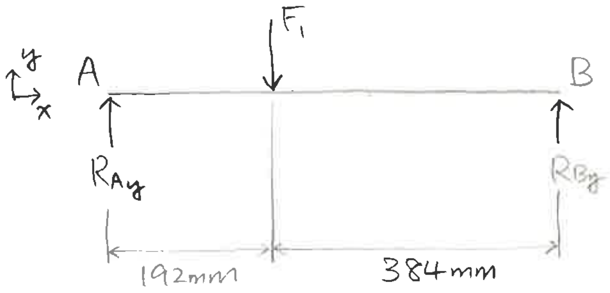
$F_1 = 9\ kN \qquad F_2 = 12\ kN$

x–y plane

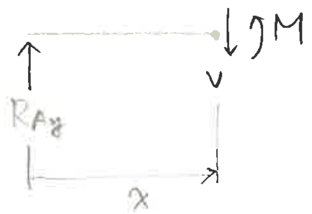


$\sum F_y = 0;\ R_{Ay} - F_1 + R_{By} = 0$

$\sum M_A = 0;\ -F_1\ (192mm) + R_{By}(576mm) = 0$

$\Rightarrow R_{Ay} = 6kN \qquad R_{By} = 3kN$

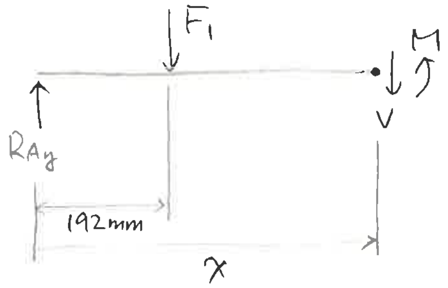
$0 \le x < 192\ mm$



$\sum M_x = 0;\ M - R_{Ay} \cdot x = 0$

$\Rightarrow M = 6x \quad (kN \cdot m)$

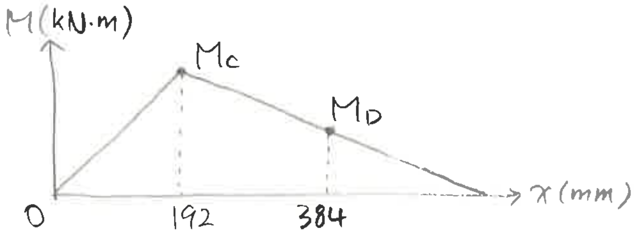
$192 \le x \le 576\ mm$



$\sum M_x = 0;\ M + F_1(x-192) - R_{Ay} x = 0$

$\Rightarrow M = -3x + 1.728 \quad (kN/m)$

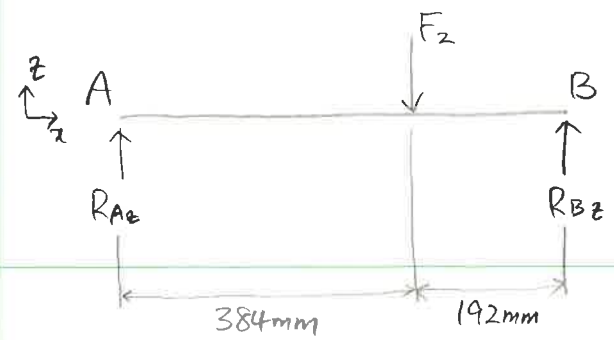
$\Rightarrow$



$M_{C_z} = 1.152\ kN/m$

$M_{D_z} = 0.576\ kN/m$

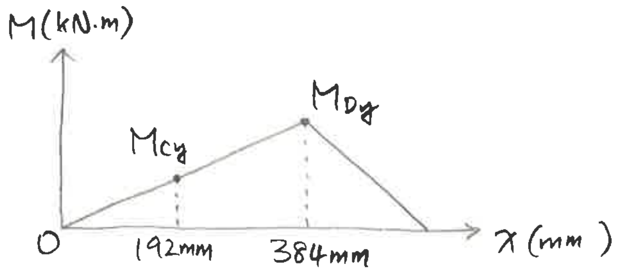
x–z plane



$\sum F_z = 0;\ R_{Az} - F_2 + R_{Bz} = 0$

$\sum M_A = 0;\ -F_2\ (384mm) + R_{Bz}(576mm) = 0$

$\Rightarrow R_{Az} = 4kN \qquad R_{Bz} = 8kN$



$0 \leq x < 384$: $M = R_{Az}\, x$

$384 \leq x \leq 576$: $M = (R_{Az} - F_2)\, x + F_2 \cdot (384\,mm)$

$M_{Cy} = 0.768$ kN/m

$M_{Dy} = 1.536$ kN·m

At $x = 192$ mm, $M = 1.385$ kN·m

$x = 384$ mm, $M = 1.640$ kN·m

Bending moment at D is higher, but

$$\sigma_C = \frac{Mc}{I} = \frac{(1.385\ kN\cdot m)(20mm)}{\frac{\pi\,(40mm)^4}{64}} = 220\,MPa$$

$$\sigma_D = \frac{Mc}{I} = \frac{(1.640\ kN\cdot m)(45/2\ mm)}{\frac{\pi\,(45mm)^4}{64}} = 183\ MPa$$

C is more critical than D is.

DET : $\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$

At C, $\sigma_x = 220\,MPa$, $\tau_{xy} = \frac{Tr}{J} = \frac{(1.2\,kN\cdot m)(20mm)}{\frac{\pi\,(40mm)^4}{32}} = 95\ MPa$, $\sigma_y = 0$

$\Rightarrow \sigma_C' = 275\,MPa.$

$n_C = \frac{S_y}{\sigma_C'} = 0.907$ Yielding occurs at C.

At D, $\sigma_x = 183\ MPa$, $\sigma_y = 0$, $\tau_{xy} = \frac{(1.2\,kN\cdot m)(45/2\,mm)}{\frac{\pi\,(45mm)^4}{32}} = 67\ MPa$

$\Rightarrow \sigma_D' = 217\ MPa$

$n_D = \frac{S_y}{\sigma_D'} = 1.15$ Yielding is not present.

At $x = 252\,mm$,

$$M_z = -3\cdot(252mm) + 1.728 = 0.972 \ kN\cdot m$$

$$M_y = 4\cdot(252mm) = 1.008 \ kN\cdot m$$

$$M = \sqrt{M_z^2 + M_y^2} = 1.4 \ kN\cdot m$$

From the figure for K_t when $\frac{r}{d} = \frac{1.5}{40} = 0.0375$ $\frac{D}{d} = \frac{45}{40} = 1.125$

$$K_t = 2.3$$

From the figure for K_t when $S_{ut} = 590MPa$ and $r = 1.5mm$

$$q = 0.76$$

$$K_f = 1 + q(K_t - 1) = 1 + (0.76)(2.3-1) = 1.988$$

$$\Rightarrow \sigma = K_f \frac{Mc}{I} = (1.988)\frac{(1.4kN\cdot m)(20mm)}{\frac{\pi(40mm)^4}{64}} = 441 \ MPa$$

At $x = 492\,mm$

$$M_z = -3(492mm) + 1.728 = 0.252 \ kN\cdot m$$

$$M_y = -4(492mm) + 0.768 = 0.672 \ kN\cdot m$$

$$\Rightarrow M = 0.718 \ kN\cdot m$$

From the figure for K_t when $\frac{r}{d} = \frac{15}{42} = 0.0357$ and $\frac{D}{d} = \frac{45}{42} = 1.071$

$$K_t = 1.96$$

$$\Rightarrow K_f = 1 + (0.76)(1.96-1) = 1.73$$

$$\Rightarrow \sigma = K_f \frac{Mc}{I} = (1.73)\frac{(0.718kN\cdot m)(21mm)}{\frac{\pi(42mm)^4}{64}} = 171 \ MPa$$

The fillet at $x = 252\,mm$ is the critical location.

$S_e' = 0.5\,S_{ut} = 0.5\,(590\text{ MPa}) = 295\text{ MPa}$

$S_e = k_a k_b k_c S_e'$

$k_a = a S_{ut}^{b} = 4.51(590)^{-0.265} = 0.832$

$k_b = (40/7.62)^{-0.107} = 0.837$

$k_c = 1.$

$\Rightarrow S_e = (0.832)(0.837)(1)(295) = 205.4\text{ MPa}$

$$\Rightarrow n_f = \frac{S_e}{\sigma} = \frac{205.4}{441} = 0.466$$

$n_f < 1 \Rightarrow$ Infinite life is not predicted.

From the figure for f when $S_{ut} = 590$ MPa, $f = 0.867$

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.867 \cdot 590)^2}{205.4} = 1274$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = -0.132$$

$$N = \left(\frac{S_f}{a}\right)^{\frac{1}{b}} = \left(\frac{441}{1274}\right)^{-\frac{1}{0.132}} = 3093 \text{ cycles.}$$