

Chapter 4

- 4-1** For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_l = F/\delta$, $\delta = F/k_l$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta$
Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l}$$

Solving for k

$$k = \frac{1}{\frac{l^2}{k_T} + \frac{1}{k_l}} = \frac{k_l k_T}{k_l l^2 + k_T} \quad \text{Ans.}$$

- 4-2** For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For each cantilever, $k_l = F/\delta_l$, $\delta_l = F/k_l$, and, $\delta_L = F/k_L$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta_l + \delta_L$.
Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l} + \frac{F}{k_L}$$

Solving for k

$$k = \frac{1}{\frac{l^2}{k_T} + \frac{1}{k_l} + \frac{1}{k_L}} = \frac{k_L k_l k_T}{k_l k_L l^2 + k_T k_L + k_T k_l} \quad \text{Ans.}$$

- 4-3 (a)** For a torsion bar, $k = T/\theta = GJ/l$.

Two springs in parallel, with $J = \pi d_i^4/32$, and $d_1 = d_2 = d$,

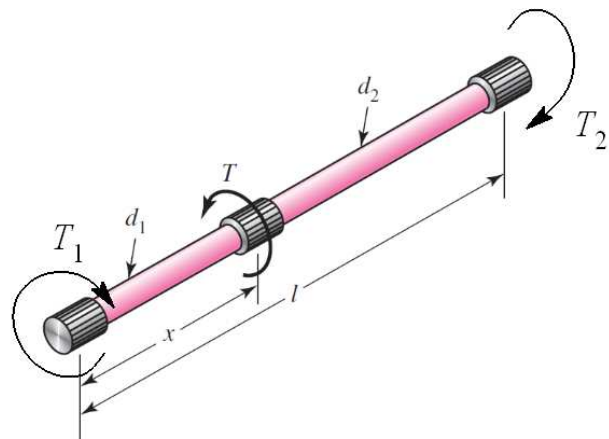
$$\begin{aligned} k &= \frac{J_1 G}{x} + \frac{J_2 G}{l-x} = \frac{\pi}{32} G \left(\frac{d_1^4}{x} + \frac{d_2^4}{l-x} \right) \\ &= \frac{\pi}{32} G d^4 \left(\frac{1}{x} + \frac{1}{l-x} \right) \quad \text{Ans.} \quad (1) \end{aligned}$$

Deflection equation,

$$\theta = \frac{T_1 x}{JG} = \frac{T_2 (l-x)}{JG}$$

results in $T_1 = \frac{T_2 (l-x)}{x} \quad (2)$

From statics, $T_1 + T_2 = T = 1500$. Substitute Eq. (2)



$$T_2 \left(\frac{l-x}{x} \right) + T_2 = 1500 \Rightarrow T_2 = 1500 \frac{x}{l} \quad \text{Ans.} \quad (3)$$

Substitute into Eq. (2) resulting in $T_1 = 1500 \frac{l-x}{l} \quad \text{Ans.} \quad (4)$

(b) From Eq. (1), $k = \frac{\pi}{32} (0.5^4) 11.5 (10^6) \left(\frac{1}{5} + \frac{1}{10-5} \right) = 28.2 (10^3) \text{ lbf} \cdot \text{in/rad} \quad \text{Ans.}$

From Eq. (4), $T_1 = 1500 \frac{10-5}{10} = 750 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$

From Eq. (3), $T_2 = 1500 \frac{5}{10} = 750 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$

From either section, $\tau = \frac{16T_i}{\pi d_i^3} = \frac{16(1500)}{\pi (0.5^3)} = 30.6 (10^3) \text{ psi} = 30.6 \text{ kpsi} \quad \text{Ans.}$

4-4 Deflection to be the same as Prob. 4-3 where $T_1 = 750 \text{ lbf} \cdot \text{in}$, $l_1 = l / 2 = 5 \text{ in}$, and $d_1 = 0.5 \text{ in}$

$$\theta_1 = \theta_2 = \theta$$

$$\frac{T_1(4)}{\frac{\pi}{32} d_1^4 G} = \frac{T_2(6)}{\frac{\pi}{32} d_2^4 G} = \frac{750(5)}{\frac{\pi}{32} (0.5^4) G} \Rightarrow \frac{4T_1}{d_1^4} = \frac{6T_2}{d_2^4} = 60 (10^3) \quad (1)$$

Or, $T_1 = 15 (10^3) d_1^4 \quad (2)$

$$T_2 = 10 (10^3) d_2^4 \quad (3)$$

Equal stress, $\tau_1 = \tau_2 \Rightarrow \frac{16T_1}{\pi d_1^3} = \frac{16T_2}{\pi d_2^3} \Rightarrow \frac{T_1}{d_1^3} = \frac{T_2}{d_2^3} \quad (4)$

Divide Eq. (4) by the first two equations of Eq.(1) results in

$$\frac{\frac{T_1}{d_1^3}}{\frac{4T_1}{d_1^4}} = \frac{\frac{T_2}{d_2^3}}{\frac{6T_2}{d_2^4}} \Rightarrow d_2 = 1.5d_1 \quad (5)$$

Statics, $T_1 + T_2 = 1500 \quad (6)$

Substitute in Eqs. (2) and (3), with Eq. (5) gives

$$15 (10^3) d_1^4 + 10 (10^3) (1.5d_1)^4 = 1500$$

Solving for d_1 and substituting it back into Eq. (5) gives
 $d_1 = 0.388 \text{ in}$, $d_2 = 0.583 \text{ in} \quad \text{Ans.}$

From Eqs. (2) and (3),

$$T_1 = 15(10^3)(0.388\ 8)^4 = 343\ \text{lbf}\cdot\text{in} \quad \text{Ans.}$$

$$T_2 = 10(10^3)(0.583\ 2)^4 = 1\ 157\ \text{lbf}\cdot\text{in} \quad \text{Ans.}$$

$$\text{Deflection of } T \text{ is} \quad \theta_1 = \frac{T_1 l_1}{J_1 G} = \frac{343(4)}{(\pi/32)(0.388\ 8^4)11.5(10^6)} = 0.053\ 18\ \text{rad}$$

$$\text{Spring constant is} \quad k = \frac{T}{\theta_1} = \frac{1500}{0.053\ 18} = 28.2(10^3)\ \text{lbf}\cdot\text{in} \quad \text{Ans.}$$

$$\text{The stress in } d_1 \text{ is} \quad \tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(343)}{\pi(0.388\ 8)^3} = 29.7(10^3)\ \text{psi} = 29.7\ \text{kpsi} \quad \text{Ans.}$$

$$\text{The stress in } d_2 \text{ is} \quad \tau_2 = \frac{16T_2}{\pi d_2^3} = \frac{16(1\ 157)}{\pi(0.583\ 2)^3} = 29.7(10^3)\ \text{psi} = 29.7\ \text{kpsi} \quad \text{Ans.}$$

- 4-5** (a) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan \alpha = (r_2 - r_1)/l$. Thus, the radius in the taper as a function of x is $r = r_1 + x \tan \alpha$, and the area is $A = \pi(r_1 + x \tan \alpha)^2$. The deflection of the tapered portion is

$$\begin{aligned} \delta &= \int_0^l \frac{F}{AE} dx = \frac{F}{\pi E} \int_0^l \frac{dx}{(r_1 + x \tan \alpha)^2} = -\frac{F}{\pi E} \frac{1}{(r_1 + x \tan \alpha) \tan \alpha} \bigg|_0^l \\ &= \frac{F}{\pi E} \left[\frac{1}{r_1 \tan \alpha} - \frac{1}{\tan \alpha (r_1 + l \tan \alpha)} \right] = \frac{F}{\pi E \tan \alpha} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \\ &= \frac{F}{\pi E \tan \alpha} \frac{r_2 - r_1}{r_1 r_2} = \frac{F}{\pi E \tan \alpha} \frac{l \tan \alpha}{r_1 r_2} = \frac{Fl}{\pi r_1 r_2 E} \\ &= \frac{4Fl}{\pi d_1 d_2 E} \quad \text{Ans.} \end{aligned}$$

(b) For section 1,

$$\delta_1 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.5^2)(30)(10^6)} = 3.40(10^{-4})\ \text{in} \quad \text{Ans.}$$

For the tapered section,

$$\delta = \frac{4}{\pi} \frac{Fl}{d_1 d_2 E} = \frac{4}{\pi} \frac{1000(2)}{(0.5)(0.75)(30)(10^6)} = 2.26(10^{-4})\ \text{in} \quad \text{Ans.}$$

For section 2,

$$\delta_2 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.75^2)(30)(10^6)} = 1.51(10^{-4}) \text{ in} \quad \text{Ans.}$$

- 4-6 (a)** Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan \alpha = (r_2 - r_1)/l$. Thus, the radius in the taper as a function of x is $r = r_1 + x \tan \alpha$, and the polar second area moment is $J = (\pi/2) (r_1 + x \tan \alpha)^4$. The angular deflection of the tapered portion is

$$\begin{aligned} \theta &= \int_0^l \frac{T}{GJ} dx = \frac{2T}{\pi G} \int_0^l \frac{dx}{(r_1 + x \tan \alpha)^4} = -\frac{1}{3} \frac{2T}{\pi G} \frac{1}{(r_1 + x \tan \alpha)^3 \tan \alpha} \bigg|_0^l \\ &= \frac{2}{3\pi} \frac{T}{G} \left[\frac{1}{r_1^3 \tan \alpha} - \frac{1}{\tan \alpha (r_1 + l \tan \alpha)^3} \right] = \frac{2}{3\pi} \frac{T}{G \tan \alpha} \left(\frac{1}{r_1^3} - \frac{1}{r_2^3} \right) \\ &= \frac{2}{3\pi} \frac{T}{G \tan \alpha} \frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi} \frac{T}{G} \left(\frac{l}{r_2 - r_1} \right) \frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi} \frac{Tl}{G} \frac{(r_1^2 + r_1 r_2 + r_2^2)}{r_1^3 r_2^3} \\ &= \frac{32}{3\pi} \frac{Tl}{G} \frac{(d_1^2 + d_1 d_2 + d_2^2)}{d_1^3 d_2^3} \quad \text{Ans.} \end{aligned}$$

(b) The deflections, in degrees, are:

Section 1,

$$\theta_1 = \frac{Tl}{GJ} \left(\frac{180}{\pi} \right) = \frac{32Tl}{\pi d_1^4 G} \left(\frac{180}{\pi} \right) = \frac{32(1500)(2)}{\pi(0.5^4)11.5(10^6)} \left(\frac{180}{\pi} \right) = 2.44 \text{ deg} \quad \text{Ans.}$$

Tapered section,

$$\begin{aligned} \theta &= \frac{32}{3\pi} \frac{Tl(d_1^2 + d_1 d_2 + d_2^2)}{G d_1^3 d_2^3} \left(\frac{180}{\pi} \right) \\ &= \frac{32}{3\pi} \frac{(1500)(2)[0.5^2 + (0.5)(0.75) + 0.75^2]}{11.5(10^6)(0.5^3)(.75^3)} \left(\frac{180}{\pi} \right) = 1.14 \text{ deg} \quad \text{Ans.} \end{aligned}$$

Section 2,

$$\theta_2 = \frac{Tl}{GJ} \left(\frac{180}{\pi} \right) = \frac{32Tl}{\pi d_2^4 G} \left(\frac{180}{\pi} \right) = \frac{32(1500)(2)}{\pi(0.75^4)11.5(10^6)} \left(\frac{180}{\pi} \right) = 0.481 \text{ deg} \quad \text{Ans.}$$

- 4-7** The area and the elastic modulus remain constant. However, the force changes with respect to x . From Table A-5, the unit weight of steel is $\gamma = 0.282 \text{ lbf/in}^3$ and the elastic modulus is $E = 30 \text{ Mpsi}$. Starting from the top of the cable (i.e. $x = 0$, at the top).

$$F = \gamma(A)(l-x)$$

$$\delta_c = \int_0^l \frac{F dx}{AE} = \frac{w}{E} \int_0^l (l-x) dx = \frac{\gamma}{E} \left(lx - \frac{1}{2} x^2 \right) \bigg|_0^l = \frac{\gamma l^2}{2E} = \frac{0.282 [500(12)]^2}{2(30)10^6} = 0.169 \text{ in}$$

From the weight at the bottom of the cable,

$$\delta_w = \frac{Wl}{AE} = \frac{4Wl}{\pi d^2 E} = \frac{4(5000)[500(12)]}{\pi(0.5^2)30(10^6)} = 5.093 \text{ in}$$

$$\delta = \delta_c + \delta_w = 0.169 + 5.093 = 5.262 \text{ in} \quad \text{Ans.}$$

The percentage of total elongation due to the cable's own weight

$$\frac{0.169}{5.262}(100) = 3.21\% \quad \text{Ans.}$$

4-8 $\Sigma F_y = 0 = R_1 - F \Rightarrow R_1 = F$
 $\Sigma M_A = 0 = M_1 - Fa \Rightarrow M_1 = Fa$
 $V_{AB} = F, M_{AB} = F(x-a), V_{BC} = M_{BC} = 0$

Section AB:

$$\theta_{AB} = \frac{1}{EI} \int F(x-a) dx = \frac{F}{EI} \left(\frac{x^2}{2} - ax \right) + C_1 \quad (1)$$

$$\theta_{AB} = 0 \text{ at } x = 0 \Rightarrow C_1 = 0$$

$$y_{AB} = \frac{F}{EI} \int \left(\frac{x^2}{2} - ax \right) dx = \frac{F}{EI} \left(\frac{x^3}{6} - a \frac{x^2}{2} \right) + C_2 \quad (2)$$

$$y_{AB} = 0 \text{ at } x = 0 \Rightarrow C_2 = 0, \text{ and}$$

$$y_{AB} = \frac{Fx^2}{6EI} (x-3a) \quad \text{Ans.}$$

Section BC:

$$\theta_{BC} = \frac{1}{EI} \int (0) dx = 0 + C_3$$

From Eq. (1), at $x = a$ (with $C_1 = 0$), $\theta = \frac{F}{EI} \left(\frac{a^2}{2} - a(a) \right) = -\frac{Fa^2}{2EI} = C_3$. Thus,

$$\theta_{BC} = -\frac{Fa^2}{2EI}$$

$$y_{BC} = -\frac{Fa^2}{2EI} \int dx = -\frac{Fa^2}{2EI} x + C_4 \quad (3)$$

From Eq. (2), at $x = a$ (with $C_2 = 0$), $y = \frac{F}{EI} \left(\frac{a^3}{6} - a \frac{a^2}{2} \right) = -\frac{Fa^3}{3EI}$. Thus, from Eq. (3)

$$-\frac{Fa^2}{2EI} a + C_4 = -\frac{Fa^3}{3EI} \Rightarrow C_4 = \frac{Fa^3}{6EI} \quad \text{Substitute into Eq. (3), obtaining}$$

$$y_{BC} = -\frac{Fa^2}{2EI} x + \frac{Fa^3}{6EI} = \frac{Fa^2}{6EI} (a - 3x) \quad \text{Ans.}$$

The maximum deflection occurs at $x = l$,

$$y_{\max} = \frac{Fa^2}{6EI} (a - 3l) \quad \text{Ans.}$$

4-9 $\Sigma M_C = 0 = F(l/2) - R_1 l \Rightarrow R_1 = F/2$

$$\Sigma F_y = 0 = F/2 + R_2 - F \Rightarrow R_2 = F/2$$

Break at $0 \leq x \leq l/2$:

$$V_{AB} = R_1 = F/2, \quad M_{AB} = R_1 x = Fx/2$$

Break at $l/2 \leq x \leq l$:

$$V_{BC} = R_1 - F = -R_2 = -F/2, \quad M_{BC} = R_1 x - F(x - l/2) = F(l - x)/2$$

Section AB:

$$\theta_{AB} = \frac{1}{EI} \int \frac{Fx}{2} dx = \frac{F}{EI} \frac{x^2}{4} + C_1$$

$$\text{From symmetry, } \theta_{AB} = 0 \text{ at } x = l/2 \Rightarrow \frac{F \left(\frac{l}{2} \right)^2}{4EI} + C_1 = 0 \Rightarrow C_1 = -\frac{Fl^2}{16EI}. \text{ Thus,}$$

$$\theta_{AB} = \frac{F}{EI} \frac{x^2}{4} - \frac{Fl^2}{16EI} = \frac{F}{16EI} (4x^2 - l^2) \quad (1)$$

$$y_{AB} = \frac{F}{16EI} \int (4x^2 - l^2) dx = \frac{F}{16EI} \left(\frac{4x^3}{3} - l^2 x \right) + C_2$$

$$y_{AB} = 0 \text{ at } x = 0 \Rightarrow C_2 = 0, \text{ and,}$$

$$y_{AB} = \frac{Fx}{48EI} (4x^2 - 3l^2) \quad (2)$$

y_{BC} is not given, because with symmetry, Eq. (2) can be used in this region. The maximum deflection occurs at $x = l/2$,

$$y_{\max} = \frac{F \left(\frac{l}{2} \right)}{48EI} \left[4 \left(\frac{l}{2} \right)^2 - 3l^2 \right] = -\frac{Fl^3}{48EI} \quad \text{Ans.}$$

4-10 From Table A-6, for each angle, $I_{1-1} = 207 \text{ cm}^4$. Thus, $I = 2(207) (10^4) = 4.14(10^6) \text{ mm}^4$

From Table A-9, use beam 2 with $F = 2500 \text{ N}$, $a = 2000 \text{ mm}$, and $l = 3000 \text{ mm}$; and beam 3 with $w = 1 \text{ N/mm}$ and $l = 3000 \text{ mm}$.

$$\begin{aligned} y_{\max} &= \frac{Fa^2}{6EI} (a - 3l) - \frac{wl^4}{8EI} \\ &= \frac{2500(2000)^2}{6(207)10^3(4.14)10^6} [2000 - 3(3000)] - \frac{(1)(3000)^4}{8(207)(10^3)(4.14)(10^6)} \\ &= -25.4 \text{ mm} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} M_o &= -Fa - (wl^2 / 2) \\ &= -2500(2000) - [1(3000^2)/2] = -9.5(10^6) \text{ N}\cdot\text{mm} \end{aligned}$$

From Table A-6, from centroid to upper surface is $y = 29 \text{ mm}$. From centroid to bottom surface is $y = 29.0 - 100 = -71 \text{ mm}$. The maximum stress is compressive at the bottom of the beam at the wall. This stress is

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5(10^6)(-71)}{4.14(10^6)} = -163 \text{ MPa} \quad \text{Ans.}$$

4-11

$$R_o = \frac{14}{20}(450) + \frac{10}{20}(300) = 465 \text{ lbf}$$

$$R_c = \frac{6}{20}(450) + \frac{10}{20}(300) = 285 \text{ lbf}$$

$$M_1 = 465(6) = 2790 \text{ lbf}\cdot\text{ft}$$

$$M_2 = 2790 + 15(4) = 2850 \text{ lbf}\cdot\text{ft}$$

$$\sigma_{\max} = \frac{M_{\max}}{Z} \Rightarrow 15 = \frac{34.2}{Z} \quad Z = 2.28 \text{ in}^3$$

For deflections, use beams 5 and 6 of Table A-9

$$y|_{x=10\text{ft}} = \frac{F_1 a [l - (l/2)]}{6EI} \left[\left(\frac{l}{2} \right)^2 + a^2 - 2l \frac{l}{2} \right] - \frac{F_2 l^3}{48EI}$$

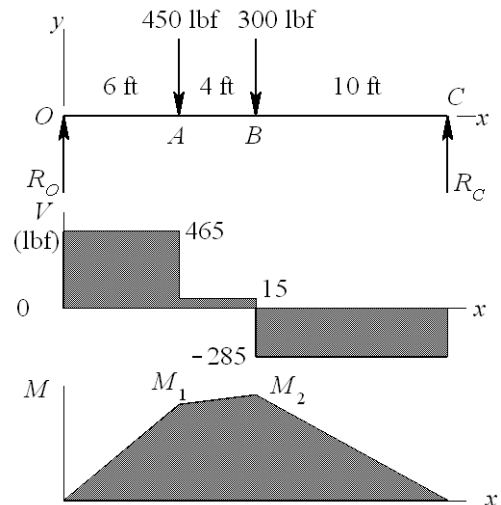
$$-0.5 = \frac{450(72)(120)}{6(30)(10^6)I(240)} (120^2 + 72^2 - 240^2) - \frac{300(240^3)}{48(30)(10^6)I}$$

$$I = 12.60 \text{ in}^4 \Rightarrow I/2 = 6.30 \text{ in}^4$$

Select two 5 in-6.7 lbf/ft channels from Table A-7, $I = 2(7.49) = 14.98 \text{ in}^4$, $Z = 2(3.00) = 6.00 \text{ in}^3$

$$y_{\text{midspan}} = \frac{12.60}{14.98} \left(-\frac{1}{2} \right) = -0.421 \text{ in}$$

$$\sigma_{\max} = \frac{34.2}{6.00} = 5.70 \text{ kpsi}$$



4-12

$$I = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4$$

From Table A-9 by superposition of beams 6 and 7, at $x = a = 15 \text{ in}$, with $b = 24 \text{ in}$ and $l = 39 \text{ in}$

$$y = \frac{Fba}{6EI} [a^2 + b^2 - l^2] + \frac{wa}{24EI} (2la^2 - a^3 - l^3)$$

$$y_A = \frac{340(24)15}{6(30)10^6(0.2485)39} [15^2 + 24^2 - 39^2] + \frac{(150/12)(15)}{24(30)10^6(0.2485)} [2(39)(15^2) - 15^3 - 39^3] = -0.0978 \text{ in} \quad \text{Ans.}$$

At $x = l/2 = 19.5 \text{ in}$

$$y = \frac{Fa[l - (l/2)]}{6EI} \left[\left(\frac{l}{2} \right)^2 + a^2 - 2l \frac{l}{2} \right] + \frac{w(l/2)}{24EI} \left[2l \left(\frac{l}{2} \right)^2 - \left(\frac{l}{2} \right)^3 - l^3 \right]$$

$$y = \frac{340(15)(19.5)}{6(30)(10^6)(0.2485)(39)} [19.5^2 + 15^2 - 39^2] \\ + \frac{(150/12)(19.5)}{24(30)(10^6)(0.2485)} [2(39)(19.5^2) - 19.5^3 - 39^3] = -0.1027 \text{ in} \quad \text{Ans.}$$

$$\% \text{ difference} = \frac{-0.1027 + 0.0978}{-0.0978} (100) = 5.01\% \quad \text{Ans.}$$

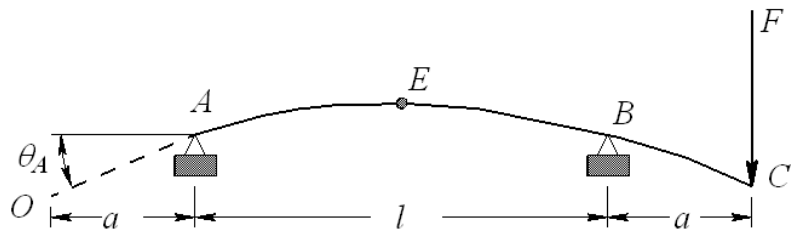
4-13 $I = \frac{1}{12} (6)(32^3) = 16.384 (10^3) \text{ mm}^4$

From Table A-9-10, beam 10

$$y_C = -\frac{Fa^2}{3EI} (l+a)$$

$$y_{AB} = \frac{Fax}{6EI} (l^2 - x^2)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EI} (l^2 - 3x^2)$$



At $x = 0$, $\frac{dy_{AB}}{dx} = \theta_A$

$$\theta_A = \frac{Fal^2}{6EI} = \frac{Fal}{6EI}$$

$$y_O = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_O = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI} (l+a) \\ = -\frac{Fa^2}{6EI} (3l+2a) = -\frac{400(300^2)}{6(207)10^3(16.384)10^3} [3(500) + 2(300)] = -3.72 \text{ mm} \quad \text{Ans.}$$

At midspan,

$$y_E = \frac{2Fa(l/2)}{6EI} \left[l^2 - \left(\frac{l}{2} \right)^2 \right] = \frac{3}{24} \frac{Fal^2}{EI} = \frac{3}{24} \frac{400(300)(500^2)}{207(10^3)16.384(10^3)} = 1.11 \text{ mm} \quad \text{Ans.}$$

4-14 $I = \frac{\pi}{64} (2^4 - 1.5^4) = 0.5369 \text{ in}^4$

From Table A-5, $E = 10.4 \text{ Mpsi}$

From Table A-9, beams 1 and 2, by superposition

$$y_B = -\frac{F_B l^3}{3EI} + \frac{F_A a^2}{6EI} (a - 3l) = \frac{-200[4(12)]^3}{3(10.4)10^6(0.5369)} + \frac{300[2(12)]^2}{6(10.4)10^6(0.5369)} [2(12) - 3(4)(12)]$$

$$y_B = -1.94 \text{ in} \quad \text{Ans.}$$

- 4-15** From Table A-7, $I = 2(1.85) = 3.70 \text{ in}^4$
 From Table A-5, $E = 30.0 \text{ Mpsi}$
 From Table A-9, beams 1 and 3, by superposition

$$y_A = -\frac{Fl^3}{3EI} - \frac{(w + w_c)l^4}{8EI} = -\frac{150(60^3)}{3(30)10^6(3.70)} - \frac{[5 + 2(5/12)](60^4)}{8(30)10^6(3.70)} = -0.182 \text{ in} \quad \text{Ans.}$$

4-16 $I = \frac{\pi}{64} d^4$

From Table A-5, $E = 207(10^3) \text{ MPa}$

From Table A-9, beams 5 and 9, with $F_C = F_A = F$, by superposition

$$y_B = -\frac{F_B l^3}{48EI} + \frac{Fa}{24EI} (4a^2 - 3l^2) \Rightarrow I = \frac{1}{48Ey_B} [-F_B l^3 + 2Fa(4a^2 - 3l^2)]$$

$$I = \frac{1}{48(207)10^3(-2)} \{-550(1000^3) + 2(375)(250)[4(250^2) - 3(1000^2)]\}$$

$$= 53.624(10^3) \text{ mm}^4$$

$$d = \sqrt[4]{\frac{64}{\pi} I} = \sqrt[4]{\frac{64}{\pi} (53.624)10^3} = 32.3 \text{ mm} \quad \text{Ans.}$$

- 4-17** From Table A-9, beams 8 (region BC for this beam with $a = 0$) and 10 (with $a = a$), by superposition

$$y_{AB} = \frac{M_A}{6EI} (x^3 - 3lx^2 + 2l^2x) + \frac{Fax}{6EI} (l^2 - x^2)$$

$$= \frac{1}{6EI} [M_A (x^3 - 3lx^2 + 2l^2x) + Fax(l^2 - x^2)] \quad \text{Ans.}$$

$$y_{BC} = \left\{ \frac{d}{dx} \left[\frac{M_A}{6EI} (x^3 - 3lx^2 + 2l^2x) \right] \right\}_{x=l} (x-l) + \frac{F(x-l)}{6EI} [(x-l)^2 - a(3x-l)]$$

$$= -\frac{M_A l}{6EI} (x-l) + \frac{F(x-l)}{6EI} [(x-l)^2 - a(3x-l)]$$

$$= \frac{(x-l)}{6EI} \{-M_A l + F[(x-l)^2 - a(3x-l)]\} \quad \text{Ans.}$$

4-18 Note to the instructor: Beams with discontinuous loading are better solved using singularity functions. This eliminates matching the slopes and displacements at the discontinuity as is done in this solution.

$$\sum M_c = 0 = R_1 l - wa \left(l - a + \frac{a}{2} \right) \Rightarrow R_1 = \frac{wa}{2l} (2l - a) \quad \text{Ans.}$$

$$\sum F_y = 0 = \frac{wa}{2l} (2l - a) + R_2 - wa \Rightarrow R_2 = \frac{wa^2}{2l} \quad \text{Ans.}$$

$$V_{AB} = R_1 - wx = \frac{wa}{2l} (2l - a) - wx = \frac{w}{2l} [2l(a - x) - a^2] \quad \text{Ans.}$$

$$V_{BC} = -R_2 = -\frac{wa^2}{2l} \quad \text{Ans.}$$

$$M_{AB} = \int V_{AB} dx = \frac{w}{2l} \left[2l \left(ax - \frac{x^2}{2} \right) - a^2 x \right] + C_1$$

$$M_{AB} = 0 \text{ at } x = 0 \therefore C_1 = 0 \Rightarrow M_{AB} = \frac{wx}{2l} [2al - a^2 - lx] \quad \text{Ans.}$$

$$M_{BC} = \int V_{BC} dx = \int -\frac{wa^2}{2l} dx = -\frac{wa^2}{2l} x + C_2$$

$$M_{BC} = 0 \text{ at } x = l \therefore C_2 = \frac{wa^2}{2} \Rightarrow M_{BC} = \frac{wa^2}{2l} (l - x) \quad \text{Ans.}$$

$$\theta_{AB} = \int \frac{M_{AB}}{EI} dx = \frac{1}{EI} \int \frac{wx}{2l} (2al - a^2 - lx) dx = \frac{1}{EI} \left[\frac{w}{2l} \left(alx^2 - \frac{1}{2} a^2 x^2 - \frac{1}{3} lx^3 \right) + C_3 \right]$$

$$y_{AB} = \int \theta_{AB} dx = \frac{1}{EI} \int \left[\frac{w}{2l} \left(alx^2 - \frac{1}{2} a^2 x^2 - \frac{1}{3} lx^3 \right) + C_3 \right] dx$$

$$= \frac{1}{EI} \left[\frac{w}{2l} \left(\frac{1}{3} alx^3 - \frac{1}{6} a^2 x^3 - \frac{1}{12} lx^4 \right) + C_3 x + C_4 \right]$$

$$y_{AB} = 0 \text{ at } x = 0 \therefore C_4 = 0$$

$$\theta_{BC} = \int \frac{M_{BC}}{EI} dx = \frac{1}{EI} \int \frac{wa^2}{2l} (l - x) dx = \frac{1}{EI} \left[\frac{wa^2}{2l} \left(lx - \frac{1}{2} x^2 \right) + C_5 \right]$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = a \therefore$$

$$\frac{1}{EI} \left[\frac{w}{2l} \left(ala^2 - \frac{1}{2} a^4 - \frac{1}{3} la^3 \right) + C_3 \right] = \frac{1}{EI} \left[\frac{wa^2}{2l} \left(la - \frac{1}{2} a^2 \right) + C_5 \right] \Rightarrow C_3 = \frac{wa^3}{6} + C_5 \quad (1)$$

$$y_{BC} = \int \theta_{BC} dx = \frac{1}{EI} \int \left[\frac{wa^2}{2l} \left(lx - \frac{1}{2} x^2 \right) + C_5 \right] dx = \frac{1}{EI} \left[\frac{wa^2}{2l} \left(\frac{1}{2} lx^2 - \frac{1}{6} x^3 \right) + C_5 x + C_6 \right]$$

$$y_{BC} = 0 \text{ at } x = l \therefore C_6 = -\frac{wa^2 l^2}{6} - C_5 l$$

$$y_{BC} = \frac{1}{EI} \left[\frac{wa^2}{2l} \left(\frac{1}{2} lx^2 - \frac{1}{6} x^3 - \frac{1}{3} l^3 \right) + C_5 (x - l) \right]$$

$$y_{AB} = y_{BC} \text{ at } x = a \therefore$$

$$\frac{w}{2l} \left(\frac{1}{3} ala^3 - \frac{1}{6} a^5 - \frac{1}{12} la^4 \right) + C_3 a = \frac{wa^2}{2l} \left(\frac{1}{2} la^2 - \frac{1}{6} a^3 - \frac{1}{3} l^3 \right) + C_5 (a - l)$$

$$C_3 a = \frac{wa^2}{24l} (3la^2 - 4l^3) + C_5 (a - l) \quad (2)$$

Substituting (1) into (2) yields $C_5 = \frac{wa^2}{24l} (-a^2 - 4l^2)$. Substituting this back into (2) gives

$$C_3 = \frac{wa^2}{24l} (4al - a^2 - 4l^2). \text{ Thus,}$$

$$y_{AB} = \frac{w}{24EI} (4alx^3 - 2a^2x^3 - lx^4 + 4a^3lx - a^4x - 4a^2l^2x)$$

$$\Rightarrow y_{AB} = \frac{wx}{24EI} [2ax^2(2l - a) - lx^3 - a^2(2l - a)^2] \quad \text{Ans.}$$

$$y_{BC} = \frac{w}{24EI} (6a^2lx^2 - 2a^2x^3 - a^4x - 4a^2l^2x + a^4l) \quad \text{Ans.}$$

This result is sufficient for y_{BC} . However, this can be shown to be equivalent to

$$y_{BC} = \frac{w}{24EI} (4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x) + \frac{w}{24EI} (x - a)^4$$

$$y_{BC} = y_{AB} + \frac{w}{24EI} (x - a)^4 \quad \text{Ans.}$$

by expanding this or by solving the problem using singularity functions.

- 4-19** The beam can be broken up into a uniform load w downward from points A to C and a uniform load w upward from points A to B . Using the results of Prob. 4-18, with $b = a$ for A to C and $a = a$ for A to B , results in

$$y_{AB} = \frac{wx}{24EI} [2bx^2(2l - b) - lx^3 - b^2(2l - b)^2] - \frac{wx}{24EI} [2ax^2(2l - a) - lx^3 - a^2(2l - a)^2]$$

$$= \frac{wx}{24EI} [2bx^2(2l - b) - b^2(2l - b)^2 - 2ax^2(2l - a) + a^2(2l - a)^2] \quad \text{Ans.}$$

$$y_{BC} = \frac{w}{24EI} [2bx^3(2l - b) - lx^4 - b^2x(2l - b)^2 - (4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x) - l(x - a)^4] \quad \text{Ans.}$$

$$\begin{aligned}
y_{CD} &= \frac{w}{24EI} \left[4blx^3 - 2b^2x^3 - lx^4 - 4b^2l^2x + 4b^3lx - b^4x + l(x-b)^4 \right] \\
&\quad - \frac{w}{24EI} \left[4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x + l(x-a)^4 \right] \\
&= \frac{w}{24EI} \left[(x-b)^4 - (x-a)^4 \right] + y_{AB} \quad \text{Ans.}
\end{aligned}$$

4-20 Note to the instructor: See the note in the solution for Problem 4-18.

$$\sum F_y = 0 = R_B - \frac{wa^2}{2l} - wa \Rightarrow R_B = \frac{wa}{2l}(2l+a) \quad \text{Ans.}$$

For region BC , isolate right-hand element of length $(l+a-x)$

$$V_{AB} = -R_A = -\frac{wa^2}{2l}, \quad V_{BC} = w(l+a-x) \quad \text{Ans.}$$

$$M_{AB} = -R_A x = -\frac{wa^2}{2l}x, \quad M_{BC} = -\frac{w}{2}(l+a-x)^2 \quad \text{Ans.}$$

$$EI\theta_{AB} = \int M_{AB} dx = -\frac{wa^2}{4l}x^2 + C_1$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x + C_2$$

$$y_{AB} = 0 \text{ at } x = 0 \Rightarrow C_2 = 0 \quad \therefore EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x$$

$$y_{AB} = 0 \text{ at } x = l \Rightarrow C_1 = \frac{wa^2l}{12} \quad \therefore$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + \frac{wa^2l}{12}x = \frac{wa^2x}{12l}(l^2 - x^2) \Rightarrow y_{AB} = \frac{wa^2x}{12EI}(l^2 - x^2) \quad \text{Ans.}$$

$$EI\theta_{BC} = \int M_{BC} dx = -\frac{w}{6}(l+a-x)^3 + C_3$$

$$EIy_{BC} = -\frac{w}{24}(l+a-x)^4 + C_3x + C_4$$

$$y_{BC} = 0 \text{ at } x = l \Rightarrow -\frac{wa^4}{24} + C_3l + C_4 = 0 \Rightarrow C_4 = \frac{wa^4}{24} - C_3l \quad (1)$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = l \Rightarrow -\frac{wa^2l}{4} + \frac{wa^2l}{12} = \frac{wa^3}{6} + C_3 \Rightarrow C_3 = -\frac{wa^2}{6}(l+a)$$

Substitute C_3 into Eq. (1) gives $C_4 = \frac{wa^2}{24}[a^2 + 4l(l+a)]$. Substitute back into y_{BC}

$$\begin{aligned}
y_{BC} &= \frac{1}{EI} \left[-\frac{w}{24}(l+a-x)^4 - \frac{wa^2}{6}x(l+a) + \frac{wa^4}{24} + \frac{wa^2l}{6}(l+a) \right] \\
&= -\frac{w}{24EI} \left[(l+a-x)^4 - 4a^2(l-x)(l+a) - a^4 \right] \quad \text{Ans.}
\end{aligned}$$

4-21 Table A-9, beam 7,

$$R_1 = R_2 = \frac{wl}{2} = \frac{100(10)}{2} = 500 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wx}{24EI} (2lx^2 - x^3 - l^3) = \frac{100x}{24(30)10^6(0.05)} [2(10)x^2 - x^3 - 10^3]$$

$$= 2.7778(10^{-6})x(20x^2 - x^3 - 1000)$$

$$\text{Slope: } \theta_{AB} = \frac{d y_{AB}}{d x} = \frac{w}{24EI} (6lx^2 - 4x^3 - l^3)$$

$$\text{At } x = l, \theta_{AB}|_{x=l} = \frac{w}{24EI} (6l^2 - 4l^3 - l^3) = \frac{wl^3}{24EI}$$

$$y_{BC} = \theta_{AB}|_{x=l} (x-l) = \frac{wl^3}{24EI} (x-l) = \frac{100(10^3)}{24(30)10^6(0.05)} (x-10) = 2.7778(10^{-3})(x-10)$$

From Prob. 4-20,

$$R_A = \frac{wa^2}{2l} = \frac{100(4^2)}{2(10)} = 80 \text{ lbf } \downarrow \quad R_B = \frac{wa}{2l} (2l+a) = \frac{100(4)}{2(10)} [2(10)+4] = 480 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wa^2 x}{12EI} (l^2 - x^2) = \frac{100(4^2)x}{12(30)10^6(0.05)} (10^2 - x^2) = 8.8889(10^{-6})x(100 - x^2)$$

$$y_{BC} = -\frac{w}{24EI} [(l+a-x)^4 - 4a^2(l-x)(l+a) - a^4]$$

$$= -\frac{100}{24(30)10^6(0.05)} [(10+4-x)^4 - 4(4^2)(10-x)(10+4) - 4^4]$$

$$= -2.7778(10^{-6}) [(14-x)^4 + 896x - 9216]$$

Superposition,

$$R_A = 500 - 80 = 420 \text{ lbf } \uparrow \quad R_B = 500 + 480 = 980 \text{ lbf } \uparrow \quad \text{Ans.}$$

$$y_{AB} = 2.7778(10^{-6})x(20x^2 - x^3 - 1000) + 8.8889(10^{-6})x(100 - x^2) \quad \text{Ans.}$$

$$y_{BC} = 2.7778(10^{-3})(x-10) - 2.7778(10^{-6})[(14-x)^4 + 896x - 9216] \quad \text{Ans.}$$

The deflection equations can be simplified further. However, they are sufficient for plotting.

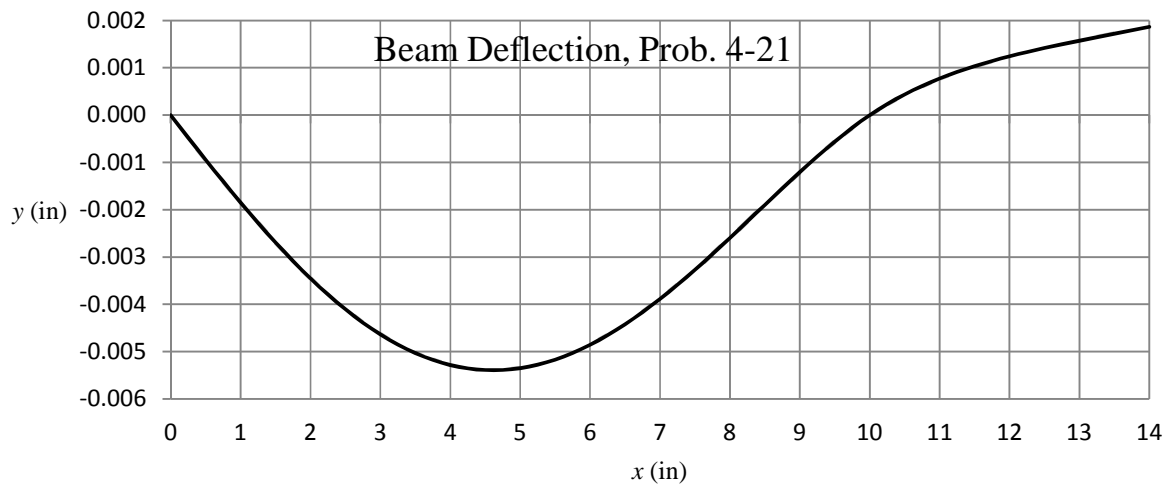
Using a spreadsheet,

| | | | | | | | | |
|-----|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| x | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
| y | 0.000000 | -0.000939 | -0.001845 | -0.002690 | -0.003449 | -0.004102 | -0.004632 | -0.005027 |

| | | | | | | | | |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| x | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| y | -0.005280 | -0.005387 | -0.005347 | -0.005167 | -0.004853 | -0.004421 | -0.003885 | -0.003268 |

| | | | | | | | | |
|-----|-----------|-----------|-----------|-----------|----------|----------|----------|----------|
| x | 8 | 8.5 | 9 | 9.5 | 10 | 10.5 | 11 | 11.5 |
| y | -0.002596 | -0.001897 | -0.001205 | -0.000559 | 0.000000 | 0.000439 | 0.000775 | 0.001036 |

| | | | | | |
|-----|----------|----------|----------|----------|----------|
| x | 12 | 12.5 | 13 | 13.5 | 14 |
| y | 0.001244 | 0.001419 | 0.001575 | 0.001722 | 0.001867 |



4-22 (a) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{1800(36^3)}{48(30)10^6} = 0.05832 \text{ in}^4$$

From $I = bh^3/12$, and $b = 10h$, then $I = 5h^4/6$, or,

$$h = \sqrt[4]{\frac{6I}{5}} = \sqrt[4]{\frac{6(0.05832)}{5}} = 0.514 \text{ in}$$

h is close to $1/2$ in and $9/16$ in, while b is close to 5.14 in. Changing the height drastically changes the spring rate, so changing the base will make finding a close solution easier. Trial and error was applied to find the combination of values from Table A-17 that yielded the closest desired spring rate.

| h (in) | b (in) | b/h | k (lbf/in) |
|----------|----------|-------|--------------|
| 1/2 | 5 | 10 | 1608 |
| 1/2 | 5½ | 11 | 1768 |
| 1/2 | 5¾ | 11.5 | 1849 |
| 9/16 | 5 | 8.89 | 2289 |
| 9/16 | 4 | 7.11 | 1831 |

$h = 1/2$ in, $b = 5 1/2$ in should be selected because it results in a close spring rate and b/h is still reasonably close to 10.

(b) $I = 5.5(0.5)^3 / 12 = 0.05729 \text{ in}^4$

$$\sigma = \frac{Mc}{I} = \frac{(Fl/4)c}{I} \Rightarrow F = \frac{4\sigma I}{lc} = \frac{4(60)10^3(0.05729)}{(36)(0.25)} = 1528 \text{ lbf}$$

$$y = \frac{Fl^3}{48EI} = \frac{(1528)(36^3)}{48(30)10^6(0.05729)} = 0.864 \text{ in} \quad \text{Ans.}$$

4-23 From the solutions to Prob. 3-68, $T_1 = 60$ lbf and $T_2 = 400$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198 \text{ in}^4$$

From Table A-9, beam 6,

$$\begin{aligned} z_A &= \left[\frac{F_1 b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right]_{x=10\text{in}} \\ &= \frac{(-575)(30)(10)}{6(30)10^6(0.1198)(40)} (10^2 + 30^2 - 40^2) \\ &\quad + \frac{460(12)(10)}{6(30)10^6(0.1198)(40)} (10^2 + 12^2 - 40^2) = 0.0332 \text{ in} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} (\theta_A)_y &= - \left(\frac{dz}{dx} \right)_{x=10\text{in}} = - \left\{ \frac{d}{dx} \left[\frac{F_1 b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=10\text{in}} \\ &= - \left\{ \frac{F_1 b_1}{6EI} (3x^2 + b_1^2 - l^2) + \frac{F_2 b_2}{6EI} (3x^2 + b_2^2 - l^2) \right\}_{x=10\text{in}} \\ &= - \frac{(575)(30)}{6(30)10^6(0.1198)(40)} [3(10^2) + 30^2 - 40^2] \\ &\quad - \frac{-460(12)}{6(30)10^6(0.1198)(40)} [3(10^2) + 12^2 - 40^2] \\ &= 6.02(10^{-4}) \text{ rad} \quad \text{Ans.} \end{aligned}$$

4-24 From the solutions to Prob. 3-69, $T_1 = 2880$ N and $T_2 = 432$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(30)^4}{64} = 39.76(10^3) \text{ mm}^4$$

The load in between the supports supplies an angle to the overhanging end of the beam. That angle is found by taking the derivative of the deflection from that load. From Table A-9, beams 6 (subscript 1) and 10 (subscript 2),

$$y_A = \left[\theta_{BC} \Big|_C (a_2) \right]_{\text{beam 6}} + (y_A)_{\text{beam 10}} \quad (1)$$

$$\begin{aligned} \theta_{BC} \Big|_C &= \left\{ \frac{d}{dx} \left[\frac{F_1 a_1 (l-x)}{6EI} (x^2 + a_1^2 - 2lx) \right] \right\}_{x=l} = \left[\frac{F_1 a_1}{6EI} (6lx - 3x^2 - a_1^2 - 2l^2) \right]_{x=l} \\ &= \frac{F_1 a_1}{6EI} (l^2 - a_1^2) \end{aligned}$$

Equation (1) is thus

$$\begin{aligned} y_A &= \frac{F_1 a_1}{6EI} (l^2 - a_1^2) a_2 - \frac{F_2 a_2^2}{3EI} (l + a_2) \\ &= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)} (510^2 - 230^2)(300) - \frac{2070(300^2)}{3(207)10^3(39.76)10^3} (510 + 300) \\ &= -7.99 \text{ mm} \quad \text{Ans.} \end{aligned}$$

The slope at A, relative to the z axis is

$$\begin{aligned} (\theta_A)_z &= \frac{F_1 a_1}{6EI} (l^2 - a_1^2) + \left\{ \frac{d}{dx} \left[\frac{F_2 (x-l)}{6EI} [(x-l)^2 - a_2(3x-l)] \right] \right\}_{x=l+a_2} \\ &= \frac{F_1 a_1}{6EI} (l^2 - a_1^2) + \frac{F_2}{6EI} [3(x-l)^2 - 3a_2(x-l) - a_2(3x-l)]_{x=l+a_2} \\ &= \frac{F_1 a_1}{6EI} (l^2 - a_1^2) - \frac{F_2}{6EI} (3a_2^2 + 2la_2) \\ &= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)} (510^2 - 230^2) \\ &\quad - \frac{2070}{6(207)10^3(39.76)10^3} [3(300^2) + 2(510)(300)] \\ &= -0.0304 \text{ rad} \quad \text{Ans.} \end{aligned}$$

4-25 From the solutions to Prob. 3-70, $T_1 = 392.16 \text{ lbf}$ and $T_2 = 58.82 \text{ lbf}$

$$I = \frac{\pi d^4}{64} = \frac{\pi (1)^4}{64} = 0.049 \text{ 09 in}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_1 b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right]_{x=8\text{in}} = \frac{(-350)(14)(8)}{6(30)10^6(0.049\ 09)(22)} (8^2 + 14^2 - 22^2) = 0.0452 \text{ in } \textit{Ans.}$$

$$z_A = \left[\frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right]_{x=8\text{in}} = \frac{(-450.98)(6)(8)}{6(30)10^6(0.049\ 09)(22)} (8^2 + 6^2 - 22^2) = 0.0428 \text{ in } \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0452^2 + 0.0428^2} = 0.0622 \text{ in } \textit{Ans.}$

$$\begin{aligned} (\theta_A)_z &= \left(\frac{d y}{d x} \right)_{x=a_1} = \left\{ \frac{d}{dx} \left[\frac{F_1 b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right] \right\}_{x=a_1} = \frac{F_1 b_1}{6EI} (3a_1^2 + b_1^2 - l^2) \\ &= \frac{(-350)(14)}{6(30)10^6(0.04909)(22)} [3(8^2) + 14^2 - 22^2] = 0.00242 \text{ rad } \textit{Ans.} \end{aligned}$$

$$\begin{aligned} (\theta_A)_y &= \left(-\frac{d z}{d x} \right)_{x=a_1} = - \left\{ \frac{d}{dx} \left[\frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=a_1} = -\frac{F_2 b_2}{6EI} (3a_1^2 + b_2^2 - l^2) \\ &= \frac{(450.98)(6)}{6(30)10^6(0.04909)(22)} [3(8^2) + 6^2 - 22^2] = -0.00356 \text{ rad } \textit{Ans.} \end{aligned}$$

The slope magnitude is $\Theta_A = \sqrt{0.00242^2 + (-0.00356)^2} = 0.00430 \text{ rad } \textit{Ans.}$

4-26 From the solutions to Prob. 3-71, $T_1 = 250 \text{ N}$ and $T_2 = 37.5 \text{ N}$

$$I = \frac{\pi d^4}{64} = \frac{\pi(20)^4}{64} = 7\ 854 \text{ mm}^4$$

$$\begin{aligned} y_A &= \left[\frac{F_{1y} b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right]_{x=300\text{mm}} = \frac{(-345 \sin 45^\circ)(550)(300)}{6(207)10^3(7\ 854)(850)} (300^2 + 550^2 - 850^2) \\ &= 1.60 \text{ mm } \textit{Ans.} \end{aligned}$$

$$\begin{aligned} z_A &= \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right]_{x=300\text{mm}} \\ &= \frac{(345 \cos 45^\circ)(550)(300)}{6(207)10^3(7\ 854)(850)} (300^2 + 550^2 - 850^2) \\ &\quad + \frac{-287.5(150)(300)}{6(207)10^3(7\ 854)(850)} (300^2 + 150^2 - 850^2) = -0.650 \text{ mm } \textit{Ans.} \end{aligned}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{1.60^2 + (-0.650)^2} = 1.73 \text{ mm } \textit{Ans.}$

$$\begin{aligned}
(\theta_A)_z &= \left(\frac{d y}{d x} \right)_{x=a_1} = \left\{ \frac{d}{dx} \left[\frac{F_{1y} b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right] \right\}_{x=a_1} = \frac{F_{1y} b_1}{6EI} (3a_1^2 + b_1^2 - l^2) \\
&= \frac{-(345 \sin 45^\circ)(550)}{6(207)10^3(7854)(850)} [3(300^2) + 550^2 - 850^2] = 0.00243 \text{ rad} \quad \text{Ans.} \\
(\theta_A)_y &= - \left(\frac{d z}{d x} \right)_{x=a_1} = - \left\{ \frac{d}{dx} \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=a_1} \\
&= - \frac{F_{1z} b_1}{6EI} (3a_1^2 + b_1^2 - l^2) - \frac{F_2 b_2}{6EI} (3a_1^2 + b_2^2 - l^2) \\
&= - \frac{(345 \cos 45^\circ)(550)}{6(207)10^3(7854)(850)} [3(300^2) + 550^2 - 850^2] \\
&\quad - \frac{-287.5(150)}{6(207)10^3(7854)(850)} [3(300^2) + 150^2 - 850^2] = 1.91 \cdot 10^{-4} \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_A = \sqrt{0.00243^2 + 0.000191^2} = 0.00244 \text{ rad} \quad \text{Ans.}$

4-27 From the solutions to Prob. 3-72, $F_B = 750 \text{ lbf}$

$$I = \frac{\pi d^4}{64} = \frac{\pi (1.25)^4}{64} = 0.1198 \text{ in}^4$$

From Table A-9, beams 6 (subscript 1) and 10 (subscript 2)

$$\begin{aligned}
y_A &= \left[\frac{F_{1y} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2y} a_2 x}{6EI} (l^2 - x^2) \right]_{x=16 \text{ in}} \\
&= \frac{(-300 \cos 20^\circ)(14)(16)}{6(30)10^6(0.1198)(30)} (16^2 + 14^2 - 30^2) + \frac{(750 \sin 20^\circ)(9)(16)}{6(30)10^6(0.1198)(30)} (30^2 - 16^2) \\
&= 0.0805 \text{ in} \quad \text{Ans.} \\
z_A &= \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2z} a_2 x}{6EI} (l^2 - x^2) \right]_{x=16 \text{ in}} \\
&= \frac{(300 \sin 20^\circ)(14)(16)}{6(30)10^6(0.1198)(30)} (16^2 + 14^2 - 30^2) + \frac{(-750 \cos 20^\circ)(9)(16)}{6(30)10^6(0.1198)(30)} (30^2 - 16^2) \\
&= -0.1169 \text{ in} \quad \text{Ans.}
\end{aligned}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0805^2 + (-0.1169)^2} = 0.142 \text{ in} \quad \text{Ans.}$

$$\begin{aligned}
(\theta_A)_z &= \left(\frac{dy}{dx} \right)_{x=a_1} = \left\{ \frac{d}{dx} \left[\frac{F_{1y}b_1x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2y}a_2x}{6EI} (l^2 - x^2) \right] \right\}_{x=a_1} \\
&= \frac{F_{1y}b_1}{6EI} (3a_1^2 + b_1^2 - l^2) + \frac{F_{2y}a_2}{6EI} (l^2 - 3a_1^2) \\
&= \frac{(-300 \cos 20^\circ)(14)}{6(30)10^6(0.1198)(30)} [3(16^2) + 14^2 - 30^2] \\
&\quad + \frac{(750 \sin 20^\circ)(9)}{6(30)10^6(0.1198)(30)} [30^2 - 3(16^2)] = 8.06(10^{-5}) \text{ rad} \quad \text{Ans.} \\
(\theta_A)_y &= - \left(\frac{dz}{dx} \right)_{x=a_1} = - \left\{ \frac{d}{dx} \left[\frac{F_{1z}b_1x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2z}a_2x}{6EI} (l^2 - x^2) \right] \right\}_{x=a_1} \\
&= - \frac{F_{1z}b_1}{6EI} (3a_1^2 + b_1^2 - l^2) - \frac{F_{2z}a_2}{6EI} (l^2 - 3a_1^2) \\
&= - \frac{(300 \sin 20^\circ)(14)}{6(30)10^6(0.1198)(30)} [3(16^2) + 14^2 - 30^2] - \frac{(-750 \cos 20^\circ)(9)}{6(30)10^6(0.1198)(30)} [30^2 - 3(16^2)] \\
&= 0.00115 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_A = \sqrt{[8.06(10^{-5})]^2 + 0.00115^2} = 0.00115 \text{ rad} \quad \text{Ans.}$

4-28 From the solutions to Prob. 3-73, $F_B = 22.8(10^3) \text{ N}$

$$I = \frac{\pi d^4}{64} = \frac{\pi(50^4)}{64} = 306.8(10^3) \text{ mm}^4$$

From Table A-9, beam 6,

$$\begin{aligned}
y_A &= \left[\frac{F_{1y}b_1x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2y}b_2x}{6EI} (x^2 + b_2^2 - l^2) \right]_{x=400\text{mm}} \\
&= \frac{[11(10^3) \sin 20^\circ](650)(400)}{6(207)10^3(306.8)10^3(1050)} (400^2 + 650^2 - 1050^2) \\
&\quad + \frac{[22.8(10^3) \sin 25^\circ](300)(400)}{6(207)10^3(306.8)10^3(1050)} (400^2 + 300^2 - 1050^2) \\
&= -3.735 \text{ mm} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
z_A &= \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2z} b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right]_{x=400\text{mm}} \\
&= \frac{[11(10^3) \cos 20^\circ](650)(400)}{6(207)10^3(306.8)10^3(1050)} (400^2 + 650^2 - 1050^2) \\
&\quad + \frac{[-22.8(10^3) \cos 25^\circ](300)(400)}{6(207)10^3(306.8)10^3(1050)} (400^2 + 300^2 - 1050^2) = 1.791 \text{ mm} \quad \text{Ans.}
\end{aligned}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{(-3.735)^2 + 1.791^2} = 4.14 \text{ mm} \quad \text{Ans.}$

$$\begin{aligned}
(\theta_A)_z &= \left(\frac{d y}{d x} \right)_{x=a_1} = \left\{ \frac{d}{dx} \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2z} b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=a_1} \\
&= \frac{F_{1y} b_1}{6EI} (3a_1^2 + b_1^2 - l^2) + \frac{F_{2y} b_2}{6EI} (3a_1^2 + b_2^2 - l^2) \\
&= \frac{[11(10^3) \sin 20^\circ](650)}{6(207)10^3(306.8)10^3(1050)} [3(400^2) + 650^2 - 1050^2] \\
&\quad + \frac{[22.8(10^3) \sin 25^\circ](300)}{6(207)10^3(306.8)10^3(1050)} [3(400^2) + 300^2 - 1050^2] \\
&= -0.00507 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_A)_y &= - \left(\frac{d z}{d x} \right)_{x=a_1} = - \left\{ \frac{d}{dx} \left[\frac{F_{1z} b_1 x}{6EI} (x^2 + b_1^2 - l^2) + \frac{F_{2z} b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=a_1} \\
&= - \frac{F_{1z} b_1}{6EI} (3a_1^2 + b_1^2 - l^2) - \frac{F_{2z} b_2}{6EI} (3a_1^2 + b_2^2 - l^2) \\
&= - \frac{[11(10^3) \cos 20^\circ](650)}{6(207)10^3(306.8)10^3(1050)} [3(400^2) + 650^2 - 1050^2] \\
&\quad - \frac{[-22.8(10^3) \cos 25^\circ](300)}{6(207)10^3(306.8)10^3(1050)} [3(400^2) + 300^2 - 1050^2] \\
&= -0.00489 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_A = \sqrt{(-0.00507)^2 + (-0.00489)^2} = 0.00704 \text{ rad} \quad \text{Ans.}$

4-29 From the solutions to Prob. 3-68, $T_1 = 60 \text{ lbf}$ and $T_2 = 400 \text{ lbf}$, and Prob. 4-23, $I = 0.1198 \text{ in}^4$. From Table A-9, beam 6,

$$\begin{aligned}
(\theta_o)_y &= -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2z}b_2x}{6EI}(x^2+b_2^2-l^2)\right]\right\}_{x=0} \\
&= -\frac{F_{1z}b_1}{6EI}(b_1^2-l^2)-\frac{F_{2z}b_2}{6EI}(b_2^2-l^2) = -\frac{-575(30)}{6(30)10^6(0.1198)(40)}(30^2-40^2) \\
&\quad -\frac{460(12)}{6(30)10^6(0.1198)(40)}(12^2-40^2) = -0.00468 \text{ rad} \quad \text{Ans.} \\
(\theta_c)_y &= -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2z}a_2(l-x)}{6EI}(x^2+a_2^2-2lx)\right]\right\}_{x=l} \\
&= -\left[\frac{F_{1z}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2z}a_2}{6EI}(6lx-2l^2-3x^2-a_2^2)\right]_{x=l} \\
&= -\frac{F_{1z}a_1}{6EI}(l^2-a_1^2)-\frac{F_{2z}a_2}{6EI}(l^2-a_2^2) \\
&= -\frac{-575(10)(40^2-10^2)}{6(30)10^6(0.1198)(40)}-\frac{460(28)(40^2-28^2)}{6(30)10^6(0.1198)(40)} = -0.00219 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

4-30 From the solutions to Prob. 3-69, $T_1 = 2\,880 \text{ N}$ and $T_2 = 432 \text{ N}$, and Prob. 4-24, $I = 39.76 (10^3) \text{ mm}^4$. From Table A-9, beams 6 and 10

$$\begin{aligned}
(\theta_o)_z &= \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_1b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_2a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=0} \\
&= \left[\frac{F_1b_1}{6EI}(3x^2+b_1^2-l^2)+\frac{F_2a_2}{6EI}(l^2-3x^2)\right]_{x=0} = \frac{F_1b_1}{6EI}(b_1^2-l^2)+\frac{F_2a_2l}{6EI} \\
&= \frac{-3\,312(280)}{6(207)10^3(39.76)10^3(510)}(280^2-510^2)+\frac{2\,070(300)(510)}{6(207)10^3(39.76)10^3} \\
&= 0.0131 \text{ rad} \quad \text{Ans.} \\
(\theta_c)_z &= \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_1a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_2a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=l} \\
&= \left[\frac{F_1a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_2a_2}{6EI}(l^2-3x^2)\right]_{x=l} = \frac{F_1a_1}{6EI}(l^2-a_1^2)-\frac{F_2a_2l}{3EI} \\
&= \frac{-3\,312(230)}{6(207)10^3(39.76)10^3(510)}(510^2-230^2)-\frac{2\,070(300)(510)}{3(207)10^3(39.76)10^3} \\
&= -0.0191 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

4-31 From the solutions to Prob. 3-70, $T_1 = 392.19 \text{ lbf}$ and $T_2 = 58.82 \text{ lbf}$, and Prob. 4-25, $I = 0.04909 \text{ in}^4$. From Table A-9, beam 6

$$\begin{aligned}
 (\theta_o)_z &= \left(\frac{d y}{d x} \right)_{x=0} = \left\{ \frac{d}{dx} \left[\frac{F_{1y} b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right] \right\}_{x=0} = \frac{F_{1y} b_1}{6EI} (b_1^2 - l^2) \\
 &= \frac{-350(14)}{6(30)10^6 (0.04909)(22)} (14^2 - 22^2) = 0.00726 \text{ rad} \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 (\theta_o)_y &= - \left(\frac{d z}{d x} \right)_{x=0} = - \left\{ \frac{d}{dx} \left[\frac{F_{2z} b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right] \right\}_{x=0} = - \frac{F_{2z} b_2}{6EI} (b_2^2 - l^2) \\
 &= - \frac{-450.98(6)}{6(30)10^6 (0.04909)(22)} (6^2 - 22^2) \\
 &= -0.00624 \text{ rad} \quad \text{Ans.}
 \end{aligned}$$

The slope magnitude is $\Theta_o = \sqrt{0.00726^2 + (-0.00624)^2} = 0.00957 \text{ rad} \quad \text{Ans.}$

$$\begin{aligned}
 (\theta_c)_z &= \left(\frac{d y}{d x} \right)_{x=l} = \left\{ \frac{d}{dx} \left[\frac{F_{1y} a_1 (l-x)}{6EI} (x^2 + a_1^2 - 2lx) \right] \right\}_{x=l} \\
 &= \left[\frac{F_{1y} a_1}{6EI} (6lx - 2l^2 - 3x^2 - a_1^2) \right]_{x=l} = \frac{F_{1y} a_1}{6EI} (l^2 - a_1^2) \\
 &= \frac{-350(8)}{6(30)10^6 (0.0491)(22)} (22^2 - 8^2) = -0.00605 \text{ rad} \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 (\theta_c)_y &= - \left(\frac{d z}{d x} \right)_{x=l} = - \left\{ \frac{d}{dx} \left[\frac{F_{2z} a_2 (l-x)}{6EI} (x^2 + a_2^2 - 2lx) \right] \right\}_{x=l} \\
 &= - \left[\frac{F_{2z} a_2}{6EI} (6lx - 2l^2 - 3x^2 - a_2^2) \right]_{x=l} = - \frac{F_{2z} a_2}{6EI} (l^2 - a_2^2) \\
 &= - \frac{-450.98(16)}{6(30)10^6 (0.04909)(22)} (22^2 - 16^2) = 0.00846 \text{ rad} \quad \text{Ans.}
 \end{aligned}$$

The slope magnitude is $\Theta_c = \sqrt{(-0.00605)^2 + 0.00846^2} = 0.0104 \text{ rad} \quad \text{Ans.}$

4-32 From the solutions to Prob. 3-71, $T_1 = 250 \text{ N}$ and $T_1 = 37.5 \text{ N}$, and Prob. 4-26, $I = 7854 \text{ mm}^4$. From Table A-9, beam 6

$$\begin{aligned}
 (\theta_o)_z &= \left(\frac{d y}{d x} \right)_{x=0} = \left\{ \frac{d}{dx} \left[\frac{F_{1y} b_1 x}{6EI} (x^2 + b_1^2 - l^2) \right] \right\}_{x=0} = \frac{F_{1y} b_1}{6EI} (b_1^2 - l^2) \\
 &= \frac{[-345 \sin 45^\circ](550)}{6(207)10^3 (7854)(850)} (550^2 - 850^2) = 0.00680 \text{ rad} \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
(\theta_o)_y &= -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2z}b_2x}{6EI}(x^2+b_2^2-l^2)\right]\right\}_{x=0} \\
&= -\frac{F_{1z}b_1}{6EI}(b_1^2-l^2)-\frac{F_{2z}b_2}{6EI}(b_2^2-l^2) = -\frac{[345\cos 45^\circ](550)}{6(207)10^3(7\,854)(850)}(550^2-850^2) \\
&\quad -\frac{-287.5(150)}{6(207)10^3(7\,854)(850)}(150^2-850^2) = 0.00316 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_o = \sqrt{0.00680^2 + 0.00316^2} = 0.00750 \text{ rad} \quad \text{Ans.}$

$$\begin{aligned}
(\theta_c)_z &= \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)\right]\right\}_{x=l} = \left[\frac{F_{1y}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)\right]_{x=l} \\
&= \frac{F_{1y}a_1}{6EI}(l^2-a_1^2) = \frac{[-345\sin 45^\circ](300)}{6(207)10^3(7\,854)(850)}(850^2-300^2) = -0.00558 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_c)_y &= -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2z}a_2(l-x)}{6EI}(x^2+a_2^2-2lx)\right]\right\}_{x=l} \\
&= -\frac{F_{1z}a_1}{6EI}(l^2-a_1^2)-\frac{F_{2z}a_2}{6EI}(l^2-a_2^2) = -\frac{[345\cos 45^\circ](300)}{6(207)10^3(7\,854)(850)}(850^2-300^2) \\
&\quad -\frac{-287.5(700)}{6(207)10^3(7\,854)(850)}(850^2-700^2) = 6.04(10^{-5}) \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_c = \sqrt{(-0.00558)^2 + [6.04(10^{-5})]^2} = 0.00558 \text{ rad} \quad \text{Ans.}$

4-33 From the solutions to Prob. 3-72, $F_B = 750 \text{ lbf}$, and Prob. 4-27, $I = 0.1198 \text{ in}^4$. From Table A-9, beams 6 and 10

$$\begin{aligned}
(\theta_o)_z &= \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2y}a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=0} \\
&= \left[\frac{F_{1y}b_1}{6EI}(3x^2+b_1^2-l^2)+\frac{F_{2y}a_2}{6EI}(l^2-3x^2)\right]_{x=0} = \frac{F_{1y}b_1}{6EI}(b_1^2-l^2)+\frac{F_{2y}a_2l}{6EI} \\
&= \frac{[-300\cos 20^\circ](14)}{6(30)10^6(0.1198)(30)}(14^2-30^2)+\frac{[750\sin 20^\circ](9)(30)}{6(30)10^6(0.1198)} = 0.00751 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_o)_y &= -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2z}a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=0} \\
&= -\left[\frac{F_{1z}b_1}{6EI}(3x^2+b_1^2-l^2)+\frac{F_{2z}a_2}{6EI}(l^2-3x^2)\right]_{x=0} = -\frac{F_{1z}b_1}{6EI}(b_1^2-l^2)-\frac{F_{2z}a_2l}{6EI} \\
&= -\frac{[300\sin 20^\circ](14)}{6(30)10^6(0.1198)(30)}(14^2-30^2)-\frac{[-750\cos 20^\circ](9)(30)}{6(30)10^6(0.1198)} = 0.0104 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_o = \sqrt{0.00751^2 + 0.0104^2} = 0.0128 \text{ rad}$ Ans.

$$\begin{aligned}
(\theta_c)_z &= \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2y}a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=l} \\
&= \left[\frac{F_{1y}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2y}a_2}{6EI}(l^2-3x^2)\right]_{x=l} = \frac{F_{1y}a_1}{6EI}(l^2-a_1^2)-\frac{F_{2y}a_2l}{3EI} \\
&= \frac{[-300\cos 20^\circ](16)}{6(30)10^6(0.1198)(30)}(30^2-16^2)-\frac{[750\sin 20^\circ](9)(30)}{3(30)10^6(0.1198)} = -0.0109 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_c)_y &= -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2z}a_2x}{6EI}(l^2-x^2)\right]\right\}_{x=l} \\
&= -\left[\frac{F_{1z}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2z}a_2}{6EI}(l^2-3x^2)\right]_{x=l} = -\frac{F_{1z}a_1}{6EI}(l^2-a_1^2)+\frac{F_{2z}a_2l}{3EI} \\
&= -\frac{[300\sin 20^\circ](16)}{6(30)10^6(0.1198)(30)}(30^2-16^2)+\frac{[-750\cos 20^\circ](9)(30)}{3(30)10^6(0.1198)} = -0.0193 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_c = \sqrt{(-0.0109)^2 + (-0.0193)^2} = 0.0222 \text{ rad}$ Ans.

4-34 From the solutions to Prob. 3-73, $F_B = 22.8 \text{ kN}$, and Prob. 4-28, $I = 306.8 (10^3) \text{ mm}^4$.
From Table A-9, beam 6

$$\begin{aligned}
(\theta_o)_z &= \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2y}b_2x}{6EI}(x^2+b_2^2-l^2)\right]\right\}_{x=0} \\
&= \frac{F_{1y}b_1}{6EI}(b_1^2-l^2)+\frac{F_{2y}b_2}{6EI}(b_2^2-l^2) = \frac{[11(10^3)\sin 20^\circ](650)}{6(207)10^3(306.8)10^3(1050)}(650^2-1050^2) \\
&\quad + \frac{[22.8(10^3)\sin 25^\circ](300)}{6(207)10^3(306.8)10^3(1050)}(300^2-1050^2) = -0.0115 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_o)_y &= -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1x}{6EI}(x^2+b_1^2-l^2)+\frac{F_{2z}b_2x}{6EI}(x^2+b_2^2-l^2)\right]\right\}_{x=0} \\
&= -\frac{F_{1z}b_1}{6EI}(b_1^2-l^2)-\frac{F_{2z}b_2}{6EI}(b_2^2-l^2) \\
&= -\frac{[11(10^3)\cos 20^\circ](650)}{6(207)10^3(306.8)10^3(1050)}(650^2-1050^2) \\
&\quad -\frac{[-22.8(10^3)\cos 25^\circ](300)}{6(207)10^3(306.8)10^3(1050)}(300^2-1050^2) = -0.00427 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_o = \sqrt{(-0.0115)^2 + (-0.00427)^2} = 0.0123 \text{ rad} \quad \text{Ans.}$

$$\begin{aligned}
(\theta_c)_z &= \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2y}a_2(l-x)}{6EI}(x^2+a_2^2-2lx)\right]\right\}_{x=l} \\
&= \left[\frac{F_{1y}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2y}a_2}{6EI}(6lx-2l^2-3x^2-a_2^2)\right]_{x=l} \\
&= \frac{F_{1y}a_1}{6EI}(l^2-a_1^2)+\frac{F_{2y}a_2}{6EI}(l^2-a_2^2) = \frac{[11(10^3)\sin 20^\circ](400)}{6(207)10^3(306.8)10^3(1050)}(1050^2-400^2) \\
&\quad +\frac{[22.8(10^3)\sin 25^\circ](750)}{6(207)10^3(306.8)10^3(1050)}(1050^2-750^2) = 0.0133 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
(\theta_c)_y &= -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EI}(x^2+a_1^2-2lx)+\frac{F_{2z}a_2(l-x)}{6EI}(x^2+a_2^2-2lx)\right]\right\}_{x=l} \\
&= -\left[\frac{F_{1z}a_1}{6EI}(6lx-2l^2-3x^2-a_1^2)+\frac{F_{2z}a_2}{6EI}(6lx-2l^2-3x^2-a_2^2)\right]_{x=l} \\
&= -\frac{F_{1z}a_1}{6EI}(l^2-a_1^2)-\frac{F_{2z}a_2}{6EI}(l^2-a_2^2) = -\frac{[11(10^3)\cos 20^\circ](400)}{6(207)10^3(306.8)10^3(1050)}(1050^2-400^2) \\
&\quad -\frac{[-22.8(10^3)\cos 25^\circ](750)}{6(207)10^3(306.8)10^3(1050)}(1050^2-750^2) = 0.0112 \text{ rad} \quad \text{Ans.}
\end{aligned}$$

The slope magnitude is $\Theta_c = \sqrt{0.0133^2 + 0.0112^2} = 0.0174 \text{ rad} \quad \text{Ans.}$

4-35 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.

In Prob. 4-29, $I = 0.1198 \text{ in}^4$, and it was found that the greater angle occurs at the bearing at O where $(\theta_o)_y = -0.00468 \text{ rad}$.

Since θ is inversely proportional to I ,

$$\theta_{\text{new}} I_{\text{new}} = \theta_{\text{old}} I_{\text{old}} \Rightarrow I_{\text{new}} = \pi d_{\text{new}}^4 / 64 = \theta_{\text{old}} I_{\text{old}} / \theta_{\text{new}}$$

or,

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

The absolute sign is used as the old slope may be negative.

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{-0.00468}{0.00105} \right| 0.1198 \right)^{1/4} = 1.82 \text{ in} \quad \text{Ans.}$$

- 4-36** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.
In Prob. 4-30, $I = 39.76 (10^3) \text{ mm}^4$, and it was found that the greater angle occurs at the bearing at C where $(\theta_C)_y = -0.0191 \text{ rad}$.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{-0.0191}{0.00105} \right| 39.76(10^3) \right)^{1/4} = 62.0 \text{ mm} \quad \text{Ans.}$$

- 4-37** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.
In Prob. 4-31, $I = 0.0491 \text{ in}^4$, and the maximum slope is $\theta_C = 0.0104 \text{ rad}$.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0104}{0.00105} \right| 0.0491 \right)^{1/4} = 1.77 \text{ in} \quad \text{Ans.}$$

- 4-38** The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.
In Prob. 4-32, $I = 7854 \text{ mm}^4$, and the maximum slope is $\theta_O = 0.00750 \text{ rad}$.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.00750}{0.00105} \right| 7\,854 \right)^{1/4} = 32.7 \text{ mm} \quad \text{Ans.}$$

4-39 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.
In Prob. 4-33, $I = 0.119\,8 \text{ in}^4$, and the maximum slope $\Theta = 0.0222 \text{ rad}$.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0222}{0.00105} \right| 0.119\,8 \right)^{1/4} = 2.68 \text{ in} \quad \text{Ans.}$$

4-40 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105 \text{ rad}$.
In Prob. 4-34, $I = 306.8 (10^3) \text{ mm}^4$, and the maximum slope is $\Theta_C = 0.0174 \text{ rad}$.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0174}{0.00105} \right| 306.8(10^3) \right)^{1/4} = 100.9 \text{ mm} \quad \text{Ans.}$$

4-41 $I_{AB} = \pi 1^4/64 = 0.04909 \text{ in}^4$, $J_{AB} = 2 I_{AB} = 0.09818 \text{ in}^4$, $I_{BC} = (0.25)(1.5)^3/12 = 0.07031 \text{ in}^4$,
 $I_{CD} = \pi (3/4)^4/64 = 0.01553 \text{ in}^4$. For Eq. (3-41), p. 116, $b/c = 1.5/0.25 = 6 \Rightarrow \beta = 0.299$.

The deflection can be broken down into several parts

1. The vertical deflection of B due to force and moment acting on B (y_1).
2. The vertical deflection due to the slope at B , θ_{B1} , due to the force and moment acting on B ($y_2 = \overline{CD} \theta_{B1} = 2 \theta_{B1}$).

3. The vertical deflection due to the rotation at B , θ_{B2} , due to the torsion acting at B ($y_3 = \overline{BC} \theta_{B1} = 5 \theta_{B1}$).
4. The vertical deflection of C due to the force acting on C (y_4).
5. The rotation at C , θ_C , due to the torsion acting at C ($y_3 = \overline{CD} \theta_C = 2 \theta_C$).
6. The vertical deflection of D due to the force acting on D (y_5).

1. From Table A-9, beams 1 and 4 with $F = -200$ lbf and $M_B = 2(200) = 400$ lbf·in

$$y_1 = -\frac{-200(6^3)}{3(30)10^6(0.04909)} + \frac{400(6^2)}{2(30)10^6(0.04909)} = 0.01467 \text{ in}$$

2. From Table A-9, beams 1 and 4

$$\begin{aligned} \theta_{B1} &= \left\{ \frac{d}{dx} \left[\frac{Fx^2}{6EI} (x-3l) + \frac{M_B x^2}{2EI} \right] \right\}_{x=l} = \left[\frac{Fx}{6EI} (3x-6l) + \frac{M_B x}{EI} \right]_{x=l} \\ &= \left\{ \frac{l}{2EI} [-Fl + 2M_B] \right\} = \frac{6}{2(30)10^6(0.04909)} [-(-200)(6) + 2(400)] = 0.004074 \text{ rad} \end{aligned}$$

$$y_2 = 2(0.004072) = 0.00815 \text{ in}$$

3. The torsion at B is $T_B = 5(200) = 1000$ lbf·in. From Eq. (4-5)

$$\theta_{B2} = \left(\frac{TL}{JG} \right)_{AB} = \frac{1000(6)}{0.09818(11.5)10^6} = 0.005314 \text{ rad}$$

$$y_3 = 5(0.005314) = 0.02657 \text{ in}$$

4. For bending of BC , from Table A-9, beam 1

$$y_4 = -\frac{-200(5^3)}{3(30)10^6(0.07031)} = 0.00395 \text{ in}$$

5. For twist of BC , from Eq. (3-41), p. 116, with $T = 2(200) = 400$ lbf·in

$$\theta_C = \frac{400(5)}{0.299(1.5)0.25^3(11.5)10^6} = 0.02482 \text{ rad}$$

$$y_5 = 2(0.02482) = 0.04964 \text{ in}$$

6. For bending of CD , from Table A-9, beam 1

$$y_6 = -\frac{-200(2^3)}{3(30)10^6(0.01553)} = 0.00114 \text{ in}$$

Summing the deflections results in

$$y_D = \sum_{i=1}^6 y_i = 0.01467 + 0.00815 + 0.02657 + 0.00395 + 0.04964 + 0.00114 = 0.1041 \text{ in } \text{Ans.}$$

This problem is solved more easily using Castigliano's theorem. See Prob. 4-71.

4-42 The deflection of D in the x direction due to F_z is from:

1. The deflection due to the slope at B , θ_{B1} , due to the force and moment acting on B ($x_1 = \overline{BC}$ $\theta_{B1} = 5 \theta_{B1}$).
2. The deflection due to the moment acting on C (x_2).

1. For AB , $I_{AB} = \pi 1^4/64 = 0.04909 \text{ in}^4$. From Table A-9, beams 1 and 4

$$\begin{aligned} \theta_{B1} &= \left\{ \frac{d}{dx} \left[\frac{Fx^2}{6EI} (x-3l) + \frac{M_B x^2}{2EI} \right] \right\}_{x=l} = \left[\frac{Fx}{6EI} (3x-6l) + \frac{M_B x}{EI} \right]_{x=l} \\ &= \left\{ \frac{l}{2EI} [-Fl + 2M_B] \right\} = \frac{6}{2(30)10^6 (0.04909)} [-(100)(6) + 2(-200)] = -0.002037 \text{ rad} \end{aligned}$$

$$x_1 = 5(-0.002037) = -0.01019 \text{ in}$$

2. For BC , $I_{BC} = (1.5)(0.25)^3/12 = 0.001953 \text{ in}^4$. From Table A-9, beam 4

$$x_2 = \frac{M_C l^2}{2EI} = \frac{2(-100)5}{2(30)10^6 (0.001953)} = -0.04267 \text{ in}$$

The deflection of D in the x direction due to F_x is from:

3. The elongation of AB due to the tension. For AB , the area is $A = \pi 1^2/4 = 0.7854 \text{ in}^2$

$$x_3 = \left(\frac{Fl}{AE} \right)_{AB} = \frac{-150(6)}{0.7854(30)10^6} = -3.82(10^{-5}) \text{ in}$$

4. The deflection due to the slope at B , θ_{B2} , due to the moment acting on B ($x_1 = \overline{BC}$ $\theta_{B2} = 5 \theta_{B2}$). With $I_{AB} = 0.04907 \text{ in}^4$,

$$\theta_{B2} = \frac{M_B l}{EI} = \frac{5(-150)6}{30(10^6)0.04909} = -0.003056 \text{ rad}$$

$$x_4 = 5(-0.003056) = -0.01528 \text{ in}$$

5. The deflection at C due to the bending force acting on C . With $I_{BC} = 0.001953 \text{ in}^4$

$$x_5 = \left(-\frac{Fl^3}{3EI} \right)_{BC} = -\frac{150(5^3)}{3(30)10^6(0.001953)} = -0.10667 \text{ in}$$

6. The elongation of CD due to the tension. For CD , the area is $A = \pi(0.75^2)/4 = 0.4418 \text{ in}^2$

$$x_6 = \left(\frac{Fl}{AE} \right)_{CD} = \frac{-150(2)}{0.4418(30)10^6} = -2.26(10^{-5}) \text{ in}$$

Summing the deflections results in

$$x_D = \sum_{i=1}^6 x_i = -0.01019 - 0.04267 - 3.82(10^{-5}) \\ - 0.01528 - 0.10667 - 2.26(10^{-5}) = -0.1749 \text{ in } \textit{Ans.}$$

4-43 $J_{OA} = J_{BC} = \pi(1.5^4)/32 = 0.4970 \text{ in}^4$, $J_{AB} = \pi(1^4)/32 = 0.09817 \text{ in}^4$, $I_{AB} = \pi(1^4)/64 = 0.04909 \text{ in}^4$, and $I_{CD} = \pi(0.75^4)/64 = 0.01553 \text{ in}^4$.

$$\theta = \left(\frac{Tl}{GJ} \right)_{OA} + \left(\frac{Tl}{GJ} \right)_{AB} + \left(\frac{Tl}{GJ} \right)_{BC} = \frac{T}{G} \left(\frac{l_{OA}}{J_{OA}} + \frac{l_{AB}}{J_{AB}} + \frac{l_{BC}}{J_{BC}} \right) \\ = \frac{250(12)}{11.5(10^6)} \left(\frac{2}{0.4970} + \frac{9}{0.09817} + \frac{2}{0.4970} \right) = 0.0260 \text{ rad } \textit{Ans.}$$

Simplified

$$\theta_s = \frac{Tl}{GJ} = \frac{250(12)(13)}{11.5(10^6)(0.09817)}$$

$$\theta_s = 0.0345 \text{ rad } \textit{Ans.}$$

Simplified is $0.0345/0.0260 = 1.33$ times greater *Ans.*

$$y_D = \frac{F_y l_{OC}^3}{3EI_{AB}} + \theta_s(l_{CD}) + \frac{F_y l_{CD}^3}{3EI_{CD}} = \frac{250(13^3)}{3(30)10^6(0.04909)} + 0.0345(12) + \frac{250(12^3)}{3(30)10^6(0.01553)} \\ y_D = 0.847 \text{ in } \textit{Ans.}$$

4-44 Reverse the deflection equation of beam 7 of Table A-9. Using units in lbf, inches

$$y = -\frac{wx}{24EI}(2lx^2 - x^3 - l^3) = -\frac{(3000/12)x}{24(30)10^6(485)}\{2(25)x^2 - x^3 - [25(12)]^3\}$$

$$= 7.159(10^{-10})x[27(10^6) - 600x^2 + x^3] \quad \text{Ans.}$$

The maximum height occurs at $x = 25(12)/2 = 150$ in

$$y_{\max} = 7.159(10^{-10})150[27(10^6) - 600(150^2) + 150^3] = 1.812 \text{ in} \quad \text{Ans.}$$

4-45 From Table A-9, beam 6,

$$y_L = \frac{Fbx}{6EI}(x^2 + b^2 - l^2)$$

$$y_L = \frac{Fb}{6EI}(x^3 + b^2x - l^2x)$$

$$\frac{dy_L}{dx} = \frac{Fb}{6EI}(3x^2 + b^2 - l^2)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb(b^2 - l^2)}{6EI}$$

Let $\xi = \left.\frac{dy_L}{dx}\right|_{x=0}$ and set $I = \frac{\pi d_L^4}{64}$. Thus,

$$d_L = \left|\frac{32Fb(b^2 - l^2)}{3\pi E l \xi}\right|^{1/4} \quad \text{Ans.}$$

For the other end view, observe beam 6 of Table A-9 from the back of the page, noting that a and b interchange as do x and $-x$

$$d_R = \left|\frac{32Fa(l^2 - a^2)}{3\pi E l \xi}\right|^{1/4} \quad \text{Ans.}$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R .

4-46 The maximum slope will occur at the left bearing. Incorporating a design factor into the solution for d_L of Prob. 4-45,

$$d = \left[\frac{32nFb(l^2 - b^2)}{3\pi El\xi} \right]^{1/4}$$

$$d = \sqrt[4]{\frac{32(1.28)(3000)(200)(300^2 - 200^2)}{3\pi(207)10^3(300)(0.001)}}$$

$$d = 38.1 \text{ mm} \quad \text{Ans.}$$

$$I = \frac{\pi(38.1^4)}{64} = 103.4(10^3) \text{ mm}^4$$

From Table A-9, beam 6, the maximum deflection will occur in BC where $dy_{BC}/dx = 0$

$$\frac{d}{dx} \left[\frac{Fa(l-x)}{6EI} (x^2 + a^2 - 2lx) \right] = 0 \Rightarrow 3x^2 - 6lx + (a^2 + 2l^2) = 0$$

$$3x^2 - 6(300)x + [100^2 + 2(300^2)] = 0 \Rightarrow x^2 - 600x + 63333 = 0$$

$$x = \frac{1}{2} \left[600 \pm \sqrt{600^2 - 4(1)63333} \right] = 463.3, 136.7 \text{ mm}$$

$x = 136.7 \text{ mm}$ is acceptable.

$$y_{\max} = \left[\frac{Fa(l-x)}{6EI} (x^2 + a^2 - 2lx) \right]_{x=136.7 \text{ mm}}$$

$$= \frac{3(10^3)100(300-136.7)}{6(207)10^3(103.4)10^3(300)} [136.7^2 + 100^2 - 2(300)136.7] = -0.0678 \text{ mm} \quad \text{Ans.}$$

4-47 $I = \pi(1.25^4)/64 = 0.1198 \text{ in}^4$. From Table A-9, beam 6

$$\delta = \sqrt{\left[\frac{F_1 a_1 (l-x)}{6EI} (x^2 + a_1^2 - 2lx) \right]^2 + \left[\frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) \right]^2}$$

$$= \left\{ \left[\frac{150(5)(20-8)}{6(30)10^6(0.1198)(20)} (8^2 + 5^2 - 2(20)(8)) \right]^2 \right.$$

$$\left. + \left[\frac{250(10)(8)}{6(30)10^6(0.1198)(20)} (8^2 + 10^2 - 20^2) \right]^2 \right\}^{1/2}$$

$$= 0.0120 \text{ in} \quad \text{Ans.}$$

4-48 $I = \pi(1.25^4)/64 = 0.1198 \text{ in}^4$. For both forces use beam 6 of Table A-9.

For $F_1 = 150 \text{ lbf}$:

$$0 \leq x \leq 5$$

$$y = \frac{F_1 b_1 x}{6EI} (x^2 + b_1^2 - l^2) = \frac{150(15)x}{6(30)10^6(0.1198)(20)} (x^2 + 15^2 - 20^2)$$

$$= 5.217(10^{-6})x(x^2 - 175) \quad (1)$$

$$5 \leq x \leq 20$$

$$y = \frac{F_1 a_1 (l-x)}{6EI} (x^2 + a_1^2 - 2lx) = \frac{150(5)(20-x)}{6(30)10^6(0.1198)(20)} [x^2 + 5^2 - 2(20)x]$$

$$= 1.739(10^{-6})(20-x)(x^2 - 40x + 25) \quad (2)$$

For $F_2 = 250 \text{ lbf}$:

$$0 \leq x \leq 10$$

$$z = \frac{F_2 b_2 x}{6EI} (x^2 + b_2^2 - l^2) = \frac{250(10)x}{6(30)10^6(0.1198)(20)} (x^2 + 10^2 - 20^2)$$

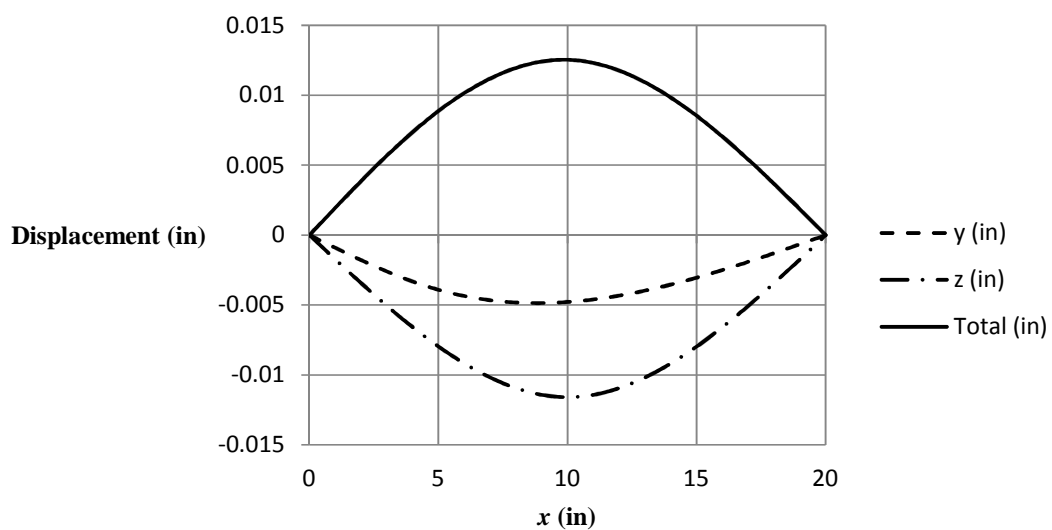
$$= 5.797(10^{-6})x(x^2 - 300) \quad (3)$$

$$10 \leq x \leq 20$$

$$z = \frac{F_2 a_2 (l-x)}{6EI} (x^2 + a_2^2 - 2lx) = \frac{250(10)(20-x)}{6(30)10^6(0.1198)(20)} [x^2 + 10^2 - 2(20)x]$$

$$= 5.797(10^{-6})(20-x)(x^2 - 40x + 100) \quad (4)$$

Plot Eqs. (1) to (4) for each 0.1 in using a spreadsheet. There are 201 data points, too numerous to tabulate here but the plot is shown below, where the maximum deflection of $\delta = 0.01255 \text{ in}$ occurs at $x = 9.9 \text{ in}$. *Ans.*



- 4-49** The larger slope will occur at the left end.
From Table A-9, beam 8

$$y_{AB} = \frac{M_B x}{6EI} (x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{M_B}{6EI} (3x^2 + 3a^2 - 6al + 2l^2)$$

With $I = \pi d^4/64$, the slope at the left bearing is

$$\left. \frac{dy_{AB}}{dx} \right|_{x=0} = \theta_A = \frac{M_B}{6E(\pi d^4/64)l} (3a^2 - 6al + 2l^2)$$

Solving for d

$$d = \sqrt[4]{\frac{32M_B}{3\pi E \theta_A l} (3a^2 - 6al + 2l^2)} = \sqrt[4]{\frac{32(1000)}{3\pi(30)10^6(0.002)(10)} [3(4^2) - 6(4)(10) + 2(10^2)]}$$

$$= 0.461 \text{ in} \quad \text{Ans.}$$

- 4-50** From Table A-5, $E = 10.4 \text{ Mpsi}$

$$\Sigma M_O = 0 = 18 F_{BC} - 6(100) \Rightarrow F_{BC} = 33.33 \text{ lbf}$$

The cross sectional area of rod BC is $A = \pi(0.5^2)/4 = 0.1963 \text{ in}^2$.

The deflection at point B will be equal to the elongation of the rod BC .

$$y_B = \left(\frac{FL}{AE} \right)_{BC} = \frac{33.33(12)}{(0.1963)30(10^6)} = 6.79(10^{-5}) \text{ in} \quad \text{Ans.}$$

- 4-51** $\Sigma M_O = 0 = 6 F_{AC} - 11(100) \Rightarrow F_{AC} = 183.3 \text{ lbf}$

The deflection at point A in the negative y direction is equal to the elongation of the rod AC . From Table A-5, $E_s = 30 \text{ Mpsi}$.

$$y_A = - \left(\frac{FL}{AE} \right)_{AC} = - \frac{183.3(12)}{[\pi(0.5^2)/4]30(10^6)} = -3.735(10^{-4}) \text{ in}$$

By similar triangles the deflection at B due to the elongation of the rod AC is

$$\frac{y_A}{6} = \frac{y_{B1}}{18} \Rightarrow y_{B1} = 3y_A = 3(-3.735)10^{-4} = -0.00112 \text{ in}$$

From Table A-5, $E_a = 10.4 \text{ Mpsi}$

The bar can then be treated as a simply supported beam with an overhang AB . From Table A-9, beam 10,

$$\begin{aligned}
y_{B2} &= (\overline{BD}) \left(\frac{dy_{BC}}{dx} \Big|_{x=l+a} \right) - \frac{Fa^2}{3EI} (l+a) = 7 \left\{ \frac{d}{dx} \left(\frac{F(x-l)}{6EI} [(x-l)^2 - a(3x-l)] \right) \right\}_{x=l+a} - \frac{Fa^2}{3EI} (l+a) \\
&= 7 \frac{F}{6EI} [3(x-l)^2 - 3a(x-l) - a(3x-l)]_{x=l+a} - \frac{Fa^2}{3EI} (l+a) = -\frac{7Fa}{6EI} (2l+3a) - \frac{Fa^2}{3EI} (l+a) \\
&= -\frac{7(100)5}{6(10.4)10^6 (0.25(2^3)/12)} [2(6)+3(5)] - \frac{100(5^2)}{3(10.4)10^6 (0.25(2^3)/12)} (6+5) \\
&= -0.01438 \text{ in}
\end{aligned}$$

$$y_B = y_{B1} + y_{B2} = -0.00112 - 0.01438 = -0.0155 \text{ in} \quad \text{Ans.}$$

4-52 From Table A-5, $E = 207 \text{ GPa}$, and $G = 79.3 \text{ GPa}$.

$$\begin{aligned}
|y_B| &= \left(\frac{Tl}{GJ} \right)_{OC} l_{AB} + \left(\frac{Tl}{GJ} \right)_{AC} l_{AB} + \frac{Fl_{AB}^3}{3EI_{AB}} = \frac{Fl_{OC}l_{AB}^2}{G(\pi d_{OC}^4/32)} + \frac{Fl_{AC}l_{AB}^2}{G(\pi d_{AC}^4/32)} + \frac{Fl_{AB}^3}{3E(\pi d_3^4/64)} \\
&= \frac{32Fl_{AB}^2}{\pi} \left[\frac{l_{OC}}{Gd_{OC}^4} + \frac{l_{AC}}{Gd_{AC}^4} + \frac{2l_{AB}}{3Ed_{AB}^4} \right]
\end{aligned}$$

The spring rate is $k = F/|y_B|$. Thus

$$\begin{aligned}
k &= \left\{ \frac{32l_{AB}^2}{\pi} \left[\frac{l_{OC}}{Gd_{OC}^4} + \frac{l_{AC}}{Gd_{AC}^4} + \frac{2l_{AB}}{3Ed_{AB}^4} \right] \right\}^{-1} \\
&= \left\{ \frac{32(200^2)}{\pi} \left[\frac{200}{79.3(10^3)18^4} + \frac{200}{79.3(10^3)12^4} + \frac{2(200)}{3(207)10^3(8^4)} \right] \right\}^{-1} \\
&= 8.10 \text{ N/mm} \quad \text{Ans.}
\end{aligned}$$

4-53 For the beam deflection, use beam 5 of Table A-9.

$$\begin{aligned}
R_1 &= R_2 = \frac{F}{2} \\
\delta_1 &= \frac{F}{2k_1}, \text{ and } \delta_2 = \frac{F}{2k_2} \\
y_{AB} &= -\delta_1 + \frac{\delta_1 - \delta_2}{l} x + \frac{Fx}{48EI} (4x^2 - 3l^3) \\
y_{AB} &= F \left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1 k_2 l} x + \frac{x}{48EI} (4x^2 - 3l^3) \right] \quad \text{Ans.}
\end{aligned}$$

For BC , since Table A-9 does not have an equation (because of symmetry) an equation will need to be developed as the problem is no longer symmetric. This can be done easily using beam 6 of Table A-9 with $a = l/2$

$$\begin{aligned} y_{BC} &= \frac{-F}{2k_1} + \frac{Fk_2 - Fk_1}{2k_1k_2l}x + \frac{F(l/2)(l-x)}{EI} \left(x^2 + \frac{l^2}{4} - 2lx \right) \\ &= F \left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{(l-x)}{48EI} (4x^2 + l^2 - 8lx) \right] \quad \text{Ans.} \end{aligned}$$

4-54

$$\begin{aligned} R_1 &= \frac{Fa}{l}, \text{ and } R_2 = \frac{F}{l}(l+a) \\ \delta_1 &= \frac{Fa}{lk_1}, \text{ and } \delta_2 = \frac{F}{lk_2}(l+a) \\ y_{AB} &= -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fax}{6EI}(l^2 - x^2) \\ y_{AB} &= F \left\{ -\frac{a}{k_1l} + \frac{x}{k_1k_2l^2} [k_2a - k_1(l+a)] + \frac{ax}{6EI}(l^2 - x^2) \right\} \quad \text{Ans.} \\ y_{BC} &= -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{F(x-l)}{6EI} [(x-l)^2 - a(3x-l)] \\ y_{BC} &= F \left\{ -\frac{a}{k_1l} + \frac{x}{k_1k_2l^2} [k_2a - k_1(l+a)] + \frac{(x-l)}{6EI} [(x-l)^2 - a(3x-l)] \right\} \quad \text{Ans.} \end{aligned}$$

4-55 Let the load be at $x \geq l/2$. The maximum deflection will be in Section AB (Table A-9, beam 6)

$$\begin{aligned} y_{AB} &= \frac{Fbx}{6EI} (x^2 + b^2 - l^2) \\ \frac{dy_{AB}}{dx} &= \frac{Fb}{6EI} (3x^2 + b^2 - l^2) = 0 \quad \Rightarrow \quad 3x^2 + b^2 - l^2 = 0 \\ x &= \sqrt{\frac{l^2 - b^2}{3}}, \quad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \quad \text{Ans.} \end{aligned}$$

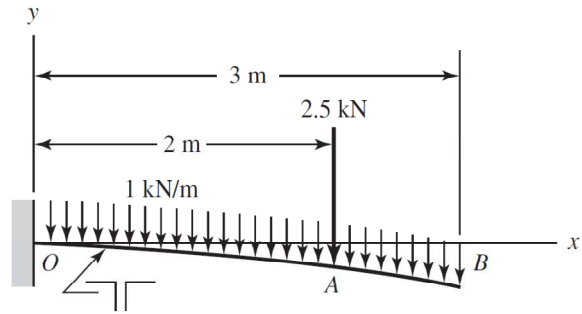
$$\text{For } x \leq l/2, \quad x_{\min} = l - 0.577l = 0.423l \quad \text{Ans.}$$

4-56

$$M_O = 1(3000)(1500) + 2500(2000) \\ = 9.5(10^6) \text{ N}\cdot\text{mm}$$

$$R_O = 1(3000) + 2500 = 5500 \text{ N}$$

$$\text{From Prob. 4-10, } I = 4.14(10^6) \text{ mm}^4$$



$$M = -9.5(10^6) + 5500x - \frac{x^2}{2} - 2500\langle x - 2000 \rangle^1$$

$$EI \frac{dy}{dx} = -9.5(10^6)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000 \rangle^2 + C_1$$

$$\frac{dy}{dx} = 0 \text{ at } x = 0 \quad \therefore C_1 = 0$$

$$EI \frac{dy}{dx} = -9.5(10^6)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000 \rangle^2$$

$$EIy = -4.75(10^6)x^2 + 916.67x^3 - \frac{x^4}{24} - 416.67\langle x - 2000 \rangle^3 + C_2$$

$$y = 0 \text{ at } x = 0 \quad \therefore C_2 = 0, \text{ and therefore}$$

$$y = -\frac{1}{24EI} \left[114(10^6)x^2 - 22(10^3)x^3 + x^4 + 10(10^3)\langle x - 2000 \rangle^3 \right]$$

$$y_B = -\frac{1}{24(207)10^3(4.14)10^6} \left[114(10^6)3000^2 - 22(10^3)3000^3 \right. \\ \left. + 3000^4 + 10(10^3)(3000 - 2000)^3 \right]$$

$$= -25.4 \text{ mm} \quad \text{Ans.}$$

$M_O = 9.5(10^6) \text{ N}\cdot\text{m}$. The maximum stress is compressive at the bottom of the beam where $y = 29.0 - 100 = -71 \text{ mm}$

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5(10^6)(-71)}{4.14(10^6)} = -163(10^6) \text{ Pa} = -163 \text{ MPa} \quad \text{Ans.}$$

The solutions are the same as Prob. 4-10.

4-57 See Prob. 4-11 for reactions: $R_O = 465 \text{ lbf}$ and $R_C = 285 \text{ lbf}$. Using lbf and inch units

$$M = 465x - 450\langle x - 72 \rangle^1 - 300\langle x - 120 \rangle^1$$

$$EI \frac{dy}{dx} = 232.5x^2 - 225\langle x-72 \rangle^2 - 150\langle x-120 \rangle^2 + C_1$$

$$EIy = 77.5x^3 - 75\langle x-72 \rangle^3 - 50\langle x-120 \rangle^3 - C_1x$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = 0 \text{ at } x = 240 \text{ in}$$

$$0 = 77.5(240^3) - 75(240-72)^3 - 50(240-120)^3 + C_1x \Rightarrow C_1 = -2.622(10^6) \text{ lbf}\cdot\text{in}^2$$

and,

$$EIy = 77.5x^3 - 75\langle x-72 \rangle^3 - 50\langle x-120 \rangle^3 - 2.622(10^6)x$$

Substituting $y = -0.5$ in at $x = 120$ in gives

$$30(10^6) I (-0.5) = 77.5 (120^3) - 75(120-72)^3 - 50(120-120)^3 - 2.622(10^6)(120)$$

$$I = 12.60 \text{ in}^4$$

Select two 5 in \times 6.7 lbf/ft channels; from Table A-7, $I = 2(7.49) = 14.98 \text{ in}^4$

$$y_{\text{midspan}} = \frac{12.60}{14.98} \left(-\frac{1}{2} \right) = -0.421 \text{ in} \quad \text{Ans.}$$

The maximum moment occurs at $x = 120$ in where $M_{\text{max}} = 34.2(10^3) \text{ lbf}\cdot\text{in}$

$$\sigma_{\text{max}} = \frac{Mc}{I} = \frac{34.2(10^3)(2.5)}{14.98} = 5710 \text{ psi} \quad \text{O.K.}$$

The solutions are the same as Prob. 4-17.

4-58 $I = \pi(1.5^4)/64 = 0.2485 \text{ in}^4$, and $w = 150/12 = 12.5 \text{ lbf/in}$.

$$R_o = \frac{1}{2}(12.5)39 + \frac{24}{39}(340) = 453.0 \text{ lbf}$$

$$M = 453.0x - \frac{12.5}{2}x^2 - 340\langle x-15 \rangle^1$$

$$EI \frac{dy}{dx} = 226.5x^2 - \frac{12.5}{6}x^3 - 170\langle x-15 \rangle^2 + C_1$$

$$EIy = 75.5x^3 - 0.5208x^4 - 56.67\langle x-15 \rangle^3 + C_1x + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = 0 \text{ at } x = 39 \text{ in} \Rightarrow C_1 = -6.385(10^4) \text{ lbf}\cdot\text{in}^2 \text{ Thus,}$$

$$y = \frac{1}{EI} \left[75.5x^3 - 0.5208x^4 - 56.67\langle x-15 \rangle^3 - 6.385(10^4)x \right]$$

Evaluating at $x = 15$ in,

$$y_A = \frac{1}{30(10^6)(0.2485)} \left[75.5(15^3) - 0.5208(15^4) - 56.67(15-15)^3 - 6.385(10^4)(15) \right]$$

$$= -0.0978 \text{ in} \quad \text{Ans.}$$

$$y_{\text{midspan}} = \frac{1}{30(10^6)(0.2485)} \left[75.5(19.5^3) - 0.5208(19.5^4) - 56.67(19.5-15)^3 - 6.385(10^4)(19.5) \right]$$

$$= -0.1027 \text{ in} \quad \text{Ans.}$$

5 % difference *Ans.*

The solutions are the same as Prob. 4-12.

4-59 $I = 0.05 \text{ in}^4$, $R_A = \frac{3(14)100}{10} = 420 \text{ lbf} \uparrow$ and $R_B = \frac{7(14)100}{10} = 980 \text{ lbf} \uparrow$

$$M = 420x - 50x^2 + 980\langle x - 10 \rangle^1$$

$$EI \frac{dy}{dx} = 210x^2 - 16.667x^3 + 490\langle x - 10 \rangle^2 + C_1$$

$$EIy = 70x^3 - 4.167x^4 + 163.3\langle x - 10 \rangle^3 + C_1x + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = 0 \text{ at } x = 10 \text{ in} \Rightarrow C_1 = -2833 \text{ lbf} \cdot \text{in}^2. \text{ Thus,}$$

$$y = \frac{1}{30(10^6)0.05} \left[70x^3 - 4.167x^4 + 163.3\langle x - 10 \rangle^3 - 2833x \right]$$

$$= 6.667(10^{-7}) \left[70x^3 - 4.167x^4 + 163.3\langle x - 10 \rangle^3 - 2833x \right] \quad \text{Ans.}$$

The tabular results and plot are exactly the same as Prob. 4-21.

4-60 $R_A = R_B = 400 \text{ N}$, and $I = 6(32^3)/12 = 16\,384 \text{ mm}^4$.

First half of beam,

$$M = -400x + 400\langle x - 300 \rangle^1$$

$$EI \frac{dy}{dx} = -200x^2 + 200\langle x - 300 \rangle^2 + C_1$$

$$\text{From symmetry, } dy/dx = 0 \text{ at } x = 550 \text{ mm} \Rightarrow 0 = -200(550^2) + 200(550 - 300)^2 + C_1$$

$$\Rightarrow C_1 = 48(10^6) \text{ N} \cdot \text{mm}^2$$

$$EIy = -66.67x^3 + 66.67\langle x - 300 \rangle^3 + 48(10^6)x + C_2$$

$$y = 0 \text{ at } x = 300 \text{ mm} \Rightarrow C_2 = -12.60(10^9) \text{ N}\cdot\text{mm}^3.$$

The term $(EI)^{-1} = [207(10^3)16\,384]^{-1} = 2.949 (10^{-10})$ Thus

$$y = 2.949 (10^{-10}) [-66.67 x^3 + 66.67 \langle x - 300 \rangle^3 + 48(10^6) x - 12.60(10^9)]$$

$$y_O = -3.72 \text{ mm} \quad \text{Ans.}$$

$$y|_{x=550 \text{ mm}} = 2.949 (10^{-10}) [-66.67 (550)^3 + 66.67 (550 - 300)^3 + 48(10^6) 550 - 12.60(10^9)] = 1.11 \text{ mm} \quad \text{Ans.}$$

The solutions are the same as Prob. 4-13.

4-61

$$\sum M_B = 0 = R_1 l + Fa - M_A \Rightarrow R_1 = \frac{1}{l} (M_A - Fa)$$

$$\sum M_A = 0 = M_A + R_2 l - F(l + a) \Rightarrow R_2 = \frac{1}{l} (Fl + Fa - M_A)$$

$$M = R_1 x - M_A + R_2 \langle x - l \rangle^1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_1 x^2 - M_A x + \frac{1}{2} R_2 \langle x - l \rangle^2 + C_1$$

$$EI y = \frac{1}{6} R_1 x^3 - \frac{1}{2} M_A x^2 + \frac{1}{6} R_2 \langle x - l \rangle^3 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = 0 \text{ at } x = l \Rightarrow C_1 = -\frac{1}{6} R_1 l^2 + \frac{1}{2} M_A l. \text{ Thus,}$$

$$EI y = \frac{1}{6} R_1 x^3 - \frac{1}{2} M_A x^2 + \frac{1}{6} R_2 \langle x - l \rangle^3 + \left(-\frac{1}{6} R_1 l^2 + \frac{1}{2} M_A l \right) x$$

$$y = \frac{1}{6EI} \left[(M_A - Fa) x^3 - 3M_A x^2 l + (Fl + Fa - M_A) \langle x - l \rangle^3 + (Fal^2 + 2M_A l^2) x \right] \quad \text{Ans.}$$

In regions,

$$\begin{aligned} y_{AB} &= \frac{1}{6EI} \left[(M_A - Fa) x^3 - 3M_A x^2 l + (Fal^2 + 2M_A l^2) x \right] \\ &= \frac{x}{6EI} \left[M_A (x^2 - 3lx + 2l^2) + Fa(l^2 - x^2) \right] \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned}
y_{BC} &= \frac{1}{6EI} \left[(M_A - Fa)x^3 - 3M_A x^2 l + (Fl + Fa - M_A)(x-l)^3 + (Fal^2 + 2M_A l^2)x \right] \\
&= \frac{1}{6EI} \left\{ M_A \left[x^3 - 3x^2 l - (x-l)^3 + 2xl^2 \right] + F \left[-ax^3 + (l+a)(x-l)^3 + axl^2 \right] \right\} \\
&= \frac{1}{6EI} \left\{ -M_A (x-l)l^2 + Fl(x-l) \left[(x-l)^2 - a(3x-l) \right] \right\} \\
&= \frac{(x-l)}{6EI} \left\{ -M_A l + F \left[(x-l)^2 - a(3x-l) \right] \right\} \quad \text{Ans.}
\end{aligned}$$

The solutions reduce to the same as Prob. 4-17.

$$\mathbf{4-62} \quad \sum M_D = 0 = R_1 l - w(b-a) \left[l - b + \frac{1}{2}(b-a) \right] \Rightarrow R_1 = \frac{w(b-a)}{2l} (2l - b - a)$$

$$M = R_1 x - \frac{w}{2} \langle x-a \rangle^2 + \frac{w}{2} \langle x-b \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_1 x^2 - \frac{w}{6} \langle x-a \rangle^3 + \frac{w}{6} \langle x-b \rangle^3 + C_1$$

$$EI y = \frac{1}{6} R_1 x^3 - \frac{w}{24} \langle x-a \rangle^4 + \frac{w}{24} \langle x-b \rangle^4 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = 0 \text{ at } x = l$$

$$C_1 = -\frac{1}{l} \left[\frac{1}{6} R_1 l^3 - \frac{w}{24} (l-a)^4 + \frac{w}{24} (l-b)^4 \right]$$

$$\begin{aligned}
y &= \frac{1}{EI} \left\{ \frac{1}{6} \frac{w(b-a)}{2l} (2l-b-a)x^3 - \frac{w}{24} \langle x-a \rangle^4 + \frac{w}{24} \langle x-b \rangle^4 \right. \\
&\quad \left. - x \left[\frac{1}{6} \frac{w(b-a)}{2l} (2l-b-a)l^3 - \frac{w}{24} (l-a)^4 + \frac{w}{24} (l-b)^4 \right] \right\} \\
&= \frac{w}{24EI} \left\{ 2(b-a)(2l-b-a)x^3 - l \langle x-a \rangle^4 + l \langle x-b \rangle^4 \right. \\
&\quad \left. - x \left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4 \right] \right\} \quad \text{Ans.}
\end{aligned}$$

The above answer is sufficient. In regions,

$$y_{AB} = \frac{w}{24EI} \left\{ 2(b-a)(2l-b-a)x^3 - x \left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4 \right] \right\}$$

$$= \frac{wx}{24EI} \left[2(b-a)(2l-b-a)x^2 - 2(b-a)(2l-b-a)l^2 + (l-a)^4 - (l-b)^4 \right]$$

$$y_{BC} = \frac{w}{24EI} \left\{ 2(b-a)(2l-b-a)x^3 - l(x-a)^4 \right. \\ \left. - x \left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4 \right] \right\}$$

$$y_{CD} = \frac{w}{24EI} \left\{ 2(b-a)(2l-b-a)x^3 - l(x-a)^4 + l(x-b)^4 \right. \\ \left. - x \left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4 \right] \right\}$$

These equations can be shown to be equivalent to the results found in Prob. 4-19.

4-63 $I_1 = \pi(1.375^4)/64 = 0.1755 \text{ in}^4$, $I_2 = \pi(1.75^4)/64 = 0.4604 \text{ in}^4$,

$$R_1 = 0.5(180)(10) = 900 \text{ lbf}$$

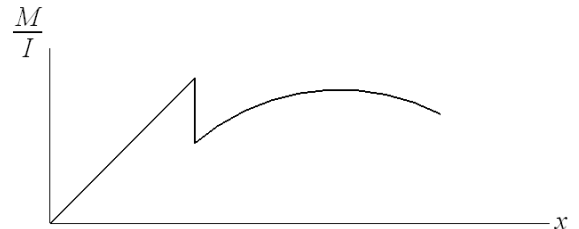
Since the loading and geometry are symmetric, we will only write the equations for the first half of the beam

For $0 \leq x \leq 8 \text{ in}$ $M = 900x - 90\langle x-3 \rangle^2$

At $x = 3$, $M = 2700 \text{ lbf}\cdot\text{in}$

Writing an equation for M/I , as seen in the figure, the magnitude and slope reduce since $I_2 > I_1$.

To reduce the magnitude at $x = 3 \text{ in}$, we add the term, $-2700(1/I_1 - 1/I_2)\langle x-3 \rangle^0$. The slope of 900 at $x = 3 \text{ in}$ is also reduced. We account for this with a ramp function, $\langle x-3 \rangle^1$. Thus,



$$\frac{M}{I} = \frac{900x}{I_1} - 2700 \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \langle x-3 \rangle^0 - 900 \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \langle x-3 \rangle^1 - \frac{90}{I_2} \langle x-3 \rangle^2$$

$$= 5128x - 9520 \langle x-3 \rangle^0 - 3173 \langle x-3 \rangle^1 - 195.5 \langle x-3 \rangle^2$$

$$E \frac{dy}{dx} = 2564x^2 - 9520 \langle x-3 \rangle^1 - 1587 \langle x-3 \rangle^2 - 65.17 \langle x-3 \rangle^3 + C_1$$

Boundary Condition: $\frac{dy}{dx} = 0$ at $x = 8 \text{ in}$

$$0 = 2564(8)^2 - 9520(8-3) - 1587(8-3)^2 - 65.17(8-3)^3 + C_1 \Rightarrow$$

$$C_1 = -68.67(10^3) \text{ lbf/in}^2$$

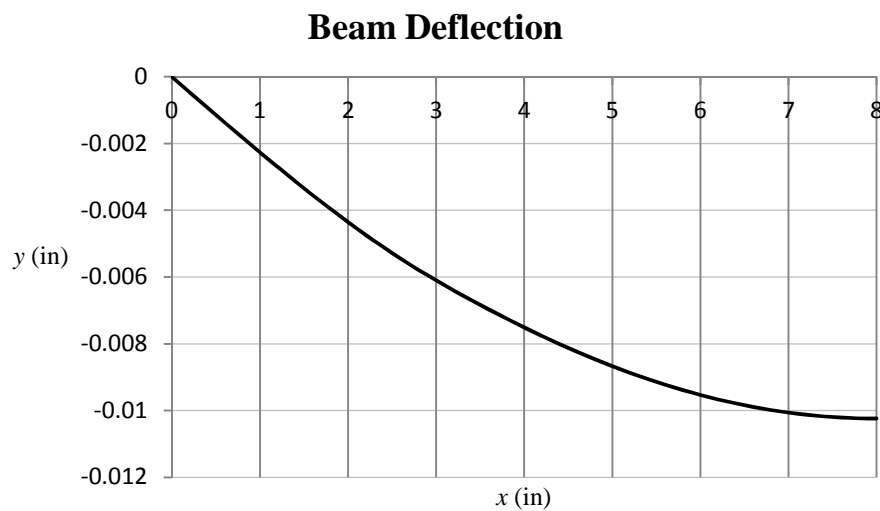
$$Ey = 854.7x^3 - 4760\langle x-3 \rangle^2 - 529\langle x-3 \rangle^3 - 16.29\langle x-3 \rangle^4 - 68.67(10^3)x + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

Thus, for $0 \leq x \leq 8 \text{ in}$

$$y = \frac{1}{30(10^6)} \left[854.7x^3 - 4760\langle x-3 \rangle^2 - 529\langle x-3 \rangle^3 - 16.29\langle x-3 \rangle^4 - 68.7(10^3)x \right] \quad \text{Ans.}$$

Using a spreadsheet, the following graph represents the deflection equation found above



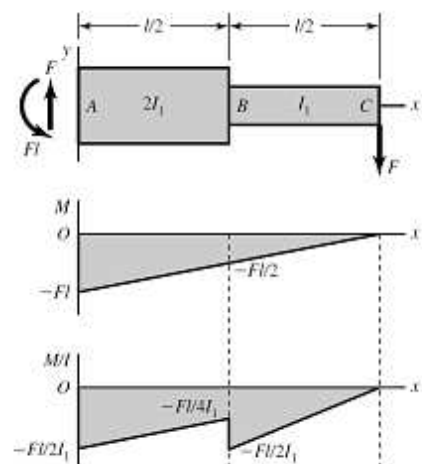
The maximum is $y_{\max} = -0.0102 \text{ in}$ at $x = 8 \text{ in}$ *Ans.*

- 4-64** The force and moment reactions at the left support are F and Fl respectively. The bending moment equation is

$$M = Fx - Fl$$

Plots for M and M/I are shown.

M/I can be expressed using singularity functions



$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2} \right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2} \right\rangle^1$$

where the step down and increase in slope at $x = l/2$ are given by the last two terms. Integrate

$$E \frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2} \right\rangle^1 + \frac{F}{4I_1}\left\langle x - \frac{l}{2} \right\rangle^2 + C_1$$

$$dy/dx = 0 \text{ at } x = 0 \Rightarrow C_1 = 0$$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x - \frac{l}{2} \right\rangle^2 + \frac{F}{12I_1}\left\langle x - \frac{l}{2} \right\rangle^3 + C_2$$

$$y = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$y = \frac{F}{24EI_1} \left(2x^3 - 6lx^2 - 3l\left\langle x - \frac{l}{2} \right\rangle^2 + 2\left\langle x - \frac{l}{2} \right\rangle^3 \right)$$

$$y|_{x=l/2} = \frac{F}{24EI_1} \left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0) \right] = -\frac{5Fl^3}{96EI_1} \quad \text{Ans.}$$

$$y|_{x=l} = \frac{F}{24EI_1} \left[2(l)^3 - 6l(l^2) - 3l\left(l - \frac{l}{2}\right)^2 + 2\left(x - \frac{l}{2}\right)^3 \right] = -\frac{3Fl^3}{16EI_1} \quad \text{Ans.}$$

The answers are identical to Ex. 4-10.

4-65 Place a dummy force, Q , at the center. The reaction, $R_1 = wl/2 + Q/2$

$$M = \left(\frac{wl}{2} + \frac{Q}{2} \right) x - \frac{wx^2}{2} \quad \frac{\partial M}{\partial Q} = \frac{x}{2}$$

Integrating for half the beam and doubling the results

$$y_{\max} = \left(2 \frac{1}{EI} \int_0^{l/2} M \left(\frac{\partial M}{\partial Q} \right) dx \right)_{Q=0} = \frac{2}{EI} \int_0^{l/2} \left[\left(\frac{wl}{2} \right) x - \frac{wx^2}{2} \right] \left(\frac{x}{2} \right) dx$$

Note, after differentiating with respect to Q , it can be set to zero

$$y_{\max} = \frac{w}{2EI} \int_0^{l/2} x^2 (l-x) dx = \frac{w}{2EI} \left(\frac{x^3 l}{3} - \frac{x^4}{4} \right) \bigg|_0^{l/2} = \frac{5w}{384EI} \quad \text{Ans.}$$

4-66 Place a fictitious force Q pointing downwards at the end. Use the variable \bar{x} originating at the free end and positive to the left

$$M = -Qx - \frac{wx^2}{2} \quad \frac{\partial M}{\partial Q} = -x$$

$$y_{\max} = \left[\frac{1}{EI} \int_0^l M \left(\frac{\partial M}{\partial Q} \right) dx \right]_{Q=0} = \frac{1}{EI} \int_0^l \left(-\frac{wx^2}{2} \right) (-x) dx = \frac{w}{2EI} \int_0^l x^3 dx$$

$$= \frac{wl^4}{8EI} \quad \text{Ans.}$$

4-67 From Table A-7, $I_{1-1} = 1.85 \text{ in}^4$. Thus, $I = 2(1.85) = 3.70 \text{ in}^4$

First treat the end force as a variable, F .
Adding weight of channels of $2(5)/12 = 0.833 \text{ lbf/in}$. Using the variable \bar{x} as shown in the figure

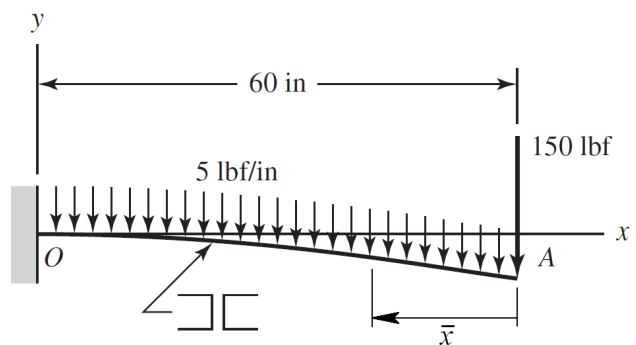
$$M = -F\bar{x} - \frac{5.833}{2}\bar{x}^2 = -F\bar{x} - 2.917\bar{x}^2$$

$$\frac{\partial M}{\partial F} = -\bar{x}$$

$$\delta_A = \frac{1}{EI} \int_0^{60} M \frac{\partial M}{\partial F} d\bar{x} = \frac{1}{EI} \int_0^{60} (F\bar{x} + 2.917\bar{x}^2)(\bar{x}) d\bar{x} = \frac{1}{EI} (F\bar{x}^3/3 + 2.917\bar{x}^4/4) \Big|_0^{60}$$

$$= \frac{(150)(60^3)/3 + (2.917)(60^4)/4}{30(10^6)(3.70)} = 0.182 \text{ in in the direction of the 150 lbf force}$$

$$\therefore y_A = -0.182 \text{ in} \quad \text{Ans.}$$



4-68 The energy includes torsion in AC, torsion in CO, and bending in AB.

Neglecting transverse shear in AB

$$M = Fx, \quad \frac{\partial M}{\partial F} = x$$

In AC and CO,

$$T = Fl_{AB}, \quad \frac{\partial T}{\partial F} = l_{AB}$$

The total energy is

$$U = \left(\frac{T^2 l}{2GJ} \right)_{AC} + \left(\frac{T^2 l}{2GJ} \right)_{CO} + \int_0^{l_{AB}} \frac{M^2}{2EI_{AB}} dx$$

The deflection at the tip is

$$\begin{aligned}\delta &= \frac{\partial U}{\partial F} = \frac{Tl_{AC}}{GJ_{AC}} \frac{\partial T}{\partial F} + \frac{Tl_{CO}}{GJ_{CO}} \frac{\partial T}{\partial F} + \int_0^{l_{AB}} \frac{M}{EI_3} \frac{\partial M}{\partial F} dx = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{1}{EI_{AB}} \int_0^{l_{AB}} Fx^2 dx \\ \delta &= \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{Fl_{AB}^3}{3EI_{AB}} = \frac{Fl_{AC}l_{AB}^2}{G(\pi d_{AC}^4/32)} + \frac{Fl_{CO}l_{AB}^2}{G(\pi d_{CO}^4/32)} + \frac{Fl_{AB}^3}{3E(\pi d_{AB}^4/64)} \\ &= \frac{32Fl_{AB}^2}{\pi} \left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4} \right) \\ k &= \frac{F}{\delta} = \frac{\pi}{32l_{AB}^2} \left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4} \right)^{-1} \\ &= \frac{\pi}{32(200)^2} \left(\frac{200}{79.3(10^3)18^4} + \frac{200}{79.3(10^3)12^4} + \frac{2(200)}{3(207)10^3(8^4)} \right)^{-1} = 8.10 \text{ N/mm} \quad \text{Ans.}\end{aligned}$$

4-69 $I_1 = \pi(1.375^4)/64 = 0.1755 \text{ in}^4$, $I_2 = \pi(1.75^4)/64 = 0.4604 \text{ in}^4$

Place a fictitious force Q pointing downwards at the midspan of the beam, $x = 8 \text{ in}$

$$R_1 = \frac{1}{2}(10)180 + \frac{1}{2}Q = 900 + 0.5Q$$

For $0 \leq x \leq 3 \text{ in}$ $M = (900 + 0.5Q)x$ $\frac{\partial M}{\partial Q} = 0.5x$

For $3 \leq x \leq 13 \text{ in}$ $M = (900 + 0.5Q)x - 90(x-3)^2$ $\frac{\partial M}{\partial Q} = 0.5x$

By symmetry it is equivalent to use twice the integral from 0 to 8

$$\begin{aligned}\delta &= \left(2 \int_0^8 \frac{M}{EI} \frac{\partial M}{\partial Q} dx \right)_{Q=0} = \frac{1}{EI_1} \int_0^3 900x^2 dx + \frac{1}{EI_2} \int_3^8 [900x - 90(x-3)^2] x dx \\ &= \frac{300x^3}{EI_1} \Big|_0^3 + \frac{1}{EI_2} \left[300x^3 - 90\left(\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right) \right] \Big|_3^8 \\ &= \frac{8100}{EI_1} + \frac{1}{EI_2} [145.5(10^3) - 25.31(10^3)] = \frac{8100}{30(10^6)0.1755} + \frac{120.2(10^3)}{30(10^6)0.4604} \\ &= 0.0102 \text{ in} \quad \text{Ans.}\end{aligned}$$

4-70 $I = \pi(0.5^4)/64 = 3.068 (10^{-3}) \text{ in}^4$, $J = 2 I = 6.136 (10^{-3}) \text{ in}^4$, $A = \pi(0.5^2)/4 = 0.1963 \text{ in}^2$.

Consider x to be in the direction of OA , y vertically upward, and z in the direction of AB . Resolve the force F into components in the x and y directions obtaining $0.6 F$ in the horizontal direction and $0.8 F$ in the negative vertical direction. The $0.6 F$ force creates strain energy in the form of bending in AB and OA , and tension in OA . The $0.8 F$ force creates strain energy in the form of bending in AB and OA , and torsion in OA . Use the dummy variable \bar{x} to originate at the end where the loads are applied on each segment,

$$\underline{0.6 F}: \quad AB \quad M = 0.6F \bar{x} \quad \frac{\partial M}{\partial F} = 0.6 \bar{x}$$

$$OA \quad M = 4.2F \quad \frac{\partial M}{\partial F} = 4.2$$

$$F_a = 0.6F \quad \frac{\partial F_a}{\partial F} = 0.6$$

$$\underline{0.8 F}: \quad AB \quad M = 0.8F \bar{x} \quad \frac{\partial M}{\partial F} = 0.8 \bar{x}$$

$$OA \quad M = 0.8F \bar{x} \quad \frac{\partial M}{\partial F} = 0.8 \bar{x}$$

$$T = 5.6F \quad \frac{\partial T}{\partial F} = 5.6$$

Once the derivatives are taken the value of $F = 15 \text{ lbf}$ can be substituted in. The deflection of B in the direction of F is*

$$\begin{aligned} (\delta_B)_F &= \frac{\partial U}{\partial F} = \left(\frac{F_a L}{AE} \right)_{OA} \frac{\partial F_a}{\partial F} + \left(\frac{TL}{JG} \right)_{OA} \frac{\partial T}{\partial F} + \frac{1}{EI} \sum \int M \frac{\partial M}{\partial F} d\bar{x} \\ &= \frac{0.6(15)15}{0.1963(30)10^6} (0.6) + \frac{5.6(15)15}{6.136(10^{-3})11.5(10^6)} (5.6) \\ &\quad + \frac{15}{30(10^6)3.068(10^{-3})} \int_0^7 (0.6\bar{x})^2 d\bar{x} + \frac{15(4.2^2)}{30(10^6)3.068(10^{-3})} \int_0^{15} d\bar{x} + \\ &\quad + \frac{15}{30(10^6)3.068(10^{-3})} \int_0^7 (0.8\bar{x})^2 d\bar{x} + \frac{15}{30(10^6)3.068(10^{-3})} \int_0^{15} (0.8\bar{x})^2 d\bar{x} \\ &= 1.38(10^{-5}) + 0.1000 + 6.71(10^{-3}) + 0.0431 + 0.0119 + 0.1173 \\ &= 0.279 \text{ in} \quad \text{Ans.} \end{aligned}$$

*Note. Be careful, this is not the actual deflection of point B . For this, dummy forces must be placed on B in the x , y , and z directions. Determine the energy due to each, take derivatives, and then substitute the values of $F_x = 9$ lbf, $F_y = -12$ lbf, and $F_z = 0$. This can be done separately and then added by vector addition. The actual deflections of B are found to be

$$\delta_B = 0.0831 \mathbf{i} - 0.2862 \mathbf{j} - 0.00770 \mathbf{k} \text{ in}$$

From this, the deflection of B in the direction of F is

$$(\delta_B)_F = 0.6(0.0831) + 0.8(0.2862) = 0.279 \text{ in}$$

which agrees with our result.

4-71 Strain energy. AB : Bending and torsion, BC : Bending and torsion, CD : Bending.

$$I_{AB} = \pi(1^4)/64 = 0.04909 \text{ in}^4, J_{AB} = 2 I_{AB} = 0.09818 \text{ in}^4, I_{BC} = 0.25(1.5^3)/12 = 0.07031 \text{ in}^4, \\ I_{CD} = \pi(0.75^4)/64 = 0.01553 \text{ in}^4.$$

For the torsion of bar BC , Eq. (3-41) is in the form of $\theta = TL/(JG)$, where the equivalent of J is $J_{eq} = \beta bc^3$. With $b/c = 1.5/0.25 = 6$, $J_{BC} = \beta bc^3 = 0.299(1.5)0.25^3 = 7.008 (10^{-3}) \text{ in}^4$.

Use the dummy variable \bar{x} to originate at the end where the loads are applied on each segment,

$$AB: \text{Bending} \quad M = F \bar{x} + 2F \quad \frac{\partial M}{\partial F} = \bar{x} + 2$$

$$\text{Torsion} \quad T = 5F \quad \frac{\partial T}{\partial F} = 5$$

$$BC: \text{Bending} \quad M = F \bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}$$

$$\text{Torsion} \quad T = 2F \quad \frac{\partial T}{\partial F} = 2$$

$$CD: \text{Bending} \quad M = F \bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}$$

$$\begin{aligned} \delta_D &= \frac{\partial U}{\partial F} = \sum \frac{Tl}{JG} \frac{\partial T}{\partial F} + \sum \frac{1}{EI} \int M \frac{\partial M}{\partial F} d\bar{x} \\ &= \frac{5F(6)}{0.09818(11.5)10^6} (5) + \frac{2F(5)}{7.008(10^{-3})11.5(10^6)} 2 + \frac{1}{30(10^6)0.04909} \int_0^6 F(\bar{x} + 2)^2 d\bar{x} \\ &\quad + \frac{1}{30(10^6)0.07031} \int_0^5 F \bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553} \int_0^2 F \bar{x}^2 d\bar{x} \\ &= 1.329(10^{-4})F + 2.482(10^{-4})F + 1.141(10^{-4})F + 1.98(10^{-5})F + 5.72(10^{-6})F \\ &= 5.207(10^{-4})F = 5.207(10^{-4})200 = 0.104 \text{ in} \quad \text{Ans.} \end{aligned}$$

4-72 $A_{AB} = \pi(1^2)/4 = 0.7854 \text{ in}^2$, $I_{AB} = \pi(1^4)/64 = 0.04909 \text{ in}^4$, $I_{BC} = 1.5(0.25^3)/12 = 1.953(10^{-3}) \text{ in}^4$, $A_{CD} = \pi(0.75^2)/4 = 0.4418 \text{ in}^2$, $I_{AB} = \pi(0.75^4)/64 = 0.01553 \text{ in}^4$. For $(\delta_D)_x$ let $F = F_x = -150 \text{ lbf}$ and $F_z = -100 \text{ lbf}$. Use the dummy variable \bar{x} to originate at the end where the loads are applied on each segment,

$$CD: \quad M_y = F_z \bar{x} \quad \frac{\partial M_y}{\partial F} = 0$$

$$F_a = F \quad \frac{\partial F_a}{\partial F} = 1$$

$$BC: \quad M_y = F \bar{x} + 2F_z \quad \frac{\partial M_y}{\partial F} = \bar{x}$$

$$F_a = F_z \quad \frac{\partial F_a}{\partial F} = 0$$

$$AB: \quad M_y = 5F + 2F_z + F_z \bar{x} \quad \frac{\partial M_y}{\partial F} = 5$$

$$F_a = F \quad \frac{\partial F_a}{\partial F} = 1$$

$$\begin{aligned} (\delta_D)_x &= \frac{\partial U}{\partial F} = \left(\frac{FL}{AE} \right)_{CD} \frac{\partial F_a}{\partial F} + \frac{1}{EI_{BC}} \int_0^5 (F \bar{x} + 2F_z) \bar{x} d\bar{x} \\ &\quad + \frac{1}{EI_{AB}} \int_0^6 (5F + 2F_z + F_z \bar{x})(5) d\bar{x} + \left(\frac{FL}{AE} \right)_{AB} \frac{\partial F_a}{\partial F} \\ &= \frac{F(2)}{0.4418(30)10^6} (1) + \frac{1}{30(10^6)1.953(10^{-3})} \left[\frac{F}{3}(5)^3 + F_z(5^2) \right] \\ &\quad + \frac{1}{30(10^6)0.04909} \left[25F(6) + 10F_z(6) + \frac{F_z}{2}(6^2)5 \right] + \frac{F(6)}{0.7854(30)10^6} (1) \\ &= 1.509(10^{-7})F + 7.112(10^{-4})F + 4.267(10^{-4})F_z + 1.019(10^{-4})F \\ &\quad + 1.019(10^{-4})F_z + 2.546(10^{-7})F = 8.135(10^{-4})F + 5.286(10^{-4})F_z \end{aligned}$$

Substituting $F = F_x = -150 \text{ lbf}$ and $F_z = -100 \text{ lbf}$ gives

$$(\delta_D)_x = 8.135(10^{-4})(-150) + 5.286(10^{-4})(-100) = -0.1749 \text{ in} \quad \text{Ans.}$$

4-73 $I_{OA} = I_{BC} = \pi(1.5^4)/64 = 0.2485 \text{ in}^4$, $J_{OA} = J_{BC} = 2 I_{OA} = 0.4970 \text{ in}^4$, $I_{AB} = \pi(1^4)/64 = 0.04909 \text{ in}^4$, $J_{AB} = 2 I_{AB} = 0.09818 \text{ in}^4$, $I_{CD} = \pi(0.75^4)/64 = 0.01553 \text{ in}^4$

Let $F_y = F$, and use the dummy variable \bar{x} to originate at the end where the loads are applied on each segment,

$$OC: \quad M = F \bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}, \quad T = 12F \quad \frac{\partial T}{\partial F} = 12$$

$$DC: \quad M = F \bar{x} \quad \frac{\partial M}{\partial F} = \bar{x}$$

$$(\delta_D)_y = \frac{\partial U}{\partial F} = \sum \left(\frac{TL}{JG} \right)_{oc} \frac{\partial T}{\partial F} + \sum \frac{1}{EI} \int M \frac{\partial M}{\partial F} d\bar{x}$$

The terms involving the torsion and bending moments in *OC* must be split up because of the changing second-area moments.

$$\begin{aligned} (\delta_D)_y &= \frac{12F(4)}{0.4970(11.5)10^6}(12) + \frac{12F(9)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.2485} \int_0^2 F \bar{x}^2 d\bar{x} \\ &\quad + \frac{1}{30(10^6)0.04909} \int_2^{11} F \bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.2485} \int_{11}^{13} F \bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553} \int_0^{12} F \bar{x}^2 d\bar{x} \\ &= 1.008(10^{-4})F + 1.148(10^{-3})F + 3.58(10^{-7})F \\ &\quad + 2.994(10^{-4})F + 3.872(10^{-5})F + 1.2363(10^{-3})F \\ &= 2.824(10^{-3})F = 2.824(10^{-3})250 = 0.706 \text{ in} \quad \text{Ans.} \end{aligned}$$

For the simplified shaft *OC*,

$$\begin{aligned} (\delta_B)_y &= \frac{12F(13)}{0.09818(11.5)10^6}(12) + \frac{1}{30(10^6)0.04909} \int_0^{13} F \bar{x}^2 d\bar{x} + \frac{1}{30(10^6)0.01553} \int_0^{12} F \bar{x}^2 d\bar{x} \\ &= 1.6580(10^{-3})F + 4.973(10^{-4})F + 1.2363(10^{-3})F = 3.392(10^{-3})F = 3.392(10^{-3})250 \\ &= 0.848 \text{ in} \quad \text{Ans.} \end{aligned}$$

Simplified is $0.848/0.706 = 1.20$ times greater *Ans.*

4-74 Place a dummy force Q pointing downwards at point B . The reaction at C is

$$R_C = Q + (6/18)100 = Q + 33.33$$

This is the axial force in member *BC*. Isolating the beam, we find that the moment is not a function of Q , and thus does not contribute to the strain energy. Thus, only energy in the member *BC* needs to be considered. Let the axial force in *BC* be F , where

$$F = Q + 33.33 \quad \frac{\partial F}{\partial Q} = 1$$

$$\delta_B = \frac{\partial U}{\partial Q} \Big|_{Q=0} = \left[\left(\frac{FL}{AE} \right)_{BC} \frac{\partial F}{\partial Q} \right]_{Q=0} = \frac{(0 + 33.33)12}{\left[\pi(0.5^2)/4 \right] 30(10^6)} (1) = 6.79(10^{-5}) \text{ in} \quad \text{Ans.}$$

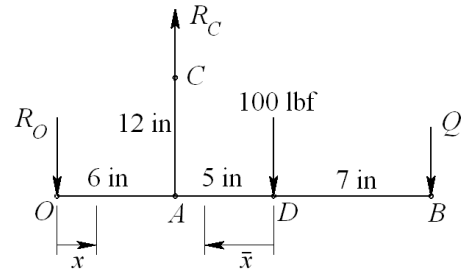
4-75 $I_{OB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$

$$A_{AC} = \pi(0.5^2)/4 = 0.1963 \text{ in}^2$$

$$\Sigma M_O = 0 = 6 R_C - 11(100) - 18 Q$$

$$R_C = 3Q + 183.3$$

$$\Sigma M_A = 0 = 6 R_O - 5(100) - 12 Q \Rightarrow R_O = 2Q + 83.33$$



Bending in OB.

BD: Bending in *BD* is only due to *Q* which when set to zero after differentiation gives no contribution.

AD: Using the variable \bar{x} as shown in the figure above

$$M = -100\bar{x} - Q(7 + \bar{x}) \quad \frac{\partial M}{\partial Q} = -(7 + \bar{x})$$

OA: Using the variable *x* as shown in the figure above

$$M = -(2Q + 83.33)x \quad \frac{\partial M}{\partial Q} = -2x$$

Axial in AC:

$$F = 3Q + 183.3 \quad \frac{\partial F}{\partial Q} = 3$$

$$\begin{aligned}
\delta_B &= \left(\frac{\partial U}{\partial Q} \right)_{Q=0} = \left[\left(\frac{FL}{AE} \right) \frac{\partial F}{\partial Q} \right]_{Q=0} + \left(\frac{1}{EI} \sum M \frac{\partial M}{\partial Q} dx \right)_{Q=0} \\
&= \frac{183.3(12)}{0.1963(30)10^6}(3) + \frac{1}{EI} \int_0^5 (100\bar{x})(7+\bar{x}) d\bar{x} + \int_0^6 2(83.33)x^2 dx \\
&= 1.121(10^{-3}) + \frac{1}{10.4(10^6)0.1667} \left[100 \int_0^5 \bar{x}(7+\bar{x}) d\bar{x} + 166.7 \int_0^6 x^2 dx \right] \\
&= 1.121(10^{-3}) + 5.768(10^{-7}) [100(129.2) + 166.7(72)] = 0.0155 \text{ in} \quad \text{Ans.}
\end{aligned}$$

4-76 There is no bending in AB . Using the variable θ , rotating counterclockwise from B

$$M = PR \sin \theta \quad \frac{\partial M}{\partial P} = R \sin \theta$$

$$F_r = P \cos \theta \quad \frac{\partial F_r}{\partial P} = \cos \theta$$

$$F_\theta = P \sin \theta \quad \frac{\partial F_\theta}{\partial P} = \sin \theta$$

$$\frac{\partial MF_\theta}{\partial P} = 2PR \sin^2 \theta$$

$$A = 6(4) = 24 \text{ mm}^2, \quad r_o = 40 + \frac{1}{2}(6) = 43 \text{ mm}, \quad r_i = 40 - \frac{1}{2}(6) = 37 \text{ mm},$$

From Table 3-4, p. 135, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489 \text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511 \text{ mm}$

From Table A-5, $E = 207(10^3) \text{ MPa}$, $G = 79.3(10^3) \text{ MPa}$

From Table 4-1, $C = 1.2$

From Eq. (4-38)

$$\begin{aligned}
\delta &= \int_0^{\frac{\pi}{2}} \frac{M}{AeE} \left(\frac{\partial M}{\partial P} \right) d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE} \left(\frac{\partial F_\theta}{\partial P} \right) d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE} \frac{\partial(MF_\theta)}{\partial P} d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG} \left(\frac{\partial F_r}{\partial P} \right) d\theta \\
&= \int_0^{\frac{\pi}{2}} \frac{P(R \sin \theta)^2}{AeE} d\theta + \int_0^{\frac{\pi}{2}} \frac{PR(\sin \theta)^2}{AE} d\theta - \int_0^{\frac{\pi}{2}} \frac{2PR \sin^2 \theta}{AE} d\theta + \int_0^{\frac{\pi}{2}} \frac{CPR(\cos \theta)^2}{AG} d\theta \\
&= \frac{\pi PR}{4AE} \left(\frac{R}{e} + 1 - 2 + \frac{EC}{G} \right) = \frac{\pi(10)(40)}{4(24)(207 \cdot 10^3)} \left(\frac{40}{0.07511} + 1 - 2 + \frac{(207 \cdot 10^3)(1.2)}{79.3 \cdot 10^3} \right) \\
\delta &= 0.0338 \text{ mm} \quad \text{Ans.}
\end{aligned}$$

- 4-77** Place a dummy force Q pointing downwards at point A. Bending in AB is only due to Q which when set to zero after differentiation gives no contribution. For section BC use the variable θ , rotating counterclockwise from B

$$M = PR \sin \theta + Q(R + R \sin \theta) \quad \frac{\partial M}{\partial Q} = R(1 + \sin \theta)$$

$$F_r = (P + Q) \cos \theta \quad \frac{\partial F_r}{\partial Q} = \cos \theta$$

$$F_\theta = (P + Q) \sin \theta \quad \frac{\partial F_\theta}{\partial Q} = \sin \theta$$

$$MF_\theta = [PR \sin \theta + QR(1 + \sin \theta)](P + Q) \sin \theta$$

$$\frac{\partial MF_\theta}{\partial Q} = PR \sin^2 \theta + PR \sin \theta(1 + \sin \theta) + 2QR \sin \theta(1 + \sin \theta)$$

But after differentiation, we can set $Q = 0$. Thus,

$$\frac{\partial MF_\theta}{\partial Q} = PR \sin \theta(1 + 2 \sin \theta)$$

$$A = 6(4) = 24 \text{ mm}^2, \quad r_o = 40 + \frac{1}{2}(6) = 43 \text{ mm}, \quad r_i = 40 - \frac{1}{2}(6) = 37 \text{ mm},$$

From Table 3-4, p.135, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489 \text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511 \text{ mm}$

From Table A-5, $E = 207(10^3) \text{ MPa}$, $G = 79.3(10^3) \text{ MPa}$

From Table 4-1, $C = 1.2$

From Eq. (4-38),

$$\begin{aligned} \delta &= \int_0^{\frac{\pi}{2}} \frac{M}{AeE} \left(\frac{\partial M}{\partial Q} \right) d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE} \left(\frac{\partial F_\theta}{\partial Q} \right) d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE} \frac{\partial(MF_\theta)}{\partial Q} d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG} \left(\frac{\partial F_r}{\partial Q} \right) d\theta \\ &= \frac{PR^2}{AeE} \int_0^{\frac{\pi}{2}} \sin \theta(1 + \sin \theta) d\theta + \frac{PR}{AE} \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta - \frac{PR}{AE} \int_0^{\frac{\pi}{2}} \sin \theta(1 + 2 \sin \theta) d\theta \\ &\quad + \frac{CPR}{AG} \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta \\ &= \left(\frac{\pi}{4} + 1 \right) \frac{PR^2}{AeE} + \frac{\pi}{4} \frac{PR}{AE} - \left(\frac{\pi}{4} + 2 \right) \frac{PR}{AE} + \frac{\pi}{4} \frac{CPR}{AG} = \frac{PR}{AE} \left[\left(\frac{\pi}{4} + 1 \right) \frac{R}{e} - 2 + \frac{\pi}{4} \frac{CE}{G} \right] \\ &= \frac{10(40)}{24(207)10^3} \left[\left(\frac{\pi}{4} + 1 \right) \frac{40}{0.07511} - 2 + \frac{\pi}{4} \frac{1.2(207)10^3}{79.3(10^3)} \right] \\ &= 0.0766 \text{ mm} \quad \text{Ans.} \end{aligned}$$

$$4-78 \quad A = 3(2.25) - 2.25(1.5) = 3.375 \text{ in}^2$$

$$R = \frac{(1+1.5)(3)(2.25) - (1+0.75+1.125)(1.5)(2.25)}{3.375} = 2.125 \text{ in}$$

Section is equivalent to the "T" section of Table 3-4, p. 135,

$$r_n = \frac{2.25(0.75) + 0.75(2.25)}{2.25 \ln[(1+0.75)/1] + 0.75 \ln[(1+3)/(1+0.75)]} = 1.7960 \text{ in}$$

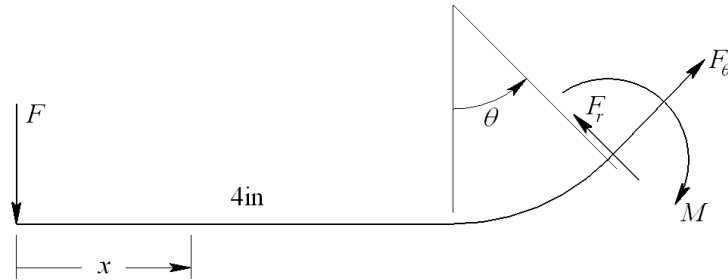
$$e = R - r_n = 2.125 - 1.7960 = 0.329 \text{ in}$$

For the straight section

$$I_z = \frac{1}{12} (2.25)(3^3) + 2.25(3)(1.5 - 1.125)^2$$

$$- \left[\frac{1}{12} (1.5)(2.25^3) + 1.5(2.25) \left(0.75 + \frac{2.25}{2} - 1.125 \right)^2 \right]$$

$$= 2.689 \text{ in}^4$$



For $0 \leq x \leq 4 \text{ in}$

$$M = -Fx \quad \frac{\partial M}{\partial F} = -x, \quad V = F \quad \frac{\partial V}{\partial F} = 1$$

For $\theta \leq \pi/2$

$$F_r = F \cos \theta \quad \frac{\partial F_r}{\partial F} = \cos \theta, \quad F_\theta = F \sin \theta \quad \frac{\partial F_\theta}{\partial F} = \sin \theta$$

$$M = F(4 + 2.125 \sin \theta) \quad \frac{\partial M}{\partial F} = (4 + 2.125 \sin \theta)$$

$$MF_\theta = F(4 + 2.125 \sin \theta)F \sin \theta \quad \frac{\partial MF_\theta}{\partial F} = 2F(4 + 2.365 \sin \theta) \sin \theta$$

Use Eqs. (4-31) and (4-24) (with $C = 1$) for the straight part, and Eq. (4-38) for the curved part, integrating from 0 to $\pi/2$, and double the results

$$\begin{aligned}\delta = \frac{2}{E} & \left\{ \frac{1}{I} \int_0^4 Fx^2 dx + \frac{F(4)(1)}{3.375(G/E)} + \int_0^{\pi/2} F \frac{(4 + 2.125 \sin \theta)^2}{3.375(0.329)} d\theta \right. \\ & + \int_0^{\pi/2} \frac{F \sin^2 \theta (2.125)}{3.375} d\theta - \int_0^{\pi/2} \frac{2F(4 + 2.125 \sin \theta) \sin \theta}{3.375} d\theta \\ & \left. + \int_0^{\pi/2} \frac{(1)F \cos^2 \theta (2.125)}{3.375(G/E)} d\theta \right\}\end{aligned}$$

Substitute $I = 2.689 \text{ in}^4$, $F = 6700 \text{ lbf}$, $E = 30 (10^6) \text{ psi}$, $G = 11.5 (10^6) \text{ psi}$

$$\begin{aligned}\delta = \frac{2(6700)}{30(10^6)} & \left\{ \frac{4^3}{3(2.689)} + \frac{4}{3.375(11.5/30)} + \frac{1}{3.375(0.329)} \left[16 \left(\frac{\pi}{2} \right) + 17(1) + 4.516 \left(\frac{\pi}{4} \right) \right] \right. \\ & \left. + \frac{2.125}{3.375} \left(\frac{\pi}{4} \right) - \frac{2}{3.375} \left[4(1) + 2.125 \left(\frac{\pi}{4} \right) \right] + \frac{2.125}{3.375(11.5/30)} \left(\frac{\pi}{4} \right) \right\} \\ = 0.0226 \text{ in} \quad & \text{Ans.}\end{aligned}$$

4-79 Since $R/h = 35/4.5 = 7.78$ use Eq. (4-38), integrate from 0 to π , and double the results

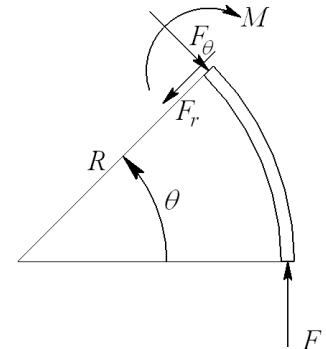
$$M = FR(1 - \cos \theta) \quad \frac{\partial M}{\partial F} = R(1 - \cos \theta)$$

$$F_r = F \sin \theta \quad \frac{\partial F_r}{\partial F} = \sin \theta$$

$$F_\theta = F \cos \theta \quad \frac{\partial F_\theta}{\partial F} = \cos \theta$$

$$MF_\theta = F^2 R \cos \theta (1 - \cos \theta)$$

$$\frac{\partial (MF_\theta)}{\partial F} = 2FR \cos \theta (1 - \cos \theta)$$



From Eq. (4-38),

$$\begin{aligned}\delta = 2 & \left[\frac{FR^2}{AeE} \int_0^\pi (1 - \cos \theta)^2 d\theta + \frac{FR}{AE} \int_0^\pi \cos^2 \theta d\theta \right. \\ & \left. - \frac{2FR}{AE} \int_0^\pi \cos \theta (1 - \cos \theta) d\theta + \frac{1.2FR}{AG} \int_0^\pi \sin^2 \theta d\theta \right] \\ = \frac{2FR}{AE} & \left(\frac{3\pi}{2} \frac{R}{e} + \frac{3\pi}{2} + 0.6\pi \frac{E}{G} \right)\end{aligned}$$

$A = 4.5(3) = 13.5 \text{ mm}^2$, $E = 207 (10^3) \text{ N/mm}^2$, $G = 79.3 (10^3) \text{ N/mm}^2$, and from Table 3-4, p. 135,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{4.5}{\ln \frac{37.25}{32.75}} = 34.95173 \text{ mm}$$

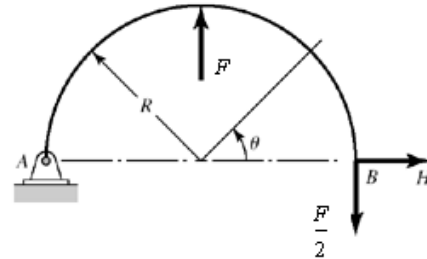
and $e = R - r_n = 35 - 34.95173 = 0.04827 \text{ mm}$. Thus,

$$\delta = \frac{2F(35)}{13.5(207)10^3} \left(\frac{3\pi}{2} \frac{35}{0.04827} + \frac{3\pi}{2} + 0.6\pi \frac{207}{79.3} \right) = 0.08583F$$

where F is in N. For $\delta = 1 \text{ mm}$, $F = \frac{1}{0.08583} = 11.65 \text{ N}$ *Ans.*

Note: The first term in the equation for δ dominates and this is from the bending moment. Try Eq. (4-41), and compare the results.

4-80 $R/h = 20 > 10$ so Eq. (4-41) can be used to determine deflections. Consider the horizontal reaction to be applied at B and subject to the constraint $(\delta_B)_H = 0$.



$$M = \frac{FR}{2}(1 - \cos \theta) - HR \sin \theta \quad \frac{\partial M}{\partial H} = -R \sin \theta \quad 0 < \theta < \frac{\pi}{2}$$

By symmetry, we may consider only half of the wire form and use twice the strain energy Eq. (4-41) then becomes,

$$\begin{aligned} (\delta_B)_H &= \frac{\partial U}{\partial H} \approx \frac{2}{EI} \int_0^{\pi/2} \left(M \frac{\partial M}{\partial H} \right) R d\theta = 0 \\ \int_0^{\pi/2} \left[\frac{FR}{2}(1 - \cos \theta) - HR \sin \theta \right] (-R \sin \theta) R d\theta &= 0 \\ -\frac{F}{2} + \frac{F}{4} + H \frac{\pi}{4} &= 0 \Rightarrow H = \frac{F}{\pi} = \frac{30}{\pi} = 9.55 \text{ N} \quad \text{Ans.} \end{aligned}$$

The reaction at A is the same where H goes to the left. Substituting H into the moment equation we get,

$$M = \frac{FR}{2\pi} [\pi(1 - \cos \theta) - 2 \sin \theta] \quad \frac{\partial M}{\partial F} = \frac{R}{2\pi} [\pi(1 - \cos \theta) - 2 \sin \theta] \quad 0 < \theta < \frac{\pi}{2}$$

$$\begin{aligned}
\delta_P &= \frac{\partial U}{\partial P} \approx \int \frac{2}{EI} \left(M \frac{\partial M}{\partial F} \right) R d\theta = \frac{2}{EI} \int_0^{\pi/2} \frac{FR^2}{4\pi^2} [\pi(1 - \cos \theta) - 2\sin \theta]^2 R d\theta \\
&= \frac{FR^3}{2\pi^2 EI} \int_0^{\pi/2} (\pi^2 + \pi^2 \cos^2 \theta + 4\sin^2 \theta - 2\pi^2 \cos \theta - 4\pi \sin \theta + 4\pi \sin \theta \cos \theta) d\theta \\
&= \frac{FR^3}{2\pi^2 EI} \left[\pi^2 \left(\frac{\pi}{2} \right) + \pi^2 \left(\frac{\pi}{4} \right) + 4 \left(\frac{\pi}{4} \right) - 2\pi^2 - 4\pi + 2\pi \right] \\
&= \frac{(3\pi^2 - 8\pi - 4) FR^3}{8\pi EI} = \frac{(3\pi^2 - 8\pi - 4)}{8\pi} \frac{(30)(40^3)}{207(10^3) \left[\pi(2^4)/64 \right]} = 0.224 \text{ mm} \quad \text{Ans.}
\end{aligned}$$

- 4-81** The radius is sufficiently large compared to the wire diameter to use Eq. (4-41) for the curved beam portion. The shear and axial components will be negligible compared to bending.

Place a fictitious force Q pointing to the left at point A.

$$M = PR \sin \theta + Q(R \sin \theta + l) \quad \frac{\partial M}{\partial Q} = R \sin \theta + l$$

Note that the strain energy in the straight portion is zero since there is no real force in that section.

From Eq. (4-41),

$$\begin{aligned}
\delta &= \left[\int_0^{\pi/2} \frac{1}{EI} \left(M \frac{\partial M}{\partial Q} \right) R d\theta \right]_{Q=0} = \frac{1}{EI} \int_0^{\pi/2} PR \sin \theta (R \sin \theta + l) R d\theta \\
&= \frac{PR^2}{EI} \int_0^{\pi/2} (R \sin^2 \theta + l \sin \theta) d\theta = \frac{PR^2}{EI} \left(\frac{\pi}{4} R + l \right) = \frac{1(5^2)}{30(10^6) \left[\pi(0.125^4)/64 \right]} \left(\frac{\pi}{4}(5) + 4 \right) \\
&= 0.551 \text{ in} \quad \text{Ans.}
\end{aligned}$$

- 4-82** Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

$$\text{Straight portion:} \quad M_{AB} = Px \quad \frac{\partial M_{AB}}{\partial P} = x$$

$$\text{Curved portion:} \quad M_{BC} = P[R(1 - \cos \theta) + l] \quad \frac{\partial M_{BC}}{\partial P} = [R(1 - \cos \theta) + l]$$

From Eq. (4-41) with the addition of the bending strain energy in the straight portion of the wire,

$$\begin{aligned}
\delta &= \int_0^l \frac{1}{EI} \left(M_{AB} \frac{\partial M_{AB}}{\partial P} \right) dx + \int_0^{\pi/2} \frac{1}{EI} \left(M_{BC} \frac{\partial M_{BC}}{\partial P} \right) R d\theta \\
&= \frac{P}{EI} \int_0^l x^2 dx + \frac{PR}{EI} \int_0^{\pi/2} [R(1 - \cos \theta) + l]^2 d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI} \int_0^{\pi/2} [R^2(1 - 2\cos \theta + \cos^2 \theta) + 2Rl(1 - \cos \theta) + l^2] d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI} \int_0^{\pi/2} [R^2 \cos^2 \theta - (2R^2 + 2Rl)\cos \theta + (R+l)^2] d\theta \\
&= \frac{Pl^3}{3EI} + \frac{PR}{EI} \left[\frac{\pi}{4} R^2 - (2R^2 + 2Rl) + \frac{\pi}{2} (R+l)^2 \right] \\
&= \frac{P}{EI} \left[\frac{l^3}{3} + \frac{\pi}{4} R^3 - R(2R^2 + 2Rl) + \frac{\pi}{2} R(R+l)^2 \right] \\
&= \frac{1}{30(10^6)\pi(0.125^4)/64} \left[\frac{4^3}{3} + \frac{\pi}{4}(5^3) - 5[2(5^2) + 2(5)(4)] + \frac{\pi}{2}(5)(5+4)^2 \right] \\
&= 0.850 \text{ in} \quad \text{Ans.}
\end{aligned}$$

4-83 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, Q , at A vertically downward. The only load in the straight section is the axial force, Q . Since this will be zero, there is no contribution.

In the curved section

$$M = PR \sin \theta + QR(1 - \cos \theta) \quad \frac{\partial M}{\partial Q} = R(1 - \cos \theta)$$

From Eq. (4-41)

$$\begin{aligned}
\delta &= \left[\int_0^{\pi/2} \frac{1}{EI} \left(M \frac{\partial M}{\partial Q} \right) R d\theta \right]_{Q=0} = \frac{1}{EI} \int_0^{\pi/2} PR \sin \theta [R(1 - \cos \theta)] R d\theta \\
&= \frac{PR^3}{EI} \int_0^{\pi/2} (\sin \theta - \sin \theta \cos \theta) d\theta = \frac{PR^3}{EI} \left(1 - \frac{1}{2} \right) = \frac{PR^3}{2EI} \\
&= \frac{1(5^3)}{2(30)10^6 [\pi(0.125^4)/64]} = 0.174 \text{ in} \quad \text{Ans.}
\end{aligned}$$

4-84 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, Q , at A vertically downward. The load in the straight section is the axial force, Q , whereas the bending moment is only a function of P and is not a function of Q . When setting $Q = 0$, there is no axial or bending contribution.

In the curved section

$$M = P[R(1 - \cos \theta) + l] - QR \sin \theta \quad \frac{\partial M}{\partial Q} = -R \sin \theta$$

From Eq. (4-41)

$$\begin{aligned} \delta &= \left[\int_0^{\pi/2} \frac{1}{EI} \left(M \frac{\partial M}{\partial Q} \right) R d\theta \right]_{Q=0} = \frac{1}{EI} \int_0^{\pi/2} P[R(1 - \cos \theta) + l](-R \sin \theta) R d\theta \\ &= -\frac{PR^2}{EI} \int_0^{\pi/2} (R \sin \theta - R \sin \theta \cos \theta + l \sin \theta) d\theta = -\frac{PR^2}{EI} \left(R + l - \frac{1}{2}R \right) = -\frac{PR^2}{2EI} (R + 2l) \\ &= -\frac{1(5^2)}{2(30)10^6 \left[\pi(0.125^4)/64 \right]} [5 + 2(4)] = -0.452 \text{ in} \end{aligned}$$

Since the deflection is negative, δ is in the opposite direction of Q . Thus the deflection is

$$\delta = 0.452 \text{ in } \uparrow \quad \text{Ans.}$$

4-85 Consider the force of the mass to be F , where $F = 9.81(1) = 9.81 \text{ N}$. The load in AB is tension

$$F_{AB} = F \quad \frac{\partial F_{AB}}{\partial F} = 1$$

For the curved section, the radius is sufficiently large to use Eq. (4-41). There is no bending in section DE . For section BCD , let θ be counterclockwise originating at D

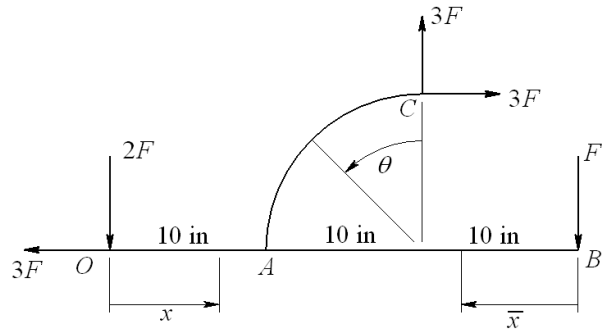
$$M = FR \sin \theta \quad \frac{\partial M}{\partial F} = R \sin \theta \quad 0 \leq \theta \leq \pi$$

Using Eqs. (4-29) and (4-41)

$$\begin{aligned} \delta &= \left(\frac{Fl}{AE} \right)_{AB} \frac{\partial F_{AB}}{\partial F} + \int_0^{\pi} \frac{1}{EI} \left(M \frac{\partial M}{\partial F} \right) R d\theta = \frac{Fl}{AE} (1) + \int_0^{\pi} \frac{FR^3}{EI} \sin^2 \theta d\theta \\ &= \frac{Fl}{AE} + \frac{\pi FR^3}{2EI} = \frac{F}{E} \left(\frac{l}{A} + \frac{\pi R^3}{2I} \right) = \frac{9.81}{207(10^3)} \left[\frac{80}{\left[\pi(2^2)/4 \right]} + \frac{\pi(40^3)}{2 \left[\pi(2^4)/64 \right]} \right] \\ &= 6.067 \text{ mm} \quad \text{Ans.} \end{aligned}$$

4-86 $A_{OA} = 2(0.25) = 0.5 \text{ in}^2$,
 $I_{OAB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$,
 $I_{AC} = \pi(0.5^4)/64 = 3.068 (10^{-3}) \text{ in}^4$

Applying a force F at point B , using statics (AC is a two-force member), the reaction forces at O and C are as shown.



OA : Axial $F_{OA} = 3F$ $\frac{\partial F_{OA}}{\partial F} = 3$

Bending $M_{OA} = -2Fx$ $\frac{\partial M_{OA}}{\partial F} = -2x$

AB : Bending $M_{AB} = -F\bar{x}$ $\frac{\partial M_{AB}}{\partial F} = -\bar{x}$

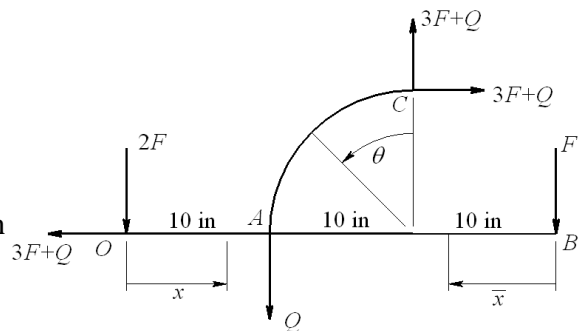
AC : Isolating the upper curved section

$M_{AC} = 3FR(\sin \theta + \cos \theta - 1)$ $\frac{\partial M_{AC}}{\partial F} = 3R(\sin \theta + \cos \theta - 1)$

$$\begin{aligned} \delta &= \left(\frac{Fl}{AE} \right)_{OA} \frac{\partial F_{OA}}{\partial F} + \frac{1}{(EI)_{OAB}} \int_0^{10} 4Fx^2 dx + \frac{1}{(EI)_{OAB}} \int_0^{20} F\bar{x}^2 d\bar{x} \\ &\quad + \frac{9FR^3}{(EI)_{AC}} \int_0^{\pi/2} (\sin \theta + \cos \theta - 1)^2 d\theta \\ &= \frac{3F(10)}{0.5(10.4)10^6} (3) + \frac{4F(10^3)}{3(10.4)10^6 (0.1667)} + \frac{F(20^3)}{3(10.4)10^6 (0.1667)} \\ &\quad + \frac{9F(10^3)}{30(10^6)3.068(10^{-3})} \int_0^{\pi/2} (\sin^2 \theta + 2\sin \theta \cos \theta - 2\sin \theta + \cos^2 \theta - 2\cos \theta + 1) d\theta \\ &= 1.731(10^{-5})F + 7.691(10^{-4})F + 1.538(10^{-3})F + 0.09778F \left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2} \right) \\ &= 0.0162F = 0.0162(100) = 1.62 \text{ in} \quad \text{Ans.} \end{aligned}$$

4-87 $A_{OA} = 2(0.25) = 0.5 \text{ in}^2$,
 $I_{OAB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$,
 $I_{AC} = \pi(0.5^4)/64 = 3.068 (10^{-3}) \text{ in}^4$

Applying a vertical dummy force, Q , at A , from statics the reactions are as shown. The dummy force is transmitted through section OA and member AC .



$$OA: F_{OA} = 3F + Q \quad \frac{\partial F_{OA}}{\partial Q} = 1$$

$$AC: M_{AC} = (3F + Q)R \sin \theta - (3F + Q)R(1 - \cos \theta) \quad \frac{\partial M_{AC}}{\partial Q} = R(\sin \theta + \cos \theta - 1)$$

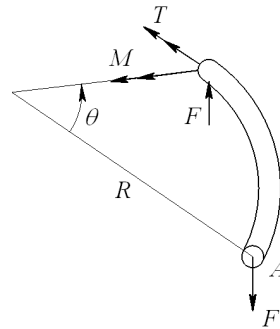
$$\begin{aligned} \delta &= \left[\left(\frac{Fl}{AE} \right)_{OA} \left(\frac{\partial F_{OA}}{\partial Q} \right) + \left(\frac{1}{EI} \right)_{AC} \int_0^{\pi/2} M_{AC} \frac{\partial M_{AC}}{\partial Q} R d\theta \right]_{Q=0} \\ &= \frac{3Fl_{OA}}{(AE)_{OA}} + \frac{3FR^3}{(EI)_{AC}} \int_0^{\pi/2} (\sin \theta + \cos \theta - 1)^2 d\theta \\ &= \frac{3(100)10}{10.4(10^6)0.5} + \frac{3(100)10^3}{30(10^6)3.068(10^{-3})} \left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2} \right) = 0.462 \text{ in} \quad \text{Ans.} \end{aligned}$$

4-88 $I = \pi(6^4)/64 = 63.62 \text{ mm}^4$

$$0 \leq \theta \leq \pi/2$$

$$M = FR \sin \theta \quad \frac{\partial M}{\partial F} = R \sin \theta$$

$$T = FR(1 - \cos \theta) \quad \frac{\partial T}{\partial F} = R(1 - \cos \theta)$$



According to Castigliano's theorem, a positive $\partial U / \partial F$ will yield a deflection of A in the negative y direction. Thus the deflection in the positive y direction is

$$(\delta_A)_y = -\frac{\partial U}{\partial F} = -\left\{ \frac{1}{EI} \int_0^{\pi/2} F(R \sin \theta)^2 R d\theta + \frac{1}{GJ} \int_0^{\pi/2} F[R(1 - \cos \theta)]^2 R d\theta \right\}$$

Integrating and substituting $J = 2I$ and $G = E / [2(1 + \nu)]$

$$\begin{aligned} (\delta_A)_y &= -\frac{FR^3}{EI} \left[\frac{\pi}{4} + (1 + \nu) \left(\frac{3\pi}{4} - 2 \right) \right] = -[4\pi - 8 + (3\pi - 8)\nu] \frac{FR^3}{4EI} \\ &= -[4\pi - 8 + (3\pi - 8)(0.29)] \frac{(250)(80)^3}{4(200)10^3(63.62)} = -12.5 \text{ mm} \quad \text{Ans.} \end{aligned}$$

4-89 The force applied to the copper and steel wire assembly is

$$F_c + F_s = 400 \text{ lbf} \quad (1)$$

Since the deflections are equal, $\delta_c = \delta_s$

$$\left(\frac{Fl}{AE}\right)_c = \left(\frac{Fl}{AE}\right)_s$$

$$\frac{F_c l}{3(\pi/4)(0.1019)^2(17.2)10^6} = \frac{F_s l}{(\pi/4)(0.1055)^2(30)10^6}$$

Yields, $F_c = 1.6046F_s$. Substituting this into Eq. (1) gives

$$1.604F_s + F_s = 2.6046F_s = 400 \Rightarrow F_s = 153.6 \text{ lbf}$$

$$F_c = 1.6046F_s = 246.5 \text{ lbf}$$

$$\sigma_c = \frac{F_c}{A_c} = \frac{246.5}{3(\pi/4)(0.1019)^2} = 10\,075 \text{ psi} = 10.1 \text{ kpsi} \quad \text{Ans.}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{153.6}{(\pi/4)(0.1055)^2} = 17\,571 \text{ psi} = 17.6 \text{ kpsi} \quad \text{Ans.}$$

$$\delta = \left(\frac{Fl}{AE}\right)_s = \frac{153.6(100)(12)}{(\pi/4)(0.1055)^2(30)10^6} = 0.703 \text{ in} \quad \text{Ans.}$$

4-90 (a) Bolt stress $\sigma_b = 0.75(65) = 48.8 \text{ kpsi} \quad \text{Ans.}$

Total bolt force $F_b = 6\sigma_b A_b = 6(48.8)\left(\frac{\pi}{4}\right)(0.5^2) = 57.5 \text{ kips}$

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = \frac{57.43}{(\pi/4)(5.5^2 - 5^2)} = -13.9 \text{ kpsi} \quad \text{Ans.}$

(b) Force from pressure

$$P = \frac{\pi D^2}{4} p = \frac{\pi(5^2)}{4}(500) = 9817 \text{ lbf} = 9.82 \text{ kip}$$

$$\Sigma F_x = 0$$

$$P_b + P_c = 9.82 \quad (1)$$

Since $\delta_c = \delta_b$,

$$\frac{P_c l}{(\pi/4)(5.5^2 - 5^2)E} = \frac{P_b l}{6(\pi/4)(0.5^2)E}$$

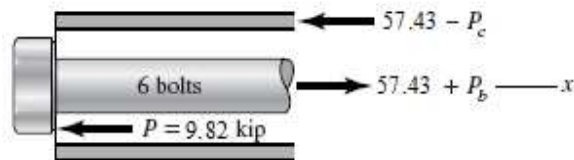
$$P_c = 3.5 P_b \quad (2)$$

Substituting this into Eq. (1)

$$P_b + 3.5 P_b = 4.5 P_b = 9.82 \Rightarrow P_b = 2.182 \text{ kip. From Eq. (2), } P_c = 7.638 \text{ kip}$$

Using the results of (a) above, the total bolt and cylinder stresses are

$$\sigma_b = 48.8 + \frac{2.182}{6(\pi/4)(0.5^2)} = 50.7 \text{ kpsi} \quad \text{Ans.}$$



$$\sigma_c = -13.9 + \frac{7.638}{(\pi/4)(5.5^2 - 5^2)} = -12.0 \text{ kpsi} \quad \text{Ans.}$$

4-91 $T_c + T_s = T \quad (1)$

$$\theta_c = \theta_s \Rightarrow \frac{T_c l}{(JG)_c} = \frac{T_s l}{(JG)_s} \Rightarrow T_c = \frac{(JG)_c}{(JG)_s} T_s \quad (2)$$

Substitute this into Eq. (1)

$$\frac{(JG)_c}{(JG)_s} T_s + T_s = T \Rightarrow T_s = \frac{(JG)_s}{(JG)_s + (JG)_c} T$$

The percentage of the total torque carried by the shell is

$$\% \text{ Torque} = \frac{100(JG)_s}{(JG)_s + (JG)_c} \quad \text{Ans.}$$

4-92 $R_O + R_B = W \quad (1)$

$$\delta_{OA} = \delta_{AB}$$

$$\left(\frac{Fl}{AE} \right)_{OA} = \left(\frac{Fl}{AE} \right)_{AB}$$

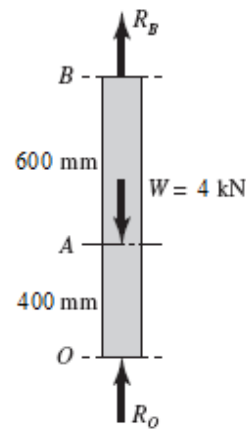
$$\frac{400R_O}{AE} = \frac{600R_B}{AE} \Rightarrow R_O = \frac{3}{2} R_B \quad (2)$$

Substitute this into Eq. (1)

$$\frac{3}{2} R_B + R_B = 4 \Rightarrow R_B = 1.6 \text{ kN} \quad \text{Ans.}$$

From Eq. (2) $R_O = \frac{3}{2} 1.6 = 2.4 \text{ kN} \quad \text{Ans.}$

$$\delta_A = \left(\frac{Fl}{AE} \right)_{OA} = \frac{2400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad \text{Ans.}$$



4-93 See figure in Prob. 4-92 solution.

Procedure 1:

1. Let R_B be the redundant reaction.

2. Statics. $R_O + R_B = 4\,000\text{ N} \Rightarrow R_O = 4\,000 - R_B$ (1)

3. Deflection of point B. $\delta_B = \frac{R_B(600)}{AE} + \frac{(R_B - 4000)(400)}{AE} = 0$ (2)

4. From Eq. (2), AE cancels and $R_B = 1\,600\text{ N}$ *Ans.*
and from Eq. (1), $R_O = 4\,000 - 1\,600 = 2\,400\text{ N}$ *Ans.*

$$\delta_A = \left(\frac{Fl}{AE} \right)_{OA} = \frac{2400(400)}{10(60)(71.7)(10^3)} = 0.0223\text{ mm} \quad \text{Ans.}$$

4-94 (a) Without the right-hand wall, the deflection of point C would be

$$\begin{aligned} \delta_C &= \sum \frac{Fl}{AE} = \frac{5(10^3)8}{(\pi/4)0.75^2(10.4)10^6} + \frac{2(10^3)5}{(\pi/4)0.5^2(10.4)10^6} \\ &= 0.01360\text{ in} > 0.005\text{ in} \therefore \text{ Hits wall} \quad \text{Ans.} \end{aligned}$$

(b) Let R_C be the reaction of the wall at C acting to the left (\leftarrow). Thus, the deflection of point C is now

$$\begin{aligned} \delta_C &= \frac{[5(10^3) - R_C]8}{(\pi/4)0.75^2(10.4)10^6} + \frac{[2(10^3) - R_C]5}{(\pi/4)0.5^2(10.4)10^6} \\ &= 0.01360 - \frac{4R_C}{\pi(10.4)10^6} \left(\frac{8}{0.75^2} + \frac{5}{0.5^2} \right) = 0.005 \end{aligned}$$

or,

$$0.01360 - 4.190(10^{-6})R_C = 0.005 \Rightarrow R_C = 2\,053\text{ lbf} = 2.05\text{ kip} \leftarrow \text{Ans.}$$

$$\Sigma F_x = 5\,000 + R_A - 2\,053 = 0 \Rightarrow R_A = -2\,947\text{ lbf}, \Rightarrow R_A = 2.95\text{ kip} \leftarrow \text{Ans.}$$

Deflection. AB is 2 947 lbf in tension. Thus

$$\delta_B = \delta_{AB} = \frac{R_A(8)}{A_{AB}E} = \frac{2947(8)}{(\pi/4)0.75^2(10.4)10^6} = 5.13(10^{-3})\text{ in} \rightarrow \text{Ans.}$$

4-95 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{JG} = \frac{T_{AB}(6)}{JG} \Rightarrow T_{OA} = \frac{3}{2}T_{AB} \quad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$\frac{3}{2}T_{AB} + T_{AB} = \frac{5}{2}T_{AB} = 200 \quad \Rightarrow \quad T_{AB} = 80 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

From Eq. (1) $T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}80 = 120 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$

$$\theta_A = \left(\frac{TL}{JG} \right)_{AB} = \frac{80(6)}{(\pi/32)0.5^4(11.5)10^6} \frac{180}{\pi} = 0.390^\circ \quad \text{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad \tau_{OA} = \frac{16(120)}{\pi(0.5^3)} = 4890 \text{ psi} = 4.89 \text{ kpsi} \quad \text{Ans.}$$

$$\tau_{AB} = \frac{16(80)}{\pi(0.5^3)} = 3260 \text{ psi} = 3.26 \text{ kpsi} \quad \text{Ans.}$$

4-96 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{(\pi/32)0.5^4 G} = \frac{T_{AB}(6)}{(\pi/32)0.75^4 G} \quad \Rightarrow \quad T_{OA} = 0.2963T_{AB} \quad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$0.2963T_{AB} + T_{AB} = 1.2963T_{AB} = 200 \quad \Rightarrow \quad T_{AB} = 154.3 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

From Eq. (1) $T_{OA} = 0.2963T_{AB} = 0.2963(154.3) = 45.7 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$

$$\theta_A = \frac{154.3(6)}{(\pi/32)0.75^4(11.5)10^6} \frac{180}{\pi} = 0.148^\circ \quad \text{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad \tau_{OA} = \frac{16(45.7)}{\pi(0.5^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \quad \text{Ans.}$$

$$\tau_{AB} = \frac{16(154.3)}{\pi(0.75^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \quad \text{Ans.}$$

4-97 Procedure 1

1. Arbitrarily, choose R_C as a redundant reaction.

2. Statics. $\Sigma F_x = 0$,

$$12(10^3) - 6(10^3) - R_O - R_C = 0$$

$$R_O = 6(10^3) - R_C \quad (1)$$

3. The deflection of point C.

$$\delta_C = \frac{[12(10^3) - 6(10^3) - R_C](20)}{AE} - \frac{[6(10^3) + R_C](10)}{AE} - \frac{R_C(15)}{AE} = 0$$

4. The deflection equation simplifies to

$$-45 R_C + 60(10^3) = 0 \Rightarrow R_C = 1\,333 \text{ lbf} = 1.33 \text{ kip} \quad \text{Ans.}$$

From Eq. (1), $R_O = 6(10^3) - 1\,333 = 4\,667 \text{ lbf} = 4.67 \text{ kip} \quad \text{Ans.}$

$$F_{AB} = F_B + R_C = 6 + 1.333 = 7.333 \text{ kips compression}$$

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{-7.333}{(0.5)(1)} = -14.7 \text{ kpsi} \quad \text{Ans.}$$

Deflection of A. Since OA is in tension,

$$\delta_A = \delta_{OA} = \frac{R_O l_{OA}}{AE} = \frac{4\,667(20)}{(0.5)(1)(30)10^6} = 0.00622 \text{ in} \quad \text{Ans.}$$



4-98 Procedure 1

1. Choose R_B as redundant reaction.

2. Statics. $R_C = wl - R_B \quad (1)$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) \quad (2)$$

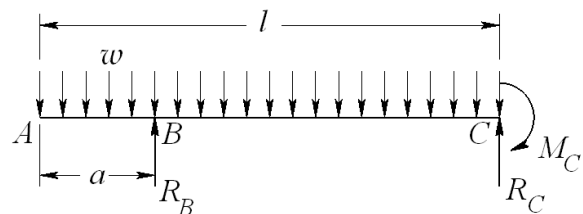
3. Deflection equation for point B. Superposition of beams 2 and 3 of Table A-9,

$$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{wl(l-a)^2}{24EI} [4l(l-a) - (l-a)^2 - 6l^2] = 0$$

4. Solving for R_B .

$$\begin{aligned} R_B &= \frac{wl}{8(l-a)} [6l^2 - 4l(l-a) + (l-a)^2] \\ &= \frac{wl}{8(l-a)} (3l^2 + 2al + a^2) \quad \text{Ans.} \end{aligned}$$

Substituting this into Eqs. (1) and (2) gives



$$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad \text{Ans.}$$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad \text{Ans.}$$

4-99 See figure in Prob. 4-98 solution.

Procedure 1

1. Choose R_B as redundant reaction.

2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) \quad (2)$$

3. Deflection equation for point B . Let the variable x start at point A and to the right. Using singularity functions, the bending moment as a function of x is

$$M = -\frac{1}{2}wx^2 + R_B \langle x-a \rangle^1 \quad \frac{\partial M}{\partial R_B} = \langle x-a \rangle^1$$

$$\begin{aligned} y_B = \frac{\partial U}{\partial R_B} &= \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial R_B} dx \\ &= \frac{1}{EI} \int_0^l -\frac{1}{2}wx^2(0) dx + \frac{1}{EI} \int_a^l \left[-\frac{1}{2}wx^2 + R_B(x-a) \right] (x-a) dx = 0 \end{aligned}$$

or,

$$-\frac{1}{2}w \left[\frac{1}{4}(l^4 - a^4) - \frac{a}{3}(l^3 - a^3) \right] + \frac{R_B}{3} [(l-a)^3 - (a-a)^3] = 0$$

Solving for R_B gives

$$R_B = \frac{w}{8(l-a)^3} [3(l^4 - a^4) - 4a(l^3 - a^3)] = \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad \text{Ans.}$$

From Eqs. (1) and (2)

$$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad \text{Ans.}$$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad \text{Ans.}$$

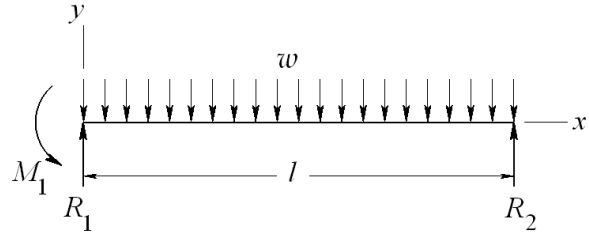
4-100 Note: When setting up the equations for this problem, no rounding of numbers was made in the calculations. It turns out that the deflection equation is very sensitive to rounding.

Procedure 2

1. Statics. $R_1 + R_2 = wl$ (1)

$$R_2 l + M_1 = \frac{1}{2} w l^2 \quad (2)$$

2. Bending moment equation.



$$M = R_1 x - \frac{1}{2} w x^2 - M_1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_1 x^2 - \frac{1}{6} w x^3 - M_1 x + C_1 \quad (3)$$

$$EI y = \frac{1}{6} R_1 x^3 - \frac{1}{24} w x^4 - \frac{1}{2} M_1 x^2 + C_1 x + C_2 \quad (4)$$

$$EI = 30(10^6)(0.85) = 25.5(10^6) \text{ lbf} \cdot \text{in}^2.$$

3. Boundary condition 1. At $x = 0$, $y = -R_1/k_1 = -R_1/[1.5(10^6)]$. Substitute into Eq. (4) with value of EI yields $C_2 = -17 R_1$.

Boundary condition 2. At $x = 0$, $dy/dx = -M_1/k_2 = -M_1/[2.5(10^6)]$. Substitute into Eq. (3) with value of EI yields $C_1 = -10.2 M_1$.

Boundary condition 3. At $x = l$, $y = -R_2/k_3 = -R_2/[2.0(10^6)]$. Substitute into Eq. (4) with value of EI yields

$$-12.75 R_2 = \frac{1}{6} R_1 l^3 - \frac{1}{24} w l^4 - \frac{1}{2} M_1 l^2 - 10.2 M_1 l - 17 R_1 \quad (5)$$

Equations (1), (2), and (5), written in matrix form with $w = 500/12 \text{ lbf/in}$ and $l = 24 \text{ in}$, are

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 24 & 1 \\ 2287 & 12.75 & -532.8 \end{pmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ M_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 12 \\ 576 \end{Bmatrix} (10^3)$$

Solving, the simultaneous equations yields

$$R_1 = 554.59 \text{ lbf}, R_2 = 445.41.59 \text{ lbf}, M_1 = 1310.1 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

For the deflection at $x = l/2 = 12 \text{ in}$, Eq. (4) gives

$$\begin{aligned}
 y|_{x=12\text{in}} &= \frac{1}{25.5(10^6)} \left[\frac{1}{6}(554.59)12^3 - \frac{1}{24} \frac{500}{12} 12^4 - \frac{1}{2}(1310.1)12^2 \right. \\
 &\quad \left. - 10.2(1310.1)12 - 17(554.59) \right] \\
 &= -5.51(10^{-3}) \text{ in} \quad \text{Ans.}
 \end{aligned}$$

4-101 Cable area, $A = \frac{\pi}{4}(0.5^2) = 0.1963 \text{ in}^2$

Procedure 2

1. Statics. $R_A + F_{BE} + F_{DF} = 5(10^3)$ (1)

$3 F_{DF} + F_{BE} = 10(10^3)$ (2)

2. Bending moment equation.

$$M = R_A x + F_{BE} \langle x - 16 \rangle^1 - 5000 \langle x - 32 \rangle^1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + \frac{1}{2} F_{BE} \langle x - 16 \rangle^2 - 2500 \langle x - 32 \rangle^2 + C_1 \quad (3)$$

$$EI y = \frac{1}{6} R_A x^3 + \frac{1}{6} F_{BE} \langle x - 16 \rangle^3 - \frac{2500}{3} \langle x - 32 \rangle^3 + C_1 x + C_2 \quad (4)$$

3. B.C. 1: At $x = 0$, $y = 0 \Rightarrow C_2 = 0$

B.C. 2: At $x = 16 \text{ in}$,

$$y_B = - \left(\frac{Fl}{AE} \right)_{BE} = - \frac{F_{BE}(38)}{0.1963(30)10^6} = -6.453(10^{-6}) F_{BE}$$

Substituting into Eq. (4) and evaluating at $x = 16 \text{ in}$

$$EI y_B = 30(10^6)(1.2)(-6.453)(10^{-6}) F_{BE} = \frac{1}{6} R_A (16^3) + C_1 (16)$$

Simplifying gives $682.7 R_A + 232.3 F_{BE} + 16 C_1 = 0$ (5)

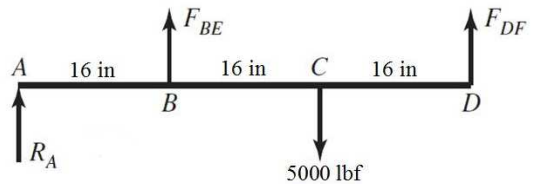
B.C. 2: At $x = 48 \text{ in}$,

$$y_D = - \left(\frac{Fl}{AE} \right)_{DF} = - \frac{F_{DF}(38)}{0.1963(30)10^6} = -6.453(10^{-6}) F_{DF}$$

Substituting into Eq. (4) and evaluating at $x = 48 \text{ in}$,

$$EI y_D = -232.3 F_{DF} = \frac{1}{6} R_A (48^3) + \frac{1}{6} F_{BE} (48 - 16)^3 - \frac{2500}{3} (48 - 32)^3 + 48 C_1$$

Simplifying gives $18\,432 R_A + 5\,461 F_{BE} + 232.3 F_{DF} + 48 C_1 = 3.413(10^6)$ (6)



Equations (1), (2), (5) and (6) in matrix form are

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 682.7 & 232.3 & 0 & 16 \\ 18432 & 5461 & 232.3 & 48 \end{pmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 5000 \\ 10000 \\ 0 \\ 3.413(10^6) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$$R_A = -970.5 \text{ lbf}, \quad F_{BE} = 3956 \text{ lbf}, \quad F_{DF} = 2015 \text{ lbf}, \quad \text{and } C_1 = -16\,020 \text{ lbf}\cdot\text{in}^2.$$

$$\sigma_{BE} = \frac{3956}{0.1963} = 20.2 \text{ kpsi}, \quad \sigma_{DF} = \frac{2015}{0.1963} = 10.3 \text{ kpsi} \quad \text{Ans.}$$

$$EI = 30(10^6)(1.2) = 36(10^6) \text{ lbf}\cdot\text{in}^2$$

$$\begin{aligned} y &= \frac{1}{36(10^6)} \left(-\frac{970.5}{6} x^3 + \frac{3956}{6} \langle x-16 \rangle^3 - \frac{2500}{3} \langle x-32 \rangle^3 - 16\,020x \right) \\ &= \frac{1}{36(10^6)} \left(-161.8x^3 + 659.3 \langle x-16 \rangle^3 - 833.3 \langle x-32 \rangle^3 - 16\,020x \right) \end{aligned}$$

$$B: x = 16 \text{ in}, \quad y_B = \frac{1}{36(10^6)} \left[-161.8(16^3) - 16\,020(16) \right] = -0.0255 \text{ in} \quad \text{Ans.}$$

$$C: x = 32 \text{ in},$$

$$\begin{aligned} y_C &= \frac{1}{36(10^6)} \left[-161.8(32^3) + 659.3(32-16)^3 - 16\,020(32) \right] \\ &= -0.0865 \text{ in} \quad \text{Ans.} \end{aligned}$$

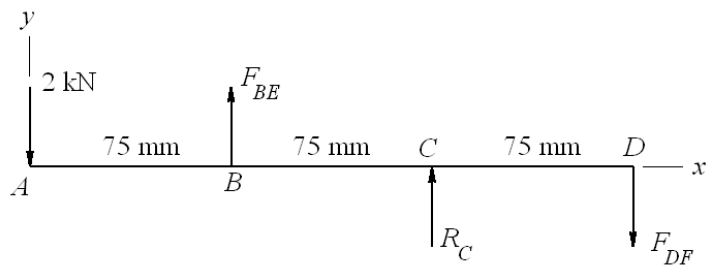
$$D: x = 48 \text{ in},$$

$$\begin{aligned} y_D &= \frac{1}{36(10^6)} \left[-161.8(48^3) + 659.3(48-16)^3 - 833.3(48-32)^3 - 16\,020(48) \right] \\ &= -0.0131 \text{ in} \quad \text{Ans.} \end{aligned}$$

4-102 Beam: $EI = 207(10^3)21(10^3)$
 $= 4.347(10^9) \text{ N}\cdot\text{mm}^2$.
 Rods: $A = (\pi/4)8^2 = 50.27 \text{ mm}^2$.

Procedure 2

1. Statics.



$$R_C + F_{BE} - F_{DF} = 2\,000 \quad (1)$$

$$R_C + 2F_{BE} = 6\,000 \quad (2)$$

2. Bending moment equation.

$$M = -2\,000x + F_{BE}\langle x - 75 \rangle^1 + R_C\langle x - 150 \rangle^1$$

$$EI \frac{dy}{dx} = -1000x^2 + \frac{1}{2}F_{BE}\langle x - 75 \rangle^2 + \frac{1}{2}R_C\langle x - 150 \rangle^2 + C_1 \quad (3)$$

$$EIy = -\frac{1000}{3}x^3 + \frac{1}{6}F_{BE}\langle x - 75 \rangle^3 + \frac{1}{6}R_C\langle x - 150 \rangle^3 + C_1x + C_2 \quad (4)$$

3. B.C 1. At $x = 75$ mm,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(50)}{50.27(207)10^3} = -4.805(10^{-6})F_{BE}$$

Substituting into Eq. (4) at $x = 75$ mm,

$$4.347(10^9)\left[-4.805(10^{-6})F_{BE}\right] = -\frac{1000}{3}(75^3) + C_1(75) + C_2$$

Simplifying gives

$$20.89(10^3)F_{BE} + 75C_1 + C_2 = 140.6(10^6) \quad (5)$$

B.C 2. At $x = 150$ mm, $y = 0$. From Eq. (4),

$$-\frac{1000}{3}(150^3) + \frac{1}{6}F_{BE}(150 - 75)^3 + C_1(150) + C_2 = 0$$

or,

$$70.31(10^3)F_{BE} + 150C_1 + C_2 = 1.125(10^9) \quad (6)$$

B.C 3. At $x = 225$ mm,

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(65)}{50.27(207)10^3} = 6.246(10^{-6})F_{DF}$$

Substituting into Eq. (4) at $x = 225$ mm,

$$4.347(10^9) \left[6.246(10^{-6}) F_{DF} \right] = -\frac{1000}{3} (225^3) + \frac{1}{6} F_{BE} (225 - 75)^3 \\ + \frac{1}{6} R_C (225 - 150)^3 + C_1 (225) + C_2$$

Simplifying gives

$$70.31(10^3) R_C + 562.5(10^3) F_{BE} - 27.15(10^3) F_{DF} + 225C_1 + C_2 = 3.797(10^9) \quad (7)$$

Equations (1), (2), (5), (6), and (7) in matrix form are

$$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 20.89(10^3) & 0 & 75 & 1 \\ 0 & 70.31(10^3) & 0 & 150 & 1 \\ 70.31(10^3) & 562.5(10^3) & -27.15(10^3) & 225 & 1 \end{pmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 2(10^3) \\ 6(10^3) \\ 140.6(10^6) \\ 1.125(10^9) \\ 3.797(10^9) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$$R_C = -2378 \text{ N}, F_{BE} = 4189 \text{ N}, F_{DF} = -189.2 \text{ N} \quad \text{Ans.} \\ \text{and } C_1 = 1.036(10^7) \text{ N}\cdot\text{mm}^2, C_2 = -7.243(10^8) \text{ N}\cdot\text{mm}^3.$$

The bolt stresses are $\sigma_{BE} = 4189/50.27 = 83.3 \text{ MPa}$, $\sigma_{DF} = -189/50.27 = -3.8 \text{ MPa}$ Ans.

The deflections are

$$\text{From Eq. (4)} \quad y_A = \frac{1}{4.347(10^9)} [-7.243(10^8)] = -0.167 \text{ mm} \quad \text{Ans.}$$

For points *B* and *D* use the axial deflection equations*.

$$y_B = -\left(\frac{Fl}{AE} \right)_{BE} = -\frac{4189(50)}{50.27(207)10^3} = -0.0201 \text{ mm} \quad \text{Ans.}$$

$$y_D = \left(\frac{Fl}{AE} \right)_{DF} = \frac{-189(65)}{50.27(207)10^3} = -1.18(10^{-3}) \text{ mm} \quad \text{Ans.}$$

*Note. The terms in Eq. (4) are quite large, and due to rounding are not very accurate for calculating the very small deflections, especially for point *D*.

4-103 (a) The cross section at *A* does not rotate. Thus, for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{FR}{2}(1 - \cos \theta) \quad \frac{\partial M}{\partial M_A} = 1$$

The rotation at A is

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI} \int_0^{\pi/2} M \frac{\partial M}{\partial M_A} R d\theta = 0$$

$$\text{Thus, } \frac{1}{EI} \int_0^{\pi/2} \left[M_A - \frac{FR}{2}(1 - \cos \theta) \right] (1) R d\theta = 0 \quad \Rightarrow \quad \left(M_A - \frac{FR}{2} \right) \frac{\pi}{2} + \frac{FR}{2} = 0$$

or,

$$M_A = \frac{FR}{2} \left(1 - \frac{2}{\pi} \right)$$

Substituting this into the equation for M gives

$$M = \frac{FR}{2} \left(\cos \theta - \frac{2}{\pi} \right) \quad (1)$$

The maximum occurs at B where $\theta = \pi/2$

$$M_{\max} = M_B = -\frac{FR}{\pi} \quad \text{Ans.}$$

(b) Assume B is supported on a knife edge. The deflection of point D is $\partial U / \partial F$. We will deal with the quarter-ring segment and multiply the results by 4. From Eq. (1)

$$\frac{\partial M}{\partial F} = \frac{R}{2} \left(\cos \theta - \frac{2}{\pi} \right)$$

Thus,

$$\begin{aligned} \delta_D &= \frac{\partial U}{\partial F} = \frac{4}{EI} \int_0^{\pi/2} M \frac{\partial M}{\partial F} R d\theta = \frac{FR^3}{EI} \int_0^{\pi/2} \left(\cos \theta - \frac{2}{\pi} \right)^2 d\theta = \frac{FR^3}{EI} \left(\frac{\pi}{4} - \frac{2}{\pi} \right) \\ &= \frac{FR^3}{4\pi EI} (\pi^2 - 8) \quad \text{Ans.} \end{aligned}$$

4-104

$$P_{\text{cr}} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi D^4}{64} (1 - K^4) \quad \text{where } K = \frac{d}{D}$$

$$P_{\text{cr}} = \frac{C\pi^2 E}{l^2} \left[\frac{\pi D^4}{64} (1 - K^4) \right]$$

$$D = \left[\frac{64P_{\text{cr}}l^2}{\pi^3 CE(1-K^4)} \right]^{1/4} \quad \text{Ans.}$$

4-105 $A = \frac{\pi}{4} D^2 (1-K^2)$, $I = \frac{\pi}{64} D^4 (1-K^4) = \frac{\pi}{64} D^4 (1-K^2)(1+K^2)$, where $K = d/D$.

The radius of gyration, k , is given by

$$k^2 = \frac{I}{A} = \frac{D^2}{16} (1+K^2)$$

From Eq. (4-46)

$$\begin{aligned} \frac{P_{\text{cr}}}{(\pi/4) D^2 (1-K^2)} &= S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 CE} = S_y - \frac{S_y^2 l^2}{4\pi^2 (D^2/16)(1+K^2) CE} \\ 4P_{\text{cr}} &= \pi D^2 (1-K^2) S_y - \frac{4S_y^2 l^2 \pi D^2 (1-K^2)}{\pi^2 D^2 (1+K^2) CE} \\ \pi D^2 (1-K^2) S_y &= 4P_{\text{cr}} + \frac{4S_y^2 l^2 (1-K^2)}{\pi (1+K^2) CE} \\ D &= \left[\frac{4P_{\text{cr}}}{\pi S_y (1-K^2)} + \frac{4S_y^2 l^2 (1-K^2)}{\pi (1+K^2) CE \pi (1-K^2) S_y} \right]^{1/2} \\ &= 2 \left[\frac{P_{\text{cr}}}{\pi S_y (1-K^2)} + \frac{S_y l^2}{\pi^2 CE (1+K^2)} \right]^{1/2} \quad \text{Ans.} \end{aligned}$$

4-106 (a) $\Sigma M_A = 0$, $(0.75)(800) - \frac{0.9}{\sqrt{0.9^2 + 0.5^2}} F_{BO}(0.5) = 0 \Rightarrow F_{BO} = 1373 \text{ N}$

Using $n_d = 4$, design for $F_{\text{cr}} = n_d F_{BO} = 4(1373) = 5492 \text{ N}$

$$l = \sqrt{0.9^2 + 0.5^2} = 1.03 \text{ m}, \quad S_y = 165 \text{ MPa}$$

In-plane:

$$k = \left(\frac{I}{A} \right)^{1/2} = \left(\frac{bh^3/12}{bh} \right)^{1/2} = 0.2887h = 0.2887(0.025) = 0.007218 \text{ m}, \quad C = 1.0$$

$$\frac{l}{k} = \frac{1.03}{0.007218} = 142.7$$

$$\left(\frac{l}{k} \right)_1 = \left(\frac{2\pi^2 (207)(10^9)}{165(10^6)} \right)^{1/2} = 157.4$$

Since $(l/k)_1 > (l/k)$ use Johnson formula.

Try 25 mm x 12 mm,

$$P_{cr} = 0.025(0.012) \left\{ 165(10^6) - \left[\frac{165(10^6)}{2\pi} (142.7) \right]^2 \frac{1}{1(207)10^9} \right\} = 29.1 \text{ kN}$$

This is significantly greater than the design load of 5492 N found earlier. Check out-of-plane.

Out-of-plane: $k = 0.2887(0.012) = 0.003464 \text{ in}$, $C = 1.2$

$$\frac{l}{k} = \frac{1.03}{0.003464} = 297.3$$

Since $(l/k)_1 < (l/k)$ use Euler equation.

$$P_{cr} = 0.025(0.012) \frac{1.2\pi^2(207)10^9}{297.3^2} = 8321 \text{ N}$$

This is greater than the design load of 5492 N found earlier. It is also significantly less than the in-plane P_{cr} found earlier, so the out-of-plane condition will dominate. Iterate the process to find the minimum h that gives P_{cr} greater than the design load.

With $h = 0.010$, $P_{cr} = 4815 \text{ N}$ (too small)

$h = 0.011$, $P_{cr} = 6409 \text{ N}$ (acceptable)

Use 25 mm x 11 mm. If standard size is preferred, use 25 mm x 12 mm. *Ans.*

$$(b) \sigma_b = -\frac{P}{dh} = -\frac{1373}{0.012(0.011)} = -10.4(10^6) \text{ Pa} = -10.4 \text{ MPa}$$

No, bearing stress is not significant. *Ans.*

4-107 This is an open-ended design problem with no one distinct solution.

4-108 $F = 1500(\pi/4)2^2 = 4712 \text{ lbf}$. From Table A-20, $S_y = 37.5 \text{ kpsi}$
 $P_{cr} = n_d F = 2.5(4712) = 11780 \text{ lbf}$

(a) Assume Euler with $C = 1$

$$I = \frac{\pi}{64} d^4 = \frac{P_{cr} l^2}{C\pi^2 E} \Rightarrow d = \left(\frac{64 P_{cr} l^2}{\pi^3 C E} \right)^{1/4} = \left[\frac{64(11780)50^2}{\pi^3(1)30(10^6)} \right]^{1/4} = 1.193 \text{ in}$$

Use $d = 1.25 \text{ in}$. The radius of gyration, $k = (I/A)^{1/2} = d/4 = 0.3125 \text{ in}$

$$\frac{l}{k} = \frac{50}{0.3125} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left(\frac{2\pi^2(1)30(10^6)}{37.5(10^3)}\right)^{1/2} = 126 \quad \therefore \text{use Euler}$$

$$P_{cr} = \frac{\pi^2(30)10^6(\pi/64)1.25^4}{50^2} = 14194 \text{ lbf}$$

Since 14 194 lbf > 11 780 lbf, $d = 1.25$ in is satisfactory. *Ans.*

(b)
$$d = \left[\frac{64(11780)16^2}{\pi^3(1)30(10^6)} \right]^{1/4} = 0.675 \text{ in, so use } d = 0.750 \text{ in}$$

$$k = 0.750/4 = 0.1875 \text{ in}$$

$$\frac{l}{k} = \frac{16}{0.1875} = 85.33 \quad \text{use Johnson}$$

$$P_{cr} = \frac{\pi}{4}(0.750^2) \left\{ 37.5(10^3) - \left[\frac{37.5(10^3)}{2\pi} 85.33 \right]^2 \frac{1}{1(30)10^6} \right\} = 12748 \text{ lbf}$$

Use $d = 0.75$ in.

(c)

$$n_{(a)} = \frac{14194}{4712} = 3.01 \quad \text{Ans.}$$

$$n_{(b)} = \frac{12748}{4712} = 2.71 \quad \text{Ans.}$$

4-109 From Table A-20, $S_y = 180$ MPa

$$4F \sin \theta = 2943$$

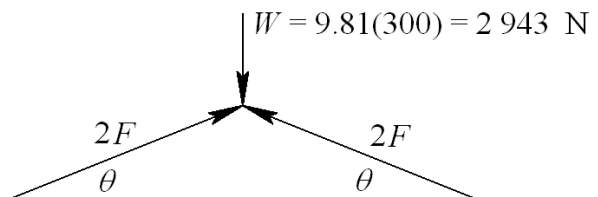
$$F = \frac{735.8}{\sin \theta}$$

In range of operation, F is maximum when $\theta = 15^\circ$

$$F_{\max} = \frac{735.8}{\sin 15^\circ} = 2843 \text{ N per bar}$$

$$P_{cr} = n_d F_{\max} = 3.50(2843) = 9951 \text{ N}$$

$$l = 350 \text{ mm, } h = 30 \text{ mm}$$



Try $b = 5$ mm. Out of plane, $k = b / \sqrt{12} = 5 / \sqrt{12} = 1.443$ mm

$$\frac{l}{k} = \frac{350}{1.443} = 242.6$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{2\pi^2 (1.4) 207 (10^9)}{180 (10^6)} \right]^{1/2} = 178.3 \quad \therefore \text{use Euler}$$

$$P_{cr} = A \frac{C\pi^2 E}{(l/k)^2} = 5(30) \frac{1.4\pi^2 (207) 10^3}{(242.6)^2} = 7\,290 \text{ N}$$

Too low. Try $b = 6$ mm. $k = 6 / \sqrt{12} = 1.732$ mm

$$\frac{l}{k} = \frac{350}{1.732} = 202.1$$

$$P_{cr} = A \frac{C\pi^2 E}{(l/k)^2} = 6(30) \frac{1.4\pi^2 (207) 10^3}{(202.1)^2} = 12\,605 \text{ N}$$

O.K. Use 25×6 mm bars *Ans.* The factor of safety is

$$n = \frac{12\,605}{2\,843} = 4.43 \quad \text{Ans.}$$

4-110 $P = 1\,500 + 9\,000 = 10\,500$ lbf *Ans.*

$$\Sigma M_A = 10\,500 (4.5/2) - 9\,000 (4.5) + M = 0$$

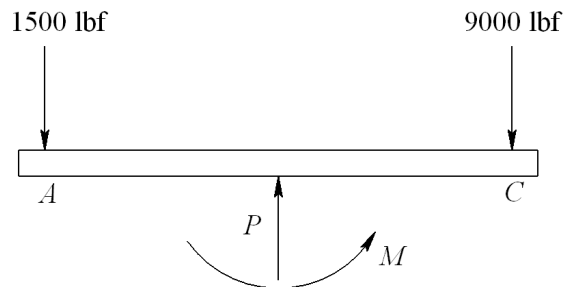
$$M = 16\,874 \text{ lbf}\cdot\text{in}$$

$$e = M / P = 16\,874 / 10\,500 = 1.607 \text{ in} \quad \text{Ans.}$$

From Table A-8, $A = 2.160 \text{ in}^2$, and $I = 2.059 \text{ in}^4$. The stresses are determined using Eq. (4-55)

$$k^2 = \frac{I}{A} = \frac{2.059}{2.160} = 0.953 \text{ in}^2$$

$$\sigma_c = -\frac{P}{A} \left(1 + \frac{ec}{k^2} \right) = -\frac{10\,500}{2.160} \left[1 + \frac{1.607(3/2)}{0.953} \right] = -17\,157 \text{ psi} = -17.16 \text{ kpsi} \quad \text{Ans.}$$



4-111 This is a design problem which has no single distinct solution.

4-112 Loss of potential energy of weight = $W(h + \delta)$

Increase in potential energy of spring = $\frac{1}{2}k\delta^2$

$$W(h + \delta) = \frac{1}{2}k\delta^2$$

or, $\delta^2 - \frac{2W}{k}\delta - \frac{2W}{k}h = 0$. $W = 30$ lbf, $k = 100$ lbf/in, $h = 2$ in yields

$$\delta^2 - 0.6\delta - 1.2 = 0$$

Taking the positive root (see discussion on p. 206 of text)

$$\delta_{\max} = \frac{1}{2} \left[0.6 + \sqrt{(-0.6)^2 + 4(1.2)} \right] = 1.436 \text{ in} \quad \text{Ans.}$$

$$F_{\max} = k \delta_{\max} = 100 (1.436) = 143.6 \text{ lbf} \quad \text{Ans.}$$

4-113 The drop of weight W_1 converts potential energy, $W_1 h$, to kinetic energy $\frac{1}{2} \frac{W_1}{g} v_1^2$.

Equating these provides the velocity of W_1 at impact with W_2 .

$$W_1 h = \frac{1}{2} \frac{W_1}{g} v_1^2 \quad \Rightarrow \quad v_1 = \sqrt{2gh} \quad (1)$$

Since the collision is inelastic, momentum is conserved. That is, $(m_1 + m_2) v_2 = m_1 v_1$, where v_2 is the velocity of $W_1 + W_2$ after impact. Thus

$$\frac{W_1 + W_2}{g} v_2 = \frac{W_1}{g} v_1 \quad \Rightarrow \quad v_2 = \frac{W_1}{W_1 + W_2} v_1 = \frac{W_1}{W_1 + W_2} \sqrt{2gh} \quad (2)$$

The kinetic and potential energies of $W_1 + W_2$ are then converted to potential energy of the spring. Thus,

$$\frac{1}{2} \frac{W_1 + W_2}{g} v_2^2 + (W_1 + W_2) \delta = \frac{1}{2} k \delta^2$$

Substituting in Eq. (1) and rearranging results in

$$\delta^2 - 2 \frac{W_1 + W_2}{k} \delta - 2 \frac{W_1^2}{W_1 + W_2} \frac{h}{k} = 0 \quad (3)$$

Solving for the positive root (see discussion on p. 206)

$$\delta = \frac{1}{2} \left[2 \frac{W_1 + W_2}{k} + \sqrt{4 \left(\frac{W_1 + W_2}{k} \right)^2 + 8 \frac{W_1^2}{W_1 + W_2} \frac{h}{k}} \right] \quad (4)$$

$W_1 = 40 \text{ N}$, $W_2 = 400 \text{ N}$, $h = 200 \text{ mm}$, $k = 32 \text{ kN/m} = 32 \text{ N/mm}$.

$$\delta = \frac{1}{2} \left[2 \left(\frac{40 + 400}{32} \right) + \sqrt{4 \left(\frac{40 + 400}{32} \right)^2 + 8 \frac{40^2}{40 + 400} \frac{200}{32}} \right] = 29.06 \text{ mm} \quad \text{Ans.}$$

$$F_{\max} = k\delta = 32(29.06) = 930 \text{ N} \quad \text{Ans.}$$

4-114 The initial potential energy of the k_1 spring is $V_i = \frac{1}{2} k_1 a^2$. The movement of the weight W the distance y gives a final potential of $V_f = \frac{1}{2} k_1 (a - y)^2 + \frac{1}{2} k_2 y^2$. Equating the two energies give

$$\frac{1}{2} k_1 a^2 = \frac{1}{2} k_1 (a - y)^2 + \frac{1}{2} k_2 y^2$$

Simplifying gives

$$(k_1 + k_2) y^2 - 2ak_1 y = 0$$

This has two roots, $y = 0$, $\frac{2k_1 a}{k_1 + k_2}$. Without damping the weight will vibrate between

these two limits. The maximum displacement is thus $y_{\max} = \frac{2k_1 a}{k_1 + k_2}$ *Ans.*

With $W = 5 \text{ lbf}$, $k_1 = 10 \text{ lbf/in}$, $k_2 = 20 \text{ lbf/in}$, and $a = 0.25 \text{ in}$

$$y_{\max} = \frac{2(0.25)10}{10 + 20} = 0.1667 \text{ in} \quad \text{Ans.}$$
