

Chapter 6

6-1 Eq. (2-21): $S_{ut} = 3.4H_B = 3.4(300) = 1020 \text{ MPa}$
 Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(1020) = 510 \text{ MPa}$
 Table 6-2: $a = 1.58, b = -0.085$
 Eq. (6-19): $k_a = aS_{ut}^b = 1.58(1020)^{-0.085} = 0.877$
 Eq. (6-20): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$
 Eq. (6-18): $S_e = k_a k_b S'_e = (0.877)(0.969)(510) = 433 \text{ MPa} \quad \text{Ans.}$

6-2 (a) Table A-20: $S_{ut} = 80 \text{ kpsi}$
 Eq. (6-8): $S'_e = 0.5(80) = 40 \text{ kpsi} \quad \text{Ans.}$
 (b) Table A-20: $S_{ut} = 90 \text{ kpsi}$
 Eq. (6-8): $S'_e = 0.5(90) = 45 \text{ kpsi} \quad \text{Ans.}$
 (c) Aluminum has no endurance limit. *Ans.*
 (d) Eq. (6-8): $S_{ut} > 200 \text{ kpsi}, S'_e = 100 \text{ kpsi} \quad \text{Ans.}$

6-3 $S_{ut} = 120 \text{ kpsi}, \sigma_{rev} = 70 \text{ kpsi}$
 Fig. 6-18: $f = 0.82$
 Eq. (6-8): $S'_e = S_e = 0.5(120) = 60 \text{ kpsi}$
 Eq. (6-14): $a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.82(120)]^2}{60} = 161.4 \text{ kpsi}$
 Eq. (6-15): $b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.82(120)}{60} \right) = -0.0716$
 Eq. (6-16): $N = \left(\frac{\sigma_{rev}}{a} \right)^{1/b} = \left(\frac{70}{161.4} \right)^{\frac{1}{-0.0716}} = 116\,700 \text{ cycles} \quad \text{Ans.}$

6-4 $S_{ut} = 1600 \text{ MPa}, \sigma_{rev} = 900 \text{ MPa}$
 Fig. 6-18: $S_{ut} = 1600 \text{ MPa} = 232 \text{ kpsi}$. Off the graph, so estimate $f = 0.77$.
 Eq. (6-8): $S_{ut} > 1400 \text{ MPa}$, so $S_e = 700 \text{ MPa}$
 Eq. (6-14): $a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.77(1600)]^2}{700} = 2168.3 \text{ MPa}$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.77(1600)}{700} \right) = -0.081838$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{900}{2168.3} \right)^{-\frac{1}{-0.081838}} = 46\,400 \text{ cycles } \textit{Ans.}$$

6-5 $S_{ut} = 230 \text{ kpsi}$, $N = 150\,000 \text{ cycles}$

Fig. 6-18, point is off the graph, so estimate: $f = 0.77$

Eq. (6-8): $S_{ut} > 200 \text{ kpsi}$, so $S'_e = S_e = 100 \text{ kpsi}$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.77(230)]^2}{100} = 313.6 \text{ kpsi}$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.77(230)}{100} \right) = -0.08274$$

$$\text{Eq. (6-13): } S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0 \text{ kpsi } \textit{Ans.}$$

6-6 $S_{ut} = 1100 \text{ MPa} = 160 \text{ kpsi}$

Fig. 6-18: $f = 0.79$

Eq. (6-8): $S'_e = S_e = 0.5(1100) = 550 \text{ MPa}$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.79(1100)]^2}{550} = 1373 \text{ MPa}$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.79(1100)}{550} \right) = -0.06622$$

$$\text{Eq. (6-13): } S_f = aN^b = 1373(150\,000)^{-0.06622} = 624 \text{ MPa } \textit{Ans.}$$

6-7 $S_{ut} = 150 \text{ kpsi}$, $S_{yt} = 135 \text{ kpsi}$, $N = 500 \text{ cycles}$

Fig. 6-18: $f = 0.798$

From Fig. 6-10, we note that below 10^3 cycles on the S - N diagram constitutes the low-cycle region, in which Eq. (6-17) is applicable.

Eq. (6-17): $S_f = S_{ut} N^{(\log f)/3} = 150(500)^{[\log(0.798)]/3} = 122 \text{ kpsi}$ *Ans.*

The testing should be done at a completely reversed stress of 122 kpsi, which is below the yield strength, so it is possible. *Ans.*

- 6-8** The general equation for a line on a $\log S_f - \log N$ scale is $S_f = aN^b$, which is Eq. (6-13). By taking the log of both sides, we can get the equation of the line in slope-intercept form.

$$\log S_f = b \log N + \log a$$

Substitute the two known points to solve for unknowns a and b . Substituting point (1, S_{ut}),

$$\log S_{ut} = b \log(1) + \log a$$

From which $a = S_{ut}$. Substituting point (10^3 , $f S_{ut}$) and $a = S_{ut}$

$$\log f S_{ut} = b \log 10^3 + \log S_{ut}$$

From which $b = (1/3) \log f$

$$\therefore S_f = S_{ut} N^{(\log f)/3} \quad 1 \leq N \leq 10^3$$

- 6-9** Read from graph: (10^3 , 90) and (10^6 , 50). From $S = aN^b$

$$\log S_1 = \log a + b \log N_1$$

$$\log S_2 = \log a + b \log N_2$$

From which

$$\begin{aligned} \log a &= \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2 / N_1} \\ &= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6 / 10^3} \\ &= 2.2095 \end{aligned}$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0 \text{ kpsi}$$

$$b = \frac{\log 50 / 90}{3} = -0.0851$$

$$(S_f)_{ax} = 162 N^{-0.0851} \quad 10^3 \leq N \leq 10^6 \text{ in kpsi} \quad \text{Ans.}$$

Check:

$$\left[(S_f)_{ax} \right]_{10^3} = 162(10^3)^{-0.0851} = 90 \text{ kpsi}$$

$$\left[(S_f)_{ax} \right]_{10^6} = 162(10^6)^{-0.0851} = 50 \text{ kpsi}$$

The end points agree.

6-10 $d = 1.5 \text{ in}$, $S_{ut} = 110 \text{ kpsi}$

Eq. (6-8): $S'_e = 0.5(110) = 55 \text{ kpsi}$

Table 6-2: $a = 2.70$, $b = -0.265$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(110)^{-0.265} = 0.777$

Eq. (6-20): $k_b = 0.879 d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$

Eq. (6-18): $S_e = k_a k_b S'_e = 0.777(0.842)(55) = 36.0 \text{ kpsi}$ *Ans.*

6-11 For AISI 4340 as-forged steel,

Eq. (6-8): $S_e = 100 \text{ kpsi}$

Table 6-2: $a = 39.9$, $b = -0.995$

Eq. (6-19): $k_a = 39.9(260)^{-0.995} = 0.158$

Eq. (6-20): $k_b = \left(\frac{0.75}{0.30} \right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi} \quad \text{Ans.}$$

For AISI 1040:

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$

$$k_a = 39.9(113)^{-0.995} = 0.362$$

$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other modifying factors is unity

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi} \quad \text{Ans.}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength. *Ans.*

6-12 $D = 1 \text{ in}$, $d = 0.8 \text{ in}$, $T = 1800 \text{ lbf}\cdot\text{in}$, $f = 0.9$, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68 \text{ kpsi}$, and $S_y = 57 \text{ kpsi}$.

(a) Fig. A-15-15: $\frac{r}{d} = \frac{0.1}{0.8} = 0.125, \frac{D}{d} = \frac{1}{0.8} = 1.25, K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-21, or from the curve-fit Eqs. (6-34) and (6-35b). Using the equations,

$$\sqrt{a} = 0.190 - 2.51(10^{-3})(68) + 1.35(10^{-5})(68)^2 - 2.67(10^{-8})(68^3) = 0.07335$$

$$q_s = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800 \text{ lbf}\cdot\text{in}$,

$$\tau_a = K_{fs} \frac{Tr}{J} = \frac{K_{fs} 16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-8): $S'_e = 0.5(68) = 34 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Eq. (6-20): $k_b = 0.879(0.8)^{-0.107} = 0.900$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18) (labeling for shear): $S_{se} = 0.883(0.900)(0.59)(34) = 15.9 \text{ kpsi}$

For purely reversing torsion, use Eq. (6-54) for the ultimate strength in shear.

Eq. (6-54): $S_{su} = 0.67 S_{ut} = 0.67(68) = 45.6 \text{ kpsi}$

Adjusting the fatigue strength equations for shear,

Eq. (6-14): $a = \frac{(f S_{su})^2}{S_{se}} = \frac{[0.9(45.6)]^2}{15.9} = 105.9 \text{ kpsi}$

Eq. (6-15): $b = -\frac{1}{3} \log \left(\frac{f S_{su}}{S_{se}} \right) = -\frac{1}{3} \log \left(\frac{0.9(45.6)}{15.9} \right) = -0.137\,27$

Eq. (6-16): $N = \left(\frac{\tau_a}{a} \right)^{\frac{1}{b}} = \left(\frac{23.3}{105.9} \right)^{-\frac{1}{0.137\,27}} = 61.7(10^3) \text{ cycles} \quad \text{Ans.}$

(b) For an operating temperature of 750°F , the temperature modification factor, from Table 6-4 is $k_d = 0.90$.

$$S_{se} = 0.883(0.900)(0.59)(0.9)(34) = 14.3 \text{ kpsi}$$

$$a = \frac{(f S_{su})^2}{S_{se}} = \frac{[0.9(45.6)]^2}{14.3} = 117.8 \text{ kpsi}$$

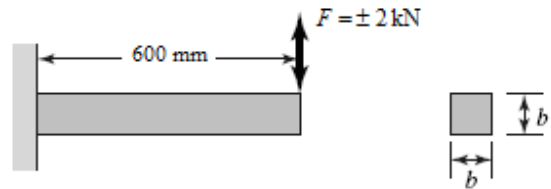
$$b = -\frac{1}{3} \log \left(\frac{f S_{su}}{S_{se}} \right) = -\frac{1}{3} \log \left(\frac{0.9(45.6)}{14.3} \right) = -0.152 \text{ 62}$$

$$N = \left(\frac{\tau_a}{a} \right)^{\frac{1}{b}} = \left(\frac{23.3}{117.8} \right)^{-\frac{1}{0.152 \text{ 62}}} = 40.9(10^3) \text{ cycles} \quad \text{Ans.}$$

- 6-13** $L = 0.6 \text{ m}$, $F_a = 2 \text{ kN}$, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770 \text{ MPa}$, $S_y = 420 \text{ MPa}$ (Table A-20)
First evaluate the fatigue strength.

$$S'_e = 0.5(770) = 385 \text{ MPa}$$

$$k_a = 57.7(770)^{-0.718} = 0.488$$



Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later.
All other modifiers are equal to one.

$$\text{Eq. (6-18): } S_e = 0.488(0.85)(385) = 160 \text{ MPa}$$

$$\text{In kpsi, } S_{ut} = 770/6.89 = 112 \text{ kpsi}$$

$$\text{Fig. 6-18: } f = 0.83$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{160} = 2553 \text{ MPa}$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.83(770)}{160} \right) = -0.2005$$

$$\text{Eq. (6-13): } S_f = aN^b = 2553(10^4)^{-0.2005} = 403 \text{ MPa}$$

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N} \cdot \text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b .

$$n = \frac{S_f}{\sigma_a} = \frac{403(10^6)}{7200/b^3} = 1.5$$

$$b = 0.0299 \text{ m} \quad \text{Select } b = 30 \text{ mm.}$$

Since the size factor was guessed, go back and check it now.

$$\text{Eq. (6-25):} \quad d_e = 0.808(hb)^{1/2} = 0.808b = 0.808(30) = 24.24 \text{ mm}$$

$$\text{Eq. (6-20):} \quad k_b = \left(\frac{24.2}{7.62} \right)^{-0.107} = 0.88$$

Our guess of 0.85 was slightly conservative, so we will accept the result of

$$b = 30 \text{ mm.} \quad \text{Ans.}$$

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3} (10^{-6}) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

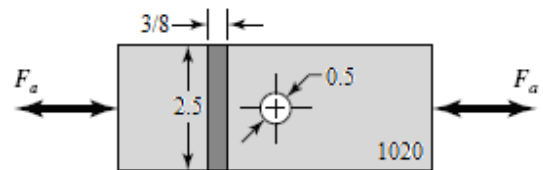
6-14 Given: $w = 2.5 \text{ in}$, $t = 3/8 \text{ in}$, $d = 0.5 \text{ in}$, $n_d = 2$. From Table A-20, for AISI 1020 CD, $S_{ut} = 68 \text{ kpsi}$ and $S_y = 57 \text{ kpsi}$.

$$\text{Eq. (6-8):} \quad S'_e = 0.5(68) = 34 \text{ kpsi}$$

$$\text{Table 6-2:} \quad k_a = 2.70(68)^{-0.265} = 0.88$$

$$\text{Eq. (6-21):} \quad k_b = 1 \text{ (axial loading)}$$

$$\text{Eq. (6-26):} \quad k_c = 0.85$$



$$\text{Eq. (6-18):} \quad S_e = 0.88(1)(0.85)(34) = 25.4 \text{ kpsi}$$

$$\text{Table A-15-1: } d/w = 0.5/2.5 = 0.2, \quad K_t = 2.5$$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). The relatively large radius is off the graph of Fig. 6-20, so we will assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(68) + 1.51(10^{-5})(68)^2 - 2.67(10^{-8})(68^3) = 0.09799$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.09799}{\sqrt{0.25}}} = 0.836$$

$$\text{Eq. (6-32):} \quad K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5 - 1) = 2.25$$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25 F_a}{(3/8)(2.5 - 0.5)} = 3 F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{25.4}{3F_a} = 2$$

$$F_a = 4.23 \text{ kips} \quad \text{Ans.}$$

6-15 Given: $D = 2 \text{ in}$, $d = 1.8 \text{ in}$, $r = 0.1 \text{ in}$, $M_{\max} = 25\,000 \text{ lbf} \cdot \text{in}$, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, $S_{ut} = 120 \text{ kpsi}$ and $S_y = 66 \text{ kpsi}$.

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(120) = 60 \text{ kpsi}$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(120)^{-0.265} = 0.76$

Eq. (6-24): $d_e = 0.370d = 0.370(1.8) = 0.666 \text{ in}$

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.76)(0.92)(1)(60) = 42.0 \text{ kpsi}$

Fig. A-15-14: $D/d = 2/1.8 = 1.11$, $r/d = 0.1/1.8 = 0.056 \Rightarrow K_t = 2.1$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). Using the equations,

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(120) + 1.51(10^{-5})(120)^2 - 2.67(10^{-8})(120^3) = 0.04770$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1 - 1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eq. (6-36): $\sigma_m = K_f \frac{\sigma_{\max} + \sigma_{\min}}{2} = (1.96) \frac{(43.7 + 0)}{2} = 42.8 \text{ kpsi}$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = (1.96) \left| \frac{(43.7 - 0)}{2} \right| = 42.8 \text{ kpsi}$$

Eq. (6-46):
$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{42.8}{42.0} + \frac{42.8}{120}$$

$$n_f = 0.73 \quad \text{Ans.}$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \quad \text{Ans.}$$

- 6-16** From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is $M = 2.1(200) = 420 \text{ kN}\cdot\text{mm}$. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{420(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: $r/d = 3/35 = 0.086$, $D/d = 50/35 = 1.43$, $K_t = 1.7$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). Using the equations, with $S_{ut} = 470 \text{ MPa} = 68.2 \text{ kpsi}$ and $r = 3 \text{ mm} = 0.118 \text{ in}$,

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(68.2) + 1.51(10^{-5})(68.2)^2 - 2.67(10^{-8})(68.2)^3 = 0.09771$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.09771}{\sqrt{0.118}}} = 0.78$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(470) = 235 \text{ MPa}$

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(470)^{-0.265} = 0.88$

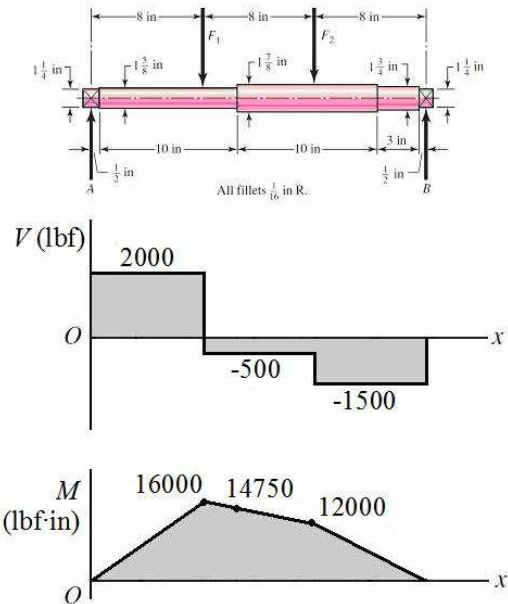
Eq. (6-24): $k_b = 1.24d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.88)(0.85)(1)(235) = 176 \text{ MPa}$

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{176}{1.55(99.8)} = 1.14 \quad \text{Infinite life is predicted.} \quad \text{Ans.}$$

6-17 From a free-body diagram analysis, the bearing reaction forces are found to be $R_A = 2000 \text{ lbf}$ and $R_B = 1500 \text{ lbf}$. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.



$$M = 16\,000 - 500(2.5) = 14\,750 \text{ lbf} \cdot \text{in}$$

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). Using the equations,

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(85) + 1.51(10^{-5})(85)^2 - 2.67(10^{-8})(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(85) = 42.5 \text{ kpsi}$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$

Eq. (6-20): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.832)(0.835)(1)(42.5) = 29.5 \text{ kpsi}$

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72(35.0)} = 0.49 \quad \text{Ans.}$$

Infinite life is not predicted. Use the S - N diagram to estimate the life.

Fig. 6-18: $f = 0.867$

Eq. (6-14): $a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$

Eq. (6-15): $b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.867(85)}{29.5} \right) = -0.1325$

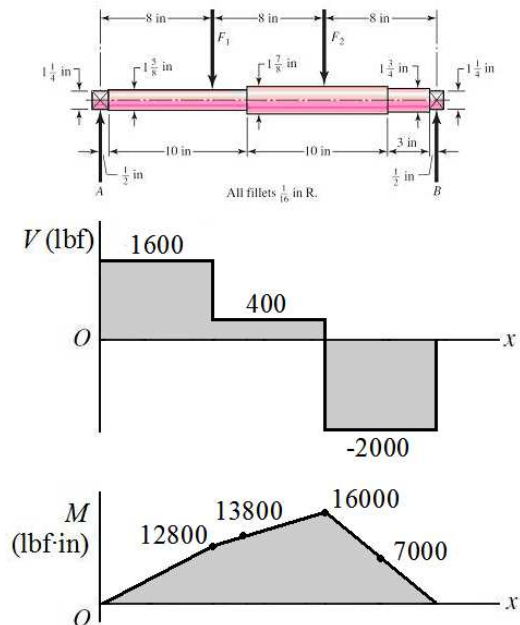
Eq. (6-16): $N = \left(\frac{K_f \sigma_{\text{rev}}}{a} \right)^{\frac{1}{b}} = \left(\frac{(1.72)(35.0)}{184.1} \right)^{-\frac{1}{0.1325}} = 4611 \text{ cycles}$
 $N = 4600 \text{ cycles} \quad \text{Ans.}$

6-18 From a free-body diagram analysis, the bearing reaction forces are found to be $R_A = 1600 \text{ lbf}$ and $R_B = 2000 \text{ lbf}$. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.

$$M = 12\,800 + 400(2.5) = 13\,800 \text{ lbf} \cdot \text{in}$$

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$



This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). Using the equations,

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(85) + 1.51(10^{-5})(85)^2 - 2.67(10^{-8})(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07690}{\sqrt{0.0625}}} = 0.76$$

$$\text{Eq. (6-32): } K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$$

$$\text{Eq. (6-8): } S'_e = 0.5S_{ut} = 0.5(85) = 42.5 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$$

$$\text{Eq. (6-20): } k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$$

$$\text{Eq. (6-26): } k_c = 1$$

$$\text{Eq. (6-18): } S_e = k_a k_b k_c S'_e = (0.832)(0.835)(1)(42.5) = 29.5 \text{ kpsi}$$

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72(32.8)} = 0.52 \quad \text{Ans.}$$

Infinite life is not predicted. Use the S - N diagram to estimate the life.

$$\text{Fig. 6-18: } f = 0.867$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.867(85)}{29.5} \right) = -0.1325$$

$$\text{Eq. (6-16): } N = \left(\frac{K_f \sigma_{\text{rev}}}{a} \right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{184.1} \right)^{\frac{1}{-0.1325}} = 7527 \text{ cycles}$$

$N = 7500 \text{ cycles} \quad \text{Ans.}$

6-19 Table A-20: $S_{ut} = 120 \text{ kpsi}$, $S_y = 66 \text{ kpsi}$

$$N = (950 \text{ rev/min})(10 \text{ hr})(60 \text{ min/hr}) = 570\,000 \text{ cycles}$$

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we will evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are $R_1 = 2 \text{ kips}$ and $R_2 = 6 \text{ kips}$. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious,

prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2 \text{ kip}(10 \text{ in}) = 20 \text{ kip} \cdot \text{in}$$

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3} \text{ kpsi}$$

Now we will get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, let us estimate a value of $q = 0.85$ from observation of Fig. 6-20, and check it later.

$$\text{Fig. A-15-9: } D/d = 1.4d/d = 1.4, \quad r/d = 0.1d/d = 0.1, \quad K_t = 1.65$$

$$\text{Eq. (6-32): } K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$$

Now, evaluate the fatigue strength.

$$S'_e = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.76$$

Since the diameter is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

$$S_e = (0.76)(0.85)(60) = 38.8 \text{ kpsi}$$

Determine the desired fatigue strength from the S - N diagram.

$$\text{Fig. 6-18: } f = 0.82$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.82(120)]^2}{38.8} = 249.6$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.82(120)}{38.8} \right) = -0.1347$$

$$\text{Eq. (6-13): } S_f = aN^b = 249.6(570\,000)^{-0.1347} = 41.9 \text{ kpsi}$$

Compare strength to stress and solve for the necessary d .

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{41.9}{(1.55)(203.7/d^3)} = 1.6$$

$$d = 2.29 \text{ in}$$

Since the size factor and notch sensitivity were guessed, go back and check them now.

$$\text{Eq. (6-20): } k_b = 0.91d^{-0.157} = 0.91(2.29)^{-0.157} = 0.80$$

From Fig. 6-20 with $r = d/10 = 0.229$ in, we are off the graph, but it appears our guess for q of 0.85 is low. Assuming the trend of the graph continues, we'll choose $q = 0.91$ and iterate the problem with the new values of k_b and q .

Intermediate results are $S_e = 36.5$ kpsi, $S_f = 39.6$ kpsi, and $K_f = 1.59$. This gives

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{39.6}{(1.59)(203.7/d^3)} = 1.6$$

$$d = 2.36 \text{ in} \quad \text{Ans.}$$

A quick check of k_b and q show that our estimates are still reasonable for this diameter.

- 6-20** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$
Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [25^2 + 3(0)^2]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(15)^2]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\text{max}} = (\sigma_{\text{max}}^2 + 3\tau_{\text{max}}^2)^{1/2} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2}$$

$$= [25^2 + 3(15^2)]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{60}{36.06} = 1.66 \quad \text{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \quad \text{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{80}{25.98} \right)^2 \left(\frac{25.00}{40} \right) \left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)} \right)^2} \right] = 1.31 \quad \text{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.00/40)^2 + (25.98/60)^2}} = 1.32 \quad \text{Ans.}$$

- 6-21** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$
Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [10^2 + 3(0)^2]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(20)^2]^{1/2} = 34.64 \text{ kpsi}$$

$$\begin{aligned}\sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2} \\ &= [10^2 + 3(20)^2]^{1/2} = 36.06 \text{ kpsi}\end{aligned}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \text{Ans.}$$

- (a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \quad \text{Ans.}$$

- (b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{80}{34.64} \right)^2 \left(\frac{10.00}{40} \right) \left\{ -1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)} \right)^2} \right\} = 1.74 \quad \text{Ans.}$$

- (c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(10.00/40)^2 + (34.64/60)^2}} = 1.59 \quad \text{Ans.}$$

- 6-22** $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$
Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [12^2 + 3(10)^2]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(15)^2]^{1/2} = 25.98 \text{ kpsi}$$

$$\begin{aligned}\sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2} \\ &= [(12+0)^2 + 3(10+15)^2]^{1/2} = 44.93 \text{ kpsi}\end{aligned}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{44.93} = 1.34 \quad \text{Ans.}$$

- (a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(21.07/40) + (25.98/80)} = 1.17 \quad \text{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{80}{25.98} \right)^2 \left(\frac{21.07}{40} \right) \left\{ -1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(21.07)} \right)^2} \right\} = 1.47 \quad \text{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(21.07/40)^2 + (25.98/60)^2}} = 1.47 \quad \text{Ans.}$$

6-23 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [0^2 + 3(30)^2]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = 0 \text{ kpsi}$$

$$\begin{aligned} \sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2} \\ &= [3(30)^2]^{1/2} = 51.96 \text{ kpsi} \end{aligned}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{51.96} = 1.15 \quad \text{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(51.96/40)} = 0.77 \quad \text{Ans.}$$

(b) Gerber criterion of Table 6-7 is only valid for $\sigma_m > 0$; therefore use Eq. (6-47).

$$n_f \frac{\sigma'_a}{S_e} = 1 \quad \Rightarrow \quad n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \quad \text{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(51.96/40)^2}} = 0.77 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the S - N diagram is applicable for σ'_a .

Fig. 6-18: $f = 0.875$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.875(80)]^2}{40} = 122.5$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.875(80)}{40} \right) = -0.08101$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{rev}}{a} \right)^{1/b} = \left(\frac{51.96}{122.5} \right)^{-\frac{1}{-0.08101}} = 39\,600 \text{ cycles } \textit{Ans.}$$

6-24 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$
Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [0^2 + 3(15)^2]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [15^2 + 3(0)^2]^{1/2} = 15.00 \text{ kpsi}$$

$$\begin{aligned} \sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2]^{1/2} \\ &= [(15)^2 + 3(15)^2]^{1/2} = 30.00 \text{ kpsi} \end{aligned}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \textit{ Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \textit{ Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2} \left(\frac{80}{15.00} \right)^2 \left(\frac{25.98}{40} \right) \left\{ -1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)} \right)^2} \right\} = 1.43 \textit{ Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.98/40)^2 + (15.00/60)^2}} = 1.44 \textit{ Ans.}$$

6-25 Given: $F_{\max} = 28$ kN, $F_{\min} = -28$ kN . From Table A-20, for AISI 1040 CD,
 $S_{ut} = 590$ MPa, $S_y = 490$ MPa,
Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \text{Ans.}$$

Determine the fatigue factor of safety based on infinite life

$$\text{Eq. (6-8):} \quad S'_e = 0.5(590) = 295 \text{ MPa}$$

$$\text{Eq. (6-19):} \quad k_a = aS_{ut}^b = 4.51(590)^{-0.265} = 0.832$$

$$\text{Eq. (6-21):} \quad k_b = 1 \quad (\text{axial})$$

$$\text{Eq. (6-26):} \quad k_c = 0.85$$

$$\text{Eq. (6-18):} \quad S_e = k_a k_b k_c S'_e = (0.832)(1)(0.85)(295) = 208.6 \text{ MPa}$$

$$\text{Fig. 6-20:} \quad q = 0.83$$

$$\text{Fig. A-15-1:} \quad d/w = 0.24, K_t = 2.44$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

Note, since $\sigma_m = 0$, the stress is completely reversing, and

$$n_f = \frac{S_e}{\sigma_a} = \frac{208.6}{324.2} = 0.64 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate the life from the S - N diagram. With $\sigma_m = 0$, the stress state is completely reversed, and the S - N diagram is applicable for σ_a .

$$S_{ut} = 590/6.89 = 85.6 \text{ kpsi}$$

$$\text{Fig. 6-18:} \quad f = 0.87$$

$$\text{Eq. (6-14):} \quad a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.87(590)]^2}{208.6} = 1263$$

$$\text{Eq. (6-15):} \quad b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.87(590)}{208.6} \right) = -0.1304$$

$$\text{Eq. (6-16):} \quad N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{324.2}{1263} \right)^{\frac{1}{-0.1304}} = 33\,812 \text{ cycles}$$

$$N = 34\,000 \text{ cycles} \quad \text{Ans.}$$

6-26 $S_{ut} = 590 \text{ MPa}$, $S_y = 490 \text{ MPa}$, $F_{\max} = 28 \text{ kN}$, $F_{\min} = 12 \text{ kN}$

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \text{Ans.}$$

Determine the fatigue factor of safety based on infinite life.

From Prob. 6-25: $S_e = 208.6 \text{ MPa}$, $K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (12\,000)}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Modified Goodman criteria:

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$$n_f = 1.20 \quad \text{Ans.}$$

Gerber criteria:

$$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \frac{\sigma_a}{S_e} \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut} \sigma_a} \right)^2} \right]$$

$$= \frac{1}{2} \left(\frac{590}{231.6} \right)^2 \frac{92.63}{208.6} \left[-1 + \sqrt{1 + \left(\frac{2(231.6)(208.6)}{590(92.63)} \right)^2} \right]$$

$$n_f = 1.49 \quad \text{Ans.}$$

ASME-Elliptic criteria:

$$n_f = \sqrt{\frac{1}{(\sigma_a / S_e)^2 + (\sigma_m / S_y)^2}} = \sqrt{\frac{1}{(92.63 / 208.6)^2 + (231.6 / 490)^2}}$$

$$= 1.54 \quad \text{Ans.}$$

The results are consistent with Fig. 6-27, where for a mean stress that is about half of the yield strength, the Modified Goodman line should predict failure significantly before the other two.

6-27 $S_{ut} = 590 \text{ MPa}, S_y = 490 \text{ MPa}$

(a) $F_{\max} = 28 \text{ kN}, F_{\min} = 0 \text{ kN}$

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \text{Ans.}$$

From Prob. 6-25: $S_e = 208.6 \text{ MPa}, K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 0}{2(10)(25-6)} \right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 0}{2(10)(25-6)} \right] = 162.1 \text{ MPa}$$

For the modified Goodman criteria,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{162.1}{208.6} + \frac{162.1}{590}$$

$$n_f = 0.95 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress. For the modified Goodman criteria, see Ex. 6-12.

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{162.1}{1 - (162.1 / 590)} = 223.5 \text{ MPa}$$

Fig. 6-18: $f = 0.87$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.87(590)]^2}{208.6} = 1263$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.87(590)}{208.6} \right) = -0.1304$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{223.5}{1263} \right)^{\frac{1}{-0.1304}} = 586\,000 \text{ cycles} \quad \text{Ans.}$$

(b) $F_{\max} = 28 \text{ kN}, F_{\min} = 12 \text{ kN}$

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \quad \text{Ans.}$$

Factor of safety based on infinite life:

$$\begin{aligned}\sigma_a &= K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 12\,000}{2(10)(25 - 6)} \right| = 92.63 \text{ MPa} \\ \sigma_m &= K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25 - 6)} \right] = 231.6 \text{ MPa} \\ \frac{1}{n_f} &= \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590} \\ n_f &= 1.20 \quad \text{Ans.}\end{aligned}$$

(c) $F_{\max} = 12 \text{ kN}$, $F_{\min} = -28 \text{ kN}$

The compressive load is the largest, so check it for yielding.

$$\begin{aligned}\sigma_{\min} &= \frac{F_{\min}}{A} = \frac{-28\,000}{10(25 - 6)} = -147.4 \text{ MPa} \\ n_y &= \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \quad \text{Ans.}\end{aligned}$$

Factor of safety based on infinite life:

$$\begin{aligned}\sigma_a &= K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{12\,000 - (-28\,000)}{2(10)(25 - 6)} \right| = 231.6 \text{ MPa} \\ \sigma_m &= K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{12\,000 + (-28\,000)}{2(10)(25 - 6)} \right] = -92.63 \text{ MPa}\end{aligned}$$

For $\sigma_m < 0$, $n_f = \frac{S_e}{\sigma_a} = \frac{208.6}{231.6} = 0.90 \quad \text{Ans.}$

Since infinite life is not predicted, estimate a life from the S - N diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress. Get a and b from part (a).

Eq. (6-16): $N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{231.6}{1263} \right)^{\frac{1}{-0.1304}} = 446\,000 \text{ cycles} \quad \text{Ans.}$

6-28 Eq. (2-21): $S_{ut} = 0.5(400) = 200 \text{ kpsi}$

Eq. (6-8): $S'_e = 0.5(200) = 100 \text{ kpsi}$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 14.4(200)^{-0.718} = 0.321$$

$$\text{Eq. (6-25): } d_e = 0.37d = 0.37(0.375) = 0.1388 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

$$\text{Eq. (6-18): } S_e = (0.321)(1)(100) = 32.1 \text{ kpsi}$$

$$F_a = \frac{40 - 20}{2} = 10 \text{ lb} \quad F_m = \frac{40 + 20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

(a) Modified Goodman criterion

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{32.1} + \frac{69.54}{200}$$

$$n_f = 0.94 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{23.18}{1 - (69.54 / 200)} = 35.54 \text{ kpsi}$$

$$\text{Fig. 6-18: } f = 0.775$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.775(200)]^2}{32.1} = 748.4$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.775(200)}{32.1} \right) = -0.228$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{35.54}{748.4} \right)^{-\frac{1}{-0.228}} = 637 \text{ 000 cycles} \quad \text{Ans.}$$

(b) Gerber criterion, Table 6-7

$$\begin{aligned}
 n_f &= \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \frac{\sigma_a}{S_e} \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut} \sigma_a} \right)^2} \right] \\
 &= \frac{1}{2} \left(\frac{200}{69.54} \right)^2 \frac{23.18}{32.1} \left[-1 + \sqrt{1 + \left(\frac{2(69.54)(32.1)}{200(23.18)} \right)^2} \right] \\
 &= 1.16 \quad \text{Infinite life is predicted} \quad \text{Ans.}
 \end{aligned}$$

6-29 $E = 207.0 \text{ GPa}$

(a) $I = \frac{1}{12}(20)(4^3) = 106.7 \text{ mm}^4$

$$y = \frac{Fl^3}{3EI} \Rightarrow F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3 \text{ N} \quad \text{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9 \text{ N} \quad \text{Ans.}$$

(b) Get the fatigue strength information.

Eq. (2-21): $S_{ut} = 3.4H_B = 3.4(490) = 1666 \text{ MPa}$

From problem statement: $S_y = 0.9S_{ut} = 0.9(1666) = 1499 \text{ MPa}$

Eq. (6-8): $S'_e = 700 \text{ MPa}$

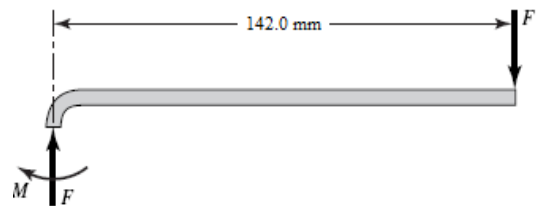
Eq. (6-19): $k_a = 1.58(1666)^{-0.085} = 0.84$

Eq. (6-25): $d_e = 0.808[20(4)]^{1/2} = 7.23 \text{ mm}$

Eq. (6-20): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-18): $S_e = 0.84(1)(700) = 588 \text{ MPa}$

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$$\begin{aligned}
 M &= 142F \text{ N}\cdot\text{mm}, \quad A = 4(20) = 80 \text{ mm}^2, \quad h = 4 \text{ mm}, \\
 r_i &= 4 \text{ mm}, \quad r_o = r_i + h = 8 \text{ mm}, \\
 r_c &= r_i + h/2 = 6 \text{ mm}
 \end{aligned}$$

Table 3-4: $r_n = \frac{h}{\ln(r_o / r_i)} = \frac{4}{\ln(8 / 4)} = 5.7708 \text{ mm}$

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$

$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$

$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-65) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$

$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$

$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$

$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$

$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$

$$(\sigma_i)_a = \left| \frac{-166.2 - (-498.6)}{2} \right| = 166.2 \text{ MPa}$$

$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$

$$(\sigma_o)_a = \left| \frac{310.8 - 103.6}{2} \right| = 103.6 \text{ MPa}$$

$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is -498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa , so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.

Inner radius:

$$\text{Since } \sigma_m < 0, n_f = \frac{S_e}{\sigma_a} = \frac{588}{166.2} = 3.54$$

Outer radius:

Since $\sigma_m > 0$, using the Modified Goodman line,

$$\begin{aligned} 1/n_f &= \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{103.6}{588} + \frac{207.2}{1666} \\ n_f &= 3.33 \end{aligned}$$

Infinite life is predicted at both inner and outer radii. The outer radius is critical, with a fatigue factor of safety of $n_f = 3.33$. *Ans.*

6-30 From Table A-20, for AISI 1018 CD, $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = 2.70(64)^{-0.265} = 0.897$$

$$\text{Eq. (6-20): } k_b = 1 \text{ (axial)}$$

$$\text{Eq. (6-26): } k_c = 0.85$$

$$\text{Eq. (6-18): } S_e = (0.897)(1)(0.85)(32) = 24.4 \text{ kpsi}$$

Fillet:

Fig. A-15-5: $D/d = 3.5/3 = 1.17$, $r/d = 0.25/3 = 0.083$, $K_t = 1.85$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q . Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(1.85 - 1) = 1.72$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{5}{3.0(0.5)} = 3.33 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{3.0(0.5)} = -10.67 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.72 \left| \frac{3.33 - (-10.67)}{2} \right| = 12.0 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.72 \left(\frac{3.33 + (-10.67)}{2} \right) = -6.31 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.67} \right| = 5.06 \quad \therefore \text{Does not yield.}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{12.0} = 2.03$$

Hole:

Fig. A-15-1: $d/w_1 = 0.4/3.5 = 0.11$ $\therefore K_t = 2.68$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q . Estimate a little high since it is off the graph, $q = 0.85$

$$K_f = 1 + 0.85(2.68 - 1) = 2.43$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{5}{0.5(3.5 - 0.4)} = 3.226 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-16}{0.5(3.5 - 0.4)} = -10.32 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.43 \left| \frac{3.226 - (-10.32)}{2} \right| = 16.5 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.43 \left(\frac{3.226 + (-10.32)}{2} \right) = -8.62 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.32} \right| = 5.23 \quad \therefore \text{does not yield}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{16.5} = 1.48$$

Thus the design is controlled by the threat of fatigue at the hole with a minimum factor of safety of $n_f = 1.48$. *Ans.*

6-31 $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = (0.897)(1)(0.85)(32) = 24.4 \text{ kpsi}$

Fillet:

Fig. A-15-5: $D/d = 2.5/1.5 = 1.67$, $r/d = 0.25/1.5 = 0.17$, $K_t \approx 2.1$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q . Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.1 - 1) = 1.94$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{16}{1.5(0.5)} = 21.3 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-4}{1.5(0.5)} = -5.33 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.94 \left| \frac{21.3 - (-5.33)}{2} \right| = 25.8 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.94 \left(\frac{21.3 + (-5.33)}{2} \right) = 15.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{21.3} = 2.54 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{25.8}{24.4} + \frac{15.5}{64}$$

$$n_f = 0.77$$

Hole:

Fig. A-15-1: $d/w_1 = 0.4/2.5 = 0.16 \quad \therefore K_t = 2.55$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q . Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.55 - 1) = 2.32$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{16}{0.5(2.5 - 0.4)} = 15.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-4}{0.5(2.5 - 0.4)} = -3.81 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.32 \left(\frac{15.2 - (-3.81)}{2} \right) = 22.1 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.32 \left(\frac{15.2 + (-3.81)}{2} \right) = 13.2 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{15.2} = 3.55 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{22.1}{24.4} + \frac{13.2}{64}$$

$$n_f = 0.90$$

Thus the design is controlled by the threat of fatigue at the fillet with a minimum factor of safety of $n_f = 0.77$ Ans.

6-32 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 6-30, the fatigue factor of safety at the hole is $n_f = 1.48$. To match this at the fillet,

$$n_f = \frac{S_e}{\sigma_a} \Rightarrow \sigma_a = \frac{S_e}{n_f} = \frac{24.4}{1.48} = 16.5 \text{ kpsi}$$

where S_e is unchanged from Prob. 6-30. The only aspect of σ_a that is affected by the fillet radius is the fatigue stress concentration factor. Obtaining σ_a in terms of K_f ,

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = K_f \left| \frac{3.33 - (-10.67)}{2} \right| = 7.00 K_f$$

Equating to the desired stress, and solving for K_f ,

$$\sigma_a = 7.00 K_f = 16.5 \Rightarrow K_f = 2.36$$

Assume since we are expecting to get a smaller fillet radius than the original, that q will be back on the graph of Fig. 6-20, so we will estimate $q = 0.8$.

$$K_f = 1 + 0.80(K_t - 1) = 2.36 \Rightarrow K_t = 2.7$$

From Fig. A-15-5, with $D/d = 3.5/3 = 1.17$ and $K_t = 2.6$, find r/d . Choosing $r/d = 0.03$, and with $d = w_2 = 3.0$,

$$r = 0.03w_2 = 0.03(3.0) = 0.09 \text{ in}$$

At this small radius, our estimate for q is too high. From Fig. 6-20, with $r = 0.09$, q should be about 0.75. Iterating, we get $K_t = 2.8$. This is at a difficult range on Fig. A-15-5 to read the graph with any confidence, but we'll estimate $r/d = 0.02$, giving $r = 0.06$ in. This is a very rough estimate, but it clearly demonstrates that the fillet radius can be relatively sharp to match the fatigue factor of safety of the hole. *Ans.*

6-33 $S_y = 60$ kpsi, $S_{ut} = 110$ kpsi

Inner fiber where $r_c = 3/4$ in

$$r_o = \frac{3}{4} + \frac{3}{16(2)} = 0.84375$$

$$r_i = \frac{3}{4} - \frac{3}{32} = 0.65625$$

Table 3-4, p. 135,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{3/16}{\ln \frac{0.84375}{0.65625}} = 0.74608 \text{ in}$$

$$e = r_c - r_n = 0.75 - 0.74608 = 0.00392 \text{ in}$$

$$c_i = r_n - r_i = 0.74608 - 0.65625 = 0.08983$$

$$A = \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = 0.035156 \text{ in}^2$$

Eq. (3-65), p. 133,

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(0.08983)}{(0.035156)(0.00392)(0.65625)} = -993.3T$$

where T is in lbf·in and σ_i is in psi.

$$\sigma_m = \frac{1}{2}(-993.3)T = -496.7T$$

$$\sigma_a = 496.7T$$

$$\text{Eq. (6-8): } S'_e = 0.5(110) = 55 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = 2.70(110)^{-0.265} = 0.777$$

$$\text{Eq. (6-25): } d_e = 0.808[(3/16)(3/16)]^{1/2} = 0.1515 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879(0.1515)^{-0.107} = 1.08 \text{ (round to 1)}$$

$$\text{Eq. (6-19): } S_e = (0.777)(1)(55) = 42.7 \text{ kpsi}$$

For a compressive midrange component, $\sigma_a = S_e / n_f$. Thus,

$$0.4967T = \frac{42.7}{3}$$

$$T = 28.7 \text{ lbf} \cdot \text{in}$$

Outer fiber where $r_c = 2.5$ in

$$r_o = 2.5 + \frac{3}{32} = 2.59375$$

$$r_i = 2.5 - \frac{3}{32} = 2.40625$$

$$r_n = \frac{3/16}{\ln \frac{2.59375}{2.40625}} = 2.49883$$

$$e = 2.5 - 2.49883 = 0.00117 \text{ in}$$

$$c_o = 2.59375 - 2.49883 = 0.09492 \text{ in}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{T(0.09492)}{(0.035156)(0.00117)(2.59375)} = 889.7T \text{ psi}$$

$$\sigma_m = \sigma_a = \frac{1}{2}(889.7T) = 444.9T \text{ psi}$$

(a) Using Eq. (6-46), for modified Goodman, we have

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{0.4449T}{42.7} + \frac{0.4449T}{110} = \frac{1}{3}$$

$$T = 23.0 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(b) Gerber, Eq. (6-47), at the outer fiber,

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}} \right)^2 = 1$$

$$\frac{3(0.4449T)}{42.7} + \left(\frac{3(0.4449T)}{110} \right)^2 = 1$$

$$T = 28.2 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{S_y}{\sigma_i} = \frac{60}{0.9933(28.2)} = 2.14 \quad \text{Ans.}$$

6-34 From Prob. 6-33, $S_e = 42.7$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 110$ kpsi

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} = \frac{6T}{(3/16)^3} = 910.2T$$

Goodman:
$$\frac{0.4551T}{42.7} + \frac{0.4551T}{110} = \frac{1}{3}$$

$$T = 22.5 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(b) Gerber:
$$\frac{3(0.4551T)}{42.7} + \left(\frac{3(0.4551T)}{110} \right)^2 = 1$$

$$T = 27.6 \text{ lbf} \cdot \text{in} \quad \text{Ans.}$$

(c)
$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{60}{0.9102(27.6)} = 2.39 \quad \text{Ans.}$$

6-35 $K_{f,\text{bend}} = 1.4$, $K_{f,\text{axial}} = 1.1$, $K_{f,\text{tors}} = 2.0$, $S_y = 300$ MPa, $S_{ut} = 400$ MPa, $S_e = 200$ MPa

Bending: $\sigma'_m = 0$, $\sigma'_a = 60$ MPa

Axial: $\sigma'_m = 20$ MPa, $\sigma'_a = 0$

Torsion: $\tau'_m = 25$ MPa, $\tau'_a = 25$ MPa

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{[1.4(60) + 0]^2 + 3[2.0(25)]^2} = 120.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{[0 + 1.1(20)]^2 + 3[2.0(25)]^2} = 89.35 \text{ MPa}$$

Using Modified Goodman, Eq. (6-46),

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{120.6}{200} + \frac{89.35}{400}$$

$$n_f = 1.21 \quad \text{Ans.}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{120.6 + 89.35} = 1.43 \quad \text{Ans.}$$

6-36 $K_{f,\text{bend}} = 1.4$, $K_{f,\text{tors}} = 2.0$, $S_y = 300$ MPa, $S_{ut} = 400$ MPa, $S_e = 200$ MPa

Bending: $\sigma'_{\max} = 150$ MPa, $\sigma'_{\min} = -40$ MPa, $\sigma'_m = 55$ MPa, $\sigma'_a = 95$ MPa

Torsion: $\tau'_m = 90$ MPa, $\tau'_a = 9$ MPa

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{[1.4(95)]^2 + 3[2.0(9)]^2} = 136.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{[1.4(55)]^2 + 3[2.0(90)]^2} = 321.1 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{136.6}{200} + \frac{321.1}{400}$$

$$n_f = 0.67 \quad \text{Ans.}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{136.6 + 321.1} = 0.66 \quad \text{Ans.}$$

Since the conservative yield check indicates yielding, we will check more carefully with σ'_{\max} obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{(\sigma_{\max})^2 + 3(\tau_{\max})^2} = \sqrt{(150)^2 + 3(90+9)^2} = 227.8 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{300}{227.8} = 1.32 \quad \text{Ans.}$$

Since yielding is not predicted, and infinite life is not predicted, we would like to estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{136.6}{1 - (321.1 / 400)} = 692.5 \text{ MPa}$$

This stress is much higher than the ultimate strength, rendering it impractical for the S - N diagram. We must conclude that the stresses from the combination loading, when increased by the stress concentration factors, produce such a high midrange stress that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

6-37 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-68, the critical stress element experiences $\sigma = 15.3 \text{ kpsi}$ and $\tau = 4.43 \text{ kpsi}$. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 15.3 \text{ kpsi}$, $\sigma_m = 0 \text{ kpsi}$, $\tau_a = 0 \text{ kpsi}$, $\tau_m = 4.43 \text{ kpsi}$. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [15.3^2 + 3(0)^2]^{1/2} = 15.3 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(4.43)^2]^{1/2} = 7.67 \text{ kpsi}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [15.3^2 + 3(4.43)^2]^{1/2} = 17.11 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{17.11} = 3.16$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8):} \quad S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19):} \quad k_a = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-20):} \quad k_b = 0.879(1.25)^{-0.107} = 0.86$$

Eq. (6-18): $S_e = 0.90(0.86)(32) = 24.8 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{15.3}{24.8} + \frac{7.67}{64}$$

$$n_f = 1.36 \quad \text{Ans.}$$

6-38 Table A-20: $S_{ut} = 440 \text{ MPa}$, $S_y = 370 \text{ MPa}$

From Prob. 3-69, the critical stress element experiences $\sigma = 263 \text{ MPa}$ and $\tau = 57.7 \text{ MPa}$. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 263 \text{ MPa}$, $\sigma_m = 0$, $\tau_a = 0 \text{ MPa}$, $\tau_m = 57.7 \text{ MPa}$. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [263^2 + 3(0)^2]^{1/2} = 263 \text{ MPa}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(57.7)^2]^{1/2} = 99.9 \text{ MPa}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [263^2 + 3(57.7)^2]^{1/2} = 281 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{281} = 1.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(220) = 170 \text{ MPa}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{263}{170} + \frac{99.9}{440}$$

$$n_f = 0.56 \quad \text{Infinite life is not predicted.} \quad \text{Ans.}$$

6-39 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-70, the critical stress element experiences $\sigma = 21.5 \text{ kpsi}$ and $\tau = 5.09 \text{ kpsi}$. The bending is completely reversed due to the rotation, and the torsion is steady, giving

$\sigma_a = 21.5$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 5.09$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [21.5^2 + 3(0)^2]^{1/2} = 21.5 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(5.09)^2]^{1/2} = 8.82 \text{ kpsi}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [21.5^2 + 3(5.09)^2]^{1/2} = 23.24 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{23.24} = 2.32$$

Obtain the modifying factors and endurance limit.

$$k_a = 2.70(64)^{-0.265} = 0.90$$

$$k_b = 0.879(1)^{-0.107} = 0.88$$

$$S_e = 0.90(0.88)(0.5)(64) = 25.3 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{21.5}{25.3} + \frac{8.82}{64}$$

$$n_f = 1.01 \quad \text{Ans.}$$

6-40 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-71, the critical stress element experiences $\sigma = 72.9$ MPa and $\tau = 20.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 72.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 20.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [72.9^2 + 3(0)^2]^{1/2} = 72.9 \text{ MPa}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(20.3)^2]^{1/2} = 35.2 \text{ MPa}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [72.9^2 + 3(20.3)^2]^{1/2} = 80.9 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{80.9} = 4.57$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(440) = 220 \text{ MPa}$$

$$\text{Eq. (6-19): } k_a = 4.51(440)^{-0.265} = 0.90$$

$$\text{Eq. (6-20): } k_b = 1.24(20)^{-0.107} = 0.90$$

$$\text{Eq. (6-18): } S_e = 0.90(0.90)(220) = 178.2 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{72.9}{178.2} + \frac{35.2}{440}$$

$$n_f = 2.04 \quad \text{Ans.}$$

6-41 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-72, the critical stress element experiences $\sigma = 35.2 \text{ kpsi}$ and $\tau = 7.35 \text{ kpsi}$. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 35.2 \text{ kpsi}$, $\sigma_m = 0 \text{ kpsi}$, $\tau_a = 0 \text{ kpsi}$, $\tau_m = 7.35 \text{ kpsi}$. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [35.2^2 + 3(0)^2]^{1/2} = 35.2 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(7.35)^2]^{1/2} = 12.7 \text{ kpsi}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [35.2^2 + 3(7.35)^2]^{1/2} = 37.4 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{37.4} = 1.44$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-20): } k_b = 0.879(1.25)^{-0.107} = 0.86$$

$$\text{Eq. (6-18): } S_e = 0.90(0.86)(32) = 24.8 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{35.2}{24.8} + \frac{12.7}{64}$$

$$n_f = 0.62 \quad \text{Infinite life is not predicted.} \quad \text{Ans.}$$

6-42 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-73, the critical stress element experiences $\sigma = 333.9$ MPa and $\tau = 126.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 333.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 126.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [333.9^2 + 3(0)^2]^{1/2} = 333.9 \text{ MPa}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(126.3)^2]^{1/2} = 218.8 \text{ MPa}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [333.9^2 + 3(126.3)^2]^{1/2} = 399.2 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{399.2} = 0.93$$

The sample fails by yielding, infinite life is not predicted. *Ans.*

The fatigue analysis will be continued only to obtain the requested fatigue factor of safety, though the yielding failure will dictate the life.

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8):} \quad S'_e = 0.5(440) = 220 \text{ MPa}$$

$$\text{Eq. (6-19):} \quad k_a = 4.51(440)^{-0.265} = 0.90$$

$$\text{Eq. (6-20):} \quad k_b = 1.24(50)^{-0.107} = 0.82$$

$$\text{Eq. (6-18):} \quad S_e = 0.90(0.82)(220) = 162.4 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{333.9}{162.4} + \frac{218.8}{440}$$

$$n_f = 0.39 \quad \text{Infinite life is not predicted.} \quad \text{Ans.}$$

6-43 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-74, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\begin{aligned}\sigma_{a,\text{bend}} &= 9.495 \text{ kpsi}, & \sigma_{m,\text{bend}} &= 0 \text{ kpsi} \\ \sigma_{a,\text{axial}} &= 0 \text{ kpsi}, & \sigma_{m,\text{axial}} &= -0.362 \text{ kpsi} \\ \tau_a &= 0 \text{ kpsi}, & \tau_m &= 11.07 \text{ kpsi}\end{aligned}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\begin{aligned}\sigma'_a &= (\sigma_a^2 + 3\tau_a^2)^{1/2} = [(9.495)^2 + 3(0)^2]^{1/2} = 9.495 \text{ kpsi} \\ \sigma'_m &= (\sigma_m^2 + 3\tau_m^2)^{1/2} = [(-0.362)^2 + 3(11.07)^2]^{1/2} = 19.18 \text{ kpsi} \\ \sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(-9.495 - 0.362)^2 + 3(11.07)^2]^{1/2} = 21.56 \text{ kpsi}\end{aligned}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{21.56} = 2.50$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-20): } k_b = 0.879(1.13)^{-0.107} = 0.87$$

$$\text{Eq. (6-18): } S_e = 0.90(0.87)(32) = 25.1 \text{ kpsi}$$

Using Modified Goodman,

$$\begin{aligned}\frac{1}{n_f} &= \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{9.495}{25.1} + \frac{19.18}{64} \\ n_f &= 1.47 \quad \text{Ans.}\end{aligned}$$

6-44 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-76, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\begin{aligned}\sigma_{a,\text{bend}} &= 33.99 \text{ kpsi}, & \sigma_{m,\text{bend}} &= 0 \text{ kpsi} \\ \sigma_{a,\text{axial}} &= 0 \text{ kpsi}, & \sigma_{m,\text{axial}} &= -0.153 \text{ kpsi} \\ \tau_a &= 0 \text{ kpsi}, & \tau_m &= 7.847 \text{ kpsi}\end{aligned}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\begin{aligned}\sigma'_a &= (\sigma_a^2 + 3\tau_a^2)^{1/2} = [(33.99)^2 + 3(0)^2]^{1/2} = 33.99 \text{ kpsi} \\ \sigma'_m &= (\sigma_m^2 + 3\tau_m^2)^{1/2} = [(-0.153)^2 + 3(7.847)^2]^{1/2} = 13.59 \text{ kpsi} \\ \sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [(-33.99 - 0.153)^2 + 3(7.847)^2]^{1/2} = 36.75 \text{ kpsi}\end{aligned}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{36.75} = 1.47$$

Obtain the modifying factors and endurance limit.

$$\begin{aligned}\text{Eq. (6-8):} \quad S'_e &= 0.5(64) = 32 \text{ kpsi} \\ \text{Eq. (6-19):} \quad k_a &= 2.70(64)^{-0.265} = 0.90 \\ \text{Eq. (6-20):} \quad k_b &= 0.879(0.88)^{-0.107} = 0.89 \\ \text{Eq. (6-18):} \quad S_e &= 0.90(0.89)(32) = 25.6 \text{ kpsi}\end{aligned}$$

Using Modified Goodman,

$$\begin{aligned}\frac{1}{n_f} &= \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{33.99}{25.6} + \frac{13.59}{64} \\ n_f &= 0.65 \quad \text{Infinite life is not predicted.} \quad \text{Ans.}\end{aligned}$$

6-45 Table A-20: $S_{ut} = 440 \text{ MPa}$, $S_y = 370 \text{ MPa}$

From Prob. 3-77, the critical stress element experiences $\sigma = 68.6 \text{ MPa}$ and $\tau = 37.7 \text{ MPa}$. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 68.6 \text{ MPa}$, $\sigma_m = 0 \text{ MPa}$, $\tau_a = 0 \text{ MPa}$, $\tau_m = 37.7 \text{ MPa}$. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\begin{aligned}\sigma'_a &= (\sigma_a^2 + 3\tau_a^2)^{1/2} = [68.6^2 + 3(0)^2]^{1/2} = 68.6 \text{ MPa} \\ \sigma'_m &= (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(37.7)^2]^{1/2} = 65.3 \text{ MPa} \\ \sigma'_{\max} &= (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [68.6^2 + 3(37.7)^2]^{1/2} = 94.7 \text{ MPa}\end{aligned}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{94.7} = 3.91$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(440) = 220 \text{ MPa}$$

$$\text{Eq. (6-19): } k_a = 4.51(440)^{-0.265} = 0.90$$

$$\text{Eq. (6-20): } k_b = 1.24(30)^{-0.107} = 0.86$$

$$\text{Eq. (6-18): } S_e = 0.90(0.86)(220) = 170 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{68.6}{170} + \frac{65.3}{440}$$

$$n_f = 1.81 \quad \text{Ans.}$$

6-46 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-79, the critical stress element experiences $\sigma = 3.46$ kpsi and $\tau = 0.882$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 3.46$ kpsi, $\sigma_m = 0$, $\tau_a = 0$ kpsi, $\tau_m = 0.882$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = [3.46^2 + 3(0)^2]^{1/2} = 3.46 \text{ kpsi}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = [0^2 + 3(0.882)^2]^{1/2} = 1.53 \text{ kpsi}$$

$$\sigma'_{\max} = (\sigma_{\max}^2 + 3\tau_{\max}^2)^{1/2} = [3.46^2 + 3(0.882)^2]^{1/2} = 3.78 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{3.78} = 14.3$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-20): } k_b = 0.879(1.375)^{-0.107} = 0.85$$

Eq. (6-18): $S_e = 0.90(0.85)(32) = 24.5 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{3.46}{24.5} + \frac{1.53}{64}$$

$$n_f = 6.06 \quad \text{Ans.}$$

6-47 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-80, the critical stress element experiences $\sigma = 16.3 \text{ kpsi}$ and $\tau = 5.09 \text{ kpsi}$. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.3 \text{ kpsi}$, $\sigma_{\min} = 0 \text{ kpsi}$, $\tau_{\max} = 5.09 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Consequently, $\sigma_m = \sigma_a = 8.15 \text{ kpsi}$, $\tau_m = \tau_a = 2.55 \text{ kpsi}$.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(64) + 1.51(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51(10^{-3})(64) + 1.35(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.38)(8.15) \right]^2 + 3 \left[(1.88)(2.55) \right]^2 \right\}^{1/2} = 13.98 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.98 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.98 + 13.98} = 1.93$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-24): } d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$$

$$\text{Eq. (6-18): } S_e = (0.90)(0.98)(32) = 28.2 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{13.98}{28.2} + \frac{13.98}{64}$$

$$n_f = 1.40 \quad \text{Ans.}$$

6-48 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-81, the critical stress element experiences $\sigma = 16.4 \text{ kpsi}$ and $\tau = 4.46 \text{ kpsi}$. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.4 \text{ kpsi}$, $\sigma_{\min} = 0 \text{ kpsi}$, $\tau_{\max} = 4.46 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Consequently, $\sigma_m = \sigma_a = 8.20 \text{ kpsi}$, $\tau_m = \tau_a = 2.23 \text{ kpsi}$.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(64) + 1.51(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

$$\text{Eq. (6-32): } K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51(10^{-3})(64) + 1.35(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

$$\text{Eq. (6-32): } K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.38)(8.20) \right]^2 + 3 \left[(1.88)(2.23) \right]^2 \right\}^{1/2} = 13.45 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.45 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.45 + 13.45} = 2.01$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-24): } d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$$

$$\text{Eq. (6-18): } S_e = (0.90)(0.98)(32) = 28.2 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{13.45}{28.2} + \frac{13.45}{64}$$

$$n_f = 1.46 \quad \text{Ans.}$$

6-49 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-82, the critical stress element experiences repeatedly applied bending, axial, and torsional stresses of $\sigma_{x,\text{bend}} = 20.2 \text{ kpsi}$, $\sigma_{x,\text{axial}} = 0.1 \text{ kpsi}$, and $\tau = 5.09 \text{ kpsi}$. Since the axial stress is practically negligible compared to the bending stress, we will simply combine the two and not treat the axial stress separately for stress concentration factor and load factor. This gives $\sigma_{\max} = 20.3 \text{ kpsi}$, $\sigma_{\min} = 0 \text{ kpsi}$, $\tau_{\max} = 5.09 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Consequently, $\sigma_m = \sigma_a = 10.15 \text{ kpsi}$, $\tau_m = \tau_a = 2.55 \text{ kpsi}$.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(64) + 1.51(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.10373}{\sqrt{0.1}}} = 0.75$$

$$\text{Eq. (6-32): } K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51(10^{-3})(64) + 1.35(10^{-5})(64)^2 - 2.67(10^{-8})(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07800}{\sqrt{0.1}}} = 0.80$$

$$\text{Eq. (6-32): } K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.38)(10.15) \right]^2 + 3 \left[(1.88)(2.55) \right]^2 \right\}^{1/2} = 16.28 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 16.28 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{16.28 + 16.28} = 1.66$$

Obtain the modifying factors and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$$

$$\text{Eq. (6-24): } d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$$

$$\text{Eq. (6-18): } S_e = (0.90)(0.98)(32) = 28.2 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{16.28}{28.2} + \frac{16.28}{64}$$

$$n_f = 1.20 \quad \text{Ans.}$$

6-50 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-83, the critical stress element on the neutral axis in the middle of the longest side of the rectangular cross section experiences a repeatedly applied shear stress of $\tau_{\max} = 14.3 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Thus, $\tau_m = \tau_a = 7.15 \text{ kpsi}$. Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{54 / 2}{14.3} = 1.89$$

Find the modifiers and endurance limit.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$$

The size factor for a rectangular cross section loaded in torsion is not readily available. Following the procedure on p. 297, we need an equivalent diameter based on the 95 percent stress area. However, the stress situation in this case is nonlinear, as described on p. 116. Noting that the maximum stress occurs at the middle of the longest side, or with a radius from the center of the cross section equal to half of the shortest side, we will simply choose an equivalent diameter equal to the length of the shortest side.

$$d_e = 0.25 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.25)^{-0.107} = 1.02$$

We will round down to $k_b = 1$.

$$\text{Eq. (6-26): } k_c = 0.59$$

$$\text{Eq. (6-18): } S_{se} = 0.9(1)(0.59)(32) = 17.0 \text{ kpsi}$$

Since the stress is entirely shear, we choose to use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67S_u = 0.67(64) = 42.9 \text{ kpsi}$.

Using Modified Goodman,

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(7.15 / 17.0) + (7.15 / 42.9)} = 1.70 \quad \text{Ans.}$$

6-51 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-84, the critical stress element experiences $\sigma = 28.0 \text{ kpsi}$ and $\tau = 15.3 \text{ kpsi}$. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 28.0 \text{ kpsi}$, $\sigma_{\min} = 0 \text{ kpsi}$, $\tau_{\max} = 15.3 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Consequently, $\sigma_m = \sigma_a = 14.0 \text{ kpsi}$, $\tau_m = \tau_a = 7.65 \text{ kpsi}$. From Table A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \quad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \quad K_{t,\text{tors}} = 1.39$$

$$\text{Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: } q_{\text{bend}} = 0.78, \quad q_{\text{tors}} = 0.82$$

$$\text{Eq. (6-32):}$$

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.78(1.60 - 1) = 1.47$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.47)(14.0) \right]^2 + 3 \left[(1.32)(7.65) \right]^2 \right\}^{1/2} = 27.0 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 27.0 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\text{max}} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{27.0 + 27.0} = 1.00$$

Since stress concentrations are included in this quick yield check, the low factor of safety is acceptable.

$$\text{Eq. (6-8): } S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19): } k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$$

$$\text{Eq. (6-24): } d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20): } k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$$

$$\text{Eq. (6-18): } S_e = (0.897)(0.978)(0.5)(64) = 28.1 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{27.0}{28.1} + \frac{27.0}{64}$$

$$n_f = 0.72 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{27.0}{1 - (27.0 / 64)} = 46.7 \text{ kpsi}$$

$$\text{Fig. 6-18: } f = 0.9$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.9(64)]^2}{28.1} = 118.07$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.9(64)}{28.1} \right) = -0.1039$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{46.7}{118.07} \right)^{-0.1039} = 7534 \text{ cycles} \approx 7500 \text{ cycles} \quad \text{Ans.}$$

6-52 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-85, the critical stress element experiences $\sigma_{x,\text{bend}} = 46.1$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}} = 46.1$ kpsi, $\sigma_{\text{min,bend}} = 0$ kpsi, $\sigma_{\text{max,axial}} = 0.382$ kpsi, $\sigma_{\text{min,axial}} = 0$ kpsi, $\tau_{\text{max}} = 15.3$ kpsi, $\tau_{\text{min}} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 23.05$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \quad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \quad K_{t,\text{tors}} = 1.39, \quad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.78(1.60 - 1) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}(K_{t,\text{axial}} - 1) = 1 + 0.78(1.75 - 1) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.47)(23.05) + (1.59) \frac{(0.191)}{0.85} \right]^2 + 3 \left[(1.32)(7.65) \right]^2 \right\}^{1/2} = 38.45 \text{ kpsi}$$

$$\sigma'_m = \left\{ \left[(1.47)(23.05) + (1.59)(0.191) \right]^2 + 3 \left[(1.32)(7.65) \right]^2 \right\}^{1/2} = 38.40 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\text{max}} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{38.45 + 38.40} = 0.70$$

Since the conservative yield check indicates yielding, we will check more carefully with σ'_{max} obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{(\sigma_{\max, \text{bend}} + \sigma_{\max, \text{axial}})^2 + 3(\tau_{\max})^2} = \sqrt{(46.1 + 0.382)^2 + 3(15.3)^2} = 53.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{53.5} = 1.01 \quad \text{Ans.}$$

This shows that yielding is imminent, and further analysis of fatigue life should not be interpreted as a guarantee of more than one cycle of life.

$$\text{Eq. (6-8):} \quad S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19):} \quad k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$$

$$\text{Eq. (6-24):} \quad d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20):} \quad k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$$

$$\text{Eq. (6-18):} \quad S_e = (0.897)(0.978)(0.5)(64) = 28.1 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{38.45}{28.1} + \frac{38.40}{64}$$

$$n_f = 0.51 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{38.45}{1 - (38.40 / 64)} = 96.1 \text{ kpsi}$$

This stress is much higher than the ultimate strength, rendering it impractical for the S - N diagram. We must conclude that the fluctuating stresses from the combination loading, when increased by the stress concentration factors, are so far from the Goodman line that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

6-53 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-86, the critical stress element experiences $\sigma_{x, \text{bend}} = 55.5 \text{ kpsi}$, $\sigma_{x, \text{axial}} = 0.382 \text{ kpsi}$ and $\tau = 15.3 \text{ kpsi}$. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\max, \text{bend}} = 55.5 \text{ kpsi}$, $\sigma_{\min, \text{bend}} = 0 \text{ kpsi}$, $\sigma_{\max, \text{axial}} = 0.382 \text{ kpsi}$, $\sigma_{\min, \text{axial}} = 0 \text{ kpsi}$, $\tau_{\max} = 15.3 \text{ kpsi}$, $\tau_{\min} = 0 \text{ kpsi}$. Consequently, $\sigma_{m, \text{bend}} = \sigma_{a, \text{bend}} = 27.75 \text{ kpsi}$, $\sigma_{m, \text{axial}} = \sigma_{a, \text{axial}} = 0.191 \text{ kpsi}$, $\tau_m = \tau_a = 7.65 \text{ kpsi}$. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \quad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \quad K_{t,\text{tors}} = 1.39, \quad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$
 Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.78(1.60 - 1) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}(K_{t,\text{axial}} - 1) = 1 + 0.78(1.75 - 1) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.47)(27.75) + (1.59)\frac{(0.191)}{0.85} \right]^2 + 3[(1.32)(7.65)]^2 \right\}^{1/2} = 44.71 \text{ kpsi}$$

$$\sigma'_m = \left\{ \left[(1.47)(27.75) + (1.59)(0.191) \right]^2 + 3[(1.32)(7.65)]^2 \right\}^{1/2} = 44.66 \text{ kpsi}$$

Since these stresses are relatively high compared to the yield strength, we will go ahead and check for yielding using the distortion energy failure theory.

$$\sigma'_{\text{max}} = \sqrt{(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}})^2 + 3(\tau_{\text{max}})^2} = \sqrt{(55.5 + 0.382)^2 + 3(15.3)^2} = 61.8 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{54}{61.8} = 0.87 \quad \text{Ans.}$$

This shows that yielding is predicted. Further analysis of fatigue life is just to be able to report the fatigue factor of safety, though the life will be dictated by the static yielding failure, i.e. $N = 1/2$ cycle. *Ans.*

$$\text{Eq. (6-8):} \quad S'_e = 0.5(64) = 32 \text{ kpsi}$$

$$\text{Eq. (6-19):} \quad k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$$

$$\text{Eq. (6-24):} \quad d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$$

$$\text{Eq. (6-20):} \quad k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$$

$$\text{Eq. (6-18):} \quad S_e = (0.897)(0.978)(0.5)(64) = 28.1 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{44.71}{28.1} + \frac{44.66}{64}$$

$$n_f = 0.44 \quad \text{Ans.}$$

- 6-54** From Table A-20, for AISI 1040 CD, $S_{ut} = 85$ kpsi and $S_y = 71$ kpsi. From the solution to Prob. 6-17 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{rev} = 35.0$ kpsi, producing $\sigma_a = 35.0$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2500(1.625/2)}{\pi(1.625^4)/32} = 2967 \text{ psi} = 2.97 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$,
 $K_{t,bend} = 1.95$, $K_{t,tors} = 1.60$
 Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{bend} = 0.76$, $q_{tors} = 0.81$
 Eq. (6-32):

$$K_{f,bend} = 1 + q_{bend}(K_{t,bend} - 1) = 1 + 0.76(1.95 - 1) = 1.72$$

$$K_{f,tors} = 1 + q_{tors}(K_{t,tors} - 1) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.72)(35.0) \right]^2 + 3 \left[(1.49)(0) \right]^2 \right\}^{1/2} = 60.2 \text{ kpsi}$$

$$\sigma'_m = \left\{ \left[(1.72)(0) \right]^2 + 3 \left[(1.49)(2.97) \right]^2 \right\}^{1/2} = 7.66 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{60.2 + 7.66} = 1.05$$

From the solution to Prob. 6-17, $S_e = 29.5$ kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{60.2}{29.5} + \frac{7.66}{85}$$

$$n_f = 0.47 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the $S-N$ diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{rev} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{60.2}{1 - (7.66 / 85)} = 66.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.867$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.867(85)}{29.5} \right) = -0.1325$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{rev}}{a} \right)^{1/b} = \left(\frac{66.2}{184.1} \right)^{\frac{1}{-0.1325}} = 2251 \text{ cycles}$$

$$N = 2300 \text{ cycles} \quad \text{Ans.}$$

6-55 From the solution to Prob. 6-18 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{rev} = 32.8$ kpsi, producing $\sigma_a = 32.8$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2200(1.625/2)}{\pi(1.625^4)/32} = 2611 \text{ psi} = 2.61 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,bend} = 1.95$, $K_{t,tors} = 1.60$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{bend} = 0.76$, $q_{tors} = 0.81$

Eq. (6-32):

$$K_{f,bend} = 1 + q_{bend} (K_{t,bend} - 1) = 1 + 0.76(1.95 - 1) = 1.72$$

$$K_{f,tors} = 1 + q_{tors} (K_{t,tors} - 1) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ [(1.72)(32.8)]^2 + 3[(1.49)(0)]^2 \right\}^{1/2} = 56.4 \text{ kpsi}$$

$$\sigma'_m = \left\{ [(1.72)(0)]^2 + 3[(1.49)(2.61)]^2 \right\}^{1/2} = 6.74 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{56.4 + 6.74} = 1.12$$

From the solution to Prob. 6-18, $S_e = 29.5$ kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{56.4}{29.5} + \frac{6.74}{85}$$

$$n_f = 0.50 \quad \text{Ans.}$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{56.4}{1 - (6.74 / 85)} = 61.3 \text{ kpsi}$$

$$\text{Fig. 6-18: } f = 0.867$$

$$\text{Eq. (6-14): } a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

$$\text{Eq. (6-15): } b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.867(85)}{29.5} \right) = -0.1325$$

$$\text{Eq. (6-16): } N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{61.3}{184.1} \right)^{-\frac{1}{0.1325}} = 4022 \text{ cycles}$$

$$N = 4000 \text{ cycles} \quad \text{Ans.}$$

6-56 $S_{ut} = 55 \text{ kpsi}$, $S_y = 30 \text{ kpsi}$, $K_{ts} = 1.6$, $L = 2 \text{ ft}$, $F_{\min} = 150 \text{ lbf}$, $F_{\max} = 500 \text{ lbf}$

$$\text{Eqs. (6-34) and (6-35b), or Fig. 6-21: } q_s = 0.80$$

$$\text{Eq. (6-32): } K_{fs} = 1 + q_s (K_{ts} - 1) = 1 + 0.80(1.6 - 1) = 1.48$$

$$T_{\max} = 500(2) = 1000 \text{ lbf} \cdot \text{in}, \quad T_{\min} = 150(2) = 300 \text{ lbf} \cdot \text{in}$$

$$\tau_{\max} = \frac{16 K_{fs} T_{\max}}{\pi d^3} = \frac{16(1.48)(1000)}{\pi(0.875)^3} = 11\,251 \text{ psi} = 11.25 \text{ kpsi}$$

$$\tau_{\min} = \frac{16 K_{fs} T_{\min}}{\pi d^3} = \frac{16(1.48)(300)}{\pi(0.875)^3} = 3375 \text{ psi} = 3.38 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{11.25 + 3.38}{2} = 7.32 \text{ kpsi}$$

$$\tau_a = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{11.25 - 3.38}{2} = 3.94 \text{ kpsi}$$

Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{30 / 2}{11.25} = 1.33$$

Find the modifiers and endurance limit.

$$\begin{aligned}
\text{Eq. (6-8):} \quad S'_e &= 0.5(55) = 27.5 \text{ kpsi} \\
\text{Eq. (6-19):} \quad k_a &= 14.4(55)^{-0.718} = 0.81 \\
\text{Eq. (6-24):} \quad d_e &= 0.370(0.875) = 0.324 \text{ in} \\
\text{Eq. (6-20):} \quad k_b &= 0.879(0.324)^{-0.107} = 0.99 \\
\text{Eq. (6-26):} \quad k_c &= 0.59 \\
\text{Eq. (6-18):} \quad S_{se} &= 0.81(0.99)(0.59)(27.5) = 13.0 \text{ kpsi}
\end{aligned}$$

Since the stress is entirely shear, we will use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67S_u = 0.67(55) = 36.9 \text{ kpsi}$.

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(3.94 / 13.0) + (7.32 / 36.9)} = 1.99 \quad \text{Ans.}$$

(b) Gerber, Table 6-7

$$\begin{aligned}
n_f &= \frac{1}{2} \left(\frac{S_{su}}{\tau_m} \right)^2 \frac{\tau_a}{S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su} \tau_a} \right)^2} \right] \\
&= \frac{1}{2} \left(\frac{36.9}{7.32} \right)^2 \left(\frac{3.94}{13.0} \right) \left[-1 + \sqrt{1 + \left(\frac{2(7.32)(13.0)}{36.9(3.94)} \right)^2} \right] \\
n_f &= 2.49 \quad \text{Ans.}
\end{aligned}$$

6-57 $S_{ut} = 145 \text{ kpsi}, S_y = 120 \text{ kpsi}$

From Eqs. (6-34) and (6-35a), or Fig. 6-20, with a notch radius of 0.1 in, $q = 0.9$. Thus, with $K_t = 3$ from the problem statement,

$$\begin{aligned}
K_f &= 1 + q(K_t - 1) = 1 + 0.9(3 - 1) = 2.80 \\
\sigma_{\max} &= -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P \\
\sigma_m &= -\sigma_a = \frac{1}{2}(-2.476P) = -1.238P \\
T_{\max} &= \frac{f P (D + d)}{4} = \frac{0.3P(6 + 1.2)}{4} = 0.54P
\end{aligned}$$

From Eqs. (6-34) and (6-35b), or Fig. 6-21, with a notch radius of 0.1 in, $q_s = 0.92$. Thus, with $K_{ts} = 1.8$ from the problem statement,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8 - 1) = 1.74$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(1.238P / 0.85)^2 + 3(1.385P)^2]^{1/2} = 2.81P$$

$$\sigma'_m = [\sigma_m^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Eq. (6-8): $S'_e = 0.5(145) = 72.5$ kpsi

Eq. (6-19): $k_a = 2.70(145)^{-0.265} = 0.722$

Eq. (6-20): $k_b = 0.879(1.2)^{-0.107} = 0.862$

Eq. (6-18): $S_e = (0.722)(0.862)(72.5) = 45.12$ kpsi

Modified Goodman: $\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{2.81P}{45.12} + \frac{2.70P}{145} = \frac{1}{3}$

$$P = 4.12 \text{ kips} \quad \text{Ans.}$$

Yield (conservative): $n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{120}{(2.81)(4.12) + (2.70)(4.12)} = 5.29 \quad \text{Ans.}$

6-58 From Prob. 6-57, $K_f = 2.80$, $K_{fs} = 1.74$, $S_e = 45.12$ kpsi

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.80 \frac{4(18)}{\pi(1.2)^2} = -44.56 \text{ kpsi}$$

$$\sigma_{\min} = -K_f \frac{4P_{\min}}{\pi d^2} = -2.80 \frac{4(4.5)}{\pi(1.2)^2} = -11.14 \text{ kpsi}$$

$$T_{\max} = f P_{\max} \left(\frac{D+d}{4} \right) = 0.3(18) \left(\frac{6+1.2}{4} \right) = 9.72 \text{ kip} \cdot \text{in}$$

$$T_{\min} = f P_{\min} \left(\frac{D+d}{4} \right) = 0.3(4.5) \left(\frac{6+1.2}{4} \right) = 2.43 \text{ kip} \cdot \text{in}$$

$$\tau_{\max} = K_{fs} \frac{16T_{\max}}{\pi d^3} = 1.74 \frac{16(9.72)}{\pi(1.2)^3} = 49.85 \text{ kpsi}$$

$$\tau_{\min} = K_{fs} \frac{16T_{\min}}{\pi d^3} = 1.74 \frac{16(2.43)}{\pi(1.2)^3} = 12.46 \text{ kpsi}$$

$$\sigma_a = \left| \frac{-44.56 - (-11.14)}{2} \right| = 16.71 \text{ kpsi}$$

$$\sigma_m = \frac{-44.56 + (-11.14)}{2} = -27.85 \text{ kpsi}$$

$$\tau_a = \frac{49.85 - 12.46}{2} = 18.70 \text{ kpsi}$$

$$\tau_m = \frac{49.85 + 12.46}{2} = 31.16 \text{ kpsi}$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(16.71 / 0.85)^2 + 3(18.70)^2]^{1/2} = 37.89 \text{ kpsi}$$

$$\sigma'_m = [\sigma_m^2 + 3\tau_m^2]^{1/2} = [(-27.85)^2 + 3(31.16)^2]^{1/2} = 60.73 \text{ kpsi}$$

Modified Goodman:
$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{37.89}{45.12} + \frac{60.73}{145}$$

$$n_f = 0.79$$

Since infinite life is not predicted, estimate a life from the S - N diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{37.89}{1 - (60.73 / 145)} = 65.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.8$

Eq. (6-14):
$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.8(145)]^2}{45.12} = 298.2$$

Eq. (6-15):
$$b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left(\frac{0.8(145)}{45.12} \right) = -0.1367$$

Eq. (6-16):
$$N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{65.2}{298.2} \right)^{\frac{1}{-0.1367}} = 67\,607 \text{ cycles}$$

$$N = 67\,600 \text{ cycles} \quad \text{Ans.}$$

6-59 For AISI 1020 CD, From Table A-20, $S_y = 390 \text{ MPa}$, $S_{ut} = 470 \text{ MPa}$. Given: $S_e = 175 \text{ MPa}$.

First Loading:
$$(\sigma_m)_1 = \frac{360 + 160}{2} = 260 \text{ MPa}, \quad (\sigma_a)_1 = \frac{360 - 160}{2} = 100 \text{ MPa}$$

Goodman: $(\sigma_a)_{e1} = \frac{(\sigma_a)_1}{1 - (\sigma_m)_1 / S_{ut}} = \frac{100}{1 - 260 / 470} = 223.8 \text{ MPa} > S_e \therefore \text{finite life}$

$$a = \frac{[0.9(470)]^2}{175} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{0.9(470)}{175} = -0.127767$$

$$N = \left(\frac{223.8}{1022.5} \right)^{-1/0.127767} = 145\,920 \text{ cycles}$$

Second loading: $(\sigma_m)_2 = \frac{320 + (-200)}{2} = 60 \text{ MPa}, \quad (\sigma_a)_2 = \frac{320 - (-200)}{2} = 260 \text{ MPa}$

$$(\sigma_a)_{e2} = \frac{260}{1 - 60 / 470} = 298.0 \text{ MPa}$$

(a) Miner's method: $N_2 = \left(\frac{298.0}{1022.5} \right)^{-1/0.127767} = 15\,520 \text{ cycles}$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \Rightarrow \frac{80\,000}{145\,920} + \frac{n_2}{15\,520} = 1 \Rightarrow n_2 = 7000 \text{ cycles} \quad \text{Ans.}$$

(b) Manson's method: The number of cycles remaining after the first loading

$$N_{\text{remaining}} = 145\,920 - 80\,000 = 65\,920 \text{ cycles}$$

Two data points: 0.9(470) MPa, 10^3 cycles
223.8 MPa, 65 920 cycles

$$\frac{0.9(470)}{223.8} = \frac{a_2 (10^3)^{b_2}}{a_2 (65\,920)^{b_2}}$$

$$1.8901 = (0.015170)^{b_2}$$

$$b_2 = \frac{\log 1.8901}{\log 0.015170} = -0.151997$$

$$a_2 = \frac{223.8}{(65\,920)^{-0.151997}} = 1208.7 \text{ MPa}$$

$$n_2 = \left(\frac{298.0}{1208.7} \right)^{1/-0.151997} = 10\,000 \text{ cycles} \quad \text{Ans.}$$

6-60 Given: $S_e = 50 \text{ kpsi}$, $S_{ut} = 140 \text{ kpsi}$, $f = 0.8$. Using Miner's method,

$$a = \frac{[0.8(140)]^2}{50} = 250.88 \text{ kpsi}$$

$$b = -\frac{1}{3} \log \frac{0.8(140)}{50} = -0.116\,749$$

$$\sigma_1 = 95 \text{ kpsi}, \quad N_1 = \left(\frac{95}{250.88} \right)^{1/-0.116\,749} = 4100 \text{ cycles}$$

$$\sigma_2 = 80 \text{ kpsi}, \quad N_2 = \left(\frac{80}{250.88} \right)^{1/-0.116\,749} = 17\,850 \text{ cycles}$$

$$\sigma_3 = 65 \text{ kpsi}, \quad N_3 = \left(\frac{65}{250.88} \right)^{1/-0.116\,749} = 105\,700 \text{ cycles}$$

$$\frac{0.2N}{4100} + \frac{0.5N}{17\,850} + \frac{0.3N}{105\,700} = 1 \Rightarrow N = 12\,600 \text{ cycles } \textit{Ans.}$$

6-61 Given: $S_{ut} = 530 \text{ MPa}$, $S_e = 210 \text{ MPa}$, and $f = 0.9$.

(a) Miner's method

$$a = \frac{[0.9(530)]^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{0.9(530)}{210} = -0.118\,766$$

$$\sigma_1 = 350 \text{ MPa}, \quad N_1 = \left(\frac{350}{1083.47} \right)^{1/-0.118\,766} = 13\,550 \text{ cycles}$$

$$\sigma_2 = 260 \text{ MPa}, \quad N_2 = \left(\frac{260}{1083.47} \right)^{1/-0.118\,766} = 165\,600 \text{ cycles}$$

$$\sigma_3 = 225 \text{ MPa}, \quad N_3 = \left(\frac{225}{1083.47} \right)^{1/-0.118\,766} = 559\,400 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{13\,550} + \frac{50\,000}{165\,600} + \frac{n_3}{559\,400} = 184\,100 \text{ cycles } \textit{Ans.}$$

(b) Manson's method:

The life remaining after the first series of cycling is $N_{R1} = 13\,550 - 5000 = 8550$ cycles. The two data points required to define $S'_{e,1}$ are $[0.9(530), 10^3]$ and $(350, 8550)$.

$$\frac{0.9(530)}{350} = \frac{a_2 (10^3)^{b_2}}{a_2 (8550)^{b_2}} \Rightarrow 1.3629 = (0.11696)^{b_2}$$

$$b_2 = \frac{\log(1.3629)}{\log(0.11696)} = -0.144280$$

$$a_2 = \frac{350}{(8550)^{-0.144280}} = 1292.3 \text{ MPa}$$

$$N_2 = \left(\frac{260}{1292.3} \right)^{-1/0.144280} = 67\,090 \text{ cycles}$$

$$N_{R2} = 67\,090 - 50\,000 = 17\,090 \text{ cycles}$$

$$\frac{0.9(530)}{260} = \frac{a_3 (10^3)^{b_3}}{a_3 (17\,090)^{b_3}} \Rightarrow 1.8346 = (0.058514)^{b_3}$$

$$b_3 = \frac{\log(1.8346)}{\log(0.058514)} = -0.213785, \quad a_3 = \frac{260}{(17\,090)^{-0.213785}} = 2088.7 \text{ MPa}$$

$$N_3 = \left(\frac{225}{2088.7} \right)^{-1/0.213785} = 33\,610 \text{ cycles} \quad \text{Ans.}$$

6-62 Given: $S_e = 45$ kpsi, $S_{ut} = 85$ kpsi, $f = 0.86$, and $\sigma_a = 35$ kpsi and $\sigma_m = 30$ kpsi for $12 (10^3)$ cycles.

Gerber equivalent reversing stress: $\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})^2} = \frac{35}{1 - (30/85)^2} = 39.98 \text{ kpsi}$

(a) Miner's method: $\sigma_{\text{rev}} < S_e$. According to the method, this means that the endurance limit has not been reduced and the new endurance limit is $S'_e = 45$ kpsi. Ans.

(b) Manson's method: Again, $\sigma_{\text{rev}} < S_e$. According to the method, this means that the material has not been damaged and the endurance limit has not been reduced. Thus, the new endurance limit is $S'_e = 45$ kpsi. Ans.

6-63 Given: $S_e = 45$ kpsi, $S_{ut} = 85$ kpsi, $f = 0.86$, and $\sigma_a = 35$ kpsi and $\sigma_m = 30$ kpsi for $12 (10^3)$ cycles.

Goodman equivalent reversing stress: $\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{35}{1 - (30/85)} = 54.09 \text{ kpsi}$

Initial cycling

$$a = \frac{[0.86(85)]^2}{45} = 116.00 \text{ kpsi}$$

$$b = -\frac{1}{3} \log \frac{0.86(85)}{45} = -0.070 \ 235$$

$$\sigma_1 = 54.09 \text{ kpsi}, \quad N_1 = \left(\frac{54.09}{116.00} \right)^{1/-0.070 \ 235} = 52 \ 190 \text{ cycles}$$

- (a) Miner's method (see discussion on p. 333): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}} = 52 \ 190 - 12 \ 000 = 40 \ 190$ cycles. The new coefficients are $b' = b$, and $a' = S_f / N^{b'} = 54.09 / (40 \ 190)^{-0.070 \ 235} = 113.89$ kpsi. The new endurance limit is

$$S'_{e,1} = a' N_e^{b'} = 113.89 (10^6)^{-0.070 \ 235} = 43.2 \text{ kpsi} \quad \text{Ans.}$$

- (b) Manson's method (see discussion on p. 334): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}} = 52 \ 190 - 12 \ 000 = 40 \ 190$ cycles. At 10^3 cycles, $S_f = 0.86(85) = 73.1$ kpsi. The new coefficients are $b' = [\log(73.1/54.09)] / \log(10^3/40 \ 190) = -0.081 \ 540$ and $a' = \sigma_1 / (N_{\text{remaining}})^{b'} = 54.09 / (40 \ 190)^{-0.081 \ 540} = 128.39$ kpsi. The new endurance limit is

$$S'_{e,1} = a' N_e^{b'} = 128.39 (10^6)^{-0.081 \ 540} = 41.6 \text{ kpsi} \quad \text{Ans.}$$
