1. $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} \alpha y_1 - \beta y_1 y_2 \\ -\gamma y_2 + \eta y_1 y_2 \end{pmatrix} = \vec{F}(t, \vec{y})$ +3

① $\vec{K}_1 = h\vec{F}(t, \vec{y})$
$\vec{K}_2 = h\vec{F}(t + \frac{h}{2}; \vec{y} + \frac{1}{2}\vec{K}_1)$ +3

i) $t = 0;\ h = 0.1,\ y_1(0) = 1100,\ y_2(0) = 120$

$$\vec{K}_1 = 0.1 \times \begin{pmatrix} \alpha \cdot y_1(0) - \beta y_1(0) y_2(0) \\ -\gamma y_2(0) + \eta y_1(0) y_2(0) \end{pmatrix} = \begin{pmatrix} -22 \\ 1.2 \end{pmatrix} = \begin{pmatrix} K_{1,1} \\ K_{1,2} \end{pmatrix}$$ +2

$$\vec{K}_2 = 0.1 \times \begin{pmatrix} \alpha(y_1(0) + \frac{1}{2}K_{1,1}) - \beta(y_1(0) + \frac{1}{2}K_{1,1})(y_2(0) + \frac{1}{2}K_{1,2}) \\ -\gamma(y_2(0) + \frac{1}{2}K_{1,2}) + \eta(y_1(0) + \frac{1}{2}K_{1,1})(y_2(0) + \frac{1}{2}K_{1,2}) \end{pmatrix}$$

$$= 0.1 \times \begin{pmatrix} 1 \cdot (1100 - \frac{1}{2}\times 22) - \cancel{\beta}\, 0.01\times (1100 - \frac{1}{2}\times 22)(120 + \frac{1}{2}\times 1.2) \\ -1 \cdot (120 + \frac{1}{2}\times 1.2) + 0.001 \cdot (1100 - \frac{1}{2}\times 22)(120 + \frac{1}{2}\times 1.2) \end{pmatrix}$$

$$= \begin{pmatrix} -22.4334 \\ 1.0733 \end{pmatrix}$$ +2

$$\vec{y}(0.1) = \vec{y}(0) + \vec{K}_2 = \begin{pmatrix} 1100 \\ 120 \end{pmatrix} + \begin{pmatrix} -22.4334 \\ 1.0733 \end{pmatrix} = \begin{pmatrix} 1077.567 \\ 121.0733 \end{pmatrix}$$ +2

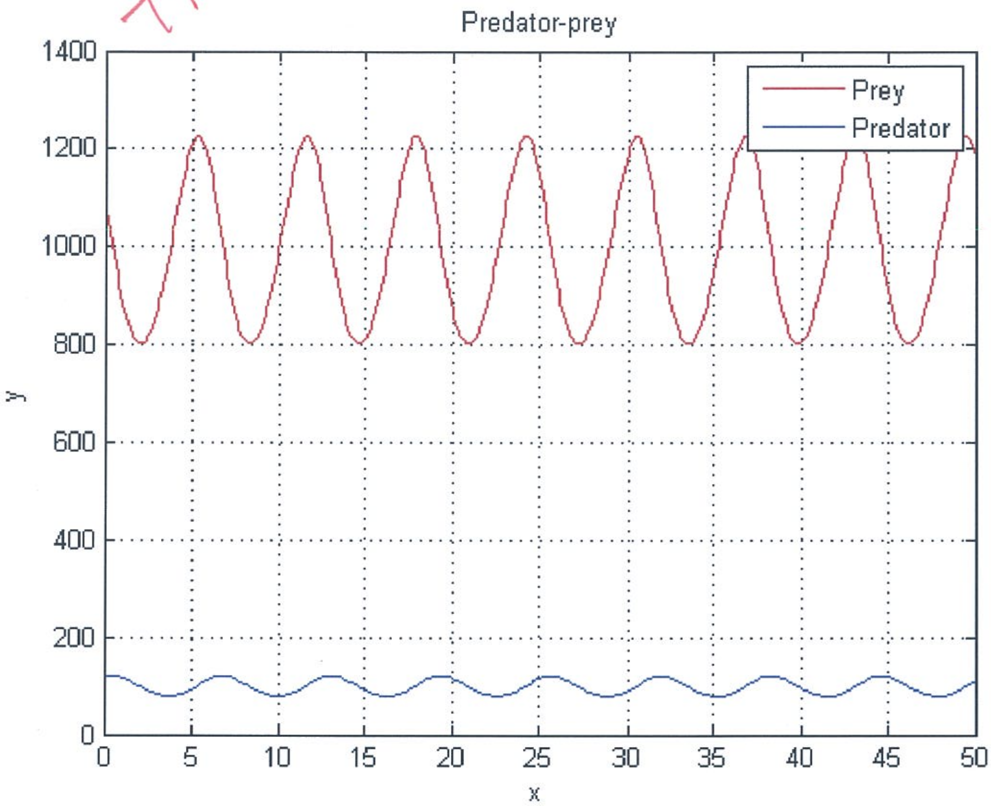
② $t = 0.1;\ h = 0.1,\ y_1(0.1) = 1077.567,\ y_2(0.1) = 121.0733$

$$\vec{K}_1 = 0.1 \cdot \begin{pmatrix} 1\times 1077.567 - 0.01\times 1077.567\times 121.0733 \\ -1\times 121.0733 + 0.001\times 1077.567\times 121.0733 \end{pmatrix} = \begin{pmatrix} -22.708 \\ 0.9391 \end{pmatrix}$$ +2

$$\vec{K}_2 = 0.1 \cdot \begin{pmatrix} (1077.567 - \frac{1}{2}\cdot 22.708) - 0.01\times(1077.567 - \frac{1}{2}\cdot 22.708)(121.0733 + 0.9391) \\ -1\times(121.0733 + 0.9391) + 0.001\times(1077.567 - \frac{1}{2}\times 22.708)(121.0733 + 0.9391) \end{pmatrix}$$

$$= \begin{pmatrix} -23.47 \\ 0.8079 \end{pmatrix}$$ +2 $\Rightarrow$ $$\vec{y}(0.2) = \begin{pmatrix} 1054.\cancel{5}87 \\ 121.88 \end{pmatrix}$$ +2

+12


Predator-prey
Prey
Predator
1400
1200
1000
800
600
400
200
0
y
0
5
10
15
20
25
30
35
40
45
50
x

2. ① a) $y_1 = f$, $y_4 = T$

$$+5 \quad \begin{cases} y_1' = y_2 \\ y_2' = y_3 \\ y_3' = -\frac{1}{2} y_1 y_3 \\ y_4' = y_5 \\ y_5' = -\frac{1}{2} P_r y_1 y_5 . \end{cases} \quad \text{w/ B.C.} \quad \begin{cases} y_1(0) = 0 \\ y_2(0) = 0 \\ y_2(7) = 1 \\ y_4(0) = 1 \\ y_4(7) = 0 . \end{cases} \quad +5$$

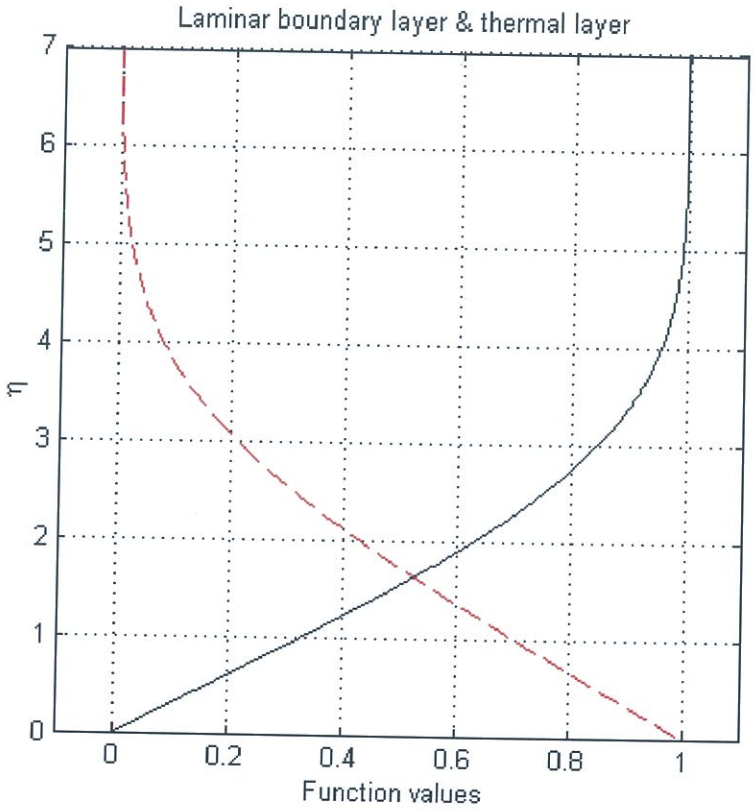
b). Let $y_3(0) = u_1$ & $y_5(0) = u_2$

```
function y = inCond(u)
  y = [0 0 u1 1 u2]          +5
end.
```

c) We need to satisfy $y_2(7) = 1$ & $y_4(7) = 0$.

```
⇒ function r = residual(u)
   r = zeros(length(u), 1);
   [xSol, ySol] = runKut4(@dEqs, x, inCond(u), xstop, h)
                                                   +1.
   lastRow = size(ySol, 1);
   r(1) = ySol(lastRow, 2) - 1;   +2
   r(2) = ySol(lastRow, 4) - 0;   +2.
end
```

+10



Laminar boundary layer & thermal layer
η
Function values
0
1
2
3
4
5
6
7
0
0.2
0.4
0.6
0.8
1

3. $y''' + \frac{1}{2} y y'' = 0, \quad y(0)=0, y'(0)=0, y'(7)=1.$

① $$\frac{-y_{i-2}+2y_{i-1}-2y_{i+1}+y_{i+2}}{2h^3} + \frac{1}{2}y_i \cdot \frac{y_{i-1}-2y_i+y_{i+1}}{h^2} = 0$$

$$-y_{i-2}+2y_{i-1}-2y_{i+1}+y_{i+2} + h \cdot y_i(y_{i-1}-2y_i+y_{i+1}) = 0 \quad (1)$$

+5.

② a) since $y(0)=0 \Rightarrow y_1 = 0$ +3

b) $y'(0) = \frac{y_2 - y_0}{2h} = 0 \Rightarrow y_0 = y_2$. $-(2)$ +2

Substituting eq.(2) into eq.(1) at $i=2$.

$$-y_0 + 2y_1 - 2y_3 + y_4 + h y_2(y_1 - 2y_2 + y_3) = 0$$

$$\Rightarrow -y_2 + 2y_1 - 2y_3 + y_4 + h y_2(y_1 - 2y_2 + y_3) = 0$$ +3

c) $y'(7)=1 \Rightarrow y'(7) = \frac{y_{n+1}-y_{n-1}}{2h} = 1 \Rightarrow y_{n+1} = y_{n-1} + 2h$

+2

Discretized eq. at $i=n-1$ is

$$-y_{n-3} + 2y_{n-2} - 2y_n + y_{n+1} + h y_{n-1}(y_{n-2} - 2y_{n-1} + y_n) = 0$$

Since $y_{n+1} = y_{n-1} + 2h$, the eq. becomes

$$-y_{n-3} + 2y_{n-2} - 2y_n + y_{n-1} + 2h + h y_{n-1}(y_{n-2} - 2y_{n-1} + y_n) = 0.$$

+3

d) $y'(7)=1 \Rightarrow y'(7) \approx \frac{y_{n-2} - 4y_{n-1} + 3y_n}{2h} = 1$

$\therefore y_{n-2} - 4y_{n-1} + 3y_n = 2h.$

+2.

③ $r(1) = y_1 - 0$

$r(2) = -y_2 + 2y_1 - 2y_3 + y_4 + h y_2 (y_1 - 2y_2 + y_3)$

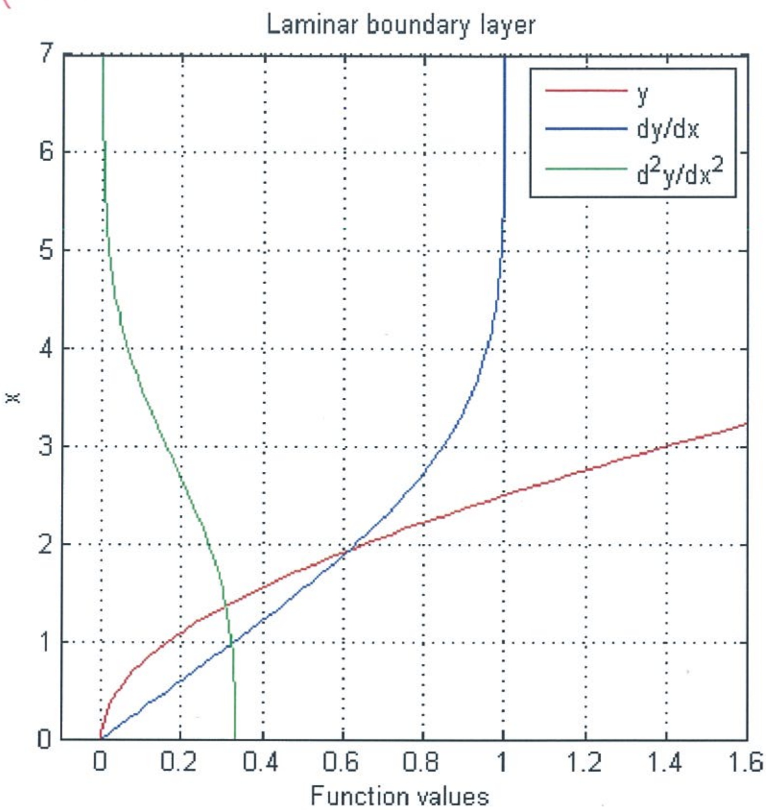
$r(i) = -y_{i-2} + 2y_{i-1} - 2y_{i+1} + y_{i+2} + h y_i (y_{i-1} - 2y_i + y_{i+1})$, $i = 3, \cdots, n-2$

$r(n-1) = -y_{n-3} + 2y_{n-2} - 2y_n + y_{n-1} + 2h + h y_{n-1} (y_{n-2} - 2y_{n-1} + y_n)$

$r(n) = y_{n-2} - 4y_{n-1} + 3y_n - 2h$

+5

+15



Laminar boundary layer
y
dy/dx
d2y/dx2
x
Function values
0
1
2
3
4
5
6
7
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6