# Intangible Investment during Sovereign Debt Crisis: Firm-level Evidence\*

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#### Abstract

This paper measures the cost of sovereign debt crises by focusing on the impact of sovereign risk on firms' intangible investment and TFP. Using Italian firm-level data, we find that small firms and high-leverage firms significantly reduce their intangible investment during the Italian sovereign debt crisis. High-leverage firms reallocate their resources from intangible capital to tangible capital to offset the tightening of financial conditions because tangible capital can be used as collateral. We analyze these patterns by developing a quantitative model incorporating sovereign default risk, financial intermediations, and firm investment decisions on both tangible and intangible capital. In the model, government default risk deteriorates banks' balance sheets, disrupting banks' ability to finance firms. Since firms depend on external funding to cover a fraction of investment, firms – especially small and high-leverage ones – reduce intangible investment, which hurts their future total factor productivity. We estimate the model using Italian data and find that the increase in sovereign risk explains the slow recovery of productivity after the debt crisis through the intangible investment channel.

**Keywords**: European debt crisis, sovereign default, intangible asset, firm heterogeneity, productivity loss

JEL classification: F34, E22, E44, G12, G15

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### 1 Introduction

During the recent European debt crisis, the spike in sovereign spreads accompanied substantial bank credit contractions to the private sector. The key open question is to understand and quantify the impacts of sovereign risk on the private sectors. This paper focuses on the effects of sovereign risk on firms' investment, especially investment in intangible assets – a key driver of productivity growth. A sovereign debt crisis has long-term impacts on firm productivity and the aggregate economy by affecting firms' intangible investment.

Using Italian firm-level data combined with sovereign debt data, we find that firms reduce their intangible investment during the Italian sovereign debt crisis, especially for small firms and high-leverage firms. Although those firms also reduce tangible asset investment, they tend to reallocate their limited resources from intangible assets towards tangible assets. Meanwhile, there is a dramatic drop in firm-level total factor productivity (TFP) during the crisis, and the recovery after the crisis is slow. Reduction in intangible asset investment is accountable for the decline in TFP and the slow recovery afterward.

To separate the sovereign debt crisis's effects on firm TFP, we develop a sovereign default model incorporating firm investment on both tangible assets and intangible assets. An increase in government default risk deteriorates banks' balance sheets, which leads to a higher loan interest rate faced by the firms. The firm-specific loan interest rate depends on each firm's collateral, and intangible assets typically can't be used as collateral. Thus, with a higher borrowing cost, firms heavily relying on external financing reduce their asset investment – especially intangible asset investment. Lower intangible investment hurts firms' productivity for the subsequent years. Our framework provides a micro-foundation for market imperfections that endogenously generate volatile and persistent shocks to TFP.

The model features a government, final goods firms, heterogeneous intermediate goods firms that accumulate intangible assets and tangible assets, financial intermediaries, and households. Final goods firms are competitive, and they convert intermediate goods to final goods. Government raises funds for expenditures and bond repayment by collecting tax revenues from the final goods firms and issuing long-term government bonds to financial intermediaries. The financial intermediaries,

owned by bankers in the households, use government repayments and household savings to lend to the intermediate goods firms and the government. When the default probability of government increases, the financial intermediaries' balance sheets deteriorate. The intermediate goods firms who borrow from the financial intermediaries to cover a fraction of investment cost would suffer higher loan interest rates. The higher firm-specific loan interest rates, which in equilibrium depend on the firm's collateral, lead to a reduction in intangible investment.

We parametrize the model using Italian firm-level data and government data. We measure firms' TFP losses from the Italian sovereign debt crisis during 2008-2012 and after the debt crisis using the calibrated model. We first feed the model with default risk shock and exogenous productivity shock to replicate the observed path of Italian sovereign risk and average firm TFP. We then use the model to construct the path of TFP in a counterfactual scenario in which the Italian economy does not experience a sovereign debt crisis. The difference between the path of TFP in the benchmark model and the no debt crisis counterfactual model isolates the endogenous TFP cost of the sovereign crisis for the Italian economy. We find that sovereign risk is accountable for Italy's slow TFP recovery. Without the sovereign debt crisis, TFP would have recovered in 2015.

**Related literature** This paper contributes to the literature regarding the impacts of sovereign risk on the private sector through financial intermediation, and the literature that studies intangible investment and financial frictions.

First, this paper is related to the literature studying the links between sovereign default risk and the private sector through financial intermediation. During a sovereign debt crisis, firms lose access to external financing cut their production, and thus, a sovereign debt crisis induces an output loss (Mendoza and Yue (2012), Arellano et al. (2019)). The link between sovereign default and private sector through banking sectors is also analyzed in Padilla (2012), Perez (2018) and D'Erasmo et al. (2019). Our paper shares the focus of studying the transmission of sovereign risk to the firms. However, we focus on firms' investment by decomposing investment into tangible

<sup>&</sup>lt;sup>1</sup>In the counterfactual no debt crisis model, we force the default risk shock to generate that sovereign spreads remain at the value in 2008, while keeping the exogenous productivity shock. This exogenous productivity shock summarizes the shocks that are not directly induced by the sovereign debt crisis, e.g., aggregate demand decline due to global recession.

investment and intangible investment. We find that the firms with a high external financing requirement reduce their intangible investment and shift their investment toward tangible assets, due to preference for pledgeable assets.

Second, this paper contributes to the literature that studies the relationship between investment in intangible assets and financial frictions. The literature does not have a consensus on intangible investment sensitivity to financial conditions. On the one hand, firms with tight financial constraints could decrease their intangible investment due to their tangibility preference. Pérez-Orive (2016) documents credit-constrained firms prefer types of capital that generate significant pledgeable output and are liquid<sup>2</sup>, since they loosen current and future credit constraints. In case of default, intangible assets are harder to seize by creditors. Thus, intangible capital faces higher financing costs, which leads to a fall in intangible investment during a crisis (Garcia-Macia (2017), Lopez and Olivella (2018)). On the other hand, Borisova and Brown (2013) shows that firms with tight financial constraints increase their intangible investment. Contributing to this open, ongoing research question, we find that firms invest heterogeneously in the intangible asset during a sovereign debt crisis, depending on their size and financial positions.

Our paper is related to Aghion et al. (2012) that discuss financial friction and the cyclicality of R&D investment. It is well documented in the literature that innovation positively correlates with firm productivity growth. <sup>3</sup> A traditional view indicates that long-term investments (R&D investment) is countercyclical due to the low opportunity cost of long-term innovative investments in recessions than in booms (Bean (1990); Aghion and Saint-Paul (1998); Bloom (2007)). However, Aghion et al. (2012) argue that this traditional view is only true for firms that are not constrained financially. Our paper focuses on the sovereign debt crisis, and its impact on firms' heterogeneous behaviors regarding intangible investment and tangible investment. We empirically present that the firms with lower dependence on external funds take advantage of

<sup>&</sup>lt;sup>2</sup>Examples for liquid capital include inventory, machinery and equipment, and commercial or industrial structures with a high resale value.

<sup>&</sup>lt;sup>3</sup>The positive effects of intangible assets on firms' economic performance are well established in the literature. This branch of literature can trace back to Griliches (1958) and research on R&D investment and firm productivity has bloomed ever since (Griliches (1979); Geroski (1989); Hall et al. (2010)). Several recent studies show that low firm-level incentives to invest in intangibles would result in TFP and output losses. The low incentives can be either caused by distortions (Ranasinghe (2014)), or monetary policy (Moran and Queralto (2018)), or equity financing shocks (Bianchi et al. (2019)), or financial crises (Queralto (2019)).

the low opportunity cost of innovation and invest more in intangibles during a debt crisis, which supports the traditional view. On the contrary, firms largely depending on external borrowing decrease their intangible investment during a debt crisis due to credit market imperfections, which echos the result in Aghion et al. (2012). Our paper includes both directions and explains firms' heterogeneous responses due to their heterogeneous dependence on external borrowing.

**Road Map** This paper proceeds as follows. Section 2 shows data and key empirical findings. Section 3 presents our model with sovereign default risk, financial intermediaries, and firm investment on both tangible and intangible capital. Section 4 presents the quantitative results of the model. Section 5 concludes.

## 2 Empirical facts

We document the impact of sovereign debt crisis on firms' intangible investment and tangible investment. Section 2.1 describes the construction of variables of interest and some statistics. Section 2.2 presents the firm-level responses in terms of intangible investment and tangible investment during sovereign debt crises. Section 2.3 provides additional empirical evidence for robustness.

## 2.1 Data Description

**Firm-level Variables** We focus on the manufacturing firms in Italy. Our main sample uses the annual firm-level data from Orbis dataset, covering the period 2006-2016. The dataset includes rich balance-sheet information on both private and public firms.

Intangible assets normally include scientific or technical knowledge, design and implementation of new processes or systems, licenses, intellectual property, market knowledge and trademarks such as brand names and publishing titles. Intangible fixed assets in Orbis dataset is defined as all intangible assets such as formation expenses, research expenses, goodwill, development expenses and all other expenses with a long term effect. To measure investment on intangible assets, we take log

difference of intangible fixed assets. We use this log-difference specification because investment is highly skewed, suggesting a log-linear rather than level-linear regression specification.<sup>4</sup> One possible concern about this log-difference measure of intangible investment stems from the discontinuity of intangible fixed assets. Using the log-difference measure of intangible investment loses some observations if firms empty the intangible fixed assets in some certain years, which may bias our baseline estimation. To deal with the observation losses of intangible investment calculated from log-difference method, we consider the extensive margin of intangible investment in Section 2.2.3 and also use alternative measures for investment in Section 2.3 as robustness checks.

We use three measures for firms' financial positions. In our baseline regression, leverage is defined as the ratio of total debt, which is the sum of short-term debt and long-term debt, over total assets. We label the leverage calculated according to this method as "total leverage". We also use short-term leverage (loan-to-asset ratio) and net leverage (the ratio of net debt to total assets, where net debt is the sum of short term loans and long term debt net of net current assets) as robustness checks. As standard in literature, we define size, liquidity, sales growth, and net current asset ratio. The detailed definitions can be found in Appendix A.

We perform standard steps to guarantee the quality of the data, we scale the intangible/tangible fixed assets by the price of intangibles and tangibles. The prices of intangibles and tangibles are calculated according to information in EUKLEMS database. <sup>5</sup> All remaining nominal variables are scaled by the producer price index. We eliminate outliers by dropping the 1st and 99th percentile of key variables. In Appendix A, we provide details on our sample selection and variables. Table 1 reports a set of summary statistics for the main variables. The median firm has a leverage of 16.3%, and negative investment during our sample period.

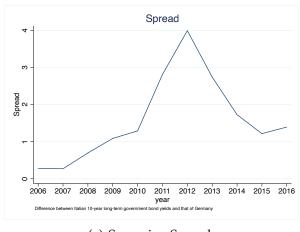
<sup>&</sup>lt;sup>4</sup>Appendix A.3 shows the aggregate distribution of wage bill and employment by firm size to display our sample representativeness.

<sup>&</sup>lt;sup>5</sup>Please see Appendix A.1 for more details on how we construct the price of intangibles and tangibles

**Table 1: Summary Statistics** 

	Obs.	Mean	St.Dev.	Min.	Max.	P25	Median	P75
Total leverage	494214	0.202	0.198	0	0.983	0	0.163	0.35
Net leverage	489840	0.071	0.391	-0.937	4.123	-0.191	0.066	0.323
Short-term leverage	497222	0.133	0.151	0	0.797	0	0.077	0.231
Size	502738	10.144	1.298	3.855	13.399	9.242	10.145	11.074
Liquidity	485069	0.074	0.11	0	0.849	0.004	0.025	0.097
Investment on tangibles (log-diff)	505188	-0.099	0.751	-2.769	3.623	-0.441	-0.177	0.083
Investment on intangibles (log-diff)	505188	-0.014	0.405	-1.877	2.484	-0.185	-0.063	0.076

**Sovereign Debt Crisis** Sovereign spread  $sp_t$ , defined as the gap between 10-year government bond yield between Italy and that of Germany, measures the severity of a sovereign debt crisis. Panel (a) of Figure 1 plots the annual sovereign spreads for Italy from 2006 to 2016. During Italian debt crisis 2011-2013, spread spikes to 2%-4%, much higher than the previous level. Sovereign debt credit rating also indicates the severity of a sovereign debt crisis. Panel (b) shows the Standard & Poor's (S&P) credit rating for Italian sovereign bonds from 2006 to 2016. In 2011, S&P revised their credit rating for Italian bonds from A+ to A, and further downgraded the rating to BBB+ in 2012, BBB in 2013 and BBB- in 2014.



Г 1 (	CAD "
End of year	S&P rating
2006	A+
2007	A+
2008	A+
2009	A+
2010	A+
2011	A
2012	BBB+
2013	BBB
2014	BBB-
2015	BBB-
2016	BBB-

(a) Sovereign Spread

(b) Credit Rating

Figure 1: Italian Sovereign Debt Crisis

## 2.2 Firm-level Responses during Sovereign Debt Crisis

We focus on how firms react differently in terms of investment during a sovereign debt crisis. In the first specification, we focus on the heterogeneous effects of sovereign risk on investment (including intangible investment and tangible investment) depending on firms' characteristics, controlling for sector-year fixed effect. To obtain the average effect, we relax the sector-year fixed effect and include more aggregate controls in the second specification. Those two specifications can be viewed as the intensive margin of changes in investment, as the log-difference measure omits the observation with zero stock of asset by construction. This issue is pronounced for intangible assets. Thus, in the third specification, we further characterize the effects of sovereign risk on the extensive margin of intangible assets, i.e., whether to continue to hold any intangible assets.

#### 2.2.1 Heterogeneous Effects

We estimate variants of the baseline empirical specification:

$$\Delta ln(assets_{i,t+1}) = \alpha(x_i \times sp_t) + Controls + \delta_i + \eta_{st} + \epsilon_{it}, \tag{1}$$

where  $\Delta ln(assets_{i,t+1}) \in \{\Delta ln(intangibles_{i,t+1}), \Delta ln(tangibles_{i,t+1})\}$  is the intangible investment or tangible investment of firm i at time t, defined as the log difference of intangible assets  $[ln(intangibles_{i,t+1}) - ln(intangibles_{it})]$  or tangible assets  $[ln(tangibles_{i,t+1}) - ln(tangibles_{it})]$ .  $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$  is the firm's size or total leverage in the year 2006. Controls contains the interaction term of  $x_i$  and GDP growth  $(\Delta GDP_t)$  and a vector of firm-level variables in time t-1, which includes size, total leverage, liquidity, sales growth, the ratio of liability to total assets, the ratio of net current assets to total assets, and the share of intangible fixed assets in total fixed assets.  $\delta_i$  is a firm fixed effect, which captures the permanent differences in investment behaviors across firms.  $\eta_{st}$  is a sector-by-year fixed effect, which captures differences in how broad sectors are exposed to aggregate shocks.  $\epsilon_{it}$  is a residual. Our main coefficient of interest is  $\alpha$ , which depicts how firms invest in response to

<sup>&</sup>lt;sup>6</sup>We use the firms' data in the first year of the sample to guarantee the variables are pre-determined. We also use other measures of leverage including net leverage and short leverage for robustness checks.

spread conditional on firm's characteristics.

Panel (a) of Table 2 reports the results from estimating the baseline specification (1) for intangible investment. We have standardized size and total leverage over the entire sample in the year 2006, so their units are in standard deviations relative to the mean. We cluster the standard errors at the firm level. Column (1) in Table 2 Panel (a) shows the result only focusing on firms' heterogeneity in size. Column (2) reports the results focusing on firms' heterogeneity in leverage. Column (3) adds both interactions. The results show that a firm with one standard deviation larger size than the average has approximately 0.0147 units *higher* semi-elasticity of intangible investment when spread increases, and a firm with one standard deviation higher leverage than the average firm has approximately a 0.0049 units *lower* semi-elasticity.

Panel (b) of Table 2 reports the results of estimating Eq (1) for tangible investment. Column (3) of Panel (b) shows that a firm with one standard deviation larger size than the average has approximately 0.0065 units *higher* semi-elasticity of tangible investment when spread increases by 1%, and a firm with one standard deviation higher leverage than the average firm has approximately a 0.0013 units *higher* semi-elasticity. Comparing Panel (a) and (b), we find that firms at each leverage level have opposite responses in terms of intangible and tangible investment: a high leverage firm would invest less on intangible assets but invest more on tangible assets compared to other firms during a sovereign debt crisis. This is because tangible assets can be used as collateral. High-leverage firms reallocate their resources from intangible assets to tangible assets in order to offset the tightening of financial conditions.

As the sector-by-year fixed effect absorbs the average effect of spread, we can only know the heterogeneous responses by estimating Eq (1). We now relax the sector-by-year fixed effect and instead include aggregate controls to compare the interaction coefficients to the average effect.

#### 2.2.2 Average Effects

To assess the economic significance of our estimated interaction coefficients  $\alpha$ , we now relax the sector-year fixed effect to obtain the average effect of sovereign debt

 $<sup>^{7}</sup>$ standardized size $_{i,2006} = \frac{size_{i,2006} - \frac{1}{N_{2006}} \sum_{i} size_{i,2006}}{std(size_{i,2006})}$ . Same standardization applies for leverage.

Table 2: Heterogeneous Responses of Firm Investment

#### (a) Intangible Investment

#### (b) Tangible Investment

	Depender	nt variable: /	$\Delta ln(intangibles_{i,t+1})$		Depender	nt variable: /	$\Delta ln(tangibles_{i,t+1})$
	(1)	(2)	(3)		(1)	(2)	(3)
$size_{i,2006} \times sp_t$	0.0140***		0.0147***	$size_{i,2006} \times sp_t$	0.0066***		0.0065***
	(0.0016)		(0.0016)		(0.0008)		(0.0008)
$lev_{i,2006} \times sp_t$		-0.0032**	-0.0049***	$lev_{i,2006} \times sp_t$		0.0020***	0.0013*
,,,		(0.0014)	(0.0014)			(0.0006)	(0.0006)
Firm controls	Yes	Yes	Yes	Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Firm FE	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	Sector-year FE	Yes	Yes	Yes
Observations	305,088	303,935	303,935	Observations	305,088	303,935	303,935
R-squared	0.066	0.066	0.066	R-squared	0.092	0.092	0.092
Number of id	60,040	59,706	59,706	Number of id	60,040	59,706	59,706

*Notes*: Results from estimation Eq (1) for intangible investment. We have normalized  $x_i \in \{size_{i,2006}, lev_{i,2006}\}$  to standardized size and leverage, so their units are in standard deviation relative to he mean. Robust standard errors (in parentheses) are clustered by firms. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

*Notes*: Results from estimation Eq (1) for tangible investment. We have normalized  $x_i \in \{size_{i,2006}, lev_{i,2006}\}$  to standardized withinfirm size and total leverage, so their units are in standard deviation relative to he mean. Robust standard errors (in parentheses) are clustered by firms. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

#### crisis by estimating

$$\Delta ln(assets_{i,t+1}) = \alpha_0 sp_t + \alpha_1(x_i \times sp_t) + Controls + AggControls + \delta_i + \epsilon_{it}, \quad (2)$$

where  $\Delta ln(assets_{i,t+1}) \in \{\Delta ln(intangibles_{i,t+1}), \Delta ln(tangibles_{i,t+1})\}$  is the intangible investment or tangible investment of firm i at time t,  $sp_t$  is the sovereign debt spread at time t, and  $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$  is firm's size or total leverage in the year 2006. Controls is a vector of firm-level variables in time t-1 as defined in Eq (1). We also include a vector of aggregate controls AggControls, which includes GDP growth  $(\Delta GDP_t)$  and its interactions with firms' characteristics  $(x_i \times \Delta GDP_t)$ .  $\delta_i$  is a firm fixed effect. Our coefficients of interest are  $\alpha_0$  and  $\alpha_1$ .  $\alpha_0$  shows the average effect of sovereign spread on firms' investment, and  $\alpha_1$  measures how semi-elasticity of investment with respect to spread depends on firm size or leverage.

Table 3 reports the results for estimating Eq (2). The coefficients for  $sp_t$  for intangible and tangible investment are around 0.004 - 0.006 in absolute value. The interaction coefficients imply an economically meaningful degree of heterogeneity. The coefficients for the interaction terms are similar to the ones in Table 2. The results show that small firms will invest less in both intangible assets and tangible assets, and high-leverage firms will reallocate their resources from intangibles to tangibles to

<sup>&</sup>lt;sup>8</sup>Appendix B.4 provides results when we add more aggregate controls, such as world GDP growth, trade openness, etc, and the results remain robust.

offset the tightening financial conditions.

Table 3: Average Responses of Firm Investment

#### (a) Intangible Investment

### (b) Tangible Investment

		Dependent	variable: $\Delta l$	n(intangibles <sub>i</sub>	t+1)	-		Dependent	variable: ∆ <i>ln</i>	$(tangibles_{it+1})$	)
	(1)	(2)	(3)	(4)	(5)	-	(1)	(2)	(3)	(4)	(5)
spt	0.00513***	0.00627***	0.00423***	0.00616***	0.00416***	$sp_t$	-0.00180***	-0.00384***	-0.00477***	-0.00395***	-0.00480***
	(0.00139)	(0.00140)	(0.00140)	(0.00140)	(0.00140)		(0.000630)	(0.000641)	(0.000680)	(0.000645)	(0.000681)
$size_{i,2006} \times sp_t$			0.0148***		0.0153***	$size_{i,2006} \times sp_t$			0.00676***		0.00653***
			(0.00158)		(0.00161)				(0.000770)		(0.000779)
$lev_{i,2006} \times sp_t$				-0.00307**	-0.00483***	$lev_{i,2006} \times sp_t$				0.00219***	0.00144**
				(0.00141)	(0.00142)					(0.000639)	(0.000641)
Firm controls	No	Yes	Yes	Yes	Yes	Firm controls	No	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	330,365	305,119	305,088	303,935	303,935	Observations	330,365	305,119	305,088	303,935	303,935
R-squared	0.008	0.055	0.055	0.055	0.056	R-squared	0.014	0.063	0.063	0.063	0.063
Number of id	62,073	60,049	60,040	59,706	59,706	Number of id	62,073	60,049	60,040	59,706	59,706

parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Notes: Results from estimation Eq (2) for intangible investment. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. \*\* p<0.05, \* p<0.1. \*\* p<0.05, \* p<0.1.

#### **Extensive Margin of Intangible Investment** 2.2.3

As our log-difference measure omits the variables with zero intangible assets by construction, here we focus on the extensive margin of intangible assets, i.e., whether to continue having intangible assets. Denote  $\mathbb{1}(intangibles_{i,t+1})$  as an indicator that equals 1 if firm i continues to hold any intangible fixed assets in period t + 1, and equals 0 if firm *i* stops to have any intangible fixed assets. Table 4 reports the results when we substitute the dependent variables with  $\mathbb{I}(intangibles_{i,t+1})$  in Eq (1) and Eq (2). Table 4 shows that large firms and low-leverage firms are more likely to continue to hold intangible assets, consistent with the intensive margin of adjusting intangible assets as shown in Section 2.2.1 and 2.2.2.

#### 2.3 **Additional Empirical Results**

This section presents several key robustness checks, including using alternative measure for investment, for sovereign debt crisis, and for firm-level leverage. More robustness checks, including taking depreciation into account, adding region-year fixed effects, adding more aggregate controls, clustering at sector level, winsorizing with alternative criteria, deflating with alternative prices, etc. can be found in Appendix B.

<sup>&</sup>lt;sup>9</sup>For tangible assets, it is less of a concern if any, because we exclude the firms with negative or zero total assets, and the remaining firms have at least positive tangible assets.

Table 4: Extensive Margin of Intangibles

			Dependen	t variable: 1(	(intangibles <sub>i,</sub>	$_{t+1})$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sp <sub>t</sub>				-0.0136***	-0.0142***	-0.0136***	-0.0142***
				(0.000366)	(0.000388)	(0.000367)	(0.000388)
$size_{i,2006} \times sp_t$	0.00438***		0.00451***		0.00463***		0.00472***
,	(0.000442)		(0.000447)		(0.000441)		(0.000446)
$lev_{i,2006} \times sp_t$		-0.000384	-0.000931**			-0.000304	-0.000893**
,		(0.000381)	(0.000384)			(0.000386)	(0.000388)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	384,511	383,121	383,121	384,570	384,511	383,121	383,121
R-squared	0.034	0.033	0.034	0.019	0.020	0.019	0.020
Number of id	71,727	71,339	71,339	71,747	71,727	71,339	71,339

Notes: Results from estimating Eq (1) and Eq (2) with the dependent variable being an indicator that equals 1 if continuing to hold any intangible assets, and equals 0 if stopping to hold any intangible assets. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Alternative measure for investment. In the baseline regression, investment is defined as the log difference of assets. The log-difference measure captures the relative change that is symmetric, additive and normed, but misses the observations with zero fixed assets. To account for the asset changes both in the intensive and extensive margin, we borrow from the job creation literature (Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996), Huber, Oberhofer, and Pfaffermayr (2013), among others) a growth measure that accounts for both entry and exit. We analogously define firm investment.

For intangible assets, firms in each year can be classified into three groups:

$$\begin{cases} \text{exiting firms} & G_x = \{i | k_{it} \neq 0, k_{i,t+1} = 0\} \\ \text{continuing firms} & G_c = \{i | k_{it} \neq 0, k_{i,t+1} \neq 0\} \\ \text{entering firms} & G_n = \{i | k_{it} = 0, k_{i,t+1} \neq 0\} \end{cases}$$

where  $k_{it}$  denotes intangible fixed assets of firm i at period t. Here "exiting" and "entering" only indicate whether firm i continues to hold intangible fixed assets. Then, investment on intangible assets (which is also the growth rate between two averages)

can be defined as:

$$g(intangibles_{i,t+1}) = \frac{k_{i,t+1} - k_{it}}{0.5(k_{i,t+1} + k_{it})} = \begin{cases} -2 & i \in G_x \\ \frac{k_{i,t+1}/k_{it} - 1}{0.5(k_{i,t+1}/k_{it} + 1)} & i \in G_c \\ 2 & i \in G_n \end{cases}$$
(3)

We refer to this measure of investment as DHS (abbreviation for Davis, Haltiwanger, and Schuh (1996)) investment. The main advantage of DHS investment is that it can accommodate both entry (into the asset market, i.e., begin to hold assets) and exit (from the asset market, i.e., do not hold assets any more). It is a second-order approximation of the log difference growth rate around 0 and it is bounded in the range [–2,2]. We estimate both empirical specifications Eq (1) and Eq (2) using DHS investment as the dependent variable.

Table 5 shows that, consistent with baseline regression results, small firms and high-leverage firms decrease their intangible investment more than other firms during a sovereign debt crisis. DHS investment for tangible asset  $g(tangibles_{i,t+1})$  can be constructed in a similar way. Table 6 reports the results.

Table 5: Alternative Measure for Intangible Investment: DHS Investment

			Dependent v	ariable: g(in	tangibles <sub>i,t+</sub>	-1)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$sp_t$				-0.0139***	-0.0162***	-0.0140***	-0.0163***
				(0.00130)	(0.00132)	(0.00131)	(0.00132)
$size_{i,2006} \times sp_t$	0.0182***		0.0189***		0.0192***		0.0198***
	(0.00150)		(0.00153)		(0.00149)		(0.00151)
$lev_{i,2006} \times sp_t$		-0.00382***	-0.00614***			-0.00367***	-0.00617***
		(0.00133)	(0.00135)			(0.00134)	(0.00135)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No	No
Observations	393,593	392,183	392,183	393,653	393,593	392,183	392,183
R-squared	0.057	0.056	0.057	0.040	0.041	0.041	0.041
Number of id	71,910	71,522	71,522	71,930	71,910	71,522	71,522

**Alternative measure for sovereign debt crisis.** Another way to measure the severity of a sovereign debt crisis is to use the credit rating for sovereign bonds from the credit rating agencies, such as Standard & Poor's (S&P). The credit rating from S&P ranges from AAA (prime grade) to D (default). For instance, A+ implies an upper

Table 6: Alternative Measure for Tangible Investment: DHS Investment

			Dependent v	variable: g(ta	$ngibles_{i,t+1}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$sp_t$				-0.0114***	-0.0125***	-0.0114***	-0.0125***
				(0.000631)	(0.000664)	(0.000632)	(0.000664)
$size_{i,2006} \times sp_t$	0.00926***		0.00927***		0.00944***		0.00934***
,	(0.000786)		(0.000795)		(0.000770)		(0.000779)
$lev_{i,2006} \times sp_t$		0.00115*	9.77e-06			0.00134**	0.000162
,		(0.000648)	(0.000653)			(0.000641)	(0.000645)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No	No
Observations	393 <i>,</i> 593	392,183	392,183	393,653	393,593	392,183	392,183
R-squared	0.084	0.083	0.084	0.054	0.055	0.054	0.055
Number of id	71,910	71,522	71,522	71,930	71,910	71,522	71,522

medium grade and B+ suggests highly speculative. As shown in Panel (b) of Figure 1, S&P credit rating for Italian sovereign bonds downgraded from A+ to A in 2011, and further downgraded to BBB+ in 2012, BBB in 2013 and BBB- in 2014.

We assign discrete numbers to represent the level of credit rating. The score is between 100 (riskless) and 0 (likely to default) as the Trading Economics credit rating. Here we list the numbers related to Italian sovereign bond credit rating from 2006 to 2016: A+ (80), A (75), BBB+ (65), BBB (60), BBB- (55).

We estimate the empirical specifications in the baseline regressions using credit rating score rather than debt spread. Table 7 shows the results. Column (1) shows the heterogeneous effect of credit rating on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t+1, 0 if firm i stops to hold any intangible fixed assets. Column (4) and (5) are the results for tangible investment. In general, during a sovereign debt crisis (lower credit rating for sovereign debt), small and high-leverage firms decrease their intangible investment more than other firms, consistent with our baseline results.

Alternative measure for leverage. In the baseline regressions, leverage is defined as the ratio of total debt to total assets (we refer to this ratio as "total leverage"),

<sup>&</sup>lt;sup>10</sup>Refer to https://tradingeconomics.com/italy/rating for full correspondence table. It is also easy to infer: AAA corresponds to 100, AA+ corresponds to 95 and each downgrade corresponds to reduction by 5 in the score.

Table 7: Alternative Measure for Debt Crisis: Sovereign Debt Credit Rating

Dependent variable	∆ln(intangil	$bles_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	∆ln(tangibl	$es_{i,t+1}$
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
rating <sub>t</sub>		0.00520***	0.00216***		0.00318***
		(0.000166)	(4.40e-05)		(8.72e-05)
$size_{i,2006} \times rating_t$	-0.00103***	-0.000953***	-0.000803***	-0.000390***	-0.000346***
,	(0.000193)	(0.000189)	(4.96e-05)	(0.000101)	(9.87e-05)
$totallev_{i,2006} \times rating_t$	0.000748***	0.000687***	-1.08e-06	-6.16e-06	-1.04e-05
,	(0.000168)	(0.000167)	(4.32e-05)	(8.07e-05)	(8.01e-05)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.066	0.060	0.025	0.092	0.071
Number of id	59,706	59,706	71,339	59,706	59,706

*Notes*: Column (1) shows the heterogeneous effect of sovereign debt credit rating on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\*\* p<0.05, \* p<0.1.

where total debt includes both short-term loans and long-term debt. Here we use net leverage and short leverage as alternative measures of leverage. Table 8 and Table 9 deliver that results are robust to these alternative leverages.

Table 8: Alternative Measure for Leverage: Net Leverage

Dependent variable	∆ln(intangib	$les_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	∆ln(tangible	$es_{i,t+1}$
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$\overline{sp_t}$		0.00385***	-0.0143***		-0.00467***
		(0.00139)	(0.000387)		(0.000673)
$size_{i,2006} \times sp_t$	0.0138***	0.0144***	0.00460***	0.00670***	0.00676***
,	(0.00161)	(0.00159)	(0.000445)	(0.000796)	(0.000777)
$netlev_{i,2006} \times sp_t$	-0.00414***	-0.00390**	-0.000948**	0.00177**	0.00218***
,	(0.00153)	(0.00152)	(0.000429)	(0.000702)	(0.000690)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,818	303,818	383,075	303,818	303,818
R-squared	0.066	0.056	0.020	0.092	0.063
Number of id	59,620	59,620	71,257	59,620	59,620

*Notes*: Estimation results here use net leverage instead of total leverage. Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Appendix B provides further robustness checks. Appendix B.1 provides results when we take depreciation rates of different assets into consideration. Appendix B.2 adopts another way to take care of the observations with 0 intangible fixed assets, where we give each observation with 0 intangible fixed assets an extra constant to keep it in the sample. Appendix B.3 controls additional region-year fixed effect or

Table 9: Alternative Measure for Leverage: Short-term Leverage

Dependent variable	∆ln(intangi	ibles <sub>it</sub> )	$\mathbb{1}(intangibles_{i,t+1})$	∆ln(tangil	$\overline{ples_{it}}$
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.00372***	-0.0141***		-0.00512***
		(0.00139)	(0.000387)		(0.000681)
$size_{i,2006} \times sp_t$	0.0138***	0.0144***	0.00463***	0.00650***	0.00662***
,	(0.00160)	(0.00158)	(0.000438)	(0.000783)	(0.000764)
$shortlev_{i,2006} \times sp_t$	-0.00328**	-0.00317**	-0.000795**	0.000838	0.000949
	(0.00141)	(0.00141)	(0.000380)	(0.000665)	(0.000660)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	305,711	305,711	385,056	305,711	305,711
R-squared	0.065	0.055	0.020	0.092	0.063
Number of id	59,912	59,912	71,535	59,912	59,912

*Notes*: Estimation results here use short-term leverage instead of total leverage. Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1). Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

province-year fixed effect, which capture the geographical differences of sample firms. Appendix B.4 includes more aggregate controls in the empirical specifications. In the baseline regression, we use continuous standardized size and leverage in 2006 as our main measure for firms' size and financial conditions. The results are robust to alternative group dummies (Appendix B.5 and B.6). For example, if we replace standardized size in 2006 with a dummy, 1 for firms with size larger than median and 0 otherwise, the baseline results are still robust. Appendix B.7 shows the results when we cluster the standard errors at the sector level. Appendix B.8 shows results hold if we alternatively winsorize the variables of interest at top and bottom 0.5% (baseline uses 1%). In the baseline regression, the intangible and tangible fixed assets are deflated by the price of intangible and tangible assets, respectively. Appendix B.9 shows that the results are robust when we deflate intangible and tangible fixed assets by Producer Price Index (PPI).

## 2.4 Slow Post-crisis TFP Recovery

The previous sections document firms intangible investment behaviors during Italian debt crisis. Recent studies show that intangible assets play an important role in productivity growth, both at the aggregate and firm levels. With growth accounting methods, research shows that intangible capital contributes to economic growth using

aggregate level data, for example, see Corrado et al. (2009) for the United States, Marrano et al. (2009) for the United Kingdom, Fukao et al. (2009) for Japan. At the firm-level, the literature finds that not only R&D contributes to firm productivity (Griliches (1979), see Hall et al. (2010) for a literature review), other types of intangible assets, such as information and communication technology (ICT), also affect firm productivity (Van Reenen et al. (2010); Piekkola (2014); Goodridge et al. (2017); Dhyne et al. (2018)). Considering the importance of intangibles to productivity, we then focus on the firm-level TFP evolution process in this section.

During the Italian debt crisis, we can observe a dramatic Total Factor Productivity (TFP) drop in Italian manufacturing sector. The recovery of TFP is slow in the aftercrisis period. We show the evolution of Italian manufacturing TFP in this section.

If we estimate the production function with a simple OLS estimation method <sup>11</sup>, the estimation is subject to simultaneity and selection biases. Firms choose the inputs based on their unobservable productivities, so that the output and variable inputs (such as labor) are determined at the same time. This section presents four widely-used TFP measures <sup>12</sup>, which potentially solve the simultaneity problem and selection biases, to document the pattern of Italian manufacturing TFP.

We first estimate the production function following Olley and Pakes (1996). This estimation strategy uses the capital investment as the proxy for unobservable productivity and the Panel (a) of Figure 2 shows the estimated TFP (abbreviated to OP measure thereafter). The second TFP measure is derived following Levinsohn and Petrin (2003), and this LP measure is shown in Panel (b). The LP measure adopts intermediate inputs, such as materials or energy, as the proxy for unobservable productivity.

Both OP measure and LP measure assume that labor input is a variable input and it can adjust freely to the productivity shock. However, Ackerberg et al. (2015) argue that OP and LP measures suffer from the functional dependence problems. Firm's optimal labor input is a function of productivity history and capital input. The variation in labor input can't identify the elasticity of labor. They propose an

<sup>&</sup>lt;sup>11</sup>We also apply the simple OLS estimation, fixed effect model, first-order difference method and second-order difference method to estimate the production function. These TFP estimation results are in Appendix C.

<sup>&</sup>lt;sup>12</sup>We show the estimation strategies for mentioned TFP measures in the Appendix C.

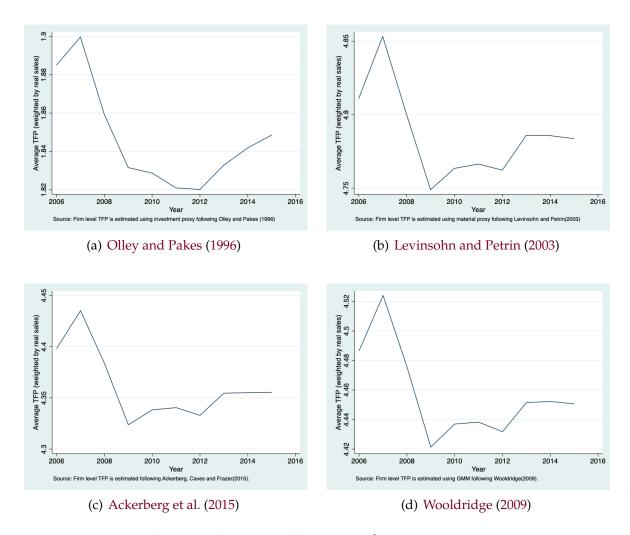


Figure 2: Average TFP in manufacturing sector

*Notes*: All the panels are the average TFP in the manufacturing sector, which is the weighted average TFP of all firms in the sample, weighted by firms' real sales. Panel (a) is estimated following Olley and Pakes (1996). Panel (b) follows Levinsohn and Petrin (2003). The estimation strategy of Panel (c) is from Ackerberg et al. (2015), while that of Panel (d) is from Wooldridge (2009).

alternative two-step estimation method, with details displayed in Appendix C. The estimation result of Garcia-Macia (2017) is in Panel (c) of Figure 2. Wooldridge (2009) comes up with a GMM framework to estimate the production function, incorporating the same moments as those in Olley and Pakes (1996) and Levinsohn and Petrin (2003). The result is shown in Panel (d) of Figure 2.

The average TFP in manufacturing sector peaks in the year of 2007, and then goes down dramatically. After 2012, we can observe mild recovery from all the panels, but the level of average TFP is far from the TFP level in the pre-crisis period.

## 2.5 Summary

The sovereign debt crisis transmits to the private sector and affects firms' investment. Distinguished from existing literature, we focus on both investment on intangible assets and tangible assets. We find that the heterogeneous effects on intangible investment are even more pronounced. Small and high-leverage firms decrease their investment on intangible assets by more than other firms. For high-leverage firms, they also tend to reallocate their resources from intangible investment to tangible investment to offset the tightening financial conditions. We observe a dramatic decline in the TFP, and the recovery of TFP is quite slow after the crisis. To rationalize the heterogeneous firms' investment behaviors and quantify the aggregate impact of the sovereign debt crisis on intangible investment and productivity, we develop a sovereign default model with heterogeneous firms who accumulate both intangible assets and tangible assets.

## 3 Model

The economy is composed of a government, final goods firms, intermediate goods firms, financial intermediaries, and households.

The government issues long-term government bonds to the financial intermediaries. The probability of a government default evolves over time according to a reduced-form stochastic process. Government collects tax revenues from the final goods firms and borrows from the financial intermediaries to finance the lump-sum transfers

to the households and services the outstanding government debt. Government can default on its debt.

There are two types of firms: final goods firms and intermediate goods firms. Final goods firms are competitive, and they convert intermediate goods to final goods. Intermediate goods firms operate under monopolistic competition, and they use tangible capital and intangible capital to produce differentiated goods. They borrow from the financial intermediaries to finance a fraction of cost for investment, and the borrowing interest rate is firm-specific depending on the firm's tangibility. The intermediate goods firms are exogenously heterogeneous in productivity and financing needs.

Households are composed of consumers and bankers, and they own intermediate goods firms. Households decide how much to consume, and they can also decide on how much to deposit their savings to the financial intermediaries. The financial intermediaries are run by the bankers who use repayments on the claims and the savings of the households to lend to the intermediate goods firms and the government.

The economy is perturbed by one aggregate shock  $\epsilon_d$ , which determines the exogenous process of government default risk. The timing of events within a period is as follows. First, the aggregate shock and the idiosyncratic shocks for intermediate goods firms are realized. Given the shocks, the government balances its budget, firms, households and financial intermediaries make their decisions, and goods and credit markets clear.

We start by describing the problems of each type of agent: the government, final goods firms, intermediate goods firms, consumers, and financial intermediaries. We then define the equilibrium for this economy.

## 3.1 The government

The government finances its expenditure  $G_t$  by issuing long-term bonds to the financial intermediaries and levying a tax rate  $\tau$  on aggregate final goods  $Y_t$ . In every period, a fraction  $\vartheta$  of debt matures and the remaining fraction remains outstanding. The government can default on its debt in every period by writing off a fraction  $f \in [0,1]$  of its outstanding obligations.

Following Bocola (2016), we assume an exogenous process of government default risk. Assume that in every period the economy is hit by a shock  $\varepsilon_d$  that follows a standard logistic distribution. The default process follows:

$$d_{t+1} = \begin{cases} 1 & \text{if } \varepsilon_{d,t+1} < s_t \\ 0 & \text{otherwise,} \end{cases}$$
 (4)

where s is an AR(1) process:

$$\log(s_t) = (1 - \rho_s)\log(s^*) + \rho_s\log(s_{t-1}) + \sigma_s\varepsilon_{st}, \quad \varepsilon_{st} \sim N(0, 1). \tag{5}$$

The probability of default is then given by:

$$p_t^d \equiv \text{Prob}(d_{t+1} = 1|s_t) = \frac{\exp(s_t)}{1 + \exp(s_t)}$$
 (6)

The government chooses  $B_{t+1}$  to maximize the government expenditure  $G_t$ , subject to the following budget constraint:

$$q_t[B_{t+1} - (1-\vartheta)(1-d_t f)B_t] + \tau Y_t = \vartheta(1-d_t f)B_t + \frac{\phi_b}{2}[B_{t+1} - (1-\vartheta)(1-d_t f)B_t]^2 + G_t,$$
(7)

where  $q_t$  is government bond price,  $B_t$  is the stock of bond in time t. Denote  $d_t = 1$  if the government defaults and  $d_t = 0$  if the government does not default.  $\phi_b$  is the parameter for bond adjustment cost.<sup>13</sup>

#### 3.2 Final Goods Firms

The final good  $Y_t$  is produced from a fixed variety of intermediate goods  $i \in [0, 1]$  using the technology:

$$Y_t \le \left[ \int (y_{it})^{\eta} di \right]^{\frac{1}{\eta}},\tag{8}$$

where the elasticity of demand is  $\frac{1}{1-\eta} > 1$ . We normalize the price of final goods to one, so the taxes paid to the government is  $\tau Y_t$ . The price of intermediate good i is  $p_{it}$ .

 $<sup>^{13}</sup>$ This is a parsimonious way to pin down government bond level. One could also set a fiscal rule, and then the government bond level is to balance the budget constraint. (see  $^{Bocola}$  (2016), ) for examples.

The final goods producers choose the intermediate goods  $\{y_{it}\}$  to solve:

$$\max_{\{y_{it}\}} (1-\tau)Y_t - \int p_{it}y_{it}di, \tag{9}$$

subject to (8). Thus, the demand function  $y_{it}$  for intermediate good i is solved as:

$$y_{it} = (\frac{1-\tau}{p_{it}})^{\frac{1}{1-\eta}} Y_t. \tag{10}$$

Demand function (10) shows that the demand for good i is negatively correlated with price  $p_{it}$ , and positively correlated with total output  $Y_t$ .

#### 3.3 Intermediate Goods Firms

A unit measure of intermediate goods producers produce differentiated goods using tangible capital and intangible capital. Each firm i produces output  $y_{it}$  with tangible capital  $k_{T,it}$  and intangible capital  $k_{L,it}$ :

$$y_{it} = z_{it} k_{T,it}^{\alpha_T} k_{I,it}^{\alpha_I} \tag{11}$$

where  $z_{it}$  is the idiosyncratic productivity shock, evolving following  $\log(z_{it}) = \rho_z \log(z_{it-1}) + \sigma_z \varepsilon_{it}$ , where  $\varepsilon_{it}$  follows a standard normal random process.  $k_{T,it}$  is the stock of tangible capital, and  $k_{I,it}$  is the stock of intangible capital.  $\alpha_T$  and  $\alpha_I$  are the income shares of tangible capital and intangible capital, respectively.  $\alpha_T + \alpha_I \leq 1$  so that the production function is non-increasing returns to scale technology. Tangible capital depreciates every period at the rate  $\delta_T$ , and there is an adjustment cost for changing the capital stock. Thus, the investment on tangible capital at period t is given by:

$$i_{T,it} = k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it}), \tag{12}$$

where  $\Theta(k_{T,it+1},k_{T,it})=\frac{\theta_T}{2}(\frac{k_{T,it+1}}{k_{T,it}}-1+\delta_T)^2k_{T,it}$  is the convex adjustment cost for tangible capital. Similarly, the investment on intangible capital at period t is given by:

$$i_{I,it} = k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it}), \tag{13}$$

where  $\delta_I$  is the depreciation rate for intangible capital, and  $\Theta(k_{I,t+1},k_{I,t}) = \frac{\theta_I}{2}(\frac{k_{I,t+1}}{k_{I,t}} - 1 + \delta_I)^2 k_{I,t}$  is the adjustment cost for investing in intangible capital.

At the beginning of the period, firm i's idiosyncratic productivity  $z_{it}$  is realized. Then firm i makes choices for next period tangible capital  $k_{T,it+1}$  and intangible capital  $k_{T,it+1}$ . We assume that firms need to borrow a fraction of their investment before production, and the financing needs  $\lambda_i$  are firm-specific and time-invariant. Heterogeneity in  $\lambda_i$  captures the heterogeneous borrowing requirement, and also captures heterogeneous leverage across the firms. The financial intermediary issues loans  $b_{it}$  to firm i at firm-specific interest rate  $R_{it}$ , and the financing requirement for firm i is:

$$b_{it} = \lambda_i (i_{T,it} + i_{L,it}), \tag{14}$$

where  $i_{T,it}$  is the investment on tangible capital, and  $i_{L,it}$  is the investment on intangible capital. We assume the distribution of  $\lambda_i$  are time-invariant, and it is denoted by  $\Lambda$ . The firm-specific interest rate  $R_{it}$  depends on the tangibility of firm i, and its form will be given in Section 3.4 where the financial intermediary's problem is introduced.

At the end of the period, production takes place. Firm i decides on the price  $p_{it}$  for its production  $y_{it}$ , taking the demand function (10) as given, and repay the debt  $R_{it}b_{it}$ . The dividend of firm i at period t is:

$$D_{it} = p_{it}z_{it}k_{T,it}^{\alpha_T}k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})] - [k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it}b_{it}.$$
(15)

Taking the aggregate demand  $Y_t$  and interest rate  $R_{it}$  as given, the problem of intermediate goods firm i is to maximize present value of the dividends  $\sum_{t=0}^{\infty} \beta^t D_{it}$  subject to the demand function (10) and the working capital constraint (14). The solution procedures are specified in Appendix D.2.

#### 3.4 Households

The representative household is composed of consumers and bankers. The household's preferences over consumption  $C_t$  is given by:

$$U = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t C_t\right],\tag{16}$$

where  $\beta \in (0,1)$  is the discount factor and  $C_t$  is consumption of the consumers in period t. Each period, the household send out bankers to run financial intermediaries, and provide them with net worth  $N_t$ . At the end of the period, the bankers bring back their returns of operations  $F_t$  to the household. Household can also save by putting one-period deposit  $M_t$  to the financial intermediary with the price  $q_t^m$ . Every period, the household also receives dividends  $D_t$  from the intermediate goods firms. The budget constraint of the household is:

$$C_t + q_t^m M_t + N_t = M_{t-1} + D_t + F_t. (17)$$

The household maximizes Eq (16) subject to Eq (17), implying the price for deposit  $q_t^m = \beta$ . The solution procedures are specified in Appendix D.3.

**Financial intermediaries.** The net worth  $N_t$  the household provides to the bankers consists of the value of government bonds that did not mature:

$$N_t = (1 - d_t f)(1 - \vartheta)q_t B_t. \tag{18}$$

The financial intermediaries use the net worth  $N_t$  and the deposits of the household, to purchase government's bonds and issue loans to firms. The budget constraint of the intermediary at the beginning of the period is:

$$q_t B_{t+1} + \int b_{it} di \le \underbrace{(1 - d_t f)(1 - \vartheta)q_t B_t}_{N_t} + q_t^m M_t. \tag{19}$$

Aside from the budget constraint, the financial intermediary are also subject to the constraint that the amount of deposits that the financial intermediaries can get from the household is bounded by their collaterals. We assume that government's bonds

can be fully pledged, while the firms' loans can only be pledged partially.  $\theta_{it}$  denotes the fraction that firm i's debt can be pledged. The fraction that the can be pledged depends on the fraction of tangible capital of firm i, as intangible capital typically can not be used as collateral. We assume the firm-specific fraction  $\theta_{it}$  that can be pledged is given by:

$$\theta_{it} = \bar{\theta} \frac{k_{T,it}}{\bar{k}} < 1, \tag{20}$$

where  $\bar{\theta}$  and  $\bar{k}$  are constant,<sup>14</sup> and  $k_{T,it}$  is the stock of tangible capital for firm i at time t.  $\bar{\theta}$  reflects that firms' loans can't be fully pledged, and  $\frac{k_{T,it}}{\bar{k}}$  reflects firm i's own tangibility. Thus, the deposit constraint for the financial intermediaries is:

$$q_t^m M_t \le q_t B_{t+1} + \int \theta_{it} b_{it} di. \tag{21}$$

Combining the budget constraint (19) and the deposit constraint (21) gives that the amount of debt that can be lend to the firms are bounded by the repayment to the financial intermediary and the fractions that can be pledged:

$$\int (1 - \theta_{it}) b_{it} di \le N_t. \tag{22}$$

At the end of the period, the financial intermediaries receives the payment from the firms and the government, and also pays back household's deposit. The returns for the financial intermediary is given by:

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t.$$
 (23)

The financial intermediary chooses  $\{M_t, B_{t+1}, b_{it}\}$  to maximize  $\mathbb{E}_t[\beta F_{t+1}]$  subject to (19) and (22). The problem of the financial intermediary gives the following pricing conditions for the government's bonds and firm's loans:

$$q_t = \mathbb{E}_t \beta [(1 - d_{t+1} f)(\vartheta + q_{t+1} (1 - \vartheta))],$$
 (24)

<sup>&</sup>lt;sup>14</sup>In the quantitative part, we set  $\bar{k}$  to the maximum capital level that firms can choose in order to guarantee  $\theta_{it} < 1$ .

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}. (25)$$

The price of long-term government bonds (24) compensates for default risk of the government. The price of firm i's loans is (25), where  $\zeta_t$  is the Lagrange multiplier on the constraint (22). The pricing function implies that firm i will pay a premium  $\frac{(1-\theta_{it})\zeta_t}{\beta}$  over the risk-free rate when the combined constraint of the financial intermediaries binds. When firms determine the investment on tangible capital and intangible capital, they consider its impact on the price of debt. The solution procedures are specified in Appendix D.4.

## 3.5 Equilibrium

Define  $S = [s, B, d, \Lambda]$  as the state variables, where s is the government default risk process, B denotes the initial level of government debt, d(s) is the default event determined by the exogenous default risk process, and  $\Lambda(z, \lambda, k_T, k_I)$  is the distribution of the intermediate firms.

The equilibrium is defined as follows. Given an aggregate state *S*, the equilibrium consists of:

- intermediate goods firms' policies for tangible capital  $k'_T(z, \lambda, k_T, k_I; S)$ , intangible capital  $k'_I(z, \lambda, k_T, k_I; S)$  and borrowing  $b(z, \lambda, k_T, k_I; S)$ , and final goods firms' output Y(S);
- policies for aggregate tangible capital  $K_T(S)$ , aggregate intangible capital  $K_I(S)$ , deposits M(S), and consumption C(S);
- price functions for firm borrowing rates  $R(k_T, S)$ , government bond price function q(s), and constant deposit price  $q^m$ ;
- the distribution of firms over idiosyncratic productivity and capitals  $\Lambda(z, \lambda, k_T, k_I)$  such that:
  - policy functions of intermediate and final goods firms satisfy their optimization problem;
  - intermediate firms borrowing rates satisfy Eq (25), and and the leverage constraint (22) holds;

- the distribution of firms is consistent with idiosyncratic shocks;
- policies for households satisfy their optimal conditions;
- government's bond for next period B'(s, B) satisfies the government budget constraint;
- government's bond price satisfy Eq (24);
- capital, firm loan, households deposit and goods markets clear.

Next, we analyze key conditions that show how changes in financial intermediaries' net worth affect firms' borrowing rates, and how they affect firms' capital choices.

Recall that the loan interest rate for firm *i* is given by

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}$$
, where  $\theta_{it} = \bar{\theta} \frac{k_{T,it}}{\bar{k}} < 1$ . (26)

When the leverage constraint (22) binds, the Lagrange multiplier  $\zeta_t > 0$ . A decline in financial intermediaries' net worth  $N_t$  reduces credit supply and it further binds the leverage constraint (22), leading to an increase in  $\zeta_t$  and the loan interest rate  $R_{it}$ . The firm loan interest rate  $R_{it}$  also depends on its tangibility of assets  $k_{T,it}/\bar{k}$  through  $\theta_{it}$ . Thus, the firms internalize the impact of tangible capital on loan interest rate when making choices for tangible investment and intangible investment. When leverage constraint (22) binds,  $\partial R_{it}/\partial k_{T,it} < 0$ , while  $\partial R_{it}/\partial k_{I,it} = 0$ . An increase in tangible capital can help to decrease firm loan interest rate, but an increase in intangible capital can not.

Eq (27) shows the first order condition for next period tangible capital choice  $k_{T,it+1}$  of firm i.<sup>15</sup> The left-hand-side is the marginal cost of increasing tangible capital, and the right-hand-side is the marginal benefit of tangible capital. Different from intangible capital, investing in tangible capital has an extra benefit of decreasing

<sup>&</sup>lt;sup>15</sup>To simplify notation, Eq (27) shows for the case when the adjustment costs equal zero.

borrowing interest rate, as shown in the bracket.

$$[1 + (R_{it} - 1)\lambda_{i}] = \beta [\eta(1 - \tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta-1}(\alpha_{T}z_{it+1}k_{T,it+1}^{\alpha_{T}-1}k_{I,it+1}^{\alpha_{I}}) + (1 - \delta_{T})[1 + (R_{it+1} - 1)\lambda_{i}]$$

$$- \left[k_{T,it+2} - (1 - \delta_{T})k_{T,it+1} + k_{I,it+2} - (1 - \delta_{I})k_{I,it+1}\right]\lambda_{i}\frac{\partial R_{it+1}}{\partial k_{T,it+1}}]$$
extra benefit of tangible capital investment (27)

During sovereign debt crises, the net worth of the financial intermediaries shrinks via Eq (18), thus tightens leverage constraint (22). When  $\zeta_t$  increases, the impact of tangible capital on borrowing interest rate is larger, i.e., higher absolute value of  $\partial R_i/\partial k_{T,i}$ . For high- $\lambda$  firm, the movement in the marginal benefit of investing in tangibles is larger. This explains our empirical finding that high-leverage firms tend to reallocate their resources from intangible capital to tangible capital during Italian sovereign debt crisis. The movements in the loan interest rate affect firms' capital choices and production decisions, which then affect the aggregate output.

Note that in the production function  $y_{it} = z_{it}k_{I,it}^{\alpha_I}k_{T,it}^{\alpha_T}$ ,  $z_{it}$  is exogenous productivity shock, and  $z_{it}k_{I,it}^{\alpha_I}$  is the effective productivity level, which determines the marginal return of tangible capital. Thus, in this model, a sovereign debt crisis affects the firms' effective productivity by affecting the choices of firms' intangible investment.

## 4 Quantitative Analysis

We now use our model to measure the impacts of sovereign default risk on the Italian economy, especially on intangible investment and the effective productivity. Section 4.1 starts by parametrizing the model and assessing its fit. Section 4.2 reports the results of our quantitative experiment, in which we use the calibrated economy to measure the effective productivity and output losses from the sovereign risk during the Italian debt crisis.

#### 4.1 Parameterization

The model is in annual frequency. There are two groups of parameters. The parameters in the first group are fixed exogeneously and are taken directly from the literature or from our estimation, and those in the second group are jointly chosen to match a set of firm and aggregate moments for the Italian economy. Table 10 lists all the parameter values.

**Fixed parameters.** Fixed parameters  $\{\alpha_T, \alpha_I, \delta_T, \delta_I, \eta, \tau, \beta, \rho_s, \sigma_s, s^*, \vartheta, f, \rho_z\}$  include technological parameters, the tax rate, preference parameters, sovereign risk process parameters, the fraction of debt maturing each year, and haircut fraction if the government defaults. The parameters  $\{\alpha_T, \alpha_I, \eta\}$  determine the shape of the production function of intermediate and final goods firms. We set  $\alpha_T$  to 0.161 and  $\alpha_I$  to 0.318 following Chen et al. (2018) and  $\eta$  to 0.75, which is the conventional values in the literature. The depreciation rate for intangible assets  $\delta_I$  is 24.3% and the depreciation rate for tangible assets  $\delta_T$  is 10.1%, according to our estimation for Italian firms in 2006 using the EUKLEMS database. The tax rate  $\tau$  is 0.22, which is the effective corporate tax rate in Italy. The persistence and standard deviation of firm productivity shock are set to be 0.9 and 0.03, following Ottonello and Winberry (2020). The discount factor  $\beta$  is set to match an annual risk-free rate of 2%. The fraction of bonds maturing is set to be 0.05. The parameters governing the persistence of sovereign risk process  $\rho_S$  are taken from Bocola (2016), as well as the haircut fraction f when the government defaults.

**Fitted parameters.** The remaining parameters in the model include parameters for investment adjustment costs  $\{\theta_T, \theta_I\}$ , parameters for working capital requirement  $\{\lambda_l, \lambda_h\}$ , a parameter  $\bar{\theta}$  capturing firm pledgeability, a parameter  $\phi_B$  measuring government adjustment cost, parameters for the sovereign risk process  $\{\sigma_s, s^*\}$ .

We target 8 sample moments that include firms, government, and banks statistics. Firms' statistics include the volatility of tangible investment relative to the volatility of real sales, the counterpart for intangible investment, the average asset tangibility (ratio of tangible asset over total assets), and the average leverage for firms within

<sup>&</sup>lt;sup>16</sup>More details on the estimation of depreciation rates, please refer to Appendix B.1.

low-leverage group and high-leverage group in 2006. The government's statistics include the ratio of government bonds to tax revenues in 2006, the average spread and the volatility of spread.

We solve the model using global methods; see Appendix E for details. Given the model policy functions, we perform simulations to obtain the model-implied counterparts of our targets. We jointly chosen the fitted parameters to match 8 sample moments. The parameters are chosen to minimize the sum of the distance between the moments in the model and their corresponding counterparts in the data.

Although we chose all parameters jointly to match the moments, we can provide a heuristic description of how the moments inform specific parameters. First, the capital adjustment costs mostly affect the investment volatility. Second, the leverage for two groups of firms pins down  $\{\lambda_l, \lambda_h\}$ ; two groups refer to firms with leverage lower than the median and firms with leverage higher than the median in 2006 in the data. Third, for the government, the ratio of government bonds to tax revenue provides information for government adjustment cost parameter  $\phi_B$ . Fourth, the firm tangibility —ratio of tangible asset over total assets —provides information for firm pledgeability parameter  $\bar{\theta}$ . Fifth, the mean and volatility of spread are informative in terms of sovereign risk prosess parameters  $\{\sigma_s, s^*\}$ .

## 4.2 Productivity Loss from Italian Debt Crisis

We now use the calibrated model to measure the firms' productivity losses from Italian sovereign debt crisis during 2008-2015. We first feed the model a sequence of  $s_t$  shocks and  $z_t$  shocks such that the model replicates the observed path of Italian sovereign risk and average firm TFP. We then use the model to construct the path of TFP in a counterfactual scenario in which the Italian economy does not experience a sovereign debt crisis. The difference between the path of TFP in the benchmark model and in the no debt crisis counterfactual model isolates the TFP effect of the sovereign crisis for the Italian economy.

Figure 3 reports the time paths for the sovereign spreads and TFP during the event. Panel (a) plots for sovereign spreads where the unit is percentage point. Panel (b) plots for the percentage change of TFP from the 2008 level. The solid red line is for

Table 10: Parameters

Parameter	Description	Value	Target/Source
Fixed param	eters -		
$\alpha_T$	Income share of tangible capital	0.25	Lopez, Olivella (2018)
$\alpha_I$	Income share of intangible capital	0.15	Lopez, Olivella (2018)
η	Markup	0.75	Conventional value
$\delta_T$	Depreciation of tangible capital	0.101	Our estimation
$\delta_I$	Depreciation of intangible capital	0.243	Our estimation
τ	Tax rate	0.22	Effective corporate tax rate
$ ho_z$	Persistence of firm productivity shock	0.90	Ottonello, Winberry (2020)
$\sigma_z$	Volatility of firm productivity shock	0.03	Ottonello, Winberry (2020)
β	Discount factor	0.98	Annual risk-free rate of 2%
v	Fraction of bonds maturing	0.05	Conventional value
$ ho_s$	Sovereign risk process	0.95	Bocola (2016)
f	Haircut fraction	0.55	Bocola (2016)
Fitted param	ieters		
$ heta_T$	Adjustment cost of tangible investment	0.1	Volatility of tangible investment
$ heta_I$	Adjustment cost of intangible investment	1	Volatility of intangible investment
$[\lambda_l, \lambda_h]$	Working capital requirements	[0.1, 1.9]	Leverage of firms
$egin{array}{c} \phi_B \ ar{ heta} \end{array}$	Bond adjustment cost	50	Government bonds/tax revenue
$ar{ heta}$	Firm pledgeability	0.8	Firm tangibility
$\sigma_{\!\scriptscriptstyle S}$	Sovereign risk process	0.56	Volatility of spread
$s^*$	Sovereign risk process	-6	Average spread

Table 11: Moments in the data and model

	Data	Model
std(tangible investment)/std(sales)	0.23	0.31
std(intangible investment)/std(sales)	0.03	0.06
mean(leverage) for low-leverage firms	0.02	0.02
mean(leverage) for high-leverage firms	0.34	0.40
government bonds/tax revenue	2.60	2.93
mean(firm tangibility)	0.87	0.58
mean(spread)	0.01	0.01
std(spread)	0.01	0.01

*Notes*: See Appendix A.4 for the construction of moments in the data.

the *Benchmark* model and the dotted black line plots for the data. In general, the model needs a negative *z* shock and a positive *s* shock to reproduce the dynamics of sovereign spreads and TFP observed in the data. By construction, the Benchmark model fits the data series for spreads and TFP. From 2008 to 2012, the sovereign spreads increase from 0.7% to 4%, and the firm average TFP decreases by 1%.

The dashed blue line is for the *No debt crisis* model, where we force  $s_t$  shocks to generate that sovereign spreads remain at the value in 2008 during the event. By construction, the sovereign spreads are constant in this counterfactual model. Under this scenario, the financial intermediaries do not suffer losses in their sovereign bond holdings, and the credit supply to the firms does not decline as much as that in the Benchmark model. Because of the more favorable firm loan interest rate, the TFP decline in the counterfactual model is not as much as that in the Benchmark model. More importantly, without debt crisis, the TFP recovers at a faster speed and almost goes back to the pre-2008 level in 2015. With debt crisis, however, the TFP is still 0.32% below the trend in 2015. This event analysis suggests that the productivity losses associated with sovereign default risk are sizable. TFP would have declined less in 2012 without the sovereign debt crisis. More strikingly, TFP would have recovered in 2015 without debt crisis. This shows that the sovereign debt crisis explains the slow TFP recovery in Italy.

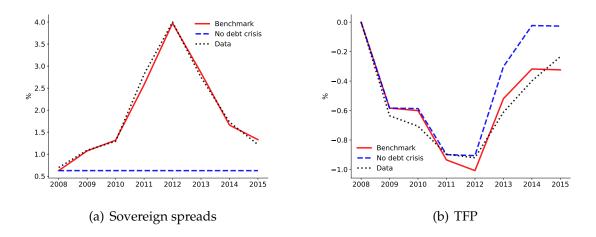


Figure 3: Measuring the productivity cost of sovereign default risk

## 5 Conclusion

Sovereign debt crises have adverse effects on banks' balance sheets, especially banks with a higher exposure to government debt. Since firms rely on external financing for capital investment, firms cut their intangible asset investment, especially those small and high-leverage firms. Especially, high-leverage firms shift their resources towards pledgeable tangible investment to loose their tight borrowing constraints. The depressed intangible investment further reduces the firm total factor productivity and lead to a slow recovery in total factor productivity even after the debt crisis.

To quantitatively measure the impacts of sovereign risk on firm investment and TFP, we developed a framework that combines sovereign debt crisis, financial intermediaries, and heterogeneous firms who invest in both tangible capital and intangible capital. The sovereign risk transmits to the firms through the financial intermediaries. The firms with higher borrowing needs suffer more from the increased sovereign risk. With higher financing costs, the firms decrease their intangible asset investment, which hurts their productivity in the following years.

Our result provides a micro-foundation for market imperfections that endogenously generate volatile and persistent shocks to TFP. It generates a fruitful future research avenue. First, it would be interesting to explore firms' behaviors in more details, such as how firms' research expenses respond during the sovereign debt crises, and which components of intangible assets play a dominant role in explaining the TFP losses. We can not observe firms' capital stock for each component of intangible assets, or tangible assets due to our data limitation. Second, with bank-firm relationships data, one can extend our framework to analyze how firm-specific exposure to sovereign debt crises affects firms' behaviors.

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### A Data

This section provides additional details related to Section 2.1, including the definition of the variables, sample selection procedures and sample distribution by size.

#### A.1 Variables

#### 1.Investment

Our baseline measure of investment in period t is defined as the log difference of intangible fixed assets between period t+1 and period  $t^{17}$ . That is to say, investment in Eq (1) denotes the investment on intangibles of firm i at the end of period t. Similarly, the tangible investment is defined as the log-difference of tangible fixed assets.

The intangible/tangible fixed assets are scaled by the price of intangibles/tangibles every year. We use EUKLEMS database <sup>18</sup> to construct the price indices for intangibles/tangibles. Figure 4 shows the asset components for intangibles/tangibles. This database reports the price for each asset type. We construct the aggregate price for intangibles as the weight average price of each component in the right square, weighted by the share of each asset.<sup>19</sup> Similar construction goes for the price of tangibles.

#### 2.Net leverage

Net leverage is measured as the ratio of firm i's net debt to total assets, where net debt is the sum of short term loans and long term debt net of net current assets.

#### 3. Short leverage

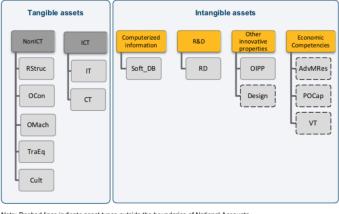
Short leverage is defined as the ratio of firm *i*'s short term loans to total assets.

### 4. Total leverage

<sup>&</sup>lt;sup>17</sup>Log difference measures the relative change that is symmetric, additive and normed. But it will not take care of the 0 observations for intangible fixed assets. As explained in the main text, we use DHS growth rate instead of log difference to deal with this concern.

<sup>&</sup>lt;sup>18</sup>Documentation for EUKLEMS database, 2019 release.

<sup>&</sup>lt;sup>19</sup>For example, Software and Database consists of 15% of intangible assets; R&D consists of 40%; Design consists of 20%, while AdvMRes constitutes 16%.



Note: Dashed lines indicate asset types outside the boundaries of National Accounts. Source: own elaboration based on Haskel and Westlake (2018).

Figure 4: Aggregates of capital services including tangible and intangible assets

Total leverage is defined as the ratio of firm i's total debt to total assets, where total debt is the sum of short term loans and long term debt.

#### 5.Size

Size is measured as the log of total assets.

#### 6.Liquidity

Liquidity is measured as the ratio of cash & cash equivalent to total assets.

#### 7.Sales growth

Sales growth is defined as the log difference of sales, i.e. sales growth<sub>it</sub> =  $log(sales_{it}) - log(sales_{it-1})$ .

#### 8. Liability ratio

Liability ratio is defined as the ratio of total liability to total assets, where total liability is the difference between total assets and share holders funds.

#### 9. Net current assets ratio

Net current assets ratio is measured as the ratio of net current assets to total assets.

#### 10.Intangible fixed assets share

Intangible fixed assets share is measured as the ratio of intangible fixed assets to the

sum of intangible and tangible fixed assets. This measure captures the relative size of firm i's intangibles.

## **A.2** Sample Selection

Our main sample excludes (in order of operation):

- 1. Firms not in manufacturing sector.
- 2. Firms with negative or zero total assets.
- 3. Firms with negative intangible/tangible fixed assets.
- 4. Firms have missing values for total assets and intangible / tangible fixed assets over the sample period.
- 5. Firms without obs in year 2006 in the sample.

After applying the sample selection operations, we winsorize the variables mentioned above at top and bottom 1% of the distribution.

## A.3 Aggregate distribution by size

Table 12: Distribution of wage bill and employment by size

Wage Bill							
	Main Sample	Gopinath et al.(2019)					
1-19 employees	0.16	0.11					
20-249 employees	0.62	0.53					
250+ employees	0.23	0.36					
	Employmen	t					
	Main Sample	Gopinath et al.(2019)					
1-19 employees	0.20	0.13					
20-249 employees	0.73	0.55					
250+ employees	0.07	0.32					

#### A.4 Moments in data

**Standard deviation ratio** For tangible/intangible investment  $i_{T,it}$  and  $i_{I,it}$ , we construct the data counterparts as:

$$i_{T,it}^d = k_{T,it+1}^d - (1 - \delta_T) k_{T,it}^d,$$
  
 $i_{Lit}^d = k_{Lit+1}^d - (1 - \delta_I) k_{Lit}^d,$ 

where  $k_{T,it}^d$  is the real tangible fixed assets and  $k_{I,it}^d$  denotes the real intangible fixed assets in the data. We use Hodrick–Prescott filter to derive the cyclical components of tangible/intangible investment, and real sales. The standard deviation of tangible investment is calculated as the average standard deviation of cyclical component across firms. Similar calculation goes for the standard deviation of intangible investment and real sales. The ratio of standard deviation of tangible investment to that of real sales is 0.231, while the counterpart for intangible investment is 0.030.

**Leverage** Firms in our sample is divided into high-/low-leverage groups by each firm's total leverage in base year 2006. The mean leverage of high-leverage group is 0.338, which that of low-leverage group is 0.020.

**Government bonds/tax revenue** The ratio of government bonds to tax revenue is calculated by the ratio of general government debt to general government revenue, which is 2.595. Data is from OECD.Stat database.

**Firm tangibility** We construct the tangibility measure for firm *i* at period t as:

$$tangibility_{it} = \frac{k_{T,it}^d}{k_{T,it}^d + k_{I,it}^d}.$$

Since the Italian data features unbalanced sample, we first take the average tangibility across sample periods to derive the average tangibility of firm i. The aggregate tangibility is then defined as mean of tangibility measures across firms.

### **B** Robustness

This section shows some robustness checks of our main empirical findings in Section 2.2.

### **B.1** Depreciation

One difference between intangible assets and tangible assets is that they depreciate at a different speed. Relatively little is known about depreciation rates for intangibles. Corrado et al. (2009) estimates the depreciation rate of R&D capital in the U.S. to be 20%. U.S. Bureau of Economic Analysis (BEA) places its central estimate of the depreciation rate for R&D at 15%. Pakes et al. (1978) get an average depreciation rate of 25%, using data for several European countries.

Fortunately, the EUKLEMS database provides depreciation rates for each asset type, which allows us to construct depreciation rates for intangible asset and tangible asset in Italy - at the aggregate level. The depreciation rate for intangible asset is the weighted average of the depreciation rates of: computer software and databases, research and development, and other intellectual property patent (IPP) assets, with the weight being the asset share. For the tangible asset, the depreciation rate is the weighted average of the depreciation rates of: computing equipment, communications equipment, transport equipment, other machinery and equipment, total non-residential investment, residential structures and cultivated assets, with the weight being the asset share. The calculated depreciation rate for intangible asset is 24.3% and depreciation rate for tangible asset is 10.1% in 2006. Our estimates are in line with the rates reported in the exiting literature.

Alternatively, we can construct the intangible investment <sup>20</sup>:

$$\Delta ln(intangibles_{dep,it}) = [ln(y_{i,t+1}) - ln((1 - dep_{intangible}) * y_{it}] * (1 - dep_{intangible}) * (1 - dep_{intangible$$

where  $y_{it}$  denotes intangible fixed assets of firm i at period t.  $dep_{intangible} = 0.243$  is the weighted average depreciation rate for intangible investment. Corresponding

<sup>&</sup>lt;sup>20</sup>intangible investment<sub>it</sub>/ $y_{it} = [y_{i,t+1} - (1 - dep_{intangible}) * y_{it}]/y_{it} \approx [ln(y_{i,t+1}) - ln((1 - dep_{intangible}) * y_{it}] * (1 - dep_{intangible})$ 

tangible investment can be constructed similarly, with depreciation rate equals 0.101. Table 13 delivers the estimation results for the investment with depreciation.

Table 13: Results for investment with depreciation

Dependent variable	$\Delta ln(intangibles_{dep,it+1})$		$\mathbb{I}(intangibles_{dep,it+1})$	$\Delta ln(tangibles_{dep,it+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.00315***	-0.0142***		-0.00432***
		(0.00106)	(0.000388)		(0.000612)
$size_{i,2006} \times sp_t$	0.0111***	0.0116***	0.00472***	0.00580***	0.00588***
	(0.00123)	(0.00122)	(0.000446)	(0.000717)	(0.000700)
$totallev_{i,2006} \times sp_t$	-0.00369***	-0.00366***	-0.000893**	0.00114*	0.00130**
,	(0.00108)	(0.00108)	(0.000388)	(0.000582)	(0.000577)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.066	0.056	0.020	0.092	0.063
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spread on intangible investment by estimating Eq (1) using  $\Delta ln(intangibles_{dep,it})$  as the dependent variable. Column (2) is the corresponding estimation of Eq (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i empties its intangible fixed assets. Column (4) and (5) are the tangible investment counterpart to Column (1) and (2). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Table 14: Intangible investment with alternative depreciation rate

Dependent variable	$\Delta ln(intangibles_{CHS2009_dep,it+1})$		$\Delta ln(intangibles_{Bl})$	$\Delta ln(intangibles_{BM2006_dep,it+1})$		$\Delta ln(intangibles_{PS1978_dep,it+1})$	
	(1) heterogeneity	(2) average	(3) heterogeneity	(4) average	(5) heterogeneity	(6) average	
$sp_t$		0.00333***		0.00341***		0.00312***	
		(0.00112)		(0.00115)		(0.00105)	
$size_{i,2006} \times sp_t$	0.0117***	0.0122***	0.0120***	0.0125***	0.0110***	0.0114***	
	(0.00130)	(0.00129)	(0.00134)	(0.00132)	(0.00122)	(0.00121)	
$totallev_{i,2006} \times sp_t$	-0.00390***	-0.00386***	-0.00399***	-0.00396***	-0.00365***	-0.00362***	
,	(0.00114)	(0.00114)	(0.00117)	(0.00117)	(0.00107)	(0.00107)	
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	
Observations	303,935	303,935	303,935	303,935	303,935	303,935	
R-squared	0.066	0.056	0.066	0.056	0.066	0.056	
Number of id	59,706	59,706	59,706	59,706	59,706	59,706	

Notes: Column (1) and (2) are results using the intangible depreciation rate 0.2 (Corrado et al. (2009)). Column (3) and (4) are results using the intangible depreciation rate 0.18 (Bernstein and Mamuneas (2006)). Column (5) and (6) are results using the depreciation rate 0.25 (Pakes et al. (1978)). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Table 14 displays the results using  $\Delta ln(intangibles_{dep,it+1})$  with alternative depreciation rates as dependent variables. The baseline results are robust to all depreciation rates.

### B.2 Keep observations with 0 intangible fixed assets

We estimate the baseline Eq (1) and Eq (2) using a dummy variable  $1(intangible_{it})$  as dependent variable, where we take the extensive margin into consideration. This section applies an alternative method to take care of the observations with 0 intangible fixed assets. Now the intangible investment is defined as:

$$\Delta ln(intangibles_{evs,it}) = ln(1 + y_{i,t+1}) - ln(1 + y_{it})$$

where  $y_{it}$  denotes intangible fixed assets of firm i at period t.

Table 15: Keep observations with 0 intangible fixed assets

Dependent variable	∆ln(intangible	$es_{eps,it+1}$	∆ln(tangib	$les_{eps,it+1})$
	(1) heterogeneity	(2) average	(3) heterogeneity	(4) average
$sp_t$		0.0187***		-0.00938***
		(0.00148)		(0.000713)
$size_{i,2006} \times sp_t$	0.00250	0.00319*	0.00839***	0.00847***
,	(0.00166)	(0.00163)	(0.000894)	(0.000873)
$totallev_{i,2006} \times sp_t$	-0.00468***	-0.00474***	0.000926	0.000968
,	(0.00154)	(0.00154)	(0.000701)	(0.000696)
Firm controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Observations	499,661	499,661	499,661	499,661
R-squared	0.049	0.047	0.071	0.051
Number of id	84,985	84,985	84,985	84,985

*Notes*: Column (1) and (2) are results for intangible investment when we keep observations with 0 intangible fixed assets. Column (3) and (4) are results for tangible investment. Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

### **B.3** Region-year Fixed Effects

We further add region-year fixed effect and province-year fixed effect into our baseline regressions, controlling for the geographical differences. For example, firms near the border may highly expose to foreign trade, which is less affected by Italian sovereign debt crisis. Table 16 and Table 17 show that our baseline results are robust to including region-year fixed effect or province-year fixed effect.

Table 16: Region-year FE

Dependent variable	∆ln(intangib	$les_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
_	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.0564	0.0210		0.00466
		(0.0398)	(0.0131)		(0.0178)
$size_{i,2006} \times sp_t$	0.0147***	0.0151***	0.00451***	0.00641***	0.00670***
	(0.00165)	(0.00163)	(0.000444)	(0.000807)	(0.000792)
$totallev_{i,2006} \times sp_t$	-0.00482***	-0.00488***	-0.00109***	0.00111*	0.00115*
	(0.00145)	(0.00144)	(0.000386)	(0.000656)	(0.000653)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Region-year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.067	0.066	0.034	0.093	0.092
Number of id	58,918	58,918	70,408	58,918	58,918

Table 17: Province-year FE

Dependent variable	∆ln(intangib	$les_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.0542	-0.0226		-0.00294
		(0.0746)	(0.0208)		(0.0443)
$size_{i,2006} \times sp_t$	0.0146***	0.0150***	0.00452***	0.00647***	0.00678***
,	(0.00165)	(0.00163)	(0.000445)	(0.000810)	(0.000796)
$totallev_{i,2006} \times sp_t$	-0.00473***	-0.00479***	-0.00104***	0.00119*	0.00123*
,	(0.00145)	(0.00145)	(0.000388)	(0.000658)	(0.000655)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Province-year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.069	0.068	0.036	0.095	0.094
Number of id	58,918	58,918	70,408	58,918	58,918

## **B.4** More Aggregate Controls

The baseline results for intangible investment hold when we add more aggregate controls, including GDP, world GDP growth and trade openness.

Table 18: Add More Aggregates

Dependent variable	$\Delta ln(intangibles_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$
$sp_t$	0.0315***	-0.00924***	0.0148***
	(0.00204)	(0.000550)	(0.000996)
$size_{i,2006} \times sp_t$	0.0149***	0.00462***	0.00640***
	(0.00161)	(0.000440)	(0.000783)
$totallev_{i,2006} \times sp_t$	-0.00487***	-0.000935**	0.00138**
, ,	(0.00142)	(0.000382)	(0.000645)
Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Observations	303,935	383,121	303,935
R-squared	0.063	0.032	0.074
Number of id	59,706	71,339	59,706

*Notes*: Column (1) shows the estimation of Eq (2) with additional aggregate controls. The dependent variable in Column (2) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i empties its intangible fixed assets. Column (3) is the tangible investment counterpart to Column (1). Robust standard errors are in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

### **B.5** Group Dummy

Instead of using continuous standardized size and total leverage in 2006, we construct dummies for size and total leverage. Eq (1) and (2) are modified as:

$$\Delta ln(assets_{i,t+1}) = \alpha(dx_i \times sp_t) + Controls + \delta_i + \eta_{st} + \epsilon_{it}, \tag{28}$$

$$\Delta ln(assets_{i,t+1}) = \alpha_0 sp_t + \alpha_1(dx_i \times sp_t) + Controls + \delta_i + \epsilon_{it}, \tag{29}$$

where  $dx_i \in \{dsize_{i,2006}, dtotallev_{i,2006}\}$  are the dummies for firm's size or total leverage in the year 2006.  $dsize_{i,2006}$  is 1 if size of firm i is larger than the median in 2006, 0 otherwise.  $dtotallev_{i,2006}$  is 1 if total leverage of firm i is higher than the median in 2006, 0 otherwise. The baseline results hold for alternative indicators for firms' size and total leverage.

Table 19: Group dummy

Dependent variable	∆ln(intangib	$les_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{it+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.00239	-0.0169***		-0.0100***
		(0.00246)	(0.000735)		(0.00130)
$dsize_{i,2006} \times sp_t$	0.0193***	0.0205***	0.00640***	0.00732***	0.00768***
	(0.00284)	(0.00282)	(0.000768)	(0.00137)	(0.00135)
$dtotallev_{i,2006} \times sp_t$	-0.00601**	-0.00586**	-0.000677	0.00320**	0.00347***
	(0.00283)	(0.00282)	(0.000738)	(0.00130)	(0.00129)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	305,111	305,111	384,567	305,111	305,111
R-squared	0.066	0.055	0.020	0.092	0.063
Number of id	60,048	60,048	71,746	60,048	60,048

## **B.6** Group Dummy for Sector Median

There are concerns for Appendix B.5 because there may be firms in some specific sector gathering in one indicator group. Therefore, we use sector median as the standard to construct dummy indicators for size and total leverage. For Eq (28) and (29),  $dsize_{i,2006}$  is 1 if size of firm i is larger than the sector median in 2006, 0 otherwise.  $dtotallev_{i,2006}$  is 1 if total leverage of firm i is higher than the sector median in 2006, 0 otherwise. The baseline results are also robust.

Table 20: Group with Sector Median

Dependent variable	$\Delta ln(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		-0.00258	-0.0168***		-0.00973***
		(0.00245)	(0.000732)		(0.00128)
$dsize_{i,2006} \times sp_t$	0.0205***	0.0203***	0.00617***	0.00774***	0.00750***
	(0.00282)	(0.00282)	(0.000767)	(0.00135)	(0.00134)
$dtotallev_{i,2006} \times sp_t$	-0.00526*	-0.00514*	-0.000532	0.00294**	0.00313**
,	(0.00282)	(0.00282)	(0.000739)	(0.00130)	(0.00129)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	304,956	304,956	384,405	304,956	304,956
R-squared	0.066	0.055	0.020	0.092	0.063
Number of id	60,022	60,022	71,721	60,022	60,022

### **B.7** Cluster at Sector-level

The baseline results are robust to clustering at sector level.

Table 21: Cluster at sector-level

Dependent variable	$\Delta ln(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.00416***	-0.0142***		-0.00480***
		(0.00147)	(0.000617)		(0.00102)
$size_{i,2006} \times sp_t$	0.0147***	0.0153***	0.00472***	0.00645***	0.00653***
,	(0.00177)	(0.00161)	(0.000440)	(0.00101)	(0.00103)
$totallev_{i,2006} \times sp_t$	-0.00487**	-0.00483**	-0.000893***	0.00127*	0.00144**
, ,	(0.00196)	(0.00199)	(0.000297)	(0.000710)	(0.000684)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.066	0.056	0.020	0.092	0.063
Number of id	59,706	59,706	71,339	59,706	59,706

## B.8 Winsorizing at 0.5%

The baseline sample winsorizes the variables of interest at top and bottom 1%. This section shows the baseline estimation results are robust if we winsorize the variables of interest at top and bottom 0.5%.

Table 22: Winsorizing 0.5%

Dependent variable	$\Delta ln(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.00189	-0.0143***		-0.00654***
		(0.00146)	(0.000380)		(0.000718)
$size_{i,2006} \times sp_t$	0.0139***	0.0148***	0.00444***	0.00693***	0.00705***
	(0.00164)	(0.00162)	(0.000414)	(0.000809)	(0.000788)
$totallev_{i,2006} \times sp_t$	-0.00476***	-0.00471***	-0.00108***	0.000351	0.000460
	(0.00151)	(0.00150)	(0.000381)	(0.000689)	(0.000680)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	341,284	341,284	406,803	341,284	341,284
R-squared	0.069	0.059	0.021	0.091	0.066
Number of id	63,631	63,631	73,770	63,631	63,631

## B.9 Deflate intangible/tangible fixed assets with PPI

The baseline estimation deflates the intangible/tangible fixed assets by the price of intangible/tangible assets. We replace the price of investment with PPI and the results remain robust.

Table 23: PPI deflation

Dependent variable	$\Delta ln(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta ln(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$sp_t$		0.00127	-0.0142***		-0.00496***
		(0.00140)	(0.000388)		(0.000680)
$size_{i,2006} \times sp_t$	0.0144***	0.0150***	0.00473***	0.00648***	0.00657***
	(0.00163)	(0.00161)	(0.000446)	(0.000798)	(0.000779)
$totallev_{i,2006} \times sp_t$	-0.00476***	-0.00471***	-0.000893**	0.00130**	0.00145**
,	(0.00143)	(0.00142)	(0.000388)	(0.000647)	(0.000642)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,941	303,941	383,121	303,941	303,941
R-squared	0.065	0.054	0.020	0.089	0.063
Number of id	59,705	59,705	71,339	59,705	59,705

## C Estimation of Total Factor Productivity (TFP)

We are interest in the productivity evolution during the Italian debt crisis. This section provides several measures wildly used in estimating the production function. We find that average TFP in the manufacturing sector drops dramatically during the Italian debt crisis, and the TFP recovery is quite slow. This result is robust to all the TFP measures mentioned in this section.

We use the same production function to summarize all the following TFP estimation methods.

$$y_{it} = \alpha + \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \eta_{it}, \tag{30}$$

where  $y_{it}$ ,  $l_{it}$  and  $k_{it}$  are the natural logarithm of firm i's value added, labor input and capital.  $\omega_{it}$  denotes the firm-specific productivity.  $\eta_{it}$  refers to either measurement error or unforecastable productivity shock. This section focuses on estimating the productivity  $w_{it}$ .

### C.1 Simple estimation methods

We show the estimation results using OLS estimation method, fixed effect model, first-order difference method and second-order method in this section. As shown in Figure 5, the averageproductivity in manufacturing sector first drops after 2007, and recover only a fraction of the pre-crisis TFP level after 2012. Notice that the recoveries in Panel (b)-(d) are relatively large in magnitude. However the simple measures here suffer from the endogeneity problem due to the correlation between inputs and productivity.

## C.2 Olley and Pakes (1996)

When we apply a simple OLS estimation method to estimate the production function, the estimation results suffer from simultaneity and selection biases. Since firm's input choices depend on unobservable productivity, the output and variable inputs (such as labor) are then determined at the same time <sup>21</sup>. Firm's entry and exit decisions

<sup>&</sup>lt;sup>21</sup>This simultaneity problem is analyzed by Marschak and Andrews (1944).

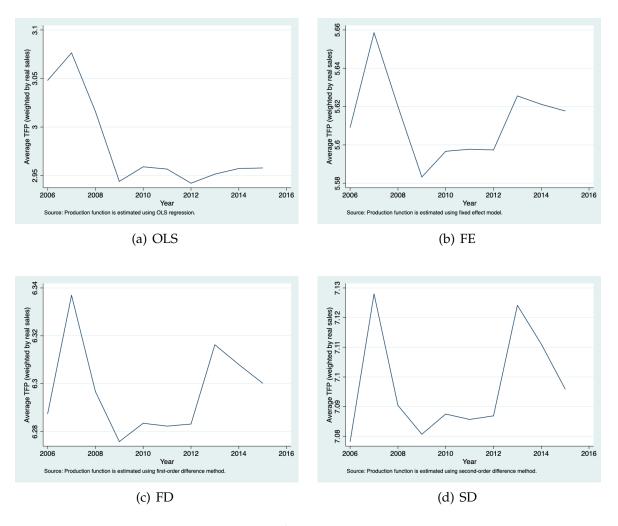


Figure 5: Average TFP in manufacturing sector - alternative measures

*Notes*: All the panels are the average TFP in the manufacturing sector, which is the weighted average TFP of all firms in the sample, weighted by firms' real sales. Panel (a) is estimated with OLS regression of Eq(30). Panel (b) is the estimated TFP adding firm fixed effect. The estimation strategy of Panel (c) is the first-order difference method, while that of Panel (d) is the second-order difference method.

highly rely on the future productivity. Only continuing firms decide on their inputs, while inefficient firms exit the market.

Olley and Pakes (1996) propose a semiparametric method, controlling for the simultaneity and selection biases when estimating the Cobb-Douglas production function. They use capital investment  $i_{it} = f(\omega_{it}, k_{it})$  to proxy the unobservable productivity  $\omega_{it}$ .

$$\omega_{it} = f^{-1}(i_{it}, k_{it}) = h(i_{it}, k_{it}). \tag{31}$$

It can be proved that investment  $i_{it}$  is strictly increasing in  $\omega_{it}$  (for every capital level  $k_{it}$ ). Substituting Eq (31) into Eq (30), we can get

$$y_{it} = \beta_l l_{it} + \Phi_{it}(k_{it}, i_{it}) + \eta_{it},$$
 (32)

where

$$\Phi_{it} = \alpha + \beta_k k_{it} + h(i_{it}, k_{it}). \tag{33}$$

Eq (32) is a partial linear model in terms of the labor input. Olley and Pakes (1996) propose a non-parametric two-step estimation method to estimate Eq (32).

First, using the moment  $\mathbb{E}[\eta_{it}|\Omega_{it}]=0$ , where  $\Omega_{it}$  is the information set at time t, we can non parametrically estimate Eq (32) by approximating  $\Phi_{it}(k_{it},i_{it})$  by a  $n^{th}$  order polynomial  $^{22}$  or a local linear regression. Then the coefficient of labor input  $\hat{\beta}_l$  and  $\hat{\Phi}_{it}$  can be consistently identified in the first step.

Assume that productivity follows a first-order Markov process:

$$\omega_{it+1} = \mathbb{E}(\omega_{it+1}|\Omega_{it}) + \xi_{it+1} = \mathbb{E}(\omega_{it+1}|\omega_{it}) + \xi_{it+1} = g(\omega_{it}) + \xi_{it+1}, \tag{34}$$

where  $\Omega_{it}$  is the information set at time t and  $\xi_{it+1}$  is the innovation in productivity  $\omega_{it+1}$ , which is not related with productivity and inputs  $E[\xi_{it+1}|\Omega_{it}] = 0$ . Then the partial linear model can be rewritten as

$$y_{it+1} - \hat{\beta}_{l}l_{it+1} = \alpha + \beta_{k}k_{it+1} + \omega_{it+1} + \eta_{it+1}$$

$$= \alpha + \beta_{k}k_{it+1} + g(\omega_{it}) + \xi_{it+1} + \eta_{it+1}$$

$$= \alpha + \beta_{k}k_{it+1} + g(\hat{\Phi}_{it} - \alpha - \beta_{k}k_{it}) + \xi_{it+1} + \eta_{it+1}.$$

<sup>&</sup>lt;sup>22</sup>They usually use 3rd order polynomial to approximate  $\Phi_{it}(k_{it}, i_{it})$ .

The second step estimation is base on the moment condition  $E[\xi_{it+1} + \eta_{it+1}|\Omega_{it}] = 0$ , together with the estimated  $\hat{\beta}_l$  and  $\hat{\Phi}_{it}$  obtained in the first step.

Firm i's TFP is then given by

$$ln(TFP_{it}) = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it}$$
(35)

### C.3 Levinsohn and Petrin (2003)

Levinsohn and Petrin (2003) contribute to the literature on estimating production functions by extending Olley and Pakes (1996). Olley and Pakes (1996) use the investment in capital as a proxy for unobservable productivity. However, this investment proxy has some potential concerns. First, there are some zero-investment observations. The investment equation  $i_{it} = f(\omega_{it}, k_{it})$  can't be inverted due to the violation of the monotonicity condition. Second, non-convex adjustment cost may affect how investment responds to the productivity. For example, investment may have an upper bond. Both cases may fail the investment proxy.

Olley and Pakes (1996) treat the labor input as the only freely variable input, while Levinsohn and Petrin (2003) add intermediate input  $\iota_{it}$ , such as materials or energy, as the other freely variable input. Now, the production function is given as:

$$y_{it} = \alpha + \beta_l l_{it} + \beta_k k_{it} + \beta_i l_{it} + \omega_{it} + \eta_{it}. \tag{36}$$

They apply intermediate inputs  $\iota_{it}$  as the proxy for productivity, as shown by:

$$\iota_{it} = \iota(\omega_{it}, k_{it}). \tag{37}$$

Similarly, the partial linear model can be written as:

$$y_{it} = \beta_l l_{it} + \Phi_{it}(k_{it}, i_{it}) + \eta_{it},$$
 (38)

where

$$\Phi_{it} = \alpha + \beta_k k_{it} + \beta_i \iota_{it} + \omega_{it} (\iota_{it}, k_{it}). \tag{39}$$

The second step then requires to estimate:

$$y_{it+1} - \hat{\beta}_{l}l_{it+1} = \alpha + \beta_{k}k_{it+1} + \beta_{\iota}l_{it+1} + \omega_{it+1} + \eta_{it+1}$$

$$= \alpha + \beta_{k}k_{it+1} + \beta_{\iota}l_{it+1} + g(\omega_{it}) + \xi_{it+1} + \eta_{it+1}$$

$$= \alpha + \beta_{k}k_{it+1} + \beta_{\iota}l_{it+1} + g(\hat{\Phi}_{it} - \alpha - \beta_{k}k_{it} - \beta_{\iota}l_{it}) + \xi_{it+1} + \eta_{it+1}.$$

Identification relys on  $\mathbb{E}[\xi_{it+1} + \eta_{it+1} | \Omega_{it}] = 0$ . Firm *i*'s TFP is similarly given by

$$ln(TFP_{it}) = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it}$$
(40)

## C.4 Wooldridge (2009)

In both Olley and Pakes (1996) and Levinsohn and Petrin (2003), the unknown function  $\Phi_{it}$  are approximated by low-order polynomials. Wooldridge (2009) show that the proxy variable (either investment or intermediate inputs) approaches can be organized into a generalized method of moments (GMM) framework by specifying different instruments for different equations in two steps, which can obtain consistent estimation of the production function. Following Fons-Rosen et al. (2019), we call this two-step GMM method as Wooldridge-Levinsohn-Petrin (WLP) estimation thereafter.

Assume the productivity is a function of proxy variables  $m_{it}$  (investment in Olley and Pakes (1996), intermediate inputs in Levinsohn and Petrin (2003)) and capital input:

$$\omega_{it} = g(m_{it}, k_{it}). \tag{41}$$

Beside, the dynamics of productivity satisfies:

$$\mathbb{E}(\omega_{it}|\omega_{it-1},...,\omega_{i1}) = \mathbb{E}(\omega_{it}|\omega_{it-1}), \qquad t = 2, 3, ..., T,$$
(42)

so that,

$$\mathbb{E}(\omega_{it}|k_{it},l_{i,t-1},k_{i,t-1},m_{i,t-1},...,l_{i,1},k_{i,1},m_{i,1}) = \mathbb{E}(\omega_{it}|\omega_{it-1}) \equiv f[g(m_{i,t-1},k_{i,t-1})]$$
(43)

Similarly define  $\xi_{it} \equiv \omega_{it} - \mathbb{E}(\omega_{it}|\omega_{it-1})$ . The two equations used to identify

 $(\beta_l, \beta_k)$  are given as:

$$y_{it} = \alpha + \beta_l l_{it} + \beta_k k_{it} + g(m_{it}, k_{it}) + \eta_{it}$$
  
$$y_{it} = \alpha + \beta_l l_{it} + \beta_k k_{it} + f[g(m_{i,t-1}, k_{i,t-1})] + \xi_{it} + \eta_{it}$$

The corresponding orthogonality conditions are given as

$$\mathbb{E}(\eta_{it}|l_{it},k_{it},m_{it},l_{i,t-1},k_{i,t-1},m_{i,t-1},...,l_{i,1},k_{i,1},m_{i,1}) = 0, \quad t = 1,2,...,T,$$
(44)

$$\mathbb{E}(\xi_{it} + \eta_{it}|k_{it}, l_{i,t-1}, k_{i,t-1}, m_{i,t-1}, ..., l_{i,1}, k_{i,1}, m_{i,1}) = 0, \quad t = 2, ..., T.$$
(45)

We estimate these equations at the two-digit industry level with time fixed effects. For g, we specify a third-degree polynomial approximation. We also use  $l_{it-1}$  and  $l_{it-2}$  as instruments for labor input  $l_{it}$ . Thus, the productivity of firm i at time t  $TFP_{it}$  is given by:

$$ln(TFP_{it}) = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it}. \tag{46}$$

## C.5 Ackerberg et al. (2015)

Identification strategies of both Olley and Pakes (1996) and Levinsohn and Petrin (2003) assume firms can adjust variable inputs (like labor) at no cost, in response to productivity shocks. Ackerberg et al. (2015) argue that if labor input  $l_{it}$  is functionally dependent on capital input  $k_{it}$ , intermediate input  $l_{it}$ , and t, the identification methods mentioned above doesn't deliver consistent estimation of production functions, which they call the functional dependence problems. Ackerberg et al. (2015) adapt the methodologies of Olley and Pakes (1996) and Levinsohn and Petrin (2003), and propose a slightly different approach using the same moments. Ackerberg et al. (2015) resolve the functional dependence problems by using a two-step estimation method without identifying any production parameters in the first step.

Based on the framework of Levinsohn and Petrin (2003), Ackerberg et al. (2015) assume labor input has dynamic implications, which means labor decisions not only affect current profits, but also future profits. Besides, firms' intermediate input is

given by  $m_{it} = \tilde{f}_t(k_{it}, l_{it}, \omega_{it})$ . Then the partial linear equation changes to:

$$y_{it} = \alpha + \beta_l l_{it} + \beta_k k_{it} + \tilde{f}_t^{-1}(k_{it}, l_{it}, m_{it}) + \eta_{it} = \tilde{\Phi}_t(k_{it}, l_{it}, m_{it}) + \eta_{it}.$$
 (47)

The first stage moment condition is defined as:

$$\mathbb{E}[\eta_{it}|\Omega_{it}] = \mathbb{E}[y_{it} - \tilde{\Phi}_t(k_{it}, l_{it}.m_{it})|\Omega_{it}] = 0. \tag{48}$$

We notice that no parameter can be identified, but the estimated  $\hat{\Phi}_t(k_{it}, l_{it}.m_{it})$  is obtained in the first stage.

The second stage moment condition is given in a similar way.

$$\mathbb{E}[\xi_{it+1} + \eta_{it+1} | \Omega_{it}] = \mathbb{E}[y_{it+1} - \alpha - \beta_l l_{it+1} - \beta_k k_{it+1} - g(\tilde{\Phi}_t(k_{it}, l_{it}, m_{it}) - \alpha - \beta_l l_{it+1} - \beta_k k_{it+1}) | \Omega_{it}] = 0.$$

where  $\tilde{\Phi}_t(k_{it}, l_{it}, m_{it})$  can be replaced by the estimated  $\hat{\Phi}_t(k_{it}, l_{it}.m_{it})$ . Firm's TFP is defined in the same manner.

# **D** Proofs

#### **D.1** Government

Given the default decision  $d_t$  and the price of government bond  $q_t$ , the government maximizes government expenditure G, subject to the budget constraint (7). The FOC in terms of  $B_{t+1}$  is given as:

$$[B_{t+1}]$$
  $q_t = \phi_b[B_{t+1} - (1 - \vartheta)(1 - d_t f)B_t]$ 

### **D.2** Intermediate Goods Firms

Each intermediate goods firm i takes the aggregate demand  $Y_t$  and interest rate  $R_{it}$  as given, and maximizes the present value of the dividends, which is subject to the demand function (10) and the working capital constraint (14).

$$\max \sum_{t=0}^{\infty} \beta^t D_{it},$$

where

$$D_{it} = p_{it}z_{it}k_{T,it}^{\alpha_T}k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})] - [k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it}b_{it},$$

subject to

$$y_{it} = \left(\frac{1-\tau}{p_{it}}\right)^{\frac{1}{1-\eta}} Y_t,$$

$$b_{it} = \lambda_i (i_{T,it} + i_{I,it}).$$

When there's no adjustment cost ( $\theta_T = \theta_I = 0$ ), the first order conditions (FOCs) are given by:

$$\begin{split} [k_{T,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = & \beta [\eta(1 - \tau)Y_{t+1}^{1 - \eta}y_{it+1}^{\eta - 1}(\alpha_T z_{it+1}k_{T,it+1}^{\alpha_T - 1}k_{I,it+1}^{\alpha_I}) \\ & \quad + (1 - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] \\ & \quad - \left[k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1}\right]\lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}}] \end{split}$$

$$\begin{aligned} [k_{I,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = & \beta[\eta(1 - \tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta - 1}(\alpha_I z_{it+1}k_{T,it+1}^{\alpha_T}k_{I,it+1}^{\alpha_I - 1}) \\ & + (1 - \delta_I)[1 + (R_{it+1} - 1)\lambda_i]] \end{aligned}$$

Combining both FOCs, we can get:

$$\eta(1-\tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta}(\frac{\alpha_T}{k_{T,it+1}} - \frac{\alpha_I}{k_{I,it+1}}) + (\delta_I - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] = \lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}} \left[ k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1} \right]$$

### D.3 Households

The problem of the households is to maximize the preference (16), subject to the budget constraint (17). The FOCs are given as:

$$[C_t] \qquad \beta^t = \xi_t \tag{49}$$

$$[M_t] \qquad \xi_{t+1} = q_t^m \xi_t \tag{50}$$

where  $\xi_t$  is the Lagrange multiplier on the budget constraint (17). Then we can get:

$$q_t^m = \beta \tag{51}$$

Notice the price for deposit are constant over time.

#### Financial intermediary **D.4**

The financial intermediary's problem

$$\max_{\{M_t, B_{t+1}, b_{it}\}} E_t[\beta F_{t+1}] \tag{52}$$

where

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t.$$

subject to

$$(\mu_t) \qquad q_t B_{t+1} + \int b_{it} di \le N_t + q_t^m M_t, \tag{53}$$

$$(\zeta_t) \qquad \int (1 - \theta_{it}) b_{it} di \le N_t. \tag{54}$$

The FOCs are:

$$[b_{it}] \beta R_{it} - \mu_t - (1 - \theta_{it})\zeta_t = 0 (55)$$

$$[B_{t+1}] \qquad \beta \mathbb{E}_t[(1 - d_{t+1}f)(\vartheta + q_{t+1}(1 - \vartheta))] - \mu_t q_t = 0$$

$$[M_t] \qquad -\beta + \mu_t q_t^m = 0$$
(56)

$$[M_t] \qquad -\beta + \mu_t q_t^m = 0 \tag{57}$$

Using the FOC from households' problem  $q_t^m = \beta$ , we can get the pricing conditions for the government bond (24) and firm loan (25).

### **E** Numerical Solution

The aggregate state of the economy includes the aggregate shock on the government default risk process s and the initial level of government debt B. The government chooses borrowing B'(s, B, d) to maximize the government expenditure, subject to budget constraint and the bond price. The exogenous default risk process determines the default event d(s) and government bond price q(s).

The aggregate state variables and government sector outcome variables are relevant for the firms' choices for tangible capital and intangible capital only because they affect the firms' loan interest rate through the slackness of leverage constraint  $\zeta$  when the net worth of the financial intermediaries N changes. Thus,  $\zeta$  and the idiosyncratic states  $\{z,\lambda,k_T,k_I\}$  are sufficient to determine the firms' choices. For any state (s,B,d), the net worth is given by  $N(s,B,d)=(1-df)(1-\vartheta)q(s)B$ . Net worth affects the firm specific loan interest rate  $R_{it}=\frac{1+(1-\theta_{it})\zeta_t}{\beta}$ , where  $\zeta_t$  is the Lagrange multiplier for the leverage constraint  $\int (1-\theta_{it})b_{it}di \leq N_t$ . When the leverage constraint does not bind,  $\zeta_t=0$ , otherwise,  $\zeta_t>0$ .

The government default risk transmits to the firms through the leverage constraint. When government defaults (d = 1) or the default risk increases so that the bond price q decreases, financial intermediaries' net worth N decreases. With lower net worth, the leverage constraint becomes binding, or the binding is tighter. Tighter leverage constraint increases the shadow price of borrowing (Lagrange multiplier  $\zeta$ ), and thus increases the firm loan interest rate R.

We now describe the computation algorithm in detail.

- 1. Create grid points for default risk process *s*, government bond *B*, and an indicate *d* to denote whether the government is in default or not.
- 2. Create grid points for productivity shock z, financing needs  $\lambda$ , tangible capital  $k_T$ , intangible capital  $k_I$ , and the Lagrange multiplier  $\zeta \in [0, \zeta_{max}]$ .
- 3. Guess the initial government bond price Q(s).
- 4. Guess the initial value function of firm  $V_0(z, \lambda, k_T, k_I, \zeta)$  and aggregate output  $Y_0(\Lambda(z, \lambda, k_T, k_I), \zeta)$ .

- 5. Update the bond price using Eq (24).
- 6. Update the value function and policy functions  $k'_T(z, \lambda, k_T, k_I, \zeta)$ ,  $k'_I(z, \lambda, k_T, k_I, \zeta)$  and  $b(z, \lambda, k_T, k_I, \zeta)$ , and compute the stationary distribution.
- 7. Update the aggregate output  $Y_{upd}(\Lambda(z,\lambda,k_T,k_I),\zeta)$  and an aggregate term that includes firm loan demand b:  $X(\Lambda(z,\lambda,k_T,k_I),\zeta) \equiv \int (1-\theta_{it})b_{it}di$ .
- 8. Iterate until the firm value function, aggregate output and the government bond price converge.
- 9. Given shocks, and government policies  $\{s, B, d\}$ , compute the net worth N(s, B, d).
- 10. Compute equilibrium Lagrange multiplier  $\zeta(s, B, d)$ : if  $X(\Lambda(z, \lambda, k_T, k_I), 0) \le N(s, B, d)$ , then  $\zeta = 0$ , otherwise,  $\zeta$  is chosen such that  $X(\Lambda(z, \lambda, k_T, k_I), \zeta) = N(s, B, d)$ .