Handout 3

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1 Show that the BLUP formula still holds true without normality assumption

Claim: Suppose we have the linear mixed model:

$$y = X\beta + Z\alpha + \epsilon,$$

where X is a $n \times p$ matrix with full column rank, $Var(\alpha) = G$, $Var(\epsilon) = R$, α and ϵ are uncorrelated. Furthermore, suppose the fixed effects β are unknown and the variance components θ are known. Note that we do not need normality assumption here.

Show that the Best Linear Unbiased Prediction (BLUP) of $\xi = a'\alpha$ is $\hat{\xi} = a'GZ'Py$, where $P = V^{-1} - V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}$.

In the textbook, we know that the BLUP of ξ is expressed as

$$\tilde{\xi} = a'GZ'V^{-1}(y - X\tilde{\beta}),$$

where $\tilde{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}y$. Actually, $\tilde{\xi}$ is equivalent to $\hat{\xi}$.

Proof:

2 An example of Best Prediction: P89 Example 2.3

Consider the one-way random effects model of Example 1.1 with normality assumption:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij},$$

for $i = 1, \cdots, m, j = 1, \cdots, k_i$, where $\alpha_i, i = 1, \cdots, m$ are random effects that are i.i.d. $N(0, \sigma^2)$; ϵ_{ij} are errors that are i.i.d. $N(0, \tau^2)$; the random effects are independent of the errors.

Show that the Best Prediction of α_i is given by

$$\hat{\alpha}_i = \frac{k_i \sigma^2}{\tau^2 + k_i \sigma^2} (\bar{y}_{i\cdot} - \mu),$$

where $\bar{y}_{i\cdot} = k_i^{-1} \sum\limits_{j=1}^{k_i} y_{ij}$.

Proof: