Due Wednesday, March 2nd, 4:00 pm in 2131 Kemper

- 1. (10 points, 1 point each) For each of the following scenarios, decide which graph algorithms would be best to use: critical path analysis, unweighted shortest path, Dijkstra's, Kruskal's, Ford-Fulkerson, Articulations Points, or none of the above.
  - a. Determine the number of people to assign to each job in a large project.
  - b. Given a map of the power grid, the production of power plants, and the power consumption of the cities, determine how best to transfer electricity from one city to another to minimize the amount of electricity transferred within the country.
  - c. Given a road map of a city, determine which bridges must be reinforced because they are the only way to get from some neighborhood to the rest of the city.
  - d. Given the road map of a city, for each house in a city, determine their nearest and second nearest elementary school.
  - e. Given the population of an endangered area, and a road map that include the number of people each road can carry in an hour, determine how long it would take to evacuate the area.
  - f. Given blueprints that show the location of the main electrical panel and power outlets, design the wiring of a building to minimize the amount of wire used.
  - g. It is 3am and you are in Manhattan where all the blocks are the same length, and have 25 mph speed limits, determine the quickest route to your destination.
  - h. Given the seven recipes, determine the length of time for you to prepare a seven-course dinner.
  - i. Given a map of all the blood vessels of the brain, determine how long it will take for blood entering from the carotid artery to reach every parts of the brain.
  - j. Given a list of the times flights will arrive at an airport, and their fuel remaining when they arrive, provide an ordering of the flight landings than ensures all will land before they run out of fuel.

## 2. (2 points) Weiss 9.12

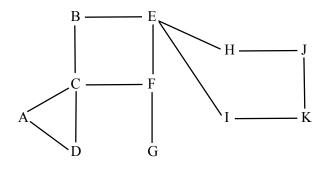
Suppose that G = (V,E) is a tree, s is the root, and we add a vertex t and edges of infinite capacity from all leaves in G to t. Give a linear-time algorithm to find a maximum flow from s to t.

## 3. (2 points) Weiss 9.39

A graph is *k*-colorable if each vertex can be given one of *k* colors, and no edge connects identically colored vertices. Give a linear-time algorithm to test a graph for two-colorability. Assume graphs are stored in adjacency list format; you must specify any additional data structures that are needed.

## 4. (2 points) Weiss 9.21

"Find all the articulation points in the graph in Figure 9.85. Show the depth-first spanning tree, and the values of *Num* and *Low* for each vertex." Figure 9.85 is below. For your DFS, start at F, and when you have a choice of adjacent vertices search them in alphabetical order. Thus, from F you would first go to C.



5. (8 points, 2 points each) Given the below set of figures between US airports apply the specified algorithms. Fill in the tables that have the layouts identical to the samples provided below, including airport ordering, so that it is easier for the TA to grade them.

	BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO
BOS				187		1258	867		2704
BWI				184		946	621		
DFW				1391	1235	1121	802		1464
JFK	187	184	1391			1090	740	144	
LAX			1235			2342			337
MIA	1258	946	1121	1090	2342				
ORD	867	621	802	740				849	1846
PVD				144			849		
SFO	2704		1464		337		1846		

- a. Assuming the numbers are miles, provide a Dijkstra table for the shortest paths from MIA.
- b. Assuming the numbers are miles, provide a Prim's table for the minimum spanning tree starting from SFO.
- c. Assuming the numbers are miles, provide the Union by Size table for Kruskal's. When two trees have the same size, have the airport with the smaller index be the new root.
- d. Assuming the numbers are the number of people possible to carry between cities, apply Ford-Fulkersons to determine the number of people that can get from SFO to JFK. If two edges with same possible remaining flow are available at the same time, select the edge that starts from an airport closer to the beginning of the alphabet.

Table format for parts a and b				Table format for part c, Kruskal's									
BOS BWI	Known	d <sub>v</sub>	p <sub>v</sub>										
DFW					0	1	2	3	4	5	6	7	8
JFK LAX					BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO
MIA													
ORD													
PVD													
SFO													

Table format for Ford-Fulkerson Solution. Number of passengers flying between cities.

		Destinations									
		BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO	
Origins	BOS	X									
	BWI		X								
	DFW			X							
	JFK	X	X	X	X	X	X	X	X	X	
	LAX					X					
	MIA						X				
	ORD							X			
	PVD								X		
	SFO									X	