# 0-1 Knapsack problem[1]

- Given $n$ items $\{1, 2, \ldots, n\}$
- Item $i$ is worth $v_i$, and weight $w_i$
- Find a most valuable subset of items with total weight $\leq W$

0-1 knapsack problem can be expressed as

*Find a subset $\mathcal{S} \subseteq \{1, 2, \ldots, n\}$ such that*

$$\text{maximize} \quad \sum_{i \in \mathcal{S}} v_i$$

$$\text{subject to} \quad \sum_{i \in \mathcal{S}} w_i \leq W$$

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[1]Have to either take an item or not take it – cann't take part of it.

# Greedy solution strategies

Three possible greedy approaches:

1. Greedy by highest value $v_i$

2. Greedy by least weight $w_i$

3. Greedy by largest value density $\dfrac{v_i}{w_i}$

*All three appraches generate feasible solutions. However, we cannot guarantee that any of them will always generate an optimal solution!*

# Example

| $i$ | $v_i$ | $w_i$ | $v_i/w_i$ |
|-----|-------|-------|-----------|
| 1   | 6     | 1     | 6         |
| 2   | 10    | 2     | 5         |
| 3   | 12    | 3     | 4         |

Total weight $W = 5$

Greedy by value density $v_i/w_i$:

- take items 1 and 2.
- value = 16, weight = 3
- Leftover capacity = 2

Optimal solution

- take items 2 and 3.
- value = 22, weight = 5
- no leftover capacity

Question: how about greedy by highest value? by least weight?