# Chapter 7

**7-1** **(a)** DE-Gerber, Eq. (7-10):

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} = \sqrt{4[(2.2)(70)]^2 + 3[(1.8)(45)]^2} = 338.4 \text{ N} \cdot \text{m}$$

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2} = \sqrt{4[(2.2)(55)]^2 + 3[(1.8)(35)]^2} = 265.5 \text{ N} \cdot \text{m}$$

$$d = \left\{ \frac{8(2)(338.4)}{\pi(210)\left(10^6\right)} \left[ 1 + \left( 1 + \left[ \frac{2(265.5)(210)\left(10^6\right)}{338.4(700)\left(10^6\right)} \right]^2 \right)^{1/2} \right] \right\}^{1/3}$$

$d = 25.85\ (10^{-3})$ m = 25.85 mm *Ans.*

**(b)** DE-elliptic, Eq. (7-12) can be shown to be

$$d = \left( \frac{16n}{\pi} \sqrt{\frac{A^2}{S_e^2} + \frac{B^2}{S_y^2}} \right)^{1/3} = \left( \frac{16(2)}{\pi} \sqrt{\frac{(338.4)^2}{\left[(210)\left(10^6\right)\right]^2} + \frac{(265.5)^2}{\left[(560)\left(10^6\right)\right]^2}} \right)^{1/3}$$

$d = 25.77\ (10^{-3})$ m = 25.77 mm *Ans.*

**(c)** DE-Soderberg, Eq. (7-14) can be shown to be

$$d = \left[ \frac{16n}{\pi} \left( \frac{A}{S_e} + \frac{B}{S_y} \right) \right]^{1/3} = \left[ \frac{16(2)}{\pi} \left( \frac{338.4}{210\left(10^6\right)} + \frac{265.5}{560\left(10^6\right)} \right) \right]^{1/3}$$

$d = 27.70\ (10^{-3})$ m = 27.70 mm *Ans.*

**(d)** DE-Goodman: Eq. (7-8) can be shown to be

$$d = \left[ \frac{16n}{\pi} \left( \frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right]^{1/3} = \left[ \frac{16(2)}{\pi} \left( \frac{338.4}{210\left(10^6\right)} + \frac{265.5}{700\left(10^6\right)} \right) \right]^{1/3}$$

$d = 27.27\ (10^{-3})$ m = 27.27 mm *Ans.*

| Criterion | $d$ (mm) | Compared to DE-Gerber | |
|---|---|---|---|
| DE-Gerber | 25.85 | | |
| DE-Elliptic | 25.77 | 0.31% Lower | Less conservative |
| DE-Soderberg | 27.70 | 7.2% Higher | More conservative |
| DE-Goodman | 27.27 | 5.5% Higher | More conservative |

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**7-2** This problem has to be done by successive trials, since $S_e$ is a function of shaft size. The material is SAE 2340 for which $S_{ut}$ = 175 kpsi, $S_y$ = 160 kpsi, and $H_B \geq 370$.

Eq. (6-19), p. 295: $k_a = 2.70(175)^{-0.265} = 0.69$

*Trial* #1: Choose $d_r$ = 0.75 in

Eq. (6-20), p. 296: $k_b = 0.879(0.75)^{-0.107} = 0.91$

Eq. (6-8), p.290: $S_e' = 0.5S_{ut} = 0.5(175) = 87.5$ kpsi

Eq. (6-18), p. 295: $S_e$ = 0.69 (0.91)(87.5) = 54.9 kpsi

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$
$$D = \frac{d_r}{0.65} = \frac{0.75}{0.65} = 1.15 \text{ in}$$
$$r = \frac{D}{20} = \frac{1.15}{20} = 0.058 \text{ in}$$

Fig. A-15-14:

$$d = d_r + 2r = 0.75 + 2(0.058) = 0.808 \text{ in}$$
$$\frac{d}{d_r} = \frac{0.808}{0.75} = 1.08$$
$$\frac{r}{d_r} = \frac{0.058}{0.75} = 0.077$$
$$K_t = 1.9$$

Fig. 6-20, p. 296: $r$ = 0.058 in, $q$ = 0.90

Eq. (6-32), p. 303: $K_f$ = 1 + 0.90 (1.9 – 1) = 1.81

Fig. A-15-15: $K_{ts}$ = 1.5

Fig. 6-21, p. 304: $r$ = 0.058 in, $q_s$ = 0.92

Eq. (6-32), p. 303: $K_{fs}$ = 1 + 0.92 (1.5 – 1) = 1.46

We select the DE-ASME Elliptic failure criteria, Eq. (7-12), with $d$ as $d_r$, and $M_m = T_a = 0$,

$$d_r = \left\{ \frac{16(2.5)}{\pi} \left[ 4\left(\frac{1.81(600)}{54.9(10^3)}\right)^2 + 3\left(\frac{1.46(400)}{160(10^3)}\right)^2 \right]^{1/2} \right\}^{1/3}$$
$$d_r = 0.799 \text{ in}$$

*Trial* #2: Choose $d_r$ = 0.799 in.

$$k_b = 0.879(0.799)^{-0.107} = 0.90$$
$$S_e = 0.69\,(0.90)(0.5)(175) = 54.3 \text{ kpsi}$$
$$D = \frac{d_r}{0.65} = \frac{0.799}{0.65} = 1.23 \text{ in}$$
$$r = D/20 = 1.23/20 = 0.062 \text{ in}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 0.799 + 2(0.062) = 0.923 \text{ in}$$
$$\frac{d}{d_r} = \frac{0.923}{0.799} = 1.16$$
$$\frac{r}{d_r} = \frac{0.062}{0.799} = 0.078$$

With these ratios only slightly different from the previous iteration, we are at the limit of readability of the figures. We will keep the same values as before.

$$K_t = 1.9, \quad K_{ts} = 1.5, \quad q = 0.90, \quad q_s = 0.92$$
$$\therefore K_f = 1.81, \quad K_{fs} = 1.46$$

Using Eq. (7-12) produces $d_r$ = 0.802 in. Further iteration produces no change. With $d_r$ = 0.802 in,

$$D = \frac{0.802}{0.65} = 1.23 \text{ in}$$
$$d = 0.75(1.23) = 0.92 \text{ in}$$

A look at a bearing catalog finds that the next available bore diameter is 0.9375 in. In nominal sizes, we select $d$ = 0.94 in, $D$ = 1.25 in, $r$ = 0.0625 in *Ans.*

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**7-3** $F \cos 20°(d / 2) = T_A$, $F = 2\, T_A / (d \cos 20°) = 2(340) / (0.150 \cos 20°) = 4824$ N.
The maximum bending moment will be at point $C$, with $M_C$ = 4824(0.100) = 482.4 N·m.
Due to the rotation, the bending is completely reversed, while the torsion is constant.
Thus, $M_a$ = 482.4 N·m, $T_m$ = 340 N·m, $M_m = T_a = 0$.

For sharp fillet radii at the shoulders, from Table 7-1, $K_t$ = 2.7, and $K_{ts}$ = 2.2. Examining Figs. 6-20 and 6-21 (pp. 303 and 304 respectively) with $S_{ut} = 560$ MPa, conservatively estimate $q$ = 0.8 and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32):
$$K_f = 1 + 0.8(2.7 - 1) = 2.4$$
$$K_{fs} = 1 + 0.9(2.2 - 1) = 2.1$$

**(a)** We will choose to include fatigue stress concentration factors even for the static analysis to avoid localized yielding.

Eq. (7-15):
$$\sigma'_{\max} = \left[ \left( \frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

Eq. (7-16): $n = \dfrac{S_y}{\sigma'_{\max}} = \dfrac{\pi d^3 S_y}{16}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{-1/2}$

Solving for *d*,

$$d = \left\{\frac{16n}{\pi S_y}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2}\right\}^{1/3}$$

$$= \left(\frac{16(2.5)}{\pi\left(420\right)\left(10^6\right)}\left\{4\left[(2.4)(482.4)\right]^2 + 3\left[(2.1)(340)\right]^2\right\}^{1/2}\right)^{1/3}$$

$d$ = 0.0430 m = 43.0 mm *Ans.*

**(b)** $k_a = 4.51(560)^{-0.265} = 0.84$

Assume $k_b$ = 0.85 for now. Check later once a diameter is known.

$S_e$ = 0.84(0.85)(0.5)(560) = 200 MPa

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(482.4)}{200\left(10^6\right)}\right)^2 + 3\left(\frac{2.1(340)}{420\left(10^6\right)}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= 0.0534 \text{ m} = 53.4 \text{ mm}$$

With this diameter, we can refine our estimates for $k_b$ and $q$.

Eq. (6-20): $k_b = 1.51 d^{-0.157} = 1.51\left(53.4\right)^{-0.157} = 0.81$

Assuming a sharp fillet radius, from Table 7-1, $r = 0.02d$ = 0.02 (53.4) = 1.07 mm.

Fig. (6-20): $q$ = 0.72
Fig. (6-21): $q_s$ = 0.77

Iterating with these new estimates,

Eq. (6-32): $K_f$ = 1 + 0.72 (2.7 – 1) = 2.2
$K_{fs}$ = 1 + 0.77 (2.2 – 1) = 1.9
Eq. (6-18): $S_e$ = 0.84(0.81)(0.5)(560) = 191 MPa
Eq. (7-12): $d$ = 53 mm *Ans.*

Further iteration does not change the results.

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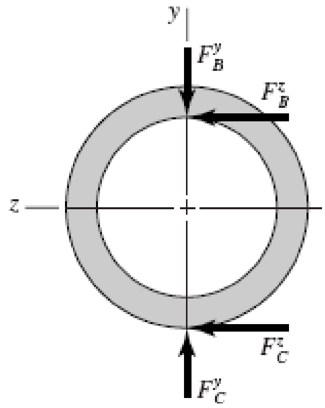
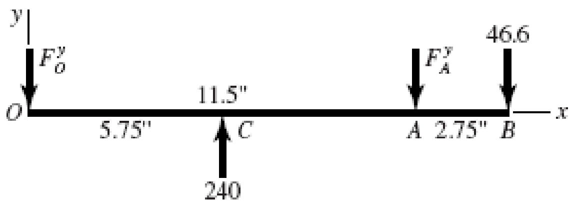
**7-4** We have a design task of identifying bending moment and torsion diagrams which are preliminary to an industrial roller shaft design. Let point $C$ represent the center of the span of the roller.




$$F_C^y = 30(8) = 240 \text{ lbf}$$
$$F_C^z = 0.4(240) = 96 \text{ lbf}$$
$$T = F_C^z(2) = 96(2) = 192 \text{ lbf} \cdot \text{in}$$
$$F_B^z = \frac{T}{1.5} = \frac{192}{1.5} = 128 \text{ lbf}$$

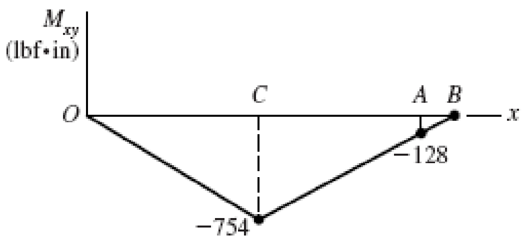
$$F_B^y = F_B^z \tan 20^\circ = 128 \tan 20^\circ = 46.6 \text{ lbf}$$

**(a)** *xy-plane*

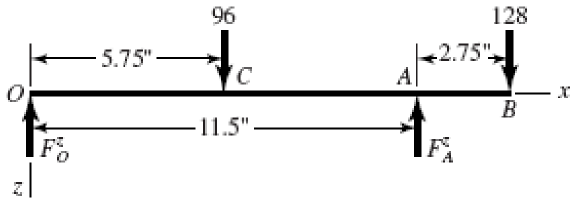



$$\Sigma M_O = 240(5.75) - F_A^y(11.5) - 46.6(14.25) = 0$$
$$F_A^y = \frac{240(5.75) - 46.6(14.25)}{11.5} = 62.3 \text{ lbf}$$
$$\Sigma M_A = F_O^y(11.5) - 46.6(2.75) - 240(5.75) = 0$$
$$F_O^y = \frac{240(5.75) + 46.6(2.75)}{11.5} = 131.1 \text{ lbf}$$

Bending moment diagram:



*xz-plane*



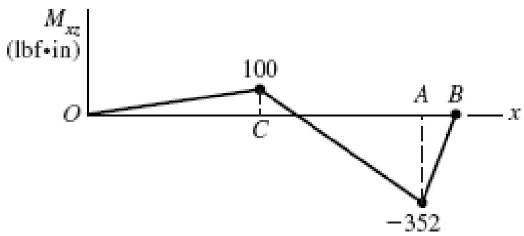

$$\Sigma M_O = 0 = 96(5.75) - F_A^z(11.5) + 128(14.25)$$

$$F_A^z = \frac{96(5.75) + 128(14.25)}{11.5} = 206.6 \text{ lbf}$$

$$\Sigma M_A = 0 = F_O^z(11.5) + 128(2.75) - 96(5.75)$$

$$F_O^z = \frac{96(5.75) - 128(2.75)}{11.5} = 17.4 \text{ lbf}$$

Bending moment diagram:

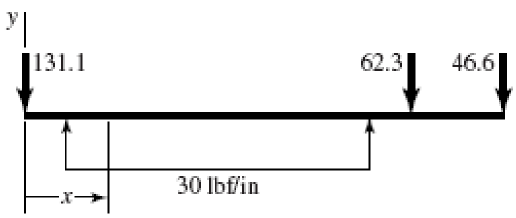



$$M_C = \sqrt{100^2 + (-754)^2} = 761 \text{ lbf} \cdot \text{in}$$

$$M_A = \sqrt{(-128)^2 + (-352)^2} = 375 \text{ lbf} \cdot \text{in}$$

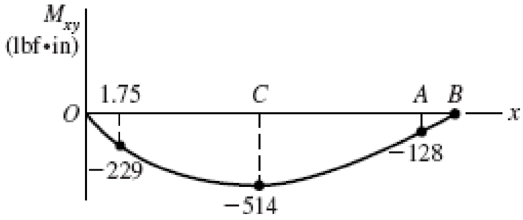
Torque: The torque is constant from *C* to *B*, with a magnitude previously obtained of 192 lbf·in.

**(b)** *xy-plane*




$$M_{xy} = -131.1x + 15\langle x - 1.75\rangle^2 - 15\langle x - 9.75\rangle^2 - 62.3\langle x - 11.5\rangle^1$$

Bending moment diagram:

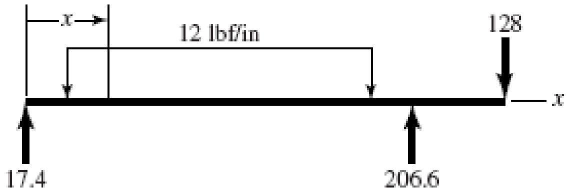



$M_{max}$ = –516 lbf · in and occurs at 6.12 in.

$$M_C = 131.1(5.75) - 15(5.75 - 1.75)^2 = 514 \text{ lbf} \cdot \text{in}$$

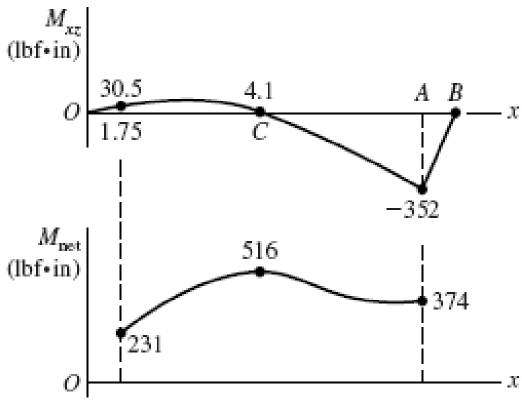
This is reduced from 754 lbf · in found in part (a). The maximum occurs at $x = 6.12$ in rather than *C*, but it is close enough.

*xz-plane*




$$M_{xz} = 17.4x - 6\langle x - 1.75\rangle^2 + 6\langle x - 9.75\rangle^2 + 206.6\langle x - 11.5\rangle^1$$

Bending moment diagram:




Let $M_{\text{net}} = \sqrt{M_{xy}^2 + M_{xz}^2}$

Plot $M_{\text{net}}(x)$, $1.75 \le x \le 11.5$ in

$M_{max}$ = 516 lbf · in at $x$ = 6.25 in

Torque: The torque rises from 0 to 192 lbf·in linearly across the roller, then is constant to *B*. *Ans.*

**7-5** This is a design problem, which can have many acceptable designs. See the solution for Prob. 7-17 for an example of the design process.

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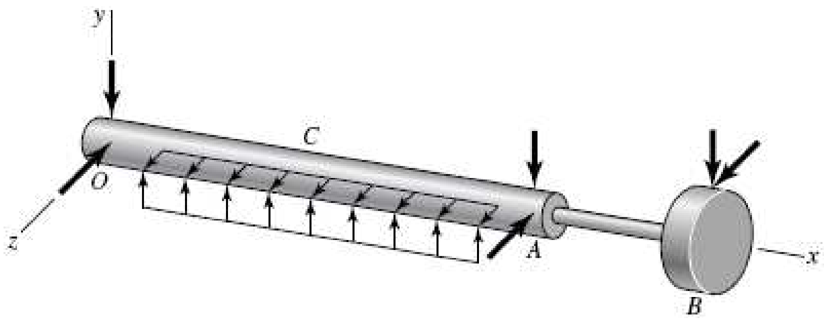
**7-6** If students have access to finite element or beam analysis software, have them model the shaft to check deflections. If not, solve a simpler version of shaft for deflection. The 1 in diameter sections will not affect the deflection results much, so model the 1 in diameter as 1.25 in. Also, ignore the step in *AB*.




From Prob. 7-4, integrate $M_{xy}$ and $M_{xz}$.

*xy plane,* with $dy/dx = y'$

$$EIy' = -\frac{131.1}{2}\left(x^2\right) + 5\langle x-1.75\rangle^3 - 5\langle x-9.75\rangle^3 - \frac{62.3}{2}\langle x-11.5\rangle^2 + C_1 \qquad (1)$$

$$EIy = -\frac{131.1}{6}\left(x^3\right) + \frac{5}{4}\langle x-1.75\rangle^4 - \frac{5}{4}\langle x-9.75\rangle^4 - \frac{62.3}{6}\langle x-11.5\rangle^3 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_1 = 1908.4 \text{ lbf}\cdot\text{in}^3$$

From (1), $x = 0$: $EIy' = 1908.4$

$x = 11.5$: $EIy' = -2153.1$

*xz plane* (treating $z\uparrow +$)

$$EIz' = \frac{17.4}{2}\left(x^2\right) - 2\langle x-1.75\rangle^3 + 2\langle x-9.75\rangle^3 + \frac{206.6}{2}\langle x-11.5\rangle^2 + C_3 \qquad (2)$$

$$EIz = \frac{17.4}{6}\left(x^3\right) - \frac{1}{2}\langle x-1.75\rangle^4 + \frac{1}{2}\langle x-9.75\rangle^4 + \frac{206.6}{6}\langle x-11.5\rangle^3 + C_3 x + C_4$$

$$z = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_4 = 0$$

$$z = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_3 = 8.975 \text{ lbf}\cdot\text{in}^3$$

From (2), $x = 0$: $EIz' = 8.975$

$x = 11.5$: $EIz' = -683.5$

At $O$: $EI\theta = \sqrt{1908.4^2 + 8.975^2} = 1908.4 \text{ lbf} \cdot \text{in}^3$

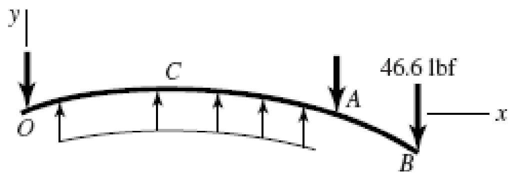
At $A$: $EI\theta = \sqrt{(-2153.1)^2 + (-683.5)^2} = 2259.0 \text{ lbf} \cdot \text{in}^3$ (dictates size)

$$\theta = \frac{2259}{30\left(10^6\right)\left(\pi/64\right)\left(1.25^4\right)} = 0.000\,628 \text{ rad}$$

$$n = \frac{0.001}{0.000\,628} = 1.59$$

At gear mesh, $B$

*xy* plane




With $I = I_1$ in section $OCA$,

$$y'_A = -2153.1 / EI_1$$

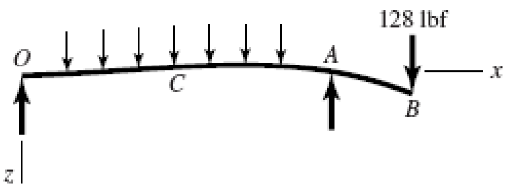
Since $y'_{B/A}$ is a cantilever, from Table A-9-1, with $I = I_2$ in section $AB$

$$y'_{B/A} = \frac{Fx(x-2l)}{2EI_2} = \frac{46.6}{2EI_2}(2.75)[2.75 - 2(2.75)] = -176.2 / EI_2$$

$$\therefore y'_B = y'_A + y'_{B/A} = -\frac{2153.1}{30\left(10^6\right)\left(\pi/64\right)\left(1.25^4\right)} - \frac{176.2}{30\left(10^6\right)\left(\pi/64\right)\left(0.875^4\right)}$$

$= -0.000\,803$ rad (magnitude greater than 0.0005 rad)

*xz* plane




$$z'_A = -\frac{683.5}{EI_1}, \qquad z'_{B/A} = -\frac{128\left(2.75^2\right)}{2EI_2} = -\frac{484}{EI_2}$$

$$z'_B = -\frac{683.5}{30\left(10^6\right)\left(\pi/64\right)\left(1.25^4\right)} - \frac{484}{30\left(10^6\right)\left(\pi/64\right)\left(0.875^4\right)} = -0.000\,751 \text{ rad}$$

$$\theta_B = \sqrt{(-0.000\,803)^2 + (-0.000\,751)^2} = 0.00110 \text{ rad}$$

Crowned teeth must be used.

Finite element results: Error in simplified model

$\theta_O = 5.47(10^{-4})$ rad 3.0%

$\theta_A = 7.09(10^{-4})$ rad 11.4%

$\theta_B = 1.10(10^{-3})$ rad 0.0%

The simplified model yielded reasonable results.

*Strength* $S_{ut} = 72$ kpsi, $S_y = 39.5$ kpsi

At the shoulder at $A$, $x = 10.75$ in. From Prob. 7-4,

$$M_{xy} = -209.3 \text{ lbf}\cdot\text{in}, \quad M_{xz} = -293.0 \text{ lbf}\cdot\text{in}, \quad T = 192 \text{ lbf}\cdot\text{in}$$

$$M = \sqrt{(-209.3)^2 + (-293)^2} = 360.0 \text{ lbf}\cdot\text{in}$$

$$S_e' = 0.5(72) = 36 \text{ kpsi}$$

$$k_a = 2.70(72)^{-0.265} - 0.869$$

$$k_b = \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$$

$$k_c = k_d = k_e = k_f = 1$$

$$S_e = 0.869(0.879)(36) = 27.5 \text{ kpsi}$$

$D / d = 1.25$, $r / d = 0.03$

Fig. A-15-8: $K_{ts} = 1.8$

Fig. A-15-9: $K_t = 2.3$

Fig. 6-20: $q = 0.65$

Fig. 6-21: $q_s = 0.70$

Eq. (6-32): $K_f = 1 + 0.65(2.3 - 1) = 1.85$

$K_{fs} = 1 + 0.70(1.8 - 1) = 1.56$

Using DE-ASME Elliptic, Eq. (7-11) with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(1^3)}\left\{4\left[\frac{1.85(360)}{27\,500}\right]^2 + 3\left[\frac{1.56(192)}{39\,500}\right]^2\right\}^{1/2}$$

$n = 3.91$

Perform a similar analysis at the profile keyway under the gear.

The main problem with the design is the undersized shaft overhang with excessive slope at the gear. The use of crowned-teeth in the gears will eliminate this problem.

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**7-7 through 7-16**

These are design problems, which can have many acceptable designs. See the solution for Prob. 7-17 for an example of the design process.

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**7-17** **(a)** One possible shaft layout is shown in part (e). Both bearings and the gear will be located against shoulders. The gear and the motor will transmit the torque through the keys. The bearings can be lightly pressed onto the shaft. The left bearing will locate the shaft in the housing, while the right bearing will float in the housing.

**(b)** From summing moments around the shaft axis, the tangential transmitted load through the gear will be

$$W_t = T/(d/2) = 2500/(4/2) = 1250 \text{ lbf}$$

The radial component of gear force is related by the pressure angle.

$$W_r = W_t \tan\phi = 1250 \tan 20^\circ = 455 \text{ lbf}$$

$$W = \left(W_r^2 + W_t^2\right)^{1/2} = \left(455^2 + 1250^2\right)^{1/2} = 1330 \text{ lbf}$$

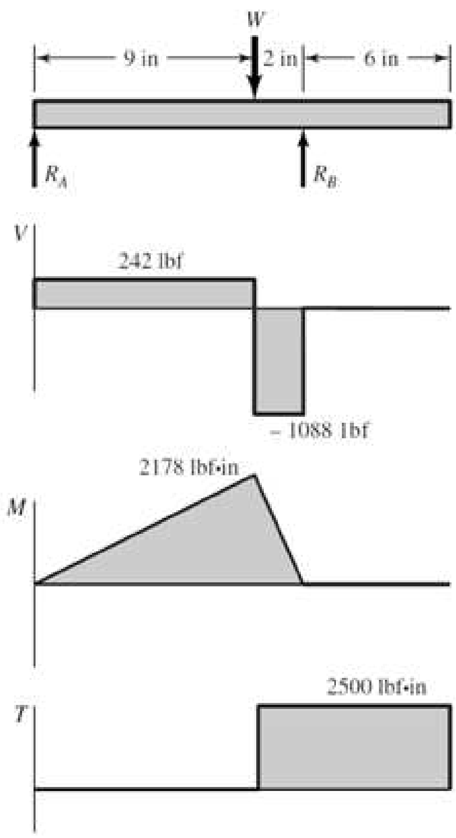
Reactions $R_A$ and $R_B$, and the load $W$ are all in the same plane. From force and moment balance,

$$R_A = 1330(2/11) = 242 \text{ lbf}$$

$$R_B = 1330(9/11) = 1088 \text{ lbf}$$

$$M_{\max} = R_A(9) = 242(9) = 2178 \text{ lbf}\cdot\text{in}$$

Shear force, bending moment, and torque diagrams can now be obtained.




*Ans.*

**(c)** Potential critical locations occur at each stress concentration (shoulders and keyways). To be thorough, the stress at each potentially critical location should be evaluated. For now, we will choose the most likely critical location, by observation of the loading situation, to be in the keyway for the gear. At this point there is a large stress concentration, a large bending moment, and the torque is present. The other locations either have small bending moments, or no torque. The stress concentration for the keyway is highest at the ends. For simplicity, and to be conservative, we will use the maximum bending moment, even though it will have dropped off a little at the end of the keyway.

**(d)** At the gear keyway, approximately 9 in from the left end of the shaft, the bending is completely reversed and the torque is steady.

$$M_a = 2178 \text{ lbf} \cdot \text{in} \qquad T_m = 2500 \text{ lbf} \cdot \text{in} \qquad M_m = T_a = 0$$

From Table 7-1, p. 365, estimate stress concentrations for the end-milled keyseat to be $K_t$ = 2.14 and $K_{ts}$ = 3.0. For the relatively low strength steel specified (AISI 1020 CD), roughly estimate notch sensitivities of $q$ = 0.75 and $q_s$ = 0.80, obtained by observation of Figs. 6-20 and 6-21, assuming a typical radius at the bottom of the keyseat of $r / d$ = 0.02, and a shaft diameter of up to 3 inches.

Eq. (6-32): $K_f = 1 + 0.75(2.14 - 1) = 1.9$

$K_{fs} = 1 + 0.8(3.0 - 1) = 2.6$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

For estimating $k_b$, guess $d = 2$ in.

Eq. (6-20) $k_b = (2/0.3)^{-0.107} = 0.816$

Eq. (6-18) $S_e = 0.883(0.816)(0.5)(68) = 24.5$ kpsi

Selecting the DE-Goodman criteria for a conservative first design,

Eq. (7-8):
$$d = \left[ \frac{16n}{\pi} \left\{ \frac{\left[4\left(K_f M_a\right)^2\right]^{1/2}}{S_e} + \frac{\left[3\left(K_{fs} T_m\right)^2\right]^{1/2}}{S_{ut}} \right\} \right]^{1/3}$$

$$d = \left[ \frac{16(1.5)}{\pi} \left\{ \frac{\left[4\left(1.9 \cdot 2178\right)^2\right]^{1/2}}{24\,500} + \frac{\left[3\left(2.6 \cdot 2500\right)^2\right]^{1/2}}{68\,000} \right\} \right]^{1/3}$$

$d = 1.57$ in *Ans.*

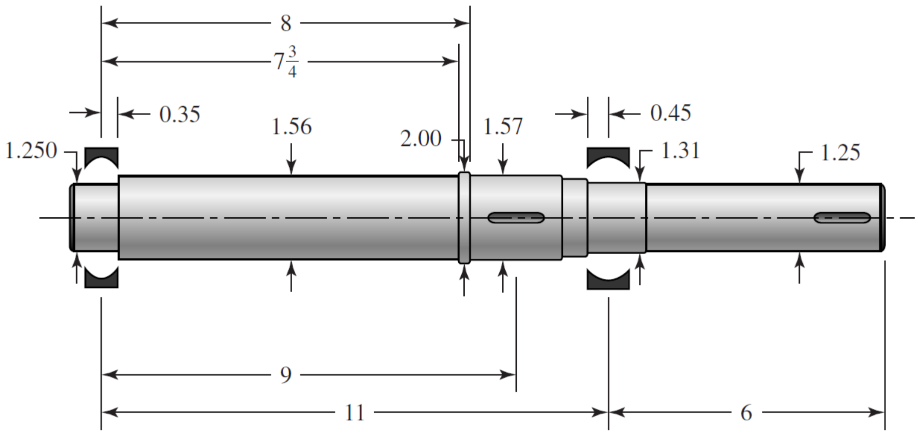
With this diameter, the estimates for notch sensitivity and size factor were conservative, but close enough for a first iteration until deflections are checked. Check yielding with this diameter.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma'_{\max} = \left[\left(\frac{32(1.9)(2178)}{\pi(1.57)^3}\right)^2 + 3\left(\frac{16(2.6)(2500)}{\pi(1.57)^3}\right)^2\right]^{1/2} = 18\,389 \text{ psi } = 18.4 \text{ kpsi}$$

$$n_y = S_y / \sigma'_{\max} = 57/18.4 = 3.1 \qquad Ans.$$

**(e)** Now estimate other diameters to provide typical shoulder supports for the gear and bearings (p. 364). Also, estimate the gear and bearing widths.




**(f)** Entering this shaft geometry into beam analysis software (or Finite Element software), the following deflections are determined:

| | |
|---|---|
| Left bearing slope: | 0.000 532 rad |
| Right bearing slope: | – 0.000 850 rad |
| Gear slope: | – 0.000 545 rad |
| Right end of shaft slope: | – 0.000 850 rad |
| Gear deflection: | – 0.001 45 in |
| Right end of shaft deflection: | 0.005 10 in |

Comparing these deflections to the recommendations in Table 7-2, everything is within typical range except the gear slope is a little high for an uncrowned gear.

**(g)** To use a non-crowned gear, the gear slope is recommended to be less than 0.0005 rad. Since all other deflections are acceptable, we will target an increase in diameter only for the long section between the left bearing and the gear. Increasing this diameter from the proposed 1.56 in to 1.75 in, produces a gear slope of – 0.000 401 rad. All other deflections are improved as well.

**7-18**

**(a)** Use the distortion-energy elliptic failure locus. The torque and moment loadings on the shaft are shown in the solution to Prob. 7-17.

Candidate critical locations for strength:

- Left seat keyway
- Right bearing shoulder
- Right keyway

Table A-20 for 1030 HR: $S_{ut} = 68 \text{ kpsi}, S_y = 37.5 \text{ kpsi}, H_B = 137$

Eq. (6-8): $S'_e = 0.5(68) = 34.0 \text{ kpsi}$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

$$k_c = k_d = k_e = 1$$

*Left keyway*

See Table 7-1 for keyway stress concentration factors,

$$\left.\begin{aligned} K_t &= 2.14 \\ K_{ts} &= 3.0 \end{aligned}\right\} \text{Profile keyway}$$

For an end-mill profile keyway cutter of 0.010 in radius, estimate notch sensitivities.

Fig. 6-20: $q = 0.51$

Fig. 6-21: $q_s = 0.57$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.57(3.0 - 1) = 2.1$

$$K_f = 1 + 0.51(2.14 - 1) = 1.6$$

Eq. (6-20): $k_b = \left(\dfrac{1.875}{0.30}\right)^{-0.107} = 0.822$

Eq. (6-18): $S_e = 0.883(0.822)(34.0) = 24.7 \text{ kpsi}$

Eq. (7-11): $\dfrac{1}{n_f} = \dfrac{16}{\pi(1.875^3)}\left\{4\left[\dfrac{1.6(2178)}{24\,700}\right]^2 + 3\left[\dfrac{2.1(2500)}{37\,500}\right]^2\right\}^{\frac{1}{2}}$

$n_f = 3.5$ *Ans.*

*Right bearing shoulder*

The text does not give minimum and maximum shoulder diameters for 03-series bearings (roller). Use $D$ = 1.75 in.

$$\frac{r}{d} = \frac{0.030}{1.574} = 0.019, \qquad \frac{D}{d} = \frac{1.75}{1.574} = 1.11$$

Fig. A-15-9: $K_t = 2.4$

Fig. A-15-8: $K_{ts} = 1.6$

Fig. 6-20: $q = 0.65$

Fig. 6-21: $q_s = 0.70$

Eq. (6-32): $K_f = 1 + 0.65(2.4-1) = 1.91$

$$K_{fs} = 1 + 0.70(1.6-1) = 1.42$$

$$M = 2178\left(\frac{0.453}{2}\right) = 493 \text{ lbf}\cdot\text{in}$$

Eq. (7-11):
$$\frac{1}{n_f} = \frac{16}{\pi(1.574^3)}\left[4\left(\frac{1.91(493)}{24\,700}\right)^2 + 3\left(\frac{1.42(2500)}{37\,500}\right)^2\right]^{1/2}$$

$n_f = 4.2$ *Ans.*

*Right keyway*

Use the same stress concentration factors as for the left keyway. There is no bending moment, thus Eq. (7-11) reduces to:

$$\frac{1}{n_f} = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3 S_y} = \frac{16\sqrt{3}(2.1)(2500)}{\pi\left(1.5^3\right)(37\,500)}$$

$n_f = 2.7$ *Ans.*

*Yielding*

Check for yielding at the left keyway, where the completely reversed bending is maximum, and the steady torque is present. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.6)(2178)}{\pi(1.875)^3}\right)^2 + 3\left(\frac{16(2.1)(2500)}{\pi(1.875)^3}\right)^2\right]^{1/2}$$

$$= 8791 \text{ psi} = 8.79 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{37.5}{8.79} = 4.3 \qquad \textit{Ans.}$$

Check in smaller diameter at right end of shaft where only steady torsion exists.

$$\sigma'_{\max} = \left[3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[3\left(\frac{16(2.1)(2500)}{\pi(1.5)^3}\right)^2\right]^{1/2}$$

$$= 13\,722 \text{ psi} = 13.7 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{37.5}{13.7} = 2.7 \qquad Ans.$$

**(b)** One could take pains to model this shaft exactly, using finite element software. However, for the bearings and the gear, the shaft is basically of uniform diameter, 1.875 in. The reductions in diameter at the bearings will change the results insignificantly. Use $E$ = 30 Mpsi for steel.

To the left of the load, from Table A-9, case 6,

$$\theta_{AB} = \frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = \frac{1449(2)(3x^2 + 2^2 - 11^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)}$$
$$= 2.4124(10^{-6})(3x^2 - 117)$$

At $x$ = 0 in: $\quad \theta = -2.823(10^{-4})$ rad

At $x$ = 9 in: $\quad \theta = 3.040(10^{-4})$ rad

To the right of the load, from Table A-9, case 6,

$$\theta_{BC} = \frac{dy_{BC}}{dx} = \frac{Fa}{6EIl}\left(-3x^2 + 6xl - 2l^2 - a^2\right)$$

At $x = l$ = 11 in:

$$\theta = \frac{Fa}{6EIl}\left(l^2 - a^2\right) = \frac{1449(9)(11^2 - 9^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)} = 4.342(10^{-4}) \text{ rad}$$

Obtain allowable slopes from Table 7-2.

*Left bearing:*

$$n_{fs} = \frac{\text{Allowable slope}}{\text{Actual slope}} = \frac{0.001}{0.000\,282\,3} = 3.5 \qquad Ans.$$

*Right bearing:*

$$n_{fs} = \frac{0.0008}{0.000\,434\,2} = 1.8 \qquad Ans.$$

*Gear mesh slope:*

Table 7-2 recommends a minimum relative slope of 0.0005 rad. While we don’t know the slope on the next shaft, we know that it will need to have a larger diameter and be stiffer. At the moment we can say

$$n_{fs} < \frac{0.0005}{0.000\,304} = 1.6 \qquad Ans.$$

**7-19** The most likely critical locations for fatigue are at locations where the bending moment is high, the cross section is small, stress concentration exists, and torque exists. The two-plane bending moment diagrams, shown in the solution to Prob. 3-72, indicate decreasing moments in both planes to the left of $A$ and to the right of $C$, with combined values at $A$ and $C$ of $M_A$ = 5324 lbf·in and $M_C$ = 6750 lbf·in. The torque is constant between $A$ and $B$, with $T$ = 2819 lbf·in. The most likely critical locations are at the stress concentrations near $A$ and $C$. The two shoulders near $A$ can be eliminated since the shoulders near $C$ have the same geometry but a higher bending moment. We will consider the following potentially critical locations:

- keyway at $A$
- shoulder to the left of $C$
- shoulder to the right of $C$

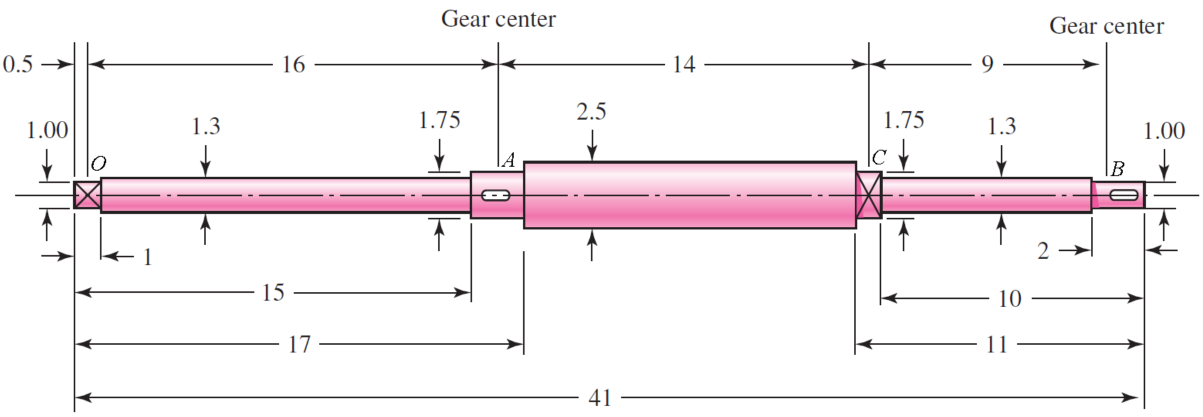



Table A-20: $S_{ut}$ = 64 kpsi, $S_y$ = 54 kpsi

Eq. (6-8): $S_e' = 0.5(64) = 32.0$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

$k_c = k_d = k_e = 1$

*Keyway at A*

Assuming $r/d$ = 0.02 for typical end-milled keyway cutter (p. 365), with $d$ = 1.75 in, $r = 0.02d$ = 0.035 in.

Table 7-1: $K_t$ = 2.14, $K_{ts}$ = 3.0

Fig. 6-20: $q$ = 0.65

Fig. 6-21: $q_s$ = 0.71

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.65(2.14 - 1) = 1.7$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.71(3.0 - 1) = 2.4$

Eq. (6-20): $k_b = \left(\dfrac{1.75}{0.30}\right)^{-0.107} = 0.828$

Eq. (6-18): $S_e = 0.897(0.828)(32) = 23.8$ kpsi

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations.
Using Eq. (7-9) with $M_m = T_a = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.7)(5324)\right]^2}=18\,102\ \text{lbf}\cdot\text{in}=18.10\ \text{kip}\cdot\text{in}$$

$$B=\sqrt{3\left(K_{fs} T_m\right)^2}=\sqrt{3\left[(2.4)(2819)\right]^2}=11\,718\ \text{lbf}\cdot\text{in}=11.72\ \text{kip}\cdot\text{in}$$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(18.10)}{\pi\left(1.75^3\right)(23.8)}\left\{1+\left[1+\left(\frac{2(11.72)(23.8)}{(18.10)(64)}\right)^2\right]^{1/2}\right\}$$

$n = 1.3$

*Shoulder to the left of C*

$r / d = 0.0625 / 1.75 = 0.036$, $D / d = 2.5 / 1.75 = 1.43$

Fig. A-15-9: $K_t = 2.2$
Fig. A-15-8: $K_{ts} = 1.8$
Fig. 6-20: $q = 0.71$
Fig. 6-21: $q_s = 0.76$
Eq. (6-32): $K_f = 1+q\left(K_t-1\right)=1+0.71(2.2-1)=1.9$

$K_{fs} = 1+q_s\left(K_{ts}-1\right)=1+0.76(1.8-1)=1.6$

Eq. (6-20): $k_b=\left(\dfrac{1.75}{0.30}\right)^{-0.107}=0.828$

Eq. (6-18): $S_e = 0.897(0.828)(32) = 23.8\ \text{kpsi}$

For convenience, we will use the full value of the bending moment at *C*, even though it will be slightly less at the shoulder. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.9)(6750)\right]^2}=25\,650\ \text{lbf}\cdot\text{in}=25.65\ \text{kip}\cdot\text{in}$$

$$B=\sqrt{3\left(K_{fs} T_m\right)^2}=\sqrt{3\left[(1.6)(2819)\right]^2}=7812\ \text{lbf}\cdot\text{in}=7.812\ \text{kip}\cdot\text{in}$$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(25.65)}{\pi\left(1.75^3\right)(23.8)}\left\{1+\left[1+\left(\frac{2(7.812)(23.8)}{(25.65)(64)}\right)^2\right]^{1/2}\right\}$$

$n = 0.96$

*Shoulder to the right of C*

$r / d = 0.0625 / 1.3 = 0.048, \; D / d = 1.75 / 1.3 = 1.35$

Fig. A-15-9: $K_t = 2.0$
Fig. A-15-8: $K_{ts} = 1.7$
Fig. 6-20: $q = 0.71$
Fig. 6-21: $q_s = 0.76$
Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.71(2.0 - 1) = 1.7$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.76(1.7 - 1) = 1.5$

Eq. (6-20): $k_b = \left(\dfrac{1.3}{0.30}\right)^{-0.107} = 0.855$

Eq. (6-18): $S_e = 0.897(0.855)(32) = 24.5$ kpsi

For convenience, we will use the full value of the bending moment at *C*, even though it will be slightly less at the shoulder. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.7)(6750)\right]^2} = 22\,950 \text{ lbf}\cdot\text{in} = 22.95 \text{ kip}\cdot\text{in}$$

$$B = \sqrt{3\left(K_{fs} T_m\right)^2} = \sqrt{3\left[(1.5)(2819)\right]^2} = 7324 \text{ lbf}\cdot\text{in} = 7.324 \text{ kip}\cdot\text{in}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(22.95)}{\pi(1.3^3)(24.5)}\left\{1 + \left[1 + \left(\frac{2(7.324)(24.5)}{(22.95)(64)}\right)^2\right]^{1/2}\right\}$$

$n = 0.45$

The critical location is at the shoulder to the right of *C*, where $n = 0.45$ and finite life is predicted. *Ans.*

Though not explicitly called for in the problem statement, a static check for yielding is especially warranted with such a low fatigue factor of safety. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.7)(6750)}{\pi(1.3)^3}\right)^2 + 3\left(\frac{16(1.5)(2819)}{\pi(1.3)^3}\right)^2\right]^{1/2} = 55\,845 \text{ psi} = 55.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_{\max}} = \frac{54}{55.8} = 0.97$$

This indicates localized yielding is predicted at the stress-concentration, though after localized cold-working it may not be a problem. The finite fatigue life is still likely to be the failure mode that will dictate whether this shaft is acceptable.

It is interesting to note the impact of stress concentration on the acceptability of the proposed design. This problem is linked with several previous problems (see Table 1-2, p. 34) in which the shaft was considered to have a constant diameter of 1.25 in. In each of the previous problems, the 1.25 in diameter was more than adequate for deflection, static, and fatigue considerations. In this problem, even though practically the entire shaft has diameters larger than 1.25 in, the stress concentrations significantly reduce the anticipated fatigue life.

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**7-20** For a shaft with significantly varying diameters over its length, we will choose to use shaft analysis software or finite element software to calculate the deflections. Entering the geometry from the shaft as defined in Prob. 7-19, and the loading as defined in Prob. 3-72, the following deflection magnitudes are determined:

| **Location** | **Slope (rad)** | **Deflection (in)** |
|---|---|---|
| Left bearing *O* | 0.00640 | 0.00000 |
| Right bearing *C* | 0.00434 | 0.00000 |
| Left Gear *A* | 0.00260 | 0.04839 |
| Right Gear *B* | 0.01078 | 0.07517 |

Comparing these values to the recommended limits in Table 7-2, we find that they are all out of the desired range. This is not unexpected since the stress analysis of Prob. 7-19 also indicated the shaft is undersized for infinite life. The slope at the right gear is the most excessive, so we will attempt to increase all diameters to bring it into compliance. Using Eq. (7-18) at the right gear,

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d \left( dy/dx \right)_{\text{old}}}{\left( \text{slope} \right)_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(0.01078)}{0.0005} \right|^{1/4} = 2.15$$

Multiplying all diameters by 2.15, we obtain the following deflections:

| **Location** | **Slope (rad)** | **Deflection (in)** |
|---|---|---|
| Left bearing *O* | 0.00030 | 0.00000 |
| Right bearing *C* | 0.00020 | 0.00000 |
| Left Gear *A* | 0.00012 | 0.00225 |
| Right Gear *B* | 0.00050 | 0.00350 |

This brings the slope at the right gear just to the limit for an uncrowned gear, and all other slopes well below the recommended limits. For the gear deflections, the values are below recommended limits as long as the diametral pitch is less than 20.

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**7-21** The most likely critical locations for fatigue are at locations where the bending moment is high, the cross section is small, stress concentration exists, and torque exists. The two-plane bending moment diagrams, shown in the solution to Prob. 3-73, indicate both planes have a maximum bending moment at $B$. At this location, the combined bending moment from both planes is $M$ = 4097 N·m, and the torque is $T$ = 3101 N·m. The shoulder to the right of $B$ will be eliminated since its diameter is only slightly smaller, and there is no torque. Comparing the shoulder to the left of $B$ with the keyway at $B$, the primary difference between the two is the stress concentration, since they both have essentially the same bending moment, torque, and size. We will check the stress concentration factors for both to determine which is critical.

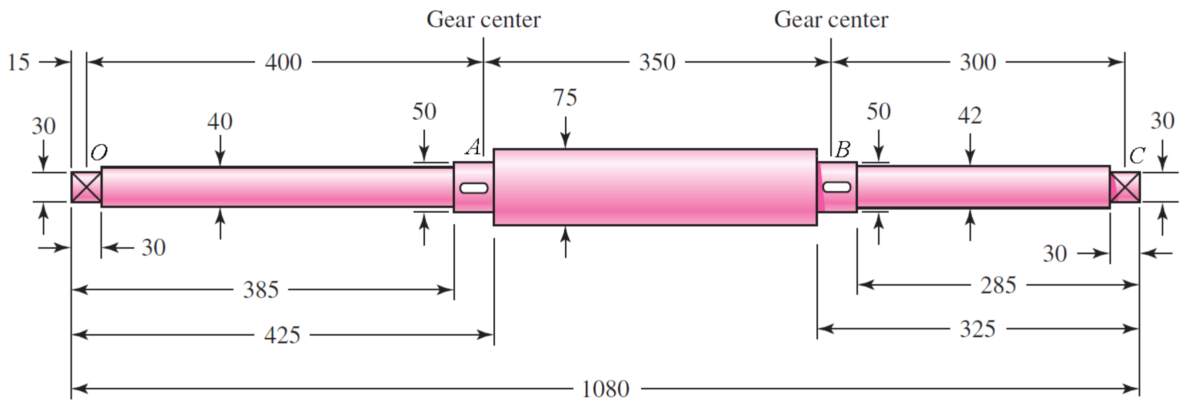



Table A-20: $S_{ut}$ = 440 MPa, $S_y$ = 370 MPa

*Keyway at A*
Assuming $r / d = 0.02$ for typical end-milled keyway cutter (p. 365), with $d$ = 50 mm, $r = 0.02d$ = 1 mm.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$
Fig. 6-20: $q = 0.66$
Fig. 6-21: $q_s = 0.72$
Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.66(2.14 - 1) = 1.8$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.72(3.0 - 1) = 2.4$$

*Shoulder to the left of B*

$$r / d = 2 / 50 = 0.04, \quad D / d = 75 / 50 = 1.5$$

Fig. A-15-9: $K_t = 2.2$

Fig. A-15-8: $K_{ts} = 1.8$
Fig. 6-20: $q = 0.73$
Fig. 6-21: $q_s = 0.78$
Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.73(2.2 - 1) = 1.9$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.78(1.8 - 1) = 1.6$$

Examination of the stress concentration factors indicates the keyway will be the critical location.

Eq. (6-8): $S'_e = 0.5(440) = 220 \text{ MPa}$
Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.899$
Eq. (6-20): $k_b = \left(\dfrac{50}{7.62}\right)^{-0.107} = 0.818$

$$k_c = k_d = k_e = 1$$

Eq. (6-18): $S_e = 0.899(0.818)(220) = 162 \text{ MPa}$

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4(K_f M_a)^2} = \sqrt{4[(1.8)(4097)]^2} = 14\,750 \text{ N}\cdot\text{m}$$

$$B = \sqrt{3(K_{fs} T_m)^2} = \sqrt{3[(2.4)(3101)]^2} = 12\,890 \text{ N}\cdot\text{m}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(14\,750)}{\pi(0.050^3)(162)(10^6)}\left\{1 + \left[1 + \left(\frac{2(12\,890)(162)(10^6)}{(14\,750)(440)(10^6)}\right)^2\right]^{1/2}\right\}$$

$n = 0.25$ Infinite life is not predicted. *Ans.*

Though not explicitly called for in the problem statement, a static check for yielding is especially warranted with such a low fatigue factor of safety. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.8)(4097)}{\pi(0.050)^3}\right)^2 + 3\left(\frac{16(2.4)(3101)}{\pi(0.050)^3}\right)^2\right]^{1/2} = 7.98(10^8) \text{ Pa} = 798 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_{\max}} = \frac{370}{798} = 0.46$$

This indicates localized yielding is predicted at the stress-concentration. Even without the stress concentration effects, the static factor of safety turns out to be 0.93. Static failure is predicted, rendering this proposed shaft design unacceptable.

This problem is linked with several previous problems (see Table 1-2, p. 34) in which the shaft was considered to have a constant diameter of 50 mm. The results here are consistent with the previous problems, in which the 50 mm diameter was found to slightly undersized for static, and significantly undersized for fatigue. Though in the current problem much of the shaft has larger than 50 mm diameter, the added contribution of stress concentration limits the fatigue life.

---

**7-22** For a shaft with significantly varying diameters over its length, we will choose to use shaft analysis software or finite element software to calculate the deflections. Entering the geometry from the shaft as defined in Prob. 7-21, and the loading as defined in Prob. 3-73, the following deflection magnitudes are determined:

| **Location** | **Slope (rad)** | **Deflection (mm)** |
|---|---|---|
| Left bearing *O* | 0.01445 | 0.000 |
| Right bearing *C* | 0.01843 | 0.000 |
| Left Gear *A* | 0.00358 | 3.761 |
| Right Gear *B* | 0.00366 | 3.676 |

Comparing these values to the recommended limits in Table 7-2, we find that they are all well out of the desired range. This is not unexpected since the stress analysis in Prob. 7-21 also indicated the shaft is undersized for infinite life. The transverse deflection at the left gear is the most excessive, so we will attempt to increase all diameters to bring it into compliance. Using Eq. (7-17) at the left gear, assuming from Table 7-2 an allowable deflection of $y_{\text{all}}$ = 0.01 in = 0.254 mm,

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d y_{\text{old}}}{y_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(3.761)}{0.254} \right|^{1/4} = 1.96$$

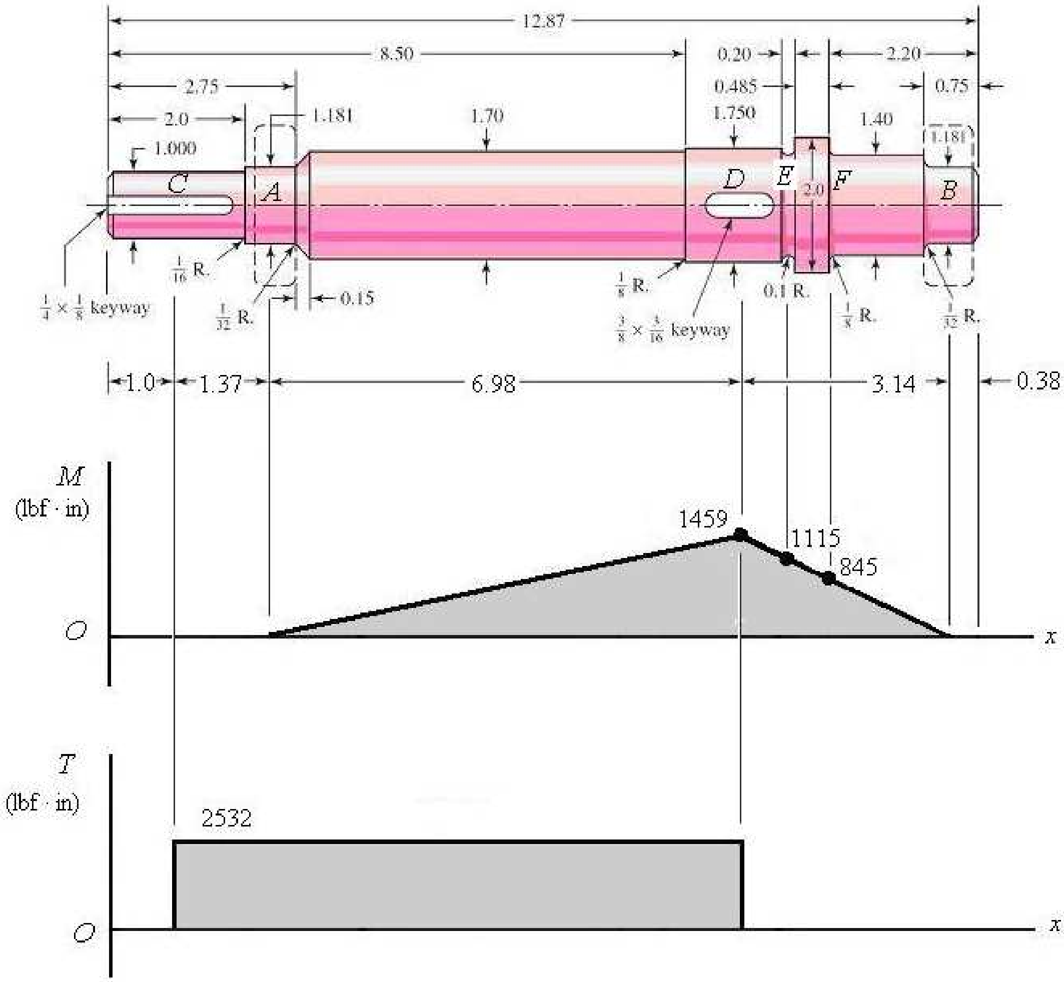
Multiplying all diameters by 2, we obtain the following deflections:

| **Location** | **Slope (rad)** | **Deflection (mm)** |
|---|---|---|
| Left bearing *O* | 0.00090 | 0.000 |
| Right bearing *C* | 0.00115 | 0.000 |
| Left Gear *A* | 0.00022 | 0.235 |
| Right Gear *B* | 0.00023 | 0.230 |

This brings the deflection at the gears just within the limit for a spur gear (assuming $P$ < 10 teeth/in), and all other deflections well below the recommended limits.

---

**7-23** **(a)** Label the approximate locations of the effective centers of the bearings as $A$ and $B$, the fan as $C$, and the gear as $D$, with axial dimensions as shown. Since there is only one gear, we can combine the radial and tangential gear forces into a single resultant force with an accompanying torque, and handle the statics problem in a single plane. From statics, the resultant reactions at the bearings can be found to be $R_A$ = 209.9 lbf and $R_B$ = 464.5 lbf. The bending moment and torque diagrams are shown, with the maximum bending moment at $D$ of $M_D$ = 209.9(6.98) = 1459 lbf·in and a torque transmitted from $D$ to $C$ of $T$ = 633 (8/2) = 2532 lbf·in. Due to the shaft rotation, the bending stress on any stress element will be completely reversed, while the torsional stress will be steady. Since we do not have any information about the fan, we will ignore any axial load that it would introduce. It would not likely contribute much compared to the bending anyway.




Potentially critical locations are identified as follows:

- Keyway at $C$, where the torque is high, the diameter is small, and the keyway creates a stress concentration.

- Keyway at $D$, where the bending moment is maximum, the torque is high, and the keyway creates a stress concentration.
- Groove at $E$, where the diameter is smaller than at $D$, the bending moment is still high, and the groove creates a stress concentration. There is no torque here, though.
- Shoulder at $F$, where the diameter is smaller than at $D$ or $E$, the bending moment is still moderate, and the shoulder creates a stress concentration. There is no torque here, though.
- The shoulder to the left of $D$ can be eliminated since the change in diameter is very slight, so that the stress concentration will undoubtedly be much less than at $D$.

Table A-20: $S_{ut}$ = 68 kpsi, $S_y$ = 57 kpsi
Eq. (6-8): $S'_e = 0.5(68) = 34.0$ kpsi
Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

*Keyway at C*
Since there is only steady torsion here, only a static check needs to be performed. We'll use the maximum shear stress theory.

$$\tau = \frac{Tr}{J} = \frac{2532\left(1.00/2\right)}{\pi\left(1.00^4\right)/32} = 12.9 \text{ kpsi}$$

Eq. (5-3): $n_y = \dfrac{S_y/2}{\tau} = \dfrac{57/2}{12.9} = 2.21$

*Keyway at D*
Assuming $r/d = 0.02$ for typical end-milled keyway cutter (p. 365), with $d$ = 1.75 in, $r = 0.02d = 0.035$ in.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$
Fig. 6-20: $q = 0.66$
Fig. 6-21: $q_s = 0.72$
Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.66(2.14 - 1) = 1.8$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.72(3.0 - 1) = 2.4$

Eq. (6-20): $k_b = \left(\dfrac{1.75}{0.30}\right)^{-0.107} = 0.828$

Eq. (6-18): $S_e = 0.883(0.828)(34.0) = 24.9$ kpsi

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations.
Using Eq. (7-9) with $M_m = T_a = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.8)(1459)\right]^2}=5252\text{ lbf}\cdot\text{in}=5.252\text{ kip}\cdot\text{in}$$

$$B=\sqrt{3\left(K_{fs} T_m\right)^2}=\sqrt{3\left[(2.4)(2532)\right]^2}=10\,525\text{ lbf}\cdot\text{in}=10.53\text{ kip}\cdot\text{in}$$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(5.252)}{\pi\left(1.75^3\right)(24.9)}\left\{1+\left[1+\left(\frac{2(10.53)(24.9)}{(5.252)(68)}\right)^2\right]^{1/2}\right\}$$

$n = 3.59$ *Ans.*

*Groove at E*

We will assume Figs. A-15-14 is applicable since the 2 in diameter to the right of the groove is relatively narrow and will likely not allow the stress flow to fully develop. (See Fig.7-9 for the stress flow concept.)

$r / d = 0.1 / 1.55 = 0.065, \quad D / d = 1.75 / 1.55 = 1.13$

Fig. A-15-14: $K_t = 2.1$

Fig. 6-20: $q = 0.76$

Eq. (6-32): $K_f = 1+q\left(K_t-1\right)=1+0.76(2.1-1)=1.8$

Eq. (6-20): $k_b=\left(\dfrac{1.55}{0.30}\right)^{-0.107}=0.839$

Eq. (6-18): $S_e = 0.883(0.839)(34)=25.2\text{ kpsi}$

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.8)(1115)\right]^2}=4122\text{ lbf}\cdot\text{in}=4.122\text{ kip}\cdot\text{in}$$

$B = 0$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(4.122)}{\pi\left(1.55^3\right)(25.2)}\left\{1+\left[1+(0)^2\right]^{1/2}\right\}$$

$n = 4.47$ *Ans.*

*Shoulder at F*

$r / d = 0.125 / 1.40 = 0.089, \quad D / d = 2.0 / 1.40 = 1.43$

Fig. A-15-9: $K_t = 1.7$

Fig. 6-20: $q = 0.78$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.5$

Eq. (6-20): $k_b = \left(\dfrac{1.40}{0.30}\right)^{-0.107} = 0.848$

Eq. (6-18): $S_e = 0.883(0.848)(34) = 25.5$ kpsi

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.5)(845)\right]^2} = 2535 \text{ lbf} \cdot \text{in} = 2.535 \text{ kip} \cdot \text{in}$$

$B = 0$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(2.535)}{\pi(1.40^3)(25.5)}\left\{1 + \left[1 + (0)^2\right]^{1/2}\right\}$$

$n = 5.42$ *Ans.*

**(b)** The deflection will not be much affected by the details of fillet radii, grooves, and keyways, so these can be ignored. Also, the slight diameter changes, as well as the narrow 2.0 in diameter section, can be neglected. We will model the shaft with the following three sections:

| **Section** | **Diameter (in)** | **Length (in)** |
|---|---|---|
| 1 | 1.00 | 2.90 |
| 2 | 1.70 | 7.77 |
| 3 | 1.40 | 2.20 |

The deflection problem can readily (though tediously) be solved with singularity functions. For examples, see Ex. 4-7, p. 173, or the solution to Prob. 7-24. Alternatively, shaft analysis software or finite element software may be used. Using any of the methods, the results should be as follows:

| **Location** | **Slope (rad)** | **Deflection (in)** |
|---|---|---|
| Left bearing *A* | 0.000290 | 0.000000 |
| Right bearing *B* | 0.000400 | 0.000000 |
| Fan *C* | 0.000290 | 0.000404 |
| Gear *D* | 0.000146 | 0.000928 |

Comparing these values to the recommended limits in Table 7-2, we find that they are all within the recommended range.

---

**7-24** Shaft analysis software or finite element software can be utilized if available. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

*Deflection*: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

*Statics*: Left support: $R_1 = 7(315-140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

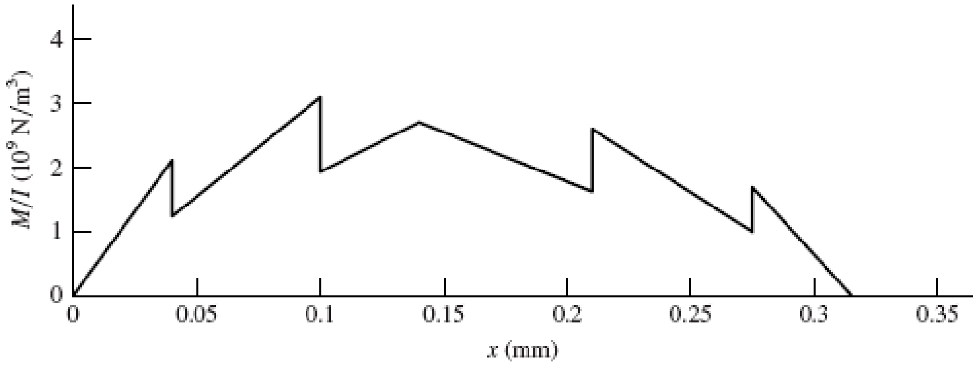
Determine the bending moment at each step.

| $x$(mm) | 0 | 40 | 100 | 140 | 210 | 275 | 315 |
|---|---|---|---|---|---|---|---|
| $M$(N · m) | 0 | 155.56 | 388.89 | 544.44 | 326.67 | 124.44 | 0 |

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})$ m$^4$, $I_{40} = 1.257(10^{-7})$ m$^4$, $I_{45} = 2.013(10^{-7})$ m$^4$

Plot $M/I$ as a function of $x$.

| $x$(m) | $M/I$ ($10^9$ N/m$^3$) | Step | Slope | ΔSlope |
|---|---|---|---|---|
| 0 | 0 | | 52.8 | |
| 0.04 | 2.112 | | | |
| 0.04 | 1.2375 | –0.8745 | 30.942 | –21.86 |
| 0.1 | 3.094 | | | |
| 0.1 | 1.932 | –1.162 | 19.325 | –11.617 |
| 0.14 | 2.705 | | | |
| 0.14 | 2.705 | 0 | –15.457 | –34.78 |
| 0.21 | 1.623 | | | |
| 0.21 | 2.6 | 0.977 | -24.769 | -9.312 |
| 0.275 | 0.99 | | | |
| 0.275 | 1.6894 | 0.6994 | -42.235 | -17.47 |
| 0.315 | 0 | | | |




The steps and the change of slopes are evaluated in the table. From these, the function *M/I* can be generated:

$$M/I = \Big[52.8x - 0.8745\langle x-0.04\rangle^0 - 21.86\langle x-0.04\rangle^1 - 1.162\langle x-0.1\rangle^0$$
$$-11.617\langle x-0.1\rangle^1 - 34.78\langle x-0.14\rangle^1 + 0.977\langle x-0.21\rangle^0$$
$$-9.312\langle x-0.21\rangle^1 + 0.6994\langle x-0.275\rangle^0 - 17.47\langle x-0.275\rangle^1\Big]10^9$$

Integrate twice:

$$E\frac{dy}{dx} = \Big[26.4x^2 - 0.8745\langle x-0.04\rangle^1 - 10.93\langle x-0.04\rangle^2 - 1.162\langle x-0.1\rangle^1$$
$$-5.81\langle x-0.1\rangle^2 - 17.39\langle x-0.14\rangle^2 + 0.977\langle x-0.21\rangle^1$$
$$-4.655\langle x-0.21\rangle^2 + 0.6994\langle x-0.275\rangle^1 - 8.735\langle x-0.275\rangle^2 + C_1\Big]10^9 \quad (1)$$

$$Ey = \Big[8.8x^3 - 0.4373\langle x-0.04\rangle^2 - 3.643\langle x-0.04\rangle^3 - 0.581\langle x-0.1\rangle^2$$
$$-1.937\langle x-0.1\rangle^3 - 5.797\langle x-0.14\rangle^3 + 0.4885\langle x-0.21\rangle^2$$
$$-1.552\langle x-0.21\rangle^3 + 0.3497\langle x-0.275\rangle^2 - 2.912\langle x-0.275\rangle^3 + C_1x + C_2\Big]10^9$$

*Boundary conditions*: $y = 0$ at $x = 0$ yields $C_2 = 0$;
$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\ 25$ N/m$^2$.

Equation (1) with $C_1 = -0.295\ 25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the results of a full model which models the 35 and 55 mm diameter steps.

| $x$ (mm) | $\theta$ (rad) | F.E. Model | Full F.E. Model |
|---|---|---|---|
| 0 | –0.001 4260 | –0.001 4270 | –0.001 4160 |
| 140 | –0.000 1466 | –0.000 1467 | –0.000 1646 |
| 315 | 0.001 3120 | 0.001 3280 | 0.001 3150 |

The main discrepancy between the results is at the gear location ($x$ = 140 mm). The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max}$ = (0.001/0.001 426)7 = 4.91 kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, apply Eq. (7-18) to the most offending deflection (at $x$ = 0) to determine a multiplier to be used for all diameters.

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d \left( dy/dx \right)_{\text{old}}}{\left(\text{slope}\right)_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(0.0014260)}{0.001} \right|^{1/4} = 1.093$$

Form a table:

| | | | | | | |
|---|---|---|---|---|---|---|
| Old $d$, mm | 20.00 | 30.00 | 35.00 | 40.00 | 45.00 | 55.00 |
| New ideal $d$, mm | 21.86 | 32.79 | 38.26 | 43.72 | 49.19 | 60.12 |
| Rounded up $d$, mm | 22.00 | 34.00 | 40.00 | 44.00 | 50.00 | 62.00 |

Repeating the full finite element model results in

$x$ = 0: $\theta = -9.30 \times 10^{-4}$ rad
$x$ = 140 mm: $\theta = -1.09 \times 10^{-4}$ rad
$x$ = 315 mm: $\theta = 8.65 \times 10^{-4}$ rad

This is well within our goal. Have the students try a goal of 0.0005 rad at the gears.

*Strength*: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

| $x$ (mm) | 0 | 15 | 40 | 100 | 110 | 140 | 210 | 275 | 300 | 330 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ (MPa) | 0 | 22.0 | 37.0 | 61.9 | 47.8 | 60.9 | 52.0 | 39.6 | 17.6 | 0 |
| $\tau$ (MPa) | 0 | 0 | 0 | 0 | 0 | 6 | 8.5 | 12.7 | 20.2 | 68.1 |
| $\sigma'$ (MPa) | 0 | 22.0 | 37.0 | 61.9 | 47.8 | 61.8 | 53.1 | 45.3 | 39.2 | 118.0 |

Table A-20 for AISI 1020 CD steel: $S_{ut}$ = 470 MPa, $S_y$ = 390 MPa

At $x$ = 210 mm:
Eq. (6-19): $k_a = 4.51(470)^{-0.265} = 0.883$

Eq. (6-20): $k_b = (40/7.62)^{-0.107} = 0.837$

Eq. (6-18): $S_e = 0.883\,(0.837)(0.5)(470) = 174$ MPa

$D/d = 45/40 = 1.125, \quad r/d = 2/40 = 0.05$

Fig. A-15-8: $K_{ts} = 1.4$

Fig. A-15-9: $K_t = 1.9$

Fig. 6-20: $q = 0.75$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_f = 1 + 0.75(1.9 - 1) = 1.68$

$K_{fs} = 1 + 0.79(1.4 - 1) = 1.32$

Choosing DE-ASME Elliptic to inherently include the yield check, from Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi\left(0.04^3\right)}\left\{4\left[\frac{1.68(326.67)}{174\left(10^6\right)}\right]^2 + 3\left[\frac{1.32(107)}{390\left(10^6\right)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm:
The von Mises stress is the highest but it comes from the steady torque only.

$D/d = 30/20 = 1.5, \quad r/d = 2/20 = 0.1$

Fig. A-15-9: $K_{ts} = 1.42$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_{fs} = 1 + 0.79(1.42 - 1) = 1.33$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi\left(0.02^3\right)}\left(\sqrt{3}\right)\left[\frac{1.33(107)}{390\left(10^6\right)}\right]$$

$$n = 2.49$$

Note that since there is only a steady torque, Eq. (7-11) reduces to essentially the equivalent of the distortion energy failure theory.

Check the other locations.

If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

---

**7-25 and 7-26** With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints – explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.

- The adequacy of their design must be demonstrated and possibly include a designer's notebook.
- Many of the fundaments of the course, based on this text and this course, are useful. The student will find them useful and notice that he/she is doing it.
- Don't let the students create a time sink for themselves. Tell them how far you want them to go.

---

**7-27** This task was once given as a final exam problem. This problem is a learning experience. Following the task statement, the following guidance was added.

- Take the first half hour, resisting the temptation of putting pencil to paper, and decide what the problem really is.
- Take another twenty minutes to list several possible remedies.
- Pick one, and show your instructor how you would implement it.

The students' initial reaction is that he/she does not know much from the problem statement. Then, slowly the realization sets in that they do know some important things that the designer did not. They knew how it failed, where it failed, and that the design wasn't good enough; it was close, though.

Also, a fix at the bearing seat lead-in could transfer the problem to the shoulder fillet, and the problem may not be solved.

To many students' credit, they chose to keep the shaft geometry, and selected a new material to realize about twice the Brinell hardness.

---

**7-28** In Eq. (7-22) set

$$I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

to obtain

$$\omega = \left(\frac{\pi}{l}\right)^2 \left(\frac{d}{4}\right)\sqrt{\frac{gE}{\gamma}} \qquad (1)$$

or

$$d = \frac{4l^2\omega}{\pi^2}\sqrt{\frac{\gamma}{gE}} \qquad (2)$$

**(a)** From Eq. (1) and Table A-5

$$\omega = \left(\frac{\pi}{0.6}\right)^2 \left(\frac{0.025}{4}\right)\sqrt{\frac{9.81(207)(10^9)}{76.5\left(10^3\right)}} = 883 \text{ rad/s} \quad \textit{Ans.}$$

**(b)** From Eq. (1), we observe that the critical speed is linearly proportional to the diameter. Thus, to double the critical speed, we should double the diameter to $d$ = 50 mm. *Ans.*

**(c)** From Eq. (2),

$$l\omega = \frac{\pi^2}{4}\frac{d}{l}\sqrt{\frac{gE}{\gamma}}$$

Since $d / l$ is the same regardless of the scale,

$$l\omega = \text{constant} = 0.6(883) = 529.8$$
$$\omega = \frac{529.8}{0.3} = 1766 \text{ rad/s} \qquad \textit{Ans.}$$

Thus the first critical speed doubles.

---

**7-29** From Prob. 7-28, $\omega = 883$ rad/s

$$A = 4.909\left(10^{-4}\right) \text{ m}^2, \quad I = 1.917\left(10^{-8}\right) \text{m}^4, \quad \gamma = 7.65\left(10^4\right) \text{ N/m}^3$$
$$E = 207(10^9) \text{ Pa}, \quad w = A\gamma l = 4.909\left(10^{-4}\right)7.65\left(10^4\right)(0.6) = 22.53 \text{ N}$$

*One element*:

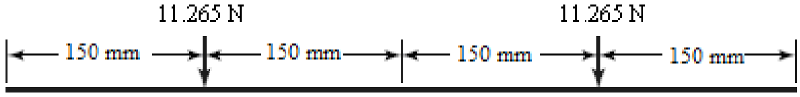
Eq. (7-24):

$$\delta_{11} = \frac{0.3(0.3)\left(0.6^2 - 0.3^2 - 0.3^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 1.134\left(10^{-6}\right) \text{m/N}$$

$$y_1 = w_1\delta_{11} = 22.53(1.134)\left(10^{-6}\right) = 2.555\left(10^{-5}\right) \text{ m}$$
$$y_1^2 = 6.528\left(10^{-10}\right)$$
$$\Sigma wy = 22.53(2.555)\left(10^{-5}\right) = 5.756\left(10^{-4}\right)$$
$$\Sigma wy^2 = 22.53(6.528)\left(10^{-10}\right) = 1.471\left(10^{-8}\right)$$

$$\omega_1 = \sqrt{g\frac{\Sigma wy}{\Sigma wy^2}} = \sqrt{9.81\frac{5.756\left(10^{-4}\right)}{1.471\left(10^{-8}\right)}} = 620 \text{ rad/s} \qquad \text{(30\% low)}$$

*Two elements:*



$$\delta_{11} = \delta_{22} = \frac{0.45(0.15)\left(0.6^2 - 0.45^2 - 0.15^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 6.379\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{12} = \delta_{21} = \frac{0.15(0.15)(0.6^2 - 0.15^2 - 0.15^2)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 4.961\left(10^{-7}\right) \text{ m/N}$$

$$y_1 = y_2 = w_1\delta_{11} + w_2\delta_{12} = 11.265(6.379)\left(10^{-7}\right) + 11.265(4.961)\left(10^{-7}\right) = 1.277\left(10^{-5}\right) \text{ m}$$

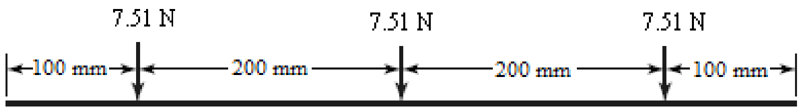
$$y_1^2 = y_2^2 = 1.632\left(10^{-10}\right)$$

$$\Sigma wy = 2(11.265)(1.277)\left(10^{-5}\right) = 2.877\left(10^{-4}\right)$$

$$\Sigma wy^2 = 2(11.265)(1.632)\left(10^{-10}\right) = 3.677\left(10^{-9}\right)$$

$$\omega_1 = \sqrt{9.81\left[\frac{2.877\left(10^{-4}\right)}{3.677\left(10^{-9}\right)}\right]} = 876 \text{ rad/s} \qquad (0.8\% \text{ low})$$

*Three elements:*




$$\delta_{11} = \delta_{33} = \frac{0.5(0.1)\left(0.6^2 - 0.5^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 3.500\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{22} = \frac{0.3(0.3)\left(0.6^2 - 0.3^2 - 0.3^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 1.134\left(10^{-6}\right) \text{ m/N}$$

$$\delta_{12} = \delta_{32} = \frac{0.3(0.1)\left(0.6^2 - 0.3^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 5.460\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{13} = \frac{0.1(0.1)\left(0.6^2 - 0.1^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 2.380\left(10^{-7}\right) \text{ m/N}$$

$$y_1 = 7.51\left[3.500\left(10^{-7}\right) + 5.460\left(10^{-7}\right) + 2.380\left(10^{-7}\right)\right] = 8.516\left(10^{-6}\right)$$

$$y_2 = 7.51\left[5.460\left(10^{-7}\right) + 1.134\left(10^{-6}\right) + 5.460\left(10^{-7}\right)\right] = 1.672\left(10^{-5}\right)$$

$$y_3 = 7.51\left[2.380\left(10^{-7}\right) + 5.460\left(10^{-7}\right) + 3.500\left(10^{-7}\right)\right] = 8.516\left(10^{-6}\right)$$

$$\Sigma wy = 7.51\left[8.516\left(10^{-6}\right) + 1.672\left(10^{-5}\right) + 8.516\left(10^{-6}\right)\right] = 2.535\left(10^{-4}\right)$$

$$\Sigma wy^2 = 7.51\left\{\left[8.516\left(10^{-6}\right)\right]^2 + \left[1.672\left(10^{-5}\right)\right]^2 + \left[8.516\left(10^{-6}\right)\right]^2\right\} = 3.189\left(10^{-9}\right)$$

$$\omega_1 = \sqrt{9.81\left[\frac{2.535\left(10^{-4}\right)}{3.189\left(10^{-9}\right)}\right]} = 883 \text{ rad/s}$$

The result is the same as in Prob. 7-28. The point was to show that convergence is rapid using a static deflection beam equation. The method works because:

- If a deflection curve is chosen which meets the boundary conditions of moment-free and deflection-free ends, as in this problem, the strain energy is not very sensitive to the equation used.
- Since the static bending equation is available, and meets the moment-free and deflection-free ends, it works.

---

**7-30** **(a)** For two bodies, Eq. (7-26) is

$$\begin{vmatrix} (m_1\delta_{11} - 1/\omega^2) & m_2\delta_{12} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) \end{vmatrix} = 0$$

Expanding the determinant yields,

$$\left(\frac{1}{\omega^2}\right)^2 - (m_1\delta_{11} + m_2\delta_{22})\left(\frac{1}{\omega_1^2}\right) + m_1 m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) = 0 \qquad (1)$$

Eq. (1) has two roots $1/\omega_1^2$ and $1/\omega_2^2$. Thus

$$\left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) = 0$$

or,

$$\left(\frac{1}{\omega^2}\right)^2 + \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\left(\frac{1}{\omega}\right)^2 + \left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = 0 \qquad (2)$$

Equate the third terms of Eqs. (1) and (2), which must be identical.

$$\frac{1}{\omega_1^2}\frac{1}{\omega_2^2} = m_1 m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) \quad \Rightarrow \quad \frac{1}{\omega_2^2} = \omega_1^2 m_1 m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})$$

and it follows that

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{g^2}{w_1 w_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})}} \qquad \textit{Ans.}$$

**(b)** In Ex. 7-5, part (*b*), the first critical speed of the two-disk shaft ($w_1$ = 35 lbf, $w_2$ = 55 lbf) is $\omega_1$ = 124.8 rad/s. From part (a), using influence coefficients,

$$\omega_2 = \frac{1}{124.8}\sqrt{\frac{386^2}{35(55)\left[2.061(3.534) - 2.234^2\right]\left(10^{-8}\right)}} = 466 \text{ rad/s} \qquad \textit{Ans.}$$

---

**7-31** In Eq. (7-22), for $\omega_1$, the term $\sqrt{I/A}$ appears. For a hollow uniform diameter shaft,

$$\omega_1 \propto \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\left(d_o^4 - d_i^4\right)/64}{\pi\left(d_o^2 - d_i^2\right)/4}} = \sqrt{\frac{1}{16}\frac{\left(d_o^2 + d_i^2\right)\left(d_o^2 - d_i^2\right)}{d_o^2 - d_i^2}} = \frac{1}{4}\sqrt{d_o^2 + d_i^2}$$

This means that when a solid shaft is hollowed out, the critical speed increases beyond that of the solid shaft of the same size. *Ans.*

By how much?

$$\frac{(1/4)\sqrt{d_o^2 + d_i^2}}{(1/4)\sqrt{d_o^2}} = \sqrt{1 + \left(\frac{d_i}{d_o}\right)^2}$$

The possible values of $d_i$ are $0 \le d_i \le d_o$, so the range of the critical speeds is

$\omega_1\sqrt{1+0}$ to about $\omega_1\sqrt{1+1}$

or from $\omega_1$ to $\sqrt{2}\,\omega_1$.

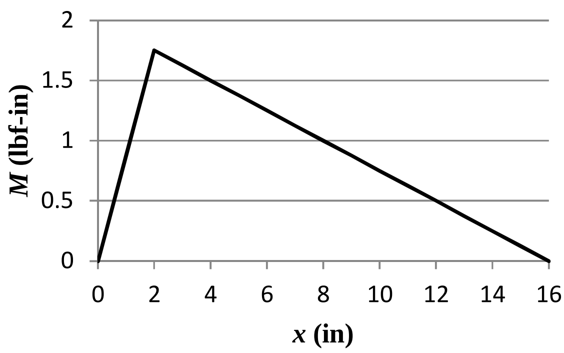
For the specific case where the inner diameter is half of the outer diameter, the ratio of the critical speeds is

$$\sqrt{1 + \left(\frac{d_i}{d_o}\right)^2} = \sqrt{1 + \left(\frac{\frac{1}{2}d_o}{d_o}\right)^2} = 1.12 \qquad \textit{Ans.}$$

---

**7-32** All steps will be modeled using singularity functions with a spreadsheet. Programming both loads will enable the user to first set the left load to 1, the right load to 0 and calculate $\delta_{11}$ and $\delta_{21}$. Then set the left load to 0 and the right to 1 to get $\delta_{12}$ and $\delta_{22}$. The spreadsheet shows the $\delta_{11}$ and $\delta_{21}$ calculation. A table for $M / I$ vs. $x$ is easy to make. First, draw the bending-moment diagram as shown with the data.

| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| *M* | 0 | 0.875 | 1.75 | 1.625 | 1.5 | 1.375 | 1.25 | 1.125 | 1 |




| *x* | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
|---|---|---|---|---|---|---|---|---|---|
| *M* | 0.875 | 0.75 | 0.625 | 0.5 | 0.375 | 0.25 | 0.125 | 0 | |

The second-area moments are:

$$0 \le x \le 1 \text{ in and } 15 \le x \le 16 \text{ in}, \quad I_1 = \pi\left(2^4\right)/64 = 0.7854 \text{ in}^4$$

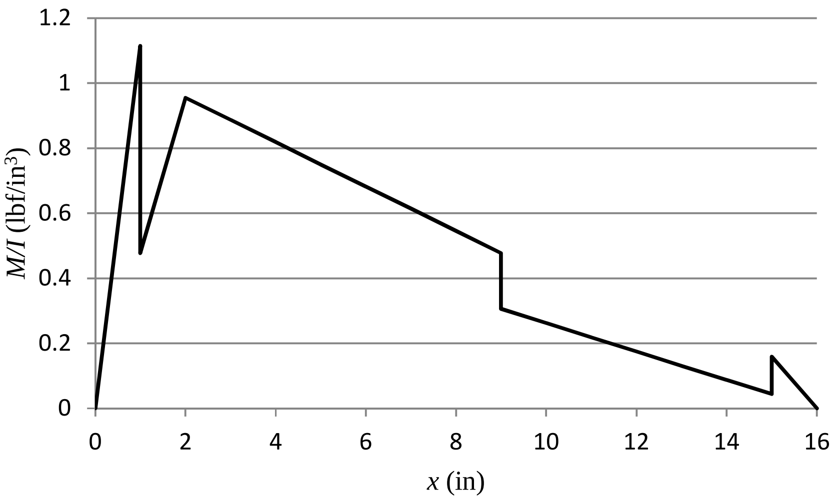
$$1 \le x \le 9 \text{ in}, \quad I_2 = \pi\left(2.472^4\right)/64 = 1.833 \text{ in}^4$$

$$9 \le x \le 15 \text{ in}, \quad I_3 = \pi\left(2.763^4\right)/64 = 2.861 \text{ in}^4$$

Divide $M$ by $I$ at the key points $x$ = 0, 1, 2, 9, 14, 15, and 16 in and plot

| *x* | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *M/I* | 0 | 1.1141 | 0.4774 | 0.9547 | 0.9547 | 0.8865 | 0.8183 | 0.7501 | 0.6819 | 0.6137 | 0.5456 |

| *x* | 9 | 9 | 10 | 11 | 12 | 13 | 14 | 14 | 15 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *M/I* | 0.4774 | 0.3058 | 0.2621 | 0.2185 | 0.1748 | 0.1311 | 0.0874 | 0.0874 | 0.0437 | 0.1592 | 0 |




From this diagram, one can see where changes in value (steps) and slope occur. Using a spreadsheet, one can form a table of these changes. An example of a step is, at $x$ = 1 in, $M/I$ goes from 0.875/0.7854 = 1.1141 lbf/in$^3$ to 0.875/1.833 = 0.4774 lbf/in$^3$, a step change of 0.4774 – 1.1141 = – 0.6367 lbf/in$^3$. A slope change also occurs at at $x$ = 1 in. The slope for $0 \leq x \leq 1$ in is 1.1141/1 = 1.1141 lbf/in$^2$, which changes to (0.9547 – 0.4774)/1 = 0.4774 lbf/in$^2$, a change of 0.4774 – 1.1141 = – 0.6367 lbf/in$^2$. Following this approach, a table is made of all the changes. The table shown indicates the column letters and row numbers for the spreadsheet.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | *x* | *M* | *M/I* | step | Slope | Δ Slope |
| 2 | 1a | 0.875 | 1.114085 | 0.000000 | 1.114085 | 0.000000 |
| 3 | 1b | 0.875 | 0.477358 | -0.636727 | 0.477358 | -0.636727 |
| 4 | 2 | 1.75 | 0.954716 | 0.000000 | 0.477358 | 0.000000 |
| 5 | 2 | 1.75 | 0.954716 | 0.000000 | -0.068194 | -0.545552 |
| 6 | 9a | 0.875 | 0.477358 | 0.000000 | -0.068194 | 0.000000 |
| 7 | 9b | 0.875 | 0.305854 | -0.171504 | -0.043693 | 0.024501 |
| 8 | 14 | 0.25 | 0.087387 | 0.000000 | -0.043693 | 0.000000 |
| 9 | 14 | 0.25 | 0.087387 | 0.000000 | -0.043693 | 0.000000 |
| 10 | 15a | 0.125 | 0.043693 | 0.000000 | -0.043693 | 0.000000 |
| 11 | 15b | 0.125 | 0.159155 | 0.115461 | -0.159155 | -0.115461 |
| 12 | 16 | 0 | 0.000000 | 0.000000 | -0.159155 | 0.000000 |

The equation for $M / I$ in terms of the spreadsheet cell locations is:

$$M / I = \mathrm{E2}\ (x) + \mathrm{D3}\ \langle x-1\rangle^0 + \mathrm{F3}\langle x-1\rangle^1 + \mathrm{F5}\langle x-2\rangle^1$$
$$+\mathrm{D7}\ \langle x-9\rangle^0 + \mathrm{F7}\ \langle x-9\rangle^1 + \mathrm{D11}\ \langle x-15\rangle^0 + \mathrm{F11}\ \langle x-15\rangle^1$$

Integrating twice gives the equation for $Ey$. Assume the shaft is steel. Boundary conditions $y = 0$ at $x = 0$ and at $x = 16$ inches provide integration constants ($C_1 = -4.906$ lbf/in and $C_2 = 0$). Substitution back into the deflection equation at $x = 2$ and 14 in provides the $\delta$'s. The results are: $\delta_{11} = 2.917(10^{-7})$ and $\delta_{12} = 1.627(10^{-7})$. Repeat for $F_1 = 0$ and $F_2 = 1$, resulting in $\delta_{21} = 1.627(10^{-7})$ and $\delta_{22} = 2.231(10^{-7})$. This can be verified by finite element analysis.

$$
\begin{aligned}
y_1 &= 18(2.917)(10^{-7}) + 32(1.627)(10^{-7}) = 1.046(10^{-5}) \\
y_2 &= 18(1.627)(10^{-7}) + 32(2.231)(10^{-7}) = 1.007(10^{-5}) \\
y_1^2 &= 1.093(10^{-10}), \quad y_2^2 = 1.014(10^{-10}) \\
\sum wy &= 5.105(10^{-4}), \quad \sum wy^2 = 5.212(10^{-9})
\end{aligned}
$$

Neglecting the shaft, Eq. (7-23) gives

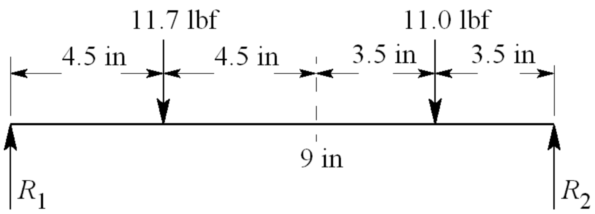
$$\omega_1 = \sqrt{386\frac{5.105(10^{-4})}{5.212(10^{-9})}} = 6149 \text{ rad/s} \quad \text{or} \quad 58\,720 \text{ rev/min} \qquad \textit{Ans.}$$

Without the loads, we will model the shaft using 2 elements, one between $0 \le x \le 9$ in, and one between $0 \le x \le 16$ in. As an approximation, we will place their weights at $x = 9/2 = 4.5$ in, and $x = 9 + (16 - 9)/2 = 12.5$ in. From Table A-5, the weight density of steel is $\gamma = 0.282$ lbf/in$^3$. The weight of the left element is

$$w_1 = \gamma\frac{\pi}{4}\sum d^2 l = 0.282\left(\frac{\pi}{4}\right)\left[2^2(1) + 2.472^2(8)\right] = 11.7 \text{ lbf}$$

The right element is

$$w_2 == 0.282\left(\frac{\pi}{4}\right)\left[2.763^2(6) + 2^2(1)\right] = 11.0 \text{ lbf}$$




The spreadsheet can be easily modified to give

$$
\begin{aligned}
&\delta_{11} = 9.605\left(10^{-7}\right), \quad \delta_{12} = \delta_{21} = 5.718\left(10^{-7}\right), \delta_{22} = 5.472\left(10^{-7}\right) \\
&y_1 = 1.753\left(10^{-5}\right), \quad y_2 = 1.271\left(10^{-5}\right) \\
&y_1^2 = 3.072\left(10^{-10}\right), \quad y_2^2 = 1.615\left(10^{-10}\right) \\
&\sum wy = 3.449\left(10^{-4}\right), \quad \sum wy^2 = 5.371\left(10^{-9}\right)
\end{aligned}
$$

$$\omega_1 = \sqrt{386\left[\frac{3.449\left(10^{-4}\right)}{5.371\left(10^{-9}\right)}\right]} = 4980 \text{ rad/s}$$

A finite element model of the exact shaft gives $\omega_1$ = 5340 rad/s. The simple model is 6.8% low.

*Combination*: Using Dunkerley's equation, Eq. (7-32):

$$\frac{1}{\omega_1^2} \approx \frac{1}{6149^2} + \frac{1}{4980^2} \quad \Rightarrow \quad \omega_1 \doteq 3870 \text{ rad/s} \quad \textit{Ans.}$$

---

**7-33** We must not let the basis of the stress concentration factor, as presented, impose a viewpoint on the designer. Table A-16 shows $K_{ts}$ as a decreasing monotonic as a function of $a/D$. All is not what it seems. Let us change the basis for data presentation to the full section rather than the net section.

$$\tau = K_{ts}\tau_0 = K'_{ts}\tau'_0$$
$$K_{ts} = \frac{32T}{\pi AD^3} = K'_{ts}\left(\frac{32T}{\pi D^3}\right)$$

Therefore

$$K'_{ts} = \frac{K_{ts}}{A}$$

Form a table:

| $(a/D)$ | $A$ | $K_{ts}$ | $K'_{ts}$ | |
|---|---|---|---|---|
| 0.050 | 0.95 | 1.77 | 1.86 | |
| 0.075 | 0.93 | 1.71 | 1.84 | |
| 0.100 | 0.92 | 1.68 | 1.83 | ← minimum |
| 0.125 | 0.89 | 1.64 | 1.84 | |
| 0.150 | 0.87 | 1.62 | 1.86 | |
| 0.175 | 0.85 | 1.60 | 1.88 | |
| 0.200 | 0.83 | 1.58 | 1.90 | |

$K'_{ts}$ has the following attributes:

- It exhibits a minimum;
- It changes little over a wide range;
- Its minimum is a stationary point minimum at $a / D \doteq 0.100$;

- Our knowledge of the minima location is
  $$0.075 \le (a / D) \le 0.125$$

We can form a design rule: In torsion, the pin diameter should be about 1/10 of the shaft diameter, for greatest shaft capacity, that is, $a \approx 0.10\ D$. *Ans.*

However, it is not catastrophic if one forgets the rule.

---

**7-34** From the solution to Prob. 3-72, the torque to be transmitted through the key from the gear to the shaft is $T$ = 2819 lbf·in. From Prob. 7-19, the nominal shaft diameter supporting the gear is 1.00 in. From Table 7-6, a 0.25 in square key is appropriate for a 1.00 in shaft diameter. The force applied to the key is

$$F = \frac{T}{r} = \frac{2819}{1.00/2} = 5638 \text{ lbf}$$

Selecting 1020 CD steel for the key, with $S_y$ = 57 kpsi, and using the distortion-energy theory, $S_{sy} = 0.577\ S_y = (0.577)(57) = 32.9$ kpsi.

Failure by shear across the key:

$$\tau = \frac{F}{A} = \frac{F}{tl}$$

$$n = \frac{S_{sy}}{\tau} = \frac{S_{sy}}{F / tl} \qquad \Rightarrow \qquad l = \frac{nF}{tS_{sy}} = \frac{1.1(5638)}{0.25(32\,900)} = 0.754 \text{ in}$$

Failure by crushing:

$$\sigma = \frac{F}{A} = \frac{F}{(t/2)l} \quad n = \frac{S_y}{\sigma} = \frac{S_y}{2F/(tl)} \qquad \Rightarrow \qquad l = \frac{2Fn}{tS_y} = \frac{2(5638)(1.1)}{0.25(57)(10^3)} = 0.870 \text{ in}$$

Select ¼-in square key, 7/8 in long, 1020 CD steel. *Ans.*

---

**7-35** From the solution to Prob. 3-73, the torque to be transmitted through the key from the gear to the shaft is $T$ = 3101 N·m. From Prob. 7-21, the nominal shaft diameter supporting the gear is 50 mm. To determine an appropriate key size for the shaft diameter, we can either convert to inches and use Table 7-6, or we can look up standard metric key sizes from the internet or a machine design handbook. It turns out that the recommended metric key for a 50 mm shaft is 14 x 9 mm. Since the problem statement specifies a square key, we will use a 14 x 14 mm key. For comparison, using Table 7-6 as a guide, for $d$ = 50 mm = 1.97 in, a 0.5 in square key is appropriate. This is equivalent to 12.7 mm. A 14 x 14 mm size is conservative, but reasonable after rounding up to standard sizes.

The force applied to the key is

$$F = \frac{T}{r} = \frac{3101}{0.050/2} = 124\left(10^3\right)\text{ N}$$

Selecting 1020 CD steel for the key, with $S_y$ = 390 MPa, and using the distortion-energy theory, $S_{sy}$ = 0.577 $S_y$ = 0.577(390) = 225 MPa.

Failure by shear across the key:

$$\tau = \frac{F}{A} = \frac{F}{tl}$$

$$n = \frac{S_{sy}}{\tau} = \frac{S_{sy}}{F/(tl)} \quad \Rightarrow \quad l = \frac{nF}{tS_{sy}} = \frac{1.1(124)\left(10^3\right)}{(0.014)(225)\left(10^6\right)} = 0.0433\text{ m} = 43.3\text{ mm}$$

Failure by crushing:

$$\sigma = \frac{F}{A} = \frac{F}{(t/2)l}$$

$$n = \frac{S_y}{\sigma} = \frac{S_y}{2F/(tl)} \quad \Rightarrow \quad l = \frac{2Fn}{tS_y} = \frac{2(124)\left(10^3\right)(1.1)}{(0.014)(390)\left(10^6\right)} = 0.0500\text{ m} = 50.0\text{ mm}$$

Select 14 mm square key, 50 mm long, 1020 CD steel. *Ans.*

---

**7-36** Choose basic size $D = d$ = 15 mm. From Table 7-9, a locational clearance fit is designated as 15H7/h6. From Table A-11, the tolerance grades are $\Delta D$ = 0.018 mm and $\Delta d$ = 0.011 mm. From Table A-12, the fundamental deviation is $\delta_F$ = 0 mm.

*Hole:*

Eq. (7-36): $D_{\max} = D + \Delta D$ = 15 + 0.018 = 15.018 mm *Ans.*

$D_{\min} = D$ = 15.000 mm *Ans.*

*Shaft:*

Eq. (7-37): $d_{\max} = d + \delta_F$ = 15.000 + 0 = 15.000 mm *Ans.*

$d_{\min} = d + \delta_F - \Delta d$ = 15.000 + 0 – 0.011 = 14.989 mm *Ans.*

---

**7-37** Choose basic size $D = d$ = 1.75 in. From Table 7-9, a medium drive fit is designated as H7/s6. From Table A-13, the tolerance grades are $\Delta D$ = 0.0010 in and $\Delta d$ = 0.0006 in. From Table A-14, the fundamental deviation is $\delta_F$ = 0.0017 in.

*Hole:*

Eq. (7-36): $D_{\max} = D + \Delta D$ = 1.75 + 0.0010 = 1.7510 in *Ans.*

$D_{min} = D = 1.7500$ in *Ans.*

*Shaft:*

Eq. (7-38): $d_{min} = d + \delta_F = 1.75 + 0.0017 = 1.7517$ in *Ans.*

$d_{max} = d + \delta_F + \Delta d = 1.75 + 0.0017 + 0.0006 = 1.7523$ in *Ans.*

---

**7-38** Choose basic size $D = d = 45$ mm. From Table 7-9, a sliding fit is designated as H7/g6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm. From Table A-12, the fundamental deviation is $\delta_F = -0.009$ mm.

*Hole:*

Eq. (7-36): $D_{max} = D + \Delta D = 45 + 0.025 = 45.025$ mm *Ans.*

$D_{min} = D = 45.000$ mm *Ans.*

*Shaft:*

Eq. (7-37): $d_{max} = d + \delta_F = 45.000 + (-0.009) = 44.991$ mm *Ans.*

$d_{min} = d + \delta_F - \Delta d = 45.000 + (-0.009) - 0.016 = 44.975$ mm *Ans.*

---

**7-39** Choose basic size $D = d = 1.250$ in. From Table 7-9, a close running fit is designated as H8/f7. From Table A-13, the tolerance grades are $\Delta D = 0.0015$ in and $\Delta d = 0.0010$ in. From Table A-14, the fundamental deviation is $\delta_F = -0.0010$ in.

*Hole:*

Eq. (7-36): $D_{max} = D + \Delta D = 1.250 + 0.0015 = 1.2515$ in *Ans.*

$D_{min} = D = 1.2500$ in *Ans.*

*Shaft:*

Eq. (7-37): $d_{max} = d + \delta_F = 1.250 + (-0.0010) = 1.2490$ in *Ans.*

$d_{min} = d + \delta_F - \Delta d = 1.250 + (-0.0010) - 0.0010 = 1.2480$ in *Ans.*

---

**7-40** Choose basic size $D = d = 35$ mm. From Table 7-9, a locational interference fit is designated as H7/p6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm. From Table A-12, the fundamental deviation is $\delta_F = 0.026$ mm.

*Hole:*

Eq. (7-36): $D_{max} = D + \Delta D = 35 + 0.025 = 35.025$ mm

$D_{min} = D = 35.000$ mm

The bearing bore specifications are within the hole specifications for a locational interference fit. Now find the necessary shaft sizes.

*Shaft:*

Eq. (7-38): $d_{min} = d + \delta_F = 35 + 0.026 = 35.026$ mm *Ans.*

$d_{max} = d + \delta_F + \Delta d = 35 + 0.026 + 0.016 = 35.042$ mm *Ans.*

---

**7-41** Choose basic size $D = d = 1.5$ in. From Table 7-9, a locational interference fit is designated as H7/p6. From Table A-13, the tolerance grades are $\Delta D = 0.0010$ in and $\Delta d = 0.0006$ in. From Table A-14, the fundamental deviation is $\delta_F = 0.0010$ in.

*Hole:*

Eq. (7-36): $D_{max} = D + \Delta D = 1.5000 + 0.0010 = 1.5010$ in

$D_{min} = D = 1.5000$ in

The bearing bore specifications exactly match the requirements for a locational interference fit. Now check the shaft.

*Shaft:*

Eq. (7-38): $d_{min} = d + \delta_F = 1.5000 + 0.0010 = 1.5010$ in

$d_{max} = d + \delta_F + \Delta d = 1.5000 + 0.0010 + 0.0006 = 1.5016$ in

The shaft diameter of 1.5020 in is greater than the maximum allowable diameter of 1.5016 in, and therefore does not meet the specifications for the locational interference fit. *Ans.*

---

**7-42** **(a)** Basic size is $D = d = 35$ mm.

Table 7-9: H7/s6 is specified for medium drive fit.

Table A-11: Tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm.

Table A-12: Fundamental deviation is $\delta_F = +0.043$ mm.

Eq. (7-36): $D_{max} = D + \Delta D = 35 + 0.025 = 35.025$ mm

$D_{min} = D = 35.000$ mm

Eq. (7-38): $d_{min} = d + \delta_F = 35 + 0.043 = 35.043$ mm *Ans.*

$d_{max} = d + \delta_F + \Delta d = 35 + 0.043 + 0.016 = 35.059$ mm *Ans.*

**(b)**

Eq. (7-42): $\delta_{min} = d_{min} - D_{max} = 35.043 - 35.025 = 0.018$ mm

Eq. (7-43): $\delta_{max} = d_{max} - D_{min} = 35.059 - 35.000 = 0.059$ mm

Eq. (7-40): $$p_{max} = \frac{E\delta_{max}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{207\left(10^9\right)\left(0.059\right)}{2\left(35^3\right)}\left[\frac{\left(60^2 - 35^2\right)\left(35^2 - 0\right)}{60^2 - 0}\right] = 115 \text{ MPa} \quad \textit{Ans.}$$

$$p_{\min} = \frac{E\delta_{\min}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{207\left(10^9\right)(0.018)}{2\left(35^3\right)}\left[\frac{\left(60^2 - 35^2\right)\left(35^2 - 0\right)}{60^2 - 0}\right] = 35.1 \text{ MPa} \qquad \textit{Ans.}$$

**(c)** For the shaft:

Eq. (7-44): $\sigma_{t,\text{shaft}} = -p = -115 \text{ MPa}$

Eq. (7-46): $\sigma_{r,\text{shaft}} = -p = -115 \text{ MPa}$

Eq. (5-13): $\sigma' = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)^{1/2}$

$$= \left[(-115)^2 - (-115)(-115) + (-115)^2\right]^{1/2} = 115 \text{ MPa}$$

$$n = S_y / \sigma' = 390/115 = 3.4 \qquad \textit{Ans.}$$

For the hub:

Eq. (7-45): $\sigma_{t,\text{hub}} = p\dfrac{d_o^2 + d^2}{d_o^2 - d^2} = 115\left(\dfrac{60^2 + 35^2}{60^2 - 35^2}\right) = 234 \text{ MPa}$

Eq. (7-46): $\sigma_{r,\text{hub}} = -p = -115 \text{ MPa}$

Eq. (5-13): $\sigma' = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)^{1/2}$

$$= \left[(234)^2 - (234)(-115) + (-115)^2\right]^{1/2} = 308 \text{ MPa}$$

$$n = S_y / \sigma' = 600/308 = 1.9 \qquad \textit{Ans.}$$

**(d)** A value for the static coefficient of friction for steel to steel can be obtained online or from a physics textbook as approximately $f = 0.8$.

Eq. (7-49) $T = (\pi/2) f p_{\min} l d^2$

$$= (\pi/2)(0.8)(35.1)\left(10^6\right)(0.050)(0.035)^2 = 2700 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

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