

This make-up homework is due at 11:59 pm on Sunday, December 14.

1. An ecologist takes data (x_i, Y_i) , $i = 1, \dots, n$, where x_i is the size of an area and Y_i is the number of moss plants in the area. We model the data by $Y_i \sim \text{Poisson}(\theta x_i)$, with the Y_i 's taken to be independent.
 - (a) Find the MLE of θ . Calculate its bias and variance. What is its asymptotic distribution?
 - (b) Find a minimal sufficient statistic for θ . Justify your answer.
 - (c) Assume $\theta \sim \text{Gamma}(a, b)$. What is the posterior distribution for θ ? Can you find a minimax rule under squared error loss?

2. Consider a decision problem with possible states of nature θ_1 and θ_2 . Four non-randomized actions a_1, a_2, a_3 , and a_4 are considered with the following risk functions:

<i>Action</i>	a_1	a_2	a_3	a_4
θ_1	1	4	1	2
θ_2	5	2	2	3

- (a) Give and plot the risk set that consists of both the non-randomized and randomized actions.
- (b) Find a non-minimax Bayes action (any of your choice). Specify the corresponding prior vector of your Bayes action.
- (c) Find a minimax action.

3. Suppose that X_1, \dots, X_n is a random sample from a uniform distribution on the interval $(0, \theta)$, and that the following hypotheses are to be tested:

$$H_0 : \theta \geq 2, \quad H_1 : \theta < 2.$$

Let $Y_n = \max\{X_1, \dots, X_n\}$, and consider a test whose critical region contains all outcomes for which $Y_n \leq 1.5$.

- (a) Determine the power function of the test.
- (b) Determine the size of the test.