University of California, Davis
Department of Statistics

Name ....
P R I N T   P L E A S E

Sample

STA 135 **Midterm II**

Instructions: 1. **WORK ALL PROBLEMS**. Please, give details and explanations and **SHOW ALL YOUR WORK** so that partial credits can be given.
2. You may use **three** sheets of **notes** and a **calculato**r but **no** other reference materials.

Points

1. Let $\underline{X}_1$, $\underline{X}_2$, $\underline{X}_3$, $\underline{X}_4$, be random samples from a p-dimensional multivariate normal distribution with mean vector $\underline{\mu}$ and covariance matrix Σ.

(a) Find the distribution of $(\underline{X}_1 + \underline{X}_2 + \underline{X}_3 + \underline{X}_4) / 4$.
(b) Find the distribution of $(\underline{X}_1 - \underline{X}_2 + \underline{X}_3 - \underline{X}_4) / 2$.
(c) Suppose the original population is not multivariate normal and we have increased the sample size from 4 to 400. What is the approximate distribution of the sample mean vector.
(d) Let S denote the sample covariance matrix. With the information given in part (c), what is the approximate distribution of $n(\underline{X} - \underline{\mu})' S^{-1} (\underline{X} - \underline{\mu})$.

(25)

2. In a study of grizzly bears the following summary statistics on head length (cm) and head width (cm) were obtained for n= 61 bears.

$\bar{\underline{x}} = [17.98 \quad 31.13]'$, $\quad S = \begin{bmatrix} 9.95 & 13.88 \\ 13.88 & 21.26 \end{bmatrix}$

(a) Obtain the large-sample 95% simultaneous confidence intervals for the means of each one of these two measurements. ($\chi^2_2(0.05) = 5.99$)

(b) Obtain the large-sample 95% confidence region for mean head length and head width. ($\chi^2_2(0.05) = 5.99$)

(c) Obtained the 95% large-sample Bonferroni confidence interval for means of these two measurements. ($Z(.0125) = 2.24$)

(25)

3. The following data matrix is observed for a two-dimensional random vector $\underline{X}$.

$$\mathbf{X} = \begin{bmatrix} 3 & 4 \\ 6 & 2 \\ 3 & 3 \end{bmatrix}$$

Assume that the population is multivariate normal with unknown mean vector $\underline{\mu}$ and unknown covariance matrix Σ.

(a) Use the Hotelling T^2 to test $H_0 : \underline{\mu} = [\,3 \quad 2\,]'$ against $H_0 : \underline{\mu} \neq [\,3 \quad 2\,]'$ at 0.05 level of significance. ($F_{2,\,1}(.05) = 200$).

(b) Construct a 95% confidence region for mean vector $\underline{\mu}$ and use that to test the hypothesis stated in (a)

(25)

4. A researcher considered three indices measuring the severity of heart attacks. All three indices were evaluated for each patient. The values of these indices for n= 40 heart attack patients arriving at a hospital emergency room produced the summary statistics:

$$\bar{\underline{x}} = [46.1 \;\; 57.3 \;\; 50.4]'$$

$$\mathbf{S} = \begin{bmatrix} 101.3 & 63.0 & 71.0 \\ 63.0 & 80.2 & 55.6 \\ 71.0 & 55.6 & 97.4 \end{bmatrix}$$

Assume that the population is multivariate normal with unknown mean vector $\underline{\mu}$ and unknown covariance matrix Σ, and test the equality of mean indices at .05 level. ($F_{2,\,38}(.05) = 3.25$)

(25)