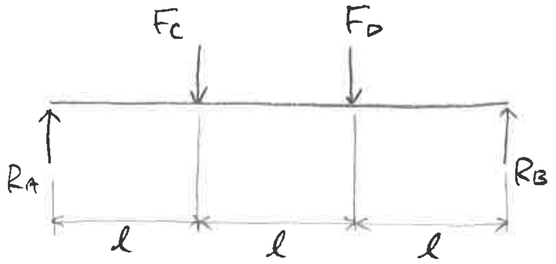
(a)



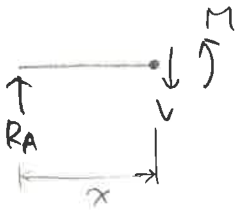
$(\ell = 192\,mm)$

$\Sigma F_y = 0;\quad R_A - F_C - F_D + R_B = 0$

$\Sigma M_A = 0;\quad -F_C \cdot \ell - F_D(2\ell) + R_B \cdot (3\ell) = 0$

$\Rightarrow R_A = 2.5\,kN,\quad R_B = 2.0\,kN$

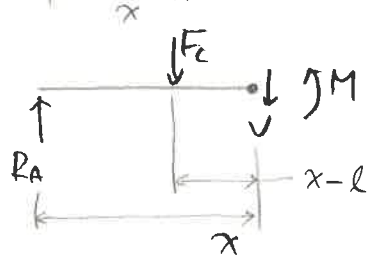
$0 \leqq x < \ell$;



$\Sigma M = 0;\quad M - R_A x = 0$

$\Rightarrow M = R_A x$

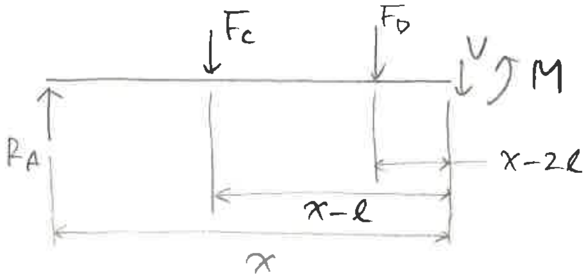
$0 \leqq x < 2\ell$;



$\Sigma M = 0;\quad M - R_A x + F_C(x-\ell) = 0$

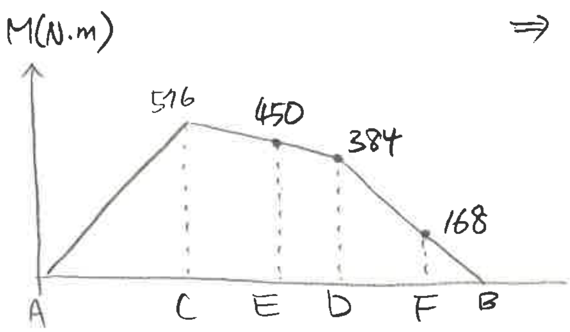
$\Rightarrow M = (R_A - F_C)x + F_C \cdot \ell$

$0 \leqq x < 3\ell$;



$\Sigma M = 0;\quad M - R_A x + F_C(x-\ell) + F_D(x-2\ell) = 0$

$\Rightarrow M = (R_A - F_C - F_D) + F_C \cdot \ell + F_D \cdot (2\ell)$



$$\sigma_C = \frac{M_C \cdot c}{I} = \frac{(516\,N\cdot m)(40/2\,mm)}{\frac{\pi(40mm)^4}{64}} = 91.7\,MPa$$

$$\sigma_D = \frac{M_D \cdot c}{I} = \frac{(384\,N\cdot m)(45/2\,mm)}{\frac{\pi(45\,mm)^4}{64}} = 42.9\,MPa$$

$$\sigma_E = K_t \frac{M_E C}{I} = (2.3)\frac{(450\,N\cdot m)(40/2\,mm)}{\frac{\pi(40mm)^4}{64}} = 164.7\,MPa$$

($K_t = 2.3$ determined from the 1st mid-term exam)

$$\sigma_F = K_t \frac{M_F C}{I} = (1.96)\frac{(168\,N\cdot m)(45/2\,mm)}{\frac{\pi(45mm)^4}{64}} = 45.3\,MPa$$

($K_t = 1.96$ at F from the 1st mid-term exam).

σ_E is the largest and thus E is the critical location.

(b) From Eq. (6-55), when $(\tau_a)_{torsion} = (\tau_m)_{torsion} = 0$,

$$\begin{cases} \sigma_a' = (K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial}\dfrac{(\sigma_a)_{axial}}{0.85} \\ \sigma_m' = (K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axial}(\sigma_m)_{axial} \end{cases}$$

Using Eqns. (7-3) and $(\sigma_a)_{axial} = \frac{P_a}{A}$ and $(\sigma_m)_{axial} = \frac{P_m}{A}$

where P_a = alternating axial load, P_m = mean axial load and

A = cross sectional area of the shaft,

$$\begin{cases} \sigma_a' = (K_f)_{bending}\left(\dfrac{32M_a}{\pi d^3}\right) + \dfrac{(K_f)_{axial}}{0.85}\dfrac{4P_a}{\pi d^2} \\ \sigma_m' = (K_f)_{bending}\left(\dfrac{32M_m}{\pi d^3}\right) + (K_f)_{axial}\dfrac{4P_m}{\pi d^2} \end{cases}$$

∴ Eq. (7-7) is rewritten as

$$\frac{1}{n} = \frac{1}{S_e}\left[(K_f)_{bending}\left(\frac{32M_a}{\pi d^3}\right) + \frac{(K_f)_{axial}}{0.85}\frac{4P_a}{\pi d^2}\right] + \frac{1}{S_{ut}}\left[(K_f)_{bending}\left(\frac{32M_m}{\pi d^3}\right) + (K_f)_{axial}\frac{4P_m}{\pi d^2}\right]$$

○ ok.

Eq. (7-8) can be modified as

$$\frac{1}{n}d^3 - (K_f)_{axial}\left[\frac{1}{S_e}\cdot\frac{4P_a}{0.85\pi} + \frac{1}{S_{ut}}\frac{4P_m}{\pi}\right]d = (K_f)_{bending}\left[\frac{1}{S_e}\left(\frac{32M_a}{\pi d^3}\right) + \frac{1}{S_{ut}}\left(\frac{32M_m}{\pi d^3}\right)\right]$$

○ ok.

(c) From Part (b)

$$\frac{1}{n} = \frac{\sigma_a'}{S_f} + \frac{\sigma_m'}{S_{ut}}$$ (σ_a' and σ_m' from part (b))

S_f ↖ not S_{ut}!

$$S_f = 1274 \times (10^4)^{-0.132}$$ (Eq. 6-13 in text book)

$$= 377.7\text{ MPa}$$ a, b from the 1st mid-term exam.

$$\sigma_a = \frac{P_a}{A}, \quad \sigma_m = \frac{P_m}{A} \qquad P_a = \frac{P_{max} - P_{min}}{2}, \quad P_m = \frac{P_{max} + P_{min}}{2}$$

$P_{max} = 50$ kN and $P_{min} = 10$ kN $\Rightarrow \begin{cases} \sigma_a = 15.9\text{ MPa} \\ \sigma_m = 23.9\text{ MPa} \end{cases}$

$M_a = M_E = 450$ N.m $M_m = 0$ (completely reversed bending)

$\therefore \left.\begin{array}{l} \sigma_a' = 176.8\text{ MPa} \\ \sigma_m' = 43.8\text{ MPa} \end{array}\right\}$ from the solution in part (b)

$(K_f)_{bending}$ from 1st mid-term exam.

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_f} = \frac{1}{n} \Rightarrow n = 1.844$$

($S_e = 205.4$ MPa from the 1st mid-term exam).

(d) $$\sigma_a' = (K_f)_{bending}\left(\frac{32M_a}{\pi d^3}\right) + \frac{(K_f)_{axial}}{0.85}\frac{4P_a}{\pi d^2}$$

Solve for P_a from $\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$ when $n = 1$

$$\Rightarrow P_a = 27.8\text{ kN}.$$

$(K_f)_{axial}$? $r/d = 0.0375$, $D/d = 45/40 = 1.125 \Rightarrow K_t = 2.1$ from Fig. A-15-7. Then, $(K_f)_{axial} = 1 + q(K_t - 1) = 1 + (0.76)(2.1 - 1) = 1.836$

(q from 1st mid-term exam)