Set 8.1)

Problem 18

$$y^{(4)} = -2yy''$$

$$y(0) = y'(0) = 0 \qquad y(4) = 0 \qquad y'(4) = 1$$

```
function p8_1_18
% Shooting method for 4th-order boundary value problem
% in Problem 18, Problem Set 1.

xStat = 0; xStop = 4;   % Range of integration.
h = 0.1;                % Step size.
freq = 5;               % Frequency of printout.
u = [0 0];              % Trial values of u(1)
                        % and u(2).
x = xStat;
u = newtonRaphson2(@residual,u,1e-10);
[xSol,ySol] = runKut4(@dEqs,x,inCond(u),xStop,h);
printSol(xSol,ySol,freq)

    function F = dEqs(x,y) % Differential equations.
    F = zeros(1,4);
    F(1) = y(2); F(2) = y(3); F(3) = y(4);
    F(4) = -2*y(1)*y(3);
    end

    function y = inCond(u) % Initial conditions; u(1)
    y = [0 0 u(1) u(2)]; % and u(2) are unknowns.
    end

    function r = residual(u) % Bounday residuals.
    r = zeros(length(u),1);
    x = xStat;
    [xSol,ySol] = runKut4(@dEqs,x,inCond(u),xStop,h);
    lastRow = size(ySol,1);
    r(1) = ySol(lastRow,1);
    r(2) = ySol(lastRow,2) - 1;
    end
end
```

```
>> p8_1_18
     x              y1            y2            y3            y4
   0.0000e+00    0.0000e+00    0.0000e+00   -3.6774e-01    2.6832e-01
   5.0000e-01   -4.0382e-02   -1.5038e-01   -2.3409e-01    2.6459e-01
   1.0000e+00   -1.3938e-01   -2.3485e-01   -1.0494e-01    2.5107e-01
   1.5000e+00   -2.6476e-01   -2.5644e-01    1.7941e-02    2.4367e-01
   2.0000e+00   -3.8559e-01   -2.1638e-01    1.4452e-01    2.7113e-01
   2.5000e+00   -4.6971e-01   -1.0713e-01    3.0041e-01    3.6666e-01
   3.0000e+00   -4.7723e-01    9.5906e-02    5.2803e-01    5.6249e-01
   3.5000e+00   -3.5014e-01    4.4173e-01    8.7983e-01    8.5484e-01
   4.0000e+00   -7.7875e-12    1.0000e+00    1.3714e+00    1.0619e+00
```

Problem 19

$$\ddot{x} = -\frac{c}{m} v\,\dot{x} \qquad \ddot{y} = -\frac{c}{m} v\,\dot{y} - g \qquad v = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$x(0) = y(0) = 0 \qquad x(10\ \mathrm{s}) = 8000\ \mathrm{m} \qquad y(10\ \mathrm{s}) = 0$$

We use the notation

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

```
function p8_1_19
% Shooting method for 4th-order boundary value problem
% in Problem 19, Problem Set 1.

xStart = 0; xStop = 10;      % Range of integration.
h = 2;                       % Step size.
freq = 0;                    % Frequency of printout.
u = [1000 600];              % Trial values of u(1)
                             % and u(2).
x = xStart;
u = newtonRaphson2(@residual,u);
[xSol,ySol] = bulStoer(@dEqs,x,inCond(u),xStop,h);
printSol(xSol,ySol,freq)

    function F = dEqs(x,y) % Differential equations.
    c = 3.2e-4; m = 20; g = 9.80665;
    v = sqrt(y(2)^2 + y(4)^2);
    F = zeros(1,4);
    F(1) = y(2); F(2) = -c/m*v*y(2);
    F(3) = y(4); F(4) = -c/m*v*y(4) - g;
    end

    function y = inCond(u) % Initial conditions; u(1)
    y = [0 u(1) 0 u(2)]; % and u(2) are unknowns.
    end

    function r = residual(u) % Bounday residuals.
    r = zeros(length(u),1);
    x = xStart;
    [xSol,ySol] = bulStoer(@dEqs,x,inCond(u),xStop,h);
    n = size(ySol,1);
    r(1) = ySol(n,1) - 8000;
    r(2) = ySol(n,3);
    end
end
```

x	y1	y2	y3	y4
0.0000e+00	0.0000e+00	8.5349e+02	0.0000e+00	5.0150e+01
1.0000e+01	8.0000e+03	7.5089e+02	1.0248e-12	-4.8051e+01

$$v_0 = \sqrt{\dot{x}_0^2 + \dot{y}_0^2} = \sqrt{853.49^2 + 50.150^2} = 854.96$$

$$\theta = \tan^{-1}\frac{50.150}{853.49} = 0.058691\ \mathrm{rad} = 3.363°$$

Set 4.1)

Problem 2

$$f(x) = x^3 - 3.23x^2 - 5.54x + 9.84$$

We begin with a root search starting at $x = 1$ and launch bisection once the root is bracketed.

x	$f(x)$	Interval
1.0	2.070	
1.2	0.269	
1.4	-1.503	$(1.2, 1.4)$
$(1.2 + 1.4)/2 = 1.3$	-0.624	$(1.2, 1.3)$
$(1.2 + 1.3)/2 = 1.25$	-0.179	$(1.2, 1.25)$
$(1.2 + 1.25)/2 = 1.225$	0.045	$(1.225, 1.25$
$(1.225 + 1.25)/2 = 1.2375$	-0.067	$(1.225, 1.2375)$
$(1.225 + 1.2375)/2 = 1.2313$	-0.012	$(1.225, 1.2313)$
$(1.225 + 1.2313)/2 = 1.2282$	0.017	$(1.2282, 1.2313)$
$(1.2282 + 1.2313)/2 = 1.2298$	0.002	$(1.2298, 1.2313)$
$(1.2298 + 1.2313)/2 = 1.2306$	-0.005	$(1.2298, 1.2306$
$(1.2298 + 1.2306)/2 = 1.2302$	-0.002	$(1.2298, 1.2302)$

The root is $x = 1.230$ ◄

Problem 3

The starting points are

$$x_1 = 4 \quad f_1 = \cosh(4)\cos(4) - 1 = -18.8499$$

$$x_2 = 5 \quad f_2 = \cosh(5)\cos(5) - 1 = 20.0506$$

Bisection yields the point

$$x_3 = 4.5 \quad f_3 = \cosh(4.5)\cos(4.5) - 1 = -10.4888$$

The improved root can be estimated with Ridder's formula:

$$x = x_3 \pm (x_3 - x_1)\frac{f_3}{\sqrt{f_3^2 - f_1 f_2}}$$

if $f_1 - f_2 < 0$, choose $+$ sign

if $f_1 - f_2 > 0$, choose $-$ sign

Here are the results of the iterations:

x_1	x_2	x_3	f_1	f_2	f_3	x	$f(x)$
4	5	4.5	-18.8499	20.0506	-10.4888	4.7374	0.4280
4.5	4.7374	4.6187	-10.4888	0.4280	-5.7415	4.7301	0.0017
4.6187	4.7301	4.6744	-5.7415	0.0017	-3.0359	4.7300	1.7308e-6
4.6744	4.7300	4.7022	-3.0359	1.7308e-6	-1.5606	4.7300	0

Hence the root $x = 4.7300$

Problem 4

Newton's formula is

$$x \leftarrow x - \frac{f(x)}{f'(x)}$$

where

$$\begin{aligned} f(x) &= \cosh x \cos x - 1 \\ f'(x) &= \sinh x \cos x - \cosh x \sin x \end{aligned}$$

Starting with $x = 4.5$, successive applications of the formula yield

$$\begin{aligned} x &\leftarrow 4.5 - \frac{-10.489}{34.52} = 4.804 \\ x &\leftarrow 4.804 - \frac{4.573}{66.31} = 4.735 \\ x &\leftarrow 4.735 - \frac{0.283}{58.20} = 4.730 \\ x &\leftarrow 4.730 - \frac{0.001}{57.65} = 4.730 \blacktriangleleft \end{aligned}$$

Problem 6

$$
\begin{aligned}
f(x) &= \sin x + 3\cos x - 2 \\
f'(x) &= \cos x - 3\sin x
\end{aligned}
$$

$$x \leftarrow x - \frac{f(x)}{f'(x)}$$

Starting with $x = -2$, successive applications of Newton's iterative formula yield

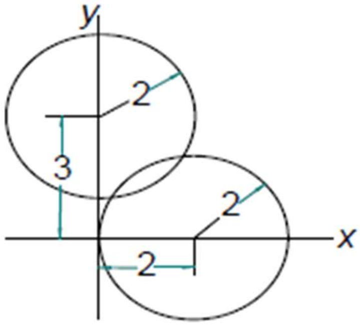
$$
\begin{aligned}
x &\leftarrow -2 - \frac{-4.1577}{2.3117} = -0.2015 \\
x &\leftarrow -0.2015 - \frac{0.7392}{1.5801} = -0.6693 \\
x &\leftarrow -0.6693 - \frac{-0.2676}{2.6456} = -0.5682 \\
x &\leftarrow -0.5682 - \frac{-0.0093}{2.4571} = -0.5644 \\
x &\leftarrow -0.5644 - \frac{0.0000}{2.445} = -0.5644 \blacktriangleleft
\end{aligned}
$$

Starting with $x = 2$, we get

$$
\begin{aligned}
x &\leftarrow 2 - \frac{-2.3391}{-3.1440} = 1.2560 \\
x &\leftarrow 1.2560 - \frac{-0.1203}{-2.5430} = 1.2087 \\
x &\leftarrow 1.2087 - \frac{-0.0021}{-2.4512} = 1.2078 \\
x &\leftarrow 1.2078 - \frac{0.0000}{-2.4495} = 1.2078 \blacktriangleleft
\end{aligned}
$$

Problem 23

$$
\begin{aligned}
f_1(x, y) &= (x-2)^2 + y^2 - 4 \\
f_2(x, y) &= x^2 + (y-3)^2 - 4
\end{aligned}
$$



The rough locations of the intersection points are $(2,2)$ and $(0,1)$. Letting $x = x_1$ and $y = x_2$, the following function defined the equations:

```
function y = p4_1_23(x)
% Equations used in Problem 23, Problem Set 4.1
y = [(x(1) - 2)^2 + x(2)^2 - 4;
x(1)^2 + (x(2) - 3)^2 - 4];
end
```

The following command returns the coordinates of the first point:

```
>> newtonRaphson2(@p4_1_23,[2;2])

ans =

    1.7206
    1.9804
```

Changing the starting point to [0; 1], we obtain the coordinates of the second point

```
>> newtonRaphson2(@p4_1_23,[0;1])

ans =

    0.2794
    1.0196
```

Problem 30

Letting $x = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & T \end{bmatrix}^T$, the equations to be solved are

```
function y = p4_1_30(x)
% Equations used in Prob. 30, Problem Set 4.1
y = [x(4)*(-tan(x(2)) + tan(x(1))) - 16;
x(4)*( tan(x(3)) + tan(x(2))) - 20;
-4*sin(x(1)) - 6*sin(x(2)) + 5*sin(x(3)) + 3;
4*cos(x(1)) + 6*cos(x(2)) + 5*cos(x(3)) - 12];
end
```

Rough estimates (starting values) of the variables are

$$\theta_1 = 0.8 \text{ rad} \qquad \theta_2 = 0.3 \text{ rad} \qquad \theta_3 = 0.4 \text{ rad} \qquad T = 20 \text{ kN}$$

The solution is obtained with the command

```
>> newtonRaphson2(@p4_1_30,[0.8;0.3;0.4;20])

ans =

    0.9358
    0.4334
    0.5800
   17.8884
```

Therefore, the solution is

$$\begin{aligned} \theta_1 &= 0.9358 \text{ rad } = 53.62^\circ \blacktriangleleft \\ \theta_2 &= 0.4334 \text{ rad } = 24.83^\circ \blacktriangleleft \\ \theta_3 &= 0.5800 \text{ rad } = 33.23^\circ \blacktriangleleft \\ T &= 17.89 \text{ kN} \end{aligned}$$