**Problems that need fixing**

**in the third draft of the California Math Framework**

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This version has corrected many issues and is much easier to read. Thanks for all of the hard work. **But there are still errors in this draft about what the CA CCSS say about standard algorithms and the centrality of multiplication and division in Grade 3.**

The Framework correctly emphasizes that students need to develop understanding of multidigit operations. But it **confuses fluent use of a standard algorithm with the first use of a standard algorithm**. It repeatedly states that standard algorithms can only be introduced/taught/learned in the grade at which the CA CCSS say that students must use a standard algorithm fluently. **The CA CCSS clearly state that students are to develop, discuss, and use efficient, accurate, and generalizable methods in the first year in which that kind of computation is introduced:**

California CCSS p 14: Grade 1 “students are to develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10.”

California CCSS p. 18: Grade 2 students “solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations.”

So in grade 1 and 2 in the California CCSS some versions of standard algorithms which are efficient, accurate, and generalizable methods can and even must be developed, but they should be done meaningfully and with drawings.

California CCSS 1.NBT.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relation- ship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

California CCSS 2.NBT.7: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.

The CMF nicely shows variations in multiplication standard algorithms as efficient, accurate, and generalizable methods. The CMF needs to do the same for grade 1 and 2 accessible versions of standard algorithms that would enable California students to develop, understand, and explain general methods. A description of these drawn from NCTM materials are below on pages 4 to 6. This material could be added earlier in Chapter 6 or just before the table about fluent use of standard algorithms.

**Therefore these changes need to be made to be consistent with the CA CCSS:**

Chapter 6 lines 1563 to 1567 need to be omitted:

*Everyday Mathematics* offers guidance for families, explaining how premature instruction in standard algorithms can often lead to erroneous and even harmful ideas. Students may come to believe that mathematics is mostly about memorizing, that mathematics problems should be solved in a few minutes, and that there is just one right way to solve a problem.

KCF: This is false. I carried out the five-year evaluation of Everyday Mathematics for Max Bell as requested by NSF. Teachers in Everyday Math allowed students to use their own methods but then taught the bad standard algorithms right before spring tests. These opinions about standard algorithms arise from bad teaching and bad algorithms not from the fact that all standard algorithms are bad.

1568 to 1594 also is false as seen above and needs to be replaced by the blue paragraph below:

Note that the CA CCSSM do not include standard algorithms in transitional kindergarten through grade three, although there are standards addressing fluencies needed for proficiency in standard algorithms in later grades. Instead, as shown in figure 6.31, the progression related to standard algorithms begins with the standard algorithm for addition and subtraction in grade four; the algorithm for multiplication is addressed in grade five; and the introduction of the standard algorithm for whole number division comes in grade six. (Chapter seven addresses grade six.)

REPLACE WITH: The CA CCSSM do not call for fluency with standard algorithms in grades TK–3, so there is time to develop meanings for accessible standard algorithms with drawings in these grades. The CA CCSSM do say that first-grade “Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10.” (California CCSS p 14). And second-grade students “solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations.” (California CCSS p 18). The CA CCSSM place fluent use of standard algorithms at the grades indicated below in the table. Fluent use means without drawings or other visual representations, so it is important to introduce sense-making with drawings **and easy variations of standard algorithms** at earlier grades.

In Chapter 3 there is now a lovely example and discussion in lines 456 to 476 about first graders using efficient, flexible, generalizable methods. But the final two sentences are wrong and need to be removed. The important and lovely discussion is correct but these sentences are false and are not needed.

“Note that while students in first grade do begin to add two-digit numbers, they do so using strategies as distinguished from formal algorithms. The CA CCSSM intentionally place the standard algorithms for addition and subtraction in fourth grade (4.NBT.4).” Not true; the CA CCSSM place fluency with standard algorithms for addition and subtraction in fourth grade (4.NBT.4)

**A second major problem that needs to be corrected is that the CMF refers to just one standard algorithm as if there is just one. This is wrong.** The NBT Progression clarifies that there is no single standard algorithm for any operation: “The Standards do not specify a particular standard algorithm for each operation. This progression gives examples of algorithms that could serve as the standard algorithm and discusses their advantages and disadvantages. p.3” and “In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. This progression gives examples of different recording methods and discusses their advantages and disadvantages. p.3” Yet the California Math Framework repeatedly talks about “the standard algorithm” as one written method and does not recognize or emphasize for the early grades this crucial variation in written methods which can arise from the thinking of different students. The CMF does recognize this for multiplication and gives good examples in Chapter 6. But it is crucial to do so for Grades 1 and 2.

These variations in how to write the standard algorithm vary in difficulty. Some are easier than others and result in fewer errors. It would be helpful if California teachers were aware of these better variations. The Framework rightfully stresses student understanding and explaining of methods and finding advantages and disadvantages of various methods. This meaning-making with variations of standard algorithms is an important part of the learning progression for multidigit computation and can occur in any grade level before fluency is expected. This point should be clear in the Framework. An NCTM publication [Focus in Grade 2 (2011). Reston, VA: The National Council of Teachers of Mathematics, Inc. pp. 87-89] has figures that show for addition and subtraction how Grade 2 students can use drawings for easier variations of standard algorithms, explain those drawings, and answer questions from classmates. It would be helpful for these examples to appear in the Framework where teachers can see them. These are not fluent uses of standard algorithms because they involve drawings. These are building meanings for steps in accessible methods that will make fluency easier at Grade 4 when the drawings may be dropped.

The California Math Framework rightfully emphasizes meaning-making approaches to computation. It now can clarify that these meaning-making approaches form a learning path in which the easier variations of standard algorithms can be discussed and used by students meaningfully in the earlier grades if students use drawings or place-value objects to support their thinking. Using these examples from NCTM is especially important because teachers need to see uses students can make of drawings and that students can explain steps in written methods. These examples can be inserted in Chapter 3 or in Chapter 6. It is important for California teachers to see examples of Grade 2 students explaining 3-digit additions and subtractions with drawings because such work is in the CA CCSS and is important to do in Grade 2. Such work enables third graders to concentrate on multiplication and division and just quickly review 3-digit adding and subtracting. **These drawings are on the next 3 pages. They might be introduced by something like this:**

“Later in the year students work with drawings or objects to develop methods for adding and subtracting 3-digit numbers (2.NBT.7). Written methods are important for many students for such larger numbers, and it is important that explanations of written methods be related to drawings. In Figure 2.21 a Grade 2 student explains their method to the class and does such relating of the written method and the drawing. This method is an easier standard algorithm invented by students in which the new groups are written below. This makes it easier to see the totals of each column as a teen number and makes it easier to add each column of place values. Classmates ask questions of the explainer. In Figure 2.22 a student explains a simpler subtraction standard algorithm in which all ungrouping is done first. This method eliminates errors students make with the traditional alternating standard algorithm in which they may forget to ungroup in a given column. In such an explaining classroom, students also explain other methods, and the class discusses advantages and disadvantages of each method.”

**Methods follow on three pages.**

Text

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Focus in Grade 2 (2011). Reston, VA: The National Council of Teachers of Mathematics, Inc. p. 87.

Graphical user interface

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A picture containing text

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Focus in Grade 2 (2011). Reston, VA: The National Council of Teachers of Mathematics, Inc. pp. 88-89

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Chapter 6 979: Figure 6.15 Grade Two Content Connections, Big Ideas, and Content Standards

Related to the above issues is a vital addition needed for the third row: NBT.9 needs to be added to the list because NBT.9 specifies explaining and explaining is crucial to understanding. Also NBT.5, 6, 7 concern multidigit addition and subtraction and not money. These are crucial standards and need to be described in this table. Add this text: Connect these money values to place values and to 2-digit and 3-digit methods of adding and subtracting and explain such methods using drawings as needed.

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Also the end of the Big Idea in the fifth table row shown above needs this added: Use these place values to develop understanding with 3-digit adding and subtracting.

NBT.9 about explaining also could be added to this row.

These additions to the Grade 2 table broaden the Big Ideas to crucially important Grade 2 concepts of meanings in multidigit addition and subtraction instead of the less important standards about dollars and cents and skip counting to 100.

**The Crucial Importance of Multiplication and Division in Grade 3**

**Chapter 6 1065: Figure 6.16: Progression Chart of Big Ideas through Grades 3–5**

**The most important concepts in Grade 3 are multiplication and division. Single-digit multiplication and division are introduced and come to fluency in Grade 3. But these do not even appear in Figure 6.16.** Two small changes in Figure 6.16 can fix this.

Change Addition and Subtraction Patterns in Figure 6.12 and Figure 6.53 to **Patterns in Four Operations.** There are manypatterns in multiplication and division, so this change includes these many vital patterns. This also reflects the header above 3.OA.8 and 9.

Add to Number flexibility to 100 for all four operations.

Both of these circles should also be bigger in Figure 6.52 Grade 3 Big Ideas because they are crucially important in Grade 3.