

Matrix 1								Matrix 2							
4	5	7	6	2	3	8	1	6	1	5	4	7	2	3	8
1	2	6	7	5	4	8	3	4	2	7	8	1	5	3	6
4	2	6	1	5	3	8	7	3	1	7	5	2	6	8	4
5	4	8	2	3	1	7	6	3	8	7	1	5	6	4	2
1	2	3	5	7	6	8	4	4	1	6	8	7	5	3	2
2	8	5	1	4	6	3	7	3	1	8	4	2	6	5	7
8	2	6	3	4	5	1	7	8	1	4	5	3	6	7	2
2	6	7	8	1	3	5	4	4	1	8	5	6	3	2	7

- Naïve Method:

```
void multiply(int A[][N], int B[][N], int C[][N])
{
    for (int i = 0; i < N; i++)          -->N=8
    {
        for (int j = 0; j < N; j++)      -->N=8
        {
            C[i][j] = 0;
            for (int k = 0; k < N; k++)  -->N=8
            {
                C[i][j] += A[i][k]*B[k][j];
            }
        }
    }
}
```

$T(n) = O(n^3) = 8*8*8 = 512;$

- Divide and Conquer:

Consider Matrix 1 is X and Matrix 2 is Y

Divide X and Y into 8 sub-matrices A, B, C, and D.

X			Y	
A	B		E	F
C	D		G	H

Do 8 matrix multiplications recursively.

Compute Z by combining results (doing 4 matrix additions).

Lets assume $n = 2^c$ for some constant c and let A, B, C and D be $n/2 \times n/2$ matrices

$$T(N) = 8T(N/2) + O(N^2) = 8*(8*(8*(8*(...1...) + (N/8)^2) + (N/4)^2) + (N/2)^2) + O(N^2)$$

$$a = 8; b = 2; d = 2; \Rightarrow a > b^d$$

$$\Rightarrow T(N) = O(N^{\log_2 8}) = O(N^3)$$

- Strassen's Method

Z	
$(S1 + S2 - S4 + S6)$	$(S4 + S5)$
$(S6 + S7)$	$(S2 + S3 + S5 - S7)$

$$S1 = (B - D) * (G + H)$$

$$S2 = (A + D) * (E + H)$$

$$S3 = (A - C) * (E + F)$$

$$S4 = (A + B) * H$$

$$S5 = A * (F - H)$$

$$S6 = D * (G - E)$$

$$S7 = (C + D) * E$$

$$T(n) = 7T(n/2) + O(n^2)$$

$$\Rightarrow T(n) = 7T(n/2) + n^2$$

$$= n^2 + 7(7T(n/2^2) + (n/2)^2)$$

$$= n^2 + (7/2^2)n^2 + 7^2T(n/2^2)$$

$$= n^2 + (7/2^2)n^2 + (7/2^2)^2n^2 + 7^3T(n/2^3)$$

$$= n^2 + (7/2^2)n^2 + (7/2^2)^2n^2 + (7/2^2)^3n^2 + \dots + (7/2^2)^{\log(n-1)}n^2 + 7^{\log n}$$

$$= n^2 * O((7/2^2)^{\log(n-1)}) + 7^{\log n}$$

$$= n^2 * O(7^{\log n} / (2^2)^{\log(n-1)}) + 7^{\log n}$$

$$= n^2 * O(7^{\log n} / n^2) + 7^{\log n}$$

$$= O(7^{\log n})$$

$$\Rightarrow T(n) = O(n^{\log 7}) = O(n^{2.8074})$$