ECS 122A Midterm Exam II, February 26, 2016 **Solutions**

1. (a) The (global) optimal solution to the problem *contains within it* an (local) optimal solutions to subproblems. For example, LCS.

   Note: this is one of the hallmarks of the applicability of dynamic programming.

   (b) $O(\underline{n \cdot 2^m})$

   $O(\underline{m \cdot n})$

   (c) $(A_1 A_2)((A_3 A_4)((A_5(A_6 A_7)))$

2. (a) **False**. It's only true for a connected graph. First, it's easy to see $|E| = O(|V|^2)$. If the graph is connected, $|E| \geq |V| - 1 = \Omega(|V|)$. Therefore, $\Theta(\lg E) = \Theta(\lg V)$.

   (b) **False**. In DFS, the vertices are processed in a LIFO order (stack).

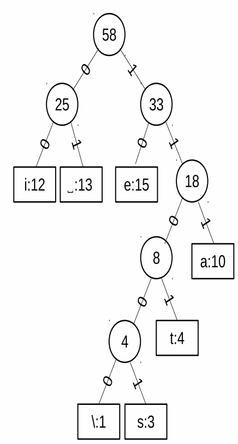
   (c) **False**. It's true for a DAG (Directed Acyclic Graph).

3. Run DFS, if find a back edge, there is a cycle. The run time in this case is $O(|V|)$, not $O(|V| + |E|)$! If ever see $|V|$ distinct edges, must have seen a back edge because in acyclic undirected forest, $|E| \leq |V| - 1$.

4.

char	frequency	code	# bits
a	10	111	3
e	15	10	2
i	12	00	2
s	3	11001	5
t	4	1101	4
⊔	13	01	2
\	1	11000	5

Show your work. When you're finished, fill in the column of the table labeled "code" with the binary codeword for each character.



5. (a) For example, for the change of $N = 55$ cents,

greedy:	$50 + 5 \cdot 1$	6 coins
optimal:	$25 + 3 \cdot 10$	4 coins

(b) $\textsc{Change}(N) = \min\{\textsc{Coins}(N), 1 + \textsc{Coins}(N - 50), 2 + \textsc{Coins}(N - 100)\}$

(c) The optimal solution has to either have 0, 1 or 2 half-dollars in it. The solution finds the best of options. Running time $= 3 \cdot O(N) = O(N)$.