hypothesis (avoiding a Type II error).

Type I and Type II Error Table

Hypothesis testing:

Actual \ Decision	H_0 true (Fail to reject)	H_0 false (Rejecting H_0)
H_0 true	TRUE NEGATIVE Correct decision: **Confidence level** (prob $1-\alpha$)	FALSE POSITIVE Type I Error: **Significance level/Size (α)** (prob α)
H_0 false	FALSE NEGATIVE Type II Error: fail to reject (prob β)	TRUE POSITIVE Correct decision: **Power** (prob $1-\beta$)

Balancing Type I and Type II Errors

- Understanding the Tradeoff

Disease prediction:

- **Type I Error:** Incorrectly diagnosing a healthy patient with a disease (false positive).
- **Type II Error:** Failing to diagnose a patient who actually has the disease (false negative).
- **Balance:** In situations where the cost of incorrectly diagnosing a healthy patient (Type I error) is high, a lower α might be used. However, if the consequences of missing a disease diagnosis (Type II error) are severe, a higher confidence level (95%) might be chosen.

Court Case:

- **Type I Error:** Convicting an innocent person (wrongful punishment).
- **Type II Error:** Letting a guilty person go free (failure to protect society).
- **Balance:** In a criminal trial, a higher confidence level may be required (e.g., 99% or more) to avoid convicting innocent people, though this may increase the risk of letting some guilty individuals go free.