The risk-free rate in heterogeneous-agent incomplete-insurance economies

Huggett, 1993, JEDC

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1 Key Question

- Why has the average real risk-free interest rate been less than one percent? representative agent model have been unsuccessful
- He investigated the conjecture that market imperfections are important for determining the risk-free rate, specifically the importance of idiosyncratic shocks and incomplete insurance
- Why this structure generates a low risk-free rate? With a credit limit, agents are restricted in the level of their indebtedness. However, agents are not restricted from accumulating large credit balances. A low risk-free rate is needed to persuade agents not to accumulate large credit balances so that the credit market can clear

2 Model

2.1 Summary

- Heterogeneous-agent incomplete-insurance models of asset pricing
- Pure **exchange economy** where agents experience (uninsurable) idiosyncratic endowment shocks and smooth consumption by holding a single risk-free asset(credit balance)
- Each agent holds a credit balance with a central credit authority
- The credit balance must always remain above a fixed credit limit
- Agent accumulates credit balances in good times and runs down credit balances in bad times

2.2 Structure

- A continuum of agents of total mass equal to one
- Each period each agent receives an endowment of the one perishable consumption good, high(e_h) or low(e_l), $E = \{e_h, e_l\}$
- Endowment follows Markov process with stationary transition probability
- Preference

$$E[\sum_{t=0}^{\infty} \beta^t u(c_t)] \text{ where } \beta \in (0,1)$$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \text{ where } \sigma > 1$$

- To obtain a credit balance of a' units next period, an agent must pay a'q goods this perid, where q is the price of next-period credit balances(Let $q > 0$ be the constant price of credit balances each period)
- Credit balances must always remain above a credit limit $\underline{a}$ ($\underline{a} < 0$)
- Budget constraint: chooses consumption c and next-period credit balances a', given credit balance a and endowment e

$$c + a'q \leq a + e \text{ where } c \geq 0 \text{ and } a' \geq \underline{a}$$

- Individual state vector and state space:
  $x = (a, e) \in X = A \times E$ where $A = [\underline{a}, \infty), E = e_h, e_l$ and $e_h > e_l$
- Bellman equation

$$v(x;q) = max_{(c,a') \in \Gamma(x;q)} u(c) + \beta \sum_{e'} v(a', e'; q)\pi(e'|e)$$

  where

$$\Gamma(x;q) = \{(c, a') : c + a'q \leq a + e; c \geq 0; a' \geq \underline{a}\}$$

3 Equilibrium

- Probability measure
  - Probability measure defined on subsets of the individual state space
  - Let ψ be a probability measure on (S, β_s) where $S = [\underline{a}, \overline{a}] \times E$ and β_s is the Borel σ - algebra
  - For $B \in \beta_s$, $\psi(B)$ indicates the mass of agents whose individual state vectors lie in B

- Since the question at hand concerns the average interest rate, a useful simplification is that the probability measure ψ and the price of credit q remain unchanged over time
- This paper adopts the stationary recursive equilibrium structure described in Lucas(1980), concentrating on stationary equilibria
- For a probability measure ψ to be stationary or unchanged over time, a transition function P, $P : S \times \beta_s \rightarrow [0, 1]$, is needed
- $P(x, B)$ is the probability that an agent with state x will have an individual state vector lying in B next period

- Definition
  - A stationary equilibrium for this economy is $(c(x), a(x), q, \psi)$ satisfying:
    1. $c(x)$ and $a(x)$ are optimal decision rules, given q
    2. Markets clear: (i) $\int_S c(x)d\psi = \int_S ed\psi$, (ii)$\int_S a(x)d\psi = 0$
    3. ψ is a stationary probability measure: $\psi(B) = \int_S P(x, B)d\psi$ for all $B \in \beta_s$
  - Conditions say
    1. Agents optimize
    2. Consumption and endowment averaged over the population are equal and that credit balances averaged over the population are zero
    3. The distribution of agents over states is unchanging

4 Computation

- e_h, e_l : earnings when employed and not employed
  calibrate the endowment process to match measures of the variability of labor earnings and the time duration in a nonemployed state
- As there are six model periods in one year, the discount factor(β) on an annual basis is 0.96
- Risk aversion coefficient is from Mehra and Prescott(1985)
- $\underline{a}$ is selected to examine the sensitivity of the results to different credit limits
- A credit limit of -5.3 is equal to one year's average endowment
- Computation Method
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Parameter	Value	-
e_h	1.0	-
e_l	0.1	-
$\pi(e_h\|e_h)$	0.925	-
$\pi(e_h\|e_l)$	0.5	-
β	0.99322	discount factor
σ	1.5	risk aversion coefficient
$\underline{a}$	$\in \{-2, -4, -6, -8\}$	credit limit

5 Result

5.1 Optimal decision rules

- This figure graphs the decision rules $a(a, e_h)$ and $a(a, e_l)$ on a 45° line diagram, where the graph of $a(a, e_h)$ always lies above the graph of $a(a, e_l)$

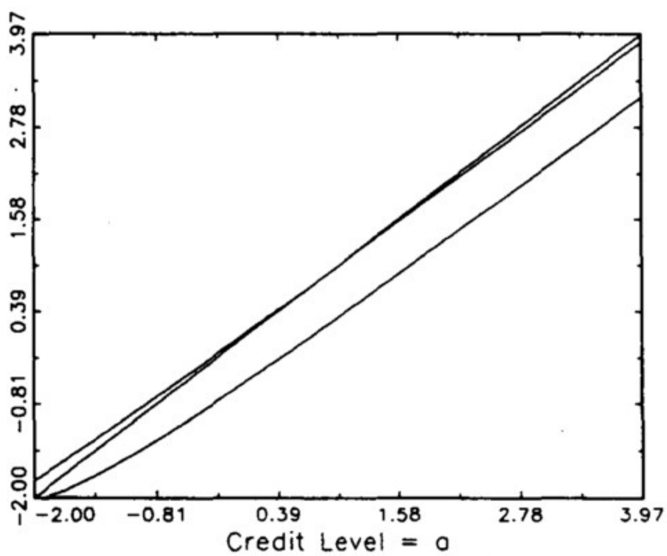

Fig. 1. Optimal decision rule.

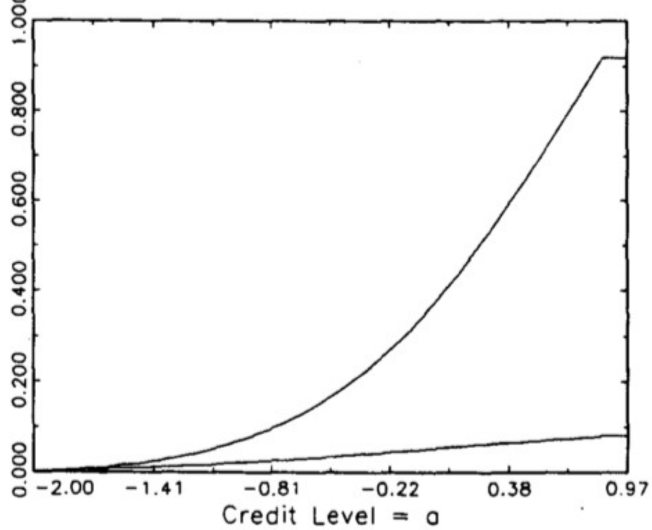

Fig. 2. Stationary distribution.

Table 1

Coefficient of relative risk aversion $\sigma = 1.5$.

Credit limit ($\underline{a}$)	Interest rate (r)	Price (q)
− 2	− 7.1%	1.0124
− 4	2.3%	0.9962
− 6	3.4%	0.9944
− 8	4.0%	0.9935

Table 2

Coefficient of relative risk aversion $\sigma = 3.0$.

Credit limit ($\underline{a}$)	Interest rate (r)	Price (q)
− 2	− 23 %	1.0448
− 4	− 2.6%	1.0045
− 6	1.8%	0.9970
− 8	3.7%	0.9940