Solution: Homework 1

19.5. The table of means is given below. You will also find the main effects of factors A and B and the interaction effects (in the brackets) for Problems 19.7 and 19.8.

Table of means

	Factor B					
Factor A	B_1	B_2	B_3	B_4	$\mu_{i.}$	α_i
A_1	250	265	268	269	263	-6
A_2	288	273	270	269	275	6
$\mu_{.j}$	269	269	269	269	$\mu_{..} = 269$	
β_j	0	0	0	0		

Table of interactions

	Factor B			
Factor A	B_1	B_2	B_3	B_4
A_1	-13	2	5	6
A_2	13	-2	-5	-6

(a) Factor B main effects are all zero: no main effect of factor B.
(b) Interactions effects are present since the curves are not parallel.
(c) Table of $\{\ln \mu_{ij}\}$

	Factor B				
Factor A	B_1	B_2	B_3	B_4	
A_1	5.5215	5.5797	5.5910	5.5947	
A_2	5.6630	5.6095	5.5948	5.5947	

Once again, the graph of $\{\ln \mu_{ij}\}$ indicate presence of interactions.

19.8.
(a)

$$
\begin{aligned}
E(MSE) &= \sigma^2 = 4^2 = 16 \\
E(MSAB) &= \sigma^2 + n \sum \sum (\alpha\beta)^2_{ij}/[(a-1)(b-1)] = 952
\end{aligned}
$$

(b) The ratio $E(MSAB)/E(MSE) = 59.5$ is quite large, i.e., $E(MSAB)$ is much larger than $E(MSE)$. Since $MSAB/MSE$ fluctuates about $E(MSAB)/E(MSE)$, any F-test for a data set from the populations with the means and σ^2 as given here will very likely result in rejecting $H_0 : (\alpha\beta)_{ij} = 0$ for all i, j (against H_1 : not all $(\alpha\beta)_{ij}$ are equal to zero) since the p-value of the F-test is likely to be very small.

19.18
(a) The fitted values are $\hat{Y}'_{ijk} = \bar{Y}'_{ij.}$. The values of the estimated means are
$\bar{Y}'_{11.} = 0.44348$, $\bar{Y}'_{12.} = 0.80997$, $\bar{Y}'_{13.} = 1.10670$, $\bar{Y}'_{21.} = 0.39823$, $\bar{Y}'_{22.} = 0.58096$, $\bar{Y}'_{23.} = 0.86639$.

The residuals $\{e'_{ijk}\}$ are

	$j=1$		$j=2$		$j=3$	
	-0.4435	0.0336	-0.3329	-0.1110	0.0974	0.1238
	0.0336	-0.4435	-0.1110	-0.2079	-0.0653	-0.2036
$i=1$	-0.1425	0.3347	0.0931	-0.5089	-0.1525	0.3847
	0.1586	0.4016	0.3040	-0.0318	-0.3286	-0.5046
	-0.4435	0.5108	0.3942	0.5123	0.3083	0.3405
	-0.3982	0.0790	0.1972	-0.2799	0.1750	0.3377
	-0.0972	0.5049	0.0211	0.0211	0.0879	-0.1674
$i=2$	-0.0972	0.3007	-0.1038	0.2641	0.2476	0.1336
	-0.3982	-0.3982	-0.5810	0.3221	-0.2643	-0.0213
	0.3007	0.2038	-0.2799	0.4190	0.0367	-0.5654

(b) The dotplot show that the residuals are roughly equally aligned with reasonable symmetry about 0. Thus it seems that the assumption of equal variance is reasonable here and the distribution of the error terms may be symmetric about zero.
(c) The normal probability plot does not indicate any obvious departure from normality. Corr=0.987.
19.19
(a) Estimated mean plot shows almost parallel lines indicating that interactions effects may be negligible. Factor B means seem to be quite different indicating the presence of factor B main effects. However, factor A means do not seem to be very different and thus it is unclear if factor A main effects are present.
(b) ANOVA table

Source	df	SS	MS
Duration	$a-1=1$	0.44129	0.44129
Weight Gain	$b-1=2$	3.20098	1.60049
Interaction	$(a-1)(b-1)=2$	0.11989	0.05995
Error	$(n-1)ab=54$	5.46770	0.10125
Totral	$nab-1=59$	9.22987	

Most of the variability in the response seem to be explained by the main effect of factor B (weight gain)
(c) $H_0 : (\alpha\beta)_{ij} = 0$ for all i and j, H_1 : not all $(\alpha\beta)_{ij}$ equal zero.
Decision rule; reject H_0 if $F^* = MSAB/MSE > F(0.95; 2, 54) = 3.17$.
Since $F^* = MSAB/MSE = 0.59 < F(0.95; 2, 54)$, we cannot reject H_0.
(d) $H_0 : \alpha_i = 0$ for all i, H_1 : not all α_i equal 0.
Decision rule: reject H_0 if $F^* = MSA/MSE > F(0.95; 1, 54) = 4.02$.
Since $F^* = MSA/MSE = 4.36$, we reject H_0.
$H_0 : \beta_j = 0$ for all j, H_1 : not all β_j equal 0.
Decision rule: reject H_0 if $F^* = MSB/MSE > F(0.95; 2, 54) = 3.17$
Since $F^* = MSB/MSE = 15.81$, we reject H_0.
(e) The overall level of significance, using Kimball inequality, has a bound

$$\alpha \leq 1 - (0.95)(0.95)(0.95) = 0.143.$$

(f) The results from parts (c) and (d) confirm the preliminary assessments in part (a)

19.34

(a) $s(\bar{Y}'_{22.}) = 0.1006$, $t(0.975; 54) = 2.005$.A 95% confidence interval for μ_{22} is $\bar{Y}'_{22.} \pm t(0.975; 54)\ s(\bar{Y}'_{22.})$, i.e., $(0.3793, 0.7827)$.

We have a 95% confidence that μ_{22} is in the interval $(0.3793, 0.7827)$.

(b) $\hat{D} = \bar{Y}'_{23.} - \bar{Y}'_{21.} = 0.4682$, $s(\hat{D}) = \sqrt{(2/n)MSE} = 0.1423$, $t(0.975; 54) = 2.005$.

A 95% confidence interval for D is $\hat{D} \pm t(0.975; 54)s(\hat{D})$, i.e., $(0.1829, 7535)$. Since this interval does not include 0, we can conclude that μ_{23} and μ_{21} are different at a 0.05 level of significance.

(c) We have $\bar{Y}'_{1..} = 0.7867$, $\bar{Y}'_{2..} = 0.6152$; $\bar{Y}'_{.1.} = 0.4209$, $\bar{Y}'_{.2.} = 0.6955$, $\bar{Y}'_{.3.} = 0.9866$.

The plots are not given here. But the values given above suggest that factor B main effects are present, whereas it is unclear if factor A main effects are present.

(d) Total number of intervals to be constructed is 4 - one for pairwise comparisons for factor A and 3 for pairwise comparisons for factor B. Since $\alpha = 0.1$, the multipliers are

$$\begin{aligned}
\text{Bonferroni:} \quad & B = t(1-\alpha/8; 54) = t(0.9875; 54) = 2.306, \\
\text{Tukey: factor A,} \quad & q(1-\alpha/2; 2, 54) = q(0.95; 2, 54) = 2.84,\ T = 2.008 \\
\text{Tukey: factor B,} \quad & q(1-\alpha/2; 3, 54) = q(0.95; 3, 54) = 3.41,\ T = 2.411, \\
\text{Scheffe} \ : \quad & S = \sqrt{(a+b-2)F(1-\alpha; a+b-2; 54)} = \sqrt{3F(0.95; 3, 54)} = 2.569.
\end{aligned}$$

Bonferroni seems to be the preferred method. Even though Tukey method seems to be the best for factor A comparisons, but it suffers from having a higher multiplier than Bonferroni for factor B comparisons.

(e) Let

$$D_1 = \mu_{1.} - \mu_{2.},\ D_2 = \mu_{.1} - \mu_{.2},\ D_3 = \mu_{.1} - \mu_{.3},\ D_4 = \mu_{.2} - \mu_{.3},$$

Note that $s(\hat{D}_1) = 0.0822$ and $s(\hat{D}_i) = 0.1006$, $i = 2, 3, 4$. The Bonferroni multiplier is $B - 2.306$. Simultaneous 90% confidence intervals (Bonferroni) are

$$\begin{aligned}
D_1 &: \ 0.1715 \pm (2.306)(0.0822), \text{ i.e., } (-0.018, 0.361), \\
D_2 &: \ -0.2746 \pm (2.306)(0.1006), \text{ i.e., } (-0.507, -0.043), \\
D_3 &: \ -0.5657 \pm (2.306)(0.1006), \text{ i.e., } (-0.798, -0.334), \\
D_4 &: \ -0.2911 \pm (2.306)(0.1066), \text{ i.e., } (-0.523, -0.059).
\end{aligned}$$

Since the first interval contains 0, presence of factor A main effect is in doubt. The last three intervals all exclude 0 and hence the evidence points to all the three factor B means are different. These are consistent with the preliminary evidence in part (c).

(f) Here

$$L = 0.3\mu_{.1} + 0.4\mu_{.2} + 0.3\mu_{.3},\ \hat{L} = 0.0415,\ t(0.975; 54) = 2.005..$$

95% confidence interval for L is $\hat{L} + t(0.975; 54)s(\hat{L})$, i.e., $(0.6172, 0.7836)$. In original units this interval is $(3.142, 5.076)$. Yes, the evidence points that the mean number of days is less than 7.

19.41. We would like to have $Bs(\hat{D}_i) \leq 0.2$, $i = 1, \ldots, 4$. Since $a = 2$ and $b = 3$, we have $s(\hat{D}_1) \leq s(\hat{D}_i), i = 2, 3, 4$. Thus we need to have $Bs(\hat{D}_i) \leq 0.2$, i.e., $B^2 s^2(\hat{D}_i) \leq (0.2)^2, i = 2, 3, 4$, i.e., (taking $s^2(\hat{D}_i) \approx 2\sigma^2/(na)$)

$$\begin{aligned} B^2 \frac{2\sigma^2}{na} &\leq (0.2)^2, \text{ i.e.,} B^2 \frac{(2)\sigma^2}{n(2)} \leq (0.2)^2, \\ \text{i.e., } n &\geq (B\sigma/0.2)^2 = 13.61. \end{aligned}$$

So we we should have $n \geq 14$.

19.47

Note that

$$\bar{Y}_{ij\cdot} - \bar{Y}_{\cdots} = \hat{\alpha}_i + \hat{\beta}_j + \widehat{(\alpha\beta)}_{ij}.$$

Now

$$SSTR = n \sum \sum (\bar{Y}_{ij\cdot} - \bar{Y}_{\cdots})^2,$$

and hence

$$\begin{aligned} \sum \sum (\bar{Y}_{ij\cdot} - \bar{Y}_{\cdots})^2 &= \sum \sum \left[\hat{\alpha}_i + \hat{\beta}_j + \widehat{(\alpha\beta)}_{ij} \right]^2 \\ &= \sum \sum \hat{\alpha}_i^2 + \sum \sum \hat{\beta}_j^2 + \sum \sum \widehat{(\alpha\beta)}_{ij}{}^2 \\ &\quad + 2 \sum \sum \hat{\alpha}_i \hat{\beta}_j + 2 \sum \sum \hat{\alpha}_i \widehat{(\alpha\beta)}_{ij} + 2 \sum \sum \hat{\beta}_j \widehat{(\alpha\beta)}_{ij}. \end{aligned}$$

Note that each of three cross product terms are zero and thus we have

$$\begin{aligned} \sum \sum (\bar{Y}_{ij\cdot} - \bar{Y}_{\cdots})^2 &= \sum \sum \hat{\alpha}_i^2 + \sum \sum \hat{\beta}_j^2 + \sum \sum \widehat{(\alpha\beta)}_{ij}{}^2 \\ &= b \sum \hat{\alpha}_i^2 + a \sum \hat{\beta}_j^2 + \sum \sum \widehat{(\alpha\beta)}_{ij}{}^2. \end{aligned}$$

The result now follows since

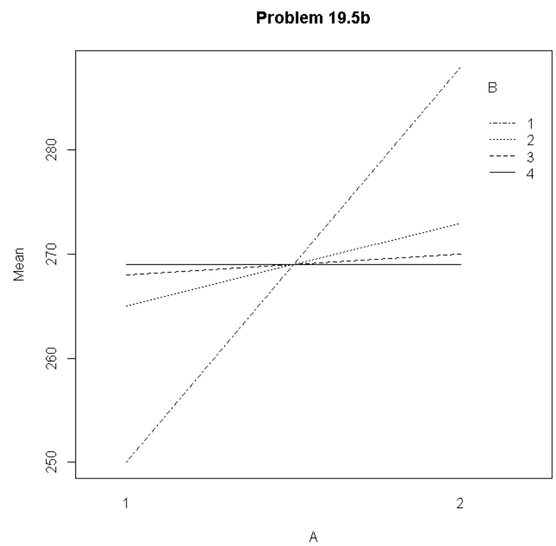
$$\begin{aligned} SSTR &= n \sum \sum (\bar{Y}_{ij\cdot} - \bar{Y}_{\cdots})^2 \\ &= nb \sum \hat{\alpha}_i^2 + na \sum \hat{\beta}_j^2 + n \sum \sum \widehat{(\alpha\beta)}_{ij}{}^2 \\ &= S \cdot SA + SSB + SSAB. \end{aligned}$$

19.48. Note that

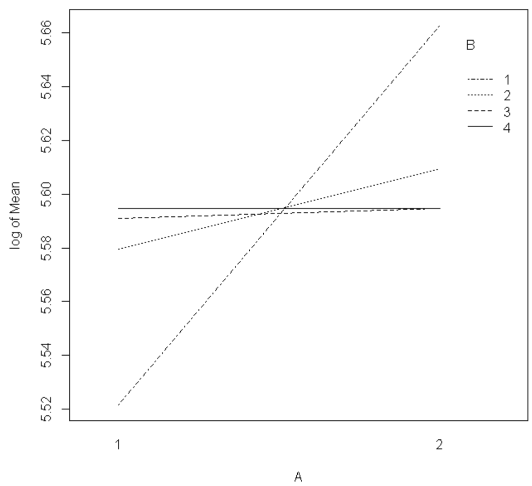
$$\begin{aligned} \bar{Y}_{\cdot j \cdot} &= \mu_{\cdot\cdot} + \beta_j + \bar{\varepsilon}_{\cdot j \cdot} = \mu_{\cdot j} + \bar{\varepsilon}_{\cdot j \cdot\cdot}, \\ \hat{L} &= \sum c_j \bar{Y}_{\cdot j \cdot} = \sum c_j \mu_{\cdot j} + \sum c_j \bar{\varepsilon}_{\cdot j \cdot} = L + \sum c_j \bar{\varepsilon}_{\cdot j \cdot}. \end{aligned}$$

Since $\{\bar{\varepsilon}_{\cdot j \cdot}\}$ are iid $N(0, \sigma^2/(na))$, we have

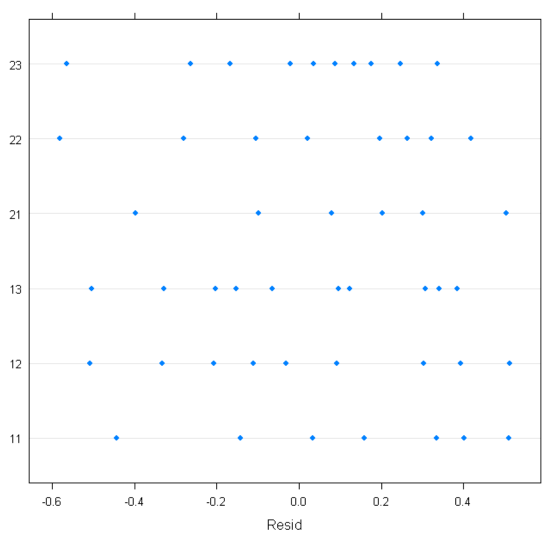
$$\begin{aligned} E(\hat{L}) &= L, \text{ and} \\ Var(\hat{L}) &= \sum c_j^2 Var(\bar{\varepsilon}_{\cdot j \cdot}) = \sum c_j^2 \sigma^2/(na) = \frac{\sigma^2}{na} \sum c_j^2. \end{aligned}$$



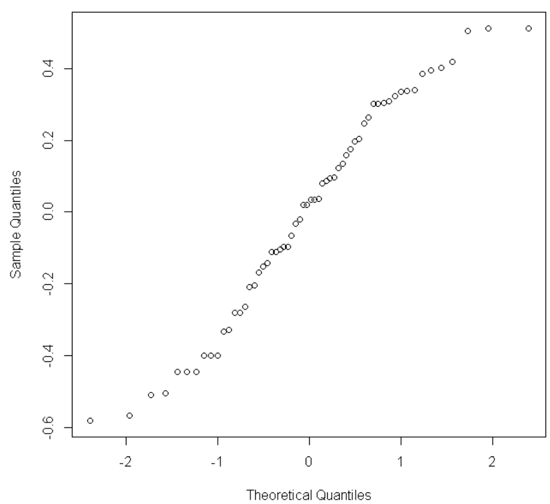
Problem 19.5c



Problem 19.18b



Problem 19.18c



Problem 19.19a