1. Consider 2-D Laplace equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = S(x, y),$$

with the boundary conditions

$$T(0, y) = T(1, y) = 0 \quad \text{for} \quad 0 \le y \le 1.$$

$$\partial T/\partial y|_{x,0} = 0;\ T(x, 1) = 0 \quad \text{for} \quad 0 \le x \le 1,$$

$$S(x, y) = -10 \ \text{ for } 0 < x < 1 \text{ and } 0 < y < 1$$

Assume that $\Delta x = \Delta y = h$.

a) Write the finite difference equations for the successive over-relaxation (SOR) iteration

b) Write the finite difference equations for the successive line over-relaxation (SLOR) method

c) (Programming) Start a guess of $T = 0$ at all points at which T is unknown and use the SOR iteration with $\omega = 1.8$. Error is defined as:

$$\text{err} \equiv \max\left(\text{abs}\left(\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{T_{i,j}^{n}} \right) \right), \quad 2 \le i \le I - 1, \ 1 \le j \le J - 1,$$

and the error tolerance is 1e-6. Use $I = J = 21$. Discuss the convergence rate based on the iteration number when the solution converges. Plot the solution using 'surf' function in MATLAB.

d) (Programming) Repeat c) using the SLOR method.

e) (Programming) Repeat c) using the Generalized Minimum Residual (GMRES) method (gmres function in MATLAB).