Practice Midterm I

STA 131B
Winter 2016
University of California, Davis

Exam Rules: This exam is closed book and closed notes. Use of calculators, cell phones or other communication devices is not allowed. You must show all of your work to receive credit. You will have 50 minutes to complete the exam.

Note: The practice exam is longer than the actual exam so you will have more to practice.

Name : _____

ID : _____

1. Suppose that the number of defects in a 1200-foot roll of magnetic recording tape has a Poisson distribution for which the value of the mean (θ) is unknown.

   a) Suppose five rolls of this tape are selected at random. Determine the joint p.d.f. $f(x_1, \ldots, x_5|\theta)$ of the five rolls.

   b) Suppose the number of defects found on the rolls are $2, 2, 6, 0$ and 3. If the prior distribution of θ is the gamma distribution with parameters $\alpha = 3$ and $\beta = 1$, find the posterior distribution of θ.

   c) Find the Bayes estimator with respect to the squared error loss function.

2. Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution for which the p.d.f. $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, suppose the value of θ is unknown, and is a fixed (non-random) value $\theta > 0$.

a) Determine the joint p.d.f. of $X_1, \ldots, X_n$.

b) Find the M.L.E. of θ.

c) Find the M.L.E. of θ^2.

3. Suppose that $X_1, \ldots, X_n$ form a random sample from the Poisson distribution with parameter θ.

    a) Determine the method of moments estimator of θ based on the first moment.

    b) Show that your estimator in part (a) is the same as the M.L.E.

    c) Find a method of moment estimator for the standard deviation of this Poisson distribution.

4. Suppose that the proportion θ of defective items in large shipment is unknown and that the prior distribution of θ is the beta distribution with parameters 1 and 10. Assume in a random sample of 20 items one find that 1 item is defective.

a) What is the expected value and variance of the prior distribution?

b) What is the posterior distribution?

c) What is the Bayes estimator for θ if one uses the quadratic loss function?

d) Find the M.L.E. for θ. Is it the same as the Bayes estimator?

e) Suppose that you change the sampling plan and will keep on sampling until you find 3 defective items. Let X be the number of non-defective items until this happens. Derive the M.L.E. and Bayes estimator again.