PROBLEM SET 4.1

Problem 1

The problem is to find the zero of $f(x) = x^3 - 75$. With $f'(x) = 3x^2$, Newton's formula is

$$x \leftarrow x - \frac{x^3 - 75}{3x^2}$$

Starting with $x = 4$, succesive iterations yield

$$\begin{aligned}
x &\leftarrow 4 - \frac{4^3 - 75}{3(4)^2} = 4.229 \\
x &\leftarrow 4.229 - \frac{4.229^3 - 75}{3(4.229)^2} = 4.217 \\
x &\leftarrow 4.217 - \frac{4.217^3 - 75}{3(4.217)^2} = 4.217 \quad \blacktriangleleft
\end{aligned}$$

Problem 2

$$f(x) = x^3 - 3.23x^2 - 5.54x + 9.84$$

We begin with a root search starting at $x = 1$ and launch bisection once the root is bracketed.

x	$f(x)$	Interval
1.0	2.070	
1.2	0.269	
1.4	−1.503	(1.2, 1.4)
(1.2 + 1.4)/2 = 1.3	−0.624	(1.2, 1.3)
(1.2 + 1.3)/2 = 1.25	−0.179	(1.2, 1.25)
(1.2 + 1.25)/2 = 1.225	0.045	(1.225, 1.25
(1.225 + 1.25)/2 = 1.2375	−0.067	(1.225, 1.2375)
(1.225 + 1.2375)/2 = 1.2313	−0.012	(1.225, 1.2313)
(1.225 + 1.2313)/2 = 1.2282	0.017	(1.2282, 1.2313)
(1.2282 + 1.2313)/2 = 1.2298	0.002	(1.2298, 1.2313)
(1.2298 + 1.2313)/2 = 1.2306	−0.005	(1.2298, 1.2306
(1.2298 + 1.2306)/2 = 1.2302	−0.002	(1.2298, 1.2302)

The root is $x = 1.230$ ◄

Problem 3

$$f(x) = \cosh x \cos x - 1$$

The starting points are

$$x_1 = 4 \qquad x_2 = 5$$

The first step is bisection, giving us the point

$$x_3 = 4.5$$

Each subsequent step uses the quadratic interpolation formula

$$x = -\frac{f_2 f_3 x_1(f_2 - f_3) + f_3 f_1 x_2(f_3 - f_1) + f_1 f_2 x_3(f_1 - f_2)}{(f_1 - f_2)(f_2 - f_3)(f_3 - f_1)}$$

to compute the improved value of x, followed by reordering of data points using the following scheme:

$$\begin{aligned} \text{if } x &< x_3 : x_2 \leftarrow x_3 \\ \text{if } x &> x_3 : x_1 \leftarrow x_3 \\ x_3 &\leftarrow x \end{aligned}$$

Here are the results of the computations:

x_1	x_2	x_3	f_1	f_2	f_3	x	$f(x)$
4.000	5.000	4.500	−18.850	20.051	−10.489	4.907	12.038
4.500	5.000	4.907	−10.489	20.051	12.038	4.716	−0.818
4.500	4.907	4.716	−10.489	12.038	−0.818	4.731	0.0582
4.716	4.907	4.731	−0.818	12.038	0.0582	4.730	

Hence the root is $x = 4.730$ ◄.

Problem 4

Newton's formula is

$$x \leftarrow x - \frac{f(x)}{f'(x)}$$

where

$$\begin{aligned} f(x) &= \cosh x \cos x - 1 \\ f'(x) &= \sinh x \cos x - \cosh x \sin x \end{aligned}$$

Starting with $x = 4.5$, successive applications of the formula yield

$$\begin{aligned}
x &\leftarrow 4.5 - \frac{-10.489}{34.52} = 4.804 \\
x &\leftarrow 4.804 - \frac{4.573}{66.31} = 4.735 \\
x &\leftarrow 4.735 - \frac{0.283}{58.20} = 4.730 \\
x &\leftarrow 4.730 - \frac{0.001}{57.65} = 4.730 \ \blacktriangleleft
\end{aligned}$$

Problem 5

$$f(x) = \tan x - \tanh x$$

x	$f(x)$	Interval
7.0	−0.129	
7.4	1.049	(7.0, 7.4)
$(7.0 + 7.4)/2 = 7.2$	0.305	(7.0, 7.2)
$(7.0 + 7.2)/2 = 7.1$	0.065	(7.0, 7.1)
$(7.0 + 7.1)/2 = 7.05$	−0.036	(7.05, 7.1)
$(7.05 + 7.1)/2 = 7.075$	0.013	(7.05, 7.075)
$(7.05 + 7.075)/2 = 7.063$	−0.011	(7.063, 7.075)
$(7.063 + 7.075)/2 = 7.069$	0.000	

$$x = 7.069 \ \blacktriangleleft$$

Problem 6

$$\begin{aligned}
f(x) &= \sin x + 3\cos x - 2 \\
f'(x) &= \cos x - 3\sin x
\end{aligned}$$

$$x \leftarrow x - \frac{f(x)}{f'(x)}$$

Starting with $x = -2$, successive applications of Newton's iterative formula yield

$$\begin{aligned}
x &\leftarrow -2 - \frac{-4.1577}{2.3117} = -0.2015 \\
x &\leftarrow -0.2015 - \frac{0.7392}{1.5801} = -0.6693 \\
x &\leftarrow -0.6693 - \frac{-0.2676}{2.6456} = -0.5682 \\
x &\leftarrow -0.5682 - \frac{-0.0093}{2.4571} = -0.5644 \\
x &\leftarrow -0.5644 - \frac{0.0000}{2.445} = -0.5644 \ \blacktriangleleft
\end{aligned}$$

Starting with $x = 2$, we get

$$\begin{aligned}
x &\leftarrow 2 - \frac{-2.3391}{-3.1440} = 1.2560 \\
x &\leftarrow 1.2560 - \frac{-0.1203}{-2.5430} = 1.2087 \\
x &\leftarrow 1.2087 - \frac{-0.0021}{-2.4512} = 1.2078 \\
x &\leftarrow 1.2078 - \frac{0.0000}{-2.4495} = 1.2078 \ \blacktriangleleft
\end{aligned}$$

Problem 7

$$f(x) = \sin x + 3\cos x - 2$$

$$x_{i+1} = x_i - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} f(x_i)$$

Start with $x_0 = -2$, $x_1 = -1.5$:

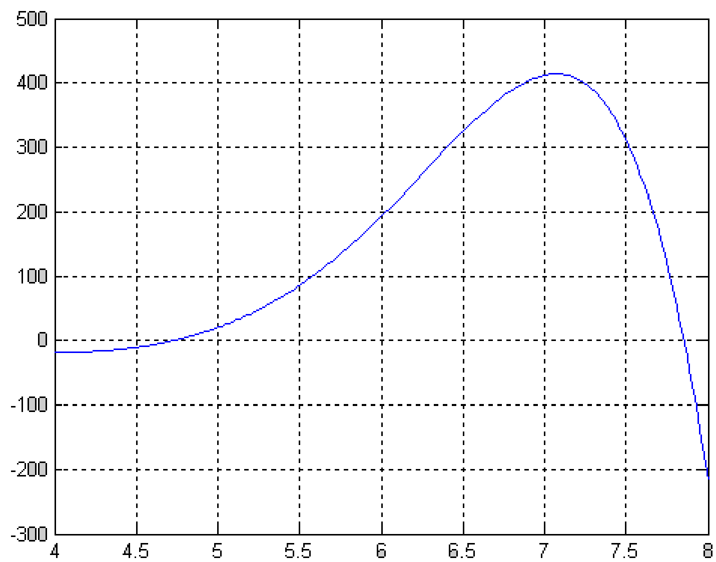
$$\begin{aligned}
x_2 &= -1.5 - \frac{-1.5 - (-2)}{-2.7853 - (-4.1577)}(-2.7853) = -0.4852 \\
x_3 &= -0.4852 - \frac{-0.4852 - (-1.5)}{0.1872 - (-2.7853)}(0.1872) = -0.5491 \\
x_4 &= -0.5491 - \frac{-0.5491 - (-0.4852)}{0.0369 - 0.1872}(0.0369) = -0.5648 \\
x_5 &= -0.5648 - \frac{-0.5648 - (-0.5491)}{-0.0013 - 0.0369}(-0.0013) = -0.5643 \\
x_6 &= -0.5643 - \frac{-0.5643 - (-0.5648)}{0.0000 - (-0.0013)}(0.0000) = -0.5643 \ \blacktriangleleft
\end{aligned}$$

Start with $x_0 = 2$, $x_1 = 1.5$:

$$\begin{aligned}
x_2 &= 1.5 - \frac{1.5 - 2}{-0.7903 - (-2.3391)}(-0.7903) = 1.2449 \\
x_3 &= 1.2449 - \frac{1.2449 - 1.5)}{-0.0921 - (-0.7903)}(-0.0921) = 1.2112 \\
x_4 &= 1.2112 - \frac{1.2112 - 1.2449}{-0.0083 - (-0.0921)}(-0.0083) = 1.2079 \\
x_5 &= 1.2079 - \frac{1.2079 - 1.2112}{-0.0001 - (-0.0083)}(-0.0001) = 1.2079 \quad \blacktriangleleft
\end{aligned}$$

Problem 8

$$f(x) = \cosh x \cos x - 1$$



(a)

We see from the plot that a root of $f(x) = 0$ is at approximately $x = 4.75$.

(b)

The first "improved" value of x predicted by the Newton-Raphson formula is at the intersection of the tangent at $x = 4$ and the x-axis. Since the tangent is almost horizontal, the intersection point is off the right end of the plot (in $x > 8$). It is clear that subsequent iterations would keep x away from the root near 4.75.

We can confirm our findings from the Newton-Raphson formula:

$$x \leftarrow x - \frac{f(x)}{f'(x)} = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{-18.85}{2.829} = 10.66$$

which is indeed "off the chart".

Problem 9

$$\begin{aligned} f(x) &= x^3 - 1.2x^2 - 8.19x + 13.23 \\ f'(x) &= 3x^2 - 2.4x - 8.19 \end{aligned}$$

In Example 4.7 it was suggested that if m is the multiplicity of the root, convergence can be improved by using the modified version

$$x \leftarrow x - m\frac{f(x)}{f'(x)}$$

of the Newton-Raphson formula (in our case $m = 2$).
Starting with $x = 2$, we get

$$\begin{aligned} x &\leftarrow 2 - 2\frac{0.05}{-0.99} = 2.1010 \\ x &\leftarrow 2.1010 - 2\frac{5.205 \times 10^{-6}}{0.0103} = 2.1000 \\ x &\leftarrow 2.1000 - 2\frac{0.000}{1.02 \times 10^{-6}} = 2.1000 \quad \blacktriangleleft \end{aligned}$$

Problem 10

The easiest way to handle this problem is to simply replace `bisect` with `ridder` in Example 4.3. We chose a slightly different approach and wrote the function `rootsRidder` loosely based on Example 4.3:

```
function rootsRidder(func,a,b,dx,tol)
% Computes all the roots of func(x) in the interval (a,b)
% with Ridder's method.
% USAGE: roots(func,a,b,dx,tol)
% func = handle of function that returns f(x)
% dx   = increment of x used in root search
% tol  = error tolerance (default is 10.e-6)
```

```
if nargin < 5; tol = 1.0e-6; end
fprintf('Roots:\n')
while 1
    [x1,x2] = rootsearch(func,a,b,dx);
    if isnan(x1)
        fprintf('Done'); break
    else
        a = x2;
        x = ridder(func,x1,x2,tol);
        if isnan(x); continue
        else fprintf('%16.6e\n', x); end
    end
end
```

The roots are now obtained with the program

```
% problem4_1_10
func = inline('x*sin(x) + 3*cos(x) - x','x');
rootsRidder(func,-6,6,0.5)

Roots:
  -4.712389e+000
  -3.208839e+000
   1.570796e+000
Done
```

Note the use of MATLAB's *in-line function*, which is passed to `rootsRidder`. An in-line function can be evaluated by feval in the same manner as a function stored in a M-file. The advantage of an in-line function is that it does not create a new M-file.

Problem 11

The algorithm listed below is similar to `rootsRidder` in Problem 10.

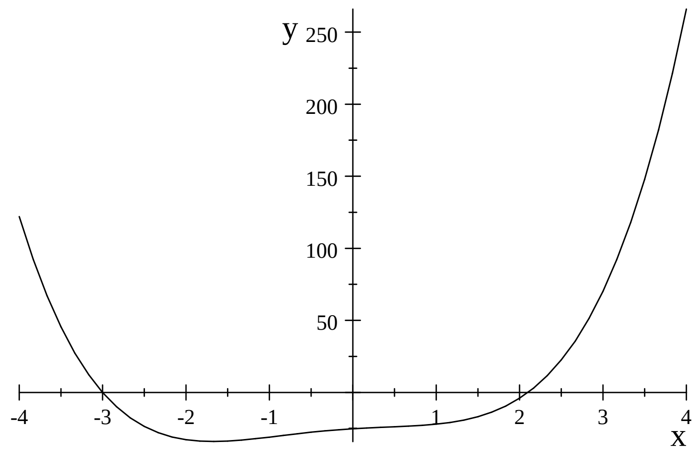
```
function rootsNewton(func,dfunc,a,b,dx,tol)
% Computes all the roots of f(x) in the interval (a,b)
% with the Newton-Raphson method.
% USAGE: rootsNewton(func,dfunc,a,b,dx,tol)
% func  = handle of function that returns f(x)
% dfunc = handle of function that returns f'(x)
% dx    = increment of x used in root search
% tol   = error tolerance (default is 10.e-6)
```

```
if nargin < 6; tol = 1.0e-6; end
fprintf('Roots:\n')
while 1
    [x1,x2] = rootsearch(func,a,b,dx);
    if isnan(x1)
        fprintf('Done'); break
    else
        a = x2;
        x = newtonRaphson(func,dfunc,x1,x2,tol);
        if isnan(x); continue
        else fprintf('%16.6e\n', x); end
    end
end

% problem4_1_11
func = inline('x*sin(x) + 3*cos(x) - x','x');
dfunc = inline('x*cos(x) - 2*sin(x) - 1','x');
rootsNewton(func,dfunc,-6,6,0.5)

>> Roots:
  -4.712389e+000
  -3.208839e+000
   1.570796e+000
Done
```

Problem 12

$$\begin{aligned} f(x) &= x^4 + 0.9x^3 - 2.3x^2 + 3.6x - 25.2 \\ f'(x) &= 4x^3 + 2.7x^2 - 4.6x + 3.6 \end{aligned}$$

Whenever possible, the function should plotted in order to gain information about its behaviour and locate its zeros.

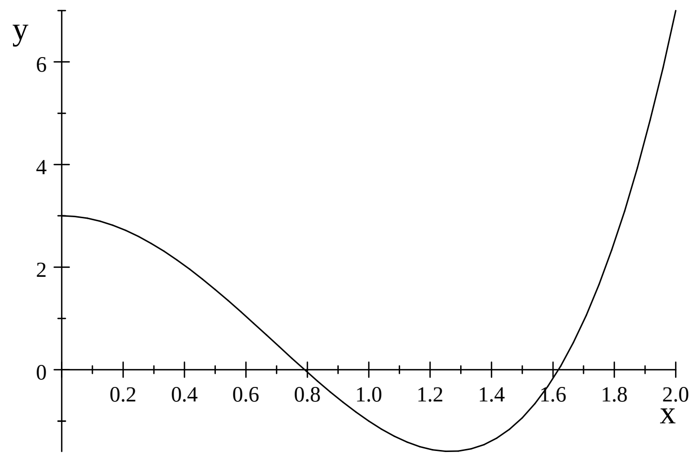


From the plot of $f(x)$ we see that there are two roots, located in $(-3.2, 2.4)$. As the derivative of the function is easily obtained, we use the Newton-Raphson method due to its superior covergence. The program below calls `rootsNewton` listed in Problem 11.

```
% problem4_1_12
func = inline('x^4 + 0.9*x^3 - 2.3*x^2 + 3.6*x - 25.2','x');
dfunc = inline('4*x^3 + 2.7*x^2 - 4.6*x + 3.6','x');
rootsNewton(func,dfunc,-3.2,2.4,2)

>> Roots:
  -3.000000e+000
   2.100000e+000
Done
```

Problem 13

$$
\begin{aligned}
f(x) &= x^4 + 2x^3 - 7x^2 + 3 \\
f'(x) &= 4x^3 + 6x^2 - 14x
\end{aligned}
$$

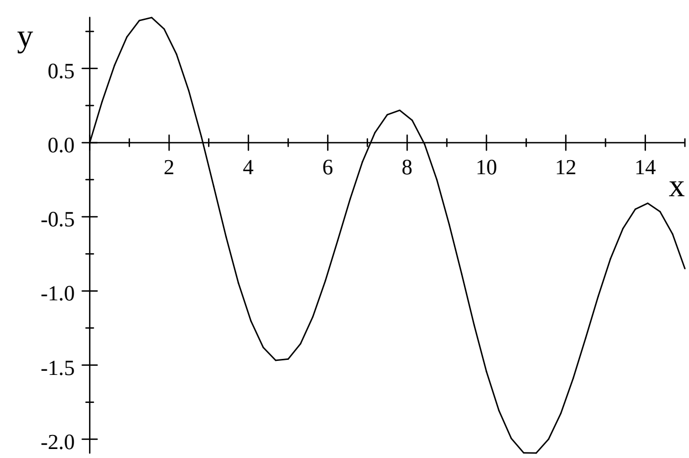


By inspection of the plot of $f(x)$ we see that the two positive roots are located in $(0.7, 1.7)$. Again we compute these roots with the function `rootsNewton` in Problem 11.

```
% problem4_1_13
func = inline('x^4 + 2*x^3 - 7*x^2 + 3','x');
dfunc = inline('4*x^3 + 6*x^2 - 14*x','x');
rootsNewton(func,dfunc,0.7,1.7,0.5)

>> Roots:
   7.912878e-001
   1.618034e+000
Done
```

Problem 14

$$f(x) = \sin x - 0.1x$$

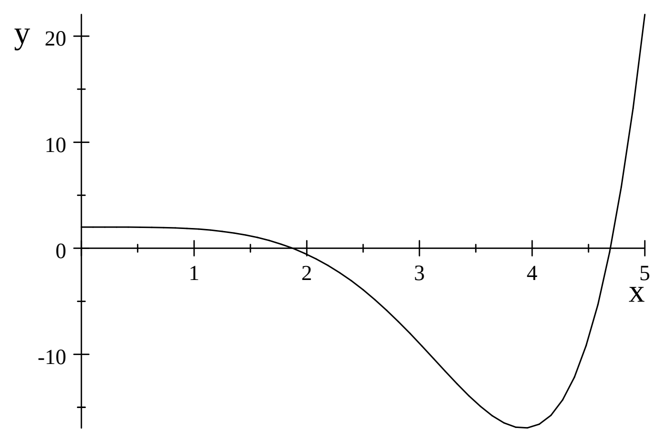


The plot shows that all the positive nonzero roots are in the interval $(2, 9)$. Here we chose to compute the roots with `rootsRidder` listed in Problem 10.

```
% problem4_1_14
func = inline('sin(x) - 0.1*x','x');
rootsRidder(func,2,9,0.5)

>> Roots:
   2.852342e+000
   7.068174e+000
   8.423204e+000
Done
```

Problem 15

$$f(\beta) = \cosh\beta\cos\beta + 1$$
$$f'(\beta) = \sinh x\cos x - \cosh x\sin x$$



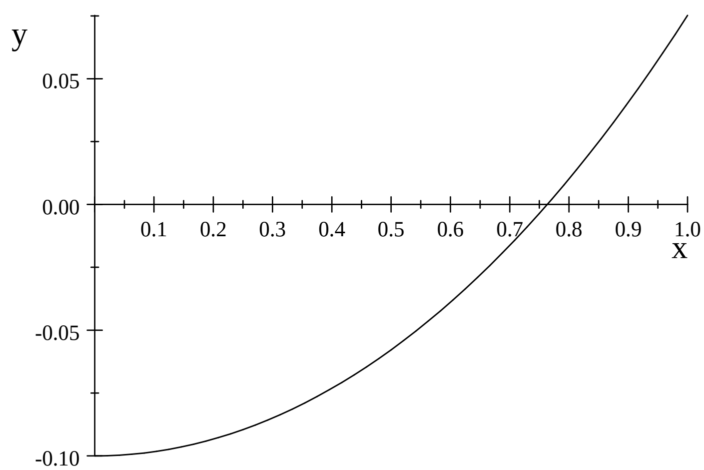
The plot of $f(\beta)$ reveals that the roots lie in $(1.8, 2.0)$ and $(4.6, 4.8)$. The following program uses `newtonRaphson` to compute these roots:

```
% problem4_1_15
func = inline('cosh(x)*cos(x) + 1','x');
dfunc = inline('sinh(x)*cos(x) - cosh(x)*sin(x)','x');
b = 0.025; h = 0.0025; rho = 7850; E = 200e9; L = 0.9;
I = b*h^3/12; m = rho*b*h*L;
C = sqrt(E*I/(m*L^3))/(2*pi);
bracket = [1.8 2; 4.6 4.8];
for i = 1:2
    beta = newtonRaphson(func,dfunc,bracket(i,1),bracket(i,2));
```

```
    freq = C*beta^2
end

>> freq =
    2.5166
freq =
   15.7713
```

Problem 16

$$
\begin{aligned}
f(\beta) &= \frac{1}{\beta}\sinh\beta - \frac{s}{L} = \frac{1}{\beta}\sinh\beta - 1.1 \\
f'(\beta) &= \frac{1}{\beta}\cosh\beta - \frac{1}{\beta^2}\sinh\beta
\end{aligned}
$$



According to the plot, the smallest positive root lies in $(0.72, 0.80)$. We use `newtonRaphson` to compute this root.

```
% problem4_1_16
func = inline('sinh(x)/x - 1.1','x');
dfunc = inline('cosh(x)/x - sinh(x)/x^2','x');
beta = newtonRaphson(func,dfunc,0.72,0.8)
gamma = 77000; L = 1000; s = 1100;
sigma0 = gamma*L/(2*beta);
max_stress = sigma0*cosh(beta)

>> beta =
 0.7634
max_stress =
 6.5855e+007
```

Problem 17

We non-dimensionalize the secant formula by dividing both sides by E:

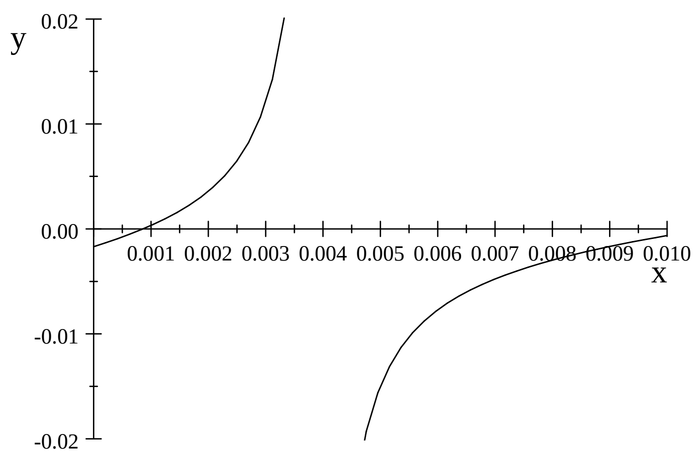
$$f\left(\frac{\bar{\sigma}}{E}\right) = \frac{\bar{\sigma}}{E}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{\bar{\sigma}}{E}}\right)\right] - \frac{\sigma_{\max}}{E}$$

Substituting

$$\begin{aligned} \frac{ec}{r^2} &= \frac{85(170)}{(142)^2} = 0.7166 \qquad \frac{L}{2r} = \frac{7100}{2(142)} = 25.0 \\ \frac{\sigma_{\max}}{E} &= \frac{120 \times 10^6}{71 \times 10^9} = 1.6901 \times 10^{-3} \end{aligned}$$

and using the notation $u = \bar{\sigma}/E$, the secant formula is

$$f(u) = u\left(1 + 0.7166\sec\left(25\sqrt{u}\right)\right) - 1.6901 \times 10^{-3}$$



The plot of $f(u)$ shows that the smallest root is in the interval $(0.0004, 0.0012)$. We used Ridder's method to compute this root:

```
% problem4_1_17
func = inline('u*(1 + 0.7166/cos(25*sqrt(u))) - 1.6901e-3','u');
u = ridder(func, 0.0004, 0.0012)

>> u =
  8.6032e-004
```

$$\begin{aligned} P &= A\bar{\sigma} = AEu = (25\,800 \times 10^{-6})(71 \times 10^9)(8.6032 \times 10^{-4}) \\ &= 1.576 \times 10^6 \text{ N} \blacktriangleleft \end{aligned}$$

Problem 18

Dividing both sides of the Bernoulli equation

$$\frac{Q^2}{2gb^2h_0^2} + h_0 = \frac{Q^2}{2gb^2h^2} + h + H$$

by h_0, we get

$$\frac{Q^2}{2gb^2h_0^3} + 1 = \frac{Q^2}{2gb^2h_0^3}\left(\frac{h_0}{h}\right)^2 + \frac{h}{h_0} + \frac{H}{h_0}$$

Introducing $u = h/h_0$ and rearranging, this becomes

$$f(u) = \frac{Q^2}{2gb^2h_0^3}\left(1 - \frac{1}{u^2}\right) - u + \left(1 - \frac{H}{h_0}\right) = 0$$

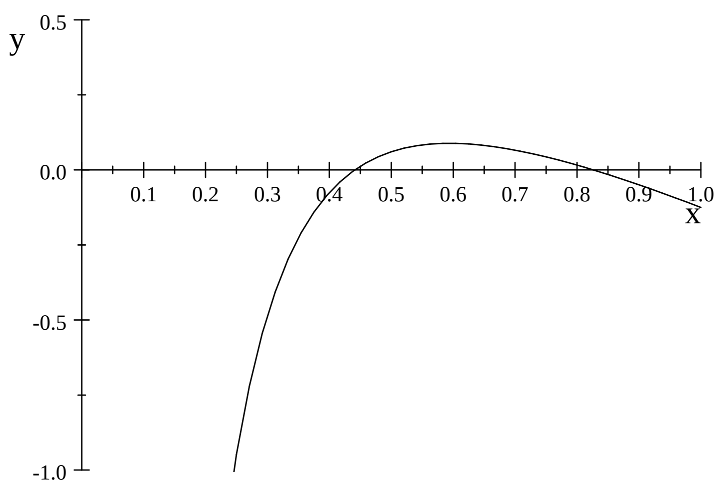
Substituting

$$\frac{Q^2}{2gb^2h_0^3} = \frac{(1.2)^2}{2(9.81)(1.8)^2(0.6)^3} = 0.10487$$

$$1 - \frac{H}{h_0} = 1 - \frac{0.075}{0.6} = 0.875$$

we obtain

$$f(u) = 0.10487\left(1 - \frac{1}{u^2}\right) - u + 0.875$$



The plot of $f(u)$ indicates that there are two roots. The smaller root, which is in $(0.4, 0.48)$, can be determined with the following program:

```
 problem4_1_18
func = inline('0.10487*(1 - 1/u^2) - u + 0.875','u');
u = ridder(func, 0.4, 0.48)

>> u =
    0.4412
```

$$h = uh_0 = 0.4412(0.6) = 0.2647 \text{ m} \blacktriangleleft$$

The larger root can be computed with the same program by changing the brackets to $(0.8, 0.84)$. It yields $u = 0.8263$, so that

$$h = 0.8263(0.6) = 0.4958 \text{ m} \blacktriangleleft$$

Evidently the fluid flow can exist in on of the two states under the given conditions.

Problem 19

$$v = u \ln \frac{M_0}{M_0 - \dot{m}t} - gt$$

We want the root of

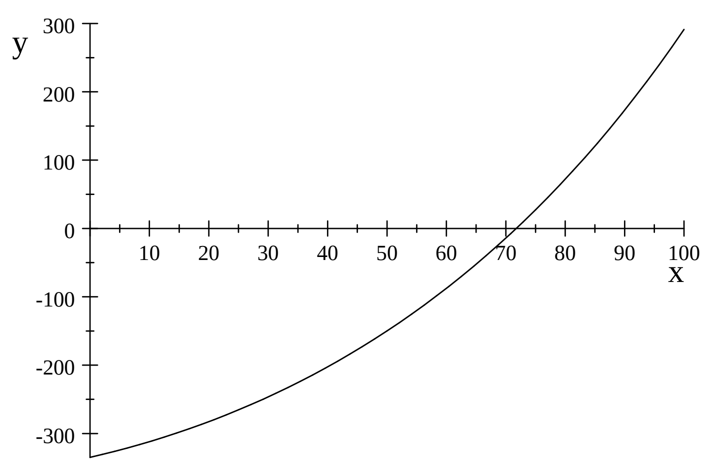
$$f(t) = u \ln \left[\frac{1}{1 - (\dot{m}/M_0)t} \right] - gt - v_{\text{sound}} = 0$$

where

$$\frac{\dot{m}}{M_0} = \frac{13.3 \times 10^3}{2.8 \times 10^6} = 0.004\,75 \text{ s}^{-1}$$

Thus

$$f(t) = 2510 \ln \left(\frac{1}{1 - 0.004\,75t} \right) - 9.81t - 335$$



The plot of $f(t)$ locates the root in $(68\ , 76)$ s. The following program was used for the computation of the root:

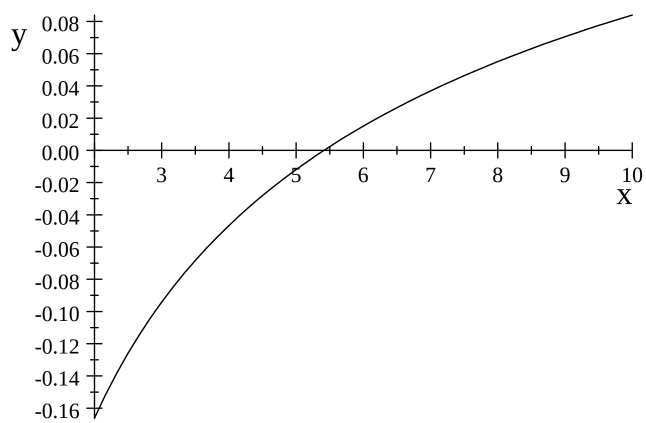
```
% problem4_1_19
func = inline('2510*log(1/(1 - 4.75e-3*t)) - 9.81*t - 335','t');
t = ridder(func, 68, 76)

>> t =
   70.8780
```

Problem 20

$$\eta = \frac{\ln(T_2/T_1) - (1 - T_1/T_2)}{\ln(T_2/T_1) + (1 - T_1/T_2)/(\gamma - 1)}$$

With $\eta = 0.3$, $\gamma - 1 = 2/3$ and the notation $u = T_2/T_1$, the equation becomes

$$f(u) = \frac{\ln u - (1 - 1/u)}{\ln u + 1.5(1 - 1/u)} - 0.3 = 0$$



From the plot of $f(u)$ we see that the root is in $(5.2, 5.6)$. We found this root with the following program:

```
% problem4_1_20
func = inline('(log(u)-(1-1/u))/(log(u)+1.5*(1-1/u))-0.3'...
             ,'u');
u = ridder(func,5.2,5.6)

>> u =
    5.4125
```

Problem 21

$$G = -RT \ln \left[(T/T_0)^{5/2} \right]$$

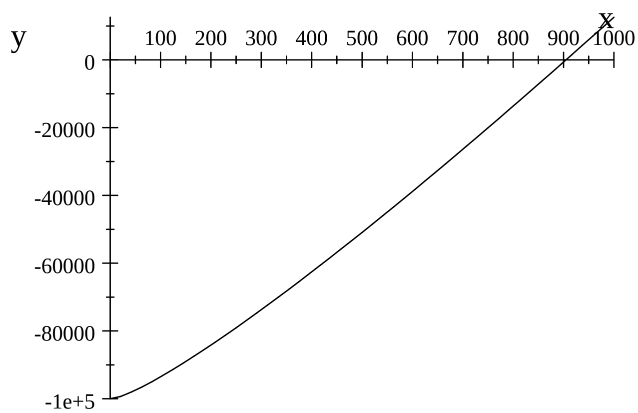
$$f(T) = G + \frac{5}{2} RT \ln(T/T_0)$$

Substituting

$$\frac{5}{2}R = \frac{5}{2}(8.314\,41) = 20.7860$$

we get

$$f(T) = -10^5 + 20.7860T \ln(T/4.444\,18)$$

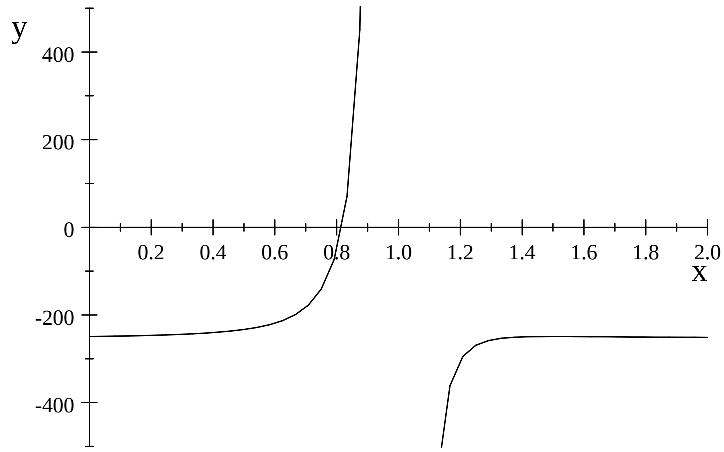


The plot of $f(T)$ shows a root in $(880, 920)$. Here is the program that computes this root:

```
% problem4_1_21
func = inline('-1.0e5 + 20.7860*T*log(T/4.44418)','T');
T = ridder(func,880,920)

>> T =
  904.9435
```

Problem 22

$$f(\xi) = \frac{\xi(3-2\xi)^2}{(1-\xi)^3} - 249.2$$

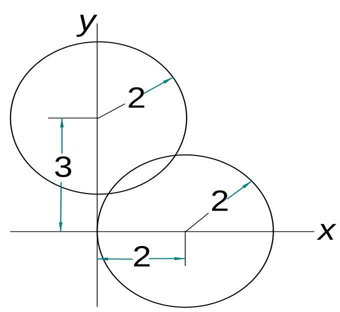


The plot of $f(\xi)$ shows a root in $(0.7, 0.9)$, which is computed with the program

```
% problem4_1_22
func = inline('x*(3 - 2*x)^2/(1 - x)^3 - 249.2','x');
xi = ridder(func,0.7,0.9)

>> xi =
    0.8171
```

Problem 23

$$\begin{aligned} f_1(x,y) &= (x-2)^2 + y^2 - 4 \\ f_2(x,y) &= x^2 + (y-3)^2 - 4 \end{aligned}$$



The rough locations of the intersection points are $(2,2)$ and $(0,1)$. Letting $x = x_1$ and $y = x_2$, the following function defined the equations:

```
function y = p4_1_23(x)
```

```
% Equations used in Problem 23, Problem Set 4.1
y = [(x(1) - 2)^2 + x(2)^2 - 4;
    x(1)^2 + (x(2) - 3)^2 - 4];
```

The following command returns the coordinates of the first point:

```
>> newtonRaphson2(@p4_1_23,[2;2])
ans =
    1.7206
    1.9804
```

Changing the starting point to $[0;1]$, we obtain the coordinates of the second point

```
>> newtonRaphson2(@p4_1_23,[0;1])
ans =
    0.2794
    1.0196
```

Problem 24

$$\begin{aligned} f_1(x,y) &= \sin x + 3\cos x - 2 \\ f_2(x,y) &= \cos x - \sin y + 0.2 \end{aligned}$$

The following function uses the notation $x = x_1$ and $y = x_2$:

```
function y = p4_1_24(x)
% Equations used in Problem 24, Problem Set 4.1
y = [sin(x(1)) + 3*cos(x(2)) - 2;
    cos(x(1)) - sin(x(2)) + 0.2];
```

The x and y-coordinates can now be obtained with the command

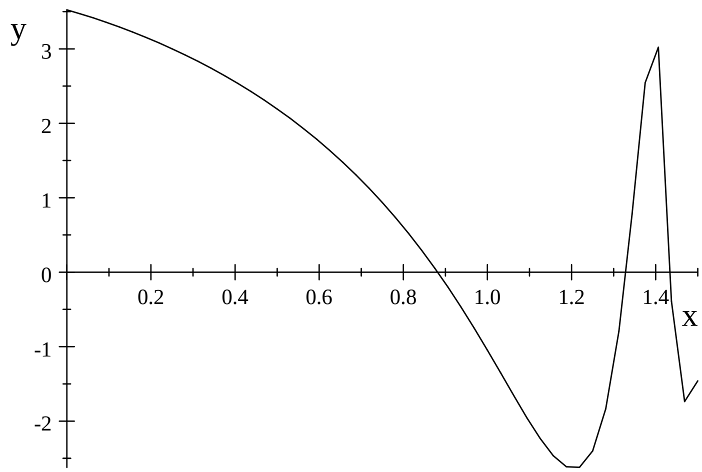
```
>> newtonRaphson2(@p4_1_24,[1;1])
ans =
    0.7912
    1.1267
```

Problem 25

$$\begin{aligned} \tan x - y &= 1 \\ \cos x - 3\sin y &= 0 \end{aligned}$$

It is not easy to search for the roots of simultaneous equations. Here we can overcome the difficulty by solving the first equation for y and substituting the result into the second equation. This gives us the single transcendental equation

$$f(x) = \cos x - 3\sin(\tan x - 1) = 0$$



From the plot of $f(x)$ we see that there are 5 roots in the interval $(0, 1.5)$. The first root is about $x = 0.88$; the other roots are closely spaced near the end of the interval (the spacing of roots becomes infinitesimal at $x = \pi/2$). The program listed below is based on `rootsRidder` in Problem 10. It searches for the roots from $x = 0.8$ to 1.5 in increments of 0.025 (the increment has to be small in order to catch all the roots).

```
% problem4_1_25
func = inline('cos(x) - 3*sin(tan(x) - 1)','x');
n = 5; a = 0.8; b = 1.5; dx = 0.025;
fprintf('Roots:\n')
while 1
    [x1,x2] = rootsearch(func,a,b,dx);
    if isnan(x1)
        fprintf('Done'); break
    else
        a = x2;
        x = ridder(func,x1,x2);
        if isnan(x); continue
        else
            y = tan(x) - 1;
            fprintf('%12.6f %12.6f\n', x,y);
        end
    end
end
```

```
>> Roots:
    0.881593     0.213595
    1.329402     3.061823
    1.435176     6.328269
    1.474872     9.392847
    1.497350    12.590833
Done
```

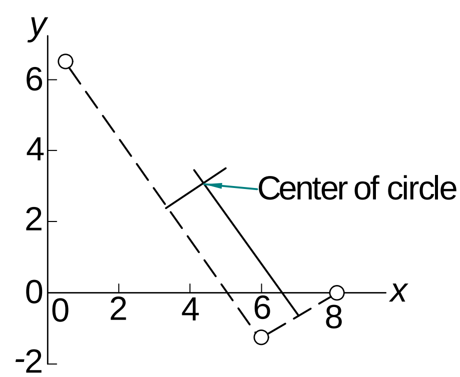
Problem 26

$$(x-a)^2 + (y-b)^2 - R^2 = 0$$

Substituting the coordinates of the given points into the above equation, we get

$$\begin{aligned}
(8.21-a)^2 + b^2 - R^2 &= 0 \\
(0.34-a)^2 + (6.62-b)^2 - R^2 &= 0 \\
(5.96-a)^2 + (-1.12-b)^2 - R^2 &= 0
\end{aligned}$$

By plotting the points, we can estimate the parameters of the circle. It appears that reasonable starting values are $a = 5$, $b = 3$ and $R = 5$.



The following function uses the notation $a = x_1$, $b = x_2$, $R = x_3$.

```
function y = p4_1_26(x)
% Equations used in Problem 26, Problem Set 4.1
y = [(8.21 - x(1))^2 + x(2)^2 - x(3)^2;
    (0.34 - x(1))^2 + (6.62 - x(2))^2 - x(3)^2;
    (5.96 - x(1))^2 + (-1.12 - x(2))^2 - x(3)^2];

>> newtonRaphson2(@p4_1_26,[5;3;5])
ans =
    4.8301
    3.9699
    5.2138
```

Problem 27

$$\frac{C}{1 + e\sin(\theta + \alpha)} - R = 0$$

After substituting the three sets of given data, we obtain the simulataneous equations

$$\begin{aligned}
\frac{C}{1 + e\sin(-\pi/6 + \alpha)} - 6870 &= 0 \\
\frac{C}{1 + e\sin(\alpha)} - 6728 &= 0 \\
\frac{C}{1 + e\sin(\pi/6 + \alpha)} - 6615 &= 0
\end{aligned}$$

The starting value $C = 6800$ seems reasonable, but e and α are not easy to guess. The orbit has some eccentricity, so that $e = 0.5$ should not be out of line ($e = 0$ will not work because it results in a singular Jacobian matrix). We also used $\alpha = 0$, which was a pure guess.

The minimum value of R is

$$R_{\min} = \frac{C}{1 + e}$$

occuring at

$$\sin(\theta + \alpha) = 1 \qquad \theta = \frac{\pi}{2} - \alpha$$

With the notation $C = x_1$, $e = x_2$ and $\alpha = x_3$, we arrive at the following program:

```
function y = p4_1_27(x)
% Equations used in Problem 27, Problem Set 4.1
y = [x(1)/(1 + x(2)*sin(-pi/6 + x(3))) - 6870;
     x(1)/(1 + x(2)*sin(x(3))) - 6728;
     x(1)/(1 + x(2)*sin(pi/6 + x(3))) - 6615];

>> format short e
>> newtonRaphson2(@p4_1_27,[6800;0.5;0])
ans =
  6.8193e+003
  4.0599e-002
  3.4078e-001
```

Problem 28

$$
\begin{aligned}
x &= (v\cos\theta)t \\
y &= -\frac{1}{2}gt^2 + (v\sin\theta)t
\end{aligned}
$$

We also need the expression for dy/dx:

$$
\begin{aligned}
\frac{dx}{dt} &= v\cos\theta \qquad \frac{dy}{dt} = -gt + v\sin\theta \\
\frac{dy}{dx} &= \frac{-gt + v\sin\theta}{v\cos\theta}
\end{aligned}
$$

Letting τ denote the time of flight, the specified end conditions are

$$
x(\tau) = 300 \text{ m} \qquad y(\tau) = 61 \text{ m} \qquad \left.\frac{dy}{dx}\right|_\tau = -1
$$

which yield the equations

$$
\begin{aligned}
(v\cos\theta)\,\tau - 300 &= 0 \\
-\frac{1}{2}g\tau^2 + (v\sin\theta)\,\tau - 61 &= 0 \\
\frac{-g\tau + v\sin\theta}{v\cos\theta} + 1 &= 0
\end{aligned}
$$

To estimate the initial values of the unknowns, we guess $\tau = 10$ s and $\theta = 45^\circ$. Then the first of the above equations yields $v = 300/\left(\tau\cos\theta\right) \approx 57$ m/s. The following program uses the notation $\tau = x_1$, $v = x_2$ and $\theta = x_3$:

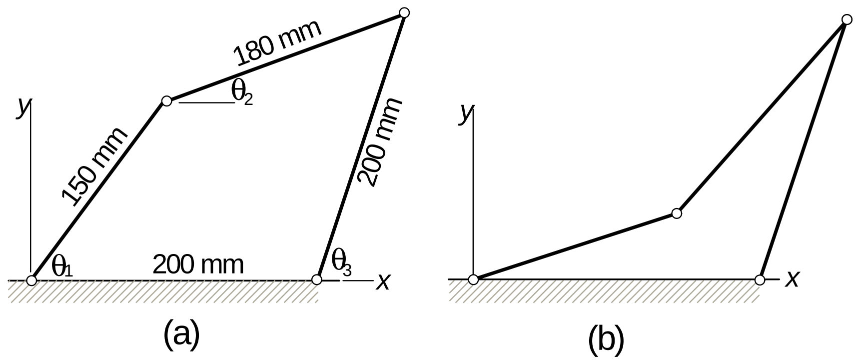
```
function y = p4_1_28(x)
% Equations used in Problem 27, Problem Set 4.1
y = [x(1)*x(2)*cos(x(3)) - 300;
     x(1)*x(2)*sin(x(3)) - 9.81*x(1)^2/2 - 61;
     (-9.81*x(1) + x(2)*sin(x(3)))/(x(2)*cos(x(3))) + 1];
```

```
>> format short e
>> newtonRaphson2(@p4_1_28,[10;57;pi/4])
ans =
  8.5789e+000
  6.0353e+001
  9.5279e-001
```

Thus $\tau = 8.579$ s, $v = 60.35$ m/s and $\theta = 0.9528$ rad $= 54.6$ deg. ◄

Problem 29

$$\begin{aligned} 150\cos\theta_1 + 180\cos\theta_2 - 200\cos\theta_3 &= 200 \\ 150\sin\theta_1 + 180\sin\theta_2 - 200\sin\theta_3 &= 0 \end{aligned}$$



Here is the function that refines defines the equations, given that $\theta_3 = 75^\circ = 5\pi/12$ rad:

```
function y = p4_1_29(x)
% Equations used in Problem 29, Problem Set 4.1
y = [150*cos(x(1)) + 180*cos(x(2)) - 200*cos(5*pi/12) - 200;
     150*sin(x(1)) + 180*sin(x(2)) - 200*sin(5*pi/12)];
```

We estimate from the figure that $\theta_1 = 60^\circ$ and $\theta_2 = 20^\circ$ in configuration (a). Using these as starting values, the solution is obtained with the command

```
>> newtonRaphson2(@p4_1_29,[pi/3;pi/9])
ans =
  9.5960e-001
  4.0148e-001
```

Therefore, $\theta_1 = 0.9596$ rad $= 55.0^\circ$, $\theta_2 = 0.4015$ rad $= 23.0^\circ$ ◀
For configuration (b) we estimate $\theta_1 = 20^\circ$ and $\theta_2 = 60^\circ$. With these starting values we have

```
>> newtonRaphson2(@p4_1_29,[pi/9;pi/3])

ans =
  3.4939e-001
  9.0752e-001
```

Here $\theta_1 = 0.3494$ rad $= 20.0^\circ$, $\theta_2 = 0.9075$ rad $= 52.0^\circ$ ◀

Problem 30

Letting $x = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & T \end{bmatrix}^T$, the equations to be solved are

```
function y = p4_1_30(x)
% Equations used in Prob. 30, Problem Set 4.1
y = [x(4)*(-tan(x(2)) + tan(x(1))) - 16;
     x(4)*( tan(x(3)) + tan(x(2))) - 20;
    -4*sin(x(1)) - 6*sin(x(2)) + 5*sin(x(3)) + 3;
     4*cos(x(1)) + 6*cos(x(2)) + 5*cos(x(3)) - 12];
```

Rough estimates (starting values) of the variables are

$$\theta_1 = 0.8 \text{ rad} \qquad \theta_2 = 0.3 \text{ rad} \qquad \theta_3 = 0.4 \text{ rad} \qquad T = 20 \text{ kN}$$

The solution is obtained with the command

```
>> newtonRaphson2(@p4_1_30,[0.8;0.3;0.4;20])

ans =
    0.9358
    0.4334
    0.5800
   17.8884
```

Therefore, the solution is

$$\begin{aligned} \theta_1 &= 0.9358 \text{ rad} = 53.62^\circ \blacktriangleleft \\ \theta_2 &= 0.4334 \text{ rad} = 24.83^\circ \blacktriangleleft \\ \theta_3 &= 0.5800 \text{ rad} = 33.23^\circ \blacktriangleleft \\ T &= 17.89 \text{ kN} \end{aligned}$$

PROBLEM SET 4.2

Problem 1

$$P_3(x) = 3x^3 + 7x^2 - 36x + 20 \qquad r = -5$$

$$\begin{aligned}
b_1 &= a_1 = 3 \\
b_2 &= a_2 + rb_1 = 7 + (-5)(3) = -8 \\
b_3 &= a_3 + rb_2 = -36 + (-5)(-8) = 4
\end{aligned}$$

$$\therefore P_2 = 3x^2 - 8x + 4 \quad \blacktriangleleft$$

Problem 2

$$P_4(x) = x^4 - 3x^2 + 3x - 1 \qquad r = 1$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = 0 + 1(1) = 1 \\
b_3 &= a_3 + rb_2 = -3 + 1(1) = -2 \\
b_4 &= a_4 + rb_3 = 3 + 1(-2) = 1
\end{aligned}$$

$$\therefore P_3 = x^3 + x^2 - 2x + 1 \quad \blacktriangleleft$$

Problem 3

$$P_5(x) = x^5 - 30x^4 + 361x^3 - 2178x^2 + 6588x - 7992 \qquad r = 6$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = -30 + 6(1) = -24 \\
b_3 &= a_3 + rb_2 = 361 + 6(-24) = 217 \\
b_4 &= a_4 + rb_3 = -2178 + 6(217) = -876 \\
b_5 &= a_5 + rb_4 = 6588 + 6(-876) = 1332
\end{aligned}$$

$$P_4(x) = x^4 - 24x^3 + 217x^2 - 876x + 1332$$

Problem 4

$$P_4(x) = x^4 - 5x^3 - 2x^2 - 20x - 24 \qquad r = 2i$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = -5 + (2i)(1) = -5 + 2i \\
b_3 &= a_3 + rb_2 = -2 + (2i)(-5 + 2i) = -6 - 10i \\
b_4 &= a_4 + rb_3 = -20 + (2i)(-6 - 10i) = -12i
\end{aligned}$$

$$P_3(x) = x^3 - (5 - 2i)x^2 - (6 + 10i)x - 12i \quad \blacktriangleleft$$

Problem 5

$$P_3(x) = 3x^3 - 19x^2 + 45x - 13 \qquad r = 3 - 2i$$

$$\begin{aligned}
b_1 &= a_1 = 3 \\
b_2 &= a_2 + rb_1 = -19 + (3 - 2i)(3) = -10 - 6i \\
b_3 &= a_3 + rb_2 = 45 + (3 - 2i)(-10 - 6i) = 3 + 2i
\end{aligned}$$

$$P_2(x) = 3x^2 - (10 + 6i)\,x + (3 + 2i) \quad \blacktriangleleft$$

Problem 6

$$P_3(x) = x^3 + 1.8x^2 - 9.01x - 13.398 \qquad r = -3.3$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = 1.8 + (-3.3)(1) = -1.5 \\
b_3 &= a_3 + rb_2 = -9.01 + (-3.3)(-1.5) = -4.06
\end{aligned}$$

$$P_2(x) = x^2 - 1.5x - 4.06$$

The roots are

$$r = \frac{1.5 \pm \sqrt{1.5^2 + 4(4.06)}}{2} = \frac{1.5 \pm 4.3}{2} = \begin{cases} 2.9 \\ -1.4 \end{cases} \quad \blacktriangleleft$$

Problem 7

$$P_3(x) = x^3 - 6.64x^2 + 16.84x - 8.32 \qquad r = 0.64$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = -6.64 + 0.64(1) = -6 \\
b_3 &= a_3 + rb_2 = 16.84 + 0.64(-6.0) = 13
\end{aligned}$$

$$P_2(x) = x^2 - 6x + 13$$

The roots are

$$r = \frac{6 \pm \sqrt{6^2 - 4(13)}}{2} = \frac{6 \pm 4i}{2} = \begin{cases} 3 + 2i \\ 3 - 2i \end{cases} \quad \blacktriangleleft$$

Problem 8

$$P_3(x) = 2x^3 - 13x^2 + 32x - 13 \qquad r = 3 - 2i$$

$$\begin{aligned}
b_1 &= a_1 = 2 \\
b_2 &= a_2 + rb_1 = -13 + (3 - 2i)\,(2) = -7 - 4i \\
b_3 &= a_3 + rb_2 = 32 + (3 - 2i)\,(-7 - 4i) = 3 + 2i
\end{aligned}$$

$$P_2(x) = 2x^2 - (7 + 4i)x + (3 + 2i)$$

Since complex roots come in conjugate pairs, we know that a zero of $P_2(x)$ is

$$r = 3 + 2i \quad \blacktriangleleft$$

$$\begin{aligned}
b_1 &= a_1 = 2 \\
b_2 &= a_2 + rb_1 = -(7 + 4i) + (3 + 2i)(2) = -1
\end{aligned}$$

$$P_1(x) = -1 + 2x$$

The zero of $P_1(x)$ is

$$r = 0.5 \quad \blacktriangleleft$$

Problem 9

$$P_4(x) = x^4 - 3x^3 + 10x^2 - 6x - 20 \qquad r = 1 + 3i$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = -3 + (1+3i)(1) = -2 + 3i \\
b_3 &= a_3 + rb_2 = 10 + (1+3i)(-2+3i) = -1 - 3i \\
b_4 &= a_4 + rb_3 = -6 + (1+3i)(-1-3i) = 2 - 6i
\end{aligned}$$

$$P_3(x) = x^3 + (-2+3i)x^2 + (-1-3i)x + (2-6i)$$

Another zero of $P_4(x)$ is the conjugate of $1 + 3i$, namely

$$r = 1 - 3i$$

$$\begin{aligned}
b_1 &= a_1 = 1 \\
b_2 &= a_2 + rb_1 = (-2+3i) + (1-3i)(1) = -1 \\
b_3 &= a_3 + rb_2 = (-1-3i) + (1-3i)(-1) = -2
\end{aligned}$$

$$P_2(x) = x^2 - x + -2$$

The roots of the quadratic are

$$r = \frac{1 \pm \sqrt{1^2 + 4(2)}}{2} = \begin{cases} \;\;2 \\ -1 \end{cases}$$

Thus the roots of $P_4(x)$ are $1 \pm 3i$, 2 and -1. ◄

Problem 10

```
>> polyroots([1 2.1 -2.52 2.1 -3.52])
ans =
  -0.0000 - 1.0000i
   1.1000
   0.0000 + 1.0000i
  -3.2000
```

Problem 11

```
>> polyroots([1 -156 -5 780 4 -624])
ans =
   1.0000
  -1.0000
  -2.0000
   2.0000
 156.0000
```

Problem 12

```
>> polyroots([1 4 -8 -34 57 130 -150])
ans =
   2.0000 - 1.0000i
   1.0000
  -3.0000
   2.0000 + 1.0000i
  -3.0000 - 1.0000i
  -3.0000 + 1.0000i
```

Problem 13

```
>> polyroots([8 28 34 -13 -124 19 220 -100])
ans =
  -2.0000
   0.5000
  -2.0000
   1.0000 - 0.5000i
   1.0000 + 0.5000i
  -1.0000 - 2.0000i
  -1.0000 + 2.0000i
```

Problem 14

```
>> polyroots([1 -7 7 25 24 -98 -472 440 800])
ans =
   2.0000
  -1.0000
  -2.0000
   3.0000 - 1.0000i
  -1.0000 + 2.0000i
   3.0000 + 1.0000i
  -1.0000 - 2.0000i
   4.0000
```

Problem 15

```
>> polyroots([1 5+1i -8+5i 30-14i -84])
ans =
   2.0000
  -0.0000 + 2.0000i
   0.0000 - 3.0000i
  -7.0000
```

Note that the complex roots do not appear in conjugate pairs if the coefficients of the polynomial are not real.

Problem 16

$$\omega^4 + 2\frac{c}{m}\omega^3 + 3\frac{k}{m}\omega^2 + \frac{c}{m}\frac{k}{m}\omega + \left(\frac{k}{m}\right)^2 = 0$$

With $c/m = 12\ \mathrm{s}^{-1}$ and $k/m = 1500\ \mathrm{s}^{-2}$, we get

$$\omega^4 + 24\omega^3 + 4500\omega^2 + 18 \times 10^3\omega + 2.25 \times 10^6 = 0$$

```
> polyroots([1 24 4500 18e3 2.25e6])
ans =
  -0.6230 -24.0302i
  -0.6230 +24.0302i
 -11.3770 +61.3545i
 -11.3770 -61.3545i
```

The two combinations of (ω_r, ω_i) are

$$\left(-0.0623\ \mathrm{s}^{-1}, 24.03\ \mathrm{s}^{-1}\right)\ \text{and}\ \left(-11.38\ \mathrm{s}^{-1}, 61.35\ \mathrm{s}^{-1}\right) \blacktriangleleft$$

Problem 17

The slope of the beam is

$$\begin{aligned} y' &= \frac{w_0}{120EI}(5x^4 - 9L^2x^2 + 6L^3x) \\ &= \frac{w_0L^4}{120EI}(5\xi^4 - 9\xi^2 + 6\xi) \end{aligned}$$

where $\xi = x/L$. Since $y' = 0$ at the point of maximum displacement, the value of ξ that we are looking for is a root of

$$P_4(\xi) = 5\xi^4 - 9\xi^2 + 6\xi$$

We could find the our roots of this equation with the function `polyroots`, but this is unnecessary. Because the slope of the beam is zero at supports, we know that two of the roots are $\xi = 0$ and $\xi = 1$. Factoring out these roots, we have

$$P_4(\xi) = \xi(\xi - 1)(b_1\xi^2 + b_2\xi + b_3)$$

The b's are obtained by Horner's algorithm:

$$\begin{aligned} b_1 &= a_1 = 5 \\ b_2 &= a_2 + rb_1 = 0 + 1(5) = 5 \\ b_3 &= a_3 + rb_2 = -9 + 1(5) = -4 \end{aligned}$$

We have now reduced the problem to finding the roots of the quadratic equation

$$5\xi^2 + 5\xi - 4 = 0$$

The positive root is

$$\xi = \frac{-5 + \sqrt{5^2 - 4(5)(-4)}}{10} = 0.5247 \blacktriangleleft$$