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Comments on 2023 Revision of CMF

Overview:

To achieve a coherent academic plan for California's students, any changes to the Math Framework should be done in a way that is easily and transparently compatible with the current CA-CCSS-M content standard, or it should be done in coordination with a revision of those standards. The 2022 and 2023 California Mathematics Framework CMF drafts do neither.

The admirable goal of each CMF draft has been to broaden engagement and success in mathematics. As a professor of mathematics at the CSU for the last 20 years, I can attest that these are critical and urgent issues that we must attend to immediately. The 2023 revision of the CMF is a significant improvement over that 2022 version; it clarifies the difference between data literacy and data science, streamlines some of the chapters, removes the MIC pathway, and reaffirms the CA-CCSS-M standards (e.g. lines 2002-2005 of chapter 8).

On the subject of CA-CCSS-M standards, it is important to note that the SBAC exam and the Intersegmental Committee of Academic Senates (ICAS) base their criterion on those standards. Moreover, the Academic Senate of the California State University System recently affirmed its support of using the CA-CCSS-M content standards for Algebra 1, Geometry, and Algebra 2 or Integrated Math 1-3 as appropriate preparation for post-secondary study (https://www.calstate.edu/csu-system/faculty-staff/academic-senate/resolutions/2022-2023/3599.pdf). Thus, transparency between the standards and the framework is critical to educational equity in the transition from high school to post-secondary study.

Unfortunately, the 2023 version still has significant flaws. First it calls for a **complex instructional structure** that overshadows the laudable goals of the framework and buries the mathematical content. Second, it calls on math teachers to take on instruction in social justice, experimental design, data management and much more. This **expanding content scope for math teachers** is in addition to the CA-CCSS-M content standards, and the result will be structurally impossible. Teachers already struggle with the current standards, and the proposed changes would be unsupportable by the current levels of investment in professional learning. Third, the current version of the CMF continues to use the term **data science** to promote important material in statistics, computer science, and social science. This risks misleading students, families, teachers, and guidance counselors about what courses are a good preparation for further studies in STEM. That risk is particularly high for under-resourced schools, where circumstances may limit the ability of staff to contextualize for each student the parts of the CMF document that attempt to clarify this point.

The complex structure of instruction called on by the CMF bears further discussion. It integrates three 'drivers for investigation' (DIs are the ‘why’) and four 'content connections’ (CIs are the what) with the existing eight CA-CCSS-M mathematical practices (SMPs are the ‘how’). Each grouping of components is compelling, and it is important that we move into a pedagogy that motivates students with open ended lesson starters that put mathematics in context and surface different understandings. It is clear from my experience in the CSU that our current practices are driving students away from STEM. However, when put together the proposal calls for balancing 15 components, and that count does not include the mathematical content contained in the ‘big ideas’ webs for each grade level with progressions between grade levels. That structure is a lot to juggle while standing in front of 35 students with diverse needs. One wonders if there isn’t a simpler framework within which the “why, how, what” approach to mathematical instruction could be scaffolded in order to broaden engagement and success in math? For example, Illustrative Mathematics has created within its curricular design a framework that works within the structure of the common core to build progressions across grade levels, contextualization with an eye to broaden engagement, and attention to professional development. Perhaps that could be a model?

In addition to the concerns above, the current draft framework leaves vague the critical entwining of instructional components (DI's, CC's, MP's) with the big ideas. For example, the big ideas are not included in Figure 1.4 of Chapter One (and elsewhere). Since this figure is central to the framework, the omission of big ideas leaves the impression that the content is a secondary concern. This is critical because the big ideas appear to replace the CA-CCSS-M content standards, at least on the student-facing side. This could make sense if it were the case that on the teacher-facing side the connections were clear. Unfortunately, the 900+ pages of the framework fail to make connections between the standards and the big ideas in the crucial and controversial area of Algebra 2 and Integrate 3. This omission sows the seeds of distrust and concern.

Given the issues above, it seems unlikely that teachers and curriculum creators will be able to smoothly integrate the various components called for in the CMF. This risks curricula that lack transparency around what students are supposed to learn, something that is critical to equitable instructional practices.

The State Board of Education would do well to read the article in the May 2023 volume of the AMS Notices. It summarizes a forum that the AMS Committee on Education hosted in June 2022 in which these issues were discussed cordially and professionally: https://www.ams.org/journals/notices/202305/noti2689/noti2689.html?adat=May%202023&trk=2689&galt=none&cat=education&pdfissue=202305&pdffile=rnoti-p798.pdf

The best course of action at this time would be to restart the framework process and have the framework revision happen in conjunction with a revision of the CA-CCSS-M and ICAS’s forthcoming revision of their Statement of Math Competencies Expected of Entering College Students.
https://icas-ca.org/wp-content/uploads/2020/05/ICAS-Statement-Math-Competencies-2013.pdf.

Comments on specific chapters:

Summary

- Support for: “While data literacy—the ability to understand and use data to answer questions—is part of data science, the field of data science also includes advanced mathematics, and computational skills that build upon—and go far beyond—the content contained in the kindergarten through grade twelve CA CCSSM. ”
- Chapter 12 Overview: How does “Such an approach reflects the goal of achieving conceptual understanding, problem-solving capacity, and procedural fluency.” relate to the DIs, SMPs, and CIs?

Chapter 1

- Line 360 Teaching the Big Ideas.
    - I like big ideas, but find that there are too many levels of content organization here: Big Ideas, Drivers of Instruction (why), Math Practices (how), Content Connections (what), and of course the "with" which are the content standards.
    - This section sets up a false choice between big ideas and content standards. If the Big ideas are to be "legal" they must subsume the content standards. They may well be a better student facing concept, but the content standards are like the molecules that make up the big idea atoms.
- Line 392 Figure 1.2 Grade Six Big Ideas.
    - Present the big ideas as both a list and the web.Also for each grade level there should be a table identifying which CA-CCSS-M content standards are within each big idea and which are not covered by any big idea.. That will help us understand which standards are being emphasized. This could be an appendix.
- Line 457 Figure 1.3 The Why, How and What of Learning Mathematics.
    - Add a column called "with" and either put the "big idea" from that grade level or the content standards that are relevant.
- Line 463 Figure 1.4 Drivers of Investigation, Standards for Mathematical Practices, and Content Connections.
    - Add a column called "with" and either put the "big idea" from that grade level or the content standards that are relevant.
- Lines 574-577 “Thus, the tendency has been to take one of two instructional approaches: cover some standards at the depth they merit while skipping

others, or try to cover all grade-level standards but compromise opportunities for students to gain a deep understanding of any one of them."
    - This is a real phenomenon and it is why the framework should be revised in conjunction with the CA-CCSS-M.
- Lines 708-710 Definition of Rigor. "In this framework, *rigor* refers to an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned so that each supports the other."
    - Does this help? The definition of rigor should help different stakeholder groups have confidence that what will be implemented will be more than just a feel good activity.
    - Footnote: it is not clear that the CMF definition of rigor is more or less demanding than the CA-CCSS-M definition.
- Lines 710-716. I support " Using this definition, conceptual understanding cannot be considered rigorous if it cannot be *used* to analyze a novel situation encountered in a real-world application or within mathematics itself (for new examples and phenomena). Computational speed and accuracy cannot be called rigorous unless it is accompanied by conceptual understanding of the strategy being used, including why it is appropriate in a given situation. And a correct answer to an application problem is not rigorous if the solver cannot explain both the ideas of the model used and the methods of calculation. "
- Line 716-721: "In other words, rigor is *not* about abstraction. In fact, a push for premature abstraction leads, for many students, to an absence of rigor. It is true that more advanced mathematics often occurs in more abstract contexts. This leads many to value more abstract subject matter as a marker of rigor. "Abstraction" in this case usually means "less connected to reality.""
    - Abstraction does in part mean that a tool or skill can be studied outside of context and thus applied in many situations. To have that transference between situations students need to practice many contexts and they need to gain fluency in the skill. It is not a choice of one over the other.
- Line 774 Add a bullet point that rigor is supported when students see mathematical skills and content in various contexts and instructors help make these reincarnations of the skills and content transparent. Rigor is also supported when students use different skills and content to solve the same problem and then teachers help facilitate a discussion on why the different approaches give the same answer. Finally rigor is supported when teachers engage the class in a "post mortem" of the work that arrived at a solution, pointing out where incorrect answers helped redirect the work towards deeper understanding and the correct answer.

Chapter 5

- Line 55-60 The majority of what is being advocated for is data literacy and statistics with the opportunity to explore in the future courses that combine CS, Math, and Statistics. Why not use the words Data Literacy and Statistical Literacy? Why engage in the hype of data science which runs the risk of confusion with preparation for STEM?
- Line 1166-1171 (with deleted lines). “Students’ prior work learning about describing and comparing distributions and random sampling comes together in high school. High school students continue to visualize and represent univariate data with dot plots, histograms, and box plots; use measures of center and spread to describe such distributions (S-ID.4); and compare distributions from different populations or samples using these representations and statistics (S-ID.1–3).”
    - This is not currently happening. Students come to college with absolutely none of the k-8 experience described in the previous sections and thus none of the standards described in this quotation.
- Lines 1178-1185 “Data exploration begins with a search for available data about a context of interest. The data set is then examined for hidden patterns and associations. At the high school level, visualization of data can illustrate unexpected structure. Any patterns or associations discovered can lead to new hypotheses or questions to investigate further. Students began this process in eighth grade and continue in high school with experiences in which they examine data sets with multiple variables that are measured for each member of the sample. They plot pairs of variables to decide which ones might show associations.”
    - Teachers will need deep professional learning to integrate this kind of instruction into their work and do so in a way that the visualizations and data collection do not swallow up all the time and muddy the big mathematical and statistical ideas. Without a revision of the CA-CCSS-M it will be impossible to do it all, just as now it is impossible so teachers skip over the statistics content.
- Line 1396 Equitable and Inclusive Instruction.
    - This entire section belongs in chapter 1 as it applies to any area of mathematics or statistics (or really any) instruction. This is particularly true of the first and third bullet points.

Chapter 8

- Overall this chapter helped to make clearer the very laudable goals of the DI, CC, SMP, Big Ideas framework for instruction.
- Lines 158-197. This is a great description of what is best about CMF. The cycle proposed recommends: 1. posing an authentic problem and asking students to ponder and wonder, 2. Soliciting questions and estimates, 3. The

development of necessary mathematics within that purpose and connecting it to other times that mathematics has arisen.
    - This is excellent and I wonder if there is a way to get here without the baggage of the DI, CI, SMP, BI baggage.
- Lines 227-246 Collect/Consider the Data:
    - I remain concerned about the fact that: We cannot do it all. Which of the material we are currently covering will be skipped? I am in favor of doing fewer things more deeply, but I also worry that this actually adds MORE to teachers' plates.
- Lines 247-261 Analyze the data
    - Concern: I have never met a HS student who did not take AP Stat who could do this. So again, how will we get there?
- Line 262. Good add "or statistics course"
- Lines 270-273 "They can explain the meaning of population estimates or other results and discuss in general terms possible sources of error such as missing data and imperfect data collection. They are able to interpret margins of error and confidence intervals graphically, demonstrating correct probabilistic understanding."
    - "In general terms" is a good add.
    - Revise second sentence to "They are introduced to the concepts of margins of error and confidence intervals graphically, practicing correct probabilistic understanding."
- Lines 519-525: Teach Toward Social Justice. (and component three of chapter 2)
    - The title of the section makes it sound like teachers MUST do this, whereas the paragraph talks about how teachers "can take a justice-oriented perspective". I think that the title should be clarified.
    - Chapter 2 lines 459-465 breaks down "teaching toward social justice into two parts that may wee more broadly accessible to teacher than "teaching for social justice" does. These are:
        - creating opportunities for students to both see themselves, as well as people from all backgrounds, as capable and successful doers of mathematics
        - empower learners with tools to examine inequities and address important issues in their lives and communities through mathematics
    - Couldn't these be further broadened to say:
        - creating opportunities for students people from all backgrounds, as capable and successful doers of mathematics

        - empower learners with tools to examine important issues in their lives and communities through mathematics
    - Why is social justice the only engaging way to talk about math that is called out in the framework? Why not a broader lens. Some teachers and student groups may have different capacities for such conversations. A given teacher may be comfortable with social justice, whereas another may hold beliefs that they feel will be unwelcome with colleagues or uncomfortable for students. Now days with the level of division in society is it fair to ask math teachers to take on this work alone?
- Lines 688-699 Combined Algebra II and Pre-calculus
    - This and the acknowledgement that there may be ways to stretch at least some algebra 1 back to 8th grade are nice adds.
    - I wonder if one approach could be to eliminate pre-calculus entirely and do non-AP calculus that covers less calculus but gets the pre-calc material in within (mostly) the context of the calculus.
- Line 720: High School Pathways to STEM Figure 8.4
    - This figure defines options not pathways, so it would be better to use that language.
    - For options one needs to understand where the CA-CCSS-M Algebra 2, Math 3 standards will show up. Financial Algebra does this. How do the others as pairs work?
- Lines 749-764 "...As such, Data Science might be either a third- or fourth-year course, depending on the content included."
    - I cannot tell the difference between 3rd and 4th year classes. Is this implying that a 3rd year class MUST be constructed to develop ALL elements of Math III or Alg II or whatever was not covered in years 1 and 2? Or is it the opposite?
- Lines 770-773 "Depending upon their post-secondary goals, students may choose different third- and fourth-year courses, and all college-intending students should complete four years of mathematics in high school to meet California State University and University of California recommendations."
    - Here again the CMF is vague on whether these 3rd and 4th year options need to satisfy the Alg 2, Math 3 standards.
- Lines 831-840: BOARS revision of Area C was done outside of UC requirements for faculty input and without input from CSU faculty whose system follows the UC guidelines. The fact that the UC and CSU use the same guidelines is important to equity. In order for us to do so, it is correct that BOARS follows the guidelines of the CA-CCSS-M and ICAS. Building a framework on a flawed UC decision making process is a bad choice.
- Line 917 Course Conent in the Grades 9-12
    - Replace this with appendix A and put this in the appendix.

    - This comes verbatim from: https://www.cde.ca.gov/ci/cr/dl/documents/dlintegrationstdsguide.pdf Which was authored by Boaler. Please add reference.
- Lines 924-926: “For the first and second courses in the pathways, readers will also find tables that show how Big Ideas (left column) relate to the Content Connection (middle column) and the CA CCSSM content standards (right column)”
    - Why not for Alg 2 and Math III?? That is what sows the seeds of distrust.
    - Add a list of the standards NOT included in the big ideas
- Lines 945-948: “The critical areas of instruction involve deepening and extending students’ understanding of linear and exponential relationships by comparing and contrasting those relationships and by applying linear models to data that exhibit a linear trend.”

    - This is great but does not match what is in Appendix A on Algebra 1 lines 28-67.

- Lines 1301-1307 “But in light of studies showing that California’s past encouragement of middle school acceleration undermined success for many students, this framework proposes instead that school districts adjust the high school curriculum by eliminating redundancies in the content of current courses. Doing so would streamline the number of courses students would need to take before Calculus and remove the need for all students to take algebra in eight grade to reach higher math levels by high school graduation.”

    - Is there evidence that this is a good approach?

Appendix A

- lines 28-67 on Algebra 1 do not match Lines 924-926 of chapter 8
- Lines 90-93 “Algebra II divide polynomials, which results in remainders, leading to the factor and remainder theorems.”
    - The depth of this may be where we differentiate between STEM and non-STEM. Maybe some of the depth can be in pre-calc or in a non-ap calc that does pre-calc as needed?
- Line 107 Conic Sections
    - Again, is this for all? Is it the most important?
- Line 110 say what the three basic functions are.
- Lines 186-192 “Mathematics III students extend their understanding of polynomials to see them as a system analogous to the integers (and rational expressions as analogous to rational numbers) and develop an understanding of the structural similarities between the two sets. They draw on analogies

between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multidigit integers, and division of polynomials with long division of integers."

  - As for Alg 2: The depth of this may be where we differentiate between STEM and non-STEM. Maybe some of the depth can be in pre-calc or in a non-ap calc that does pre-calc as needed?

- Lines 195-198 "they build on these experiences to derive the Laws of Sines and Cosines, develop the notion of radian measure for angles, and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena."
  - I am not sure who needs law of sines in advance of using it as prep for STEM when you do trig or vector distances.
- Lines 249-271: **Be familiar with modeling with functions**.
  - These are great.
- Line 282 "Mathematical problems can often be solved in a variety of ways, and it is both legitimate and important to often allow students the choice of solving problems in ways that make the most sense to them."
  - And, students who have approached a problem differently should be encouraged to understand how the different approaches are similar and different and why they arrive at the same answer or answers.
- Line 331 "**6. Be able to work with and solve equations (and inequalities)**."
  - And, students must understand what a solution to an equation means, and what a solution to a system of equations means. They must further understand why certain algebraic manipulations preserve solutions whereas others (e.g. multiplying both sides by zero) do not.
- Line 357 "Attention to units of measurement is necessary for meaningful answers to quantitative questions about the world."
  - Yes!
- Line 376 "Although traditionally done in the context of plane geometry, such justification can also be done with algebra (e.g., mathematical induction to establish some formulas). "
  - I would a different example. I have never met a CSU student who remembered induction from HS. How about proving that if y=quadratic in x, then the graph is a shift/stretch of y=x^2.