senator_pca

February 19, 2026

1 PCA and senate voting data

In this problem, we are given the $n \times d$ data matrix X with entries in $\{-1, 0, 1\}$, where each row corresponds to a senator and each column to a bill. We first import this data, print some relevant values, and normalize it as necessary to ready it for further computation.

1.0.1 Places you will need to modify this code are enclosed in a `#TODO` block. You should not need to modify code outside these blocks to complete the problems. Questions that you are expected to answer in text are marked in red. For solution files, solutions will be presented in blue.

[1]:
```python
# import the necessary packages for data manipulation, computation and PCA
import pandas as pd
import numpy as np
import scipy as sp
from numpy import linalg as LA
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
%matplotlib inline

np.random.seed(7)
```

[2]:
```python
# import the data matrix
!gdown 1De3PwqrmFsMyKOVeejMs4PSoRiJIS8D1 -O senator_pca_data_matrix.csv
!gdown 1uYVNV1ZV7wx_4KiSFxyRZPbG2P5r4bMB -O senator_pca_politician_labels.txt
senator_df =  pd.read_csv('senator_pca_data_matrix.csv')
affiliation_file = open('senator_pca_politician_labels.txt', 'r')
affiliations = [line.split('\n')[0].split(' ')[1] for line in affiliation_file.
  ↪readlines()]
X = np.array(senator_df.values[:, 3:].T, dtype='float64') # transpose to get␣
  ↪senators as rows
print('X.shape: ', X.shape)
n = X.shape[0] # number of senators
d = X.shape[1] # number of bills

# this is just used for plotting, feel free to ignore
assert set(affiliations) == {"Red", "Blue", "Yellow"}
# assign a marker and hatch to each affiliation
```

```
markers = [("Red", "o", "/"), ("Blue", "^", "-"), ("Yellow", "D", "+")]
```

```
Downloading…
From: https://drive.google.com/uc?id=1De3PwqrmFsMyKOVeejMs4PSoRiJIS8D1
To: /content/senator_pca_data_matrix.csv
100% 270k/270k [00:00<00:00, 30.2MB/s]
Downloading…
From: https://drive.google.com/uc?id=1uYVNV1ZV7wx_4KiSFxyRZPbG2P5r4bMB
To: /content/senator_pca_politician_labels.txt
100% 1.18k/1.18k [00:00<00:00, 4.19MB/s]
X.shape:  (100, 542)
```

We observe that the number of rows, n, is the number of senators and is equal to 100. The number of columns, d, is the number of bills and is equal to 542.

[3]:
```
# print an example row of the data matrix
typical_row = X[0]
print(typical_row.shape)
print(typical_row)
```

```
(542,)
[ 1.  1.  1. -1. -1.  1.  1.  1.  1. -1.  1. -1. -1.  1.  1. -1.  1.  1.
  1.  1.  1. -1.  1.  1.  1. -1.  1. -1.  1.  1.  1.  1.  1. -1.  1. -1.
 -1. -1. -1.  1.  1. -1. -1. -1. -1.  1.  1.  1. -1.  1.  1. -1.  1.  1.
 -1.  1.  1.  1.  1. -1.  1. -1. -1. -1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1. -1.  0. -1.  1.  1.  1. -1. -1.  1.  1. -1. -1.  1.  1.  1. -1.
  1. -1.  1. -1.  1.  1. -1. -1. -1.  1.  1.  1. -1. -1. -1. -1. -1. -1.
  1. -1.  1.  1. -1. -1. -1.  1. -1.  1. -1.  1.  0.  0.  1.  1. -1.  1.
  1. -1.  1.  1. -1.  1. -1. -1.  1.  1.  1.  1.  0. -1. -1.  1.  1. -1.
  1.  1.  1.  1.  1.  0.  1.  0.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.
 -1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  0.  1. -1. -1.  1.  1.  1.
  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1. -1.
  1.  1.  0.  1.  0. -1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1.  1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1. -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1. -1.
  1.  1.  1.  1.  1.  1. -1. -1. -1.  1.  1. -1.  1. -1. -1.  1.  1.  1.
 -1.  1.  1.  1. -1.  1. -1.  1. -1. -1.  1. -1. -1.  1.  1.  1. -1.  1.
  1.  1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. -1. -1.
  1. -1.  1. -1. -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1.  1.  1. -1.  1. -1.
  1.  1.  1.  1.  1.  1. -1.  1. -1. -1. -1. -1.  1.  1.  1.  1.  1.  1.
  1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1. -1. -1.  1. -1.  1.  1.  1.
  1.  1. -1.  1. -1.  1. -1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  0.  1. -1.  1.  1.  1.  1.  1. -1. -1. -1.  1.  1.  0.  1.  1.  1.
  1.  1.  1.  1. -1. -1.  0.  0.  0.  0.  0.  0.  0.  1.  1. -1. -1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1. -1.  1.  1.  1.  1. -1. -1.
  1.  1.  1.  1. -1. -1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1.
 -1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1. -1. -1. -1.  1.  1.  1.  1. -1. -1.  1.  1. -1.  1.  1.  1.  1.
  1.  1.]
```

A row of X consists of 542 entries -1 (senator voted against), 1 (senator voted for), or 0 (senator abstained), one for each bill.

```
[4]: # print an example column of the data matrix
typical_column = X[:,0]
print(typical_column.shape)
print(typical_column)
```

```
(100,)
[ 1.  1.  1.  1.  1.  1.  1. -1.  1. -1.  1. -1.  1. -1. -1. -1.  1.  1.
 -1.  1.  1. -1.  1. -1.  1.  1.  1. -1. -1.  1.  1.  1. -1.  1.  1.  1.
 -1. -1. -1. -1.  1. -1. -1.  1.  1. -1. -1. -1. -1. -1.  1.  1. -1. -1.
  1.  1. -1. -1. -1. -1. -1.  1.  1.  1.  1.  1. -1. -1. -1.  1. -1. -1.
  1. -1. -1.  1.  1.  1. -1. -1. -1.  1.  1. -1.  1. -1.  1.  1.  1. -1.
 -1. -1. -1. -1.  1.  1.  1. -1. -1. -1.]
```

A column of X consists of 100 entries in {-1, 0, 1}, one for each senator that voted on the bill.

```
[ ]: # compute the mean vote on each bill
X_mean = np.mean(X, axis = 0)
plt.plot(X_mean)
plt.title('means of each column of X')
plt.xlabel('column/bill')
plt.ylabel('mean vote')
plt.show()
```

We observe that the mean of the columns is not zero, so we center the data by subtracting the mean of each bill's vote from its respective column.

```
[10]: # center the data matrix
X_original = X.copy() # save a copy for part (d) and (e)
X = X - np.mean(X, axis = 0)
```

```
107.5
```

1.1 a) Maximizing $\sigma^2(f(X, \vec{a}))$

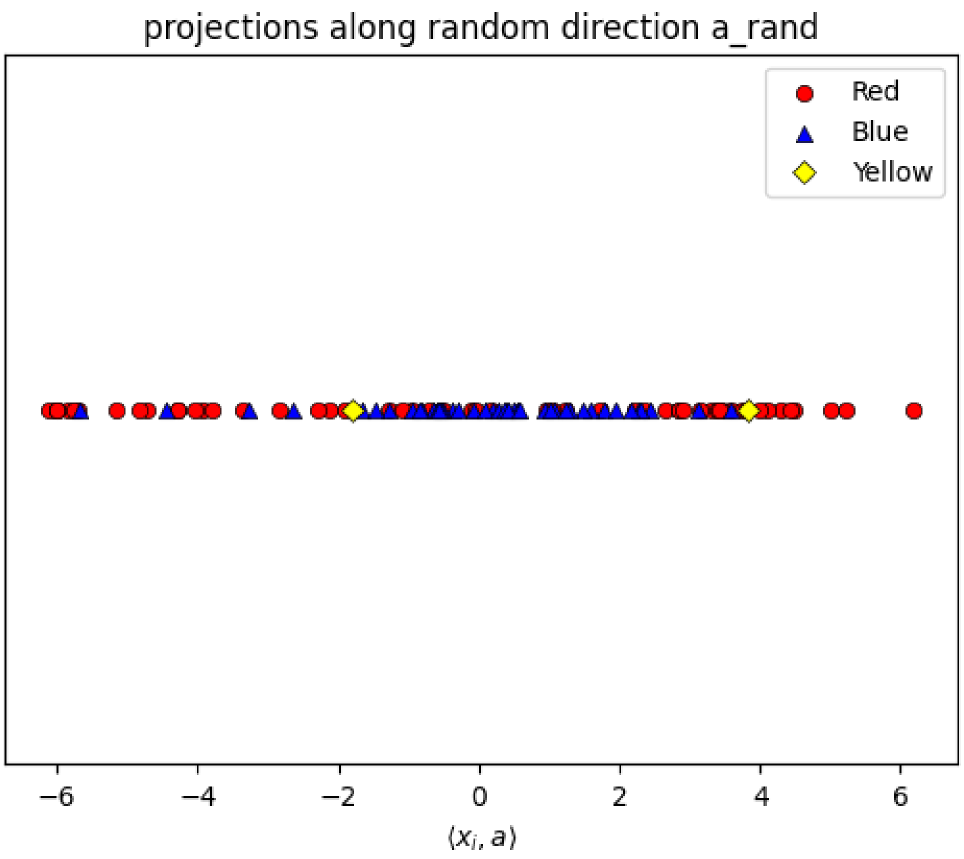
In this problem, you are asked to find a unit-norm vector $\vec{a} \in \mathbb{R}^d$ maximizing the empirical variance $\sigma^2(f(X, \vec{a}))$.

We first provide a function to calculate the scores, $f(X, \vec{a})$.

```
[22]: # define score function
def f(X, a):
    return X @ a
```

Before we calculate the $\vec{a}$ that maximizes variance, let's observe what the scalar projections on a random direction $\vec{a}$ look like.

[12]:
```python
# generate a random direction and normalize the vector
a_rand = np.random.rand(d)
a_rand = a_rand/np.linalg.norm(a_rand)

# compute associated scores along a_rand
scores_rand = f(X, a_rand)

# visualize the scores along a_rand, coloring them by party affiliation
for aff, marker, _ in markers:
    plt.scatter(
        scores_rand[np.array(affiliations) == aff],
        np.zeros_like(scores_rand[np.array(affiliations) == aff]),
        c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
    )
plt.legend()
plt.title('projections along random direction a_rand')
plt.xlabel('$\\langle x_i, a\\rangle$')
cur_axes = plt.gca()
cur_axes.axes.get_yaxis().set_visible(False)
plt.show()

print('variance along random direction a_rand: ', scores_rand.var())
```

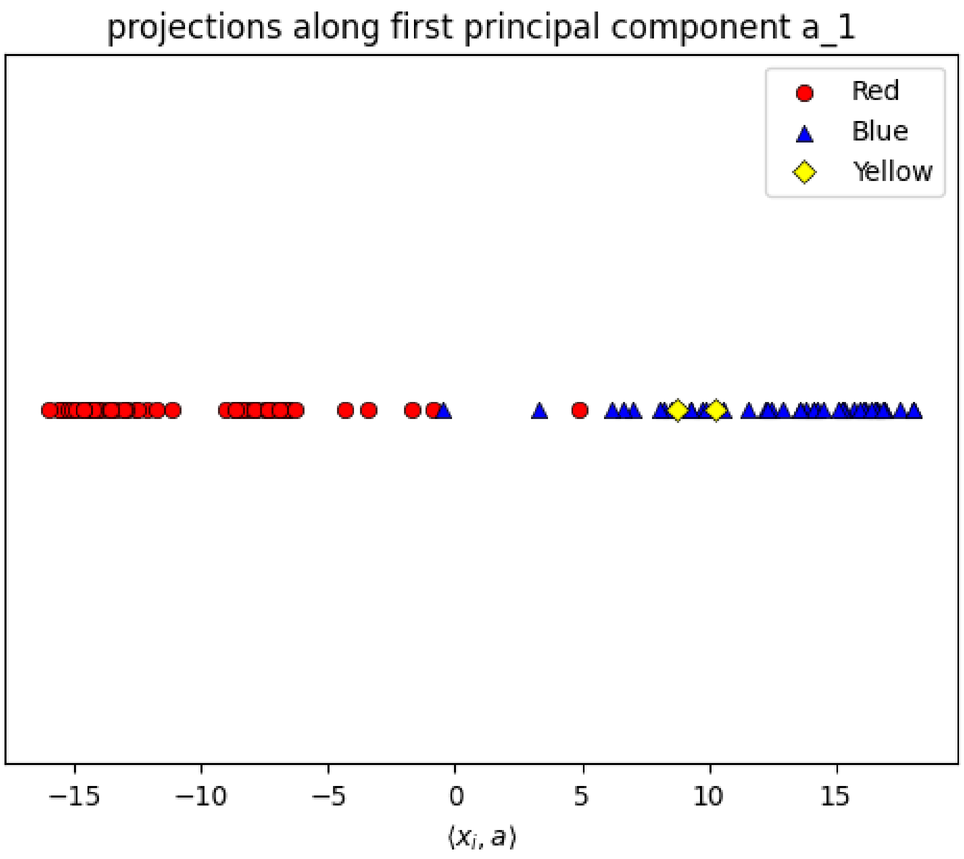


```
variance along random direction a_rand:  9.26745439089334
```

Note here that projecting along the random vector `a_rand` does not explain much variance at all — data points are clustered together and intermixed across parties. It is clear that this direction does not give us any information about the senators' affiliations.

```
[21]: ################################################################################
      ### TODO: Calculate a_1, the first principal component of X.
      # Hint: The PCA package imported from sklearn.decomposition will be useful here,
      # in particular the function pca.fit(). What should the dimensions of a_1 be?
      pca = PCA(n_components=1)
      pca.fit(X)
      a_1 = pca.components_[0]
      assert a_1.shape == (d,)

      ### end TODO
      ################################################################################

      a_1 = a_1/np.linalg.norm(a_1)
      # compute and visualize the scores along a_1
      scores_a_1 = f(X, a_1)

      for aff, marker, _ in markers:
          plt.scatter(
              scores_a_1[np.array(affiliations) == aff],
              np.zeros_like(scores_a_1[np.array(affiliations) == aff]),
              c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
          )
      plt.legend()
      plt.title('projections along first principal component a_1')
      plt.xlabel('$\\langle x_i, a \\rangle$')
      cur_axes = plt.gca()
      cur_axes.axes.get_yaxis().set_visible(False)
      plt.show()

      print('variance along first principal component: ', scores_a_1.var())
```

```
(542,)
```



```
variance along first principal component:  149.74896507620733
```

If you computed `a_1` correctly, you should observe that the variance is much higher than the `a_rand` projection, and that blue and red dots are now spread in two clusters. This makes sense: the first principal component is the direction along which data varies most, and that is often along party lines. You just found a mathematical model for partisanship!

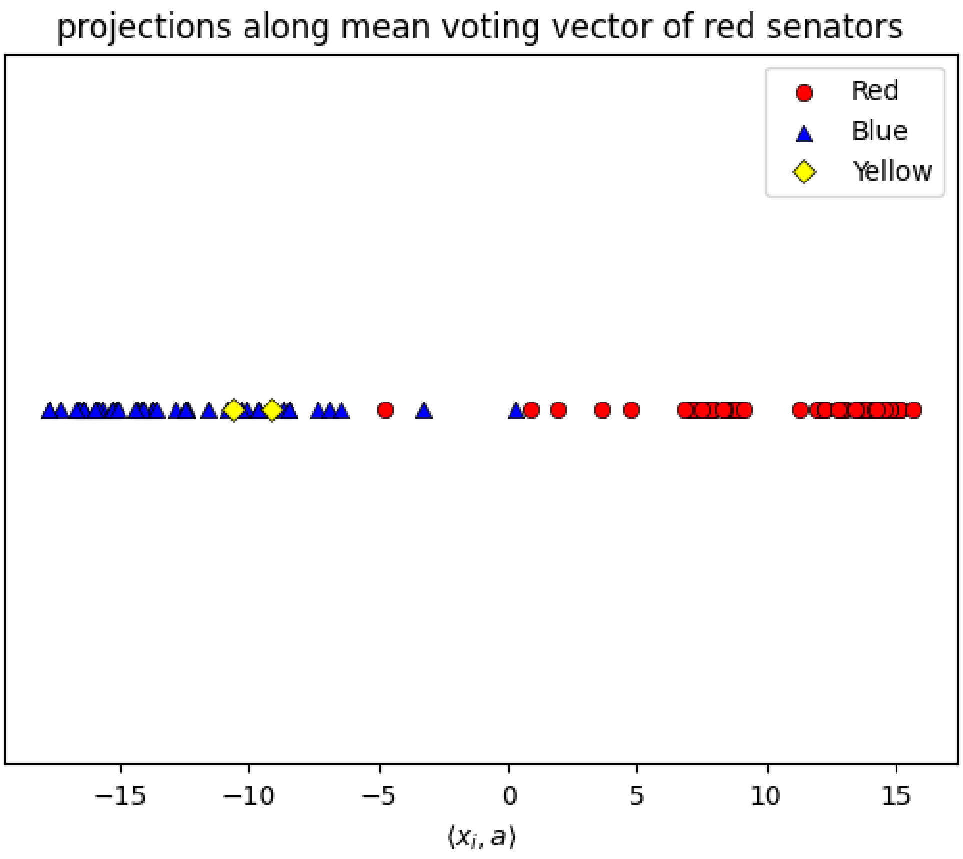
1.2 b) Comparison to party averages

We observed above that the direction of maximum variance appears to be determined by party alignment; we now want to quantify how true that is by computing variance along vectors that describe the average position of each party. Specifically, we will compute variance along the following two vectors:

- `a_mean_red`: unit vector along the mean of rows of `X` corresponding to 'Red' senators
- `a_mean_blue`: unit vector along the mean of rows of `X` corresponding to 'Blue' senators

Fill in the code as indicated below to calculate these values and compute their relationships to `a_1` and each other, then answer the interpretation question that follows.

[30]:
```
##############################################################################
### TODO: Calculate mu_red, the array of dimension (542, ) whose values
### are the mean across rows of X corresponding to 'Red' senators only.
# Hint: Print out the 'affiliations' variable and observe its contents.
mask = np.array(affiliations) == 'Red'

mu_red = np.mean(X[mask], axis = 0)

### end TODO
################################################################################

# normalize the vector to generate unit a_mean_red
a_mean_red = mu_red/np.linalg.norm(mu_red)

# compute and visualize the scores along a_mean_red
scores_mean_red = f(X, a_mean_red)

for aff, marker, _ in markers:
    plt.scatter(
        scores_mean_red[np.array(affiliations) == aff],
        np.zeros_like(scores_mean_red[np.array(affiliations) == aff]),
        c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
    )
plt.legend()
plt.title('projections along mean voting vector of red senators')
plt.xlabel('$\\langle x_i, a \\rangle$')
cur_axes = plt.gca()
cur_axes.axes.get_yaxis().set_visible(False)
plt.show()

print('variance along mean voting vector of red senators: ', scores_mean_red.
 ↪var())
```

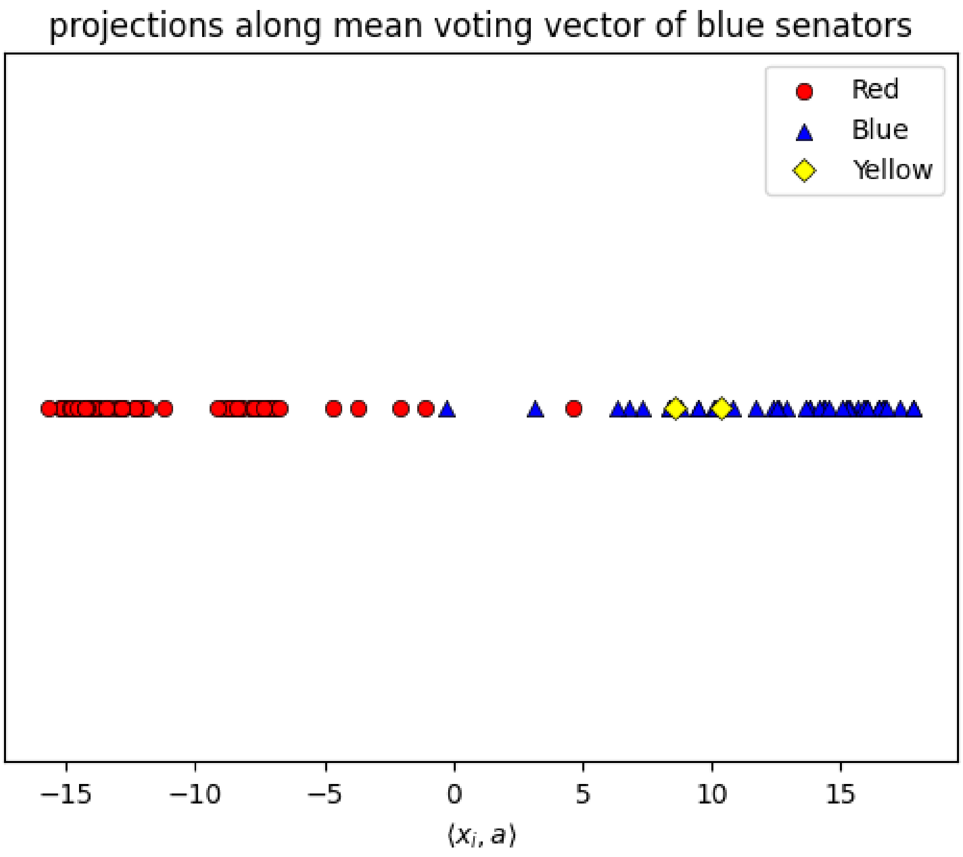


```
variance along mean voting vector of red senators:  148.80699963205723
```

[31]:

```
###########################################################################
### TODO: Calculate mu_blue, the array of dimension (542, ) whose values
### are the mean across rows of X corresponding to 'RBlue' senators only.
# Hint: Print out the 'affiliations' variable and observe its contents.
# print(len(affiliations))
# print(affiliations)

mask = np.array(affiliations) == 'Blue'

mu_blue = np.mean(X[mask], axis = 0)
### end TODO
###########################################################################

# normalize the vector to generate unit a_mean_blue
a_mean_blue = mu_blue/np.linalg.norm(mu_blue)

# compute and visualize the scores along a_mean_blue
scores_mean_blue = f(X, a_mean_blue)
```

```
for aff, marker, _ in markers:
    plt.scatter(
        scores_mean_blue[np.array(affiliations) == aff],
        np.zeros_like(scores_mean_blue[np.array(affiliations) == aff]),
        c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
    )
plt.legend()
plt.title('projections along mean voting vector of blue senators')
plt.xlabel('$\\langle x_i, a \\rangle$')
cur_axes = plt.gca()
cur_axes.axes.get_yaxis().set_visible(False)
plt.show()

print('variance along mean voting vector of blue senators: ', scores_mean_blue.
 ↪var())
```



```
variance along mean voting vector of blue senators:  148.9088414400461
```

```
[32]: # compute dot product of and angle between a_mean_red and a_mean_blue:
      dot_product_blue_red = a_mean_blue.T @ a_mean_red
      angle_blue_red = np.arccos(dot_product_blue_red) * 180/np.pi

      print('dot product of a_mean_blue and a_mean_red: ', dot_product_blue_red)
      print('angle between a_mean_blue and a_mean_red (degrees): ', angle_blue_red)
```

```
dot product of a_mean_blue and a_mean_red:  -0.9992350984093116
angle between a_mean_blue and a_mean_red (degrees):  177.75886458298177
```

1.2.1 TODO: Interpretation

Comment on the relationships between a_mean_red and a_mean_blue above based on their dot products and relative angles.

TODO: The two vectors are almost on the opposite direction to each other, i.e. $\mu_{blue} \approx -\mu_{red}$.

Next, we will see how aligned the mean voting vectors are with the first principal component of the data.

```
[33]: # check angle between mean voting vector of red senators and the first␣
       ↪principal component as well as that of blue senators and the first principal␣
       ↪component
      dot_product_red_a1 = a_mean_red.T @ a_1
      angle_red_a1 = np.arccos(dot_product_red_a1) * 180/np.pi

      print('dot product of a_mean_red and a_1: ', dot_product_red_a1)
      print('angle between a_mean_red and a_1 (degrees): ', angle_red_a1)

      dot_product_blue_a1 = a_mean_blue.T @ a_1
      angle_blue_a1 = np.arccos(dot_product_blue_a1) * 180/np.pi

      print('dot product of a_mean_blue and a_1: ', dot_product_blue_a1)
      print('angle between a_mean_blue and a_1 (degrees): ', angle_blue_a1)
```

```
dot product of a_mean_red and a_1:  -0.9965356912813021
angle between a_mean_red and a_1 (degrees):  175.22941782780575
dot product of a_mean_blue and a_1:  0.9969831227823048
angle between a_mean_blue and a_1 (degrees):  4.451697983372387
```

1.2.2 TODO: Interpretation

Comment on the relationships between Red and Blue senators to partisanship based on the two products of a_mean_blue and a_mean_red with a_1, the top principal component of the covariance, i.e., the maximum variance direction.

TODO: The blue vector is almost in the same direction with the first principle components, which indicates that the first principle component $\approx -\mu_{red}$.

1.3 c) Computing total variance

We now wish to observe the variance of the data along the first two principal component axes.

Fill in the code below to calculate the total variance of the data along the first two principal components a_1 and a_2 and to plot the data on the corresponding axes.

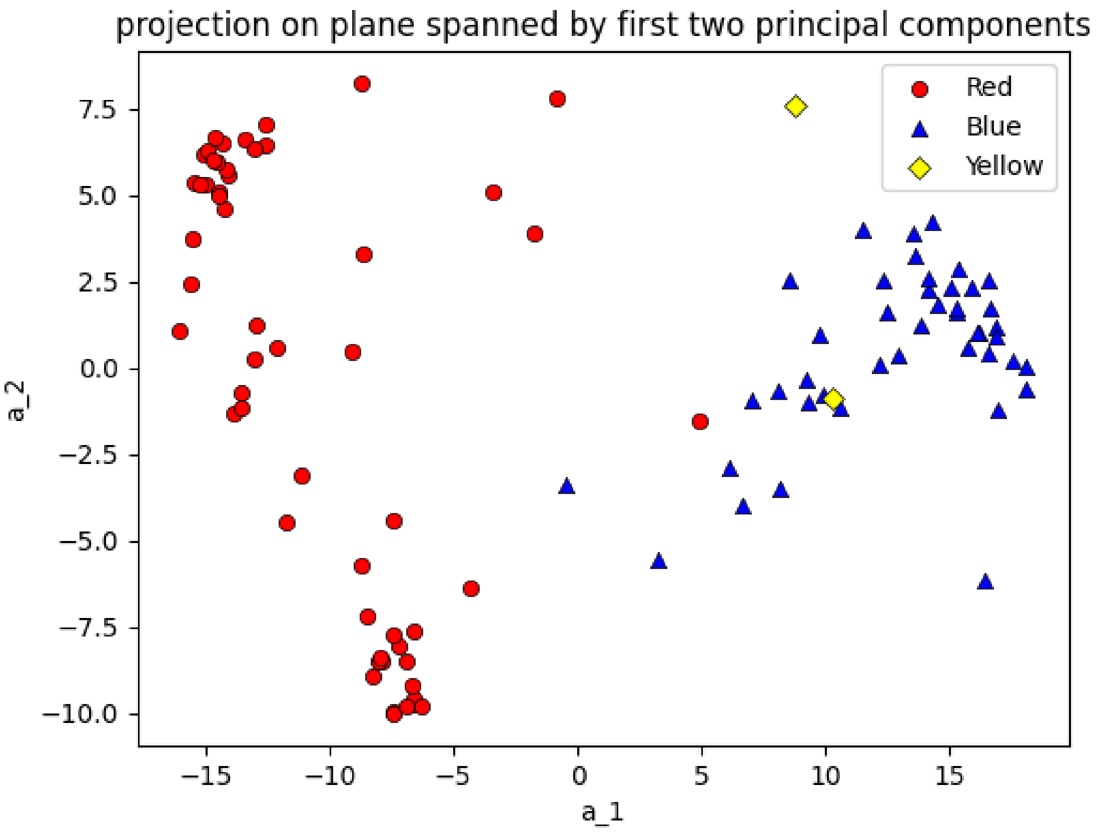
[41]:
```
################################################################################
### TODO: Calculate the Sigma matrix (defined in LaTeX problem) and the total␣
 ↪variance across a_1 and a_2.
# Hint: The latter value is equal to the sum of the two largest eigenvalues of␣
 ↪Sigma. You can use either the PCA library or the numpy.linalg library that␣
 ↪were already imported at the start of this notebook.
Sigma = X.T @ X / n
pca = PCA(n_components=2)
pca.fit(X)
total_variance = np.sum(pca.explained_variance_) * (n-1) / n
### end TODO
################################################################################
print(Sigma)
print('total variance explained by first two principal components: ',␣
 ↪total_variance)
```

```
[[ 1.     -0.78   -0.8    … 0.14   -0.24   -0.23  ]
 [-0.78    0.9996  0.8592 … -0.0892  0.298   0.2886]
 [-0.8     0.8592  0.9984 … -0.0984  0.296   0.2872]
 …
 [ 0.14   -0.0892 -0.0984 … 0.2684  0.024   0.0178]
 [-0.24    0.298   0.296  … 0.024   0.47    0.173 ]
 [-0.23    0.2886  0.2872 … 0.0178  0.173   0.4651]]
total variance explained by first two principal components:  175.17116160523148
```

Next, we calculate and plot the projection onto the plane spanned by the first two principal components.

[42]:
```
# calculate projected data matrix and observe its shape
pca = PCA(n_components=2)
projected = pca.fit_transform(X)
print(projected.shape)

# plot projected data matrix
for aff, marker, _ in markers:
    plt.scatter(
        projected[np.array(affiliations) == aff, 0],
        projected[np.array(affiliations) == aff, 1],
        c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
    )
plt.legend()
plt.xlabel('a_1')
plt.ylabel('a_2')
plt.title('projection on plane spanned by first two principal components')
plt.show()
```

```
(100, 2)
```



1.4 d) Finding bills that are the most/least contentious

We now wish to observe which bills are the most and least contentious — i.e., those for which most senators voted unanimously, and those for which support was most varied. We consider one possible way of quantifying this relationship mathematically.

We can compute the variance of each column of X — each of which corresponds to a particular bill — and use this variance as a measure of "contentiousness" (i.e., the more contentious a bill, the higher its variance in terms of senator vote count). Note that the variance of a particular bill in column j can be viewed as the variance of scores along $\vec{e}_j$, where $\vec{e}_j$ is a basis vector whose j^{th} entry is 1 and all others 0.

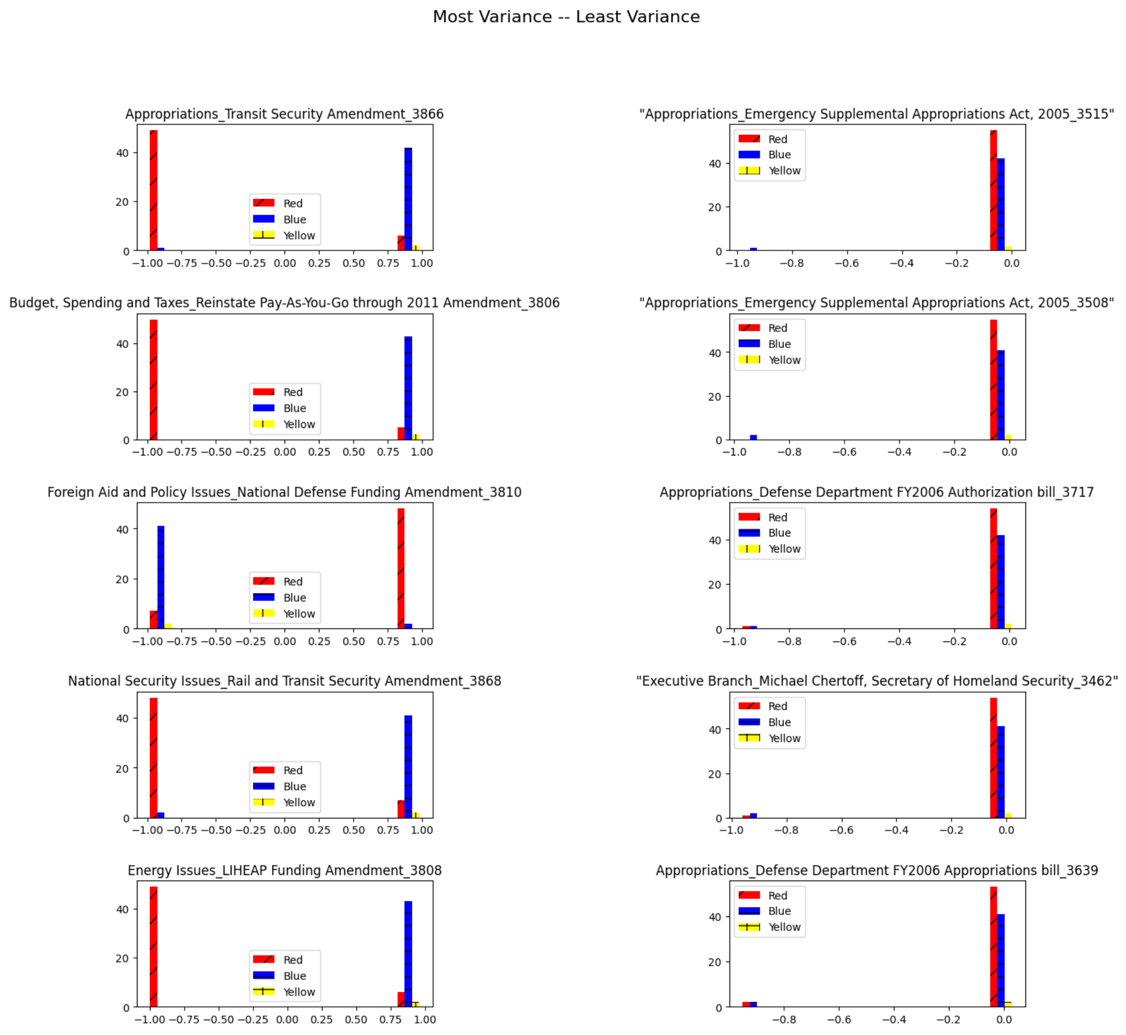
Fill in the code below to calculate the variance of X, extract the most and least contentious bills, and plot their vote counts, commenting on your results where indicated.

[46]:
```python
# calculate the variance of each column
list_variances = X.var(axis=0)
bills = senator_df['bill_type bill_name bill_ID'].values

##########################################################################
### TODO: Compute sorted_idx_variances, an np.array of shape (542,) containing
### integer entries that are the indices of variance scores in list_variances in
### decreasing order of variance. For example, if list_variances = [1, 3, 2, 4],
### then sorted_idx_variances = np.array([3,1,2,0]).
# Hint: Use np.argsort().
sorted_idx_variances = np.argsort(list_variances)[::-1]

### end TODO
##########################################################################

print(sorted_idx_variances.shape)
```

```
(542,)
```

Using this sorted index, we can now plot the vote counts for the top 5 highest and lowest variance bills.

[47]:
```python
# retrieve the bills with the 5 highest and lowest variances
top_5 = [bills[sorted_idx_variances[i]] for i in range(5)]
bot_5 = [bills[sorted_idx_variances[-1-i]] for i in range(5)]

# set up figure with all desired subplots
fig, axes = plt.subplots(5,2, figsize=(15,15))

# plot highest variance bills
for i in range(5):
    idx = sorted_idx_variances[i]

    # retrieve vote counts from original uncentered data matrix
    Xs = []
    colors = []
    labels = []
    hatches = []
    for color, _, hatch in markers:
        Xs.append(X_original[np.array(affiliations) == color, idx])
        colors.append(color)
        labels.append(color)
        hatches.append(hatch)

    _, _, patches = axes[i, 0].hist(
        Xs,
        color=colors,
        label=labels,
    )
    for patch_set, hatch in zip(patches, hatches):
        for patch in patch_set.patches:
            patch.set_hatch(hatch)

    axes[i,0].legend()
    axes[i,0].set_title(bills[idx])

# plot lowest variance bills
for i in range(1,6):
    idx2 = sorted_idx_variances[-i]

    # retrieve vote counts from original uncentered data matrix
    Xs = []
    colors = []
    labels = []
    hatches = []
    for color, _, hatch in markers:
        Xs.append(X_original[np.array(affiliations) == color, idx2])
        colors.append(color)
        labels.append(color)
        hatches.append(hatch)

    _, _, patches = axes[i-1, 1].hist(
        Xs,
        color=colors,
        label=labels,
    )
    for patch_set, hatch in zip(patches, hatches):
        for patch in patch_set.patches:
            patch.set_hatch(hatch)

    axes[i-1,1].legend()
    axes[i-1,1].set_title(bills[idx2])

plt.subplots_adjust(hspace=0.5, wspace = 1)
fig.suptitle('Most Variance -- Least Variance', fontsize=16)
plt.show()
```



1.4.1 TODO: Interpretation

Comment on the voting trends you observe in the plots above. In general, if a vote is contentious, what do you expect the plots to look like? What about if a vote is uncontentious?

TODO: If a vote is contentious, the majority of red and blue should be in different values. Otherwise, they share the same value. .

2 e) Infer political affiliation using top two PCA directions

We now consider a strategy to infer the political affiliation of two senators based on how they voted for the bills and considering the projection of their votes onto the two principal components.

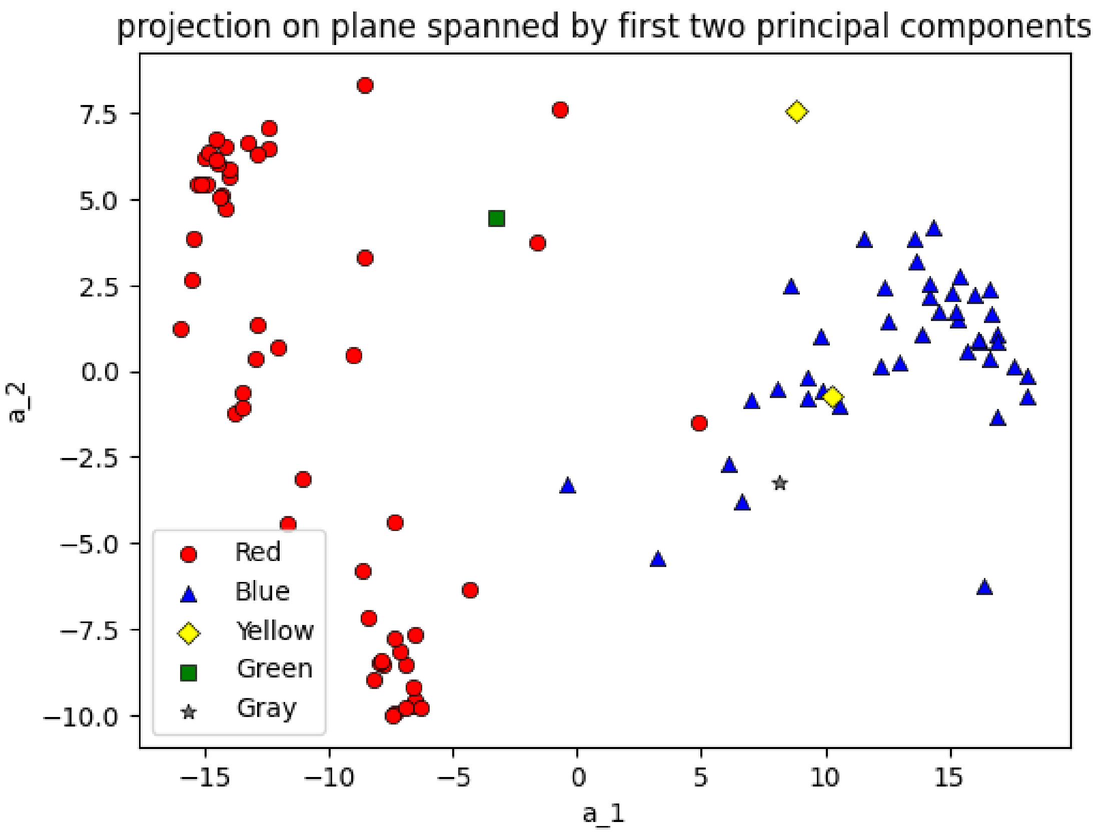
In this part, we will compute the top two PCA directions for our given data after removing two specific senators. Then we will plot all the senators in 2D based on their projections on the top two directions as in part c). The two senators whose affiliation needs to be inferred are marked in green and gray. Can you infer the political affiliation (Red or Blue) of the green and gray senator by looking at the points in this 2D plane?

[48]:
```
X_train=np.delete(X,[0,1],0)
affiliations_train=affiliations[:]
affiliations_train[0]='Green'
affiliations_train[1]='Gray'



# this is just used for plotting, feel free to ignore
assert set(affiliations_train) == {"Red", "Blue", "Yellow", "Green", "Gray"}
# assign a marker and hatch to each affiliation
markers_train = [("Red", "o", "/"), ("Blue", "^", "-"), ("Yellow", "D", "+"),␣
 ↪("Green", "s", "x"), ("Gray", "*", ".")]
```

[49]:
```
# calculate projected data matrix and observe its shape
pca = PCA(n_components=2)
pca.fit(X_train)
projected = pca.transform(X)
print(projected.shape)
```

```
(100, 2)
```

[50]:
```
# plot projected data matrix
for aff, marker, _ in markers_train:
    plt.scatter(
        projected[np.array(affiliations_train) == aff, 0],
        projected[np.array(affiliations_train) == aff, 1],
        c=aff, marker=marker, edgecolors="black", linewidth=0.5, label=aff
    )
plt.legend()
plt.xlabel('a_1')
plt.ylabel('a_2')
plt.title('projection on plane spanned by first two principal components')
plt.show()
```



2.0.1 TODO: Interpretation

Based on the plot above, what is the likely affiliation of the Green senator? What is the likely affiliation of the Grey senator?

TODO: The green senator is more likely to be from red affiliation. And the grey from blue affiliation.

2.1 f) Finding extreme senators

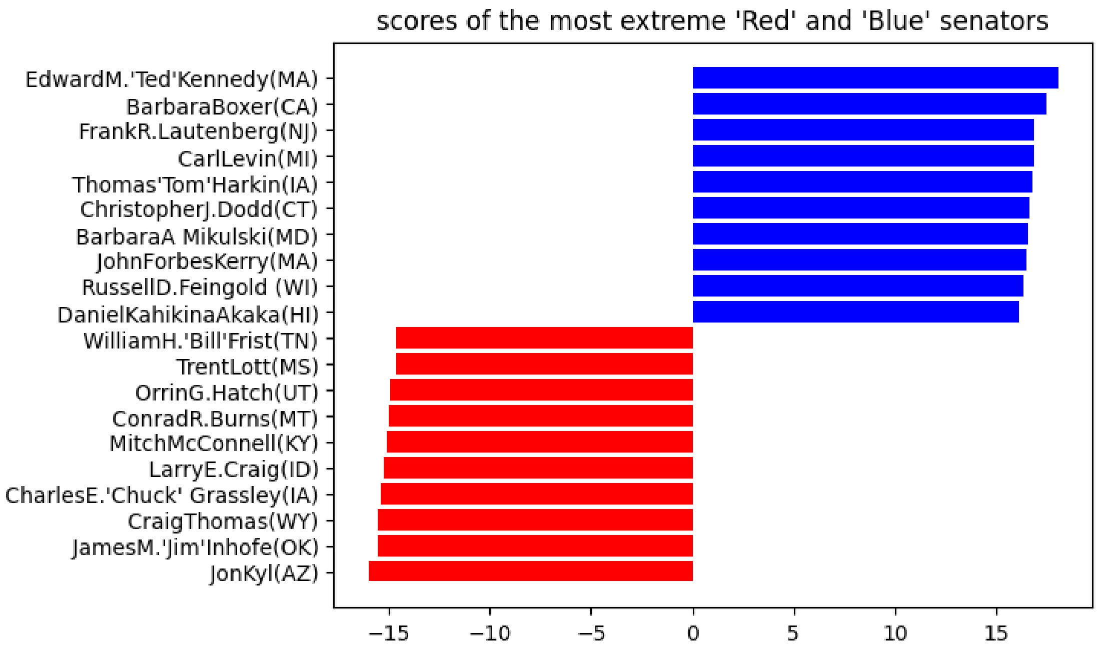
Lastly, let us return to our initial definition of $f(X, \vec{a})$, which assigns each senator a score. We will now use this value computed along the first principal component $\vec{a}$ to assign the following classifications to our senators:

- Senators with the top 10 most positive scores and top 10 most negative scores are classified as *most extreme*.
- Senators with the 20 scores closest to 0 are classified as *least extreme*.

In the final subproblem, we observe these scores and how they relate to party affiliation.

We first compute the most extreme senators:

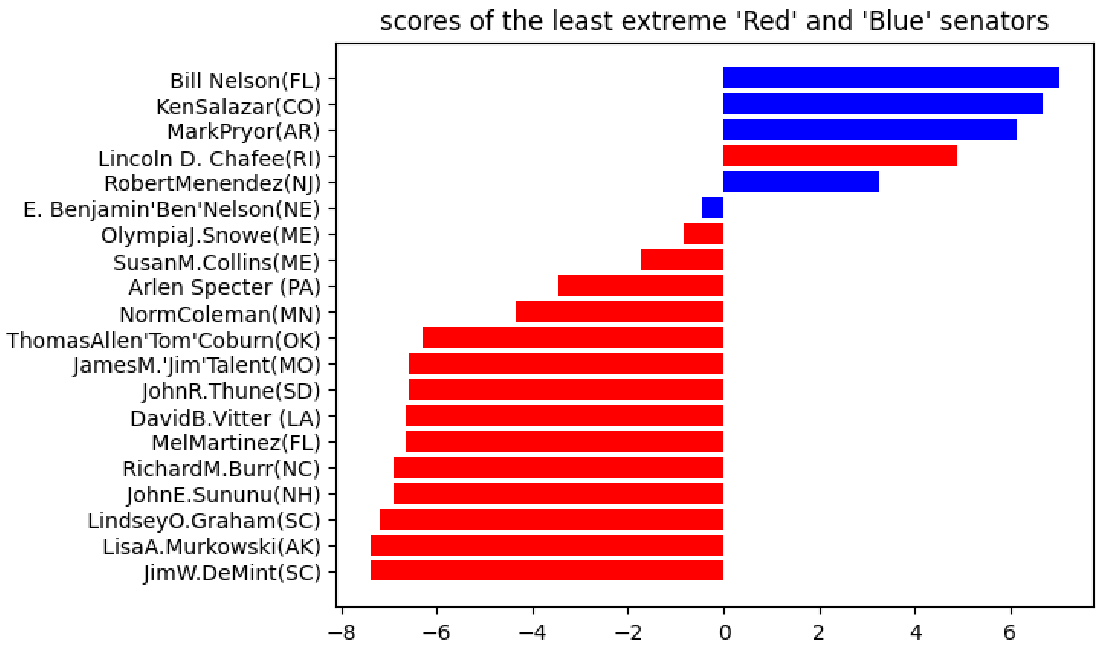
```
[51]: # extract senator names
      senators = senator_df.columns.values[3:]

      # compute and sort senators scores and corresponding affiliations
      senator_scores = f(X,a_1)
      complete_sort_indices = np.argsort(senator_scores)
      sort_indices = np.hstack([complete_sort_indices[:10], complete_sort_indices[-11:
       ↪-1]])
      senators_sorted = senators[sort_indices]
      senator_scores_sorted = senator_scores[sort_indices]
      affiliations = np.array(affiliations)
      affiliations_sorted = affiliations[sort_indices]

      plt.barh(y = senators_sorted, width = senator_scores_sorted, color =␣
       ↪affiliations_sorted)
      plt.title('scores of the most extreme \'Red\' and \'Blue\' senators')
      plt.show()
```



And the least extreme senators:

```
[52]: # compute and sort senators scores and corresponding affiliations
      senator_scores = f(X,a_1)
      complete_sort_indices = np.argsort(np.abs(senator_scores))[:20]
      senator_scores_le= senator_scores[complete_sort_indices]
      senators_le = senators[complete_sort_indices]
      affiliations = np.array(affiliations)
```

```
affiliations_le = affiliations[complete_sort_indices]
sort_indices = np.argsort(senator_scores_le)
senators_sorted = senators_le[sort_indices]
senator_scores_sorted = senator_scores_le[sort_indices]
affiliations_sorted = affiliations_le[sort_indices]

plt.barh(y = senators_sorted, width = senator_scores_sorted, color =␣
 ↪affiliations_sorted)
plt.title('scores of the least extreme \'Red\' and \'Blue\' senators')
plt.show()
```



2.1.1 TODO: Interpretation

Comment on the sign of senators' scores and what they say about party affiliation for both the most and least extreme senators.

TODO: Most of the senators in blue have positive first principle component values while the ones in red have negative values. When the absolute value is large, the color is purely blue or red. But when it is close to zero, the red and blue start to intersect.