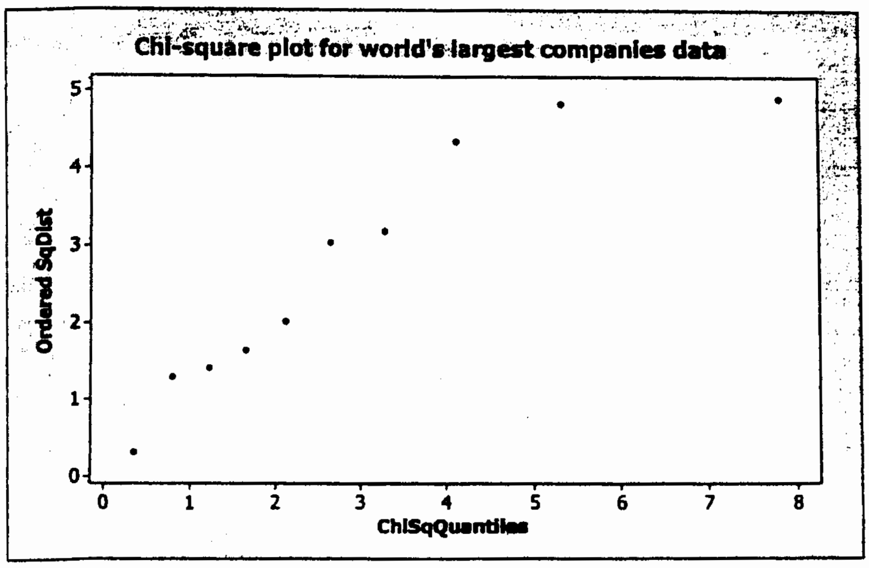
Homework #5

4.25 The chi-square plot for the world's largest companies data is shown below. The plot is reasonably straight and it would be difficult to reject multivariate normality given the small sample size of $n = 10$. Information leading to the construction of this plot is also displayed.



$$\bar{\mathbf{x}} = \begin{bmatrix} 155.6 \\ 14.7 \\ 710.9 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 7476.5 & 303.6 & -35576 \\ 303.6 & 26.2 & -1053.8 \\ -35576 & -1053.8 & 237054 \end{bmatrix}$$

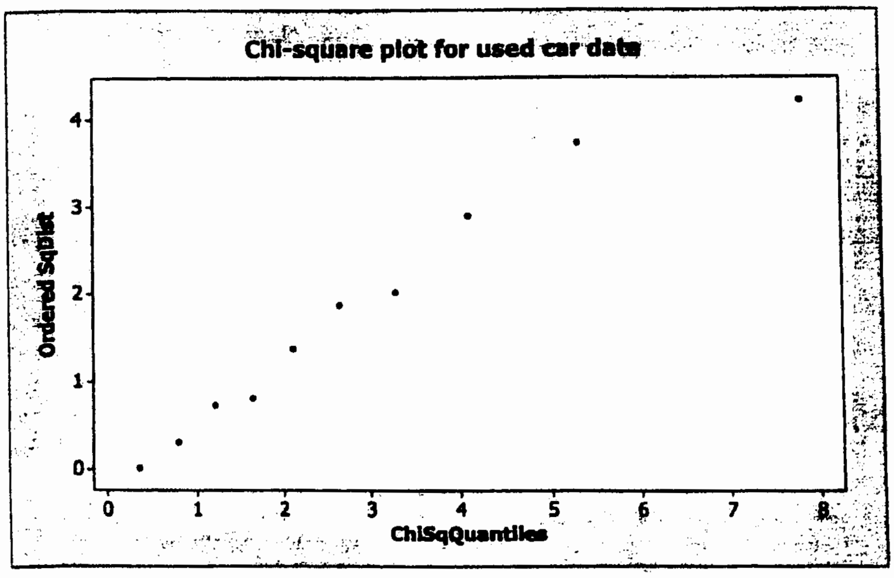
Ordered SqDist	Chi-square Quantiles
.3142	.3518
1.2894	.7978
1.4073	1.2125
1.6418	1.6416
2.0195	2.1095
3.0411	2.6430
3.1891	3.2831
4.3520	4.1083
4.8365	5.3170
4.9091	7.8147

4.26 (a) $\bar{\mathbf{x}} = \begin{bmatrix} 5.20 \\ 12.48 \end{bmatrix}$, $\mathbf{S} = \begin{bmatrix} 10.6222 & -17.7102 \\ -17.7102 & 30.8544 \end{bmatrix}$, $\mathbf{S}^{-1} = \begin{bmatrix} 2.1898 & 1.2569 \\ 1.2569 & .7539 \end{bmatrix}$

Thus d_j^2 = 1.8753, 2.0203, 2.9009, .7353, .3105, .0176, 3.7329, .8165, 1.3753, 4.2153

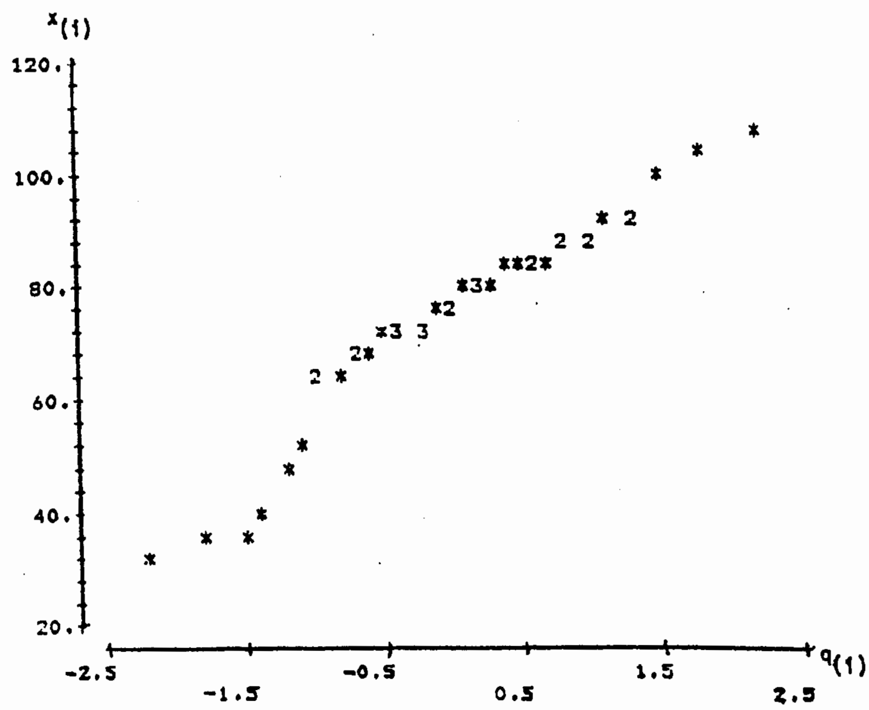
(b) Since $\chi_2^2(.5) = 1.39$, 5 observations (50%) are within the 50% contour.

(c) The chi-square plot is shown below.



(d) Given the results in parts (b) and (c) and the small number of observations (n = 10), it is difficult to reject bivariate normality.

4.28 Q-Q plot is shown below.



Since r_Q = .970 < .972 (See Table 4.2 for n = 40 and α = .05), we would reject the hypothesis of normality at the 5% level.

4.29

(a).

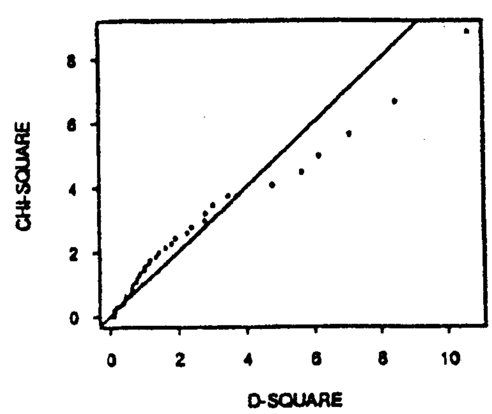
$$\bar{x} = \begin{pmatrix} 10.046719 \\ 9.4047619 \end{pmatrix}, \quad S = \begin{pmatrix} 11.363531 & 3.126597 \\ & 30.978513 \end{pmatrix}.$$

Generalized distances are as follows;

0.4607	0.6592	2.3771	1.6283	0.4135	0.4761	1.1849
10.6392	0.1388	0.8162	1.3566	0.6228	5.6494	0.3159
0.4135	0.1225	0.8988	4.7647	3.0089	0.6592	2.7741
1.0360	0.7874	3.4438	6.1489	1.0360	0.1388	0.8856
0.1380	2.2489	0.1901	0.4607	1.1472	7.0857	1.4584
0.1225	1.8985	2.7783	8.4731	0.6370	0.7032	1.8014

(b). The number of observations whose generalized distances are less than $\chi_2(0.5) = 1.39$ is 26. So the proportion is 26/42=0.6190.

(c). CHI-SQUARE PLOT FOR (X1 X2)



5.1 a) $\bar{\mathbf{x}} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$; $S = \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix}$

$T^2 = 150/11 = 13.64$

b) T^2 is $3F_{2,2}$ (see (5-5))

c) $H_0: \boldsymbol{\mu}' = [7,11]$

$\alpha = .05$ so $F_{2,2}(.05) = 19.00$

Since $T^2 = 13.64 < 3F_{2,2}(.05) = 3(19) = 57$; do not reject H_0 at the $\alpha = .05$ level

5.2. Let $\underline{Y} = C\underline{X}$. Then,

the new data matrix is $CX = \begin{bmatrix} -3 & 15 \\ 4 & 16 \\ 5 & 11 \end{bmatrix}$.

$\bar{\underline{y}} = \begin{bmatrix} 2 \\ 14 \end{bmatrix}$ and $S_{\underline{Y}} = \begin{bmatrix} 19 & -5 \\ -5 & 7 \end{bmatrix}$

This leads to $S_{\underline{Y}}^{-1} = \begin{bmatrix} 7/108 & 5/108 \\ 5/108 & 19/108 \end{bmatrix}$, $\mu_{o\underline{Y}} = C\mu_o = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$

$$T^2 = n(\bar{\underline{y}} - \underline{\mu}_{o\underline{Y}})' S^{-1} (\bar{\underline{y}} - \underline{\mu}_{oY})$$

$$= 3\,[-2 \quad 0] \begin{bmatrix} 7/108 & 5/108 \\ 5/108 & 19/108 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \end{bmatrix} = \frac{3}{108}[-14 \quad -10]\begin{bmatrix} -2 \\ 0 \end{bmatrix} = \frac{3(28)}{108} = \frac{7}{9}$$

5.3 a) $$T^2 = \frac{(n-1)\left|\sum_{j=1}^{n}(\mathbf{x}_j-\boldsymbol{\mu}_0)(\mathbf{x}_j-\boldsymbol{\mu}_0)'\right|}{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\bar{\mathbf{x}})(\mathbf{x}_j-\bar{\mathbf{x}})'\right|} - (n-1) = \frac{3(244)}{44} - 3 = 13.64$$

b) $$\Lambda = \left(\frac{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\bar{\mathbf{x}})(\mathbf{x}_j-\bar{\mathbf{x}})'\right|}{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\boldsymbol{\mu}_0)(\mathbf{x}_j-\boldsymbol{\mu}_0)'\right|}\right)^{n/2} = \left(\frac{44}{244}\right)^2 = .0325$$

Wilks' lambda $= \Lambda^{2/n} = \Lambda^{1/2} = \sqrt{.0325} = .1803$