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1. (2 points) Prove that equation $T(n) = 2^n$ is true, given the equation $T(n) = 1 + \sum_{j=0}^{n-1} T(j)$ and the initial condition $T(0) = 1$ of the rod-cutting problem. Use the inductive proof for your solution.

   Hint: $\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$

when $n=0$ $T(0)=1$.

and $n=1$ $T(1) = 1 + \sum_{j=0}^{0} T(j) = 1 + T(0) = 2$ $T(1) = 2^1 = 2$

$\Rightarrow$ when $n=1$ $T(1)=2$, So $T(n) = 2^n$ also satisfy

And then we need to show if $T(n) = 2^n$ is true

then $T(n+1) = 2^{n+1}$ is also true

Given equation, $T(n+1) = 1 + \sum_{j=0}^{n} T(j) = 1 + \sum_{j=0}^{n} 2^j$

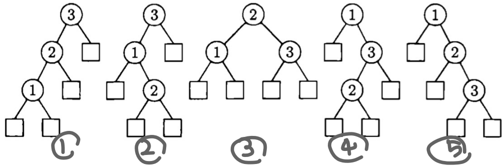
$= 1 + \frac{2^{n+1}-1}{2-1} = 2^{n+1}$ (by hint)

$T(0)$, $T(1)$ is true, and If $T(n)$ is true, then

$T(n+1)$ also true. So we can say $T(n) = 2^n$

2. (3 points) Let's assume that there is a binary search tree having three nodes, and their keys $K_1 < K_2 < K_3$ have respective probabilities 0.2, 0.5, and 0.3 for searching (i.e., we search for the first, second, and third node of the probabilities of 0.2, 0.5, and 0.3). What is the expected number of comparisons required to search for a node?

Hint: Given three nodes with their keys $K_1 < K_2 < K_3$, there are five possible trees as shown below (by assuming they are equally likely). In the figure, the circles denote each node, and the rectangles denote NIL. For example, when we search for node 1 in the first tree, the number of comparisons required to find it is 3. Similarly, when we search for node 3 in the fourth tree, the number of comparisons is 2.



not $\left(\frac{1}{6}\ \frac{1}{6}\ \frac{2}{6}\ \frac{1}{6}\ \frac{1}{6}\right)$

by assuming they are equally likely, so each tree probability $\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}$

$K_1 = 1 \quad K_2 = 2 \quad K_3 = 3$

$(0.2) \quad (0.5) \quad (0.3)$ (probabilyty)

Total Expect value $= \sum$ (tree probability $\times$ key proba $\times$ expect value)

① $= \frac{1}{5} \times 0.2 \times 3 + \frac{1}{5} \times 0.5 \times 2 + \frac{1}{5} \times 0.3 \times 1 = \frac{1.9}{5}$

② $= \frac{1}{5}(0.2 \times 2 + 0.5 \times 3 + 0.3 \times 1) = \frac{2.2}{5}$

③ $= \frac{1}{5}(0.2 \times 2 + 0.5 \times 1 + 0.3 \times 2) = \frac{1.5}{5}$

④ $= \frac{1}{5}(0.2 \times 1 + 0.5 \times 3 + 0.3 \times 2) = \frac{2.3}{5}$

⑤ $= \frac{1}{5}(0.2 \times 1 + 0.5 \times 2 + 0.3 \times 3) = \frac{2.1}{5}$

① + ② + ③ + ④ + ⑤ $= \frac{10}{5} = 2$

- What is the maximum number of edges in an undirected graph with V vertices and no parallel edges? Parallel edges (also called multiple edges or a multi-edge), are, in an undirected graph, two or more edges that are incident to the same two vertices.

Let's assume n vertices Graph.

First vertice can have (n-1) edges and second has (n-2) edses (because no parallel edges) then (n-3), (n-4) ..., to 0.

So, the maximum number of edses is

$$(n-1) + (n-2) + \cdots + 1 + 0 = \frac{n(n-1)}{2}$$

- What is the minimum number of edges in an undirected graph with V vertices, none of which are isolated?

none of isolated. So When every Vertice has one edses, the edses number is minimum.

So the minimum number of edses In undirected graph is $n-1$