DGE OLG model summary

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1 Household

- N_t: total population at t
- l_t^s: individual labor supply
- L_t: effective labor ($= \theta e \overline{y}^s l_t^s$)
- $N_{t+1}(1) = (1+n_t)N_t(1)$
- H.H comprises one worker
- maximizes expected utility at the beginning of age 1

$$max \sum_{s=1}^{T} \beta^{s-1} (\Pi_{j=1}^{s} \phi_{t+j-1}^{j-1}) E_t[u(c_{t+s-1}^s, l_{t+s-1}^s) + w(g_{t+s-1})]$$

where l: labor, $1-l$: leisure, g_t: govt. consumption per capita, $w(g_t)$: utility from govt. consumption

$$u(c, 1-l) = \frac{(c^{\gamma}(1-l)^{1-\gamma})^{1-\eta}}{1-\eta}$$

where $\frac{1}{\eta}$: IES 다른 기간 소비의 대체 탄력성

- total gross labor income $= A_t w_t \epsilon(s, \theta, e) l_t^s = A_t w_t \theta e \overline{y}^s l_t^s$
- $\theta e \overline{y}^s l_t^s$: individual effective labor supply
  where A_t: aggregate productivity, w_t: wage per efficiency unit
- net non-capital income

$$y_t^s = \begin{cases} (1-\tau_t^l - \tau_t^p)\epsilon(s,\theta,e)A_t w_t l_t^s & s = 1, ..., T^w \\ pen_t & s = T^w + 1, ..., T \end{cases}$$

- budget constraint

$$(1+\tau^c)c_t^s + k_{t+1}^{s+1} + b_{t+1}^{s+1} = y_t^s + (1-\tau^k)(r_t - \delta)k_t^s + k_t^s + R_t^b b_t^s + tr_t$$

where $k_t^1 = k_t^{T+1} = 0, b_t^1 = b_t^{T+1} = 0$
non-negative constraint on assets: $b_t^s \geq 0, k_t^s \geq 0$

- Bellman equation
  In equilibrium, the allocation to two assets (k, b) is indeterminate since the after tax returns on both assets are equal
  Assume that all households hold both assets in the same proportion $\frac{K}{K+B}, \frac{B}{K+B}$
  $a_t^s \equiv k_t^s + b_t^s, z \equiv (s, \theta, e, a)$

$$V_t(z) = max_{c,l,a'} u(c, 1-l) + w(g) + \beta\phi_t^s \sum_{\theta'} prob(\theta'|\theta)V_{t+1}(z)$$

$$\text{s.t } (1+\tau^c)c = y + [1 + (1-\tau^k)(r-\delta)]a + tr - a', a \geq 0$$

2 Technology

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$
$$w_t = (1-\alpha)K_t^\alpha (A_t L_t t)^{-\alpha}$$
$$r_t = \alpha K_t^{\alpha-1} (A_t L_t)^{1-\alpha}$$
$$\text{maximize profit } \Pi_t = Y_t - r_t K_t - w_t A_t L_t$$

3 Government

- confiscate all accidental bequests Beq_t
- budget constraint : $Tr_t + G_t + r_t^b B_t - Tax_t - Beq_t = B_{t+1} - B_t$

$$Tax_t = \tau^l A_t L_t w_t + \tau^k (r_t - \delta)K_t + \tau^c C_t$$

$$Pen_t = \tau^p A_t L_t w_t$$

4 Equilibrium

- divide individual variables (except l_t^s) by aggregate productivity A_t

$$x \in \{c, y, b, k\} \rightarrow \tilde{x_t^s} \equiv \frac{x_t^s}{A_t}$$

- divide aggregate variables (except L_t) by $A_t \times N_t$

$$X \in \{Pen, Tr, G, B, Beq, Tax, Y, K, C, \Omega\} \rightarrow \tilde{X}_t \equiv \frac{X_t}{A_t N_t}$$

- stationary aggregate labor

$$\tilde{L_t} \equiv \frac{L_t}{N_t}\text{(aggregate effective labor supply)}$$

- Bellman equation in stationary form

$$\tilde{V}_t = \frac{V_t}{A_t^{\gamma(1-\eta)}},\ \tilde{z} = (s,\theta,e,\tilde{a}), \tilde{u}(\tilde{c},1-l) = \frac{u(c,l)}{A_t^{\gamma(1-\eta)}} = \frac{(\tilde{c}^{\gamma}(1-l)^{1-\gamma})^{1-\eta}}{1-\eta}$$

$$\tilde{V}_t(\tilde{z}) = \max_{\tilde{c},l,\tilde{a}'}\{\tilde{u}(\tilde{c},l) + (1+g_A)^{\gamma(1-\eta)}\beta\phi_t^s \sum_{\theta'} prob(\theta'|\theta)\tilde{V}_{t+1}(\tilde{z}_{s+1})\}$$

$$(1+\tau^c)\tilde{c} = \tilde{y} + [1+(1-\tau^k)(r_t-\delta)]\tilde{a} + \tilde{tr} - (1+g_A)\tilde{a}'$$

with

$$\tilde{y} = \begin{cases} (1-\tau_t^l - \tau_t^p)\epsilon(s,\theta,e)w_t l & s = 1,...,T^w \\ \tilde{pen}_t & s = T^w+1,...,T \end{cases}$$

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$$\tilde{Tax}_t = \tau^l w\tilde{L}_t + \tau^k(r_t-\delta)\tilde{K}_t + \tau^c\tilde{C}_t$$

$$\frac{\tilde{Beq}_t}{(1+n)(1+g_A)} = \sum_{s=2}^{T}\sum_{i_\theta=1}^{n_\theta}\sum_{j=1}^{2}\sum_{i_a=1}^{n_a}(1-\phi^s)[1+(1-\tau^k)(r-\delta)]\tilde{a}'(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})f(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})$$

$$\tilde{Tr}_t = \tilde{tr}_t = \tilde{Tax}_t + \tilde{Beq}_t - \tilde{G}_t + [(1+g_A)(1+n) - (1-r^b)]\tilde{B}_t$$

$$pen_t = repl \times w_t times \tilde{L}_t$$

$$\tilde{Pen}_t = \tilde{pen}_t \times sum(mass[nw : nage])$$

$$\tilde{Pen}_t = \tau^p\tilde{L}_t w_t \to \tau^p = \frac{\tilde{Pen}}{\tilde{L}_t w_t}$$

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$$\tilde{L}_t = \sum_{s=1}^{T^w}\sum_{i_\theta=1}^{n_\theta}\sum_{j=1}^{2}\sum_{i_a=1}^{n_a}\epsilon(s,\theta_{i_\theta},e_j,)l(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})f(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})$$

$$\tilde{\Omega}_t = \sum_{s=1}^{T}\sum_{i_\theta=1}^{n_\theta}\sum_{j=1}^{2}\sum_{i_a=1}^{n_a}\tilde{a}_{i_a}f(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})$$

$$\tilde{\Omega}_t = \tilde{B}_t + \tilde{K}_t$$

$$w = (1-\alpha)\tilde{K}^{\alpha}\tilde{L}^{-\alpha}$$

$$r = \alpha\tilde{K}^{\alpha-1}\tilde{L}^{1-\alpha}$$

$$r_t^b = (1-\tau_r^k)(r_t-\delta)$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{G}_t + (1+g_A)(1+n)\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t$$

$$\tilde{C}_t = \sum_{s=1}^{T}\sum_{i_\theta=1}^{n_\theta}\sum_{j=1}^{2}\sum_{i_a=1}^{n_a}\tilde{c}(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})f(s,\theta_{i_\theta},e_j,\tilde{a}_{i_a})$$