Homework #6

5.5 $H_0: \underline{\mu}' = [.55, .60]$; $T^2 = 1.17$

$\alpha = .05$; $F_{2,40}(.05) = 3.23$

Since $T^2 = 1.17 < \frac{2(41)}{40} F_{2,40}(.05) = 2.05(3.23) = 6.62$,

we do not reject H_0 at the $\alpha = .05$ level. The result is consistent with the 95% confidence ellipse for $\underline{\mu}$ pictured in Figure 5.1 since $\underline{\mu}' = [.55, .60]$ is <u>inside</u> the ellipse.

5.8 $$\underline{a} = S^{-1}(\bar{\underline{x}} - \underline{\mu}_0) = \begin{bmatrix} 227.273 & -181.818 \\ -181.818 & 212.121 \end{bmatrix} \left(\begin{bmatrix} .564 \\ .603 \end{bmatrix} - \begin{bmatrix} .55 \\ .60 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2.636 \\ -1.909 \end{bmatrix}$$

$$t^2 = \frac{n(\underline{a}'(\bar{\underline{x}} - \underline{\mu}_0))^2}{\underline{a}' S \underline{a}} = \frac{42\left([2.636 \;\; -1.909] \begin{bmatrix} .014 \\ .003 \end{bmatrix}\right)^2}{[2.636 \;\; -1.909] \begin{bmatrix} .0144 & .0117 \\ .0117 & .0146 \end{bmatrix} \begin{bmatrix} 2.636 \\ -1.909 \end{bmatrix}} = 1.31 = T^2$$

5.10 a) 95% T^2 simultaneous confidence intervals:

Lngth2: (130.65, 155.93) Lngth4: (160.33, 185.95)
Lngth3: (127.00, 191.58) Lngth5: (155.37, 198.91)

b) 95% T^2 simultaneous intervals for change in length (ΔLngth):

ΔLngth2-3: (–21.24, 53.24)
ΔLngth3-4: (–22.70, 50.42)
ΔLngth4-5: (–20.69, 28.69)

c) 95% confidence region determined by all μ_{2-3}, μ_{4-5} such that

$$\left(16 - \mu_{2-3}, 4 - \mu_{4-5}\right) \begin{bmatrix} .011024 & .009386 \\ .009386 & .025135 \end{bmatrix} \begin{pmatrix} 16 - \mu_{2-3} \\ 4 - \mu_{4-5} \end{pmatrix} \le 72.96/7 = 10.42$$

where μ_{2-3} is the mean increase in length from year 2 to 3, and μ_{4-5} is the mean increase in length from year 4 to 5.

Beginning at the center $\underset{\sim}{\bar{x}}' = (16,4)$, the axes of the 95% confidence ellipsoid are:

$$\text{major axis} \qquad \pm\sqrt{157.8}\sqrt{72.96}\begin{pmatrix} .895 \\ -.447 \end{pmatrix}$$

$$\text{minor axis} \qquad \pm\sqrt{33.53}\sqrt{72.96}\begin{pmatrix} .447 \\ .895 \end{pmatrix}$$

(See confidence ellipsoid in part e.)

d) Bonferroni 95% simultaneous confidence intervals ($m = 7$):

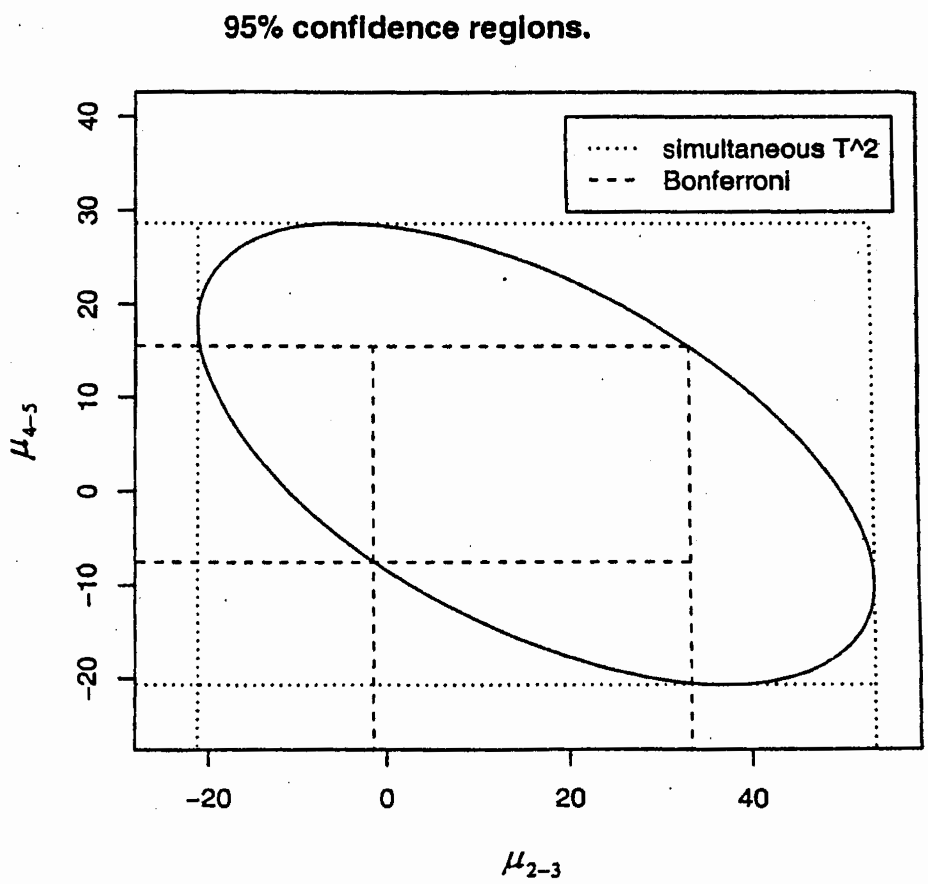
Lngth2: (137.37, 149.21)
Lngth3: (144.18, 174.40)

Lngth4: (167.14, 179.14)
Lngth5: (166.95, 187.33)

ΔLngth2-3: (−1.43, 33.43)
ΔLngth3-4: (−3.25, 30.97)

ΔLngth4-5: (−7.55, 15.55)

e) The Bonferroni 95% confidence rectangle is much smaller and more informative than the 95% confidence ellipse.



5.15

(a).

$$\begin{aligned} E(X_{ij}) &= (1)p_i + (0)(1-p_i) = p_i. \\ Var(X_{ij}) &= (1-p_i)^2 p_i + (0-p_i)^2(1-p_i) = p_i(1-p_i) \end{aligned}$$

(b). $Cov(X_{ij}, X_{kj}) = E(X_{ij}X_{ik}) - E(X_{ij})E(X_{kj}) = 0 - p_i p_k = -p_i p_k.$

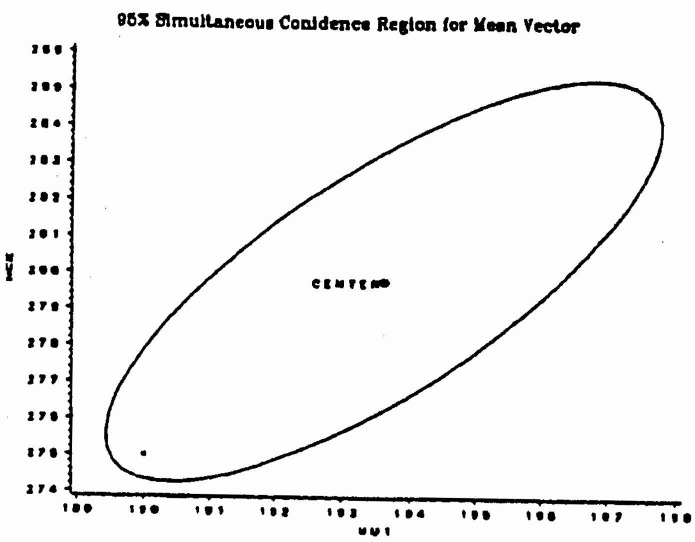
5.16

(a). Using $\hat{p}_j \pm \sqrt{\chi_4^2(0.05)}\sqrt{\hat{p}_j(1-\hat{p}_j)/n}$, the 95 % confidence intervals for p_1, p_2, p_3, p_4, p_5 are

(0.221, 0.370), (0.258, 0.412), (0.098, 0.217), (0.029, 0.112), (0.084, 0.198) respectively.

(b). Using $\hat{p}_1 - \hat{p}_2 \pm \sqrt{\chi_4^2(0.05)}\sqrt{(\hat{p}_1(1-\hat{p}_1) + \hat{p}_2(1-\hat{p}_2) - 2\hat{p}_1\hat{p}_2)/n}$, the 95 % confidence interval for $p_1 - p_2$ is $(-0.118, 0.0394)$. There is no significant difference in two proportions.

5.20 **(a).** Yes, they are plausible since the hypothesized vector $\underset{\sim}{\mu_0}$ (denoted as * in the plot) is inside the 95% confidence region.

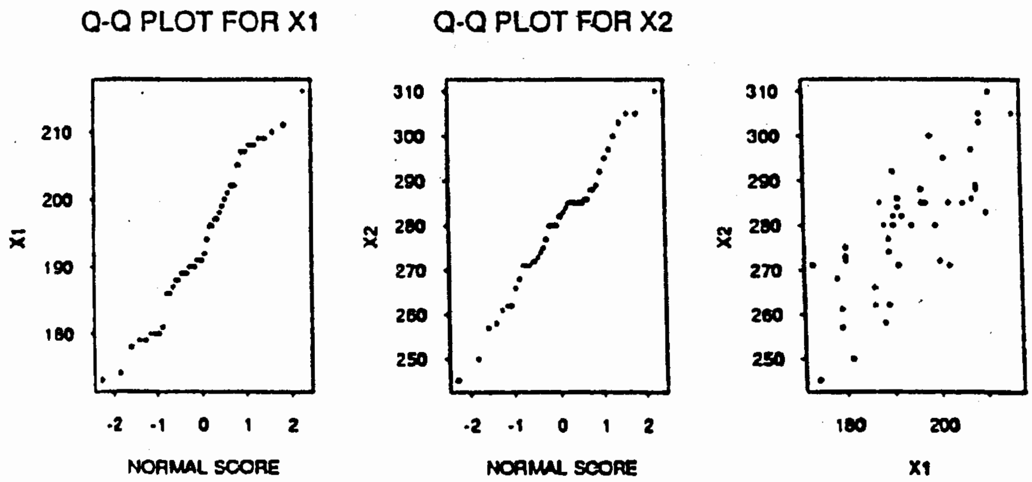


(b).

	LOWER	UPPER
Bonferroni C. I.:	189.822	197.423
	274.782	284.774
Simultaneous C. I.:	189.422	197.823
	274.256	285.299

Simultaneous confidence intervals are larger than Bonferroni's confidence intervals. Simultaneous confidence intervals will touch the simultaneous confidence region from outside.

(c). Q-Q plots suggests non-normality of (X_1, X_2). Could try transforming X_1.



6.5 a) $H_0: C\underset{\sim}{\mu} = \underset{\sim}{0}$ where $C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$, $\underset{\sim}{\mu}' = [\mu_1, \mu_2, \mu_3]$.

$$C\underset{\sim}{\bar{x}} = \begin{bmatrix} -11.2 \\ 6.9 \end{bmatrix}, \quad CSC' = \begin{bmatrix} 55.5 & -32.6 \\ -32.6 & 66.4 \end{bmatrix}$$

$$T^2 = n(C\underset{\sim}{\bar{x}})'(CSC')^{-1}(C\underset{\sim}{\bar{x}}) = 90.4; \quad n = 40; \quad q = 3$$

$$\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(.05) = \frac{(39)2}{38}(3.25) = 6.67$$

Since $T^2 = 90.4 > 6.67$ reject $H_0: C\underset{\sim}{\mu} = \underset{\sim}{0}$

b) 95% simultaneous confidence intervals:

$\mu_1 - \mu_2$: $(46.1 - 57.3) \pm \sqrt{6.67}\sqrt{\frac{55.5}{40}} = -11.2 \pm 3.0$

$\mu_2 - \mu_3$: 6.9 ± 3.3

$\mu_1 - \mu_3$: -4.3 ± 3.3

The means are all different from one another.