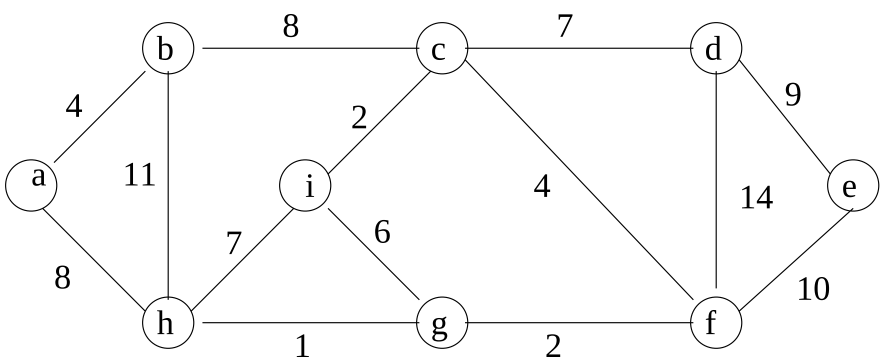
ECS122A Homework Assignment #7 **Due: 4:00pm, March 9, 2016**

1. Let $G = (V, E)$ be a connected undirected graph with distinct edge weights. Prove that G has a unique minimum spanning tree.

2. Run Prim's algorithm with the root vertex a for finding the minimum spanning tree on the following graph; whenever there a choice of nodes, always use alphabetic ordering. Mark on the graph showing the intermediate values, and list the final minimum spanning tree.

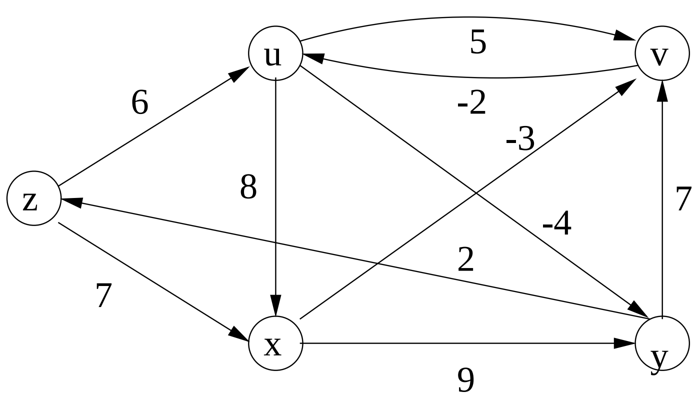


3. Run Kruskal's algorithm for finding the minimum spanning tree on the graph shown in Problem 2. List the order of edges adding to the MST, i.e., the set A.

4. Prove or disprove: Prim's MST algorithm will work correctly even if weights may be negative.

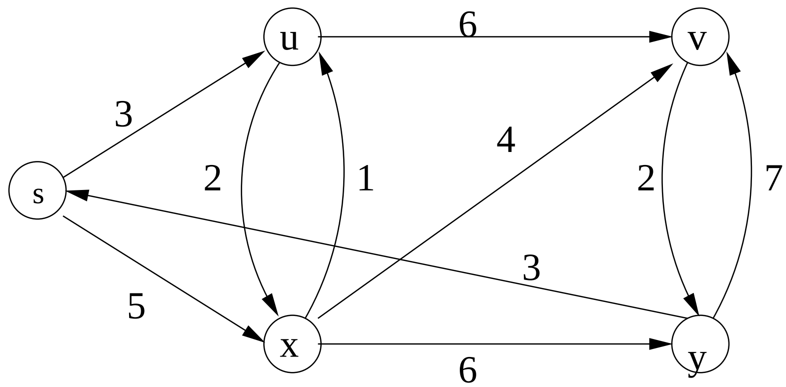
5. Show how to find the *maximum* spanning tree of a graph , that is, the spring tree of largest total weight.

6. (a) Run the Bellman-Ford algorithm on the following directed graph, using vertex y as the source. Relax edges in lexicographic order in each pass. Draw a table showing the d and π values after each pass.



(b) Change the weight of edge (y, v) to 4 and run the algorithm again, using z as the source.

7. Run Dijkstra's algorithm on the following directed graph, (a) first using vertex s as the source, and (b) then using vertex y as the source. Mark on the graph showing the intermediate values, and list the final shortest-path.



8. Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers.

9. We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Given an efficient algorithm to find the most reliable path between two given vertices.

10. Consider a undirected graph $G = (V, E)$ with nonnegative weights $w(u, v) \geq 0$ on its edges $(u, v) \in E$. Assume you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all vertices from a particular vertex $s \in V$. Now suppose we change the weights on every edge by adding 1 to each of them. The new weights are $w'(u, v) = w(u, v) + 1$ for every $(u, v) \in E$.

    - Would the minimum spanning tree change due to the change in weights? Give an example where it changes or prove that it cannot change.
    - Would the shortest paths change due to the change in weights? Give an example where it changes or prove that it cannot change.