MEN35101 – Machine Element Design, Fall Term 2019

1st Midterm Exam

October 8, 2019

10:30 am to Noon

You must clearly show all work in detail and answers on the answer sheets provided.

Put your name and student ID on the cover of the answer sheets.

Consider the shaft carrying two belt pulleys as shown in Figure 1 on the back of the paper. Assume the belt tension on the loose side at B is 20 percent of the tension on the tight side. The shaft is also subjected to the axial load, $P = -5\ \text{kN}$. The two bearings at O and C act as simple supports. The shaft is made of 1018 CD steel, of which the yield strength, S_y, is $370\ \text{MPa}$.

(a) **(10pts)** Determine the tensions in the belt on the pulley B and the bearing reaction forces at O and C, assuming that the shaft is in the static equilibrium state.

(b) **(20pts)** Draw shear force, bending-moment and torque diagrams.

(c) **(40pts)** From the bending moment diagram you found in (b), identify the location where the resultant bending moment is the maximum. At the corresponding cross section, find all the non-zero stresses at the four points as indicated in Figure 2.

(d) **(10pts)** Identify the critical location based on the result in (c). Compute the principal stresses (σ_1, σ_2 and σ_3) and the maximum shear stress ($\tau_{\max}$) at the location.

(e) **(10pts)** Find the safety factors based on the maximum shear stress theory (MSST) and distortion energy theory (DET), respectively.

(f) **(10pts)** How much should you increase the diameter in order to achieve the minimum safety factor of 3.0 based on MSST?

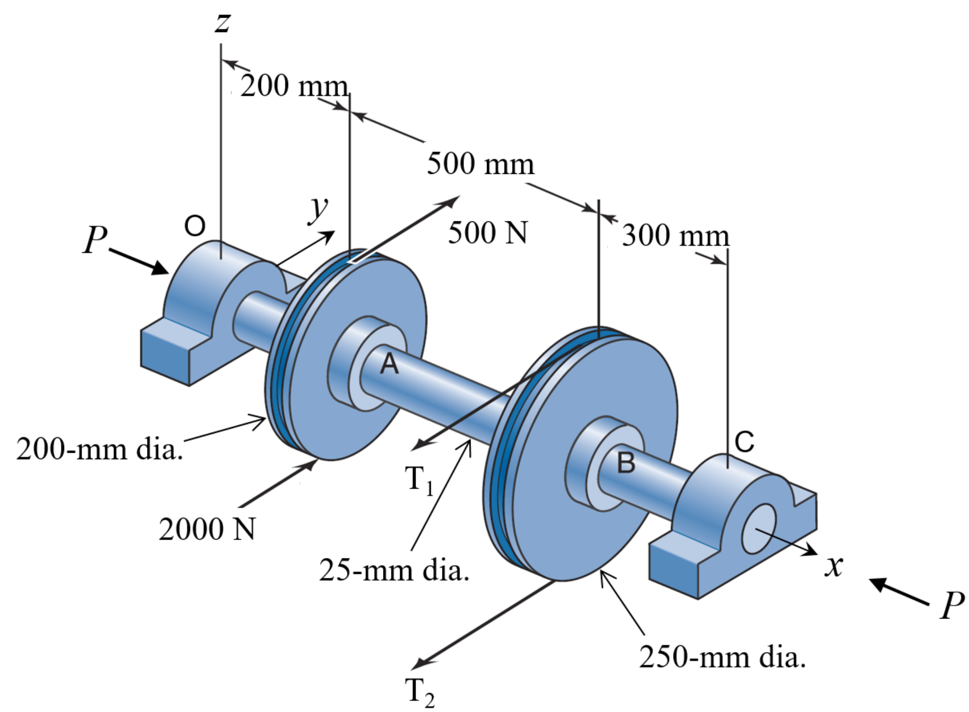
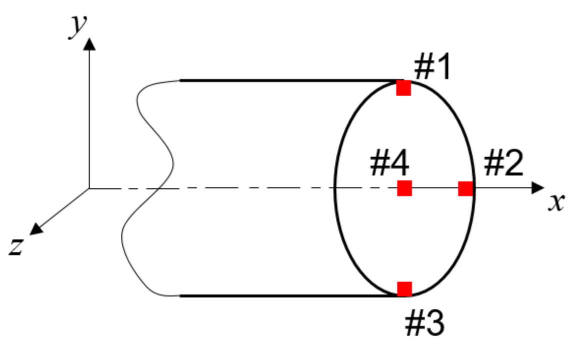


Figure 1



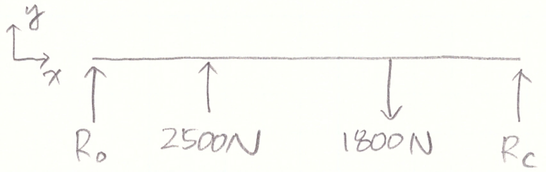
#1: $y = d/2,\ z = 0$
#2: $y = 0,\ z = -d/2$
#3: $y = -d/2,\ z = 0$
#4: $y = 0,\ z = 0$
d: diameter of the shaft

Figure 2

(a) $T_1 = 0.2T_2$

$\sum T = 0$; $(2000N - 500N)(0.1m) + (T_1 - T_2)(0.125m) = 0$

$\Rightarrow \boxed{T_1 = 300N, \quad T_2 = 1500N}$

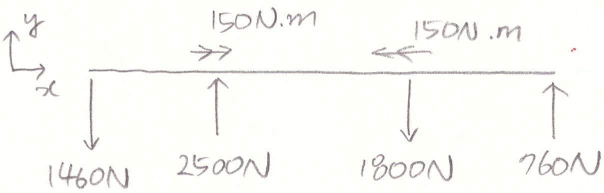


$\sum F_y = 0$; $R_o + 2500 - 1800 + R_c = 0$

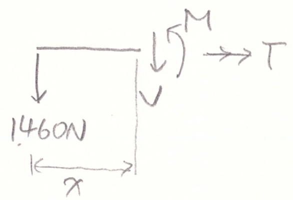
$\sum M_o = 0$; $(0.2m)(2500N) - (0.7m)(1800N)$
$+ (1m) \cdot R_c = 0$

$\Rightarrow \boxed{R_o = -1460N, \quad R_c = 760N}$

(b)



① $0 \leq x < 200mm$

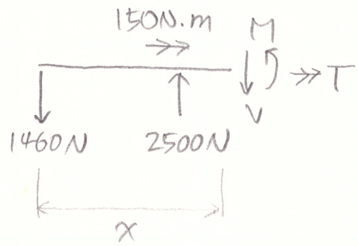


$\sum F_y = 0$; $-1460 - V = 0 \Rightarrow V = -1460N$

$\sum M_x = 0$; $M + 1460x = 0 \Rightarrow M = -1460x$ N.m

$\sum T_x = 0$; $T = 0$

② $200 \leq x < 700mm$



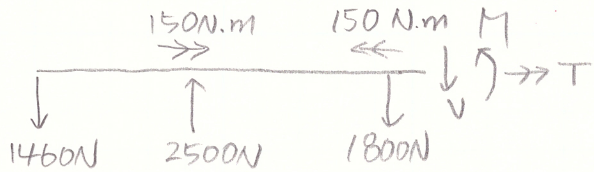
$\sum F_y = 0$; $-1460 + 2500 - V = 0 \Rightarrow V = 1040N$

$\sum M_x = 0$; $M + 1460x - 2500(x - 0.2) = 0$

$\Rightarrow M = 1040x - 500$ (N.m)

$\sum T_x = 0$; $T + 150 = 0 \Rightarrow T = -150$ N.m

③ $700 \leq x \leq 1000mm$



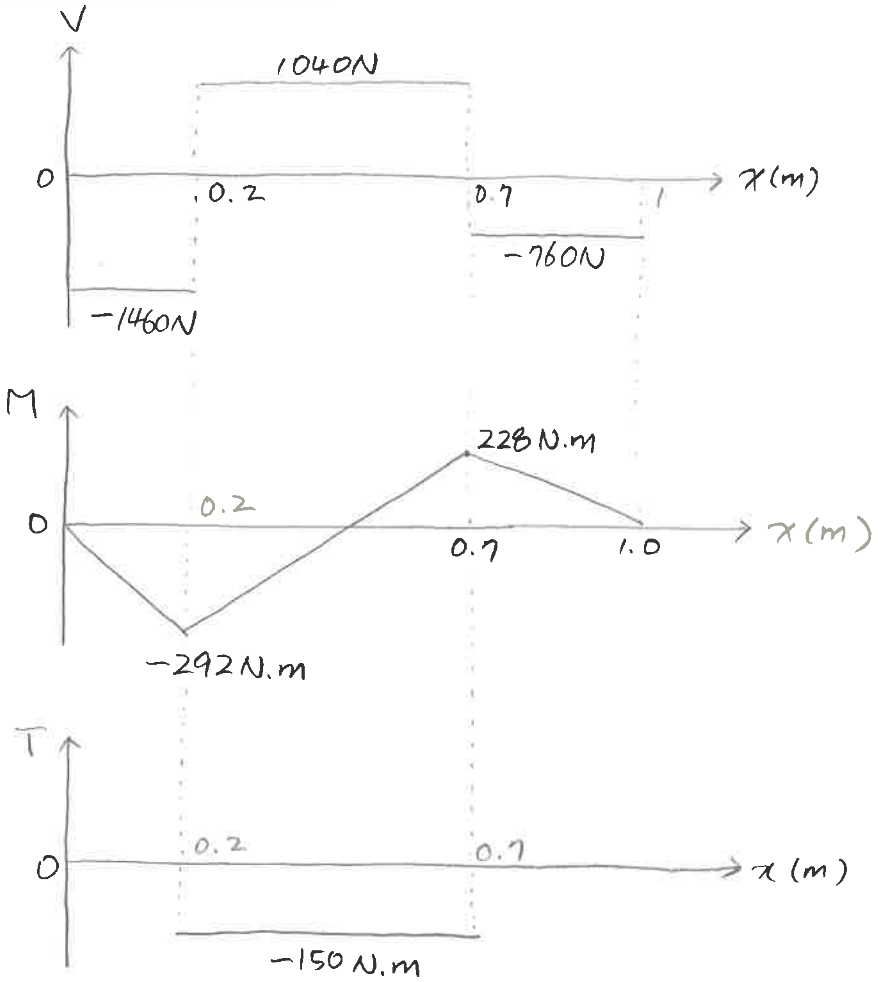
$\sum F_y = 0$; $-1460 + 2500 - 1800 - V = 0$

$\Rightarrow V = -760N$

$\sum M_x = 0$; $M = 760x - 760$ (N.m)

$\sum T_x = 0$; $T + 150 - 150 = 0$

$\Rightarrow T = 0$



(c) $\sigma_{bending} = \dfrac{Mc}{I} = \dfrac{(292\ N.m)(12.5mm)}{\pi(25mm)^4/64} = 190.354 MPa$

$\sigma_{axial} = \dfrac{P}{A} = \dfrac{5000\ N}{\pi(12.5\ mm)^2} = 10.186\ MPa$

$\tau_{torque} = \dfrac{Tc}{J} = \dfrac{(150\ N.m)(12.5\ mm)}{\pi(25mm)^4/32} = 48.892\ MPa$

$\tau_{transverse} = \dfrac{4V}{3A} = \dfrac{4}{3} \cdot \dfrac{1040N}{\pi(12.5mm)^2} = 2.825 MPa$

#1 : $\sigma_{xx} = +\sigma_{bending} - \sigma_{axial} = 180.168 MPa$

$\tau_{xz} = -\tau_{torque} = -48.892 MPa$

#2 : $\sigma_{xx} = -\sigma_{axial} = -10.186\ MPa$

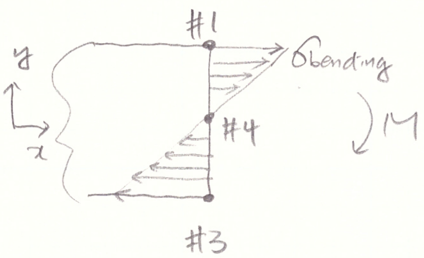
$\tau_{xy} = -\tau_{torque} - \tau_{trans} = -51.717\ MPa$

#3: $\sigma_{xx} = -\sigma_{bending} - \sigma_{axial} = -200.54\ \text{MPa}$

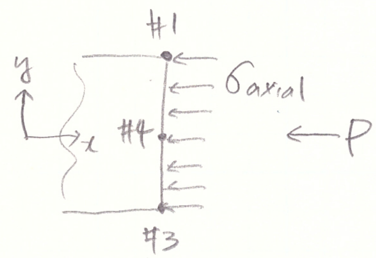
$\tau_{xz} = \tau_{torque} = 48.892\ \text{MPa}.$

#4: $\sigma_{xx} = -\sigma_{axial} = -10.186\ \text{MPa}$

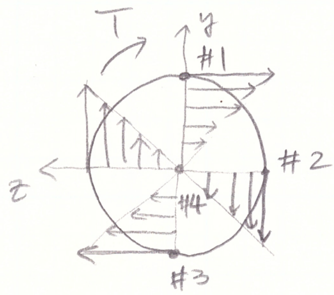
$\tau_{xy} = -\tau_{transverse} = -2.825\ \text{MPa}$



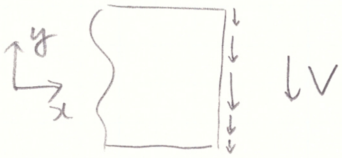
<Bending moment>



<Axial loading>



<Torque>



<Transverse shear>

(d) #3 is the critical location.

$\sigma_1 = 11.285\ \text{MPa}, \quad \sigma_2 = 0, \quad \sigma_3 = -211.825\ \text{MPa}.$

$\tau_{max} = 111.555\ \text{MPa}$

(e) $\text{MSST} = \dfrac{S_y}{\sigma_1 - \sigma_3} = 1.658$

$\text{DET} = \dfrac{S_y}{(\sigma_1^2 - \sigma_1 \cdot \sigma_3 + \sigma_3^2)^{1/2}} = 1.700$

(f). $\sigma = \dfrac{Mc}{I} = \dfrac{M(d/2)}{\pi d^4/64} = \dfrac{32M}{\pi d^3} \Rightarrow \sigma \propto \dfrac{1}{d^3}$

The normal stress due to bending is dominant in this problem and the bending stress is inversely proportional to d^3.

For the minimum safety of 3.0, the minimum safety factor of 1.658 should be increased by a factor of $3/1.658 = 1.8$. Therefore, the diameter should be increased by a factor of $1.8^{1/3}$ or $d = 25 \times 1.8^{1/3} \approx 30.5\text{mm}$