1. The bin packing decision problem is that given an unlimited number of bins, each of capacity 1, and n objects with sizes $s_1, s_2, \ldots, s_n$, where $0 < s_i \leq 1$, do the objects fit in k bins? where k is a given integer.

   The bin packing optimization problem is to find the smallest number of bins into which the objects can be packed.

   Show that if the decision problem can be solved in polynomial time, then the optimization problem can also be solved in polynomial time.

2. Show that if the hamiltonian cycle decision problem can be solved, then the problem of listing the vertices of a hamiltonian cycle in order is also solvable.

3. Suppose that we had a polynomial-time subprogram TSP to solve the traveling saleperson decision problem (i.e., given a complete weighted graph and an integer k, it determines whether there is a tour of total weight at most k.)

   (a) Show how to use the TSP subprogram to determine the weight of an optimal tour in polynomial time.

   (b) Show how to use the TSP subprogram to find an optimal tour in polynomial time.

4. A graph $G = (V, E)$ is said to be k-colorable if there is a way to paint its vertices using k different colors such that no adjacent vertices are painted the same color. When k is a number, by k-COLOR we denote the decision problem of k-colorable graphs.

   (a) Give an efficient algorithm to determine a 2-coloring of a graph if one exists.

   (b) The 3-COLOR problem is NP-complete (You may assume this). Use this to prove that the 4-COLOR is NP-complete.

5. The Set-Partition (SP) problem takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and $\bar{A} = S - A$ such that

   $$\sum_{x \in A} x = \sum_{x \in \bar{A}} x.$$

   Show that the SP problem is NP-complete by reducing from the Subset-Sum problem.