Homework 2
(Due on Monday, Jan 25)

Problems (from App. Lin. Stat. Model): 18.15*, 18.16, 21.7, 21.8, 21.19 and the questions given below.

***Add part (f) to Problem 18.15.**
18.15 (f). First add 1 to each Y value and then use the Bartlett's and Hartley's statistics (as given in Supplementary Note 1) to obtain the transformations that make the variances as equal as possible. Plot the values of these statistics against λ. Are your findings similar to those in part (e)? Comment.

[You may form a group of 2 students (including yourself) registered in this course. Only one work per group needs to be submitted. Please write down the names of the group-members on the first page. The first page of submitted homework should contain the names of the students in the group, **but all work should start from page 2**.]
Note that we will not grade all the problems.

A. Consider a two factor model with one observation for each treatment combination

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + \varepsilon_{ij}, j = 1, \ldots, b, i = 1, \ldots, a,$$

where $\{\varepsilon_{ij}\}$ are iid $N(0, \sigma^2)$, and $\{\alpha_i\}$ and $\{\beta_j\}$ satisfy the constraints $\sum \alpha_i = 0$ and $\sum \beta_j = 0$. Let $\hat{\mu}_{..}$, $\{\hat{\alpha}_i\}$, $\{\hat{\beta}_j\}$, $SSTO$, SSA, MSE etc. be defined as in the lecture notes.
(i) Show that $SSTO = SSA + SSB + SSE$.
(ii) Show that $E(MSE) = \sigma^2$.
(iii) Show that $E(MSB) = \sigma^2 + a \sum \beta_j^2/(b-1)$.
(iv) Let $L = \sum c_i \alpha_i$ be a contrast of $\{\alpha_i\}$ and let $\hat{L} = \sum c_i \hat{\alpha}_i$ be its least squares estimate. Show that $L \sim N(L, \sigma^2(\hat{L}))$, where $\sigma^2(\hat{L}) = b^{-1} \sum c_i^2 \sigma^2$.