

# Intangible Investment during Sovereign Debt Crisis: Firm-level Evidence<sup>\*</sup>

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July 24, 2021  
[Latest Version]

## *Abstract*

This paper measures the output and TFP costs of sovereign risk by focusing on firm-level investment. Combining Italian firm-level and aggregate-level data, we show that firms reduce their investment and reallocate their resources away from intangible assets and towards tangible assets during the sovereign debt crisis. This asset reallocation is more pronounced among small and high-leverage firms, indicating the role of financial constraints. In our model, sovereign risk deteriorates banks' balance sheets, disrupting banks' ability to finance firms. Firms with greater external financing needs are more exposed to sovereign risk. Facing tightening financial constraints, firms internalize that tangibles can be used as collateral while intangibles can not, thus reallocating resources towards tangible investment. In a counterfactual analysis, we find that elevated sovereign risk explains 86% of output losses and 72% of TFP losses during the 2011-2013 Italian sovereign debt crisis.

**Keywords:** Sovereign debt crisis, intangible asset, output loss, TFP loss

**JEL classification:** F34, E22, E44, G12, G15

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<sup>\*</sup>We thank Yan Bai, George Alessandria, Gaston Chaumont, Stepan Gordeev, Armen Khederlarian, Zhiyuan Chen, Xiaomei Sui, Min Fang, José Villegas, Zibin Huang, and Pablo Guerron for helpful comments, and participants at several seminars and conferences for valuable insights. Minjie Deng acknowledges the financial support of SSHRC-SFU Institutional Grant.

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# 1 Introduction

The recent European sovereign debt crisis is associated with substantial declines in credit to private sector and real economic activities. Recent literature emphasizes the pass-through of sovereign risk to the private sector through the banking sector (Gennaioli et al. (2014), Perez et al. (2015), Bocola (2016), Arellano et al. (2019), Bottero et al. (2020)). Because banks are often the main creditor of the government, sovereign risk deteriorates banks' balance sheets and disrupt the financing to the firms. A key open question is to quantify the real economic impacts of sovereign risk on the private sector. This paper quantifies the impact of sovereign risk on firm investment and endogenizes the costs of sovereign risk through the private sector.

We use Italian firm-level data to estimate the impact of sovereign risk on firm investment. Importantly, besides tangible investment, we additionally focus on investment on intangible assets, which was often ignored in previous sovereign default literature. Investment in intangible assets accounts for an increasing proportion than previous decades (EU KLEMS database). Furthermore, the positive effects of intangible assets on firms' productivity and economic performance are well established in the literature (Griliches (1958), Griliches (1979), Geroski (1989), Hall et al. (2010)). Declines in investment in intangible assets (hereafter, intangible investment) during the sovereign debt crisis affect firms' productivity and output.

We empirically document the impact of sovereign risk on firms' intangible and tangible investment. First, during Italian sovereign debt crisis, firms reduce their investment both in intangible assets and tangible assets on average. Second, although firms also reduce tangible investment, they reallocate their limited resources away from intangible assets and towards tangible assets. Third, the asset reallocation pattern is more pronounced within small and high-leverage firms. Our results are robust to alternative measures for investment, sovereign risk, firm-level variables, and are also robust to different empirical specifications.

Heterogeneous asset reallocation during the sovereign debt crisis points out the role of financial constraints. We build a sovereign default model incorporating firm investment on both tangible and intangible assets, to measure the aggregate output and TFP costs of sovereign risk. In our framework, an increase in government default risk deteriorates banks' balance sheets, which leads to a higher loan interest rate

faced by the firms. The firm-specific loan interest rate also depends on each firm's collateral. Banks accept tangible assets, instead of intangible assets, as collateral. As a result, firms reduce intangible investment and reallocate to tangible investment to offset the tightening borrowing constraint. The declines in intangible investment hurt firms' future productivity and output. Firms are not equally affected by the elevated sovereign risk and higher loan interest rate: firms highly relying on the external borrowing from the banks are more exposed to the sovereign risk.

Our framework incorporates heterogeneous firms with both intangible and tangible investment into an otherwise canonical general equilibrium model of sovereign debt and default. The economy is composed of final goods firms, heterogeneous intermediate goods firms, financial intermediaries, households, and a central government. The final goods firms are competitive and they convert intermediate goods to final goods. The intermediate goods firms need to borrow from the financial intermediaries to finance a fraction of their investment. Those firms differ in their productivity and external financing needs. The financial intermediaries play a key role in transmitting sovereign risk to the firms: they use their net worth, which includes unmatured government bonds, to purchase long-term government bond and provide loans to firms. The government collects tax revenues from the final goods firms and borrows from the financial intermediaries to finance the lump-sum transfers to the households and service the outstanding government bond. The government may default on its bond repayment.

We parametrize the model using Italian data to assess the output and productivity losses due to sovereign risk. We target sample moments that include firms, banks, and government statistics from 2006 to 2015. Using the estimated model, we show the impact of a sovereign risk shock by plotting the impulse response functions (IRFs) to a one standard deviation increase in sovereign spread. IRFs show that firms decrease their future intangible and tangible assets, but the ratio of tangible assets to intangible assets goes up, which are consistent with the empirical findings. With lower capital inputs, firms' output decline. The decline in intangible investment further decreases firms' future TFP. The model endogenously generates the output decline and TFP decline when sovereign spread increases, as opposed to the previous literature that assumes exogenous declines in endowment, or output, or TFP when the government defaults.

We then feed the model with exogenous shocks to replicate the observed path of Italian sovereign risk and real GDP during 2006-2016. Using the 2006-2016 model-simulated sample, we run the regression counterpart. We show that the model can replicate the baseline empirical findings: firms increase their tangibles-to-intangibles ratio during sovereign debt crisis. Moreover, small and high-leverage firms would reallocate their assets by more, compared with other firms.

To measure the output and TFP costs directly due to the sovereign risk, we specifically focus on the debt crisis period and construct a counterfactual scenario in which the Italian economy does not experience debt crisis. We then compare the results of our benchmark model and the counterfactual model with no debt crisis. The differences between the paths of key variables in the benchmark model and those in the no debt crisis counterfactual model isolate the impact of the sovereign crisis on the Italian economy. We find that the losses associated with sovereign default risk are sizable. During 2011-2013, output would have declined only 0.6% without the sovereign debt crisis, instead of 4.3% in the benchmark model. TFP would have declined only 0.7% without the sovereign debt crisis, instead of 2.5% in the benchmark model. Our quantitative results suggest that the sovereign risk explains 86% of the output losses and 72% of the TFP losses during the 2011-2013 Italian debt crisis.

**Related literature.** Our paper measures the output and TFP costs of sovereign risk by focusing on the firm-level responses, thus combining the elements of the sovereign default literature with those of the literature on the impact of financial frictions on firms. We also contribute to the growing literature on intangible assets and financial frictions.

The model builds on the sovereign default models pioneered by [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), and [Arellano \(2008\)](#). Most sovereign default literature assumes exogenous endowment or output declines when sovereign defaults (e.g., [Arellano \(2008\)](#), [Chatterjee and Eyigungor \(2012\)](#), [Mendoza and Yue \(2012\)](#)), and some literature features a production economy and assumes exogenous TFP decline when sovereign defaults (e.g., [Arellano et al. \(2018\)](#), [Alessandria et al. \(2020\)](#), [Deng \(2019\)](#)). Different from above approaches, the model in this paper adopts a reduced-form exogenous shock that determines the government default probability,

which is similar to [Bocola \(2016\)](#).<sup>1</sup> Our model endogenously generates the output decline and TFP decline when sovereign spread increases. The TFP decline through intangible investment channel is novel in the sovereign default literature.

Recent papers in the sovereign debt and default literature study the links between sovereign default risk and the private sector through financial intermediation. During a sovereign debt crisis, firms lose access to external financing, cut their production and generate output losses ([Mendoza and Yue \(2012\)](#)). The link between sovereign default and private sector through banking sectors is also analyzed in [Sosa-Padilla \(2018\)](#), [Perez et al. \(2015\)](#), [Arellano et al. \(2019\)](#), and [D’Erasmus et al. \(2020\)](#). Our paper shares the focus of studying the transmission of sovereign risk to the firms through the financial intermediation. Consistent with [Arellano et al. \(2019\)](#), an increase in the future default probability can transmit to the private sector through financial intermediation, even when the government keeps repaying the debt. We contribute by focusing on both firms’ intangible and tangible investment, and we are able to measure the losses directly due to the rising default risk by incorporating intangible channel.

Our empirical findings also relate to literature that uses micro data to test the impacts of banks’ balance sheet on firms’ credit ([Bofondi et al. \(2018\)](#), [Acharya et al. \(2018\)](#), [Bottero et al. \(2020\)](#)), sales ([Arellano et al. \(2019\)](#)), and investment ([Kalemli-Özcan et al. \(2018\)](#)). We use a similar approach and focus on the firm-level investment both in tangible assets and intangible assets. We also explore the heterogeneous asset reallocation among the firms and motivate the theoretical model.

This paper relates to the literature that studies the relationship between credit constraints and firms’ intangible investment behavior over the business cycle. [Lopez and Olivella \(2018\)](#) finds that intangible capital, which cannot be used by financially constrained firms as collateral, is key to generate labor market volatility in response to financial shocks. Several literature also finds declines in intangible investment due to financial frictions during recessions or normal times ([Garcia-Macia \(2017\)](#), [Demmou et al. \(2020\)](#)). Similar to these papers, the key to our mechanism is the firm-level collateral and asset tangibility. We contribute by quantifying the impact of sovereign risk on firm intangible investment and the role of financial frictions. We also quantify

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<sup>1</sup>As mentioned in [Bocola \(2016\)](#), this shock captures empirical evidence that much variations in Italian sovereign spreads during the European debt crisis was driven by factors orthogonal to domestic fundamentals.

the aggregate costs of a sovereign debt crisis through our theoretical model.

Our model implication on TFP costs of sovereign debt crisis relates to an extensive literature that studies the financial friction and R&D investment. The positive effects of intangible assets on firms' productivity and economic performance are well established in the literature.<sup>2</sup> Although traditional literature shows that R&D investment is countercyclical due to the low opportunity cost of long-term innovative investments in recessions than in booms (Bean (1990), Aghion and Saint-Paul (1998)), more recent literature, such as Aghion et al. (2012), show that this traditional view is only true for firms that are not constrained financially. Several literature also highlights the key role of credit constraints in R&D investment (Brown et al. (2012), Hall et al. (2016), Peia and Romelli (2020), Xue et al. (2021)).

**Road map.** This paper proceeds as follows. Section 2 shows the data and key empirical findings. Section 3 presents our model with sovereign default risk, financial intermediaries, and firm investment on both tangible and intangible assets. Section 4 calibrates the model and uses the model to measure the output and TFP costs of sovereign risk. Section 5 concludes.

## 2 Empirical facts

This section documents the empirical results about the impact of sovereign risk on firms' intangible investment and tangible investment. Section 2.1 describes the construction of variables of interest and provides summary statistics. Section 2.2 shows the firm-level responses in terms of both intangible investment and tangible investment during sovereign debt crises. Section 2.3 provides additional empirical evidence for robustness.

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<sup>2</sup>This branch of literature can trace back to Griliches (1958); research on R&D investment and firm productivity has bloomed ever since (Griliches (1979); Geroski (1989); Hall et al. (2010)). Several recent studies show that low firm-level incentives to invest in intangibles would result in TFP and output losses. The low incentives can be either caused by distortions (Ranasinghe (2014)), or monetary policy (Moran and Queralto (2018)), or equity financing shocks (Bianchi et al. (2019)), or financial crises (Queralto (2019)).

## 2.1 Data description

**Firm-level variables.** Our main sample uses the annual firm-level data from Orbis dataset, covering the period 2006-2016. The dataset covers a large majority of Italian firm including both private and public firms and includes rich balance-sheet information.<sup>3</sup> The core variables in our analysis are firm investment (investment on intangible fixed assets and tangible fixed assets), key balance sheet indicators (total assets, short-term debt and long-term debt), and additional firm-level variables.

We perform standard steps to guarantee the quality of the data. For nominal variables, we scale them by the producer price index. For intangible asset and tangible assets, the prices of intangibles and tangibles are calculated according to price information in EU KLEMS database. We further restrict the sample to the firms that already exist in 2006 and exclude firms that are not in the manufacturing sector. In Appendix A, we provide details on variables and sample selections.

Intangible fixed assets on firms' balance sheets in Orbis dataset are defined as all intangible assets such as formation expenses, research expenses, goodwill, development expenses and all other expenses with a long-term effect. To measure investment on intangible assets, we take log difference of intangible fixed assets. We use this log-difference specification because investment is highly skewed, suggesting a log-linear rather than level-linear regression specification. One possible concern about this log-difference measure of intangible investment stems from the discontinuity of intangible fixed assets. Using the log-difference measure of intangible investment loses some observations if firms empty the intangible fixed assets in some certain years, which may bias our baseline estimation. To deal with the observation losses of intangible investment calculated by the log-difference method, we consider the extensive margin of intangible investment and also use alternative measures for investment as robustness checks.

In our baseline regression, leverage is defined as the ratio of total debt over total assets, where total debt is the sum of short-term debt and long-term debt. We label the leverage calculated according to this method as "total leverage". We also use short-term leverage (loan-to-asset ratio) and net leverage (the ratio of net debt to

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<sup>3</sup>Appendix A.3 shows the aggregate distribution of wage bill and employment by firm size to display our sample representativeness.

total assets, where net debt is the sum of short term loans and long term debt net of net current assets) as robustness checks. As standard in literature, we define size, liquidity, sales growth, and net current asset ratio.<sup>4</sup> Table 1 reports a set of summary statistics for the main variables.

Table 1: Summary statistics

	Obs.	Mean	St.Dev.	Min.	Max.	P25	Median	P75
Net leverage	389560	0.053	0.37	-0.937	3.963	-0.205	0.054	0.309
Short-term leverage	389560	0.131	0.148	0	0.796	0	0.078	0.229
Total leverage	389560	0.202	0.195	0	0.983	0	0.165	0.349
Size	389560	10.123	1.203	4.565	13.323	9.283	10.145	11.011
Liquidity	389560	0.075	0.111	0	0.848	0.004	0.025	0.1
Investment on tangibles (log-diff)	389560	-0.124	0.746	-2.769	3.623	-0.462	-0.197	0.046
Investment on intangibles (log-diff)	389560	-0.04	0.397	-1.877	2.484	-0.204	-0.086	0.041

**Sovereign debt crisis.** The Italian economy was hit by the global financial crisis in 2008-2009 and recovered slightly in 2010. Then the economy experienced a second deep recession featuring sovereign debt crisis in 2011-2013, when real GDP further declined by 4.4%. Figure 1 plots the monthly sovereign spreads for Italy. Sovereign spread is defined as the gap between 30-year government bond yield between Italy and that of Germany.<sup>5</sup> During 2011-2013, spread spikes to about 4.5%. Sovereign debt credit rating also indicates the severity of a sovereign debt crisis. In 2011, the Standard & Poor's (S&P) revised their credit rating for Italian bonds from A+ to A, and further downgraded the rating to BBB+ in 2012, BBB in 2013 and BBB- in 2014.

## 2.2 Firm-level responses during sovereign debt crisis

We focus on how firms' investments react differently during the sovereign debt crisis. In the first specification, we focus on how the heterogeneous effects of sovereign risk on investment (including intangible and tangible investment) depend on firms' characteristics, controlling for sector-year fixed effect. To obtain the average effect of sovereign risk, we relax the sector-year fixed effect and include more aggregate controls in the second specification. This specification allows us to include government spread in the regression and examine its coefficients. After examining the responses

<sup>4</sup>The detailed definitions can be found in Appendix A.1.

<sup>5</sup>Similar patterns hold with sovereign spread data with different maturities.





Figure 1: Sovereign spread

Notes: Italian sovereign spread (monthly data). Spread is defined as the gap between 30-year government bond yield and that of Germany. Data is obtained from GFDFinæon.

of intangible investment and tangible investment, we test the asset reallocation from intangible assets to tangible assets during the sovereign debt crisis.

### 2.2.1 Heterogeneous effects

To estimate the responses of firm-level investment to sovereign risk and how the responses depend on firm-level characteristics, we estimate variants of the baseline empirical specification:

$$\Delta \log(\text{assets}_{i,t+1}) = \alpha(x_i \times sp_t) + \text{Controls} + \delta_i + \eta_{st} + \epsilon_{it}, \quad (1)$$

where *assets* represent two types of assets: intangible assets and tangible assets.  $\Delta \log(\text{assets}_{i,t+1}) \in \{\Delta \log(\text{intangibles}_{i,t+1}), \Delta \log(\text{tangibles}_{i,t+1})\}$  denotes intangible investment or tangible investment of firm *i* at time *t*, which are defined as the log-difference of intangible assets  $[\log(\text{intangibles}_{i,t+1}) - \log(\text{intangibles}_{it})]$  or the log-difference of tangible assets  $[\log(\text{tangibles}_{i,t+1}) - \log(\text{tangibles}_{it})]$ .  $sp_t$  is the sovereign spread at time *t*.  $x_i$  represents firm-level characteristics and we focus on two key characteristics: size and leverage. Thus,  $x_i \in \{\text{size}_{i,2006}, \text{totallev}_{i,2006}\}$  is firm's

size or total leverage in the year 2006.<sup>6</sup> *Controls* contains the interaction term of  $x_i$  and GDP growth ( $\Delta GDP_t$ ) and a vector of firm-level variables in time  $t - 1$ , which includes size, total leverage, liquidity, sales growth, the ratio of liability to total assets, and the ratio of net current assets to total assets.  $\delta_i$  controls for firm fixed effect, which captures the permanent differences in investment behaviors across firms.  $\eta_{st}$  is a sector-year fixed effect, which captures differences in sectoral exposure to aggregate shocks.  $\epsilon_{it}$  is a residual. Our main coefficient of interest is  $\alpha$ , which depicts how firms invest in response to sovereign spread, conditional on firm's characteristics.

Panel (a) of Table 2 reports the results from estimating the baseline specification (1) for intangible investment. We have standardized size and total leverage over the entire sample in the year 2006, so their units are in standard deviations relative to the mean.<sup>7</sup> We cluster the standard errors at the firm level. Column (1) in Table 2 Panel (a) shows the result when only focusing on firms' heterogeneity in size. Column (2) reports the results when focusing on firms' heterogeneity in total leverage. Column (3) adds both interactions. The results show that a firm with one standard deviation larger size than the average has approximately 1.998 units *higher* semi-elasticity of intangible investment when spread increases, and a firm with one standard deviation higher leverage than the average firm has approximately a 0.652 units *lower* semi-elasticity.

Panel (b) of Table 2 reports the results of estimating Eq (1) for tangible investment. Column (3) of Panel (b) shows that a firm with one standard deviation larger size than the average has approximately 0.652 units *higher* semi-elasticity of tangible investment when spread increases, and a firm with one standard deviation higher leverage than the average firm has approximately a 0.129 units *higher* semi-elasticity. Comparing Panel (a) and (b), we find that firms at each leverage level have opposite responses in terms of intangible and tangible investment: a high leverage firm would invest less on intangible assets but invest more on tangible assets compared to other firms during a sovereign debt crisis.

As the sector-year fixed effect absorbs the average effect of spread, we can only

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<sup>6</sup>We use firms' characteristics in the first year of the sample to guarantee the variables are pre-determined. We also use other measures of leverage including net leverage and short leverage for robustness checks.

<sup>7</sup>standardized size <sub>$i,2006$</sub>  =  $\frac{size_{i,2006} - \frac{1}{N_{2006}} \sum_i size_{i,2006}}{std(size_{i,2006})}$ . Same standardization applies for leverage.

know the heterogeneous responses by estimating Eq (1). We now relax the sector-year fixed effect and instead include aggregate controls, which allows us to compare the heterogeneous responses to the average effects.

Table 2: Heterogeneous responses of firm investment

(a) Intangible investment				(b) Tangible investment			
	$\Delta \log(intangibles_{i,t+1})$				$\Delta \log(tangibles_{i,t+1})$		
	(1)	(2)	(3)		(1)	(2)	(3)
$size_{i,2006} \times sp_t$	1.905*** (0.189)		1.998*** (0.191)	$size_{i,2006} \times sp_t$	0.671*** (0.091)		0.652*** (0.092)
$totallev_{i,2006} \times sp_t$		-0.426*** (0.165)	-0.652*** (0.166)	$totallev_{i,2006} \times sp_t$		0.202*** (0.074)	0.129* (0.074)
Firm controls	Yes	Yes	Yes	Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Firm FE	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	Sector-year FE	Yes	Yes	Yes
Observations	303,935	303,935	303,935	Observations	303,935	303,935	303,935
R-squared	0.026	0.025	0.026	R-squared	0.068	0.068	0.068
Number of id	59,706	59,706	59,706	Number of id	59,706	59,706	59,706

Notes: Results from estimation Eq (1) for intangible investment. We have normalized  $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$  to standardized size and leverage, so their units are in standard deviation relative to the mean. Robust standard errors (in parentheses) are clustered by firms. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Notes: Results from estimation Eq (1) for tangible investment. We have normalized  $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$  to standardized within-firm size and total leverage, so their units are in standard deviation relative to the mean. Robust standard errors (in parentheses) are clustered by firms. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

## 2.2.2 Average effects

To assess the economic significance of our estimated interaction coefficients  $\alpha$ , we now relax the sector-year fixed effect to obtain the average effect of sovereign risk by estimating

$$\Delta \log(assets_{i,t+1}) = \alpha_0 sp_t + \alpha_1 (x_i \times sp_t) + Controls + AggControls + \delta_i + \epsilon_{it}, \quad (2)$$

where  $\Delta \log(assets_{i,t+1}) \in \{\Delta \log(intangibles_{i,t+1}), \Delta \log(tangibles_{i,t+1})\}$  denotes intangible or tangible investment of firm  $i$  at time  $t$ ,  $sp_t$  is the sovereign spread at time  $t$ , and  $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$  is firm's size or total leverage in the year 2006.  $Controls$  is a vector of firm-level variables in time  $t - 1$  as defined in Eq (1). We also include a vector of aggregate controls  $AggControls$ , which includes GDP growth ( $\Delta GDP_t$ ) and its interactions with firms' characteristics ( $x_i \times \Delta GDP_t$ ).<sup>8</sup>  $\delta_i$  is a firm

<sup>8</sup>Appendix B.3 provides results when we add more aggregate controls, such as world GDP growth, trade openness, etc, and the results remain robust.

fixed effect. Our coefficients of interest are  $\alpha_0$  and  $\alpha_1$ .  $\alpha_0$  shows the average effect of sovereign spread on firms' investment, and  $\alpha_1$  measures how semi-elasticity of investment with respect to spread depends on firm size or total leverage.

Table 3 reports the results for estimating Eq (2). The coefficient of  $sp_t$  for intangible (tangible) investment is around 0.1-0.3 (0.6-0.7) in absolute value. The interaction coefficients imply an economically meaningful degree of heterogeneity. The coefficients for the interaction terms are similar to the ones in Table 2. There are several observations. First, on average, firms will invest less in both intangible assets and tangible assets. Second, firms are heterogeneous in their investment responses. Specially, compared with other firms, high-leverage firms will invest more on tangible assets and invest less on intangible assets, indicating a higher extent of asset reallocation. In the next section, we use the tangibles-to-intangibles ratio as the dependent variable to show the asset reallocation.

Table 3: Average responses of firm investment

(a) Intangible investment					(b) Tangible investment				
	$\Delta \log(intangibles_{it+1})$					$\Delta \log(tangibles_{it+1})$			
	(1)	(2)	(3)	(4)		(1)	(2)	(3)	(4)
$sp_t$	-0.117 (0.163)	-0.376** (0.164)	-0.109 (0.163)	-0.375** (0.164)	$sp_t$	-0.689*** (0.074)	-0.777*** (0.078)	-0.693*** (0.074)	-0.778*** (0.078)
$size_{i,2006} \times sp_t$		1.984*** (0.187)		2.073*** (0.189)	$size_{i,2006} \times sp_t$		0.683*** (0.089)		0.663*** (0.089)
$totallev_{i,2006} \times sp_t$			-0.376** (0.164)	-0.617*** (0.166)	$totallev_{i,2006} \times sp_t$			0.219*** (0.073)	0.142* (0.074)
Firm controls	Yes	Yes	Yes	Yes	Firm controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Firm FE	Yes	Yes	Yes	Yes
Observations	303,935	303,935	303,935	303,935	Observations	303,935	303,935	303,935	303,935
R-squared	0.012	0.013	0.012	0.013	R-squared	0.042	0.042	0.042	0.043
Number of id	59,706	59,706	59,706	59,706	Number of id	59,706	59,706	59,706	59,706

Notes: Results from estimation Eq (2) for intangible investment. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Notes: Results from estimation Eq (2) for tangible investment. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

### 2.2.3 Reallocation towards tangibles

When firms cut more intangible investment than tangible investment, their asset allocation changes. To better visualize the reallocation pattern indicated by the previous results, we replace the dependent variable in Eq (1) and (2) with the ratio of tangible fixed assets to intangible fixed assets. The estimation results are shown in Table 4. The positive sign for  $sp_t$  shows that on average, firms increase their tangibles-to-intangibles ratios when sovereign spread goes up. This asset reallocation

depends on firms' characteristics: the negative sign for  $size \times spread$  interaction term and the positive sign for  $leverage \times spread$  show that small firms and high-leverage firms reallocate resources towards tangible assets by more. In Section 4.2, we would compare the estimation results with those using the model-simulated data.

Table 4: Responses of tangibles-to-intangibles ratio

	$\log(\text{tangibles}_{i,t+1}) / \log(\text{intangibles}_{i,t+1})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$sp_t$				1.686*** (0.150)	1.453*** (0.140)	1.678*** (0.151)
$size_{i,2006} \times sp_t$	-1.009*** (0.166)		-1.064*** (0.169)	-0.986*** (0.165)		-1.042*** (0.167)
$totallev_{i,2006} \times sp_t$		0.195 (0.143)	0.323** (0.145)		0.199 (0.143)	0.327** (0.145)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No
Observations	380,769	380,769	380,769	380,769	380,769	380,769
R-squared	0.145	0.144	0.145	0.141	0.141	0.141
Number of id	73,697	73,697	73,697	73,697	73,697	73,697

Notes: Results from estimation Eq (1) and (2) for tangibles-to-intangibles ratios. Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## 2.3 Additional empirical results

This section presents several key robustness checks, including using alternative measures for investment, for sovereign debt crisis, and for firm-level leverage. More robustness checks, including taking depreciation into account, adding region-year fixed effects, adding more aggregate controls, clustering at sector level, winsorizing the sample with alternative criteria, deflating with alternative price indices, using alternative maturities for government spread, etc. can be found in Appendix B.

In the baseline regressions, we use log-difference of assets to measure investment. The log-difference measure captures the relative change that is symmetric, additive and normed. However, one potential concern is that the log-difference measure omits the observation with zero stock of asset by construction, which could be more pronounced for intangible assets. To deal with this concern, we further characterize

the effects of sovereign risk on the extensive margin of intangible investment and also use alternative measure for investment.

**Extensive margin of intangible investment.** Here we focus on the extensive margin of intangible assets<sup>9</sup>, i.e., whether to continue having intangible assets. Denote  $\mathbb{1}(intangibles_{i,t+1})$  as an indicator that equals 1 if firm  $i$  continues to hold any intangible fixed assets in period  $t + 1$ , and equals 0 if firm  $i$  stops having any intangible fixed assets. Table 5 reports the results when we substitute the dependent variables with  $\mathbb{1}(intangibles_{i,t+1})$  in Eq (1) and Eq (2). Table 5 shows that large firms and low-leverage firms are more likely to continue to hold intangible assets, consistent with the intensive margin results of adjusting intangible assets as shown in Section 2.2.1 and 2.2.2.

Table 5: Extensive margin of intangibles

	$\mathbb{1}(intangibles_{i,t+1})$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$sp_t$				-1.789*** (0.041)	-1.866*** (0.043)	-1.788*** (0.041)	-1.866*** (0.043)
$size_{i,2006} \times sp_t$	0.607*** (0.050)		0.621*** (0.050)		0.630*** (0.050)		0.643*** (0.050)
$totallev_{i,2006} \times sp_t$		-0.017 (0.043)	-0.093** (0.043)			-0.002 (0.043)	-0.083* (0.043)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	383,121	383,121	383,121	383,121	383,121	383,121	383,121
R-squared	0.029	0.028	0.029	0.017	0.018	0.017	0.018
Number of id	71,339	71,339	71,339	71,339	71,339	71,339	71,339

Notes: Results from estimating Eq (1) and Eq (2) with the dependent variable being an indicator that equals 1 if continuing to hold any intangible assets, and equals 0 if stopping to hold any intangible assets. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

**Alternative measure for investment.** To account for the asset changes both in the intensive and extensive margin, we borrow from the job creation literature (Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996), Huber, Oberhofer, and

<sup>9</sup>For tangible assets, using log-difference to measure investment is less of a concern (if any), because we exclude the firms with negative or zero total assets, and the remaining firms have at least positive tangible assets.

Pfaffermayr (2013), among others) a growth measure that accounts for both intensive and extensive margin. We analogously define firm investment.

For intangible assets, firms in each year can be classified into three groups:

$$\begin{cases} \text{exiting firms} & G_x = \{i | k_{it} \neq 0, k_{i,t+1} = 0\} \\ \text{continuing firms} & G_c = \{i | k_{it} \neq 0, k_{i,t+1} \neq 0\} \\ \text{entering firms} & G_n = \{i | k_{it} = 0, k_{i,t+1} \neq 0\} \end{cases}$$

where  $k_{it}$  denotes intangible fixed assets of firm  $i$  at period  $t$ . Here "exiting" and "entering" only indicate whether firm  $i$  continues to hold intangible fixed assets. Then, investment on intangible assets (which is also the growth rate between two averages) can be defined as:

$$g(\text{intangibles}_{i,t+1}) = \frac{k_{i,t+1} - k_{it}}{0.5(k_{i,t+1} + k_{it})} = \begin{cases} -2 & i \in G_x \\ \frac{k_{i,t+1}/k_{it} - 1}{0.5(k_{i,t+1}/k_{it} + 1)} & i \in G_c \\ 2 & i \in G_n \end{cases} \quad (3)$$

We refer to this measure of investment as DHS (abbreviation for Davis, Haltiwanger, and Schuh (1996)) investment. The main advantage of DHS investment is that it can accommodate both entry (into the asset market, i.e., begin to hold assets) and exit (from the asset market, i.e., do not hold assets any more). It is a second-order approximation of the log difference growth rate around 0 and it is bounded in the range  $[-2, 2]$ . We estimate both empirical specifications Eq (1) and Eq (2) using DHS investment as the dependent variable.

Table 6 shows that, consistent with baseline regression results, small firms and high-leverage firms decrease their intangible investment more than other firms during a sovereign debt crisis.

**Alternative measure for sovereign debt crisis.** Another way to measure the severity of a sovereign debt crisis is to use the credit rating for sovereign bonds from the credit rating agencies, such as Standard & Poor's (S&P). The credit rating from S&P ranges from AAA (prime grade) to D (default). For instance, A+ implies an upper medium grade and B+ suggests highly speculative. As shown in Table 7, S&P credit

Table 6: Alternative measure for intangible investment: DHS investment

	g(intangibles <sub>i,t+1</sub> )						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$sp_t$				-2.672*** (0.150)	-2.964*** (0.152)	-2.671*** (0.150)	-2.975*** (0.152)
$size_{i,2006} \times sp_t$	2.469*** (0.174)		2.589*** (0.175)		2.580*** (0.171)		2.697*** (0.173)
$totallev_{i,2006} \times sp_t$		-0.465*** (0.154)	-0.786*** (0.155)			-0.415*** (0.154)	-0.758*** (0.155)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No	No
Observations	392,201	392,201	392,201	392,201	392,201	392,201	392,201
R-squared	0.032	0.031	0.032	0.013	0.014	0.013	0.015
Number of id	71,523	71,523	71,523	71,523	71,523	71,523	71,523

rating for Italian sovereign bonds downgraded from A+ to A in 2011, and further downgraded to BBB+ in 2012, BBB in 2013 and BBB- in 2014.

We assign discrete numbers to represent the level of credit rating. The score is between 100 (riskless) and 0 (likely to default) as the Trading Economics credit rating.<sup>10</sup> Here we list the numbers related to Italian sovereign bond credit rating from 2006 to 2016: A+ (80), A (75), BBB+ (65), BBB (60), BBB- (55).

Table 7: Credit rating for Italy by S&amp;P

Year (end of year)	S&P	Assigned number
2006	A+	80
2007	A+	80
2008	A+	80
2009	A+	80
2010	A+	80
2011	A	75
2012	BBB+	65
2013	BBB	60
2014	BBB-	55
2015	BBB-	55
2016	BBB-	55

We estimate the empirical specifications in the baseline regressions using credit rating score rather than debt spread. Table 8 shows the results. Column (1) shows the heterogeneous effect of credit rating on intangible investment by estimating Eq

<sup>10</sup>Refer to <https://tradingeconomics.com/italy/rating> for full correspondence table. It is also easy to infer: AAA corresponds to 100, AA+ corresponds to 95 and each downgrade corresponds to reduction by 5 in the score.



(1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t + 1$ , 0 if firm  $i$  stops holding any intangible fixed assets. Column (4) and (5) are the results for tangible investment. In general, during the sovereign debt crisis (lower credit rating for sovereign debt), small and high-leverage firms decrease their intangible investment more than other firms, consistent with our baseline results.

Table 8: Alternative measure for debt crisis: sovereign debt credit rating

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$rating_t$		-0.031*** (0.001)	-0.010*** (0.000)		-0.013*** (0.000)
$size_{i,2006} \times rating_t$	0.008*** (0.001)	0.007*** (0.001)	0.004*** (0.000)	0.001* (0.001)	0.001 (0.000)
$totallev_{i,2006} \times rating_t$	-0.003*** (0.001)	-0.003*** (0.001)	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.019	0.020	0.068	0.047
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of sovereign debt credit rating on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

**Alternative measure for leverage.** In the baseline regressions, leverage is defined as the ratio of total debt to total assets (we refer to this ratio as "total leverage"), where total debt includes both short-term loans and long-term debt. Here we use net leverage and short leverage as alternative measures of leverage. Table 9 and Table 10 show that the baseline results are robust to these alternative leverages.

Appendix B further provides some robustness checks. Appendix B.1 provides results when we take depreciation rates of different assets into consideration. Appendix B.2 controls additional region-year fixed effect or province-year fixed effect, which capture the geographical differences of sample firms. Appendix B.3 includes more aggregate controls in the empirical specifications. In the baseline regression, we use continuous standardized size and leverage in 2006 as our main measure for firm's size and financial condition. The results are robust to alternative group dummies (Appendix B.4 and B.5). For example, if we replace standardized size in 2006 with a

Table 9: Alternative measure for leverage: net leverage

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.402** (0.163)	-1.879*** (0.043)		-0.767*** (0.077)
$size_{i,2006} \times sp_t$	1.886*** (0.189)	1.961*** (0.187)	0.628*** (0.050)	0.674*** (0.091)	0.682*** (0.089)
$netlev_{i,2006} \times sp_t$	-0.523*** (0.179)	-0.433** (0.177)	-0.089* (0.048)	0.150* (0.081)	0.184** (0.079)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,715	303,715	382,908	303,715	303,715
R-squared	0.026	0.013	0.019	0.068	0.042
Number of id	59,583	59,583	71,209	59,583	59,583

Notes: Estimation results here use net leverage instead of total leverage. Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table 10: Alternative measure for leverage: short-term leverage

Dependent variable	$\Delta \log(intangibles_{it})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{it})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.403** (0.164)	-1.861*** (0.043)		-0.797*** (0.078)
$size_{i,2006} \times sp_t$	1.916*** (0.189)	1.991*** (0.186)	0.629*** (0.049)	0.644*** (0.091)	0.655*** (0.088)
$shortlev_{i,2006} \times sp_t$	-0.493*** (0.165)	-0.465*** (0.165)	-0.067 (0.043)	0.122 (0.077)	0.133* (0.076)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	304,704	304,704	383,904	304,704	304,704
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	59,699	59,699	71,295	59,699	59,699

Notes: Estimation results here use short-term leverage instead of total leverage. Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

dummy, 1 for firms with size larger than median and 0 otherwise, the baseline results are still robust. Appendix B.6 shows the results when we cluster the standard errors at the sector level. Appendix B.7 shows results hold if we alternatively winsorize the variables of interest at top and bottom 0.5% (baseline uses 1%). In the baseline regression, the intangible and tangible fixed assets are deflated by the price of intangible and tangible assets, respectively. Appendix B.8 shows that the results are robust when we deflate intangible and tangible fixed assets by Producer Price Index (PPI). We then replace the baseline sovereign spread with aggregate-level firm spread (Appendix B.9), and spreads with different maturities (Appendix B.10). Our baseline results remain robust.

## 2.4 Summary

The sovereign debt crisis transmits to the private sector and affects firms' investment. Distinguished from existing literature, we focus on firms' investment both on intangible assets and tangible assets. During the sovereign debt crisis, firms reduce both of their intangible and tangible investment, but small and high-leverage firms reduce their investment on intangible assets by more than other firms. To rationalize firms' heterogeneous investment behaviors and quantify the aggregate impact of the sovereign debt crisis on investment, output, and productivity, we develop a sovereign default model with heterogeneous firms who accumulate both intangible assets and tangible assets.

## 3 Model

The economy is composed of a central government, firms, financial intermediaries, and households. The government issues long-term government bonds to the financial intermediaries. The probability of a government default evolves over time according to a reduced-form stochastic process. Government collects tax revenues from the final goods firms and borrows from the financial intermediaries to finance the lump-sum transfers to the households and services the outstanding government debt. Government can default on its debt.

There are two types of firms: final goods firms and intermediate goods firms. Final goods firms are competitive, and they convert intermediate goods to final goods. Intermediate goods firms operate under monopolistic competition, and they use tangible capital and intangible capital to produce differentiated goods. They borrow from the financial intermediaries to finance a fraction of cost for investment, and the borrowing interest rate is firm-specific depending on the firm's tangibility. The intermediate goods firms are exogenously heterogeneous in productivity and financing needs.

Households are composed of consumers and bankers, and they own intermediate goods firms. Households decide on their consumption and savings to the financial intermediaries. The financial intermediaries are run by the bankers who use repayments on the claims and the savings of the households to lend to the intermediate goods firms and the government.

The economy is perturbed by an aggregate shock which exogenously determines the process of government default risk. We start by describing the problems of each type of agent: the government, final goods firms, intermediate goods firms, consumers, and financial intermediaries. We then define the equilibrium for this economy.

### 3.1 The government

The government provides transfers to the households. It finances the transfers  $T_t$  by issuing long-term bonds to the financial intermediaries and levying a tax rate  $\tau$  on aggregate final goods  $Y_t$ . In every period, a fraction  $\vartheta$  of debt matures and the remaining fraction remains outstanding. The government can default on its debt in every period by writing off a fraction  $f \in [0, 1]$  of its outstanding obligations.

Following [Bocola \(2016\)](#), we assume an exogenous process of government default risk. Assume that in every period the economy is hit by a shock  $\varepsilon_d$  that follows a standard logistic distribution. The default process follows:

$$d_{t+1} = \begin{cases} 1 & \text{if } \varepsilon_{d,t+1} < s_t \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

where  $s$  is an AR(1) process:<sup>11</sup>

$$\log(s_t) = (1 - \rho_s) \log(s^*) + \rho_s \log(s_{t-1}) + \sigma_s \varepsilon_{st}, \quad \varepsilon_{st} \sim N(0, 1). \quad (5)$$

The probability of default is then given by:

$$p_t^d \equiv \text{Prob}(d_{t+1} = 1 | s_t) = \frac{\exp(s_t)}{1 + \exp(s_t)}. \quad (6)$$

Every period, the government maximizes transfer  $T_t$  by choosing new bond level  $B_{t+1}$ , subject to its budget constraint:

$$q_t[B_{t+1} - (1 - \vartheta)(1 - d_t f)B_t] + \tau Y_t = \vartheta(1 - d_t f)B_t + \frac{\phi_b}{2}[B_{t+1} - (1 - \vartheta)(1 - d_t f)B_t]^2 + T_t, \quad (7)$$

where  $q_t$  is government bond price,  $B_t$  is the stock of bond in time  $t$ . When the government defaults,  $d_t = 1$  and a fraction  $f \in [0, 1]$  of its outstanding obligations is written off.  $\phi_b$  is the parameter for bond adjustment cost.<sup>12</sup>

### 3.2 Final goods firms

The final good  $Y_t$  is produced from a fixed variety of intermediate goods  $i \in [0, 1]$  using the technology:

$$Y_t \leq \left[ \int (y_{it})^\eta di \right]^{\frac{1}{\eta}}, \quad (8)$$

where the elasticity of demand is  $\frac{1}{1-\eta} > 1$ . We normalize the price of final goods to one, so the taxes paid to the government is  $\tau Y_t$ . The price of intermediate good  $i$  is  $p_{it}$ . The final goods producers choose the intermediate goods  $\{y_{it}\}$  to solve:

$$\max_{\{y_{it}\}} (1 - \tau) Y_t - \int p_{it} y_{it} di, \quad (9)$$

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<sup>11</sup>As explained in [Bocola \(2016\)](#), this formulation of default process is consistent with literature showing that self-fulfilling beliefs were key drivers of sovereign risk during the European debt crisis. It also allows us to isolate the economic mechanisms underlying the propagation of sovereign default risk.

<sup>12</sup>This is a parsimonious way to pin down government bond level. Alternatively, one could set a reduced-form fiscal rule, and then the government bond level is to balance the budget constraint as in [Bocola \(2016\)](#).

subject to (8). Thus, the demand function  $y_{it}$  for intermediate good  $i$  is solved as:

$$y_{it} = \left( \frac{1 - \tau}{p_{it}} \right)^{\frac{1}{1-\eta}} Y_t. \quad (10)$$

Demand function (10) shows that the demand for good  $i$  is negatively correlated with price  $p_{it}$ , and positively correlated with total output  $Y_t$ .

### 3.3 Intermediate goods firms

There is a unit measure of intermediate goods firms producing differentiated goods.<sup>13</sup> Each firm  $i$  produces output  $y_{it}$  with tangible capital  $k_{T,it}$  and intangible capital  $k_{I,it}$ :

$$y_{it} = z_{it} k_{T,it}^{\alpha_T} k_{I,it}^{\alpha_I} \quad (11)$$

where  $z_{it}$  is the idiosyncratic productivity shock, evolving following  $\log(z_{it}) = \rho_z \log(z_{it-1}) + \sigma_z \varepsilon_{it}$ , where  $\varepsilon_{it}$  follows a standard normal random process.  $k_{T,it}$  is the stock of tangible capital, and  $k_{I,it}$  is the stock of intangible capital.  $\alpha_T$  and  $\alpha_I$  are the income shares of tangible capital and intangible capital, respectively.  $\alpha_T + \alpha_I \leq 1$  so that the production function is non-increasing returns to scale technology.  $z_{it} k_{I,it}^{\alpha_I}$  determines the marginal return of tangible capital, which we refer to as TFP in the model. Tangible capital depreciates every period at the rate  $\delta_T$ , and there is an adjustment cost for changing the capital stock. Thus, the investment on tangible capital at period  $t$  is given by:

$$i_{T,it} = k_{T,it+1} - (1 - \delta_T) k_{T,it} + \Theta(k_{T,it+1}, k_{T,it}), \quad (12)$$

where  $\Theta(k_{T,it+1}, k_{T,it}) = \frac{\theta_T}{2} \left( \frac{k_{T,it+1}}{k_{T,it}} - 1 + \delta_T \right)^2 k_{T,it}$  is the convex adjustment cost for tangible capital. Similarly, the investment on intangible capital at period  $t$  is given by:

$$i_{I,it} = k_{I,it+1} - (1 - \delta_I) k_{I,it} + \Theta(k_{I,it+1}, k_{I,it}), \quad (13)$$

where  $\delta_I$  is the depreciation rate for intangible capital, and  $\Theta(k_{I,t+1}, k_{I,t}) = \frac{\theta_I}{2} \left( \frac{k_{I,t+1}}{k_{I,t}} - 1 + \delta_I \right)^2 k_{I,t}$  is the adjustment cost for investing in intangible capital.

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<sup>13</sup>We abstract from firm entry and exit for simplicity.

At the beginning of the period, firm  $i$ 's idiosyncratic productivity  $z_{it}$  is realized. Then firm  $i$  makes choices for next period tangible capital  $k_{T,it+1}$  and intangible capital  $k_{I,it+1}$ . We assume that firms need to borrow a fraction of their investment before production, and the financing needs  $\lambda_i$  are firm-specific and time-invariant. Heterogeneity in  $\lambda_i$  captures the heterogeneous borrowing requirement. The financial intermediaries provide loans  $b_{it}$  to firm  $i$  at a firm-specific interest rate  $R_{it}$ , and the financing requirement for firm  $i$  is:

$$b_{it} = \lambda_i(i_{T,it} + i_{I,it}), \quad (14)$$

where  $i_{T,it}$  is the investment on tangible capital, and  $i_{I,it}$  is the investment on intangible capital. We assume the distribution of  $\lambda_i$  are time-invariant, and it is denoted by  $\Lambda$ . The firm-specific interest rate  $R_{it}$  depends on the tangibility of firm  $i$ . The financial intermediaries' problem in Section 3.4 will introduce the functional form of the firm-specific interest rate.

At the end of the period, production takes place. Firm  $i$  decides on the price  $p_{it}$  for its production  $y_{it}$ , taking the demand function (10) as given, and repay the debt  $R_{it}b_{it}$ . The dividend of firm  $i$  at period  $t$  is:

$$\begin{aligned} D_{it} = & p_{it}z_{it}k_{T,it}^{\alpha_T}k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})] \\ & - [k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it}b_{it}. \end{aligned} \quad (15)$$

Taking the aggregate demand  $Y_t$  and interest rate  $R_{it}$  as given, the intermediate goods firm  $i$  chooses intangible investment and tangible investment to maximize the present value of the dividends  $\sum_{t=0}^{\infty} \beta^t D_{it}$ , subject to the demand function (10) and the working capital constraint (14). Appendix C.1 provides the analytical optimality conditions under a simplified case with no capital adjustment costs.

### 3.4 Households

The representative household is composed of consumers and bankers. The household's preferences over consumption  $C_t$  is given by:

$$U = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t C_t \right], \quad (16)$$

where  $\beta \in (0, 1)$  is the discount factor and  $C_t$  is consumption of the consumers in period  $t$ . The assumption of the linearity of the preferences over consumption simplifies the problem. Each period, the household sends out bankers to run financial intermediaries, and provides them with net worth  $N_t$ . At the end of the period, the bankers bring back their returns of operations  $F_t$  to the household. Household can also save by putting one-period deposit  $M_t$  to the financial intermediary with the price  $q_t^m$ .

Every period, the household also receives dividends  $D_t$  from the intermediate goods firms and lump-sum transfer  $T_t$  from the government. The budget constraint of the household is:

$$C_t + q_t^m M_t + N_t = M_{t-1} + D_t + F_t + T_t. \quad (17)$$

The household maximizes Eq (16) subject to Eq (17). The optimality conditions indicate that the price for deposit is given by  $q_t^m = \beta$ , which is constant over time.

**Financial intermediaries.** The bankers run the financial intermediaries. The financial intermediaries use the net worth  $N_t$  and the deposits of the household, to purchase government's bonds and issue loans to firms. The financial intermediaries are competitive.

The net worth  $N_t$  the household provides to the bankers consists of a constant term  $\bar{n}$  and the value of government bonds that did not mature:

$$N_t = \bar{n} + (1 - d_t f)(1 - \vartheta) q_t B_t. \quad (18)$$

The evolution of government default risk gives rise to dynamics of government



bond price  $q_t$ , as well as actual default  $d_t$ , which changes the value of the net worth for the financial intermediaries.

The *budget constraint* of the financial intermediaries at the beginning of the period is:

$$q_t B_{t+1} + \int b_{it} di \leq \underbrace{\bar{n} + (1 - d_t f)(1 - \vartheta) q_t B_t + q_t^m M_t}_{N_t}. \quad (19)$$

Aside from the budget constraint, the financial intermediaries are also subject to the *deposit constraint* that limits the amount of deposits that the financial intermediaries can get from the households:

$$q_t^m M_t \leq q_t B_{t+1} + \int \theta_{it} b_{it} di. \quad (20)$$

Here we assume that government's bonds can be fully pledged, while the firms' loans can only be pledged partially.  $\theta_{it}$  denotes the fraction that firm  $i$ 's debt can be pledged. This fraction  $\theta_{it}$  depends on the firm  $i$ 's share of tangible capital, as intangible capital typically can not be used as collateral. We assume the firm-specific fraction  $\theta_{it}$  that can be pledged is given by:

$$\theta_{it} = \frac{k_{T,it}}{\bar{k}} < 1, \quad (21)$$

where  $\bar{k}$  is constant,<sup>14</sup> and  $k_{T,it}$  is the stock of tangible capital for firm  $i$  at time  $t$ .  $\theta_{it} < 1$  reflects that firms' loans can't be fully pledged.

Combining the budget constraint (19) and the deposit constraint (20) gives that the amount of debt that can be lent to the firms (adjusted by  $\theta_{it}$ ) are bounded by the financial intermediaries' net worth, and we label it as the *leverage constraint*:

$$\int (1 - \theta_{it}) b_{it} di \leq N_t. \quad (22)$$

At the end of the period, the financial intermediaries receive the payment from the firms and the government, and also pay back household's deposits. The returns for

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<sup>14</sup>In the quantitative part, we set  $\bar{k}$  to the maximum capital level that firms can choose in order to guarantee  $\theta_{it} < 1$ .

the financial intermediaries equals:

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t. \quad (23)$$

The financial intermediary chooses  $\{M_t, B_{t+1}, b_{it}\}$  to maximize the expected return  $\mathbb{E}_t[\beta F_{t+1}]$  subject to (19) and (22). The optimality conditions give the following pricing conditions for the government's bonds and firm's loans:

$$q_t = \mathbb{E}_t\beta[(1 - d_{t+1}f)(\vartheta + q_{t+1}(1 - \vartheta))], \quad (24)$$

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}, \quad (25)$$

where  $\zeta_t$  is the Lagrange multiplier on the leverage constraint (22). The price for firm  $i$ 's loans (25) implies that firm  $i$  will pay a premium  $\frac{(1 - \theta_{it})\zeta_t}{\beta}$  over the risk-free rate when the leverage constraint of the financial intermediaries binds. When firms determine the investment on tangible capital and intangible capital, they are aware of its impact on the price of their loans.

### 3.5 Equilibrium

We now define the equilibrium for this economy. Define  $S = [s, B, d, \Lambda]$  as the state variables, where  $s$  is the government default risk process,  $B$  denotes the initial level of government debt,  $d(s)$  is the default event determined by the exogenous default risk process, and  $\Lambda(z, \lambda, k_T, k_I)$  is the distribution of the intermediate firms. We omit the time subscript  $t$  and use  $x'$  to denote a variable  $x$  in the next period.

Given an aggregate state  $S$ , the equilibrium consists of: (i) intermediate goods firms' policies for tangible capital  $k'_T(z, \lambda, k_T, k_I; S)$ , intangible capital  $k'_I(z, \lambda, k_T, k_I; S)$  and borrowing  $b(z, \lambda, k_T, k_I; S)$ , and final goods firms' output  $Y(S)$ ; (ii) policies for aggregate tangible capital  $K_T(S)$ , aggregate intangible capital  $K_I(S)$ , and consumption  $C(S)$ ; (iii) price functions for firm borrowing rates  $R(k_T, S)$ , government bond price function  $q(s)$ , and constant deposit price  $q^m$ ; and (iv) the distribution of firms over idiosyncratic productivity and capitals  $\Lambda(z, \lambda, k_T, k_I)$  such that: (i) policy functions of intermediate and final goods firms satisfy their optimization problem; (ii) intermedi-

ate firms borrowing rates satisfy Eq (25), and the leverage constraint (22) holds; (iii) the distribution of firms is consistent with idiosyncratic shocks; (iv) policies for households satisfy their optimal conditions; (v) government's bond for next period satisfies the government budget constraint; (vi) government's bond price satisfies Eq (24); and (vii) capital, goods and bond markets clear.

Next, we analyze key conditions that explain how an increase in sovereign risk affects firms' loan interest rates and their investment choices.

Recall that the financial intermediaries hold government bonds and face a leverage constraint which could be binding. When sovereign spread increases, the value of the government bonds in the financial intermediaries' balance sheet falls, which leads to a lower net worth  $N_t$ .

The loan interest rate for firm  $i$  is given by

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}, \text{ where } \theta_{it} = \frac{k_{T,it}}{\bar{k}} < 1. \quad (26)$$

The Lagrange multiplier  $\zeta_t > 0$  when the leverage constraint (22) binds. A decline in financial intermediaries' net worth  $N_t$  reduces credit supply and it further binds the leverage constraint, leading to an increase in  $\zeta_t$  and thus the loan interest rate  $R_{it}$ . The firm loan interest rate  $R_{it}$  also depends on its tangible assets, which can be used as collateral, through  $\theta_{it}$ . Thus, the firms internalize the impact of tangible capital on loan interest rate when choosing tangible investment and intangible investment. Tangible investment helps offset the tightening financial conditions by decreasing firm's loan interest rate, while intangible investment can not ( $\partial R_{it} / \partial k_{T,it} < 0$  and  $\partial R_{it} / \partial k_{I,it} = 0$  when the leverage constraint binds).

Eq (27) shows the first order condition for next period tangible capital choice  $k_{T,it+1}$  of firm  $i$ .<sup>15</sup> The left-hand-side is the marginal cost of increasing tangible capital, and the right-hand-side is the marginal benefit of tangible capital. Different from intangible capital, investing in tangible capital has an extra benefit of decreasing

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<sup>15</sup>To simplify notation, Eq (27) shows for the case when the adjustment costs equal zero. This simplification is only for analytical purpose. We solve for the general case quantitatively.

borrowing interest rate, as shown in the bracket.

$$\begin{aligned}
[1 + (R_{it} - 1)\lambda_i] = & \beta[\eta(1 - \tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta-1}(\alpha_T z_{it+1} k_{T,it+1}^{\alpha_T-1} k_{I,it+1}^{\alpha_I}) \\
& + (1 - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] \\
& - \underbrace{\left[ k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1} \right]}_{\text{extra benefit of tangible capital investment}} \lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}}]
\end{aligned} \tag{27}$$

During sovereign debt crises, the net worth of the financial intermediaries shrinks, thus tightening the leverage constraint. When  $\zeta_t$  increases, the impact of tangible capital on borrowing interest rate is larger, i.e., higher absolute value of  $\partial R_i / \partial k_{T,i}$ . For firms who largely rely on external financing (high- $\lambda$  firms), the marginal benefit of investing in tangibles is larger. This explains our empirical finding that high-leverage firms reallocate their resources from intangible capital to tangible capital by more during Italian sovereign debt crisis. The movements in the loan interest rate affect firms' capital choices and production decisions, which then affect the aggregate TFP and output.

## 4 Quantitative Analysis

We now fit the model to Italian data. This section proceeds in three steps. Section 4.1 describes our strategy to parametrize the model, reports the parameters of the model, and assesses the model fit. Section 4.2 studies the impulse responses of firm investment, output and TFP to an increase in government default risk, as well as heterogeneous responses across firms. Section 4.3 reports the results of our quantitative experiment, in which we use the model to measure the output and TFP losses during the Italian debt crisis.

## 4.1 Parameterization

The model is in annual frequency. There are two groups of parameters. The parameters in the first group are fixed exogenenously and are taken directly from the literature or from our estimation, and those in the second group are jointly chosen to match a set of firm and aggregate moments of Italian economy. Table 11 lists all the parameter values.

**Fixed parameters.** Fixed parameters are  $\{\alpha_T, \eta, \delta_T, \delta_I, \tau, \rho_z, \sigma_z, \beta, \vartheta, \rho_s, f\}$ . The parameters  $\{\alpha_T, \eta\}$  affect the shape of the production function of intermediate and final goods firms. We set  $\alpha_T$  to 0.36 following Pérez-Orive (2016).  $\eta$  is set to be 0.75, which is the conventional value in the literature. The depreciation rate for intangible assets  $\delta_I$  is 24.3% and the depreciation rate for tangible assets  $\delta_T$  is 10.1%, according to our estimation for Italian depreciation rates in 2006 using the EU KLEMS database.<sup>16</sup> The tax rate  $\tau$  is 0.24, which is the corporate tax rate in Italy. The persistence and standard deviation of firm productivity shock are set to be 0.9516 and 0.0033, following Lopez and Olivella (2018). The discount factor  $\beta$  is set to match an annual risk-free rate of 2%. The fraction of bonds maturing is set to be 0.05. The parameters governing the persistence of sovereign risk process  $\rho_s$  are taken from Bocola (2016). The haircut fraction  $f$  is consistent with empirical evidence in Cruces and Trebesch (2013).

**Fitted parameters.** The remaining parameters in the model include parameters for income share of the intangible capital  $\alpha_I$ , investment adjustment costs  $\{\theta_T, \theta_I\}$ , parameters for working capital requirement  $\{\lambda_I, \lambda_h\}$ , constant net worth  $\bar{n}$  for the financial intermediaries, a parameter  $\phi_B$  measuring government bond adjustment cost, and parameters for the sovereign risk process  $\{\sigma_s, s^*\}$ .

We target 9 sample moments that include statistics of firms, banks, and government. Firms' statistics include the volatility of tangible capital relative to the volatility of real sales, the counterpart for intangible capital, the average leverage for firms within low-leverage group and high-leverage group in 2006 and the average asset tangibility (ratio of tangible asset over total assets). The banks' statistics are the ratios of credit to non-financial corporations to credit to the government (=0.644), which is the average

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<sup>16</sup>For more details on the estimation of depreciation rates, please refer to Appendix B.1.

value from 2006 to 2015. The government's statistics include the ratio of government bonds to tax revenues in 2006, the average spread and the volatility of spread from 2006 to 2015.<sup>17</sup>

We solve the model using global methods.<sup>18</sup> Given the model policy functions, we perform simulations to obtain the model-implied counterparts of our targets. We jointly chosen the fitted parameters to match 9 sample moments. The parameters are chosen to minimize the sum of the distance between the moments in the model and their corresponding counterparts in the data.

Although we choose all parameters jointly to match the moments, we can provide a heuristic description of how the moments inform specific parameters. First, the income share of intangible capital and the capital adjustment costs mostly affect firm tangibility and capital volatility. Second, the average leverages for two groups of firms mainly pin down  $\{\lambda_l, \lambda_h\}$ ; two groups refer to firms with leverage lower than the median and firms with leverage higher than the median in 2006 in the data. Third, there is a tight relationship between  $\bar{n}$  and the ratio of the credit to non-financial corporations to the credit to the government. In the data, the average ratio of total credit to the private non-financial sector to total credit to the government sector is 0.644 from 2006 to 2015 according to Bank for International Settlements statistics. In the model, this ratio is given by  $\int b_i di / B$ . Fourth, the ratio of government bonds to tax revenue provides information for government adjustment cost parameter  $\phi_B$ . Finally, the mean and volatility of spread are informative in terms of sovereign risk process parameters  $\{\sigma_s, s^*\}$ . Table 12 reports the moments in the data and in the model. The model generates similar statistics to the ones in the data.

## 4.2 Effects of elevated sovereign risk

During sovereign debt crises, firms reduce their investment because of the higher borrowing cost. To offset the tightening financial conditions, firms reallocate their investment from intangible capital to tangible capital. That is to say, although they reduce both of their tangible and intangible investment, they cut their intangible

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<sup>17</sup>Here we use 30-year government bond spreads data. Using spread data with alternative maturities does not affect the main results.

<sup>18</sup>See Appendix D for the computational algorithm.

Table 11: Parameters

Parameter	Description	Value	Target/Source
<i>Fixed parameters</i>			
$\alpha_T$	Income share of tangible capital	0.36	Pérez-Orive (2016)
$\eta$	Markup parameter	0.75	Conventional value
$\delta_T$	Depreciation of tangible capital	0.101	Our estimation
$\delta_I$	Depreciation of intangible capital	0.243	Our estimation
$\tau$	Tax rate	0.24	Corporate tax rate
$\rho_z$	Persistence of firm productivity shock	0.9516	Lopez and Olivella (2018)
$\sigma_z$	Volatility of firm productivity shock	0.0033	Lopez and Olivella (2018)
$\beta$	Discount factor	0.98	Annual risk-free rate of 2%
$\vartheta$	Fraction of bonds maturing	0.05	Conventional value
$\rho_s$	Sovereign risk process	0.95	Bocola (2016)
$f$	Haircut fraction	0.37	Cruces and Trebesch (2013)
<i>Fitted parameters</i>			
$\alpha_I$	Income share of intangible capital	0.11	Firm tangibility
$\theta_T$	Adjustment cost of tangible investment	3.82	Vol(tangible capital)/Vol(sales)
$\theta_I$	Adjustment cost of intangible investment	0.003	Vol(intangible capital)/Vol(sales)
$[\lambda_l, \lambda_h]$	Working capital requirements	[0.11, 1.95]	Average leverage of firms
$\bar{n}$	Constant net worth	0.03	credit to firms/credit to government
$\phi_B$	Bond adjustment cost	49.6	Average government bonds/tax revenue
$\sigma_s$	Sovereign risk process	0.255	Volatility of spread
$s^*$	Sovereign risk process	-3.35	Average spread

Table 12: Moments in the data and model

	Data	Model
std(tangible capital)/std(sales)	1.606	1.592
std(intangible capital)/std(sales)	3.026	2.825
mean(leverage) for low-leverage firms	0.020	0.021
mean(leverage) for high-leverage firms	0.338	0.338
government bonds/tax revenue	2.595	2.550
credit to firms/credit to government	0.644	0.656
mean(firm tangibility)	0.865	0.850
mean(spread)	0.017	0.017
std(spread)	0.011	0.011

Notes: See Appendix A.4 for the construction of moments in the data.

investment by more.

To see this in the model, we plot the firms' impulse response functions (IRFs) to a positive  $s$  shock to the government so that the government spread increases by one standard deviation. We simulate 40,000 paths for the model for 200 periods. From periods 1 to 100, the aggregate  $s$  shock follows its underlying Markov chain. In period 101, there is a positive shock on  $s$  so that the government spread increases by one standard deviation. From period 101 on, the  $s$  shocks follow the conditional Markov process. The impulse responses plot the average, across the 40,000 paths, of the variables for the last 100 periods.

Figure 2 shows the impulse responses for the firms when there is a one standard deviation increase in sovereign spreads (Panel (a)). When sovereign spreads increase, the balance sheets of the financial intermediaries deteriorate. With lower net worth, the financial intermediaries' leverage constraint binds, increasing firm loan interest rate. Face a higher borrowing cost, the firms lower their investment. Thus both the tangible asset (Panel (b)) and intangible asset (Panel (c)) decrease. However, firms will reduce their intangible investment by more because the intangible asset can't be used as collateral. Tangible asset, as collateral, can help lower the loan interest rate. Panel (d) shows this asset reallocation pattern where the ratio of tangible asset to intangible asset increases following the shock. Since capital decreases, firms' output decreases in response (Panel (e)). Because firms decrease their intangible investment, their TFP also decreases (Panel (f)). Note that the only shock here is  $s$  shock. Thus, the model *endogenously* generates the output decline and TFP decline when sovereign spread increases.

The model can also generate the heterogeneous asset reallocation for firms with different size and leverage as in the empirical section. We use the calibrated model to mimic the Italian economy and simulate to generate model-simulated data. Consistent with the sample length in our empirical section, we focus on the Italian economy from 2006 to 2016. We feed the model a sequence of  $s_t$  shocks and  $z_t$  shocks<sup>19</sup> such that the model replicates the observed path of Italian sovereign risk and real GDP. Then we simulate to generate different firms. Using the model-simulated data, we run the same regressions as those in Table 4, where the dependent variable is the ratio

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<sup>19</sup>This exogenous productivity shock summarizes the shocks that are not directly induced by the sovereign debt crisis, e.g., aggregate demand decline due to global recession.



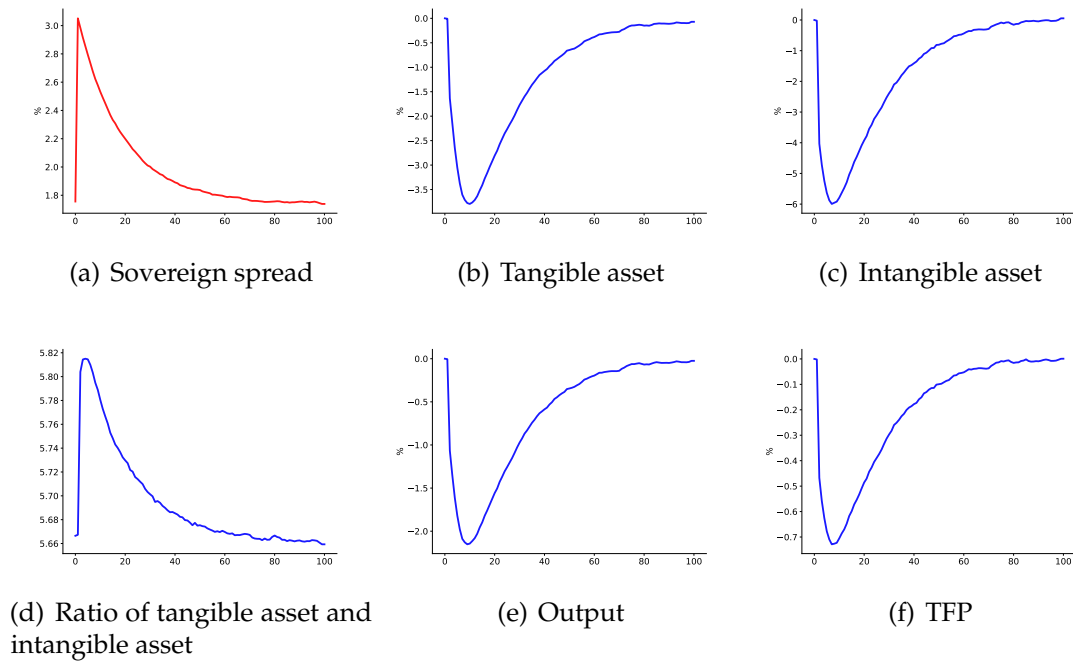


Figure 2: IRFs to one standard deviation increase in sovereign spread

*Notes:* Firm impulse response functions to a positive  $s$  shock (so that the sovereign spread increases by one standard deviation). Before the shock, the aggregate  $s$  follows its underlying Markov chain. In period 1 in the figure, there is a positive shock on  $s$  so that the government spread increases by 1 standard deviation. From period 1 on, the  $s$  shocks follow the conditional Markov process. The impulse responses plot the average across the firms.

of tangible asset to intangible asset in the next period. We investigate how sovereign debt crisis affects this ratio and how firms' characteristics (size and leverage) affect the magnitude of the reallocation.

Table 13 compares the estimation coefficients using real Italian data and those using the model-simulated data. The coefficients from the real data are the taken from column (6) in Table 4. The model can generate a similar asset reallocation pattern as in the data: firms increase their tangibles-to-intangibles ratio during sovereign debt crisis. Moreover, small and high-leverage firms would reallocate by more, compared with other firms.

Table 13: Regression results: data and model

	Data	Model
$sp_t$	1.678	0.536
$size_{i,2006} \times sp_t$	-1.042	-0.317
$totallev_{i,2006} \times sp_t$	0.327	0.611

*Notes:* Regression coefficients for the data and the model. The coefficients from the data are the taken from column (6) in Table 4. The model regression specification mimics the data regression as much as possible. For example, the model regression has firms controls, firm fixed effect, time fixed effect as in the data regression. Also, the sample time length is consistent with data regression.

### 4.3 Output and TFP losses from Italian debt crisis

In this section, we quantify the output and TFP losses from the debt crisis during 2011-2013. First, as in the previous section, we feed the model with a sequence of  $s_t$  shocks and  $z_t$  shocks such that the model replicates the observed path of Italian sovereign risk and real GDP. Then we construct a counterfactual scenario in which the Italian economy does not experience a sovereign debt crisis. We then compare the result of our benchmark model and that of the counterfactual model with no debt crisis. The differences between the paths of key variables in the benchmark model and those in the no debt crisis counterfactual model isolate the impact of the sovereign crisis on the Italian economy.

Figure 3 reports the time paths for the sovereign spreads, GDP and TFP during 2006-2016. The black dotted lines plot the paths in the data, the red solid lines plot

the result of the benchmark model, and the blue dashed lines plot the result of the counterfactual scenario where there was no debt crisis. The unit of sovereign spreads in Panel (a) is percentage point. Panel (b) and Panel (c) plot the percentage changes of GDP and TFP from the 2006 level.

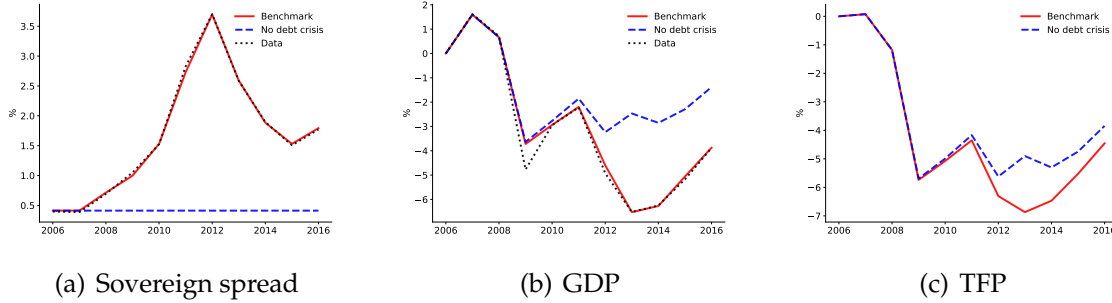


Figure 3: Measuring the costs of sovereign default risk

*Notes:* Paths for Italian sovereign spreads, GDP and TFP during 2006-2016. The black dotted lines plot for the data, the red solid lines plot for the benchmark model result, and the blue dashed lines plot for the results from the counterfactual scenario where there was no debt crisis.

By construction, the benchmark model (red solid lines) matches the sovereign spreads and GDP in the data (black dotted lines). In general, the model needs a negative  $z$  shock and a positive  $s$  shock to reproduce the dynamics of sovereign spreads and GDP observed in the data. The sovereign spread increases from 0.4% in 2006 to 3.7% in 2012. The real GDP decreases by 4.7% from 2006 to 2009, recovers slightly in 2010, and then decreases another 4.3% during 2011-2013.

The blue dashed lines show the corresponding result in the counterfactual no debt crisis case, where we adjust  $s_t$  shocks to fix the sovereign spreads at the spread level in 2006 during the event. Thus, the sovereign spreads are constant in this counterfactual case. Under this scenario, there is no increase in sovereign default risk and there is no transmission of sovereign risk to the financial intermediaries and the firms.

Comparing the red solid lines and blue dashed lines, we can isolate the output losses and TFP losses directly due to the 2011-2013 Italian sovereign debt crisis. The losses associated with sovereign default risk are sizable. During 2011-2013, output would have declined only 0.6% without the sovereign debt crisis, instead of 4.3% in the benchmark model. TFP would have declined only 0.7% without the sovereign debt crisis, instead of 2.5% in the benchmark model. Model indicates that the sovereign

default risk explains 86% of output losses and 72% of TFP losses during the 2011-2013 Italian debt crisis.

## 5 Conclusion

Sovereign debt crises have adverse effects on firms' investment. Empirical evidence shows that firms reduce their investment both in intangible assets and tangible assets. Furthermore, although firms also reduce tangible investment, they reallocate their investment away from intangible assets and towards tangible assets. This asset reallocation pattern is more pronounced in small and high-leverage firms.

We build a sovereign default model incorporating firm investment on both tangible and intangible assets to explain the empirical findings and measure the aggregate output and TFP costs due to sovereign risk. Firms internalize that tangibles can be used as collateral and thus could help offset the tightening financial constraints. When sovereign risk transmits to the firms through the financial intermediaries, the firms would lower their investment, especially investment on intangible assets. Quantitatively, sovereign risk explains a large fraction of output losses and TFP losses during 2011-2013 Italian debt crisis.

We focus on firms' investment and the approach could be generalized along other dimensions. The sovereign risk could impact the firms through different channels. For example, sovereign debt crisis may affect the entry and exit decisions of the firms or import and export decisions. A fruitful research avenue is to use firm-level data to estimate the impact of sovereign risk and explore the mechanism using firm-level data variations.

Due to data limitation, we do not observe further details of the intangible assets. It would be interesting to decompose the intangible assets and explore if certain part of the intangible assets plays the key role in explaining firms' outcomes. Understanding firms' heterogeneous investment behaviors during the crises, especially investment that is beneficial for the long-run future, provides key information for policy makers. We leave these applications to future research.

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# ONLINE APPENDIX TO “INTANGIBLE INVESTMENT DURING SOVEREIGN DEBT CRISIS: FIRM-LEVEL EVIDENCE”

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## A Data

This section provides additional details related to Section 2.1, including the definition of the variables, sample selection procedures and sample distribution by size.

### A.1 Variables

#### 1. *Investment*

Our baseline measure of investment in period  $t$  is defined as the log difference of intangible fixed assets between period  $t + 1$  and period  $t$ . That is to say,  $\text{investment}_{it}$  in Eq (1) denotes the investment on intangibles of firm  $i$  at the end of period  $t$ . Similarly, the tangible investment is defined as the log-difference of tangible fixed assets.

The intangible and tangible fixed assets are scaled by the price of intangibles and tangibles (respectively) every year. We use EU KLEMS database<sup>20</sup> to construct the price indices for intangibles/tangibles. Figure A.1 shows the asset components for intangibles/tangibles. This database reports the price for each asset type. We construct the aggregate price for intangibles as the weight average price of each component in the right square, weighted by the share of each asset.<sup>21</sup> Similar construction goes for the price of tangibles.

#### 2. *Net leverage*

Net leverage is measured as the ratio of firm  $i$ 's net debt to total assets, where net debt is the sum of short term loans and long term debt net of net current assets.

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<sup>20</sup>Documentation for EU KLEMS database, 2019 release.

<sup>21</sup>For example, Software and Database consists of 15% of intangible assets; R&D consists of 40%; Design consists of 20%, while AdvMRes constitutes 16%.

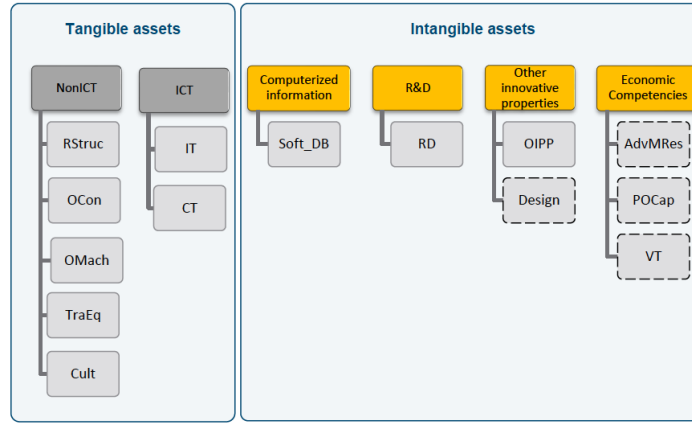


Figure A.1: Aggregates of capital services

Notes: Dashed lines indicate asset types outside the boundaries of National Accounts. Source: Report on methodologies and data construction for the EU KLEMS Release 2019 (Stehrer et al. (2019)).

### 3. Short leverage

Short leverage is defined as the ratio of firm  $i$ 's short term loans to total assets.

### 4. Total leverage

Total leverage is defined as the ratio of firm  $i$ 's total debt to total assets, where total debt is the sum of short term loans and long term debt.

### 5. Size

Size is measured as the log of total assets.

### 6. Liquidity

Liquidity is measured as the ratio of cash & cash equivalent to total assets.

### 7. Sales growth

Sales growth is defined as the log difference of sales, i.e.  $\text{sales growth}_{it} = \log(\text{sales}_{it}) - \log(\text{sales}_{it-1})$ .

### 8. Liability ratio

Liability ratio is defined as the ratio of total liability to total assets, where total liability

is the difference between total assets and share holders funds.

#### **9. *Net current assets ratio***

Net current assets ratio is measured as the ratio of net current assets to total assets.

## **A.2 Sample selection**

Our main sample excludes (in order of operation):

1. Firms not in manufacturing sector.
2. Firms with negative or zero total assets.
3. Firms with negative intangible fixed assets or tangible fixed assets.
4. Firms have missing values for total assets, intangible fixed assets, or tangible fixed assets over the sample period.
5. Firms without obs in year 2006 in the sample.

After applying the sample selection operations, we winsorize the variables mentioned above at top and bottom 1% of the distribution.

### A.3 Aggregate distribution by size

Table A.1: Distribution of wage bill and employment by size

Wage Bill		
	Main Sample	Gopinath et al.(2019)
1-19 employees	0.16	0.11
20-249 employees	0.62	0.53
250+ employees	0.23	0.36
Employment		
	Main Sample	Gopinath et al.(2019)
1-19 employees	0.20	0.13
20-249 employees	0.73	0.55
250+ employees	0.07	0.32

### A.4 Moments in data

**Standard deviation ratio** For tangible/intangible investment  $i_{T,it}$  and  $i_{I,it}$ , we construct the data counterparts as:

$$i_{T,it}^d = k_{T,it+1}^d - (1 - \delta_T)k_{T,it}^d,$$

$$i_{I,it}^d = k_{I,it+1}^d - (1 - \delta_I)k_{I,it}^d,$$

where  $k_{T,it}^d$  is the real tangible fixed assets and  $k_{I,it}^d$  denotes the real intangible fixed assets in the data. We use Hodrick–Prescott filter to derive the cyclical components of tangible/intangible investment, and real sales. The standard deviation of tangible investment is calculated as the average standard deviation of cyclical component across firms. Similar calculation goes for the standard deviation of intangible investment and real sales. The ratio of standard deviation of tangible investment to that of real sales is 0.231, while the counterpart for intangible investment is 0.030.

**Leverage** Firms in our sample is divided into high-/low-leverage groups by each firm's total leverage in base year 2006. The mean leverage of high-leverage group is

0.338, which that of low-leverage group is 0.020.

**Government bonds/tax revenue** The ratio of government bonds to tax revenue is calculated by the ratio of general government debt to general government revenue, which is 2.595. Data is from OECD.Stat database.

**Firm tangibility** We construct the tangibility measure for firm  $i$  at period  $t$  as:

$$tangibility_{it} = \frac{k_{T,it}^d}{k_{T,it}^d + k_{I,it}^d}.$$

Since the Italian data features unbalanced sample, we first take the average tangibility across sample periods to derive the average tangibility of firm  $i$ . The aggregate tangibility is then defined as mean of tangibility measures across firms.

## B Robustness

This section shows some robustness checks of our main empirical findings in Section 2.

### B.1 Depreciation

One difference between intangible assets and tangible assets is that they depreciate at a different speed. Relatively little is known about depreciation rates for intangibles. Corrado et al. (2009) estimates the depreciation rate of R&D capital in the U.S. to be 20%. U.S. Bureau of Economic Analysis (BEA) places its central estimate of the depreciation rate for R&D at 15%. Pakes et al. (1978) get an average depreciation rate of 25%, using data for several European countries.

Fortunately, the EU KLEMS database provides depreciation rates for each asset type, which allows us to construct depreciation rates for intangible asset and tangible asset in Italy - at the aggregate level. The depreciation rate for intangible asset is the weighted average of the depreciation rates of: computer software and databases, research and development, and other intellectual property patent (IPP) assets, with the weight being the asset share. For the tangible asset, the depreciation rate is the weighted average of the depreciation rates of: computing equipment, communications equipment, transport equipment, other machinery and equipment, total non-residential investment, residential structures and cultivated assets, with the weight being the asset share. The calculated depreciation rate for intangible asset is 24.3% and depreciation rate for tangible asset is 10.1% in 2006. Our estimates are in line with the rates reported in the exiting literature.

Alternatively, we can construct the intangible investment <sup>22</sup>:

$$\Delta \log(intangibles_{dep,it}) = [\log(y_{i,t+1}) - \log((1 - dep_{intangible}) * y_{it})] * (1 - dep_{intangible})$$

where  $y_{it}$  denotes intangible fixed assets of firm  $i$  at period  $t$ .  $dep_{intangible} = 0.243$  is the weighted average depreciation rate for intangible investment. Corresponding

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<sup>22</sup>intangible investment<sub>it</sub>/y<sub>it</sub> = [y<sub>i,t+1</sub> - (1 - dep<sub>intangible</sub>) \* y<sub>it</sub>]/y<sub>it</sub> ≈ [log(y<sub>i,t+1</sub>) - log((1 - dep<sub>intangible</sub>) \* y<sub>it</sub>)] \* (1 - dep<sub>intangible</sub>)

tangible investment can be constructed similarly, with depreciation rate equals 0.101. Table B.2 delivers the estimation results for the investment with depreciation.

Table B.2: Results for investment with depreciation

Dependent variable	$\Delta \log(intangibles_{dep,it+1})$		$\mathbb{I}(intangibles_{dep,it+1})$	$\Delta \log(tangibles_{dep,it+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.284** (0.124)	-1.866*** (0.043)		-0.699*** (0.070)
$size_{i,2006} \times sp_t$	1.513*** (0.145)	1.569*** (0.143)	0.643*** (0.050)	0.586*** (0.082)	0.596*** (0.080)
$totallev_{i,2006} \times sp_t$	-0.494*** (0.126)	-0.467*** (0.126)	-0.083* (0.043)	0.116* (0.067)	0.128* (0.066)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.018	0.068	0.043
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1) using  $\Delta \log(intangibles_{dep,it})$  as the dependent variable. Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table B.3: Intangible investment with alternative depreciation rate

Dependent variable	$\Delta \log(intangibles_{CHS2009,dep,it+1})$		$\Delta \log(intangibles_{BM2006,dep,it+1})$		$\Delta \log(intangibles_{PS1978,dep,it+1})$	
	(1) heterogeneity	(2) average	(3) heterogeneity	(4) average	(5) heterogeneity	(6) average
$sp_t$		-0.300** (0.131)		-0.307** (0.135)		-0.281** (0.123)
$size_{i,2006} \times sp_t$	1.599*** (0.153)	1.658*** (0.151)	1.639*** (0.157)	1.700*** (0.155)	1.499*** (0.143)	1.555*** (0.142)
$totallev_{i,2006} \times sp_t$	-0.522*** (0.133)	-0.494*** (0.133)	-0.535*** (0.136)	-0.506*** (0.136)	-0.489*** (0.125)	-0.463*** (0.124)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	303,935	303,935	303,935	303,935
R-squared	0.026	0.013	0.026	0.013	0.026	0.013
Number of id	59,706	59,706	59,706	59,706	59,706	59,706

Notes: Column (1) and (2) are results using the intangible depreciation rate 0.2 (Corrado et al. (2009)). Column (3) and (4) are results using the intangible depreciation rate 0.18 (Bernstein and Mamuneas (2006)). Column (5) and (6) are results using the depreciation rate 0.25 (Pakes et al. (1978)). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table B.3 displays the results using  $\Delta \log(intangibles_{dep,it+1})$  with alternative depreciation rates as dependent variables. The baseline results are robust to all depreciation rates.

## B.2 Region-year fixed effects

We further add region-year fixed effect and province-year fixed effect into our baseline regressions, controlling for the geographical differences. For example, firms near the border may highly expose to foreign trade, which is less affected by Italian sovereign debt crisis. Table B.4 and Table B.5 show that our baseline results are robust to including region-year fixed effect or province-year fixed effect.

Table B.4: Region-year FE

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{I}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		7.988 (5.932)	3.050 (1.921)		-0.147 (2.702)
$size_{i,2006} \times sp_t$	1.986*** (0.194)	2.058*** (0.191)	0.619*** (0.050)	0.646*** (0.093)	0.674*** (0.091)
$totallev_{i,2006} \times sp_t$	-0.659*** (0.169)	-0.649*** (0.168)	-0.116*** (0.043)	0.110 (0.075)	0.104 (0.075)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Region_year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.026	0.025	0.029	0.069	0.068
Number of id	58,918	58,918	70,408	58,918	58,918

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1) with additional region-year fixed effects. Column (2) is the corresponding estimation of Eq (2) with region-year fixed effects. The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table B.5: Province-year FE

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{I}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		8.997 (11.169)	-3.493 (3.070)		-1.324 (6.614)
$size_{i,2006} \times sp_t$	1.977*** (0.194)	2.046*** (0.191)	0.621*** (0.050)	0.653*** (0.093)	0.683*** (0.091)
$totallev_{i,2006} \times sp_t$	-0.655*** (0.169)	-0.647*** (0.169)	-0.110** (0.044)	0.121 (0.075)	0.116 (0.075)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Province-year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.029	0.028	0.032	0.071	0.070
Number of id	58,918	58,918	70,408	58,918	58,918

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1) with additional province-year fixed effects. Column (2) is the corresponding estimation of Eq (2) with region-year fixed effects. The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .



### B.3 More aggregate controls

The baseline results for intangible investment hold when we add more aggregate controls, including GDP, world GDP growth and trade openness.

Table B.6: Add more aggregates

Dependent variable	$\Delta \log(intangibles_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$
$sp_t$	4.101*** (0.241)	-1.160*** (0.062)	1.386*** (0.114)
$size_{i,2006} \times sp_t$	2.054*** (0.189)	0.634*** (0.049)	0.652*** (0.090)
$totallev_{i,2006} \times sp_t$	-0.641*** (0.166)	-0.094** (0.043)	0.129* (0.074)
Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Observations	303,935	383,121	303,935
R-squared	0.022	0.028	0.050
Number of id	59,706	71,339	59,706

Notes: Column (1) shows the estimation of Eq (2) with additional aggregate controls. The dependent variable in Column (2) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (3) is the tangible investment counterpart to Column (1). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.4 Group dummy

Instead of using continuous standardized size and total leverage in 2006, we construct dummies for size and total leverage. Eq (1) and (2) are modified as:

$$\Delta \log(\text{assets}_{i,t+1}) = \alpha(dx_i \times sp_t) + \text{Controls} + \delta_i + \eta_{st} + \epsilon_{it}, \quad (\text{B.1})$$

$$\Delta \log(\text{assets}_{i,t+1}) = \alpha_0 sp_t + \alpha_1(dx_i \times sp_t) + \text{Controls} + \text{AggControls} + \delta_i + \epsilon_{it}, \quad (\text{B.2})$$

where  $dx_i \in \{dsize_{i,2006}, dtotallev_{i,2006}\}$  are the dummies for firm's size or total leverage in the year 2006.  $dsize_{i,2006}$  is 1 if size of firm  $i$  is larger than the median in 2006, 0 otherwise.  $dtotallev_{i,2006}$  is 1 if total leverage of firm  $i$  is higher than the median in 2006, 0 otherwise. The baseline results hold for alternative indicators for firms' size and total leverage.

Table B.7: Group dummy

Dependent variable	$\Delta \log(\text{intangibles}_{i,t+1})$		$\mathbb{1}(\text{intangibles}_{i,t+1})$	$\Delta \log(\text{tangibles}_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-1.329*** (0.291)	-2.267*** (0.082)		-1.294*** (0.149)
$dsize_{i,2006} \times sp_t$	2.697*** (0.332)	2.852*** (0.329)	0.873*** (0.086)	0.706*** (0.157)	0.744*** (0.154)
$dtotallev_{i,2006} \times sp_t$	-0.812** (0.330)	-0.748** (0.329)	-0.026 (0.082)	0.341** (0.149)	0.365** (0.148)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	305,109	305,109	384,564	305,109	305,109
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	60,048	60,048	71,745	60,048	60,048

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.5 Group dummy for sector median

There are concerns for Appendix B.4 because there may be firms in some specific sector gathering in one indicator group. Therefore, we use sector median as the standard to construct dummy indicators for size and total leverage. For Eq (B.1) and (B.2),  $dsize_{i,2006}$  is 1 if size of firm  $i$  is larger than the sector median in 2006, 0 otherwise.  $dtotallev_{i,2006}$  is 1 if total leverage of firm  $i$  is higher than the sector median in 2006, 0 otherwise. The baseline results are also robust.

Table B.8: Group with sector median

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-1.324*** (0.289)	-2.253*** (0.082)		-1.267*** (0.147)
$dsize_{i,2006} \times sp_t$	2.832*** (0.330)	2.782*** (0.329)	0.846*** (0.086)	0.745*** (0.154)	0.724*** (0.154)
$dtotallev_{i,2006} \times sp_t$	-0.710** (0.329)	-0.668** (0.329)	-0.019 (0.083)	0.308** (0.149)	0.332** (0.148)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	304,954	304,954	384,402	304,954	304,954
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	60,022	60,022	71,720	60,022	60,022

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.6 Cluster at sector-level

The baseline results are robust to clustering at sector level.

Table B.9: Cluster at sector-level

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{I}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.375** (0.169)	-1.866*** (0.072)		-0.778*** (0.118)
$size_{i,2006} \times sp_t$	1.998*** (0.218)	2.073*** (0.193)	0.643*** (0.050)	0.652*** (0.112)	0.663*** (0.114)
$total_{i,2006} \times sp_t$	-0.652*** (0.232)	-0.617** (0.236)	-0.083** (0.036)	0.129* (0.069)	0.142** (0.069)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.018	0.068	0.043
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.7 Winsorizing at 0.5%

The baseline sample winsorizes the variables of interest at top and bottom 1%. This section shows the baseline estimation results are robust if we winsorize the variables of interest at top and bottom 0.5%.

Table B.10: Winsorizing 0.5%

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.749*** (0.172)	-1.882*** (0.042)		-0.972*** (0.082)
$size_{i,2006} \times sp_t$	1.983*** (0.193)	2.091*** (0.190)	0.607*** (0.046)	0.670*** (0.093)	0.681*** (0.090)
$totallev_{i,2006} \times sp_t$	-0.625*** (0.176)	-0.589*** (0.175)	-0.109** (0.043)	0.018 (0.079)	0.024 (0.078)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	341,283	341,283	406,802	341,283	341,283
R-squared	0.026	0.014	0.019	0.066	0.044
Number of id	63,631	63,631	73,770	63,631	63,631

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.8 Deflate intangible and tangible fixed assets with PPI

The baseline estimation deflates the intangible (tangible) fixed assets by the price of intangible (tangible) assets. We replace the price of investment with PPI and the results remain robust.

Table B.11: PPI deflation

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.772*** (0.164)	-1.866*** (0.043)		-0.719*** (0.078)
$size_{i,2006} \times sp_t$	1.981*** (0.191)	2.052*** (0.189)	0.643*** (0.050)	0.651*** (0.092)	0.660*** (0.089)
$totallev_{i,2006} \times sp_t$	-0.640*** (0.166)	-0.603*** (0.166)	-0.083* (0.043)	0.133* (0.074)	0.142* (0.074)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,941	303,941	383,121	303,941	303,941
R-squared	0.025	0.012	0.018	0.065	0.042
Number of id	59,705	59,705	71,339	59,705	59,705

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## B.9 Alternative measure for debt crisis: firm spread

Figure B.2 plots the Italian firm-level spread which is defined as the gap between the interest rate for loans (other than bank overdrafts) to non-financial corporations and the risk-free interest rate. The nominal risk-free rate is given by the Eurosystem main refinancing operations interest rate. Data is accessed via the Bank of Italy Statistical Database. During the Italian sovereign debt crisis, the interest rate spread for firms also goes up. We replace the sovereign spread with firm spread and the baseline results do not vary too much.

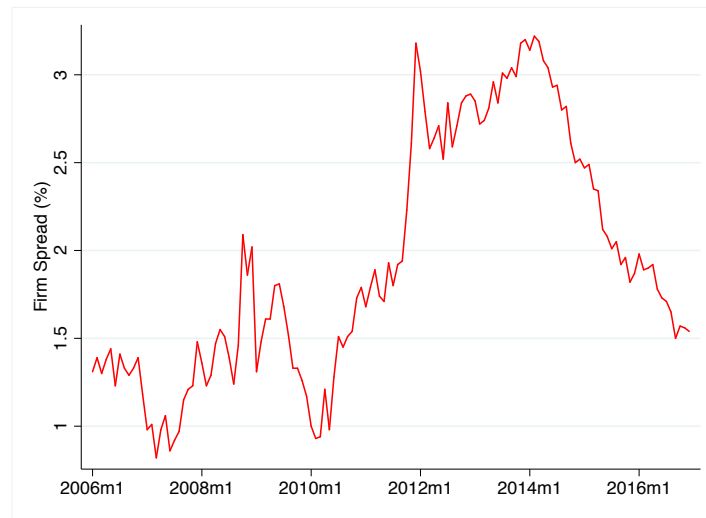


Figure B.2: Italy, firm spread

*Notes:* Interest rate spreads for firms. The series is given by the spread over the risk-free rate of the interest rate for Italian non-financial corporations on non-overdraft loans (total maturity). The nominal risk-free rate is given by the Eurosystem main refinancing operations interest rate. Data source: Bank of Italy Statistical Database.

Table B.12: Alternative measure for debt crisis: firm spread

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-2.552*** (0.159)	-0.912*** (0.041)		-0.523*** (0.079)
$size_{i,2006} \times sp_t$	0.950*** (0.186)	0.730*** (0.181)	0.102** (0.046)	0.186** (0.092)	0.016 (0.090)
$totallev_{i,2006} \times sp_t$	-0.351** (0.160)	-0.279* (0.158)	-0.050 (0.040)	0.127* (0.073)	0.143** (0.073)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.014	0.013	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .



## B.10 Alternative measure for debt crisis: spread using yields of government bonds with different maturities

We replace the baseline 30-year government bond spread with 1-year/5-year/10-year government bond spreads, and our baseline results are robust to alternative spreads.

Table B.13: Spread using 1-year government bond yield

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		2.021*** (0.200)	-1.696*** (0.053)		0.220** (0.095)
$size_{i,2006} \times sp_t$	1.964*** (0.230)	2.189*** (0.229)	0.636*** (0.061)	0.758*** (0.112)	0.853*** (0.110)
$totallev_{i,2006} \times sp_t$	-0.591*** (0.201)	-0.608*** (0.202)	-0.115** (0.053)	0.140 (0.091)	0.140 (0.090)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.014	0.015	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table B.14: Alternative measure for debt crisis: spread using 5-year government bond yield

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.684*** (0.134)	-1.178*** (0.036)		-0.240*** (0.064)
$size_{i,2006} \times sp_t$	1.440*** (0.155)	1.544*** (0.153)	0.393*** (0.041)	0.523*** (0.075)	0.546*** (0.073)
$totallev_{i,2006} \times sp_t$	-0.419*** (0.135)	-0.418*** (0.135)	-0.084** (0.035)	0.128** (0.060)	0.139** (0.060)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.015	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

Table B.15: Alternative measure for debt crisis: spread using 10-year government bond yield

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average		(4) heterogeneity	(5) average
$sp_t$		0.228 (0.146)	-1.405*** (0.039)		-0.401*** (0.070)
$size_{i,2006} \times sp_t$	1.644*** (0.170)	1.723*** (0.169)	0.451*** (0.045)	0.570*** (0.082)	0.574*** (0.080)
$totallev_{i,2006} \times sp_t$	-0.497*** (0.148)	-0.484*** (0.148)	-0.088** (0.039)	0.142** (0.066)	0.155** (0.065)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.016	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm  $i$  continues to hold intangible fixed assets in period  $t$ , 0 if firm  $i$  empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

## C Proofs

### C.1 Intermediate Goods Firms

Each intermediate goods firm  $i$  takes the aggregate demand  $Y_t$  and interest rate  $R_{it}$  as given, and maximizes the present value of the dividends, which is subject to the demand function (10) and the working capital constraint (14).

$$\max \sum_{t=0}^{\infty} \beta^t D_{it},$$

where

$$\begin{aligned} D_{it} = & p_{it} z_{it} k_{T,it}^{\alpha_T} k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})] \\ & - [k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it}b_{it}, \end{aligned}$$

subject to

$$\begin{aligned} y_{it} &= \left( \frac{1 - \tau}{p_{it}} \right)^{\frac{1}{1-\eta}} Y_t, \\ b_{it} &= \lambda_i (i_{T,it} + i_{I,it}). \end{aligned}$$

When there's no adjustment cost ( $\theta_T = \theta_I = 0$ )<sup>23</sup>, the first order conditions (FOCs) are given by:

$$\begin{aligned} [k_{T,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = & \beta [\eta(1 - \tau) Y_{t+1}^{1-\eta} y_{it+1}^{\eta-1} (\alpha_T z_{it+1} k_{T,it+1}^{\alpha_T-1} k_{I,it+1}^{\alpha_I}) \\ & + (1 - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] \\ & - [k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1}] \lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}}] \end{aligned}$$

$$\begin{aligned} [k_{I,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = & \beta [\eta(1 - \tau) Y_{t+1}^{1-\eta} y_{it+1}^{\eta-1} (\alpha_I z_{it+1} k_{T,it+1}^{\alpha_T} k_{I,it+1}^{\alpha_I-1}) \\ & + (1 - \delta_I)[1 + (R_{it+1} - 1)\lambda_i]] \end{aligned}$$

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<sup>23</sup>This simplification is only for analytical purpose. We solved for the general case in the quantitative section.

Combining both FOCs, we can get:

$$\eta(1-\tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta}\left(\frac{\alpha_T}{k_{T,it+1}} - \frac{\alpha_I}{k_{I,it+1}}\right) + (\delta_I - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] = \lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}} \left[ k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1} \right]$$

## C.2 Households

The problem of the households is to maximize the preference (16), subject to the budget constraint (17). The FOCs are given as:

$$[C_t] \quad \beta^t = \xi_t \quad (C.3)$$

$$[M_t] \quad \xi_{t+1} = q_t^m \xi_t \quad (C.4)$$

where  $\xi_t$  is the Lagrange multiplier on the budget constraint (17). Then we can get:

$$q_t^m = \beta \quad (C.5)$$

Notice the price for deposit are constant over time.

## C.3 Financial intermediary

The financial intermediary's problem

$$\max_{\{M_t, B_{t+1}, b_{it}\}} E_t[\beta F_{t+1}] \quad (C.6)$$

where

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t.$$

subject to

$$(\mu_t) \quad q_t B_{t+1} + \int b_{it}di \leq N_t + q_t^m M_t, \quad (C.7)$$

$$(\xi_t) \quad \int (1 - \theta_{it})b_{it}di \leq N_t. \quad (C.8)$$

The FOCs are:

$$[b_{it}] \quad \beta R_{it} - \mu_t - (1 - \theta_{it})\zeta_t = 0 \quad (\text{C.9})$$

$$[B_{t+1}] \quad \beta \mathbb{E}_t[(1 - d_{t+1}f)(\vartheta + q_{t+1}(1 - \vartheta))] - \mu_t q_t = 0 \quad (\text{C.10})$$

$$[M_t] \quad -\beta + \mu_t q_t^m = 0 \quad (\text{C.11})$$

Using the FOC from households' problem  $q_t^m = \beta$ , we can get the pricing conditions for the government bond (24) and firm loan (25).

## D Numerical Solution

The aggregate state of the economy includes the aggregate shock on the government default risk process  $s$  and the initial level of government debt  $B$ . The government chooses borrowing  $B'(s, B, d)$  to maximize the government expenditure, subject to budget constraint and the bond price. The exogenous default risk process determines the default event  $d(s)$  and government bond price  $q(s)$ .

The aggregate state variables and government sector outcome variables are relevant for the firms' choices for tangible capital and intangible capital only because they affect the firms' loan interest rate through the slackness of leverage constraint  $\zeta$  when the net worth of the financial intermediaries  $N$  changes. Thus,  $\zeta$  and the idiosyncratic states  $\{z, \lambda, k_T, k_I\}$  are sufficient to determine the firms' choices. For any state  $(s, B, d)$ , the net worth is given by  $N(s, B, d) = \bar{n} + (1 - df)(1 - \vartheta)q(s)B$ . Net worth affects the firm specific loan interest rate  $R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}$ , where  $\zeta_t$  is the Lagrange multiplier for the leverage constraint  $\int (1 - \theta_{it})b_{it}di \leq N_t$ . When the leverage constraint does not bind,  $\zeta_t = 0$ , otherwise,  $\zeta_t > 0$ .

The government default risk transmits to the firms through the leverage constraint. When government defaults ( $d = 1$ ) or the default risk increases so that the bond price  $q$  decreases, financial intermediaries' net worth  $N$  decreases. With lower net worth, the leverage constraint becomes binding, or the binding is tighter. Tighter leverage constraint increases the shadow price of borrowing (Lagrange multiplier  $\zeta$ ), and thus increases the firm loan interest rate  $R$ .

We now describe the computation algorithm in detail.

1. Create grid points for default risk process  $s$ , government bond  $B$ , and an indicate  $d$  to denote whether the government is in default or not.
2. Create grid points for productivity shock  $z$ , financing needs  $\lambda$ , tangible capital  $k_T$ , intangible capital  $k_I$ , and the Lagrange multiplier  $\zeta \in [0, \zeta_{max}]$ .
3. Guess the initial government bond price  $Q(s)$ .
4. Guess the initial value function of firm  $V_0(z, \lambda, k_T, k_I, \zeta)$  and aggregate output  $Y_0(\Lambda(z, \lambda, k_T, k_I), \zeta)$ .

5. Update the bond price using Eq (24).
6. Update the value function and policy functions  $k'_T(z, \lambda, k_T, k_I, \zeta)$ ,  $k'_I(z, \lambda, k_T, k_I, \zeta)$  and  $b(z, \lambda, k_T, k_I, \zeta)$ , and compute the stationary distribution.
7. Update the aggregate output  $Y_{upd}(\Lambda(z, \lambda, k_T, k_I), \zeta)$  and an aggregate term that includes firm loan demand  $b$ :  $X(\Lambda(z, \lambda, k_T, k_I), \zeta) \equiv \int (1 - \theta_{it}) b_{it} di$ .
8. Iterate until the firm value function, aggregate output and the government bond price converge.
9. Given shocks, and government policies  $\{s, B, d\}$ , compute the net worth  $N(s, B, d)$ .
10. Compute equilibrium Lagrange multiplier  $\zeta(s, B, d)$ : if  $X(\Lambda(z, \lambda, k_T, k_I), 0) \leq N(s, B, d)$ , then  $\zeta = 0$ , otherwise,  $\zeta$  is chosen such that  $X(\Lambda(z, \lambda, k_T, k_I), \zeta) = N(s, B, d)$ .