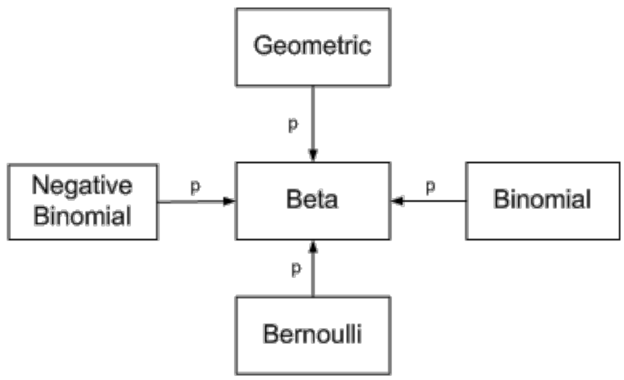
Conjugate prior relationships

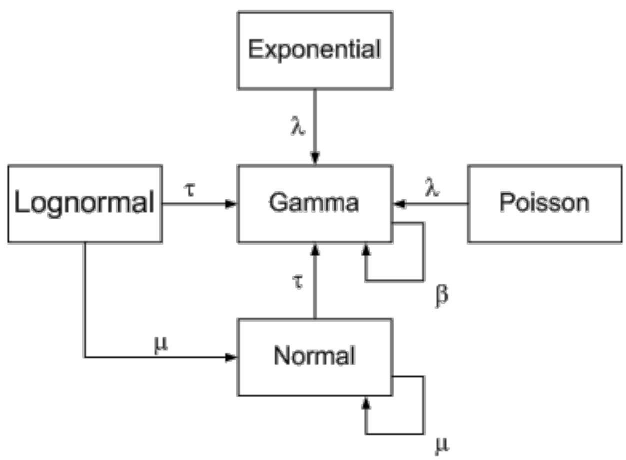
The following diagram summarizes conjugate prior relationships for a number of common sampling distributions.

Arrows point from a sampling distribution to its conjugate prior distribution. The symbol near the arrow indicates which parameter the prior is unknown.

These relationships depends critically on choice of parameterization, some of which are uncommon. This page uses the parameterizations that make the relationships simplest to state, not necessarily the most common parameterizations. See footnotes below.

Click on a distribution to see its parameterization. **Click on an arrow** to see posterior parameters.





See this page for more diagrams on this site including diagrams for probability and statistics, analysis, topology, and category theory. Also, please contact me if you're interested in Bayesian statistical consulting.

Parameterizations

Let $C(n, k)$ denote the binomial coefficient(n, k).

The **geometric** distribution has only one parameter, p, and has PMF $f(x) = p\,(1-p)^x$.

The **binomial** distribution with parameters n and p has PMF $f(x) = C(n, x)\, p^x(1-p)^{n-x}$.

The **negative binomial** distribution with parameters r and p has PMF $f(x) = C(r + x - 1, x)\, p^r(1-p)^x$.

The **Bernoulli** distribution has probability of success p.

The **beta** distribution has PDF $f(p) = \Gamma(\alpha + \beta)\, p^{\alpha-1}(1-p)^{\beta-1} / (\Gamma(\alpha)\, \Gamma(\beta))$.

The **exponential** distribution parameterized in terms of the rate λ has PDF $f(x) = \lambda \exp(-\lambda\, x)$.

The **gamma** distribution parameterized in terms of the rate has PDF $f(x) = \beta^\alpha\, x^{\alpha-1}\exp(-\beta\, x) / \Gamma(\alpha)$.

The **Poisson** distribution has one parameter λ and PMF $f(x) = \exp(-\lambda)\, \lambda^x/ x!$.

The **normal** distribution parameterized in terms of precision τ ($\tau = 1/\sigma^2$)
has PDF $f(x) = (\tau/2\pi)^{1/2} \exp(-\tau(x - \mu)^2/2)$.

The **lognormal** distribution parameterized in terms of precision τ has PDF $f(x) = (\tau/2\pi)^{1/2} \exp(-\tau(\log(x) - \mu)^2/2) / x$.

Posterior parameters

For each sampling distribution, assume we have data $x_1, x_2, \ldots, x_n$.

If the sampling distribution for x is **binomial**(m, p) with m known, and the prior distribution is **beta**(α, β), the posterior distribution for p is **beta**($\alpha + \Sigma x_i$, $\beta + mn - \Sigma x_i$). The **Bernoulli** is the special case of the binomial with $m = 1$.

If the sampling distribution for x is **negative binomial**(r, p) with r known, and the prior distribution is **beta**(α, β), the posterior distribution for p is **beta**($\alpha + nr$, $\beta + \Sigma x_i$). The **geometric** is the special case of the negative binomial with $r = 1$.

If the sampling distribution for x is **gamma**(α, β) with α known, and the prior distribution on β is gamma(α_0, β_0), the posterior distribution for β is **gamma**($\alpha_0 + n$, $\beta_0 + \Sigma x_i$). The **exponential** is a special case of the gamma with $\alpha = 1$.

If the sampling distribution for x is **Poisson**(λ), and the prior distribution on λ is **gamma**(α_0, β_0), the posterior on λ is **gamma**($\alpha_0 + \Sigma x_i$, $\beta_0 + n$).

If the sampling distribution for x is **normal**(μ, τ) with τ known, and the prior distribution on μ is **normal**(μ_0, τ_0), the posterior distribution on μ is **normal**($(\mu_0\, \tau_0 + \tau\, \Sigma x_i)/(\tau_0 + n\tau)$, $\tau_0 + n\tau$).

If the sampling distribution for x is **normal**(μ, τ) with μ known, and the prior distribution on τ is **gamma**(α, β), the posterior distribution on τ is **gamma**($\alpha + n/2$, $(n-1)S^2$) where S^2 is the sample variance.

If the sampling distribution for x is **lognormal**(μ, τ) with τ known, and the prior distribution on μ is **normal**(μ_0, τ_0), the posterior distribution on μ is **normal**(($\mu_0 \tau_0 + \tau \Pi x_i)/(\tau_0 + n\tau)$, $\tau_0 + n\tau$).

If the sampling distribution for x is **lognormal**(μ, τ) with μ known, and the prior distribution on τ is **gamma**(α, β), the posterior distribution on τ is **gamma**($\alpha + n/2$, $(n-1)S^2$) where S^2 is the sample variance.

References

A compendium of conjugate priors by Daniel Fink.

See also Wikipedia's article on conjugate priors.

Working together

Call or email me to discuss how I may be able to help you with Bayesian statistics.

Work together