CSE23201: Discrete Mathematics
Assignment 1: Suggested answers

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1. Let $A = \{0, 1\}$ and $B = \{1, 2\}$. What are $A \cup B$, $A \cap B$, $A - B$, $A \times B$ and $\mathcal{P}(B)$?

Answer.

$$\begin{aligned} A \cup B &= \{0, 1, 2\} \\ A \cap B &= \{1\} \\ A - B &= \{0\} \\ A \times B &= \{(0, 1), (0, 2), (1, 1), (1, 2)\} \\ \mathcal{P}(B) &= \{\emptyset, \{1\}, \{2\}, \{1, 2\}\} \end{aligned}$$

2. Use the set builder notation to give a description of the set below.

$$S = \left\{-1, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1\right\}$$

Answer.

$$S = \left\{\frac{n}{3} \mid n \in \mathbb{Z} \text{ and } -3 \leqslant n \leqslant 3\right\}.$$

3. For any positive integer p, let $p\mathbb{N} = \{0, p, 2p, 3p, \dots\}$ be the set of the multiples of p. What is $2\mathbb{N} \cap 3\mathbb{N}$? No justification is needed.

Answer. $2\mathbb{N} \cap 3\mathbb{N} = 6\mathbb{N}$

4. Consider the identity $(A - B) \cap C = (A \cap C) - B$.

(a) Explain why this identity is true using Venn diagrams.

(b) Show that this identity is true using the set identities given in Lecture 1.

Answer a. See Figure 1.

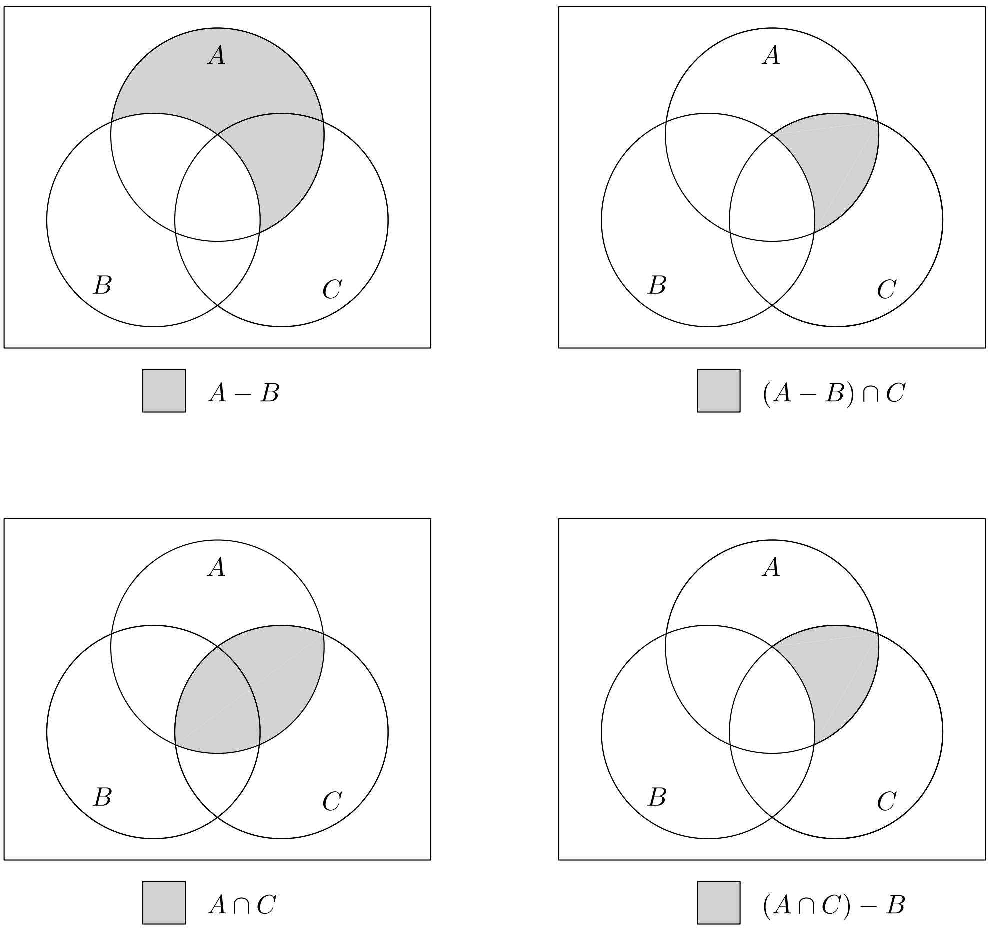

Figure 1: The two shaded areas on the right side are identical, which shows that $(A - B) \cap C = (A \cap C) - B$

Answer b.

$$
\begin{aligned}
(A - B) \cap C &= (A \cap \bar{B}) \cap C && \text{by the identity on Slide 29} \\
&= C \cap (A \cap \bar{B}) && \text{by commutative law} \\
&= (C \cap A) \cap \bar{B} && \text{by associative law} \\
&= (A \cap C) \cap \bar{B} && \text{by commutative law} \\
&= (A \cap C) - B && \text{by the identity on Slide 29}
\end{aligned}
$$

5. Let A, B and C be three sets. Is the identity below true? Justify your answer.

$$(A \cap C) \times (B \cap C) = (A \times B) \cap C^2$$

Answer. Yes, this identity is true. Here is a proof.

$$
\begin{aligned}
(A \cap C) \times (B \cap C) &= \{(x, y) \mid x \in A \cap C \text{ and } y \in B \cap C\} \\
&= \{(x, y) \mid x \in A \text{ and } x \in C \text{ and } y \in B \text{ and } y \in C\} \\
&= \{(x, y) \mid x \in A \text{ and } y \in B \text{ and } x \in C \text{ and } y \in C\} \\
&= \{(x, y) \mid (x, y) \in A \times B \text{ and } (x, y) \in C^2\} \\
&= (A \times B) \cap C^2
\end{aligned}
$$

6. Let $S = \{p, q, r\}$ be a set. Determine whether the following statements are true (T) or false (F).

- $\{p, r\} \in \mathcal{P}(S)$
- $\{p, r\} \in S$
- $\{p, r\} \not\subseteq \mathcal{P}(S)$
- $\{\emptyset\} \in \mathcal{P}(S)$

Answer. We see that the power set $\mathcal{P}(S) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, \{p, r\}, \{p, q, r\}\}$.
1. T: Since $\{p, r\} \subset S$ and the power set $\mathcal{P}(S)$ is the set of all the subsets of S, we have $\{p, r\} \in \mathcal{P}(S)$
2. F: $\{p, r\}$ is not an element of S
3. T: $\{p, r\}$ is not a subset of $\mathcal{P}(S)$.
4. F: $\{\emptyset\}$ is not an element of $\mathcal{P}(S)$.

7. In the Discrete mathematics class, there are 73 students. 40 of them take the Algorithms course, 23 take Basic circuit theory, and 31 take Computer graphics. 13 of them take Algorithms and Basic circuit theory, 9 take Basic circuit theory and Computer graphics, and 17 take Computer graphics and Algorithms. 13 of them take neither Algorithms, Basic circuit theory, nor Computer graphics.

How many students of the Discrete mathematics class take Algorithms, Basic circuit theory and Computer graphics?

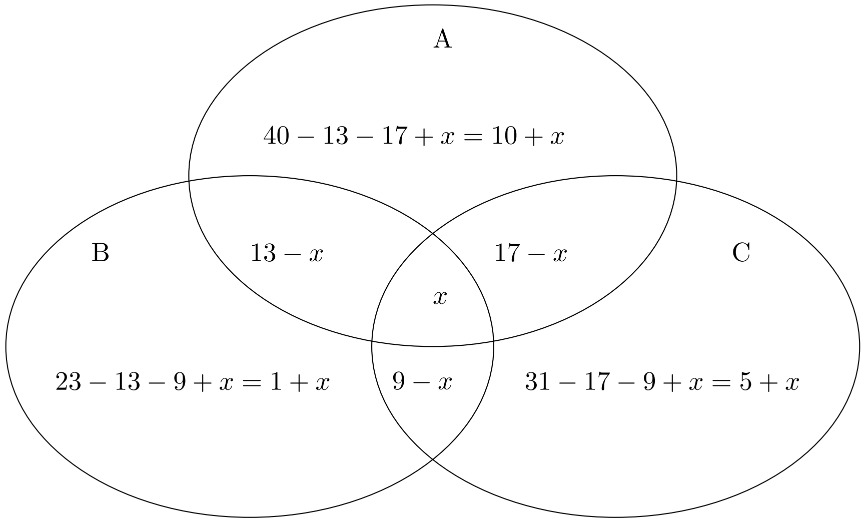

Figure 2: Answer to Question 6.

Answer. Let A, B and C denote the set of students in the Discrete mathematics class who take Algorithms, Basic circuit theory, and Computer graphics, respectively.

From the problem statement, we know that:

$$\begin{aligned}|A| &= 40\\ |B| &= 23\\ |C| &= 31\\ |A\cap B| &= 13\\ |B\cap C| &= 9\\ |C\cap A| &= 17\\ |A\cup B\cup C| &= 73-13=60\end{aligned}$$

Let $x=|A\cap B\cap C|$. We now draw the Venn diagram of A, B and C, and write the size of each region inside of it. (See Figure 2.) If we sum up these sizes, we obtain

$$60 = x+13-x+17-x+9-x+10+x+5+x+1+x = 55+x$$

and thus the answer is $x=5$.