

Algorithm HW 2

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1. partition algorithm pseudocode is below.
The subarray size $n = r - p + 1$

PARTITION (A, p, r)

- ① $x = A[r]$ \rightarrow Const time
 - ② $i = p - 1$ \rightarrow Const time
 - ③ for $j = p$ to $r - 1$ \rightarrow for loop $O(n)$
 - ④ if $A[j] \leq x$ \rightarrow Const time
 - ⑤ $i = i + 1$ \rightarrow Const time
 - ⑥ exchange $A[i]$ with $A[j]$ \rightarrow Const time
 - ⑦ exchange $A[i + 1]$ with $A[r]$ \rightarrow Const time
 - ⑧ return $i + 1$ \rightarrow Const time
- for loop

$$\begin{aligned} \text{Time complex} &= ① + ② + ③ \times (④ + ⑤ + ⑥) + ⑦ + ⑧ \\ &= n_1 \Theta(1) + n_2 \Theta(n) \end{aligned}$$

We can discard multiplicative constant and low degree.
So, PARTITION procedure of quick sort on a subarray of size n is $\Theta(n)$

2. Chebyshev's inequality is

$$P[|X - \mu| \geq k] \leq \frac{\sigma^2}{k^2} \Leftrightarrow P[\mu - k\sigma < X < \mu + k\sigma] \geq 1 - \frac{1}{k^2}$$

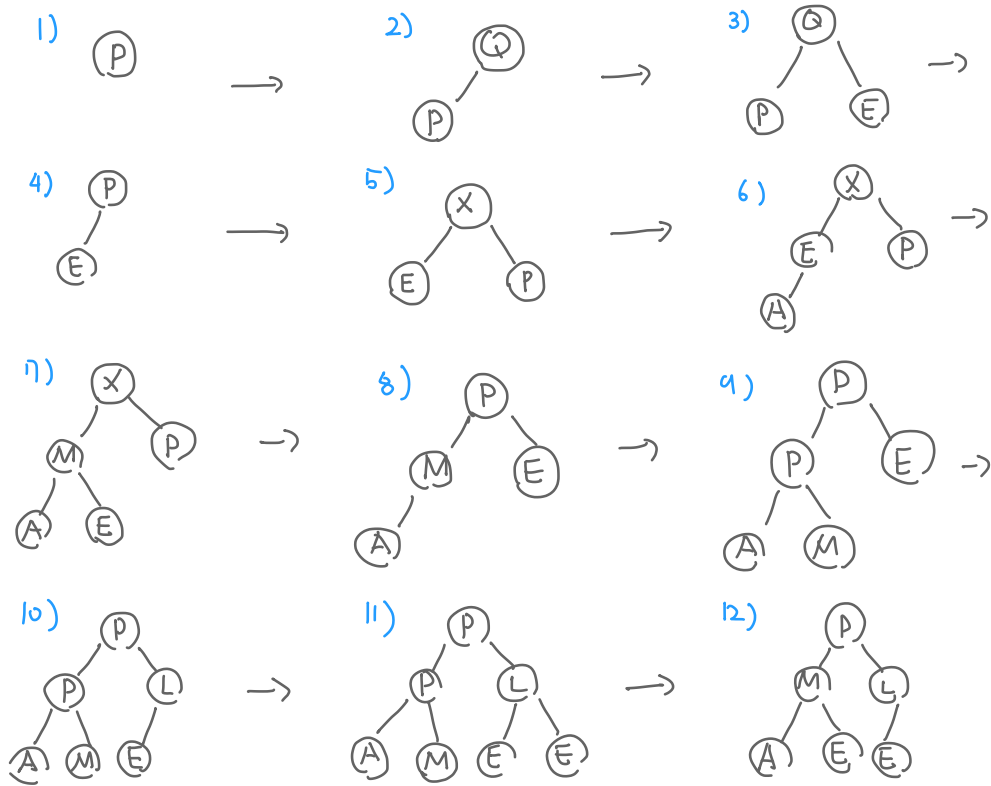
μ (number of Quick sort): $2N \ln N = 27631621.12 = 2.76 \times 10^7$
 σ (standard deviation): $0.65N = 6.5 \times 10^5$

$$\begin{aligned} & \underbrace{\hspace{10em}}_{10^8} \Rightarrow P[|X - 2.76 \times 10^7| \geq \overset{k}{7.24 \times 10^7}] \\ & = (P[X \geq 10^8] \text{ and } P[X \leq -4.48 \times 10^7]) \leq \frac{\sigma^2}{k^2} \end{aligned}$$

$$P[X \leq -4.48 \times 10^7] = 0,$$

$$\text{So } P[X \geq 10^8] \leq \frac{(6.5 \times 10^5)^2}{(7.24 \times 10^7)^2} = \underline{8.06 \times 10^{-5}}$$

3. First I draw tree and draw priority queue.



Let's draw priority queue by reference tree. (Priority queue doesn't use index 0.)
 ↳ (I used array. Linked list also okay)

