Breadth-First Search (BFS)

- An archetype for many important graph algorithms
- Input: Given $G = (V, E)$ and a source vertex s,

  Output: $d[v] =$ distance from s to v for all $v \in V$.
- distance $=$ fewest number of edges $=$ shortest path
- BFS basic idea: discovers all vertices at distance k from the source vertex before discovering any vertices at distance $k + 1$
  or expanding frontier – "greedy" – propagate a wave 1 edge-distance at a time.

Review: queue and stack data structure

- Queues and stacks are dynamic sets in which the elements removed from the set by the delete operation is prescribed.
- The queue implements a First-In-First-Out (FIFO) policy.
  The stack implements a Last-In-First-Out (LIFO) policy.
- Queue supports the following operations:
  Enqueue(Q,v): insert element v into the queue Q
  Dequeue(Q,v): delete element v from the queue Q
- There are several way efficient ways to implement queues and stacks
  In section 10.1 of the textbook, it describes a way how to use a simple array to implement each.

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Pseudocode

```
BFS(G,s)
// Breadth-First Search
for each vertex u in V-{s}
   d[u] = +infty
endfor
d[s] = 0
Q = empty  // create FIFO queue
Enqueue(Q, s)
while Q not empty
   u = Dequeue(Q)
   for each v in Adj[u]
       if d[v] = +infty,
          d[v] = d[u] + 1
          Enqueue(Q, v)
       endif
   endfor
endwhile
return d
```

Breadth-First Search (BFS)

- Breadth-First spanning tree
- Correctness of BFS
  shortest path proof – see pp.597-600 of [CLRS,3rd ed.]
  similar with weighted edges – Dijkstra's algorithm
- Running time: $O(|V| + |E|)$

  $O(|V|)$: because every vertex enqueued at most once

  $O(|E|)$: because every vertex dequeued at most once and we examine (u, v) only when u is dequeued at most once if directed, at most twice if undirected.

  Note: not $\Theta(|V| + |E|)$!