QGAN Investigation

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1 Summary on results

We use a quantum GAN (QGAN) implementation [1] in QISKIT library. The QGAN consists of a *quantum generator* and a *classical* discriminator to capture the probability distribution of *classical* training samples. The quantum generator is implemented by a variational form, i.e. a parametrised quantum circuit. The goal of a QGAN is to train the generic probability distribution.

The relative entropy quantifies the difference between two probability distributions, and it is used to evaluate the performance of machine. When the machine is well optimised, we observe that it has a value of about 5%.

Table 1: Number of qubits of quantum generator and relative entropy from different distributions.

	Number of qubits	Relative entropy [%]
$1d$ Gaussian (Fig. 2)	2	0.33
$2d$ Gaussian (Fig. 3)	4	2.26
$2d$ Copula (Fig. 5)	4	0.09
$2d$ Copula (Fig. 6)	6	5.98

In the following section, we optimise a QGAN to train the $1d$ and $2d$ Gaussian distribution, and $2d$ joint distribution of copula space. Additionally, we also train classical GAN to generate $2d$ Gaussian distribution. All codes can be found here.

2 Results

2.1 1d Gaussian distribution

For the $1d$ probability distribution, a parametrised quantum circuit of the quantum generator is trained to generate an n-qubit output state from a given n-qubit initial state $|\psi_{\text{in}}\rangle$

$$G_\theta|\psi_{\text{in}}\rangle = \sum_{i=0}^{2^n-1} \sqrt{p_\theta(i)}|i\rangle\,, \tag{1}$$

where $p_\theta(i)$ is the occurrence probabilities of the basis states $|i\rangle$.

We consider a $1d$ Gaussian distribution with mean 2 and variance 1. The training data set $x_i \in \{0, 1, 2, 3\}$ is constructed by rounding the sampled values to integer, and then truncated to [0, 3]. As a parametrised quantum circuit with the $n = 2$ qubits, we use the TwoLocal circuit with 1 depth. As the number of qubits n increases, the depth of TwoLocal circuit should increase to optimise. We train a QGAN with 300 epochs and 100 batch size. For each setting, we repeat the training 10 times to obtain the minimum relative entropy whose value is approximately 0.33%. The training process and results are shown in Fig. 1 and Fig. 2, respectively.

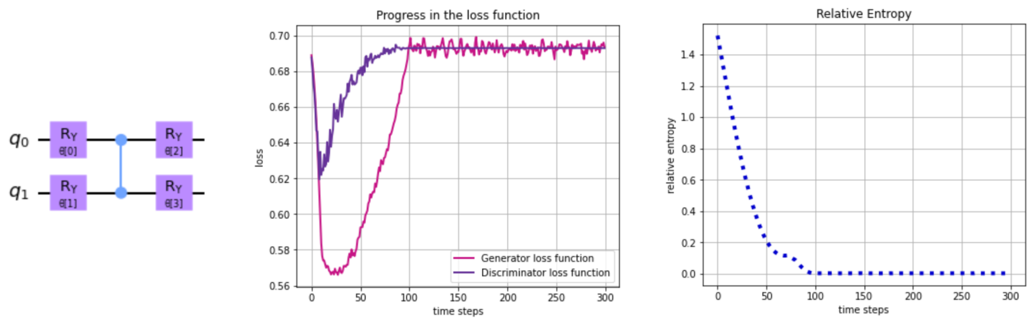

Figure 1: The QGAN with $n = 2$ training of $1d$ Gaussian distribution with mean 2 and variance 1. Here we use 2 qubits for a quantum generator. To obtain this result, we train a QGAN 20 times and choose it with the minimum relative entropy. **(Left)** A TwoLocal circuit as a parametrised quantum circuit. **(Middle)** The loss function as a function of time steps. **(Right)** The relative entropy as a function of time steps. The relative entropy at final time steps is used as a performance metric of a QGAN.

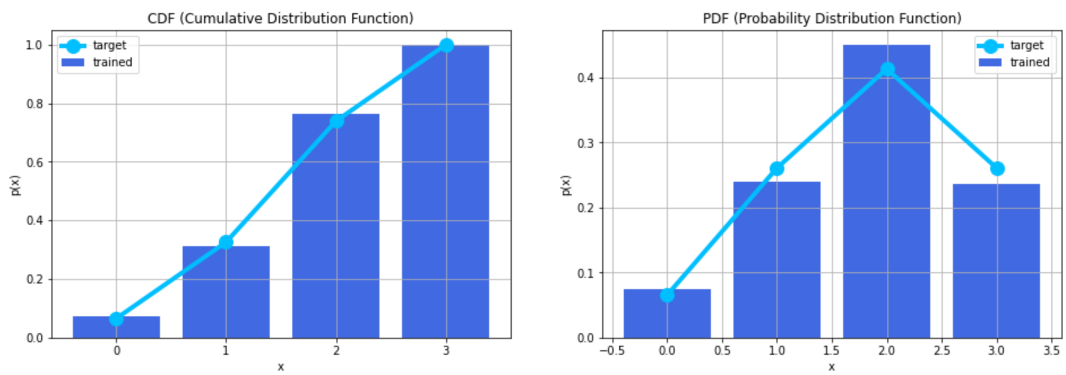

Figure 2: Results of a QGAN with $n = 2$ training of $1d$ Gaussian distribution with mean 2 and variance 1. The value of relative entropy is approximately 0.01. We generate 10^4 samples from the generator. **(Left)** CDF. **(Right)** PDF.

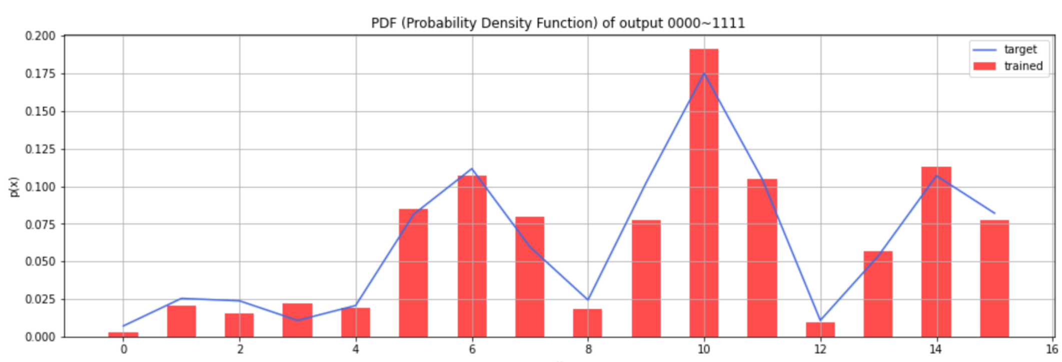

Figure 3: PDF of a QGAN with $n = 4$ training of $2d$ Gaussian distribution with mean (2, 2) and covariance ((1, 0.2), (0.2, 1)). The value of relative entropy is approximately 0.0226. The x-axis represents the decimal number of the output state of the quantum circuit from 0000 to 1111..

2.2 2d Gaussian distribution

We consider the $2d$ Gaussian distribution with mean (2, 2) and covariance ((1, 0.2), (0.2, 1)) with the $n = 4$ qubits. The training data set (x_i, y_i) is constructed by rounding the sampled values to integer, and then truncated to $x_i \in [0, 3]$ and $y_i \in [0, 3]$. As a parametrised quantum circuit, we use the

TwoLocal circuit with 2 depth. We train a QGAN with 800 epochs and 200 batch size. The minimum relative entropy is 0.0226, which indicates that the machine is well optimised shown in Fig. 3.

2.3 2d copula space

The training data are the joint distribution of copula space of daily return of AAPL and MSFT between 2010-2018. For the $2d$ probability distribution with the even integer $n = 2m$, a parametrised quantum circuit is written as

$$G_\theta|\psi_{\rm in}\rangle = \sum_{i=0}^{2^m-1}\sum_{j=0}^{2^m-1}\sqrt{p_\theta(i,j)}|i\rangle|j\rangle\,. \tag{2}$$

Let us consider the $2d$ joint probability distribution with the n-qubit output state in Ref. [2].

1. Each coordinate of the original training copula data (x, y) is the real value between zero and one. We multiply 2^n at each coordinate and then discritise them by truncating the fractional part of the number.

2. For the resulting training data (x', y') where $x' = \lfloor x\times 2^n\rfloor \in \{0, 1, \cdots, 2^n-1\}$ and $y' = \lfloor y\times 2^n\rfloor \in \{0, 1, \cdots, 2^n - 1\}$, we train a QGAN to generate the probability distribution. For $n = 6$, an 6-qubit output state from a given 6-qubit Bell initial state $|\psi_{\rm in}\rangle = \frac{1}{\sqrt{2}}(|000000\rangle + |111111\rangle)$ is

$$\begin{aligned} G_\theta|\psi_{\rm in}\rangle &= \sum_{i=0}^{7}\sum_{j=0}^{7}\sqrt{p_\theta(i,j)}|i\rangle|j\rangle \\ &= \sqrt{p_\theta(0,0)}|000000\rangle + \sqrt{p_\theta(0,1)}|000001\rangle + \cdots + \sqrt{p_\theta(7,7)}|111111\rangle\,, \end{aligned} \tag{3}$$

   where the parametrised quantum circuits are shown in Fig. 4.

3. Once the QGAN is optimised, we sample the output data (x'', y'') from the probability $p_\theta(x'', y'')$.

4. We compare the original distribution $p_\theta(x', y')$ with the generated distribution $p_\theta(x'', y'')$.

For $n = 2$ and $n = 3$, we train a QGAN with 1000 epochs and 100 batch size. The EfficientU2 ansatz and Bell state ansatz (Fig. 4) are used for $n = 2$ and $n = 3$ quantum registers, respectively. For $n = 2$, the minimum relative entropy is 0.0009, which indicates that the machine is well optimised shown in Fig. 5. On the other hand, for $n = 3$, the minimum relative entropy is 0.0598 shown in Fig. 6.

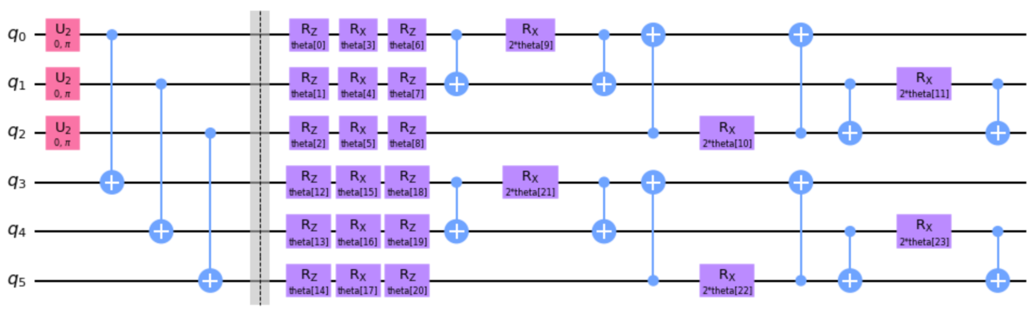

Figure 4: The generator circuit for 2 random copula variables with $n = 6$. The above and below 3-qubit denote x' and y', respectively. The left side before the dotted barrier represents the initial state $|\psi_{\rm in}\rangle$ as the bell states. The right side after the dotted barrier represents ansatz circuits G_θ. All gates of right side are parametrised by angles which are optimized during the learning process. The structure can be repeated for multiple layers each with different parameters.

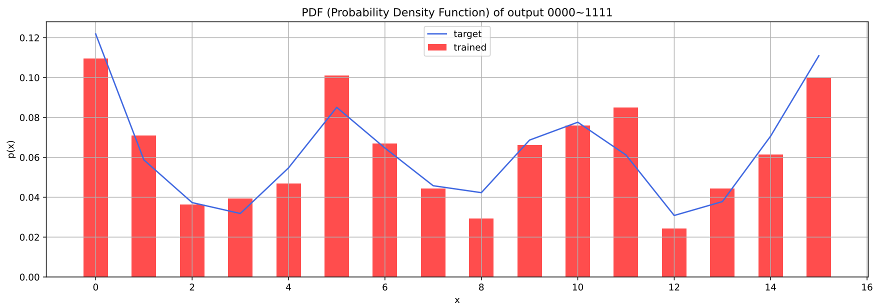

Figure 5: PDF of a QGAN with $n = 4$ training of $2d$ copula distribution with the copula space of daily return of AAPL and MSFT between 2010-2018. The value of relative entropy is approximately 0.0009. The x-axis represents the decimal number of the output state of the quantum circuit from 0000 to 1111.

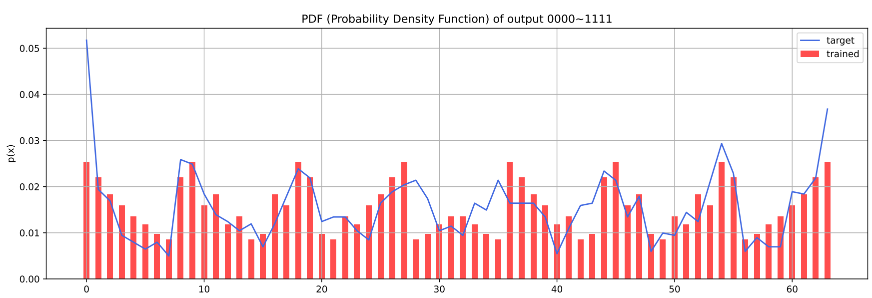

Figure 6: PDF of a QGAN with $n = 6$ training of $2d$ copula distribution with the copula space of daily return of AAPL and MSFT between 2010-2018. The value of relative entropy is approximately 0.0598. The x-axis represents the decimal number of the output state of the quantum circuit from 000000 to 111111.

References

[1] Christa Zoufal, Aurélien Lucchi, and Stefan Woerner. Quantum generative adversarial networks for learning and loading random distributions. *npj Quantum Information*, 5(1):103, 2019.

[2] Elton Yechao Zhu, Sonika Johri, Dave Bacon, Mert Esencan, Jungsang Kim, Mark Muir, Nikhil Murgai, Jason Nguyen, Neal Pisenti, Adam Schouela, Ksenia Sosnova, and Ken Wright. Generative quantum learning of joint probability distribution functions, 2021.