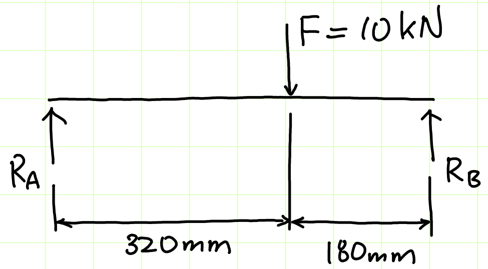
(a)

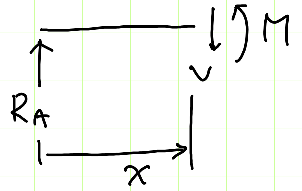


$\sum F_y = 0;\ R_A + R_B - F = 0$

$\sum M_A = 0;\ -(320\text{mm})F + (500\text{mm})R_B = 0$

$\Rightarrow R_A = 3.6\text{kN},\quad R_B = 6.4\text{kN}$

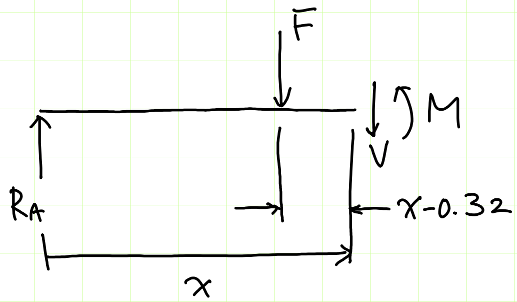
(i) $0 \leqq x < 320\text{mm}$



$\sum F_y = 0;\ R_A - V = 0 \Rightarrow V = 3.6\text{kN}$

$\sum M_x = 0;\ M - R_A x = 0 \Rightarrow M = 3.6x\ \text{kN.m}$

(ii) $320\text{mm} \leqq x \leqq 500\text{mm}$

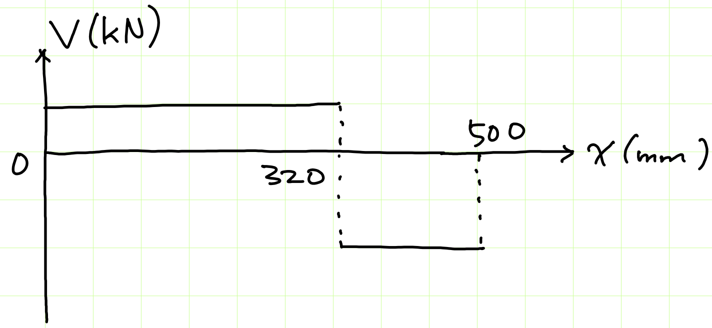


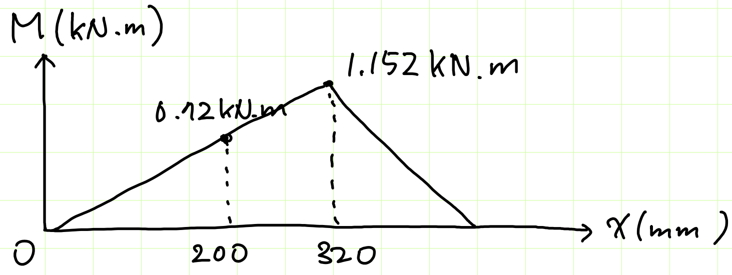
$\sum F_y = 0;\ R_A - F - V = 0$

$\Rightarrow V = R_A - F = -6.4\text{kN}$

$\sum M_x = 0;\ M - R_A x + (x - 0.32)F$

$\Rightarrow M = -6.4x + 3.2\ \text{kN.m}$





(b) Bending stress at C:

$$\sigma_c = \frac{M_c C_c}{I_c} = \frac{(1.152\ kN.m)(40mm/2)}{\pi (40\ mm)^4/64} = 183.3\ MPa$$

Shear stress due to torque at C

$$\tau_c = \frac{T r_c}{J} = \frac{(4.0\ N.m)(40mm/2)}{\pi (40mm)^4/32} = 0.318\ MPa$$

von Mises stress at C:

$$\sigma_c' = (\sigma_c^2 + 3\tau_c^2)^{1/2} = 183.3\ MPa$$

Bending stress at D:

$$\sigma_D = \frac{M_D C_D}{I_D} = \frac{(0.72 kN.m)(32\ mm/2)}{\pi (32mm)^4/64} = 223.8\ MPa$$

Shear stress due to torque at D:

$$\tau_D = \frac{T r_D}{J_D} = \frac{(4.0N)(32mm/2)}{\pi (32mm)^4/32} = 0.622\ MPa$$

von Mises stress at D:

$$\sigma_D' = (\sigma_D^2 + 3\tau_D^2)^{1/2} = 223.8\ MPa$$

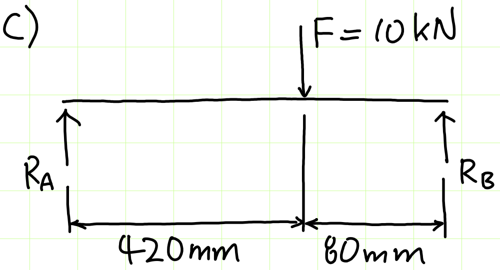
$\sigma_c' < \sigma_D'$ $\therefore$ D is the critical location.

$$n_D = \frac{S_Y}{\sigma_D'} = \frac{210\ MPa}{223.8 MPa} = 0.94 < 1$$

$$n_C = \frac{S_Y}{\sigma_c'} = \frac{210 MPa}{183.3 MPa} > 1.$$

$\Rightarrow$ Yielding failure is predicted at the critical location.

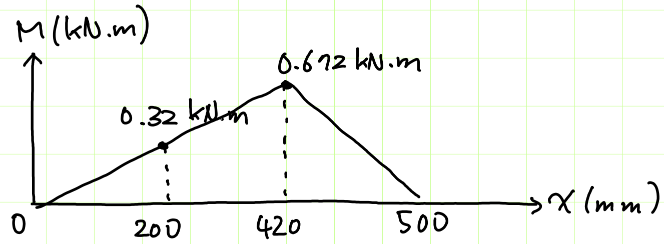
(c)



$\sum F_y = 0;\ R_A + R_B - F = 0$

$\sum M_A = 0;\ -(420\text{mm})F + (500\text{mm})R_B = 0$

$\Rightarrow R_A = 1.6\text{kN},\ R_B = 8.4\text{kN}$



Bending stress at C:

$$\sigma_C = \frac{M_C C_C}{I_C} = \frac{(0.672\text{ kN.m})(40\text{mm}/2)}{\pi(40\text{mm})^4/64} = 107.0\text{ MPa}$$

Bending stress at D:

$$\sigma_D = K_t \frac{M_D C_D}{I_D} = (1.6764)\frac{(0.32\text{ kN.m})(32\text{ mm}/2)}{\pi(32\text{mm})^4/64} = 166.8\text{MPa}$$

$\Rightarrow$ D is the critical location!

K_t is obtained from Figue A-15-9 when D=40 mm, d=32mm, and r=3mm or

$$\frac{D}{d} = 1.25,\quad \frac{r}{d} = 0.09375$$

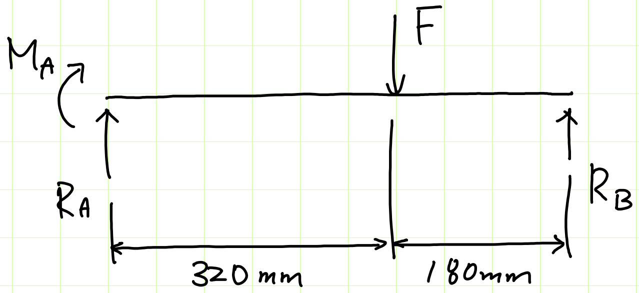
$$\Rightarrow K_t = 1.6764$$

$$n_D = \frac{S_{ut}}{\sigma_D} = \frac{150\text{MPa}}{166.8\text{MPa}} = 0.90 < 1.$$

$\therefore$ Fracture failure is predicted at D.

(d) Since the shaft is sufficiently long, the transverse shear stress due to bending is ignored here.

A fictitious moment is placed at the support A.



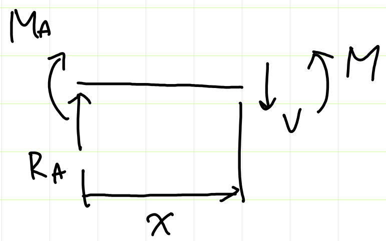
$$\sum F_y = 0\ ;\quad R_A + R_B - F = 0$$

$$\sum M_A = 0\ ;\ -M_A - 0.32F + 0.5R_B = 0$$

$$\Rightarrow R_B = 0.64F + 2M_A$$

$$R_A = -R_B + F = 0.36F - 2M_A$$

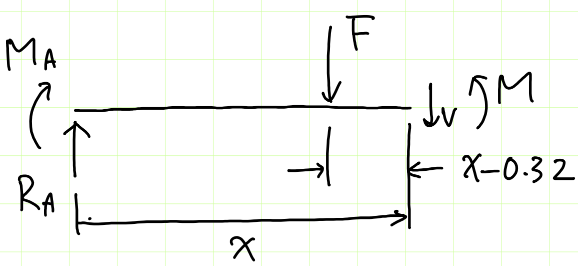
(i) $0 \leqq x < 320\text{mm}$



$$\sum M_x = 0\ ;\quad M - M_A - R_A x = 0$$

$$\Rightarrow M = M_A + R_A x$$

(ii) $320 \leqq x \leqq 500\text{ mm}$



$$\sum M_x = 0\ ;$$

$$M - M_A - R_A x + F(x - 0.32) = 0$$

$$\Rightarrow M = M_A + R_A x - F(x - 0.32)$$

Strain energy due to bending :

$$U = \frac{1}{2}\int_0^{l_D} \frac{M^2}{EI_1}\,dx + \frac{1}{2}\int_{l_D}^{l_c} \frac{M^2}{EI_2}\,dx + \frac{1}{2}\int_{l_c}^{L} \frac{M^2}{EI_2}\,dx$$

$$l_D = 200\,\text{mm}, \quad l_c = 320\,\text{mm}, \quad L = 500\,\text{mm}$$

$$I_1 = \frac{\pi(32\,\text{mm})^4}{64}, \quad I_2 = \frac{\pi(40\,\text{mm})^4}{64}$$

$E = 71.7$ GPa from Table A-5

$$U = \frac{1}{2}\int_0^{l_D} \frac{(M_A + R_A x)^2}{EI_1}\,dx + \frac{1}{2}\int_{l_D}^{l_c} \frac{(M_A + R_A x)^2}{EI_2}\,dx$$

$$+ \frac{1}{2}\int_{l_c}^{L} \frac{\left[M_A + R_A x - F(x - 0.32)\right]^2}{EI_2}\,dx$$

$$\theta_A = \left.\frac{dU}{dM_A}\right|_{M_A=0} = \int_0^{l_D} \frac{\cancelto{0}{M_A} + R_A x}{EI_1}(1 - 2x)\,dx$$

$$\boxed{\begin{aligned} &R_A = 0.36F - 2M_A \\ &\frac{dR_A}{dM_A} = -2 \end{aligned}}$$

$$+ \int_{l_D}^{l_c} \frac{\cancelto{0}{M_A} + R_A x}{EI_2}(1 - 2x)\,dx$$

$$+ \int_{l_c}^{L} \frac{\cancelto{0}{M_A} + R_A x - F(x - 0.32)}{EI_2}(1 - 2x)\,dx$$

$$= \int_0^{l_D} \frac{(0.36F)x}{EI_1}(1 - 2x)\,dx$$

$$+ \int_{l_D}^{l_c} \frac{(0.36F)x}{EI_2}(1 - 2x)\,dx$$

$$+ \int_{l_D}^{L} \frac{(0.36F)x - F(x - 0.32)}{EI_2}(1 - 2x)\,dx$$

$$= 0.0229\ \text{rad} = 1.3^\circ$$