To: California State Board of Education

From: Alan H. Schoenfeld, U.C. Berkeley ([alans@berkeley.edu](mailto:alans@berkeley.edu))

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Re: Adoption of the *2023 Mathematics Framework for California Schools*.

Dear SBE members:

The current version of the *2023 Mathematics Framework for California Schools* represents a significant improvement over the previous draft, and we support its adoption. We do, however, wish to note one point that needs attention.

Chapter 8 of the Framework sets forth two primary pathways through the secondary mathematics curriculum: the traditional and the integrated pathways. Given that both pathways are intended to address the same content and practices (those delineated in the Common Core)[[1]](#footnote-1), we expected to see closely parallel descriptions of the mathematical content to be covered in the two approaches. This was not the case regarding perhaps the most fundamental concept in mathematics, the concept of function, including the use of functions in mathematical modeling. Discrepancies can be seen easily by looking at Figures 8.5 and 8.8, describing the traditional pathway, as compared to Figures 8.13 and 8.16, describing the integrated pathway. In both pathways we find “modeling with functions” and some structural investigations (e.g., “systems of equations”) – but, in Figures 8.13 and 8.16 we also find “comparing models,” “function representations,” “functions in the world,” and “equations to predict and model.”

As we see it, the Framework’s description of the integrated pathway provides a much richer – and more appropriate, given our understanding of the Common Core – description of the ways in which students should encounter functions and modeling. The description of functions and modeling in the traditional pathway should be comparably rich.

In what follows we suggest a few ways in which this difference can be addressed.

• Early in the chapter, it might be good to stress the central role of functions and modeling across the entire secondary curriculum. This might be done by adding text like the following after line 130:

One fundamental use of mathematics is to model real-world situations, for example the costs of different cellphone plans, the changing values of used cars, or the different sizes and comparative costs of different pizzas. What all these situations have in common is that the objects and relationships between them can be expressed in mathematical terms, often with one such object being expressible as a function of another. Once a situation has been described in that way, it can be analyzed mathematically to discover relationships, find patterns, and make predictions.

As students progress through the secondary curriculum, they encounter increasingly complex mathematical functions and relationships, and increasingly sophisticated ways to represent and analyze data. They begin by working with linear functions, sets of linear functions, and some polynomial families of functions (for example quadratics). Later on they encounter logarithmic, exponential, and trigonometric functions. The mathematical objects or analytic methods they encounter may be new – but the processes of mathematizing and sensemaking are the same. It’s like writing essays using a larger vocabulary and with an improved capacity for making complex arguments. The goal, whether for applications or the study of mathematical objects and relations in their own right, is to develop robust understandings and habits of sensemaking, with an increasingly large toolkit of concepts and functions.

• There is also room for expansion in the brief description of Algebra I in lines 944-952. At minimum, the opening sentence beginning on line 944 can be expanded to say “The main purpose of Algebra I is to develop students’ *understanding of and* fluency with linear, quadratic, and exponential functions, *and their use to model real-world phenomena.”* (The words in italics are additions.)

• Might it be possible to expand Figures 8.5 and 8.8 to include some of the key ideas in Figures 8.13 and 8.16?

•• In addition, we note that the big ideas for geometry fail to include deductive reasoning. That is, formalizing and proof are presented as good in themselves. In fact, the fundamental goal is reasoning – in the early NCTM Standards they were conjoined as reasoning and proof. It would be good to have a statement somewhere along the following lines: “Producing a proof should not be seen as a way to meet abstract requirements regarding the ways that mathematical claims should be presented, but rather as the end product of reasoning and sensemaking, organized and presented in ways that make it easier to convey the resulting understandings.”

Respectfully submitted,

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1. “This Framework serves as a guide to implementing the California Common Core State Standards for Mathematics (CA CCSSM or the Standards), adopted in 2010 and updated in 2013” (Introduction, lines 37-39). [↑](#footnote-ref-1)