Inequality, Taxation, and Sovereign Default Risk*

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Abstract

This paper studies the impact of income inequality on sovereign spreads under elastic labor and endogenous taxation. We first document that high pre-tax income inequality is associated with high spreads both across countries and across U.S. states. We then develop a sovereign default model with endogenous progressive taxation and heterogeneous labor in productivity and migration cost. The government chooses the optimal combination of tax and debt, considering their interaction. Progressive taxes redistribute income but discourage labor supply and induce emigration, eroding the tax base and the government's ability to repay debt. Default risk increases sovereign spreads and borrowing costs. Thus, the government faces a trade-off between redistribution and spreads. In more unequal economies, the government opts for more redistribution and higher spreads. With the model parameterized to state-level data, we find that income inequality is an important determinant of spreads, generating more than 20% higher spreads compared with a model without income inequality. In a recession, more unequal economies suffer a larger increase in spreads.

Keywords: Sovereign default risk; income inequality; migration; optimal taxation
JEL Codes: F34, F41, E62, H63, H74

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1 Introduction

Governments often collect taxes to repay their debt. The amount of taxes that can be collected depends crucially on the income distribution and the tax system. Income inequality generates redistribution incentives, affects the desired tax progressivity, the resulted taxable income, and hence, the governments' capacity to repay their debt. This paper studies sovereign default incorporating the role of income inequality, which has been largely ignored in the sovereign default literature originated in the seminal work of Eaton and Gersovitz (1981). To study the role of income inequality on taxation and default risk, we integrate heterogeneous labor and endogenous progressive taxation into a model of sovereign debt with default risk. A progressive tax redistributes income, but distorts labor supply both in the intensive and extensive margin. To reflect the labor distortions, the model features elastic labor supply and migration choices. We use the model to study the role of income inequality on sovereign spreads theoretically and quantitatively. In addition, the model provides insights into the effects of income inequality during recessions.

Empirical evidence emphasizes the role of income inequality on sovereign spreads. High pre-tax income inequality is associated with high government spreads both across countries and across U.S. states. Using cross-country panel data containing 36 countries, we find that an increase in the Gini index by 0.1 is associated with a 0.5 percentage points increase in government spreads. Using data from 19 states in the U.S., we find that increasing the Gini index by 0.1 is associated with an increase of about 0.8 percentage points in the state government spreads. Most existing sovereign default literature can not explain the observed empirical finding because it assumes lump-sum taxes and homogeneous households. This paper provides a unified framework to study government debt and progressive taxation in a heterogeneous-household context.

The model features heterogeneous labor in productivity and migration cost, as well as endogenous taxation. The households supply labor elastically and consume after-tax labor income. They can also emigrate by paying an idiosyncratic migration cost. The migration will change the distribution of households in the economy. Given a certain distribution of households, the redistributive government chooses the tax scheme and government debt. In the tax scheme, the marginal tax rate varies with the income level, and the government decides on the progressivity of the tax. The government can also borrow externally and has the option to default on its debt. The optimal combination of taxation and debt policies depend on the inequality level of the economy, and determines the government default risk. The mechanism in the model that replicates the empirical finding is as follows. A progressive tax redistributes income but reduces labor supply and increases the emigration of the high-income labor, eroding the tax base and the government's ability to repay its debt. Higher debt spreads ensue. Thus, a redistributive government is facing a *redistribution-spread* trade-off when making policies. In a more unequal economy, the government opts for more redistribution and higher spreads.

We parametrize the model using U.S. state-level data. The key moments include pre-tax income inequality level, state-to-state migration flows, the average and volatility of government spreads. We use the model to answer the following question: how much does inequality account for government spreads? We conduct three kinds of exercises to answer this question. In the first exercise, we compare the statistics of the model and a reference model without inequality. To be more specific, we compare the average government spread in the benchmark model and the average spread in a reference model with no income inequality. The comparison shows that income inequality accounts for 23 percent of the state government spreads. In the second exercise, we study how income inequality affects the government choices and debt spreads during economic downturns. Facing a negative productivity shock, the government lowers tax progressivity to encourage labor supply and reduce emigration. The spread increases on impact because lower productivity increases the probability that the government will default and spread increases to compensate for such default risk. With lower inequality, the government can decrease tax progressivity further, thus having a less increased spread than the benchmark economy. The results of the first two exercises show that income inequality does not only serve as a constraint for the government policies in normal times, but also an even stronger constraint during recessions. In the third exercise, we study how labor elasticity affects the magnitude of the effect of inequality on government spreads. Recall that the reason that a progressive tax increases government spreads is that labor is elastic both in the intensive margin and extensive margin. A version of the model without migration also predicts a positive correlation between inequality and government spreads, but 34 percent less in the magnitude. It shows that the extensive margin of labor distortion is an important component in the model to capture the empirical fact.

We validate the model using two methods. First, we run the same regression of government spreads on income inequality as in the empirical motivation using model-simulated data. To generate a sample resembling the data, we re-estimate the model parameters for each US state. Using the model-generated data, we run the regression of government spreads on inequality, controlling for debt and total output. The model can generate a similar magnitude of the regression coefficient as in the data. Another way to validate the model is that we provide empirical evidence that supports the model mechanism and implications. The first two pieces of empirical evidence are to validate the model mechanism. Recall that the main mechanism in the model that links inequality and government spread is endogenous tax progressivity. In the data, we show that, first, the state government with higher inequality is more likely to impose a more progressive income tax system, and second, a more progressive tax is associated with higher debt spreads. The next three empirical evidence supports the model implications. First, one model implication is that the government with larger redistribution preference adopts a more progressive tax and suffers higher spreads, which is consistent with data finding. In the data, we proxy government redistribution preference using state partisan composition, which is a set of dummies including Republican, Democratic, and Split. Second, the model features a distortionary progressive tax to labor supply not only in the intensive margin but also in the extensive margin, which is also a realistic feature. Using state-to-state migration data, we also show that workers are likely to migrate to a state with lower income tax progressivity. At last, the model predicts that the net transfers to low-income households are larger in a high-inequality economy, which is also true in the data. We find that high-inequality states spend more on public welfare programs such as Medicaid.

Related literature. The model builds on sovereign default model pioneered by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). Most works in the literature are abstract from heterogeneous labor and endogenous taxation. This paper contributes to this literature by incorporating the role of income inequality, redistribution, and endogenous taxation into government default risk. The framework in this paper provides a tool to study the interactions among income distribution, taxation, borrowing, and default risk, which applies to both national and sub-national governments. Cuadra, Sanchez, and Sapriza (2010) study external debt with distortionary taxation, where they study representative households and there are no redistribution incentives involved. Pouzo and Presno (2014) and Karantounias (2019) study domestic sovereign default with distortionary taxes in a closed economy. We share the emphasis on explicit default options and distortionary taxes, but we focus on government redistribution motives in an open economy with external debt and cross-border labor mobility.

A growing body of sovereign default literature focuses on the distributional issues of default decisions. D'Erasmo and Mendoza (2016) study an environment with heterogeneous holdings of public debt across households, where government default has a distributional effect. We show that default has de facto redistribution effects even if the debt is external because of endogenous progressive taxation. With both domestic and external debt, D'Erasmo and Mendoza (2018) study optimal debt and default on domestic and foreign creditors by balancing distributional incentives versus the social value of debt. We contribute to this literature by including the optimal distortionary taxation. This paper is closely related to Jeon and Kabukcuoglu (2018), Andreasen, Sandleris, and Van der Ghote (2018) and Ferriere (2014), who study models in which income inequality plays a role in the determination of external defaults. Different from this paper, Jeon and Kabukcuoglu (2018) consider an endowment economy with exogenous taxation. Andreasen, Sandleris, and Van der Ghote (2018) propose a political economy model of sovereign default where the government needs voters' support to implement a given fiscal program. By developing a heterogeneous-agent overlapping generation model, Dovis, Golosov, and Shourideh (2016) focus on analyzing the interactions of inequality and external debt in an economy without commitment. Related to this paper, Ferriere (2014) also studies the effect of inequality and taxation on sovereign default risk. There are three key differences between this paper and Ferriere (2014). First, this paper relaxes the assumption of linear taxation while still keeping tractability. Second, we focus on endogenous progressive taxation and the effect of pre-tax inequality on government default risk. Third, we also investigate the extensive margin of labor distortions to progressive taxes, and show that the extensive margin is an important component to capture the empirical fact for the states in the US.

This paper also relates to the optimal taxation literature. By allowing the government to default on its debt, the optimal tax progressivity potentially changes because of the default risk. Aiyagari et al. (2002) endogenize the tax-smoothing results of Barro (1979) and analyze optimal taxation under incomplete markets when there is commitment to both tax policies and debt repayment. The literature has relaxed commitment assumption to past tax policies, but typically retains commitment to repayment. Debortoli and Nunes (2013) and Krusell et al. (2006) analyze time-consistent taxation and debt in deterministic setups without default. Bhandari et al. (2016) allow the government to trade an exogenously specified set of risky securities. Sleet and Yeltekin (2006) endogenize the incompleteness of government debt markets through limited commitment and private information, and considers the optimal design of fiscal policy. The model in this paper allows the government to default, thus endogenizing the ad hoc government debt limits, as well as the debt prices. Battaglini and Coate (2008) and Aguiar and Amador (2016) also analyze fiscal policy by solving the optimal taxation problem of a government with debt and distortionary taxes. In contrast, this paper studies an environment with government default and labor heterogeneity.

This paper relates broadly to the literature that focuses on inequality and debt dynamics. Azzimonti, De Francisco, and Quadrini (2014) show that when rising income inequality is associated with an increase in individual income risk, higher risk results in more public debt. Arawatari and Ono (2017) show that higher inequality increases pressure on politicians to shift the fiscal burden from the present generation to future generations, thus incentivizing politicians to finance a part of government expenditure by issuing public debt. The key difference of this paper is that we focus on government default risk and debt spreads, rather than the debt level. This paper is also related to papers that study migration and government borrowing (e.g., Alessandria, Bai, and Deng (2020), Gordon and Guerron-Quintana (2019)).

Empirically, this paper contributes by providing empirical evidence on the correlation between income inequality and government default risk. A few papers, using cross-country data, show that higher income inequality is associated with a higher default risk. Berg and Sachs (1988) find that higher income inequality is a significant predictor of a higher probability of debt rescheduling in a cross-section of middle-income countries. Aizenman and Jinjarak (2012) find that higher income inequality is associated with a lower tax base, lower de facto fiscal space, and higher sovereign spreads using data from 50 countries in 2007, 2009, and 2011; Jeon and Kabukcuoglu (2018) find that the Gini index is negatively associated with a country's creditworthiness in the next period. To our knowledge, this paper is the first to provide empirical evidence of the relationship between government spreads and income inequality using U.S. cross-state data. Some research has been done with the spreads on the general obligation debt of the state government. For example, Ang and Longstaff (2013) study systemic sovereign credit risk using credit default swaps spreads for the U.S. Treasury, individual U.S. states, and major Eurozone countries; and Arellano, Atkeson, and Wright (2016) documents sharp increases in spreads on government debt in Europe and the U.S. states.

Layout. The rest of this paper is organized as follows. Section 2 provides the empirical motivation. Section 3 presents the full model. Section 4 analyzes a simplified one-period version of the model to highlight the mechanism. Section 5 takes the full model to the data, where we parametrize the model with U.S. state-level data, conduct counterfactual exercises, and describe the time-series dynamics. Section 6 provides empirical evidence that validates the model mechanism and supports the model implications. Finally, section 7 concludes. The Appendix contains proofs, numerical examples, and the computational algorithm.

2 Empirical Motivation

In this section, we provide empirical findings of the relationship between government default risk and income inequality using data both across countries and across states in the United States. Figure 1 shows that government default risks are positively correlated with income inequality. Section 2.1 provides further regression results for cross-country sample, and Section 2.2 provides evidence for cross-state sample.

Figure 1 plots government default risks on the y-axis and income inequality on the x-axis. Panel (a) plots for the country averages of government bond default risk and inequality obtained from the raw data, spanning from 1954 to 2017 for 36 countries.[1] The government default risk on the y-axis is measured by 10-year government bond spread, which is the gap between the government bond interest rate and that of the US. The income inequality on the x-axis is measured by pre-tax Gini index. The data of government bond interest rate is from OECD Database, and the data of Gini index is obtained from the World Income Inequality Database (WIID4). Panel (b) of Figure 1 plots the state averages. The government default risk is measured by credit default swaps spreads obtained from Bloomberg, and the income inequality is proxied by pre-tax Gini index from U.S. Census Bureau and American Community Survey. For each variable, Figure 1 uses all available data in the above-mentioned databases, and the sample sizes are mostly constrained by data availability of government default risk. Panel (a) covers 1954–2017 and contains 36 countries. Panel (b) covers 2009–2019 and contains 19 states.

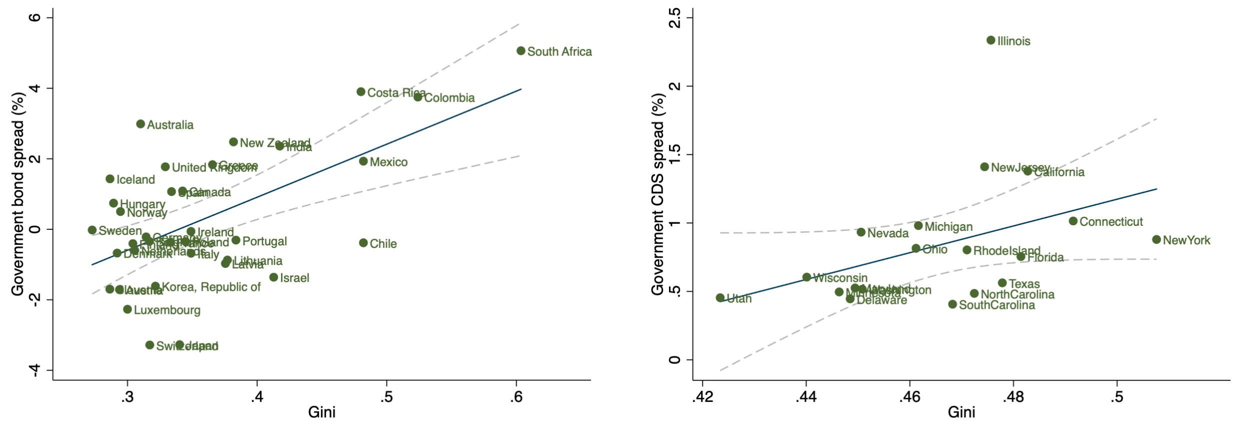
2.1 Cross-country evidence

To explore the correlation between government bond default risk and income inequality, we use the following empirical specification:

$$spread_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \tag{1}$$

where $spread_{jt}$ is the government bond spread of country j in period t. Spread here is defined as the 10-year government bond interest rate of country j in period t minus that of the U.S. for the same period; $ineq_{j,t-1}$ is the income inequality for country j in period $t-1$. Here we use two measures for income inequality: pre-tax Gini index and the gap between the income shares of top 20 percent and that of bottom 20 percent. $Z_{j,t-1}$ is a vector of

[1] The pattern is similar when using shorter time periods.



(a) Cross-country sample

(b) Cross-state sample

Figure 1: Government default risk and pre-tax income inequality

Notes: Figure 1 plots for the correlation between government bond default risk and pre-tax income inequality. In Panel (a), government default risk is measured by 10-year government bond spread; source: OECD Database. Income inequality is measured by pre-tax Gini index; source: World Income Inequality Database (WIID4). In Panel (b), government default risk is measured by credit default swaps spreads; source: Bloomberg. Income inequality is measured by pre-tax Gini index; source: U.S. Census Bureau and American Community Survey. The gray dashed lines show the 95% confidence intervals for the fitted lines.

country-level controls, including GDP and debt-to-GDP ratio. α_t is the time fixed effect.[2] The Gini index and income shares by quintile groups are from World Income Inequality Database (WIID4). GDP is converted to US dollars using purchasing power parity rates and divided by total population, and then we take its logarithm for the regression. The data on GDP is included in WIID4. We use central government debt as the percentage of GDP provided by IMF to measure debt-to-GDP ratios.

Table 1 shows the results of specification (1). Columns (1) and (2) use the Gini index as the measure of income inequality, and columns (3) and (4) use the gap between income share of the top 20 percent and that of the bottom 20 percent to measure inequality. The results show that high inequality is associated with high default risk in the next period. Increasing the Gini index by 0.1 (e.g., Sweden to Portugal) is associated with government bond spread increases of about 0.496 percentage points. For the control variables, a decrease in GDP is associated with higher default risk, and the debt-to-GDP ratio positively correlates with default risk. Those two findings are standard results in the sovereign default literature.

[2]The main variation comes from cross-country differences. We do not focus on the effect of inequality that varies over time. It also applies to the cross-state evidence.

Table 1: Regression of spread on inequality (cross-country)

	(1)	(2)	(3)	(4)
Gini	12.29***	4.96***		
	(1.32)	(1.59)		
topbottomgap			11.96***	4.84***
			(1.34)	(1.53)
Year FE	Yes	Yes	Yes	Yes
Controls		Yes		Yes
N	688	540	604	486
R^2	0.293	0.475	0.312	0.468

Standard errors in parentheses

* p<.1, ** p<0.05, *** p<0.01

Notes: For income inequality, columns (1) and (2) use the Gini index; columns (3) and (4) use the gap between income shares of the top 20% and that of the bottom 20%.

2.2 Cross-state evidence

States are sovereigns under the U.S. Constitution. The states can formulate and implement their own tax systems, as well as issue bonds to finance operations.[3] The degree of tax progressivity, debt levels, and bond spreads vary widely across states. We use the variations across states to test how income inequality affects government bond spreads.

Compared with cross-country analysis, there are two advantages of using state-level data. First, the measures for inequality across states are more comparable and consistent over time.[4] Second, comparable measures for income tax progressivity and migration flows across states allow us to validate the model mechanism.[5]

We proxy state government default risk using five-year credit default swaps (CDS) spreads. A CDS is a derivative contract in which the buyer purchases default protection on an underlying security from a seller. State government debt in this paper refers to

[3]The states can repudiate their debts without bondholders being able to claim assets in a bankruptcy process. Ang and Longstaff (2013) points out that the states within the U.S. have sovereign immunity just as countries within the Eurozone. For more about contrasting the credit risk of sovereigns within the U.S. with that of the sovereign issues within Europe, see Ang and Longstaff (2013).

[4]There is no agreed basis of definition for the construction of distribution data. Sources and methods across countries may vary. In their influential paper on the use of secondary data in studies of income distribution, Atkinson and Brandolini (2001) discuss quality and consistency issues related to income distribution data, and they show that both levels and trends in distributional data can be affected by data choices.

[5]Section 6 validates the model mechanism using cross-state data.

bonded debt. The CDS spreads in the data are tied to default events on the underlying bond, not potential missed pension payments.[6] There are large variations in spreads across states. The CDS spreads on the debt of Illinois, for example, have been relatively high and exceeded 400 basis points in 2016. Figure 2 shows the CDS spreads on the debt of California, Connecticut, New York, Ohio, and Utah from 2009 through 2018. On average, the spread of Utah is lower than one half of that of Connecticut.

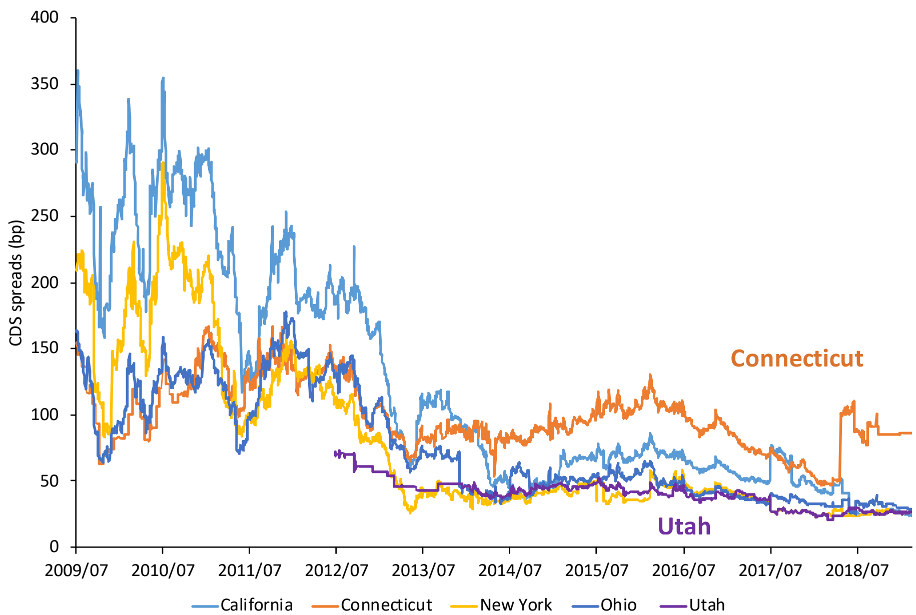


Figure 2: Government spreads in U.S. states

To estimate the correlation between income inequality and government bond spread, we use the following specification, which is similar to cross-country empirical analysis (1):

$$spread_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \tag{2}$$

where $spread_{jt}$ is the CDS spread for state j in year t; $ineq_{j,t-1}$ is proxied by the state pre-tax Gini index; $Z_{j,t-1}$ is a vector of control variables, including state partisan composition, state total output, and debt-to-output ratio; and α_t is a time fixed effect. The daily spreads

[6]Debt is positively correlated with unfunded pension liabilities across states, so the results are very likely to go through or are even stronger if including unfunded pension liabilities. There are two main reasons that prevent this paper from including unfunded pension liabilities in the regression. First, only since fiscal year 2015, state governments began to adopt the new standards (GASB 67, GASB 68) that require them to report unfunded pension liabilities on their balance sheets, so the data sample covers a very short period. Second, pension liabilities are often discounted under risk-free discount rate. Although GASB 67 provides new guidance for state governments in selecting the discount rate used to measure the present value of their unfunded liabilities, that is a recent development and there is still no proper price measure for pension liabilities.

on five-year maturity CDS obtained from Bloomberg span from 2009 to 2019. There are 19 states in the sample.[7] Although the number of states is not ideally large, it almost doubles the number of states used in Ang and Longstaff (2013).[8] Household income used here for the state Gini index is defined as income received regularly (exclusive of certain money receipts such as capital gains) before payments for personal income taxes, social security, union dues, and Medicare deductions. State partisan composition is a set of dummy variables: Democratic, Split, and Republican.[9] The state partisan composition is included in the regression for two reasons. First, it serves as a control variable for the political power in a certain state. Second, since the parties hold different views toward redistribution, the coefficients of the dummies also reflect the effect of redistribution preferences on government bond spreads. We obtain information on partisan composition of state legislatures from National Conference of State Legislatures. Data on state total output is obtained from U.S. Bureau of Economic Analysis, and data on state government debt is from Census State Finances. After merging all variables, the final panel data spans from 2009 to 2017.

Table 2 shows the results for empirical specification (2). The regression uses annual spreads. Columns (1) and (2) use the average spread in each year, and columns (3) and (4) use the last daily observation in each year. The results are robust to both measures. Increasing the Gini index by 0.1 (e.g., Utah to Connecticut) is associated with CDS spreads increases of about 0.8 percentage points. This effect is quite large. The average CDS spread in the sample is 0.86 percentage points. A one standard deviation increase in the Gini index is associated with CDS spread increases of 0.16 percentage points, which is about a 19 percent increase from the mean. The result also shows that the states with stronger redistribution motives are more likely to have higher bond spreads. The coefficients of other control variables are consistent with standard predictions of sovereign default models: total output is negatively correlated with spreads, and a higher debt-to-output ratio is associated with higher spreads.

[7]States with valid data are: California, Connecticut, Delaware, Florida, Illinois, Maryland, Michigan, Minnesota, Nevada, New Jersey, New York, North Carolina, Ohio, Rhode Island, South Carolina, Texas, Utah, Washington, and Wisconsin.

[8]Ang and Longstaff (2013) use CDS spreads for ten states.

[9]"Democratic" indicates that both legislative chambers have Democratic majorities, "Split" indicates that neither party has majorities in both legislative chambers, and "Republican" indicates both legislative chambers have Republican majorities.

Table 2: Regression of spread on inequality (cross-state)

	(1)	(2)	(3)	(4)
Gini	8.08***	8.13***	7.71***	7.96***
	(2.26)	(2.70)	(2.29)	(2.76)
Pref (="Split")		0.25		0.29
		(0.18)		(0.19)
Pref (="Democratic")		0.46***		0.44***
		(0.13)		(0.13)
Year FE	Yes	Yes	Yes	Yes
Controls		Yes		Yes
N	147	147	147	147
R^2	0.324	0.436	0.418	0.507

Standard errors in parentheses

* $p<.1$, ** $p<0.05$, *** $p<0.01$

Notes: For annual spreads, columns (1) and (2) use the average spread in each year; columns (3) and (4) use the last daily observation in each year.

2.3 Remarks

We have shown a positive correlation between income inequality and government default risk using both cross-country data and cross-state data. This finding suggests that besides well-known factors such as total output and indebtedness that would affect sovereign default risk, inequality also plays a crucial role in determining government default risk. The model in Section 3 will explain why and how inequality affects government default risk, and we will estimate the model parameters to match state-level data and quantify the effect of inequality in Section 5.

3 The Model

The model builds on the class of models in the standard framework of Eaton and Gersovitz (1981). The novel components in this model are as follows. First, workers are heterogeneous in labor productivity, which permits more realistic and detailed analysis of how income inequality plays a role in sovereign default risk. Second, the government can choose the tax schedule with a tax function as in Heathcote, Storesletten, and Violante (2017). It is a parsimonious but sufficiently flexible way to approximate the actual U.S. tax system. Third, households supply labor elastically and they are allowed to migrate, which the government will internalize when making policies. By introducing these components in a sovereign default model, this paper enriches the considerations and constraints when government makes borrowing and default decisions.

We will first provide a overview of the model, and then describe the tax function and the timing. Then we describe the households, the firms, and the government maximization problems. Finally we define the equilibrium for the model economy.

Consider a small open economy consisting of heterogeneous households, competitive firms, and a government. *Households* consume after-tax labor income. There are I types of households. Households with the same type are ex ante homogeneous before the realization of idiosycratic migration cost. To save notation, we will use subscript i to refer to a household with type i. When necessary, we will use subscript ij to refer to a household j with type i. Households are heterogeneous in labor productivity z_i and they live hand-to-mouth. Labor income is given by $y_i = wz_i\ell_i$, where w is the equilibrium wage rate per unit of labor and ℓ_i is the elastic labor supply. The households can migrate to other states if their outside options provide higher utility than their home state. Migration involves an idiosyncratic migration cost δ_{ij}, which depends on the household type as well as idiosyncratic shock. Migration changes the distribution of heterogeneous households in the economy. *Firms* produce competitively using the aggregate effective labor supply $L = \sum_{i=1}^{I} n_i z_i \ell_i$ subject to an aggregate productivity shock A, where n_i is the mass of i type households. The aggregate output in the economy is $Y = AL$. *Government* maximizes the social welfare function, which is the sum of the utility of all residents with a set of Pareto weights. The government chooses an income tax function, borrows from external creditors, and decides whether to repay the debt. The creditors recognize that the government may default and set the government bond price to break even. Thus, the bond price is endogenously determined and reflects the government default risk. If the government defaults, the economy suffers from a productivity loss and is temporarily excluded from the credit market.

3.1 Tax/transfer function

The government imposes a distortionary income tax/transfer system to redistribute income. Following Heathcote, Storesletten, and Violante (2017), the net taxes as a function of labor income y_i are given by the function:

$$T_i(y_i) = y_i - \lambda y_i^{1-\mu}, \tag{3}$$

where the parameter μ determines the degree of progressivity of the tax system. Given μ, the second parameter λ shifts the tax function and determines the average tax level. When $\mu > 0$, $T'(y) > \frac{T(y)}{y}$, the tax system is progressive. Higher μ implies that the tax rate increases faster with income, and thus the tax system is more progressive. When $\mu = 1$, there is full redistribution with an after-tax income of λ for everyone. When $\mu = 0$, $T'(y) = \frac{T(y)}{y} = 1 - \lambda$, there is no redistribution with a flat tax rate $1 - \lambda$. Conversely, the tax system is regressive when $\mu < 0$. At the break-even labor income level $y^0 = \lambda^{\frac{1}{\mu}} > 0$, the average tax rate is 0. If the tax system is progressive, households with income lower than y^0 obtain net transfers, while households with income higher than y^0 pay taxes. We restrict the search for optimal tax progressivity within a given parametric class of tax structure in the spirit of Ramsey (1927).

3.2 Timing

The timing of the model is as follows. At the beginning of the period, the aggregate productivity shock A and idiosyncratic migration cost shocks δ_{ij} for household j with type i are observed. The households decide whether to emigrate to other places. If they migrate, they leave the economy and obtain their outside values. After the migration decision, the government makes policies. If the government has access to the financial market, it decides whether to default, how much to borrow B', and the tax system $\{\lambda, \mu\}$. If the government is in financial autarky, it can only choose the tax system $\{\lambda, \mu\}$. The government in financial autarky regains access to the financial market with probability θ. Then the households staying in the economy choose labor supply l_i and consume c_i.

3.3 Households

There are I types of households each of mass n_i for $i \in \{1, 2, ..., I\}$. Denote the aggregate state as $S = (B, A, \Phi, aut)$, where B is the outstanding government debt, A is the aggregate productivity shock, and $\Phi = \{n_i\}$ denotes the distribution of the households. $aut = 1$ denotes the government is in financial autarky, and otherwise $aut = 0$.

Households are heterogeneous in labor productivity z_i. After the idiosyncratic migration cost shock δ_{ij} is observed, household j with type i decides to stay or migrate to maximize lifetime utility:

$$W_{ij}(S, \delta_{ij}) = \max\{W_i^s(S), W_i^m - \delta_{ij}\}, \tag{4}$$

where $W_i^s(S)$ is the value of staying in the home state and W_i^m is the value of emigrating. The idiosyncratic migration cost δ_{ij} follows an exponential distribution with cumulative distribution function $F(\delta) = 1 - e^{-\zeta_i \delta}$. Note that the parameter ζ_i in the cumulative distribution function is type dependent, thus the mean and variance of migration cost are different across types. In the quantitative analysis, we discipline $\{\zeta_i\}$ to match migration rates of households by income. If households emigrate, they leave the model economy and get utility W_i^m, which is independent of the home economy variables. Households will choose to stay in the home state if and only if:

$$W_i^s(S) \geq W_i^m - \delta_{ij}. \tag{5}$$

The probability of staying in the home state for the household of type i is then given by:

$$Pr(\delta_{ij} \geq W_i^m - W_i^s(S)) = e^{-\zeta_i (W_i^m - W_i^s(S))}. \tag{6}$$

Thus, the law of motion for mass of households of type i is given by:

$$n_i'(S) = e^{-\zeta_i (W_i^m - W_i^s(S))} n_i, \tag{7}$$

where n_i' is the mass of households of type i in the next period.

The households staying in the home state choose labor supply and consumption to maximize utility. Thus, the value of staying for household j of type i is given by:

$$W_i^s(S) = \max_{c_i, \ell_i} \{u(c_i, \ell_i) + \beta E W_{i,j}(S', \delta_{ij}')\}, \tag{8}$$

s.t.

$$c_i \leq \underbrace{\lambda (w z_i \ell_i)^{1-\mu}}_{\text{after-tax income}},$$

where c_i is consumption, ℓ_i is labor supply, $y_i = w z_i \ell_i$ is labor income, and $(\lambda,\, \mu)$ are variables in the tax function which are chosen by the utilitarian government.

The utility function takes the separable form:

$$u(c_i, l_i) = \frac{c_i^{1-\sigma}}{1-\sigma} - \frac{\ell_i^{1+\gamma}}{1+\gamma},$$

where $\sigma > 0$ and $\gamma > 0$. The Frisch elasticity of labor supply is $1/\gamma$.

3.4 Firms

There is a continuum of competitive firms producing using labor. We can consider a single (but nonetheless competitive) representative firm produces output Y using stochastic aggregate productivity A and aggregate labor supply L using production function $Y = AL$.

3.5 Government

The redistributive government maximizes the social welfare function, which is the sum of the utility of domestic households with a set of Pareto weights $\{\omega_i\}$ ($\omega_i > 0, \sum_{i=1}^{I} \omega_i = 1$):

$$W = \sum_{i=1}^{I} u(c_i, \ell_i) n_i' \omega_i, \tag{9}$$

where n_i' is the mass of households with type i after migration choices. The government issues one-period bonds B' and sells them to the external risk-neutral lenders at a price $q(B', A, \Phi')$. The bond price is given by the break-even condition of the lenders, and the functional form will be given later in Section 3.6. The government collects income taxes to finance debt repayment. The net tax revenue T collected from all households is given by:

$$T = \sum_{i=1}^{I} n_i'(y_i - \lambda y_i^{1-\mu}). \tag{10}$$

If the government repays, the budget constraint for the government is $B \leq T + qB'$. If the government defaults, it enters financial autarky and cannot borrow (and has no debt). The budget constraint in financial autarky is $0 \leq T$. The government regains the ability to borrow with a probability of θ. Once the economy randomly regains market access, it is assumed that the economy is with zero debt. In the case of default, as is standard in the sovereign debt literature, we impose an exogenous cost that reduces the aggregate productivity: $A^d = f(A) < A$.

Formally, the government's recursive optimization problem is as follows. In every period in which the government is not excluded from the credit market, it chooses whether to repay or default on its debt:

$$V(B, A, \Phi') = \max\{V^c(B, A, \Phi'), V^d(A, \Phi')\}, \tag{11}$$

where Φ' is the distribution of households after households' migration choices, $V^c(B, A, \Phi')$ is the repayment value, and $V^d(A, \Phi')$ is the default value.

If the government repays, it chooses a fiscal program $\{B', \mu, \lambda\}$ to maximize the social welfare function for domestic households. The repayment value is given by:

$$V^c(B, A, \Phi') = \max_{B', \mu, \lambda} \{ \sum_{i=1}^{I} u(c_i, \ell_i) n_i' \omega_i + \beta EV(B', A', \Phi'') \} \tag{12}$$

subject to

$$B \leq \sum_{i=1}^{I} n_i'(y_i - \lambda y_i^{1-\mu}) + q(B', A, \Phi')B',$$

$$n_i''(B', A', \Phi') = e^{-\zeta_i(W_i^m - W_i^s(B', A', \Phi', aut'))} n_i' \quad , i \in \{1, 2, ..., I\}.$$

If the government defaults, it can only choose a partial fiscal program $\{\mu, \lambda\}$ to maximize the social welfare function. The default value is given by:

$$V^d(A, \Phi') = \max_{\mu, \lambda} \{ \sum_{i=1}^{I} u(c_i^d, \ell_i^d) n_i' \omega_i + \beta [\theta EV(0, A', \Phi''_{aut=0}) + (1-\theta) EV^d(A', \Phi''_{aut=1})] \} \tag{13}$$

subject to

$$0 \leq \sum_{i=1}^{I} n_i'(y_i - \lambda y_i^{1-\mu}),$$

$$n''_{i,aut=0} = e^{-\zeta_i(W_i^m - W_i^s(0, A', \Phi', aut=0))} n_i' \quad , i \in \{1, 2, ..., I\},$$

$$n''_{i,aut=1} = e^{-\zeta_i(W_i^m - W_i^s(A', \Phi', aut=1))} n_i' \quad , i \in \{1, 2, ..., I\},$$

where $u(c_i^d, \ell_i^d)$ is the utility of type i households when the economy is in financial autarky, $n''_{i,aut=0} = e^{-\zeta_i(W_i^m - W_i^s(0, A', \Phi', aut=0))} n_i'$ is the law of motion of mass of type i households when the government has access to the financial market, and $n''_{i,aut=1} = e^{-\zeta_i(W_i^m - W_i^s(A', \Phi', aut=1))} n_i'$ is the law of motion of mass of type i households when the government is in financial autarky.

The government internalizes the feedback that its choices have on the private equilibrium. The government default policy can be characterized by default sets and repayment sets: the default set is $D(B, A, \Phi') = \{V^c(B, A, \Phi') < V^d(A, \Phi')\}$ and the repayment set is $R(B, A, \Phi') = \{V^c(B, A, \Phi') \geq V^d(A, \Phi')\}$.

3.6 External lenders

The external lenders are competitive and risk-neutral. They are willing to lend to the government as long as they break even in expected value. The lenders are aware of the government's incentives to default on the bonds. Thus, in equilibrium, the bond price $q(B', A, \Phi')$ captures sovereign default risk:

$$q(B', A, \Phi') = \frac{\mathbb{E}[1 - D(B', A', \Phi''(B', A', \Phi'))]}{1 + r}, \tag{14}$$

where $D(B, A, \Phi') = 1$ denotes default.

3.7 Equilibrium

We now formally define a Markov equilibrium for this economy. We start by characterizing the equilibrium conditions for the private sector, taking the government policies as given. Then we describe the recursive problem of the government.

The aggregate state of the economy includes the level of government debt B, the aggregate shocks A, the distribution of households Φ, and an indicator variable aut that denotes whether the government is in financial autarky ($aut = 1$) or not ($aut = 0$). Given the aggregate state, when the government is not in finanicial autarky, the government makes choices for borrowing, tax system, and whether to default, with decision rules given by $B' = H_B(B, A, \Phi)$, $\mu = H_\mu(B, A, \Phi)$, $\lambda = H_\lambda(B, A, \Phi)$, and $D = H_D(B, A, \Phi)$. Denote the set of aggregate state and the choices of the government together as $\hat{S} = (B, A, \Phi, aut, B', \mu, \lambda, D)$. The private sector equilibrium is defined in Definition 1. After defining the private equilibrium, the recursive equilibrium is defined in Definition 2.

Definition 1 (Private sector equilibrium). *Given an aggregate state $\{B, A, \Phi, aut\}$, government policies for borrowing, tax system, and default $\{B', \mu, \lambda, D\}$, future government decision rules $H_B = B''(B', A', \Phi')$, $H_\mu = \mu'(B', A', \Phi')$, $H_\lambda = \lambda'(B', A', \Phi')$, and $H_D = D'(B', A', \Phi')$, the private equilibrium is defined as a set of policy functions for (i) household migration choices $m(\hat{S}, z, \delta)$; (ii) household consumption $c(\hat{S}, z)$ and labor supply $\ell(\hat{S}, z)$; (iii) wage rate $w(\hat{S})$; (iv) aggregate labor demand $L(\hat{S})$ such that*

1. *The firm policy functions for wages and aggregate labor demand satisfy their optimization problem.*

2. *The household policy functions for migration, consumption, and labor supply satisfy their optimization problem.*

3. *The labor market clears:* $L = \sum_{i=1}^{I} n_i z_i \ell_i$.

Definition 2 (Markov recursive equilibrium). *Given the private equilibrium, the Markov recursive equilibrium is defined as (i) the policy functions for borrowing* $B'(B, A, \Phi')$*, tax system* $\{\mu(B, A, \Phi'), \lambda(B, A, \Phi')\}$*, and default set* $D(B, A, \Phi')$*; (ii) the value functions* $V(B, A, \Phi')$*,* $V^c(B, A, \Phi')$*, and* $V^d(A, \Phi')$ *such that*

1. *The government policy and value functions satisfy its optimization problem.*

2. *The bond price schedule reflects the government default probability and satisfies the external lenders' break-even condition.*

3. *The functions* H_B*,* H_μ*,* H_λ*, and* H_D *are consistent with the government policies.*

4. *The private equilibrium is satisfied.*

5. *The law of motions of labor are consistent with households and government policies.*

4 A Simplified Model

Before turning to the numerical solution of the full model, it is useful to illustrate the mechanism with a one-period simplified model. Assume for now the government has exogenous debt stock B_0, where $0 < B_0 \leq Y$. The government can choose the tax system and whether to default on debt B_0. The households choose labor supply and consumption. There is no migration in this simplified model.

There are two types ($i = L, H$) of households with equal mass. Let $z_L = \bar{z} - \sigma_z$ and $z_H = \bar{z} + \sigma_z$, with $0 < \sigma_z < \bar{z}$. σ_z measures labor productivity heterogeneity without changing the mean level of labor productivity. Higher σ_z corresponds to higher inequality. Assume the utility function is $u(c, \ell) = \log c - \frac{\ell^{1+\gamma}}{1+\gamma}$. Under the logarithmic utility assumption, we can obtain closed-form solutions for optimal labor choices and use the solutions to establish some important properties relating tax progressivity and default risk. The optimal labor and consumption choices for households of type i are:

$$\ell_i = (1-\mu)^{\frac{1}{1+\gamma}}, \tag{15}$$

$$c_i = \lambda(wz_i\ell_i)^{1-\mu}, \tag{16}$$

where λ and μ are chosen by the government and satisfy the government budget constraint. λ can be solved from the government budget constraint:

$$\lambda = \frac{wz_L\ell_L + wz_H\ell_H - B_0}{(wz_L\ell_L)^{1-\mu} + (wz_H\ell_H)^{1-\mu}}. \tag{17}$$

The functional form for labor supply (15) indicates that high tax progressivity, μ, discourages labor supply. Note that with logarithmic utility, the tax level parameter λ has no impact on labor supply. Appendix B derives the optimal choices of labor supply under constant relative risk aversion (CRRA) utility and shows that the main results stay unchanged.

The government has outstanding debt B_0 at the beginning of the period. The government chooses a tax system $\{\mu, \lambda\}$ and whether to repay debt B_0. If the government decides to repay, it collects tax revenues to finance the repayment, and the budget constraint is $T = B_0$, where T is the net tax revenue. If the government defaults, there is no repayment of the outstanding debt, and the budget constraint becomes $T = 0$.

Assume equal weights in the government social welfare function. The repayment value is given by:

$$V^c(B_0, A) = \max_{\mu,\lambda}\{\frac{1}{2}u(c_L, \ell_L) + \frac{1}{2}u(c_H, \ell_H)\} \tag{18}$$

subject to

$$T_L + T_H = B_0,$$

where T_L and T_H are the taxes (transfers, if negative) collected from households of type L and type H, respectively. The defaulting value is given by:

$$V^d(A) = \max_{\mu^d,\lambda^d}\{\frac{1}{2}u(c^d_L, \ell^d_L) + \frac{1}{2}u(c^d_H, \ell^d_H)\} \tag{19}$$

subject to

$$T^d_L + T^d_H = 0.$$

Compared with the budget constraint under repayment, the government under default can tax purely for redistribution without service of debt. Denote $\alpha \equiv (z_L^{1-\mu})/(z_L^{1-\mu} + z_H^{1-\mu})$ and $\alpha^d \equiv (z_L^{1-\mu^d})/(z_L^{1-\mu^d} + z_H^{1-\mu^d})$. After substituting the constraints and applying the assumed utility functional form, the government's payoff under repayment can be expressed as:

$$V^c(B_0, A) = \max_{\mu} \Big\{ \underbrace{\log\left(A\bar{z}\ell(\mu) - B_0\right)}_{\text{consumption}} - \underbrace{\frac{1-\mu}{1+\gamma}}_{\text{disutility from working}} + \underbrace{\frac{1}{2}\log[\alpha(1-\alpha)]}_{\text{redistribution}} \Big\}. \quad (20)$$

Each term of the value function has an economic interpretation and captures one of the forces determining the optimal tax progressivity μ^*. The first component $\log(A\bar{z}\ell(\mu) - B_0)$ represents the total consumption. High tax progressivity discourages labor supply and thus decreases the total output and consumption. Thus, the first term of (20) is *decreasing* in μ. The second term $\frac{1-\mu}{1+\gamma}$ shows the disutility from working. Higher tax progressivity discourages labor supply, thus brings lower disutility from working. Thus, the second term together with the negative sign is *increasing* in μ. The first two terms show the trade-off between consumption and leisure: high tax progressivity μ discourages labor supply, lowers consumption, but reduces disutility from working. With redistribution incentives, high tax progressivity μ brings extra benefit shown as the third term in (20). When $\mu = 1$, which implies $\alpha = 1/2$, there is highest welfare from redistribution. The government determines the optimal tax progressivity by equaling the marginal cost and the marginal benefit of a progressive tax. When the outstanding debt B_0 is high, or the aggregate productivity A is low, the marginal cost of increasing tax progressivity μ is high, leading to a less progressive tax in equilibrium. The monotonicity properties of each term with respect to tax progressivity μ in (20) are summarized in Proposition 1, and the effects of higher debt or lower productivity are summarized in Proposition 2. All proofs are in the Appendix.

Proposition 1 (Value function decomposition). *The repayment value of the government is given by:*

$$V^c(B_0, A) = \max_{\mu} \Big\{ \underbrace{\log\left(A\bar{z}\ell(\mu) - B_0\right)}_{\textit{consumption}} - \underbrace{\frac{1-\mu}{1+\gamma}}_{\textit{disutility from working}} + \underbrace{\frac{1}{2}\log[\alpha(1-\alpha)]}_{\textit{redistribution}} \Big\}, \quad (20)$$

where $\alpha \equiv (z_L^{1-\mu})/(z_L^{1-\mu} + z_H^{1-\mu})$, *and* $z_L = \bar{z} - \sigma_z$, $z_H = \bar{z} + \sigma_z$, *with* $0 < \sigma_z < \bar{z}$. *Then the following monotonicity properties hold: (i) Consumption is decreasing in* μ. *(ii) Disutility from working is decreasing in* μ. *(iii) Redistribution is increasing in* μ.

Proposition 2 (Effect of higher debt or lower productivity). *With higher outstanding debt* B_0 *or lower aggregate productivity* A, *the equilibrium tax progressivity* μ^* *is lower.*

Similarly, the value function of defaulting can also be decomposed into three terms:

$$V^d(A^d) = \max_{\mu} \Big\{ \underbrace{\log\left(A^d \bar{z}\ell(\mu)\right)}_{\text{consumption}} - \underbrace{\frac{1-\mu}{1+\gamma}}_{\text{disutility from working}} + \underbrace{\frac{1}{2}\log[\alpha(1-\alpha)]}_{\text{redistribution}} \Big\}, \tag{21}$$

where $A^d < A$, but there is no debt repayment. The government is facing a similar trade-off when choosing the tax progressivity: higher tax progressivity distorts labor, lower consumption, but reduces disutility from working and increases welfare from redistribution. Comparing repayment value (20) and defaulting value (21), we can find that the marginal cost of high μ on consumption is higher with debt repayment B_0, while the marginal benefits of high μ are the same under repayment and default. Thus, the optimal tax progressivity is higher under default. We can also see this property by deriving the first-order condition with respect to tax progressivity μ. Formally, the optimal tax progressivity μ^* satisfies the first-order condition:

$$\frac{1}{2}\frac{(z_H^{1-\mu^*} - z_L^{1-\mu^*})(\ln z_H - \ln z_L)}{z_L^{1-\mu^*} + z_H^{1-\mu^*}} + \frac{1}{1+\gamma} = \frac{\bar{z}\frac{1}{1+\gamma}(1-\mu^*)^{\frac{1}{1+\gamma}-1}}{\bar{z}(1-\mu^*)^{\frac{1}{1+\gamma}} - \frac{B_0}{A}}, \tag{22}$$

where $\frac{B_0}{A} > 0$. The left-hand side of (22) is a decreasing function of μ and the right-hand side of (22) is increasing in μ. When government defaults, the debt B_0 is wiped out, and the aggregate producitivity A is reduced to A^d. The left-hand side of (22) keeps unchanged, and the right-hand side of (22) is decreased because $\frac{B_0}{A} > 0$. Thus, the optimal tax progressivity under default is higher than that under repayment. This property is summarized in Proposition 3:

Proposition 3 (Effect of default). *The equilibrium tax progressivity μ^* is higher under default than that under repayment.*

Proposition 3 shows that the presence of government debt constitutes a force toward lower tax progressivity. To repay the debt, the government has to encourage labor supply to finance the debt repayment. By defaulting on its debt, the government can avoid this force and implement a more progressive tax. The model shows that besides standard benefit and cost considerations when making repayment/default decisions, the government faces another trade-off between redistribution and default.[10] This trade-off varies with different

[10] In standard sovereign default literature, the costs of defaulting include productivity losses and enforced temporary financial autarky, while the benefit of defaulting is the debt being wiped out. With endogenous taxation as in this paper, extra benefit of defaulting is that the government can implement a more progressive tax to achieve more redistribution.

levels of inequality.

Now let us explore the role of income inequality in detail. Recall that $\alpha \equiv (z_L^{1-\mu})/(z_L^{1-\mu} + z_H^{1-\mu})$. When the economy is more unequal initially, the gap between z_H and z_L, i.e., $z_H - z_L = 2\sigma_z$ increases. Thus, the redistribution benefit $\frac{1}{2}\log[\alpha(1-\alpha)]$ increases, leading to a higher tax progressivity μ. With debt repayment B_0, the increase in μ is smaller than under default. Thus, higher inequality makes the government more likely to default. We can also see this property by exploring the first-order condition (22) and then deriving the default set. Higher inequality means a larger gap between z_H and z_L. Thus, the left-hand side of (22) is increased with higher inequality, while the right-hand side (functional form, rather than the value) does not change with inequality. Thus, a higher pre-tax inequality results in a higher optimal tax progressivity. Appendix A.5 derives the default set and shows that the default set is larger with higher inequality. This property is summarized in Proposition 4:

Proposition 4 (Effect of inequality). *With higher inequality, the government is more like to default on its debt.*

This simplified version of the model is instructive in that it provides closed-form solutions to understand the main mechanism. The main insights are as follows. First, with larger debt to repay or lower aggregate productivity, the optimal tax progressivity is lower. Second, the optimal tax progressivity under default is higher than that under debt repayment. Third, income inequality increases government default risk. The results of this simplified model extend to a broader class of CRRA utility function of which the logarithmic specification is a special case. Appendix B.1 provides a numerical example to illustrate the Propositions graphically, and Appendix B.2 shows that the main results stay unchanged with a CRRA utility function.

5 Quantitative Analysis

Having highlighted the main mechanism of this paper using a simplified model, we now take the full model to the data. We first describe how to normalize the model into per-capita terms to solve the model. Then we parametrize the model with U.S. state-level data. The model matches both targeted and non-targeted moments well. We then conduct counterfactual exercises using the parametrized model.

5.1 Solving the model[11]

Note that except for labor productivity, all variables are *time-variant*. The mass of households with type i is n_i. The total population (in period t) is $N = \sum_{i=1}^{I} n_i$. We normalize the model by transforming the problem into per-capita terms. For a variable X, define the per-capita term as $x = \frac{X}{N}$. There are two types of households with labor productivity (z_L, z_H). With two types, it is enough to keep track of the fraction of n_L in each period.[12]

Recall that the aggregate state in the model is $S = (B, A, \Phi, aut)$. Now let the state be $s = (b, A, f, aut)$, where $b = B/N$ is the per-capita government bond, $f = n_L/N$ is the fraction of L-type households, and $N = n_L + n_H$. Denote the growth rate of the mass of L-type households as $g_L(s) = n'_L/n_L$ and the growth rate of the mass of H-type households as $g_H(s) = n'_H/n_H$. Now we transform the model with per-capita terms.

The government's transformed problem is as follows.[13] In every period in which the government is not excluded from the credit market, it chooses whether to repay or default depending on the per-capita value of repayment $v^c(b, A, f)$ and defaulting $v^d(A, f)$:

$$v(b, A, f) = \max\{v^c(b, A, f), v^d(A, f)\}. \tag{23}$$

Let the default decision be $d(b, A, f) = 1$ if $v^c(b, A, f) < v^d(A, f)$. The repayment value is:

$$\begin{aligned} v^c(b, A, f) = \max_{b', \mu, \lambda} \{ & u(c_L, l_L)\omega_L f g_L + u(c_H, l_H)\omega_H (1-f) g_H \\ & + \beta E v(b', A', f')[f g_L + (1-f) g_H]\}, \end{aligned} \tag{24}$$

and the budget constraint is:

$$b \leq f g_L(y_L - c_L) + (1-f) g_H(y_H - c_H) + q b'(f g_L + (1-f) g_H).$$

The defaulting value is:

$$\begin{aligned} v^d(A, f) = \max_{\mu, \lambda} \{ & u(c^d_L, l^d_L)\omega_L f g_L + u(c^d_H, l^d_H)\omega_H (1-f) g_H \\ & + \beta[\theta E v(0, A', f'_{aut=0}) + (1-\theta) E v^d(A', f'_{aut=1})][f g_L + (1-f) g_H]\}, \end{aligned} \tag{25}$$

[11] Readers who are not interested in how to solve the model can skip to Section 5.2.

[12] Solving this model with more labor types is feasible. However, the code running time is significantly longer because the households' problems are also dynamic (besides that government's problem is dynamic).

[13] The equivalence of the transformed problem and the original problem is proved in Appendix A.8.

and the budget constraint under default is:

$$0 \leq f g_L(y_L - c_L) + (1-f) g_H(y_H - c_H).$$

The bond price is given by:

$$q(b', A, f) = \frac{\mathbb{E}[1 - d(b', A', f')]}{1+r} \tag{26}$$

5.2 Parameterization

The model is in annual frequency. The utility function has the additively separable form $u(c_i, \ell_i) = \log c_i - \frac{\ell_i^{1+\gamma}}{1+\gamma}$. The aggregate productivity A follows a log-normal AR(1) process: $\log(A_t) = \rho \log(A_{t-1}) + \eta \epsilon_t$, where ϵ_t has a standard Normal distribution. If the government defaults, the economy suffers a productivity loss. Following Chatterjee and Eyigungor (2012), the productivity loss takes a quadratic form $A_d = h(A) = A - \max\left\{d_1 A + d_2 A^2, 0\right\}$. Table 3 provides all parameter values. The model has twelve parameters, with three of them assigned and nine of them set to match data moments. For the assigned parameters, $\gamma = 2$ so that the Frisch elasticity $(1/\gamma)$ is 0.5. This value is in line with microeconomic evidence (e.g., survey by Keane (2011)) and estimation by Heathcote, Storesletten, and Violante (2014). The return parameter θ is chosen to be 0.25 so that defaulting economies are excluded from financial markets for four years on average.[14] The rest of the parameters are set in the moment-matching exercise. We use the same data sample as in section 2.2 to calculate the data moments. The moments are the averages for the 19 states.[15]The productivity persistence ρ and volatility η are estimated to match state-level output process. The discount factor β and two parameters in the productivity loss function, d_1 and d_2, are jointly estimated to match debt-to-output volatility, the average spread, and the volatility of the spread. Labor productivity for low-income and high-income households are estimated to match the average Gini index and the ratio of tax revenue to income. The parameters in the migration cost distribution for the low-income ζ_L and for the high-income ζ_H are estimated to match the average outflow rate of the low-income and high-income in the

[14]Although there is very limited sample of actual defaults for the state governments, the governments do suffer from financial exclusions after a default. As an example, Arkansas defaulted on $146 million of debt in 1933. Even Arkansas' own banks were not allowed to invest in the state's bonds until 1937; large financial centers remained closed to the state for a decade or more; state banks and trusts in New York and Pennsylvania could not invest in Arkansas bonds until 1944, and not until 1954 in Massachusetts and Connecticut.

[15]The model can also be parameterized to country level.

Internal Revenue Service (IRS) Migration Dataset.[16] The model-generated moments are calculated from simulations. We simulate 3,000 paths for the model for 500 periods, then drop the 100 initial periods. we then take the average of the statistics across the 3,000 paths conditional on the economy not defaulting, since default events in the data are rare.

Table 3: Parameters values

Concept	Symbol	Value	Source/Target
Assigned Parameters			
Risk-free rate	r	4%	standard value
1/Frisch elasticity	γ	2	Heathcote et al. (2014)
Return probability	θ	0.25	Gelos et al. (2011)
Fitted Parameters			
Productivity persistence	ρ	0.9	output persistence
Productivity volatility	η	0.02	output volatility
Discount factor	β	0.87	debt-to-output volatility
Productivity loss in default	d_1	-0.4	spread mean
	d_2	0.475	spread volatility
Labor heterogeneity	$\bar{z}$	0.45	Gini index
	σ_z	0.414	income tax revenue over income
Migration cost distribution	ζ_L	0.0027	outflow rate of low-income
	ζ_H	0.0044	outflow rate of high-income

Table 4 report the key moments in the model and the data. The model generates similar statistics to the ones in the data. The standard deviation of output is 0.04 in the model, close to 0.03 in the data. The model average interest rate spreads on government debt and its standard deviation of 0.81 percent and 0.61 percent are similar to the data counterparts of 0.83 percent and 0.40 percent, although the standard deviation is larger than that in the data. The averaged debt-to-GDP ratio of 19 percent is similar to 18 percent in the data. The Gini index of 0.46 is exactly matched. The ratio of income tax revenue to income in the model of 1.35 percent is slightly lower than the one in the data of 1.8 percent. The outflow rate of the low-income and the high-income of 3.96 percent and 2.84 percent are similar to the data counterparts of 4.0 percent and 2.8 percent.

[16]The interstate migration data produced by IRS do not include households that do not file tax returns, thus missing 13% of the population. However, it is quite precise because they are not based on survey data, such as the migration data produced by U.S. Census Bureau. The outflow rates used in the calibration are calculated using data in 2016.

Table 4: Moments in data and model

	Data	Model
Std. GDP	0.03	0.04
Avg. spread (%)	0.83%	0.81%
Std. spread (%)	0.40%	0.61%
Avg. debt-to-GDP	0.18	0.19
Gini index	0.46	0.46
Avg. income tax revenue/income	1.8%	1.35%
Avg. outflow rate of low-income	4.0%	3.96%
Avg. outflow rate of high-income	2.8%	2.84%

Notes: GDP in the table refers to per capita GDP.

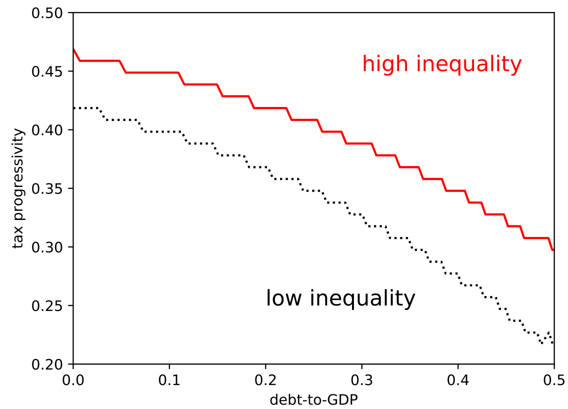
5.3 Decision rules

In Section 4 with the simplified model, we have shown in Proposition 1 that the equilibrium tax progressivity μ is higher with higher outstanding debt or lower aggregate productivity. A key difference between the simplified model and the full model is that the government can choose how much to borrow in the full model. Figure 3 shows Proposition 1 in the full model. Panel (a) plots the decision rules for tax progressivity with different debt-to-GDP ratios, and panel (b) plots the decision rules with different levels of aggregate productivity A. Higher debt b or lower productivity A decreases tax progressivity μ. Figure 3 also compares the decision rules with different inequality levels. The red solid line plots for a high inequality economy (high σ_z) and the black dotted line plots for a low inequality economy. It shows that high inequality increases the tax progressivity.

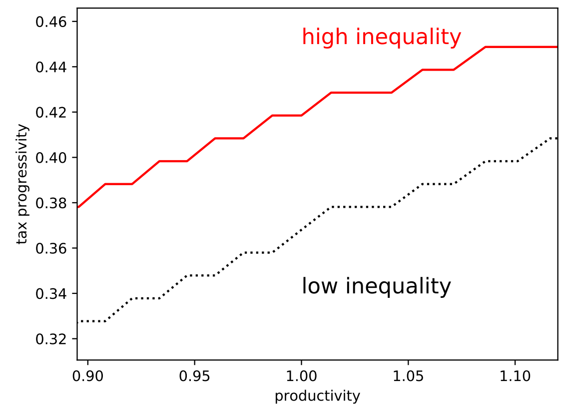
The government faces a trade-off between redistribution and default risk, which can be seen clearly from Figure 4. Panel (a) plots for the off-equilibrium borrowing with different levels of tax progressivity, and panel (b) plots for the corresponding spreads. With a more progressive tax, the government borrows more and is more likely to default, leading to higher spreads.

5.4 Impulse responses

We now describe the time series dynamics of the model by presenting impulse response functions (IRFs) of government polices and the aggregates to a negative productivity shock. We simulate 3,000 paths for the model for 500 periods. From periods 1 to 400, the aggregate productivity shock follows its underlying Markov chain. In period 401 (period 1 in the plots), there is a 5 percent negative productivity shock. From period 401 on, the



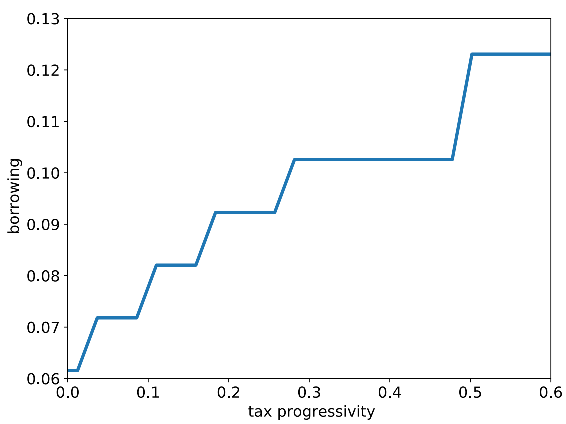
(a) Varying b



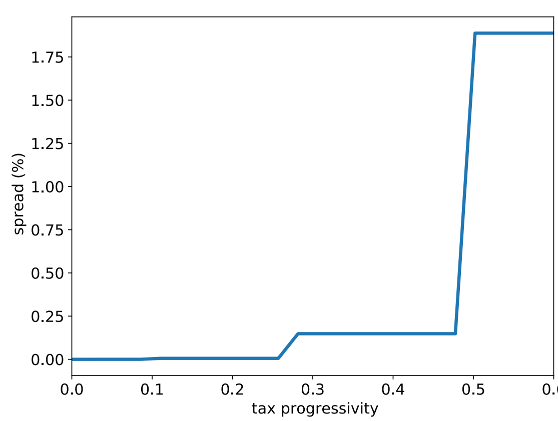
(b) Varying A

Figure 3: Decision rules

Notes: Panel (a) plots the decision rules for tax progressivity with different levels of debt b, and panel (b) plots the decision rules with different levels of aggregate productivity A. The red solid line plots for a high inequality economy and the black dotted line plots for a low inequality economy.



(a) Borrowing



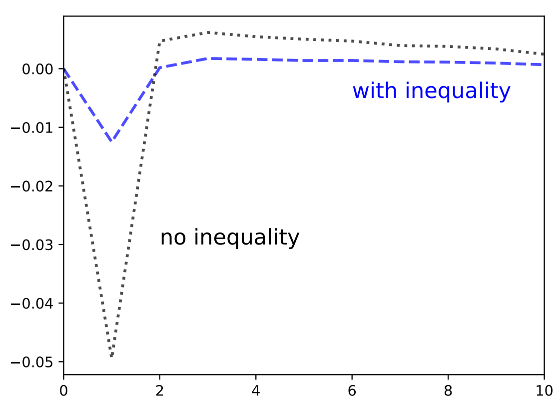
(b) Spreads

Figure 4: Spreads and tax progressivity

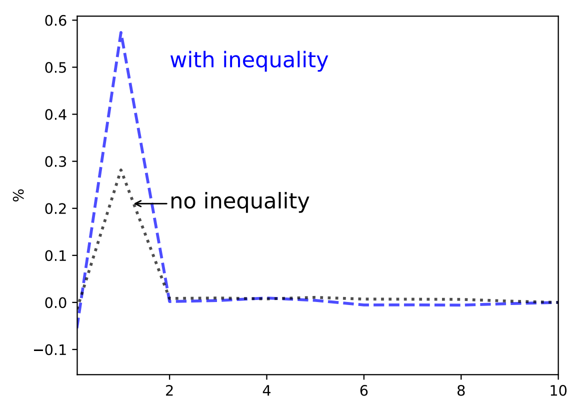
Notes: Panel (a) plots for the off-equilibrium borrowing with different levels of tax progressivity, and panel (b) plots for the corresponding spreads.

productivity shocks follow the conditional Markov process. The impulse responses plot the average, across the 3,000 paths, of the variables conditional on the economy not defaulting.

Figure 5 plots the IRFs for the benchmark model and a reference model without inequality ($\sigma_z = 0$). Panel (a) plots for tax progressivity, and panel (b) plots for spreads. The dashed blue lines are for the *benchmark* model, and the dotted black lines are for *no-inequality* reference model. We normalize each series by its long-run value. When there is a bad shock, government decreases tax progressivity to encourage the labor supply. With inequality (dashed blue line), the government only decreases tax progressivity by 0.012. Absent of inequality (dotted black line), the government could decreases tax progressivity by 0.05, about 4 times larger than the benchmark case. Although government decreases tax progressivity, spreads still go up. This is because the productivity shock is negative and persistent. Lower productivity increases the probability that the government will default. The spreads rise to compensate for such default risk. With no inequality, the spreads increase by 0.3%. For the benchmark economy, since the government lacks of capacity to decrease tax progressivity to stimulate the labor supply as shown in Panel (a), the spreads increase even further to 0.6%.



(a) Tax Progressivity

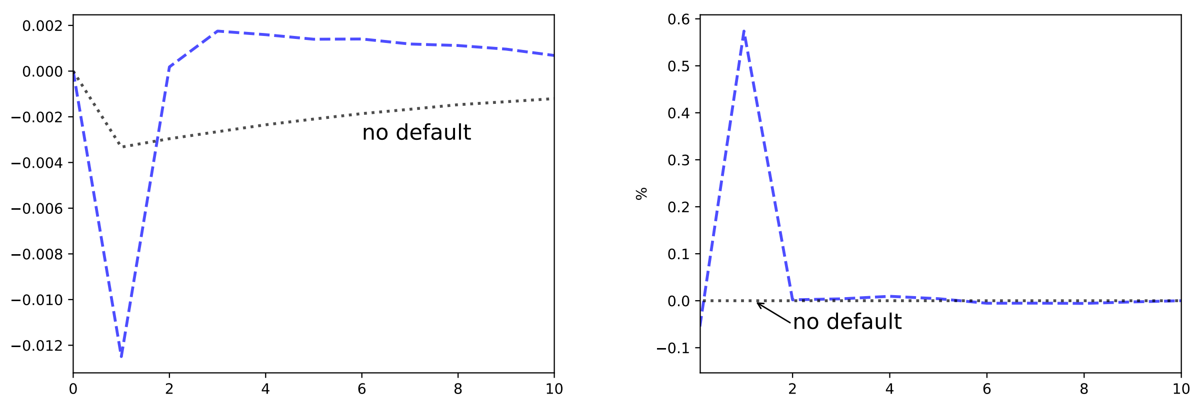


(b) Spreads

Figure 5: IRFs to a decline in productivity: role of inequality

Notes: Panel (a) and (b) plot for the responses of tax progressivity and spreads when there is a decline in productivity. The dashed blue lines plot for the benchmark economy with inequality, and the dotted black lines plot for the economy without inequality.

To disentangle the role of default risk, we compare the benchmark model with a reference model without default risk. The *no-default* model has little financial frictions and is close to Mendoza (1991). The no-default model shares the same parameter values as the benchmark except that the no-default model has a high default cost $d_2 = 0.7$. Figure 6 compares the responses of the benchmark model and a reference *no-default* model following a negative productivity shock. The dashed blue lines plot for the benchmark model, and the black dotted lines present the *no-default* reference model. With default risk, borrow is more costly, and the marginal cost of high tax progressivity μ is higher. Thus the government imposes a more regressive tax and redistribution is further weakened. During the recent European sovereign debt crisis, the Greek government adopted rather regressive austerity measures (Matsaganis and Leventi (2014)), which raised concerns of fiscal burden on low-income households.



(a) Tax Progressivity (b) Spreads

Figure 6: IRFs to a decline in productivity: role of default risk

Notes: Panel (a) and (b) plot for the responses of tax progressivity and spreads when there is a decline in productivity. The dashed blue lines plot for the benchmark economy, and the dotted black lines are for the no-default reference model.

5.5 Inequality and spreads

To explore the quantitative impact of inequality on spreads, we conduct two exercises. In the first one, we shut down inequality ($\sigma_z = 0$) in benchmark model, then simulate the model with everything else equal. The simulation process is similar to that in Section 5.4, except that here we do not introduce the negative productivity shock. Without inequality, the average spread is 0.62 percentage points. Compared with the average spread of 0.81 percentage points in the benchmark model, inequality accounts for 23 percent of the model spreads. In the second exercise, we run the same regression as in the empirical part using the model-simulated data. To generate an inequality sample resembling the data, we vary inequality level to generate observed Gini indices. Then we recalibrate model shock process and default cost to match each US state's output fluctuation and average spreads. We simulate 3,000 paths for each US state. Using the model-generated data, we regress spreads on inequality, controlling for output and debt-to-output ratio:

$$spread_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \epsilon_{jt}, \tag{27}$$

where $spread_{jt}$ are the model-generated bond spreads, $ineq_{j,t-1}$ are the exogenously matched inequality level, and $Z_{j,t-1}$ includes output and the debt-to-output ratio. Then coefficient β_1 measures the elasticity of spread on inequality. Table 5 reports the regression coefficients using model-simulated data. The full model predicts that increasing the Gini index by 0.1 is associated with government bond spread increases of 0.886 percentage points, a similar magnitude as in the data. We then shut down migration choices in the model and run the same regression. The regression coefficient of the model without migration is also reported in Table 5. The model without migration also predicts a positive correlation between government spreads and income inequality, but approximately 34 percent less in the magnitude. It shows that labor distortion in extensive margin accounts for 34 percent of the effect of inequality on spreads.

Table 5: Regression of spread on inequality

Model	Regression coef.
Data	8.13
Full model	8.86
No migration	5.85

5.6 Pareto weights

Some governments have a stronger preference for redistribution than other governments. Empirical evidence in Section 2 shows that governments with stronger redistribution preference are more likely to have higher bond spreads. Here we explore the effects of redistribution preferences in the model by varying the Pareto weights in the government social welfare function. Let the Pareto weights be $\omega_i = \frac{z_i^\eta}{\sum_i^I z_i^\eta}$, where smaller η indicates a higher redistribution preference. Table 6 compares the statistics of model-simulated data under different Pareto weights. With smaller η, the government assigns higher weights on the low-income households and imposes a tax system with higher progressivity.

Progressive tax discourages labor supply and increases the emigration of workers with high productivity, leading to a lower per-capita output and a higher debt spread.

Table 6: Experiments with Pareto weights

	tax progressivity	labor supply	outflow rate(L)	outflow rate(H)	spread
$\eta = 0$	0.59	0.74	3.96%	2.84%	0.81%
$\eta = 0.4$	0.41	0.83	4.58%	2.38%	0.79%
$\eta = 0.7$	0.18	0.93	5.52%	2.07%	0.62%

5.7 Frisch elasticity

The elasticity of labor supply is one of the crucial parameters in the model. It determines the response of hours worked to changes in the tax system and determines the degree of distortions that taxes introduce. The value of this elasticity, however, is well known to be controversial. On the one hand, researchers who look at micro data typically estimate relatively small labor supply elasticities, while on the other hand, researchers who use representative agent models to study aggregate outcomes typically employ parameterizations that imply relatively large aggregate labor supply elasticities. In the benchmark model, we choose the Frisch elasticity to be 0.5 ($\gamma = 2$), which is consistent with micro data. We also explore the model outcomes with alternative Frisch elasticities. Given inequality level (Gini index = 0.46) and redistribution preference ($\eta = 0.7$)[17], Table 7 shows the average of key variables from 3,000 simulations with alternative values for Frisch elasticities. It is not surprising that with higher Frisch elasticity, the tax progressivity is lower. This is because the government would not impose a very progressive tax system when the degree of distortions is large.

Table 7: Experiments with alternative Frisch elasticity $(1/\gamma)$

	tax progressivity	labor supply	outflow rate(L)	outflow rate(H)	spread
$\gamma = 2$	0.18	0.93	5.52%	2.07%	0.62%
$\gamma = 1$	0.15	0.92	5.89%	2.53%	0.66%
$\gamma = 0.5$	0.13	0.91	6.21%	2.98%	0.76%

[17] Heathcote, Storesletten, and Violante (2017) estimates that the current income tax progressivity in the U.S. is $\mu = 0.181$, which corresponds to $\eta = 0.7$ in this model. We also compare the outcomes with alternative Frisch elasticities under different values of η. The results are available upon request.

6 Empirical Evidence

In this section, we provide empirical evidence on U.S. state governments. The first part of empirical evidence validates the model mechanism and the second part of empirical evidence tests the model implications.

6.1 Validating model mechanism

A novel mechanism in this paper that generates the positive correlation between spreads and income inequality is endogenous tax progressivity. Here we use data to test the following two model predictions. First, with higher inequality, a government tends to impose a more progressive income tax system; and second, more progressive tax is associated with higher spread.

In most states, individual income taxes are a major source of state government revenue, accounting for 37 percent of state tax collections.[18] Income tax is the major instrument for the government to fulfill redistribution; other taxes (including federal payroll and excise taxes and state sales taxes) are either less progressive or regressive. Thus, we look at income tax progressivity in the data, which is consistent with the model. The degree of progressivity varies widely across the states. For instance, the state marginal income tax rates in California range from 1 percent to 13.3 percent in 2019, while in North Dakota they range from 1.1 percent to 2.9 percent. We use maximum state income tax rate to measure income tax progressivity. Data is from NBER's calculations using TAXSIM model (Feenberg and Coutts).[19]

The empirical specification that explores the relationship between tax progressivity and income inequality is:

$$prog_{jt} = \beta_0 + \beta_1 ineq_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \tag{28}$$

where $prog_{jt}$ is the income tax progressivity in state j in year t; $ineq_{j,t-1}$ is pre-tax income inequality proxied by Gini index for state j in year $t-1$; $Z_{j,t-1}$ is a vector of control variables, including dummies for state partisan composition, state total output, and debt-to-output

[18] Source: U.S. Census Bureau, "State and Local Government Finance," the Fiscal Year 2016. States that have no state income tax: Alaska, Florida, Nevada, New Hampshire, South Dakota, Tennessee, Texas, Washington, and Wyoming.

[19] Data Source: http://www.nber.org/~taxsim

ratio. α_t is a time fixed effect. Data covers 49 states[20] from 2006 to 2017.

Table 8 shows the result for regression (28). It shows that a more unequal state tends to impose a more progressive income tax system, controlling for redistribution preferences (proxied by state partisan composition).

Table 8: Regression of tax progressivity on inequality

	(1)	(2)
L.gini	26.78***	16.38*
	(7.64)	(8.33)
L.pref (="Split")		1.55***
		(0.47)
L.pref (="Democratic")		3.10***
		(0.36)
Year FE	Yes	Yes
Controls	Yes	Yes
N	408	392
R^2	0.052	0.200

Standard errors in parentheses
* p<.1, ** p<0.05, *** p<0.01

To explore the correlation between government bond spreads and tax progressivity, we use the following empirical specification:

$$spread_{jt} = \beta_0 + \beta_1 prog_{j,t-1} + \Gamma' Z_{j,t-1} + \alpha_t + \epsilon_{jt}, \tag{29}$$

where $spread_{jt}$ is CDS spread for state j in year t. Table 9 shows the regression results. A more progressive tax is associated with higher government bond spreads. Since spread data is available for 19 states, the size of observation is smaller than that in regression (28).

6.2 Testing model implications

6.2.1 High-inequality states have larger transfers to low-income households.

One of the model results is that with higher inequality, the net transfer (negative net tax revenue) to L-type households is larger. Now we check whether it is the case in the data. We classify 50 states into three groups according to their Gini index in 2016, and then compare their spending on public welfare programs such as Medicaid and Temporary

[20]Nebraska does not have partisan composition data since it is a non-partisan unicameral legislature.

Table 9: Regression of spreads on tax progressivity

	(1)	(2)
L.progressivity	0.03**	0.02**
	(0.01)	(0.01)
L.pref (="Split")		0.33**
		(0.16)
L.pref (="Democratic")		0.29**
		(0.11)
Year FE	Yes	Yes
Controls	Yes	Yes
N	109	109
R^2	0.551	0.584

Standard errors in parentheses
* p<.1, ** p<0.05, *** p<0.01

Assistance for Needy Families (TANF).[21] The three groups are classified as follows: *Low-inequality states* are Alaska, Utah, New Hampshire, Wyoming, Hawaii, Iowa, Nebraska, South Dakota, Minnesota, Wisconsin, Maryland, Idaho, Maine, Delaware, Indiana, North Dakota, and Vermont; *Middle-inequality states* are Kansas, Nevada, Oregon, Colorado, Washington, Oklahoma, Missouri, Montana, Ohio, Pennsylvania, Michigan, Virginia, West Virginia, Arizona, Arkansas, and South Carolina; *High-inequality states* are New Mexico, North Carolina, Rhode Island, Massachusetts, Tennessee, Texas, Illinois, Kentucky, New Jersey, Georgia, Mississippi, Alabama, Florida, California, Connecticut, Louisiana, and New York.

Table 10 lists the mean and median level of total expenditures on Medicaid and TANF by inequality group. It shows that high-inequality states are more likely to spend more on Medicaid and TANF. To control for the fact that states are different in size, Table 11 shows the ratio of total expenditures on Medicaid and TANF to total output. It shows that high-inequality states are still more likely to spend more on Medicaid and TANF relative to their output.

Not all funds for programs targeted of low-income households come from the state governments' pocket. Some programs, including TANF, Supplemental Security Income (SSI), and the Federal Low Income Home Energy Assistance Program, are mostly funded at the federal level. At the state level, health spending, and particularly payments to health providers through Medicaid, represents the largest share of state welfare costs.

[21] We also classify 50 states into five groups, and also compare for different years. The pattern stays similar in both cases.

Table 10: Total expenditures on Medicaid and TANF (in billions)

	Medicaid		TANF	
Inequality groups	mean	median	mean	median
Low	3.05	1.65	0.16	0.08
Middle	7.24	5.02	0.42	0.27
High	15.73	8.80	1.08	0.49

Table 11: Total expenditure on Medicaid and TANF over total output (%)

	Medicaid		TANF	
Inequality groups	mean	median	mean	median
Low	2.4	2.2	0.12	0.11
Middle	2.6	2.5	0.13	0.12
High	2.9	2.7	0.15	0.14

Expenditures primarily dedicated to low-income households are classified as "public welfare" on the government financial statements. The Census Bureau's classification of public welfare funding includes Medicaid, TANF, child welfare services, and a range of other assistance programs mostly for low-income individuals. According to data from the 2015 Annual Survey of State Government Finances, federal aid made up nearly two-thirds (64%) of states' public welfare general expenditures. Table 12 shows total public welfare spending and the federal share. High-inequality states spend a lot more on public welfare than low-inequality states. The federal share, however, does not necessarily increase with the inequality level of the state. As a result, although the federal government helps to fund part of the spending, the states with high inequality still spend more on welfare programs relative to low-inequality states.

Table 12: Public welfare spending (in billions) and federal shares

	Public welfare spending		Federal share	
Inequality groups	mean	median	mean	median
Low	4.60	2.48	0.62	0.60
Middle	9.77	8.16	0.69	0.70
High	22.0	12.2	0.65	0.62

6.2.2 Income tax progressivity positively correlates with outward migration.

In the model, a progressive tax redistributes income but discourages labor supply and immigration, eroding the tax base. There are empirical evidence that supports this model implication.

In 2012, California enacted legislation that increased marginal income tax rates especially for high-income households. Using data from the California Franchise Tax Board for all taxpayers, Rauh and Shyu (2019) find that high-income earners increased their rate of out-migration from California by 0.8 percentage point in response to the tax increase. They also find a substantial decrease in taxable income, which appears in 2012 and persists through the last year of their analysis in 2014.

Here we provide further evidence using data from all states. We focus on the effect of income tax progressivity on outward migration. The empirical specification is as follows:

$$\ln x_{ijt} = \beta_0 + \beta_1 \Delta \ln prog_{ijt} + \Gamma_1' \Delta \ln Z_{ijt} + \Gamma_2' \ln Z_{it} + \alpha_t + \alpha_i + \alpha_j + \epsilon_{ijt}, \qquad (30)$$

where x_{ijt} is the outflow from state i to state j in year t, $\Delta \ln prog_{ijt}$ is the gap between income tax progressivity in state i and state j. ΔZ_{ijt} is a vector of controls including the gap of unemployment rate, real personal income, regional price parities between state i and state j, and the geographical distance between state i and j. Z_{it} includes the levels of the controls in the origin state (state i), as well as the level of tax progressivity in the state i. α_t is the time fixed effect; α_i is the origin state fixed effect; and α_j is the destination state fixed effect. Data on state-to-state outflows is obtained from IRS Migration Database. Unemployment rate data is from Local Area Unemployment Statistics (LAUS); Real personal income and regional price parities data by state are from Bureau of Economic Analysis (BEA). The sample covers 2009–2016. The coefficient β_1 estimates the effect of income tax progressivity on out-migration. Table 13 shows the results for empirical specification (30). It shows that people are likely to migrate to a state with lower income tax progressivity.

Table 13: Regression of outflow on income tax progressivity

	(1)	(2)	(3)	(4)
ln prog	0.275***	0.265***	0.252***	0.171***
	(0.04)	(0.03)	(0.03)	(0.05)
Level controls	Yes	Yes	Yes	Yes
Year FE		Yes	Yes	Yes
Origin state FE			Yes	Yes
Destination state FE				Yes
N	12451	12451	12451	12451
R^2	0.363	0.405	0.648	0.928

Standard errors in parentheses

* p<.1, ** p<0.05, *** p<0.01

7 Conclusion

Income inequality affects fiscal policies of taxation, government borrowings, and default. We provide empirical evidence that pre-tax income inequality is positively correlated with government debt spreads both across countries and U.S. states. We build a model of sovereign default, endogenous taxation, elastic labor, and migration to capture and explain the interactions between tax, debt, and income redistribution. With high inequality and redistribution preference, the government proposes progressive taxation, which distorts labor supply, eroding the tax base. Facing a trade-off between redistribution and low spreads, the government is more likely to choose redistribution over low spreads in a society with high inequality. We find that more than 20 percent of state-level spreads is due to income inequality.

The standard sovereign default literature usually assumes homogeneous agents and lump-sum transfers. Thus it is silent on the government's distributional incentives and its impact on government policies. Moreover, under lump-sum transfers, there are no distortions, and default only involves wealth effect on the domestic agents. By introducing heterogeneous workers and endogenous taxation, this paper provides a framework to consider inequality and a rich set of fiscal policies including taxation, government borrowings, and default. A fruitful of research can be done along this path. For example, the framework can be used to evaluate welfare gain or loss of austerity plans during a debt crisis. The proponents of austerity argue that by reducing the government's transfer, the country has a larger capacity to repay its debt, which in turn reduces sovereign spreads and alleviates the debt crisis. The opponents on austerity, on the other hand, argue that austerity hurts low-income workers and reduces equality. An interesting future step is to address these two views with the model framework.

The connection between the sovereign debt crisis and heterogeneous households is a major open question for macroeconomics. This paper helps to understand how does income inequality constrains government policies, including taxation, borrowing, and default decisions. An important area for future work is to understand the details and channels for financial and fiscal links between sovereign debt crises and the labor market.

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A Proofs

A.1 Proof of Proposition 1

Proof. (i)

$$\frac{\partial \log(Y - B_0)}{\partial \mu} = -\frac{A\bar{z}\frac{1}{1+\gamma}(1-\mu)^{\frac{1}{1+\gamma}-1}}{A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}} - B_0} < 0$$

(ii)

$$\frac{\partial \frac{1-\mu}{1+\gamma}}{\partial \mu} = -\frac{1}{1+\gamma} < 0$$

(iii)

$$\frac{\partial \frac{1}{2}\log[\alpha(1-\alpha)]}{\partial \mu} = \frac{1}{2}\frac{(z_H^{1-\mu} - z_L^{1-\mu})(\ln z_H - \ln z_L)}{z_L^{1-\mu} + z_H^{1-\mu}} > 0$$

Thus, in the repayment value function, total consumption is decreasing in μ, disutility from working is decreasing in μ, and redistribution is increasing in μ.

A.2 Proof of footnote 10

Proof. (i) The derivative of (20) with respect to μ gives:

$$\begin{aligned}
\frac{\partial V^c}{\partial \mu} &= \frac{1}{Y - B_0}\frac{\partial Y}{\partial \mu} + \frac{1}{1+\gamma} + \frac{1}{2}\frac{1}{\alpha(1-\alpha)}\frac{\partial[\alpha(1-\alpha)]}{\partial \mu} \\
&= -\frac{A\bar{z}\frac{1}{1+\gamma}(1-\mu)^{\frac{1}{1+\gamma}-1}}{A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}} - B_0} + \frac{1}{1+\gamma} + \frac{1}{2}\frac{(z_H^{1-\mu} - z_L^{1-\mu})(\ln z_H - \ln z_L)}{z_L^{1-\mu} + z_H^{1-\mu}}
\end{aligned}$$

$\frac{\partial V^c}{\partial \mu} = 0$ gives (22).

(ii) Take the derivative of the left-hand side (LHS) of (22) with respect to μ:

$$\begin{aligned}
\frac{\partial(LHS)}{\partial \mu} &= \frac{z_L^{1-\mu} z_H^{1-\mu}(lnz_L - lnz_H)(lnz_H - lnz_L)}{(z_L^{1-\mu} + z_H^{1-\mu})^2} \\
&= \frac{-z_L^{1-\mu} z_H^{1-\mu}(lnz_H - lnz_L)^2}{(z_L^{1-\mu} + z_H^{1-\mu})^2} < 0
\end{aligned}$$

Take the derivative of the right-hand side (RHS) of (22) with respect to μ:

$$\frac{\partial(RHS)}{\partial\mu} = \frac{A\bar{z}\frac{1}{1+\gamma}(1-\mu)^{\frac{-2\gamma}{1+\gamma}}[A\bar{z} - B_0\frac{\gamma}{1+\gamma}(1-\mu)^{\frac{-1}{1+\gamma}}]}{(A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}} - B_0)^2} > 0$$

$(B_0 \leq A\bar{z}l \Rightarrow B_0 < \frac{1+\gamma}{\gamma}A\bar{z}l \Rightarrow A\bar{z} - B_0\frac{\gamma}{1+\gamma}(1-\mu)^{\frac{-1}{1+\gamma}} > 0)$

Thus, the left-hand side of (22) is a decreasing function of μ and the right-hand side of (22) is increasing in μ.

A.3 Proof of Proposition 3

Proof. The RHS of (22) in the case of repayment is

$$RHS_r = \frac{A\bar{z}\frac{1}{1+\gamma}(1-\mu)^{\frac{1}{1+\gamma}-1}}{A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}} - B_0},$$

and in default is

$$RHS_d = \frac{A^d\bar{z}\frac{1}{1+\gamma}(1-\mu)^{\frac{1}{1+\gamma}-1}}{A^d\bar{z}(1-\mu)^{\frac{1}{1+\gamma}}}.$$

Dividing RHS_r by RHS_d, we have:

$$\frac{RHS_r}{RHS_d} = \frac{A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}}}{A\bar{z}(1-\mu)^{\frac{1}{1+\gamma}} - B_0} > 1.$$

Thus, when government defaults, the RHS of (22) decreases. Since the LHS of (22) remains unchanged, and combined with the monotonic properties proved in A.2, the optimal μ is larger under default than that under repayment.

A.4 Proof of Proposition 4

Proof. Substitute z_L and z_H into the LHS of (22) with $z_L = \bar{z} - \sigma_z$ and $z_H = \bar{z} + \sigma_z$, and after some algebra:

$$LHS = \frac{1}{2}\frac{((\bar{z}+\sigma_z)^{1-\mu} - (\bar{z}-\sigma_z)^{1-\mu})(\ln(\bar{z}+\sigma_z) - \ln(\bar{z}-\sigma_z))}{(\bar{z}+\sigma_z)^{1-\mu} + (\bar{z}-\sigma_z)^{1-\mu}} + \frac{1}{1+\gamma}$$

Take the derivative of the LHS with respect to σ_z:

$$\frac{\partial(LHS)}{\partial\sigma_z} = \frac{(1-\mu)[\ln(\bar{z}+\sigma_z)-\ln(\bar{z}-\sigma_z)][(\bar{z}+\sigma_z)^{1-2\mu}+(\bar{z}-\sigma_z)^{1-2\mu}]}{[(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}]^2}$$
$$+\frac{1}{2}\frac{[(\bar{z}+\sigma_z)^{1-\mu}-(\bar{z}-\sigma_z)^{1-\mu}][(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}](\frac{1}{\bar{z}+\sigma_z}+\frac{1}{\bar{z}-\sigma_z})}{[(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}]^2} > 0$$

Thus, the LHS of (22) is increasing in σ_z. Since the LHS of (22) is increased with higher inequality (higher σ_z), while the RHS does not change with inequality, combined with the monotonic properties proved in A.2, the optimal μ is larger with higher σ_z.

Next I prove that the default set is larger under higher inequality.

Proof. The government's productivity threshold $\bar{A}$ that satisfies $V^d(A) = V^c(B_0, A)$ is given by:

$$\bar{A} = \frac{B_0}{\bar{z}(l-\Theta l^d)}$$

where

$$\Theta = \exp(-\frac{1}{2}\log\frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)} - \frac{\mu-\mu^d}{1+\gamma}),$$

and

$$\alpha \equiv \frac{(\bar{z}-\sigma_z)^{1-\mu}}{(\bar{z}-\sigma_z)^{1-\mu}+(\bar{z}+\sigma_z)^{1-\mu}},$$

$$\alpha^d \equiv \frac{(\bar{z}-\sigma_z)^{1-\mu^d}}{(\bar{z}-\sigma_z)^{1-\mu^d}+(\bar{z}+\sigma_z)^{1-\mu^d}}.$$

Lemma 1. *Θ is increasing in x.*

Proof. See Appendix A.5.
Thus, $\frac{\partial\bar{A}}{\partial\sigma_z} > 0$. That is to say, higher inequality (a higher value for σ_z) would lead to a higher productivity threshold $\bar{A}$, and thus a larger default set. Alternatively, one can write down the borrowing threshold and show that a higher σ_z leads to a lower borrowing threshold $\bar{B}_0$.

A.5 Proof of Lemma 1

Proof. Take the derivative of Θ with respect to σ_z:

$$\frac{\partial \Theta}{\partial x} = \Theta \frac{\partial[-\frac{1}{2}\log\frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)}]}{\partial \sigma_z}$$

where

$$\frac{\alpha(1-\alpha)}{\alpha^d(1-\alpha^d)} = [(\bar{z}-\sigma_z)(\bar{z}+\sigma_z)]^{\mu^d-\mu}[\frac{(\bar{z}-\sigma_z)^{1-\mu^d}+(\bar{z}+\sigma_z)^{1-\mu^d}}{(\bar{z}-\sigma_z)^{1-\mu}+(\bar{z}+\sigma_z)^{1-\mu}}]^2,$$

then

$$\frac{\partial \Theta}{\partial x} = \frac{\Theta \bar{z}}{\ln(10)(\bar{z}-\sigma_z)(\bar{z}+\sigma_z)}[(1-\mu)\frac{(\bar{z}+\sigma_z)^{1-\mu}-(\bar{z}-\sigma_z)^{1-\mu}}{(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}} - (1-\mu^d)\frac{(\bar{z}+\sigma_z)^{1-\mu^d}-(\bar{z}-\sigma_z)^{1-\mu^d}}{(\bar{z}+\sigma_z)^{1-\mu^d}+(\bar{z}-\sigma_z)^{1-\mu^d}}]$$

Lemma 2. *$f(\mu) = (1-\mu)\frac{(\bar{z}+\sigma_z)^{1-\mu}-(\bar{z}-\sigma_z)^{1-\mu}}{(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}}$ is decreasing in μ.*

Proof. See Appendix A.6.
Since $\mu^d > \mu$, we have:

$$\frac{\partial \Theta}{\partial \sigma_z} = \frac{\Theta \bar{z}}{\ln(10)(\bar{z}-\sigma_z)(\bar{z}+\sigma_z)}[(1-\mu)\frac{(\bar{z}+\sigma_z)^{1-\mu}-(\bar{z}-\sigma_z)^{1-\mu}}{(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}} - (1-\mu^d)\frac{(\bar{z}+\sigma_z)^{1-\mu^d}-(\bar{z}-\sigma_z)^{1-\mu^d}}{(\bar{z}+\sigma_z)^{1-\mu^d}+(\bar{z}-\sigma_z)^{1-\mu^d}}] > 0$$

Thus, Θ is increasing in σ_z.

A.6 Proof of Lemma 2

Proof. Take the derivative of $f(\mu)$ with respect to μ:

$$f'(\mu) = \frac{-(z_H^{1-\mu}+z_L^{1-\mu})(z_H^{1-\mu}-z_L^{1-\mu}) + 2(1-\mu)(\ln z_L - \ln z_H)z_L^{1-\mu}z_H^{1-\mu}}{[z_H^{1-\mu}+z_L^{1-\mu}]^2} < 0$$

Thus, $f(\mu) = (1-\mu)\frac{(\bar{z}+\sigma_z)^{1-\mu}-(\bar{z}-\sigma_z)^{1-\mu}}{(\bar{z}+\sigma_z)^{1-\mu}+(\bar{z}-\sigma_z)^{1-\mu}}$ is decreasing in μ.

A.7 Equivalence

I normalize the model by transforming the government's problem into per-capita terms. Here I prove the equivalence of the transformed problem and the original problem.

Recall that $S = (B, A, \Phi, aut)$. Let the state be $s = (b, A, f, aut)$, where $b = B/N$ is the per-capita government bond, $f = n_L/N$ is the fraction of L-type households, and $N = n_L + n_H$.

The following relations hold:

$$W_L^s(S) = W_L^s(s), \quad W_H^s(S) = W_H^s(s),$$

$$W_L(S, \delta_L) = W_L(s, \delta_L), \quad W_H(S, \delta_H) = W_H(s, \delta_H),$$
$$\frac{n'_L}{n_L} \equiv g_L(S) = e^{-\zeta_L(W_L^m - W_L^s(S))} = e^{-\zeta_L(W_L^m - W_L^s(s))} \equiv g_L(s),$$
$$\frac{n'_H}{n_H} \equiv g_H(S) = e^{-\zeta_H(W_H^m - W_H^s(S))} = e^{-\zeta_H(W_H^m - W_H^s(s))} \equiv g_H(s),$$
$$\frac{N'}{N} = \frac{n'_L + n'_H}{n_L + n_H} = f g_L(s) + (1-f) g_H(s),$$
$$\frac{V(B, A, \Phi')}{N} = v(b, A, f),$$
$$\frac{V^c(B, A, \Phi')}{N} = v^c(b, A, f),$$
$$\frac{V^d(A, \Phi')}{N} = v^d(A, f).$$

Also note that
$$\frac{B'}{N} = \frac{B'}{N'}\frac{N'}{N} = b'\frac{N'}{N} = b'(f g_L(s) + (1-f) g_H(s)).$$

In the original problem, the government chooses whether to repay or default:

$$V(B, A, \Phi') = \max\{V^c(B, A, \Phi'), V^d(A, \Phi')\}$$

Divide both sides of the default decision by N:

$$\frac{V(B, A, \Phi')}{N} = \max\{\frac{V^c(B, A, \Phi')}{N}, \frac{V^d(A, \Phi')}{N}\},$$

which implies
$$v(b, A, f) = \max\{v^c(b, A, f), v^d(A, f)\}.$$

Thus the default decisions satisfy

$$D(B, A, \Phi') = d(b, A, f).$$

Let the default decision be $d(b, A, f) = 1$ if $v^c(b, A, f) < v^d(A, f)$. Thus, for the bond price, we have:

$$
\begin{aligned}
q(B, A, \Phi') &= \frac{1 - Pr[D(B', A', \Phi'')]}{1+r} \\
&= \frac{1 - Pr[d(b', A', f')]}{1+r} \\
&= q(b, A, f).
\end{aligned}
$$

Now I derive the repayment value in the transformed problem. The repayment value function in the original problem is:

$$
V^c(B, A, \Phi') = \max_{B', \mu, \lambda} \{u(c_L, l_L) n'_L \omega_L + u(c_H, l_H) n'_H \omega_H + \beta E V(B', A', \Phi'')\}.
$$

Divide both sides by N:

$$
\begin{aligned}
\frac{V^c(B, A, \Phi')}{N} &= \max_{B', \mu, \lambda} \{u(c_L, l_L) \frac{n'_L}{n_L} \frac{n_L}{N} \omega_L + u(c_H, l_H) \frac{n'_H}{n_H} \frac{n_H}{N} \omega_H + \beta \frac{N'}{N} \frac{1}{N'} E V(B', A', \Phi'')\} \\
&= \max_{B', \mu, \lambda} \{u(c_L, l_L) g_L(s) f \omega_L + u(c_H, l_H) g_H(s)(1-f) \omega_H \\
&+ \beta (f g_L(s) + (1-f) g_H(s)) \frac{1}{N'} E V(B', A', \Phi'')\},
\end{aligned}
$$

which implies:

$$
v^c(b, A, f) = \max_{b', \mu, \lambda} \{u(c_L, l_L) g_L(s) f \omega_L + u(c_H, l_H) g_H(s)(1-f) \omega_H + \beta E v(b', A', f')[f g_L(s) + (1-f) g_H(s)]\}.
$$

The budget constraint in the original problem is:

$$
B \leq T + qB'.
$$

Divide both sides by N:

$$
\frac{B}{N} \leq \frac{n'_L}{n_L} \frac{n_L}{N} (y_L - c_L) + \frac{n'_H}{n_H} \frac{n_H}{N} (y_H - c_H) + q \frac{B'}{N'} \frac{N'}{N},
$$

which implies:

$$
b \leq g_L(s) f (y_L - c_L) + g_H(s)(1-f)(y_H - c_H) + q b' (f g_L(s) + (1-f) g_H(s)).
$$

The derivation of the defaulting value function in the transformed problem follows similar steps. The defaulting value function in the original problem is:

$$V^d(A,\Phi') = \max_{\mu,\lambda}\{u(c_L^d, l_L^d)n'_L\omega_L + u(c_H^d, l_H^d)n'_H\omega_H + \beta[\theta EV(0, A', \Phi''_{aut=0}) + (1-\theta)EV^d(A', \Phi''_{aut=1})]\}$$

Divide both sides by N:

$$\begin{aligned}
\frac{V^d(A,\Phi')}{N} &= \max_{\mu,\lambda}\{u(c_L^d, l_L^d)\frac{n'_L}{n_L}\frac{n_L}{N}\omega_L + u(c_H^d, l_H^d)\frac{n'_H}{n_H}\frac{n_H}{N}\omega_H \\
&+ \beta[\theta\frac{N'}{N}\frac{1}{N'}EV(0, A', \Phi''_{aut=0}) + (1-\theta)\frac{N'}{N}\frac{1}{N'}EV^d(A', \Phi''_{aut=1})]\} \\
&= \max_{\mu,\lambda}\{u(c_L^d, l_L^d)g_L(s)f\omega_L + u(c_H^d, l_H^d)g_H(s)(1-f)\omega_H \\
&+ \beta[\theta Ev(0, A', f'_{aut=0}) + (1-\theta)Ev^d(A', f'_{aut=1})][fg_L(s) + (1-f)g_H(s)]\}
\end{aligned}$$

The budget constraint under default in the original problem is:

$$0 \leq T.$$

Divide both sides by N:

$$0 \leq \frac{n'_L}{n_L}\frac{n_L}{N}(y_L - c_L) + \frac{n'_H}{n_H}\frac{n_H}{N}(y_H - c_H),$$

which implies:

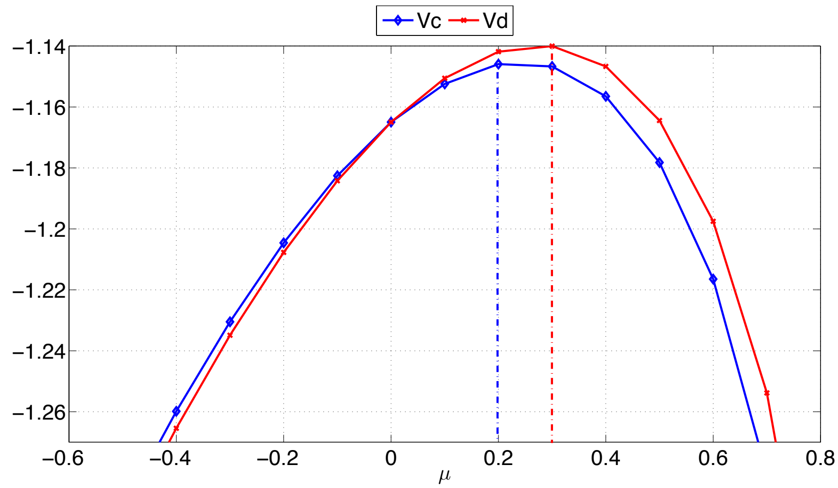
$$0 \leq g_L(s)f(y_L - c_L) + g_H(s)(1-f)(y_H - c_H).$$

B Numerical Example

Appendix B.1 illustrates Propositions in Section 4 graphically. Appendix B.2 shows that the main results stay unchanged with a CRRA utility function.

B.1 Graphical illustration

To this end, we use a numerical example with $A = 1$, $z_L = 0.3$, and $z_H = 0.7$. Figure B.1 plots the repayment value function as a function of tax progressivity as the bell-shaped blue curve, and the default value function as the bell-shaped red curve. The optimal tax progressivity in the case of repayment is $\mu^* = 0.2$, while the optimal tax progressivity in default is $\mu^{d*} = 0.3 > \mu^*$. In both cases, the optimal μ is positive, reflecting redistribution motives.

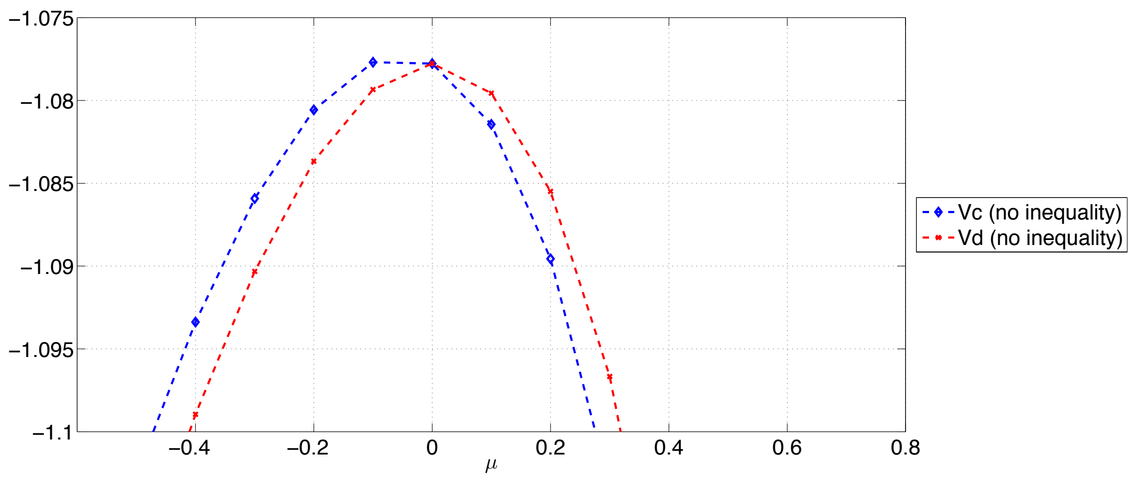


Notes: Figure plots for $b = 0.05$, $A = 1$, $z_L = 0.3$, $z_H = 0.7$.

Figure B.1: Value functions and tax progressivity

We also plot the value functions when there is no inequality ($z_L = z_H = 0.5$); see Figure B.2. When there is no inequality (i.e., representative households), there are no redistribution incentives. If the government defaults, the optimal tax progressivity is $\mu^{d*} = 0$. If the government wants to repay the debt, it would choose a regressive tax ($\mu^* = -0.1$ in this numerical example) to encourage labor supply to finance debt repayment. This result coincides with Heathcote, Storesletten, and Violante (2017) where the benevolent government in the representative agent economy chooses regressive taxes to equate social and private returns to work because the private agents do not internalize public expenditures. Figure B.3 combines Figure B.1 and B.2 to visualize the comparison between the case with inequality and the case with no inequality. Figure B.4 plots the value functions with different levels of inequality. We use $z_L = z_H = 0.5$ for the case with no inequality; $z_L = 0.4$, $z_H = 0.6$ for the case with low inequality; and $z_L = 0.3$, $z_H = 0.7$ for the case with high inequality. Figures B.1–4 show that (1) defaulting leads to higher optimal tax progressivity, and (2) higher inequality leads to higher optimal tax progressivity.

Next we show that the redistribution gain from defaulting increases with inequality. Recall that $z_L = \bar{z} - \sigma_z$, $z_H = \bar{z} + \sigma_z$, with $0 < \sigma_z < \bar{z}$. Parameter σ_z measures the inequality level. We plot the gap between defaulting value and repayment value as a function of σ_z. Note that the first two components of the value functions do not change with inequality level σ_z. The first component ("consumption") is independent of σ_z because of the assumption that the total labor productivity is mean-preserved. The second component



Notes: Figure plots for $b = 0.05$, $A = 1$, $z_L = 0.5$, $z_H = 0.5$.

Figure B.2: Value functions and tax progressivity (no inequality)

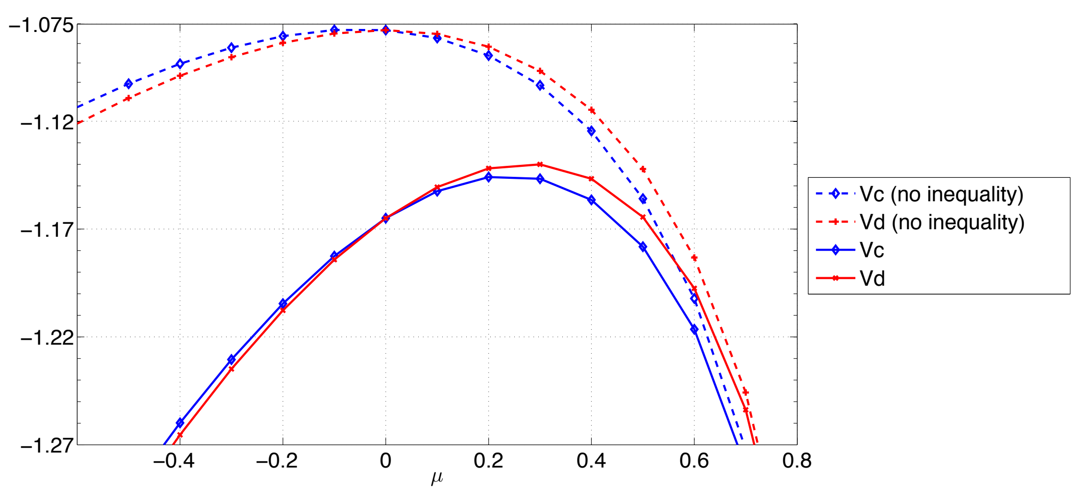


Figure B.3: Value functions and tax progressivity (with and without inequality)

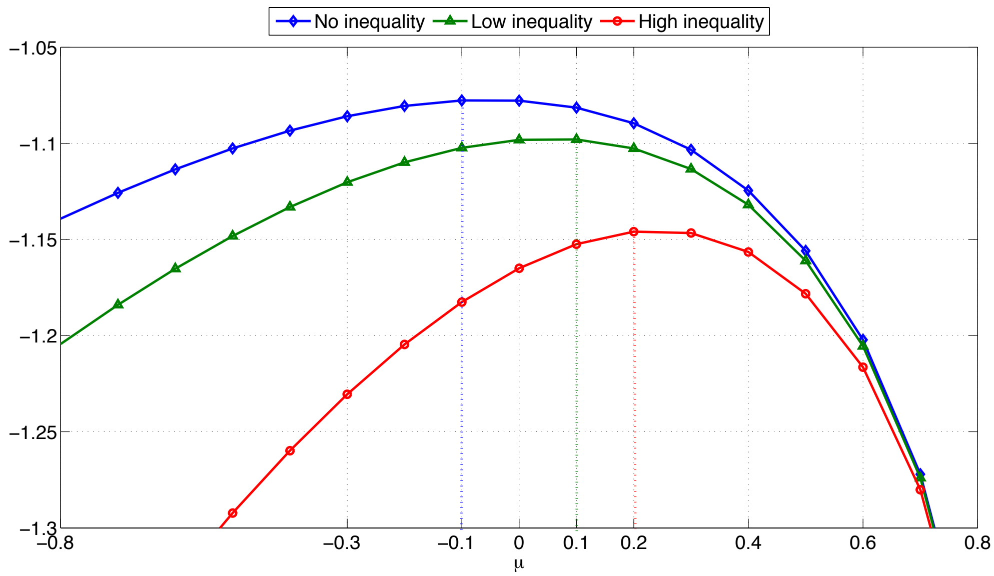


Figure B.4: Value functions and tax progressivity (different inequality level)

("disutility from working") is also independent of σ_z. Thus, the gap between two value functions is essentially the gap between the third components ("redistribution") of the two value functions.

$$V^c(B_0, A) = \max_{\mu}\{\underbrace{\log(Y - B_0)}_{\text{total consumption}} - \underbrace{\frac{1-\mu}{1+\gamma}}_{\text{disutility from working}} + \underbrace{\frac{1}{2}\log[\alpha(1-\alpha)]}_{\text{redistribution}}\},$$

$$V^d(A) = \max_{\mu^d}\{\underbrace{\log(Y^d)}_{\text{total consumption}} - \underbrace{\frac{1-\mu^d}{1+\gamma}}_{\text{disutility from working}} + \underbrace{\frac{1}{2}\log[\alpha^d(1-\alpha^d)]}_{\text{redistribution}}\},$$

Figure B.5 plots the third components in the repayment value $V^c(B_0, A)$ and the default value $V^d(A)$. With higher inequality, the redistribution value gap between default and repayment is larger. Thus, the redistribution gain from defaulting is larger for the economy with high inequality.

B.2 CRRA utility

This subsection derives the optimal labor supply choices under a constant relative risk aversion (CRRA) utility function and shows that the main results stay unchanged.

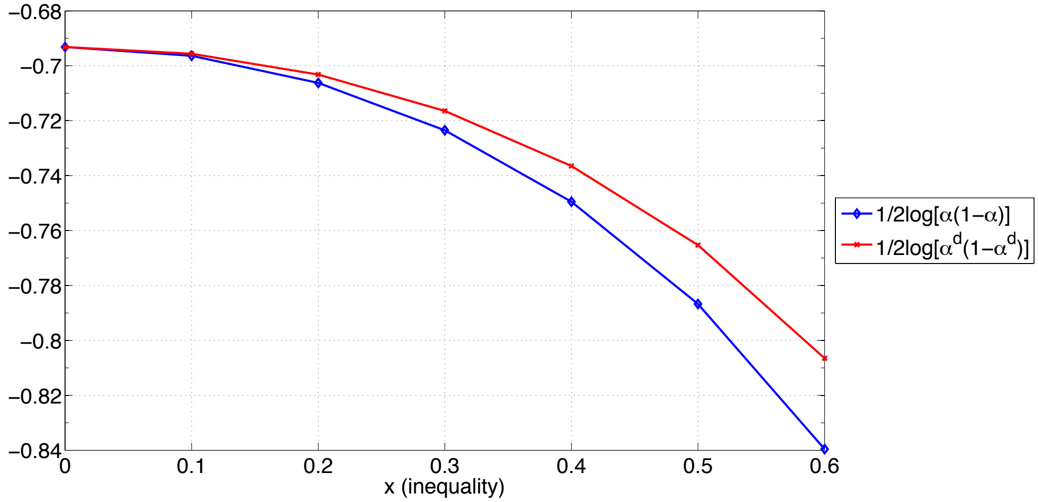


Figure B.5: Redistribution gain and inequality

Assume the utility of household i is given by:

$$u(c_i, \ell_i) = \frac{c_i^{1-\sigma}}{1-\sigma} - \frac{\ell_i^{1+\gamma}}{1+\gamma},$$

where σ is the parameter for risk aversion ($\sigma = 1$ gives logarithmic utility). The optimal choice of labor supply for household i satisfies:

$$\ell_i^{\sigma-\mu\sigma+\mu+\gamma} = (1-\mu)\lambda^{1-\sigma}(wz_i)^{1-\sigma+\mu\sigma-\mu}.$$

We calculate the optimal labor supply and λ under the following set of parameters: $A = 1$, $z_L = 0.3$, $z_H = 0.7$, and $\sigma = 2$. Then we calculate and plot the social welfare functions under different μ. The tax progressivity that maximizes the value function gives the optimal solution. To be more specific, we have three unknowns: ℓ_L, ℓ_H, and λ, as well as three nonlinear equations:

$$\ell_L^{\sigma-\mu\sigma+\mu+\gamma} - (1-\mu)\lambda^{1-\sigma}(wz_L)^{1-\sigma+\mu\sigma-\mu} = 0,$$

$$\ell_H^{\sigma-\mu\sigma+\mu+\gamma} - (1-\mu)\lambda^{1-\sigma}(wz_H)^{1-\sigma+\mu\sigma-\mu} = 0,$$

$$\lambda - \frac{wz_L\ell_L + wz_H\ell_H - B_0}{(wz_L\ell_L)^{1-\mu} + (wz_H\ell_H)^{1-\mu}} = 0.$$

For each given value of μ, we can solve for a set of $\{\ell_L, \ell_H, \lambda\}$. Then we solve for output, tax revenue, and consumption. Given consumption and labor choices, we calculate the value of the social welfare function. Finally, we plot the social welfare function as a function of tax progressivity under the following cases: with inequality versus without inequality, repayment versus default.

Figure B.6 plots the value function with inequality as a function of μ as the blue curve with no markers, and the value function without inequality as a function of μ as the red curve with triangle markers. It shows that the optimal tax progressivity is higher if the economy is unequal, which is consistent with Proposition 2.

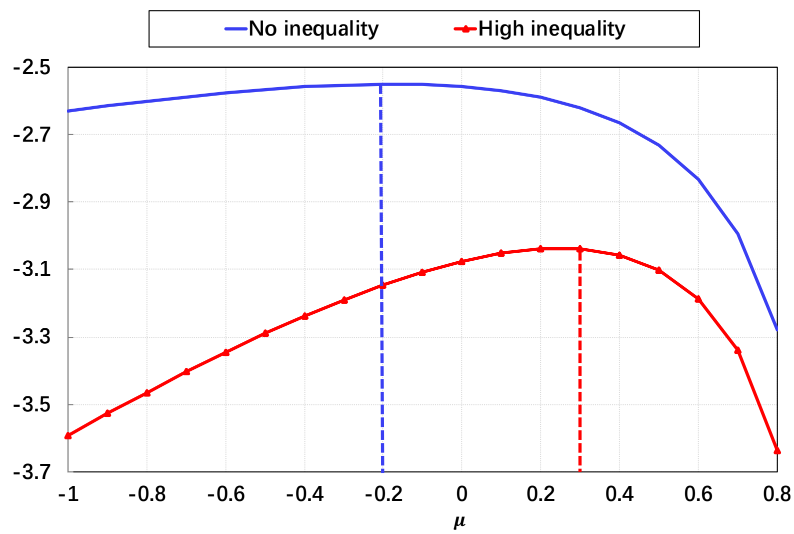


Figure B.6: Value functions and inequality under CRRA utility with $\sigma = 2$

Figure B.7 plots the repayment value function as a function of tax progressivity as the bell-shaped blue curve with round markers, and the default value function as the bell-shaped red curve. Consistent with Proposition 3, defaulting leads to higher optimal tax progressivity.

Recall that with logarithmic utility, tax progressivity μ discourages labor. We show in Figure B.8 that it is still the case with CRRA utility. The yellow dashed line plots for the total effective labor. The total effective labor is decreasing in μ, and thus the total output is decreasing in μ. Figure B.9 plots for the tax revenue from L-type households as a function of μ with the dashed blue line, tax revenue from H-type households with the solid red line, and the ratio of consumption by the L-type households and that of H-type households with the line with diamond markers. With a more progressive tax, L-type households pay less taxes, H-type households pay more taxes, and the relative consumption of L-type to that of H-type increases.

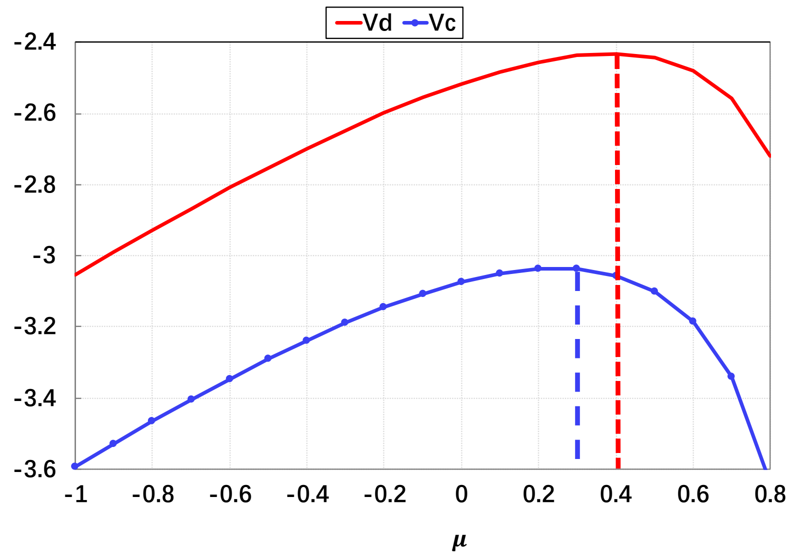


Figure B.7: Value functions under CRRA utility with $\sigma = 2$

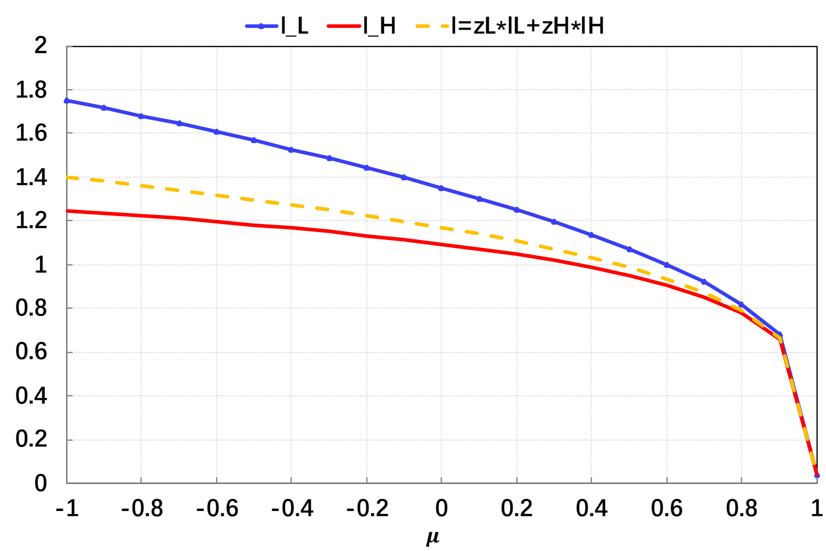


Figure B.8: Labor supply under CRRA utility with $\sigma = 2$

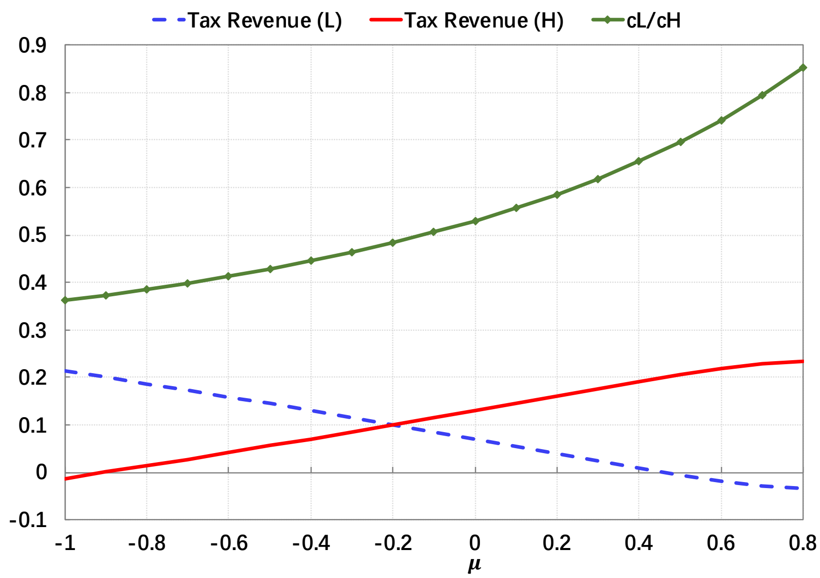


Figure B.9: Tax revenue and relative consumption under CRRA utility with $\sigma = 2$

C Computation Algorithm

We compute the problems of the utilitarian government, L-type households, and H-type households using value function iteration. In the government's problem, the state space for periods with financial market access is given by (b, A, f), or (A, f, aut) if the government defaulted. $aut = 1$ denotes the economy is in financial autarky, and $aut = 0$ otherwise. In i-type households' problem, the state space is (b, A, f, aut). The AR(1) process for aggregate productivity shock A is discretized using 21 equally spaced grid points with Tauchen's method. The government makes a borrowing decision b' and tax progressivity choice μ if not in default, but makes only a tax progressivity choice μ if in default (λ will be determined by the government budget constraint). For bonds, we use a grid with 120 equally spaced points on $b \in [0, 0.3]$. For tax progressivity, I use a grid with 120 equally spaced points on $\mu \in [-0.6, 0.6]$. For the fraction of $L-$type households f, we use a grid with 11 equally spaced points on $f \in [0, 1]$. We restrict these choice variables to be on the grid. Given optimal government policies, households determine whether to migrate or not. If they do not migrate, households choose labor supply and consumption to maximize lifetime utility. Given the decisions of the households, the government updates the repayment value and default value, and decides whether to default. For each iteration, we update the value of the government and the value of each type of household. The code stops running when the value function of the government and the value function for each type of household converge.

Here is a more detailed description of the algorithm:

1. Create grids and discretize Markov process for productivity shock A. Create grids for bonds b, and tax progressivity μ. Create grids for the fraction of L-type households f.
2. Guess the initial value function of government $V_0(b, A, f, aut)$ and the price function $q_0(b, A, f)$; guess the initial value function of L-type households $W_0^L(b, A, f, aut)$ and that of H-type households $W_0^H(b, A, f, aut)$.
3. Update the repayment value $V^c(b, A, f)$ and the default value $V^d(A, f, aut)$.
4. Compare $V^c(b, A, f)$ and $V^d(A, f, aut)$, update the defaulting rule, price function, and the value function of the government $V(b, A, f, aut)$.
5. Compute the optimal policy of the government with and without access to the credit market. With access to the credit market, the optimal policies are $\{b'(b, A, f), \mu(b, A, f)\}$; without access, the optimal policy is $\{\mu(A, f)\}$.
6. Given government policies, update the value of staying for each type of household: $W^{L,s}(b, A, f, aut)$ and $W^{H,s}(b, A, f, aut)$.
7. Compare the value of staying $W^{L,s}(b, A, f, aut)$ and the value of migrating $W^{L,m}$ for L-type households, then update the value function $W^L(b, A, f, aut)$.
8. Compare the value of staying $W^{H,s}(b, A, f, aut)$ and the value of migrating $W^{H,m}$ for H-type households, then update the value function $W^H(b, A, f, aut)$.
9. Check the distance $dist_g$ between the updated value function of the government and the one from the last iteration, the distance $dist_l$ between the updated value function of L-type households and the one from last iteration, and the distance $dist_h$ between the updated value function of H-type households and the one from last iteration. If any of these distances are larger than tolerance, then go back to 3. Otherwise, stop.