STA141_hw3

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Since in this assignment, I will mostly focus on the functions and algorithms I write, and talk about how to implement them.

As you can see, there is little global variables in my R script. The only entrance for my R script is the `main` function at the end of this script. It will call the `main_one` and `main_two` function, and each of them will accomplish the task required by question 1 and question 2.

Question One

First let me talk about the function calling logic.

The `main_one` function will call the `main_one_intermediate` function by rendering different metrics and combine them together to plot the mse figure.

The `main_one_intermediate` function will call `knn_classifier` to compute knn statistics in different k values, and then `knn_mse_analysis` will use this result to compare mse and then return the model with the smallest mse. After that, the `knn_compute` will be called to reclassify the model with the best k value to recalculate the confusion matrix and other statistics.

The `create_dist_matrix` is prior to any functions, since it calculates the distance matrix which will be used for all the subsequent functions.

The `knn_classifier` function is a combination of some small functions: `get_cv_dist_matrix`, `knn_compute` and `knn_error`. `get_cv_dist_matrix` will subset the initial dataset and distance matrix to the required one by cross-validation. `knn_compute` will calculate the predicted labels, and `knn_error` will use the predicted label to calculate mse, confusion matrix and false predicted indices.

In case of tie situations, I create a function called `break_tie`, and `knn_compute` will call it to find the optimal value when tie exists.

Finally, the `ggplot_image` function will plot the false classified images. It uses the result from `knn_mse_analysis`, which indicates the best_model and its corresponding false classified matrix, and `ggplot_image` will combine the first 64 indices to draw a plot to illustrate the point.

The `mse_plot` will plot a figure based on different metrics for mse versus k values.

Now let's see the code with comments:

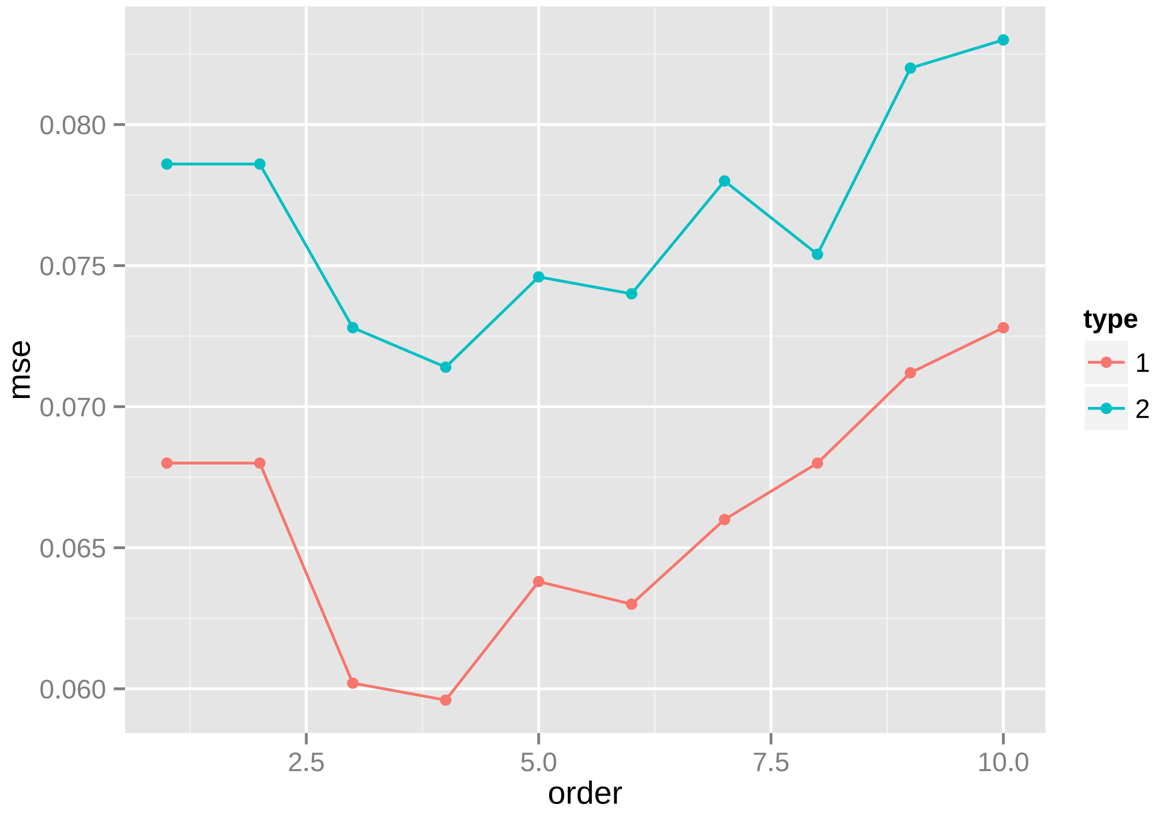
```r
digit_image <- read.csv("~/Desktop/digitsTrain.csv")
set.seed(2)
digit_image <- digit_image[sample(1:5000), ]
metrics <- c("euclidean", "manhattan")

create_dist_matrix <- function(metric, data = digit_image, y_identifier = 1) {
    # knowing the y idetifier if possible, and then calculate the
    # distance result
    dist_result <- dist(data[, -1], method = metric)
    # initialize the dist matrix
    dist_matrix <- as.matrix(dist_result)
}
```

```r
knn_compute <- function(position, k, dist_matrix, label_real,
    cv_flag = T) {
    # k is the number of knn number This is the function that
    # calculates the labels output for one part of
    # cross-validation.

    # get the row number of distance matrix
    data_N <- dim(dist_matrix)[1]
    # This is a little trick. Since the indices we extract will
    # be the index in this particular distance matrix, but this
    # matrix's columns are lack of a group of column numbers. So
    # it is necessary to transform the indices for the smallest
    # value to the normal indices. For example, if in the fourth
    # group, 3001:4000 are missing, then if a index is 2899, do
    # nothing to it, but if a index is 3445, we need to add 1000,
    # which is the size, data_N. the position_expansion is the
    # threshold to justify whether it should be transformed. Any
    # index that is larger than this variable must be tranformed.
    if (cv_flag)
        position_expansion <- (position - 1) * data_N else position_expansion <- position

    # By calling break_tie function, the prediction value will be
    # presented. Note the name of this function is due to it will
    # handle circumstances of tie, and the method of handling
    # this case will be discussed in that function.
    label_pred <- sapply(1:data_N, break_tie, dist_matrix, k,
        label_real, position_expansion, data_N, cv_flag)
    # return value
    label_pred
}

break_tie <- function(i, dist_matrix, k, label_real, position_expansion,
    data_N, cv_flag = T) {
    # This is the function that calculates the output and check
    # whether this tie. If there is, this function will check all
    # the labels' reciprocal distance with the most frequency,
    # and the one with the most value in reciprocal distance win.

    # order the first k distances
    if (cv_flag == T)
        indices <- order(dist_matrix[i, ])[1:k] else indices <- order(dist_matrix[i, ])[2:(k + 1)]

    # transfer the indices to the standard real ones
    indices_real <- ifelse(indices > position_expansion, indices +
        data_N, indices)

    # extract the real label, by using the real indices derived
    # above
    reg_labels <- label_real[indices_real]

    # extract all the values with the most frequency
    reg_labels_table <- table(reg_labels)
    reg_labels_max_freq <- max(reg_labels_table)
    result <- as.numeric(names(reg_labels_table[reg_labels_table ==
        reg_labels_max_freq]))

    if (length(result) != 1) {
        # first create the data for further use. This data frame has
        # labels versus its corresponding distances with reciprocals.
        indices_dist <- dist_matrix[i, indices]
        indices_data <- data.frame(labels = reg_labels, dist_reciprocal = 1/indices_dist)

        # first we only need the labels that are in the result, say
        # most frequent ones
        indices_data <- subset(indices_data, labels %in% result)
        # split into groups by label
        indices_data_split <- split(indices_data, indices_data$labels)
        # calculate their sum
        break_tie_vec <- sapply(indices_data_split, function(x) sum(x$dist_reciprocal))
        # select the one with the most sum value
        result <- as.numeric(names(indices_data_split)[which.max(break_tie_vec)])
    }

    result
}

knn_error <- function(label_real, label_pred) {
    # This function computes the mse (error) for subsequent
    # compare, false_indices for subsequent drawing falsely
    # classified pictures, and confusion matrix for insepcting.
    # This function accepts the labels predicted by knn_compute
    # function.

    # initialize the label dataframe to compare
    label <- data.frame(real = label_real, pred = label_pred)
    # add error variable. if it is 1, then error exists; 0, no
    # error
    label$error <- ifelse(label$real != label$pred, 1, 0)
    # calculate the mse
    mse <- sum(label$error)/length(label_pred)
    # extract false predict ones
    false_indices <- which(label$error == 1)
    # recognition matrix
    confusion_matrix <- with(label, table(real, pred))
    # return value
    result <- list(label_matrix = label, mse = mse, false_indices = false_indices,
        confusion_matrix = confusion_matrix)
}

get_cv_dist_matrix <- function(cv, dist_matrix) {
    # This function split the distance matrix into cv groups used
    # for knn prediction

    each_size <- dim(dist_matrix)[1]/cv
    cv_dist_matrix <- lapply(1:cv, function(i) {
        # First create a variable used to identify which rows to use
        # and which column to drop, then subset this index
        position_test <- (each_size * (i - 1) + 1):(each_size *
            i)
        dist_matrix[position_test, -position_test]
    })
    # return value
    cv_dist_matrix
}

knn_classfier <- function(dist_matrix, k, cv = 5, ori_data) {
    # This function is the main knn algorithm implementation
    # function. It returns different values by supplying
    # different k value to it. It first spilit the data by the
    # cross-validation folds, and then give the splitted distance
    # matrix to knn_compute to calculate the predicted values.
    # After that, calling the knn_error function to compute the
    # summary statistics for each k value, including mse,
    # false_indices and confusion_matrix.

    # get the cv split matrix and the real label
    cv_dist_matrix <- get_cv_dist_matrix(cv = cv, dist_matrix = dist_matrix)
    label_real <- ori_data$label

    # predict the output
    label_pred <- unlist(lapply(1:cv, function(i) {
        knn_compute(i, k, cv_dist_matrix[[i]], label_real)
    }))

    # calculate summary statistics and return it
    knn_result <- knn_error(label_real, label_pred)
}

knn_mse_analysis <- function(knn_result, k_max) {
    # This function combines the summary statistics from many k
    # values, from 1 to k_max, which is normally 10, and then
    # return the model with the smallest mse and its
    # corresponding statistics.

    # first extract the mse from each model, and give them names,
    # print the result
    knn_mse_total <- unlist(sapply(knn_result, `[`, "mse"))
    names(knn_mse_total) <- 1:k_max
    print(knn_mse_total)
    # select the best model with the least mse value
    best_model <- order(knn_mse_total)[1]
    # extract the best model's confustion_matrix and
    # false_indices
    confusion_matrix <- knn_result[[best_model]]$confusion_matrix
    false_indices <- knn_result[[best_model]]$false_indices
    # return value
    result <- list(mse = knn_mse_total, best_model = best_model,
        confusion_matrix = confusion_matrix, false_indices = false_indices)
}
```

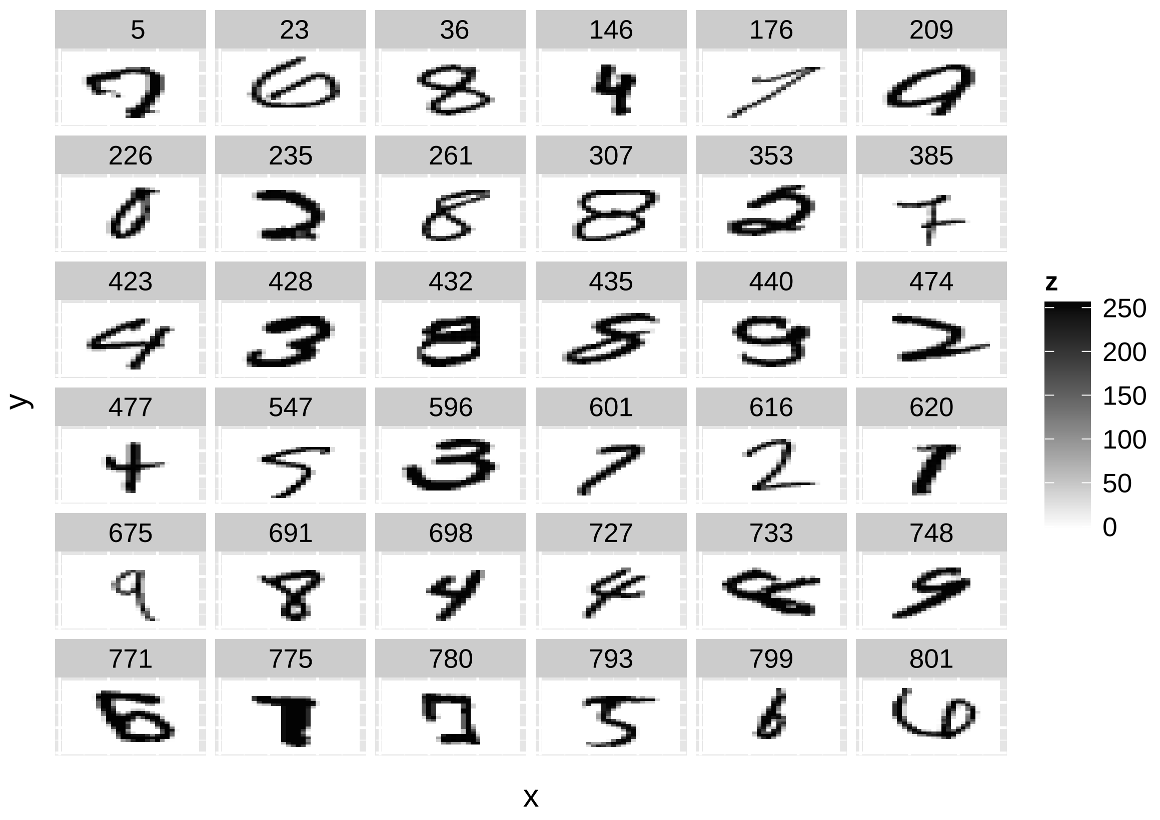
```r
ggplot_image <- function(model_best_model_false_indices) {
    # This function draws the image by giving it an index as
    # input using ggplot.  It is smart since when you give it
    # multiple inputs, it will draw the number of indices you
    # give it and draw a plot with all the images on one canvas.

    # It first extract the image information by the index from
    # the original dataset.  If there are multiple indices, it
    # will classify them by the indices by creating a type
    # variable
    data_for_ggplot_list <- lapply(model_best_model_false_indices,
        function(i) {
            data.frame(x = rep(1:28), y = rep(28:1, each = 28),
                z = as.numeric(digit_image[i, -1]), type = i)
        })
    data_for_ggplot <- do.call("rbind", data_for_ggplot_list)

    # ggplot main function
    p <- ggplot(data = data_for_ggplot, aes(x, y, fill = z)) +
        geom_raster() + scale_fill_gradient(low = "white", high = "black") +
        facet_wrap(~type) + theme(axis.ticks.x = element_blank(),
        axis.text.x = element_blank(), axis.ticks.y = element_blank(),
        axis.text.y = element_blank())
    print(p)
}

mse_plot <- function(model_mse_total, metric_length, k_max) {
    # This function accepts model_mse_total as input and then
    # plot it by split different metric. This is a plot for each
    # metric by mse versus k value.

    # creates the mse_matrix for plot based on model_mse_total
    mse_matrix <- data.frame(mse = unname(unlist(model_mse_total)),
        order = rep(1:k_max, times = metric_length), type = as.factor(rep(1:metric_length,
            each = k_max)))
    # draw the plot using ggplot
    mse_plot <- ggplot(mse_matrix, aes(x = order, y = mse, group = type,
        color = type)) + geom_point() + geom_line()
    print(mse_plot)
}

false_digit_analysis <- function(i, k = 4, digit_dist_list) {
    # This function does the job of showing misclassified digits
    # by giving the plot of the original digit and the digits
    # that predict it

    # extract the needed distance
    dist_matrix <- digit_dist_list[[1]][i, ]
    # predict, get its indices
    predict_digit <- order(dist_matrix)[1:(k + 1)]
    # creating a dataframe for each index
    false_digit_total_list <- lapply(predict_digit, function(i) {
        data.frame(x = rep(1:28), y = rep(28:1, each = 28), z = as.numeric(digit_image[i,
            -1]), type = i)
    })
    # combine the dataframes into a big one
    false_digit_total_data <- do.call(rbind, false_digit_total_list)

    # ggplot main function
    p <- ggplot(data = false_digit_total_data, aes(x, y, fill = z)) +
        geom_raster() + scale_fill_gradient(low = "white", high = "black") +
        facet_wrap(~type) + theme(axis.ticks.x = element_blank(),
        axis.text.x = element_blank(), axis.ticks.y = element_blank(),
        axis.text.y = element_blank())
    print(p)
}

# ----------------------------------------------------------------------------

main_one_intermediate <- function(digit_dist_matrix, test_number = 5000,
    k_max, print_flag) {
    # This is the intermediate function used to do the knn
    # analysis for each metric.  It first calls the
    # knn_classifier function to do the knn analysis for each k,
    # and then calls the knn_mse_analysis function to return the
    # best model and its corresponding statistics.

    # knn analysis for each k
    knn_result <- lapply(1:k_max, function(i) {
        knn_classfier(digit_dist_matrix[1:test_number, 1:test_number],
            k = i, ori_data = digit_image[1:test_number, ])
    })

    # print flag for each metric
    cat("\n")
    if (print_flag == 1)
        cat("This is for euclidean") else cat("This is for manhattan")
    cat("\n")
    cat("\n")

    cat("mse:")
    cat("\n")
    # do the knn_mse_analysis to return the best model and its
    # corresponding statistics
    knn_model_select_result <- knn_mse_analysis(knn_result, k_max)
    cat("\n")

    # show the best model
    best_model <- knn_model_select_result$best_model
    cat("The best model is k = ", best_model)
    cat("\n")
    cat("\n")

    # return value
    knn_model_select_result
}
```

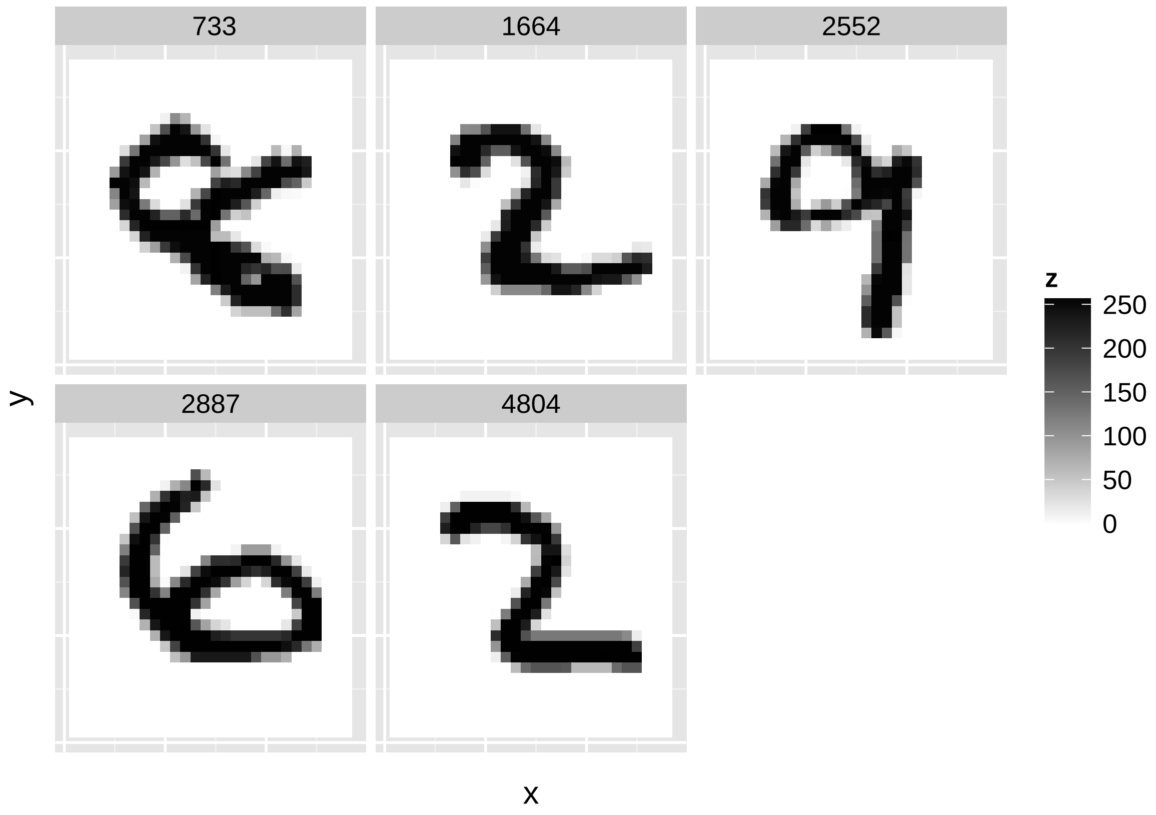
```r
main_one <- function(digit_dist_list, k_max = 10) {
    # This is the function to accomplish all the tasks for
    # question one. It first calls the main_one_intermediate to
    # find the final statistics for each metric, then extract the
    # mse and plot them on the same plot to compare the k value
    # and metric performance. Finally, plot the false classified
    # images for each metric.

    # calculate the length of metrics
    metric_length <- length(digit_dist_list)

    # return the final statisitcs for each metric by calling
    # main_one_intermediate
    knn_model_select_result <- lapply(1:metric_length, function(i) {
        main_one_intermediate(digit_dist_list[[i]], print_flag = i,
            k_max = k_max)
    })
    knn_model_select_result <- setNames(knn_model_select_result,
        metrics)

    # extract mse
    model_mse_total <- lapply(1:metric_length, function(i) knn_model_select_result[[i]]$mse)
    # draw mse plot
    mse_plot(model_mse_total, metric_length, k_max)

    # compute the mse with the best model at no cross-validation
    cat("The overall best model is k = 4 at eculidean metric")
    cat("\n")
    cat("\n")
    knn_best_result <- knn_compute(5000, 4, digit_dist_list[[1]],
        digit_image$label, cv_flag = F)
    knn_best_mse <- knn_error(label_real = digit_image$label,
        label_pred = knn_best_result)
    cat("The corresponding confusion matrix is")
    cat("\n")
    cat("\n")
    print(knn_best_mse$confusion_matrix)

    # plot best model false indices' images for each metric. I
    # only select the first 36 indices, since I think it is
    # sufficient to explain the problem.
    ggplot_image(knn_best_mse$false_indices[1:36])

    # This expression shows plots more information of falsely
    # classified digits
    ggplot_image_false <- lapply(c(733, 440), false_digit_analysis,
        4, digit_dist_list)
}

main <- function() {
    # This is the entrace for this assignment. It first reads
    # into data, then calls main_one to do question one, calls
    # main_two to do question two.
    digit_dist_list <- lapply(metrics, create_dist_matrix)
    main_one(digit_dist_list)

    # main_two(digit_image)
}

main()
```

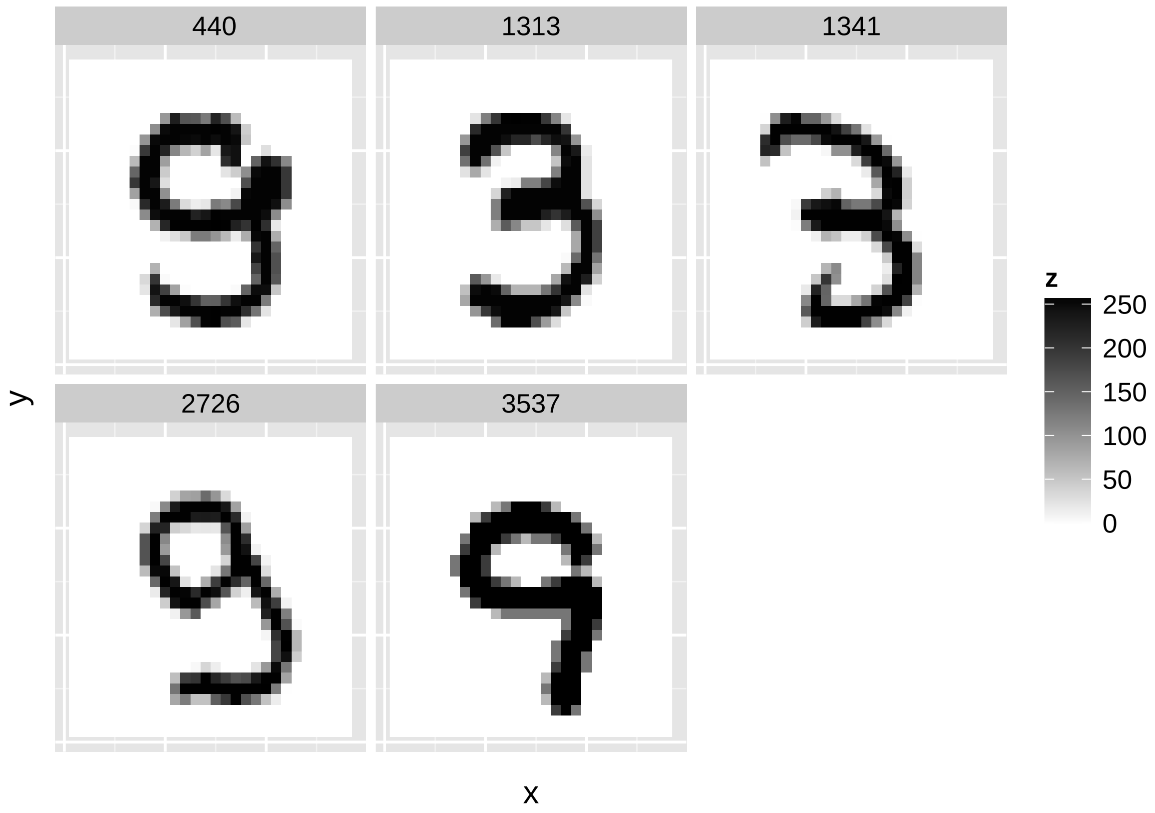
```
##
## This is for euclidean
##
## mse:
##      1      2      3      4      5      6      7      8
## 0.0680 0.0680 0.0602 0.0596 0.0638 0.0630 0.0660 0.0680
##      9     10
## 0.0712 0.0728
##
## The best model is k =  4
##
##
## This is for manhattan
##
## mse:
##      1      2      3      4      5      6      7      8
## 0.0786 0.0786 0.0728 0.0714 0.0746 0.0740 0.0780 0.0754
##      9     10
## 0.0820 0.0830
##
## The best model is k =  4
```



```
## The overall best model is k = 4 at eculidean metric
##
## The corresponding confusion matrix is
##
##     pred
## real   0   1   2   3   4   5   6   7   8   9
##    0 487   0   0   0   0   0   3   0   0   0
##    1   0 588   1   1   2   0   1   1   0   0
##    2   5  13 470   8   1   0   1  11   3   2
##    3   1   1   3 502   2   9   0   1   1   3
##    4   1   8   0   0 442   0   1   2   0  22
##    5   2   3   0  12   1 414   8   0   4   4
##    6   5   3   1   0   1   3 481   0   2   0
##    7   0  13   1   0   0   0   0 528   0  12
##    8   1   9   1  12   2  13   1   6 374   7
##    9   3   2   1   6   9   0   0   7   1 450
```


5
23
36
146
176
209
226
235
261
307
353
385
423
428
432
435
440
474
477
547
596
601
616
620
675
691
698
727
733
748
771
775
780
793
799
801
z
250
200
150
100
50
0
y
x


733
1664
2552
2887
4804
y
x
z
250
200
150
100
50
0



Note: I calculate the k values from 1 to 10, and uses 5-fold cross-validation.

Now let's see the results:

1. The best model for euclidean metric is model 4, and the best model for manhattan metric is also model 4.
2. cross-validation misclassification rate versus k and the distance metrics is the first map of R output. In the plot we can see the whole trend of mse: It first decreases, and at order 4 it has the smallest value, and then, increases afterwards. The metric manhattan is worse than the metric euclidean, but has almost the same trend.
3. The best model is clearly metric euclidean with k = 4. The confusion matrix is already shown above.
4. From the confusion matrix, we can see 0 is predicted best, only 3 misclassified, and 8 is predicted worst, there are 52 misclassified.
5. The most confused group is misclassify 4 as 9, 7 as 1, 8 as 5, 2 as 1, 8 as 3.
6. I have plotted the image for the first 36 misclassified digits in each metric. As is seen, some of them are a little hard to predict, since some of them are really hard to recognize even by our eyes. For example, label 733, it is hard to recognize it as digit '8', since it is so different from standard '8'. Another is label 440, I recognize it as '4', but the result turns out to be '9'. These digits are very hard to recognize even by our eyes, and needless to say how hard they are for machines to recognize.

The last two plots reveal more information about label 733 and 440. As is seen, label 733 is close to digit '2', '9', '6', '2', although its real digit is '8'. For label 440, it is predicted as '3', '3', '9', '9', although it is indeed '9'. This justify why our knn classifier does not do very well in some situations.

Question Two

First let me talk about the function calling logic as well.

The `main_two` function is the entrance function for question 2. It supplies different metrics and different cv fold values to `get_means_pred`, and calculates the result. After that, calls `means_mse_analysis` to analyze the result.

The `get_means_pred` function predicts the values for each cross-validation train and test combination. It first calls `get_digit_mean_pixel` to calculate the mean value for each train set, then calls `get_means_dist` to calculate the combining distance matrix, then use these two results to calculate the predict value.

The `get_digit_mean_pixel` function just split the train data and calculates the mean for each , and `get_means_dist` use the `dist` function to calculate the combining train and test datset distance.

The `means_mse_analysis` function, will compare the predict value to the real value, and calculate mse.

Now let's see the code:

```r
get_digit_mean_pixel <- function(ori_data) {
    # This is the function that from train data calculating the
    # mean for each label

    # split the data by its labels
    digit_dist_matrix_split_label <- split(ori_data, ori_data$label)
    # calculate the mean for each label
    digit_mean_pixel <- t(sapply(digit_dist_matrix_split_label,
        function(x) {
            apply(x, 2, mean)
        }))
    # return value
    digit_mean_pixel
}

get_means_dist <- function(ori_data, digit_mean_pixel, metric) {
    # This function gets the distance matrix for the test set and
    # the train set.

    # calculate the length of test data and mean of train data
    ori_data_N <- dim(ori_data)[1]
    digit_mean_pixel_N <- dim(digit_mean_pixel)[1]

    # stack the test data and the mean of train data together
    means_matrix <- rbind(digit_mean_pixel, ori_data)

    # calculate the distance matrix and select the values that
    # are useful to us
    means_dist <- as.matrix(dist(means_matrix, method = metric))[11:(ori_data_N +
        digit_mean_pixel_N), 1:10]

    # return value
    means_dist
}

get_means_pred <- function(cv_flag, ori_data, metric) {
    # This function predicts the values for each cross-validation
    # train and test combination. Since it knows which cv group
    # it is in now, it will select the corresponding train and
    # test dataset and then supplies them to the
    # get_digit_mean_pixel and get_means_dist function to
    # calculate the distance.  Then it will return the smallest
    # distance as its predict value.

    # this calculates the test_position for extracting data.
    test_position <- (1000 * (cv_flag - 1) + 1):(1000 * cv_flag)

    # Extract the data for test and train and then supplies them
    # to the needed function
    digit_mean_pixel <- get_digit_mean_pixel(ori_data[-test_position,
        ])
    means_dist <- get_means_dist(ori_data[test_position, ], digit_mean_pixel,
        metric)

    # predict the label
    means_label_pred <- apply(means_dist, 1, which.min) - 1
}

means_mse_analysis <- function(means_pred, means_real) {
    # This function analyze the predict value by calculating mse.

    # calculates mse
    means_mse <- lapply(means_pred, function(x) {
        sum(means_real != x)/(length(means_real))
    })
    means_mse
}

main_two <- function(ori_data, cv = 5) {
    # This function calculates the mse for different metrics. It
    # first calls the means_pred function by suppling different
    # metrics and then in each metric, calculates predict value
    # in each cv fold. At last, it calculates mse, and makes the
    # output friendly to print.

    # print instruction
    cat("Now come to k-means classification")
    cat("\n")

    # calculates predict value for each metric
    means_pred <- lapply(metrics, function(x) {
        temp <- lapply(1:cv, function(i) {
            get_means_pred(i, ori_data, x)
        })
        unlist(temp)
    })
    # extract real value
    means_real <- ori_data$label

    # calculates mse and give it names
```

```r
    means_mse <- means_mse_analysis(means_pred, means_real)
    means_mse <- setNames(means_mse, metrics)
    print(means_mse)

    # return value
    means_mse
}

# ------------------------------------------------------------------------

main <- function() {
    # This is the entrace for this assignment. It first reads
    # into data, then calls main_one to do question one, calls
    # main_two to do question two.

    # digit_image <- read.csv('~/Desktop/digitsTrain.csv')
    # metrics <- c('euclidean', 'manhattan') digit_dist_list <-
    # lapply(metrics, create_dist_matrix)
    # main_one(digit_dist_list)

    main_two(digit_image)
}

main()
```

```
## Now come to k-means classification
## $euclidean
## [1] 0.1908
##
## $manhattan
## [1] 0.3418
```

```
## $euclidean
## [1] 0.1908
##
## $manhattan
## [1] 0.3418
```

The result, mse, is in the output. The best result with this method is euclidean metric, and the mse is 0.1908.

When comparing it to the knn algorithm, it is much worse.

So knn with metric euclidean and k = 4 is our choice.