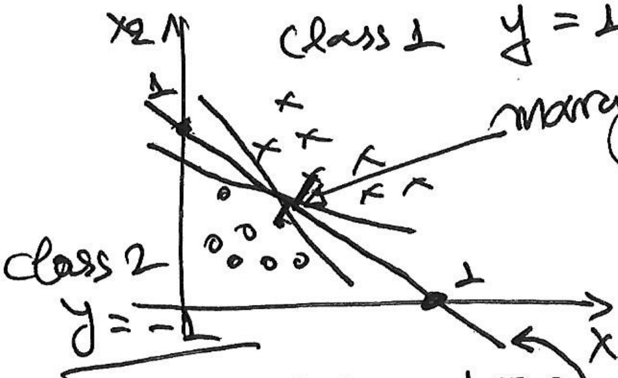
SVM



$x_2 = 1 - x_1$

$\boxed{x_1 + x_2 = 1}$

$1\cdot x_1 + 1\cdot x_2 - 1 = 0$

$\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + (-1) = 0$

$\underline{w^T}\cdot x + \underline{b} = 0$

decision boundary.

$y \in \{-1, 1\}$.

assign output $y^{(i)}$ (class).

$$f(w^T x^{(i)} + b) = \begin{cases}1 & \text{if } \underline{w^T x^{(i)} + b \geq 0}\\ -1 & \text{if } \underline{w^T x^{(i)} + b < 0}\end{cases}$$

separating hyperplane

$\boxed{w^T\cdot x + b = 0}$

$y^{(i)}(w^T x^{(i)} + b) \geq 0.$

Assume the problem is "linearly separable" (i.e. there is a line that perfectly discriminates class 1 v.s. class 2).

Functional margin

$$\hat\gamma^{(i)} = y^{(i)}(w^T x^{(i)} + b) \quad (1)$$

$\forall i = 1 \dots m$.

Problem: (w, b) can be arbitrarily large, which will not move the boundary but will have a large $\hat\gamma$ ("fake")

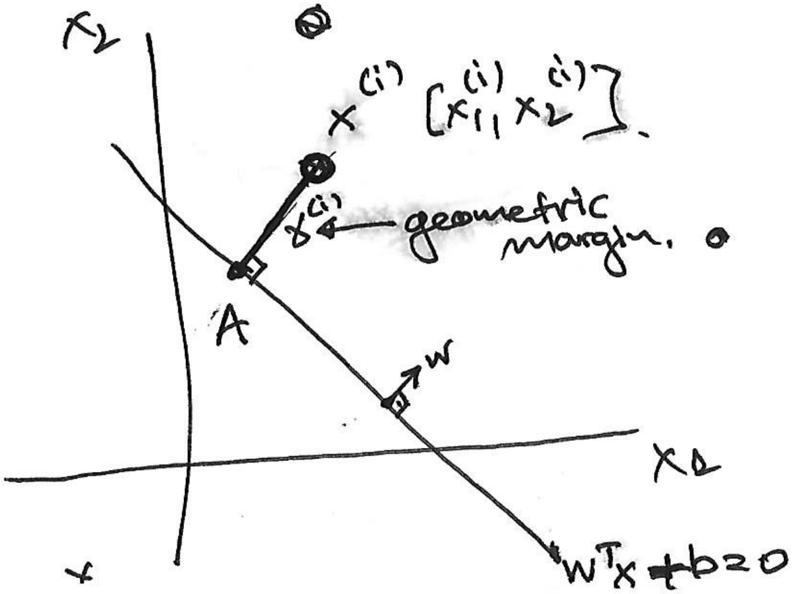
$(w', b') = (10w, 10b)$

$10w^T\cdot x + 10b = 0 \Rightarrow \boxed{w^T x + b = 0}$

but for $\hat\gamma' = y^{(i)}(10 w^T x^{(i)} + 10b) = 10\hat\gamma$

Geometric margin:

$$\gamma^{(i)} = \frac{\hat\gamma^{(i)}}{|w|} \overset{(1)}{=} y^{(i)}\left(\frac{w^T}{|w|}\cdot x^{(i)} + \frac{b}{|w|}\right) \quad (2)$$



For A: $x^{(i)} - \gamma^{(i)} \frac{w}{\|w\|} \cdot y^{(i)}$

But A is on the hyperplane (Decision Boundary)

So: $w^T \cdot x^{(A)} + b = 0 \Rightarrow$.

$$\Rightarrow w^T \cdot \left(x^{(i)} - \gamma^{(i)} \frac{w\, y^{(i)}}{\|w\|}\right) + b = 0 \Rightarrow$$

$$\Rightarrow w^T \cdot x^{(i)} - y^{(i)} \gamma^{(i)} \frac{\boxed{w^T \cdot w}}{\|w\|} + b = 0 \Rightarrow$$

$$w^T \cdot w = \|w\|^2$$

$$\Rightarrow w^T \cdot x^{(i)} - \gamma^{(i)} \cdot \frac{\|w\|^{\cancel{2}}}{\cancel{\|w\|}} y^{(i)} + b = 0 \Rightarrow$$

$$\Rightarrow \boxed{\gamma^{(i)} = \frac{1}{y^{(i)}}\left(\frac{w^T}{\|w\|} \cdot x^{(i)} + \frac{b}{\|w\|}\right)}$$

$+/- 1$

$$\cancel{y^{(i)}} \cdot \left(w^T \cdot x^{(A)} + b\right) = 0.$$

remember $y \in \{-1, 1\}$ so $y^2 = 1 \Rightarrow y^{(i)} = \frac{1}{y^{(i)}}$

$$\boxed{\gamma^{(i)} = y^{(i)} \cdot \left(\frac{w^T}{\|w\|} x^{(i)} + \frac{b}{\|w\|}\right)} \quad (3)$$

$$\hat{\gamma} = \min_i \hat{\gamma}^{(i)}$$

$$\boxed{\gamma = \min_i \gamma^{(i)}}$$

In class, I omitted $y^{(i)}$, which gives the sign of the vector to find the projection of the ith point to the hyperplane.