Notion of graphs

- Graph $G = (V, E)$
  $V = \{v_i\}$ = set of vertices
  E = set of edges = a subset of $V \times V = \{(v_i, v_j)\}$
- $|E| = O(|V|^2)$
  dense graph: $|E| \approx |V|^2$
  sparse graph: $|E| \approx |V|$
- If G is connected, then $|E| \geq |V| - 1$.
- Some variants
  - undirected: edge $(u, v) = (v, u)$
  - directed: (u, v) is edge from u to v.
  - weighted: weight on either edge or vertex
  - multigraph: multiple edges between vertices
- Further reading: Appendix B.4, pp.1168-1172 of [CLRS,3rd ed.]

Notion of graphs

Representing graph by Adjacency Matrix

- $A = (a_{ij})$ is a $|V| \times |V|$ matrix, where

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E \\ 0, & \text{otherwise} \end{cases}$$

- If G is undirected, A is symmetric, i.e., $A^T = A$.
- A is typically very sparse – use a sparse storage scheme in practice

Notion of graphs

Representing graph by Incidence Matrix

- $B = (b_{ij})$ is a $|V| \times |E|$ matrix, where

$$b_{ij} = \begin{cases} 1, & \text{if edge } e_j \text{ enters vertex } v_i \\ -1, & \text{if edge } e_j \text{ leaves vertex } v_i \\ 0, & \text{otherwise} \end{cases}$$

Notion of graphs

Representing graph by Adjacency List

- For each vertex v,

$$\text{Adj}[v] = \{ \text{ vertices adjacent to } v \ \}$$

- Variation: could also keep second list of edges coming into vertex.
- How much storage is needed?
  Answer: $\Theta(|V| + |E|)$ ("sparse representation")

Notion of graphs

- Degree of a vertex of a undirected graph = the number of incident edges
- For a digraph: Out-degree and In-degree
- For undirected graph:

$$\# \text{ of items in the adj. list} = \sum_{v \in V} \text{degree}(V) = 2|E|$$

- For digraph:

$$\# \text{ of items in the adj. list} = \sum_{v \in V} \text{out-degree}(V) = |E|$$