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1.

① $A_i = (f_{i+1} + f_i) \cdot (x_{i+1} - x_i)/2 \quad (i = 1, 2, \cdots, N-1)$

+5

② $\int_a^b f(x)\,dx \approx \sum_{i=1}^{N-1} A_i$ +5

③ Programming +10

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2. $y' = F(x, y)$

① $y(x+h) = y(x) + C_0 F(x,y)h + C_1 F[x+ph, y+qhF(x,y)]h$

Taylor series of $F[x+ph, y+qhF(x,y)]$ is.

$$F[x+ph, y+qhF(x,y)] = F(x,y) + \frac{\partial F}{\partial x}ph + qh\sum_{i=1}^{n}\frac{\partial F}{\partial y_i}F_i(x,y) + O(h^2)$$

$$y(x+h) = y(x) + C_0F(x,y)h + C_1\left(F(x,y)h + \frac{\partial F}{\partial x}ph^2 + qh^2\sum_{i=1}^{n}\frac{\partial F}{\partial y_i}F_i(x,y)\right) + O(h^3) \cdots (*)$$

+5

② $y(x+h) = y(x) + y'(x)h + \frac{1}{2!}y''(x)h^2 + O(h^3)$

$= y(x) + F(x,y)h + \frac{1}{2}F'(x,y)h^2 + O(h^3)$

$$\left(F'(x,y) = \frac{\partial F}{\partial x} + \sum_{i=1}^{n}\frac{\partial F}{\partial y_i}y_i' = \frac{\partial F}{\partial x} + \sum_{i=1}^{n}\frac{\partial F}{\partial y_i}F_i(x,y)\right)$$

$$= y(x) + F(x,y)h + \frac{1}{2}\left(\frac{\partial F}{\partial x} + \sum_{i=1}^{n}\frac{\partial F}{\partial y_i}F_i(x,y)\right)h^2 + O(h^3) \quad \cdots (**)$$

+5

③ Comparing equation $*$ and $**$. then.

$C_0 + C_1 = 1 \quad C_1 p = \frac{1}{2}, \quad C_1 q = \frac{1}{2}$

+3

④ let $C_0 = 0 \rightarrow C_1 = 1 \rightarrow p = q = \frac{1}{2}$

+2

⑤

$$\frac{dy_1}{dt} = y_2 \quad \frac{dy_2}{dt} = -y_1$$

the modified Euler's method

$$\left( \begin{matrix} k_1 = hF(x,y) \\ k_2 = hF\left(x+\frac{h}{2}, y+\frac{k_1}{2}\right) \end{matrix} \Rightarrow y(x+h) = y(x) + k_2 \right)$$ +3

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad F(x,y) = \begin{bmatrix} y_2 \\ -y_1 \end{bmatrix}$$

1st

$$k_1 = 0.1\pi \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.314 \end{bmatrix}$$

$$k_2 = 0.1\pi\, F\left(x+\frac{h}{2},\ y+\frac{k_1}{2}\right) = 0.1\pi \begin{bmatrix} 0 + \frac{-0.314}{2} \\ -\left(1+\frac{0}{2}\right) \end{bmatrix} = \begin{bmatrix} -0.0493 \\ -0.314 \end{bmatrix}$$

$$y(0.1\pi) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0493 \\ -0.314 \end{bmatrix} = \begin{bmatrix} 0.9507 \\ -0.314 \end{bmatrix} \leftarrow \text{Ans}$$

+1

2nd

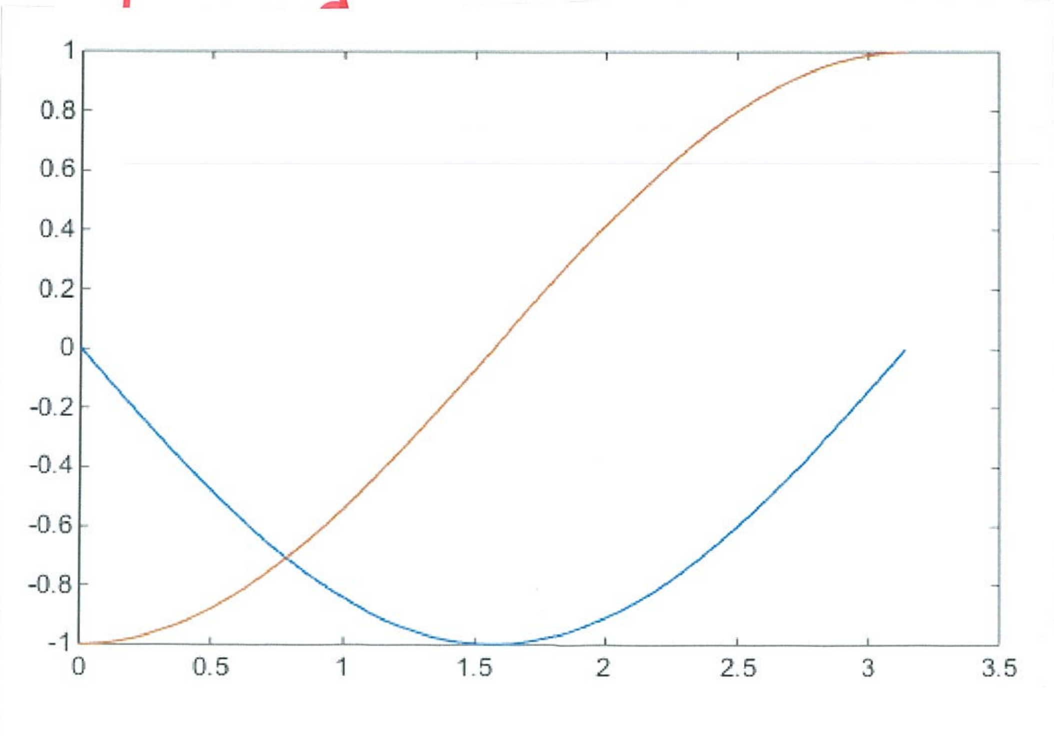
$$k_1 = 0.1\pi \begin{bmatrix} -0.314 \\ -0.9507 \end{bmatrix} = \begin{bmatrix} -0.0986 \\ -0.2987 \end{bmatrix}$$

$$k_2 = 0.1\pi\, F\left(x+\frac{h}{2},\ y+\frac{k_1}{2}\right) = 0.1\pi \begin{bmatrix} -0.314 + \frac{-0.2987}{2} \\ -\left(0.9507 + \frac{-0.0986}{2}\right) \end{bmatrix} = \begin{bmatrix} -0.1456 \\ -0.2832 \end{bmatrix}$$

$$y(0.2\pi) = \begin{bmatrix} 0.9507 \\ -0.314 \end{bmatrix} + \begin{bmatrix} -0.1456 \\ -0.2832 \end{bmatrix} = \begin{bmatrix} 0.8051 \\ -0.5972 \end{bmatrix} \leftarrow \text{Ans.}$$

+1

⑥ Programming +10



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3. a) $y = y_1$

$y' = y_1' = Da(1-y_1)\exp\left(\frac{\gamma y_2}{\gamma + y_2}\right)$ +2

$\theta = y_2$

$\theta' = y_2' = BDa(1-y_1)\exp\left(\frac{\gamma y_2}{\gamma + y_2}\right) - \beta(y_2 - \theta_c)$ +2

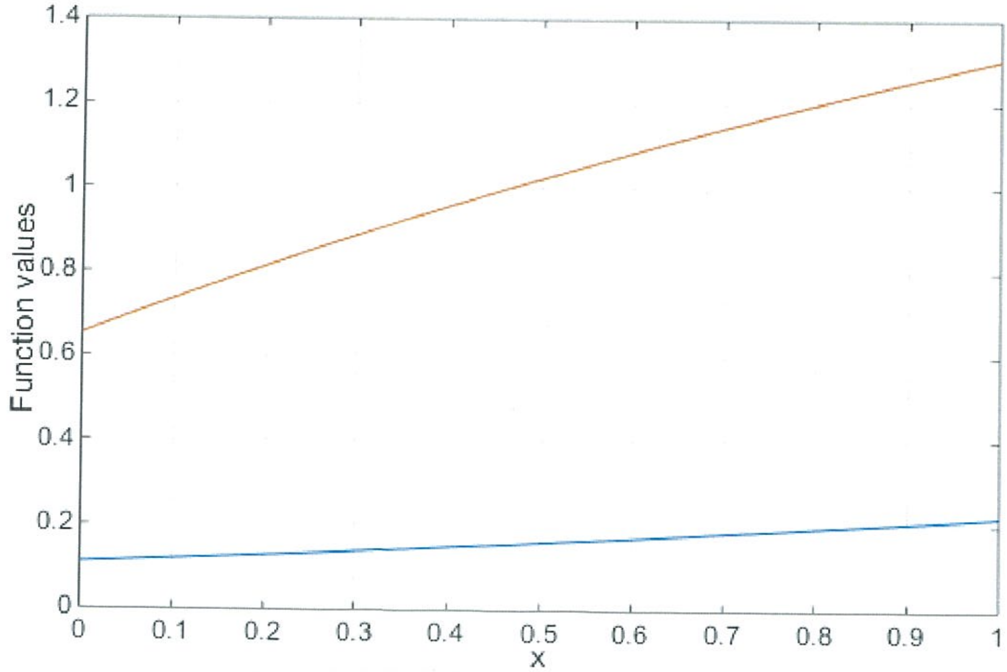
BCs.

$y_1(0) = (1-\lambda)y_1(1)$ , $y_2(0) = (1-\lambda)y_2(1)$ +2

b)

$r(1) = y_1(0) - (1-\lambda)y_1(1)$ +2 and $r(2) = y_2(0) - (1-\lambda)y_2(1)$. +2

② Programming +20



4. ①

a) $$\frac{y_{i+1}-2y_i+y_{i-1}}{\Delta r^2}+\frac{2}{r_i}\frac{y_{i+1}-y_{i-1}}{2\Delta r}+e^{y_i}=0 \quad \text{at } r=r_i$$

+5

b) $$\lim_{r\to 0}\frac{\frac{dy}{dr}}{r} \Rightarrow \frac{\frac{d^2y}{dr^2}}{1}$$

$$\Rightarrow 3\frac{d^2y}{dr^2}+e^y=0$$

+5

c) from boundary condition $y'(0)=0$.

$$\frac{y_2-y_0}{2\Delta r}=0 \qquad \therefore y_0=y_2$$

from b)

$$3\cdot\frac{y_2-2y_1+y_0}{\Delta r^2}+e^{y_1}=0$$

$$\therefore 6\cdot\frac{y_2-y_1}{\Delta r^2}+e^{y_1}=0 \quad \text{at } r=r_1$$

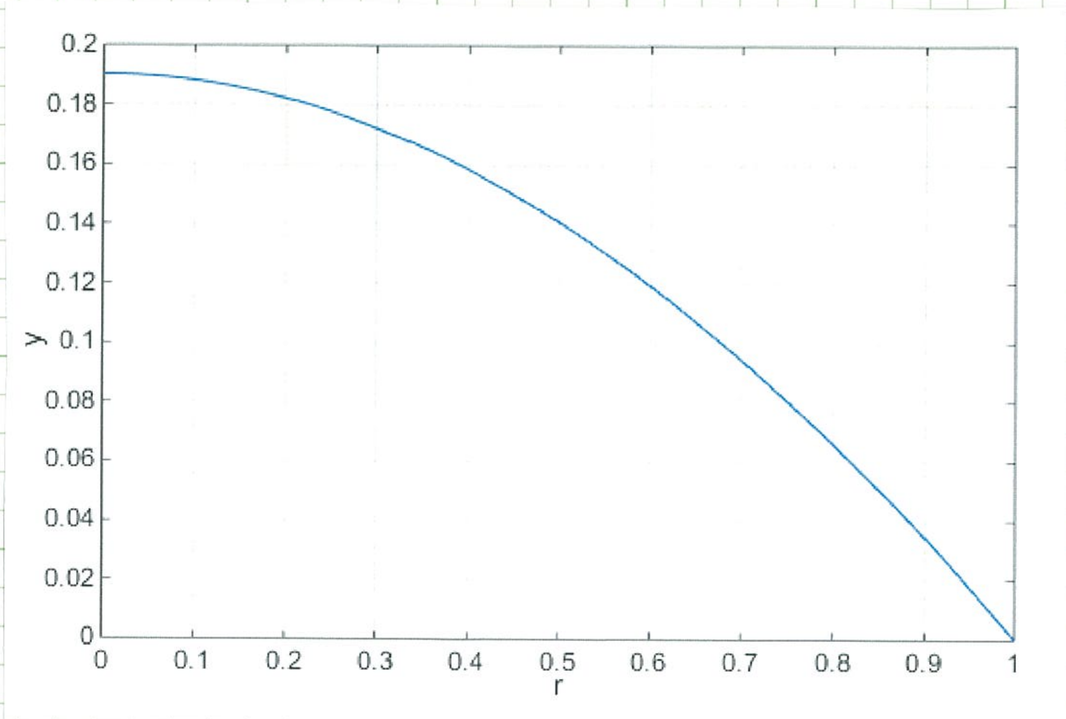
+3

d) from boundary condition $y(R=1)=0$

$$y_N=0 \quad \text{at } r=r_N=R$$

+2

② Programming +25



0.2
0.18
0.16
0.14
0.12
y
0.1
0.08
0.06
0.04
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r