1. Consider 1-D heat equation,

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + S(x),$$

with the boundary and initial conditions

$$\text{B.C.: } \left.\frac{\partial T}{\partial x}\right|_{0,t} = 0 \quad \text{and} \quad T(1,t) = 0 \quad \text{for} \quad 0 < t \leq 1,$$

$$\text{I.C.: } T(x,0) = 0 \quad \text{for} \quad 0 \leq x \leq 1,$$

For convenience, we choose $\alpha = 0.1$, $S = 1.0$, $\Delta x = 0.1$ and $\Delta t = 0.05$

① Solve the equation numerically using the explicit method (FTCS method) and plot the solution at $t = 1.0$ with 'plot' function.

② Solve the equation numerically using the implicit method (BTCS method) and plot the solution at $t = 1.0$ with 'plot' function.

③ Solve the equation numerically using the Crank-Nicolson method and plot the solution at $t = 1.0$ with 'plot' function.

2. Consider 2-D heat equation,

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S(x,y),$$

with the boundary and initial conditions

$$\text{B.C.: } \left.\frac{\partial T}{\partial x}\right|_{1,y,t} = 0 \quad \text{and} \quad T(0,y,t) = T(x,0,t) = T(x,1,t) = 0 \quad \text{for} \quad 0 < t \leq 1,$$

$$\text{I.C.: } T(x,y,0) = 0 \quad \text{for} \quad 0 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq 1.$$

For convenience, we choose $\alpha = 0.1$, $S = 1.0$, $\Delta x = \Delta y = 0.1$ and $\Delta t = 0.025$.

① Solve the equation numerically using the explicit method (FTCS method) and plot the solution at $t = 1.0$ with 'surf' function.

② Solve the equation numerically using the ADI method.

A. Derive cx, dx, ex, and bx considering the *x*-directional boundary conditions.

B. For i = I, what are cy, dy, ey, and by? (Hint: you need to apply the *x*-directional boundary condition at this point too).

C. Plot the solution at $t = 1.0$ with 'surf' function.