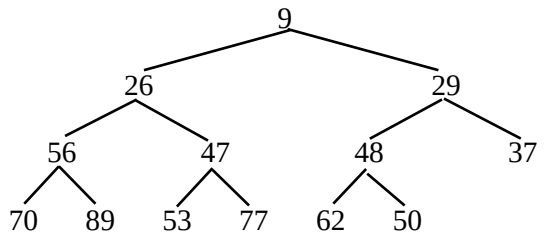
ECS 60 Written Homework #5: Priority Queues and Disjoint Sets (18 points) Winter 2016

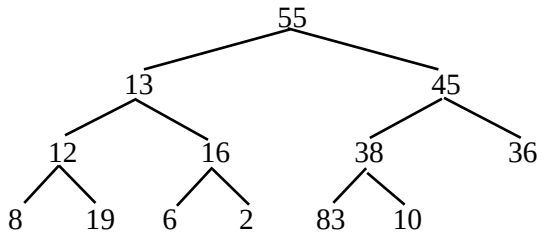
Due Friday, February 19th, 4:00 pm in 2131 Kemper

1. (4 points, 2 points each) With the following binary heap as a starting point for each of the following, show the resulting heap after completing both of the specified operations. You should draw only one heap for each part.



   a. Insert 20, insert 4
   b. DeleteMin, deleteMin.

2. (2 points) Assuming that the following values are in an array, use buildHeap described in section 6.3 (pp. 255 -257) in the text to create a binary heap. You need only show the final binary heap.



3. (2 points) "Suppose the internal nodes of two binary trees, T_1 and T_2 respectively, hold items that satisfy the heap-order property. Describe a method for combining these two trees into a tree T, whose internal nodes hold the union of te items in T_1 and T_2, and also satisfy the heap-order property. Your algorithm should run in time $O(h_1 + h_2)$ where h_1 and h_2 are the respective heights of T_1 and T_2." (from Michael T. Goodrich, Roberto Tamassia, and David Mount, *Data Structures & Algorithms, Second Edition*, Hoboken, NJ, John Wiley & Sons, 201., p.365.) Note that these are simply binary trees with pointers, and not heaps implemented as complete binary trees in vectors.

4. (8 points, 2 points each) Given the following set of operations, show the final disjoint set tree for each of the following union strategies.
   union(A, D), union(C, B), union (F, E), union (G, C), union(D, G), find(A), union(H, E), union(E, G), find(E)

   a. Arbitrary, where the set specified first in the union will always be the root of the combined set.
   b. Union by size. When the sets are the same size, the set with a root that is closer to the beginning of the alphabet should be the root of the merged set.
   c. Union by height. When the sets are the same height, the set with a root that is closer to the beginning of the alphabet should be the root of the merged set.
   d. Arbitrary with path compression. The set specified first in the union will always be the root of the merged set.

5. (2 points) Exercise 8.8 from page 375 of our text. "Prove that for the mazes generated by the algorithm in Section 8.7, the path from the starting to ending points is unique."