Handout 1

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1 Balanced Mixed ANOVA Models

Definition:

$$y = X\beta + Z\alpha + \epsilon,$$

where $Z\alpha = Z_1\alpha_1 + \cdots + Z_s\alpha_s$.

$$X = \otimes_{l=1}^{w+1} 1_{n_l}^{a_l}, Z_i = \otimes_{l=1}^{w+1} 1_{n_l}^{b_{i,l}}.$$

where $(a_1, \cdots, a_{w+1}) \in S_{w+1} = \{0,1\}^{w+1}$, and $(b_{i,1}, \cdots, b_{i,w+1}) \in S_{w+1}$. w denotes the number of factors in the model. n_l represents the number of levels for factor $l, (1 \leq l \leq w)$. n_{l+1} denotes the number of repetitions. $a_{w+1} = 1$ and $b_{i,w+1} = 1$ for all i. Example 1.2, i.e., Two-way random effects model is a balanced mixed ANOVA model.

2 Some Important Results

Let A be a matrix whose elements are functions of θ, where θ is a vector valued variable.

1. If A is nonsingular, then
$$\frac{\partial A^{-1}}{\partial \theta_i} = -A^{-1}\frac{\partial A}{\partial \theta_i}A^{-1}.$$

   Proof:

2. If A is positive definite, then
$$\frac{\partial}{\partial \theta_i} log(|A|) = tr\left(A^{-1}\frac{\partial A}{\partial \theta_i}\right).$$

   Proof:

3. Let ξ be a random vector such that $E(\xi) = \mu$ and $Var(\xi) = \Sigma$. Then for any nonrandom symmetric matrix A, we have

$$E(\xi' A\xi) = \mu' A\mu + tr(A\Sigma).$$

Proof:

4. Let $V = aI_k + bJ_k$, where I_k is a $k \times k$ identity matrix and J_k is a $k \times k$ matrix of all ones. Then the following two equations hold true:

$$|V| = a^{k-1}(a + kb),$$

and

$$V^{-1} = \frac{1}{a} \cdot (I_k - \frac{b}{a + kb} J_k).$$

This result may be used in Problem 1.7.

Proof: