ECS 122A Midterm Exam I Winter 2016 **Solution**

1. (a) (15 pts)

$$O(g(n)) = \{f(n) : \exists c, n_0 \quad \text{such that} \quad 0 \le f(n) \le cg(n) \quad \text{for} \quad \forall n \ge n_0\}$$

(b) (20 pts) Need to find constants c and n_0 such that $(n+3)^2 \le cn^2$ for all $n \ge n_0$.

For $n \ge 1$, we have

$$(n+3)^2 = n^2 + 6n + 9 \le n^2 + 6n^2 + 9n^2 = 16n^2$$

Therefore, we can choose $c = 16$ and $n_0 = 1$. **Other pairs (c, n_0) are possible.**

2. (25 pts) Here is the ordering

$$\left(\frac{3}{4}\right)^n, \quad \lg^2 n, \quad n^3 = 8^{\lg n}, \quad n!$$

3. (a) (10 pts) By the iteration

$$T(n) = 3 \cdot T(n-3) = 3^2 T(n - 3 \cdot 2) = \cdots = 3^k T(n - 3 \cdot k) = \Theta(3^k) = \Theta(3^{n/3})$$

(b) (10 pts) Since $\frac{n^{\log_2 3}}{n} = n^{\log_2 3 - 1}$ and $\log_2 3 - 1 > 0$, by case 1 of the master theorem, we have $T(n) = \Theta(n^{\log_2 3})$

(c) (10 pts) Since

$$\frac{\sqrt{n}}{n^{\log_4 2}} = \frac{\sqrt{n}}{n^{1/2}} = 1.$$

By case 2 of the master theorem, we have $T(n) = \Theta(n^{\log_4 2} \lg n) = \Theta(\sqrt{n} \lg n)$.

4. (a) (10 pts) This is false. Since when $n \bmod 2 \ne 0$, $T(n)$ is $\Theta(n^2)$, which cannot be bounded by $O(n \log n)$.

(b) (10 pts) This is true. $\Omega(n \log n)$ is an asymptotic lower bound for `HybridSort`.

(c) (10 pts) This is false. Since when $n \bmod 2 \ne 0$, $T(n)$ is $\Theta(n^2)$, which cannot be bounded by $\Theta(n \log n)$.

5. (a) (20 pts) The important part of this question is figuring out what the **OR** operation does in the worst case. If the first `FIND` returns `FALSE`, then the second `FIND` must be run to determine the final value returned. In the worst case, both `FIND` functions are executed. The `if ... else ..` statement is a constant time operation for each iteration

$$T(n) = 2T(n/2) + c.$$

where c is a constant.

(b) (10 pts) By the master theorem,

$$T(n) = \Theta(n).$$