**4.18** By Result 4.11 we know that the maximum likelihood estimates of $\underset{\sim}{\mu}$ and $\Sigma$ are $\bar{\underset{\sim}{x}} = [4,6]'$ and

$$\frac{1}{n}\sum_{j=1}^{n}(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})' = \frac{1}{4}\left\{\left(\begin{bmatrix}3\\6\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)\left(\begin{bmatrix}3\\6\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)'+\left(\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)\left(\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)'\right.$$

$$\left.+\left(\begin{bmatrix}5\\7\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)\left(\begin{bmatrix}5\\7\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)'+\left(\begin{bmatrix}4\\7\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)\left(\begin{bmatrix}4\\7\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}\right)'\right\}$$

$$= \frac{1}{4}\left\{\begin{bmatrix}-1\\0\end{bmatrix}\begin{bmatrix}-1 & 0\end{bmatrix}+\begin{bmatrix}0\\-2\end{bmatrix}\begin{bmatrix}0 & -2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix}\right\}$$

$$= \frac{1}{4}\begin{bmatrix}2 & 1\\1 & 6\end{bmatrix}$$

**4.19** a) By Result 4.7 we know that $(\underset{\sim}{X}_1-\underset{\sim}{\mu})'\,\Sigma^{-1}(\underset{\sim}{X}_1-\underset{\sim}{\mu}) \sim \chi^2_6$

b) From (4-23), $\bar{\underset{\sim}{X}} \sim N_6(\underset{\sim}{\mu},\frac{1}{20}\Sigma)$. Then $\bar{\underset{\sim}{X}}-\underset{\sim}{\mu} \sim N_6(\underset{\sim}{0},\frac{1}{20}\Sigma)$ and finally $\sqrt{20}\,(\bar{\underset{\sim}{X}}-\underset{\sim}{\mu}) \sim N_6(\underset{\sim}{0},\Sigma)$

c) From (4-23), 19S has a Wishart distribution with 19 d.f.

4.21 (a) $\overline{X}$ is distributed $N_4(\mu, n^{-1}\Sigma)$

(b) $X_1-\mu$ is distributed $N_4(0,\Sigma)$ so $(X_1-\mu)'\Sigma^{-1}(X_1-\mu)$ is distributed as chi-square with p degrees of freedom.

(c) Using Part a),

$$(\overline{X}-\mu)'(n^{-1}\Sigma)^{-1}(\overline{X}-\mu) = n(\overline{X}-\mu)'\Sigma^{-1}(\overline{X}-\mu)$$

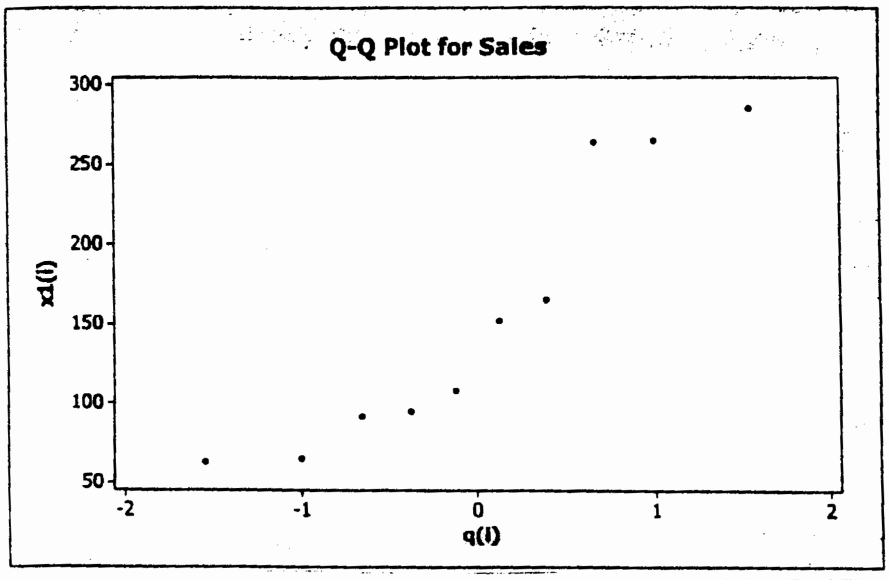
is distributed as chi-square with p degrees of freedom.

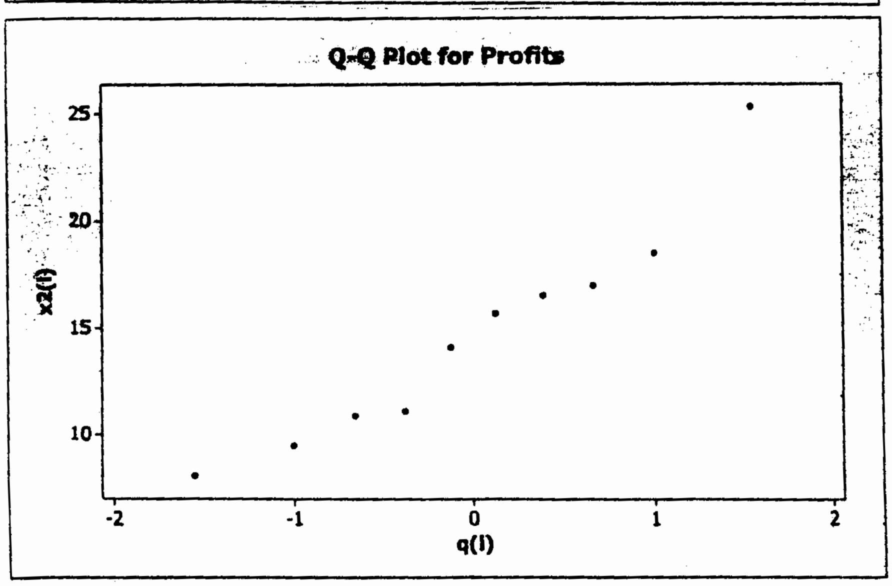
(d) Approximately distributed as chi-square with p degrees of freedom. Since the sample size is large, $\Sigma$ can be replaced by S.

**4.22** a) We see that $n = 75$ is a sufficiently large sample (compared with p) and apply Result 4.13 to get $\sqrt{n}(\underset{\sim}{X}-\underset{\sim}{\mu})$ is approximately $N_p(\underset{\sim}{0},\Sigma)$ and that $\bar{\underset{\sim}{X}}$ is approximately $N_p(\underset{\sim}{\mu},\frac{1}{n}\Sigma)$.

b) By (4-28) we conclude that $\sqrt{n}(\underset{\sim}{X}-\underset{\sim}{\mu})'S^{-1}(\bar{\underset{\sim}{X}}-\underset{\sim}{\mu})$ is approximately $\chi^2_p$.

4.24 (a) $Q$-$Q$ plots for sales and profits are given below. Plots not particularly straight, although $Q$-$Q$ plot for profits appears to be "straighter" than plot for sales. Difficult to assess normality from plots with such a small sample size ($n$ = 10).




(b) The critical point for $n$ =10 when $\alpha$ = .10 is .9351. For sales, $r_Q$ =.940 and for profits, $r_Q$ =.968. Since the values for both of these correlations are greater than .9351, we cannot reject normality in either case.