Sovereign Risk and Intangible Investment[*]

Minjie Deng

Simon Fraser University

Chang Liu

University of Rochester

Current version July 2022. First version February 2021.

Abstract

This paper measures the output and TFP losses from sovereign risk, considering firm-level intangible investment. Using Italian firm-level data, we show that firms reallocated from intangible assets to tangible assets during the recent sovereign debt crisis. This asset reallocation is more pronounced among small firms and high-leverage firms. This reallocation affects aggregate output and TFP. We build a sovereign default model incorporating firm intangible investment to explain the empirical findings. Using the model, we find that elevated sovereign risk explains 40% of the observed output losses and 22% of TFP losses.

Keywords: Sovereign debt crisis, intangible asset, firm investment, TFP loss
JEL classification: F34, E22, E44, G12, G15

1 Introduction

The recent European sovereign debt crisis was associated with substantial declines in real economic activities, emphasizing the pass-through of sovereign risk to the real sector. One important transmission channel, especially for the European countries, is the banking sector (Gennaioli et al. (2014), Perez et al. (2015), Bocola (2016), Arellano et al. (2019), Bottero et al. (2020)). Because banks are often the main creditors of governments, sovereign risk deteriorates bank balance sheets and disrupts private lending to firms. Tightening financing conditions for the firms depress firm activities and thus the real economy. We quantify the the real economic impacts of sovereign risk on the private sector by considering firm-level investment, especially investment in intangible assets.

Investment in intangible assets accounts for an increasing proportion of total investment.[1] Furthermore, the positive effects of intangible assets on firm productivity and performance are well-established in the literature (Griliches (1958), Griliches (1979), Geroski (1989), Hall et al. (2010)). Declines in investment in intangible assets (hereafter, intangible investment) affect firm productivity and output. However, intangible investment has often been ignored in previous sovereign default literature, thus missing a key component of output and productivity losses.

We start by empirically documenting the impact of sovereign risk on firms' intangible and tangible investment. First, during the Italian sovereign debt crisis, firms reduced their investment in both intangible assets and tangible assets. Second, intangible investment fell more than tangible investment as firms reallocated assets towards tangible investments. We call this behavior as *asset reallocation*. Third, the asset reallocation pattern is more pronounced within small firms and high-leverage firms. Our results are robust to alternative measures for investment, sovereign risk, and firm-level variables, and are also robust to different empirical specifications.

To explain the investment decline, asset reallocation and measure the aggregate costs of sovereign risk, we build a sovereign default model incorporating firm intangible investment. In our framework, an increase in government default risk results in deteriorating bank balance sheets, which leads to a higher loan interest rate for firms. The firm-specific loan interest rate also depends on each firm's collateral. Banks

---

[1]See EU KLEMS database.

accept tangible assets, but not intangible assets, as collateral. As a result, firms reduce intangible investment much more than tangible investment to offset the tightening borrowing constraint. The declines in intangible investment hurt future productivity and output. Firms are not equally affected by the elevated sovereign risk and higher loan interest rate: firms heavily relying on external borrowing from banks are more exposed to sovereign risk and reallocate by more.

Our framework incorporates heterogeneous firms with intangible investment into an otherwise canonical general equilibrium model of sovereign debt and default. The economy is composed of final goods firms, heterogeneous intermediate goods firms, financial intermediaries, households, and a central government. The government collects tax revenues from the final goods firms and borrows from the financial intermediaries to finance lump-sum transfers to the households and service the outstanding government debt. The government may default on its bonds, following an exogenous process. The final goods firms are competitive and they convert intermediate goods to final goods. The intermediate goods firms need to borrow from the financial intermediaries to finance a fraction of their investment and they differ in their productivity and external financing needs.

The financial intermediaries play a key role in transmitting sovereign risk to the firms: they use their net worth to purchase government bonds and provide loans to firms. An elevated sovereign default risk deteriorates the financial position of intermediaries and hence their private lending to the firms. Tightening financing conditions for the firms depress both intangible and tangible investment. However, since intangible assets can not be used as collateral, firms reduce their intangible investment by more. Lower intangible investment hurts firms' future productivity and output.

We parametrize the model using Italian data to highlight the role of intangible investment in assessing the output and productivity losses due to sovereign risk. We target sample moments that pertain to the behavior of firms, banks, and government. Using the calibrated model, we show that with increase in sovereign spreads, firms' future intangible and tangible assets decline, but intangible assets decline by more. With fewer capital inputs, firm output declines. The decline in intangible investment further decreases future firm TFP. The model endogenously generates the output decline and the TFP decline when the sovereign spread increases, as opposed to a

large previous literature that assumes exogenous declines in endowments, output, or TFP when the government defaults.

We then feed the model with a series of exogenous shocks to replicate the observed path of Italian sovereign risk and real GDP from 2006 to 2016. Using the 2006-2016 model-simulated sample, we run the same regression as in the empirical part. We show that the model can replicate the empirical findings: firms increase their tangibles-to-intangibles ratio during a sovereign debt crisis. Moreover, small firms and high-leverage firms reallocate their assets more aggressively compared to other firms.

Our model includes intangible investment that endogenously generate TFP losses, thus allowing us to measure TFP losses (and output losses) due to sovereign risk. We construct a scenario in which the Italian economy does not experience a debt crisis. We then compare the results of our benchmark model and the counterfactual model with no debt crisis. The differences between the economies of the benchmark model and the *no-debt-crisis* counterfactual model isolate the impact of the sovereign crisis on the Italian economy. We find that the losses associated with sovereign default risk are sizable. During 2011-2016, on average, output is 4.86% below trend, while it would have been 2.95% below trend without the sovereign debt crisis. As for TFP, on average, TFP is 5.5% below trend, while it would have been 4.27% without the debt crisis. Our quantitative results show that elevated sovereign risk was responsible for 40% of the observed output losses and 22% of the TFP losses from 2011 to 2016.

We also construct two reference models to highlight the role of intangible assets. In the first reference model, we eliminate intangible assets and denote this as the *no-intangible-asset* model. In the second reference model, we fix intangible assets at the median level of the invariant distribution from the benchmark model and call this the *fixed-intangible-asset* model. We recalibrate both of the reference models. The comparison between the benchmark model and the reference models isolates the role of endogenous intangible investment. Amid a sovereign debt crisis, firms reduce their intangible investment, thus reducing measured TFP, further reducing output. Without intangible investment, our reference models are silent on the TFP decline and generate less decline in output during a sovereign debt crisis. With one standard deviation increase in spread, output declines by 1.52% in the benchmark model, 0.54% in the no-intangible-asset model, and 0.47% in the fixed-intangible-asset model.

Related literature. Our paper measures TFP and output losses of sovereign risk by focusing on firm-level intangible investment responses, thus combining elements of the sovereign default literature with those of the literature on the impact of firm financial frictions.

The model builds on the sovereign default models pioneered by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). Most sovereign default literature assumes exogenous endowment declines when a sovereign defaults, while most of the rest features a production economy and assumes an exogenous TFP decline when a sovereign defaults (e.g., Arellano et al. (2018), Alessandria et al. (2020), Deng (2019)). By introducing firm intangible investment, our model endogenously generates the TFP and output declines during a sovereign debt crisis.

Recent papers in the sovereign debt and default literature study the links between sovereign default risk and the private sector through financial intermediation. During a sovereign debt crisis, firms lose access to external financing and cut their production, leading to reduced output (Mendoza and Yue (2012)). The link between sovereign default and the private sector through banking and finance is also analyzed in Perez et al. (2015), Sosa-Padilla (2018), Arellano et al. (2019), D'Erasmo et al. (2020), and Moretti (2020). Empirical findings using micro data also highlight the impacts of bank balance sheet on firm-level variables, such as credit (Bofondi et al. (2018), Acharya et al. (2018), Bottero et al. (2020)), sales (Arellano et al. (2019)), and tangible investment (Kalemli-Özcan et al. (2018). Our paper shares the focus of studying the transmission of sovereign risk to the firms through the financial intermediation. Our contribution is to examine the impacts of sovereign risk on firm investment—including both tangible and intangible investment—and quantify the aggregate losses from sovereign risk.

This paper also connects to the literature that studies the real effects of credit constraints, especially on investment. For example, Fakos et al. (2022) found that reduction in credit supply has significant real effects, explaining 11–32% of the investment slump during the Greek depression. We study both tangible and intangible investment. Our finding of declines in intangible investment due to financial frictions is consistent with Garcia-Macia (2017), Demmou et al. (2020), and Lopez and Olivella (2018). We further document the asset reallocation pattern in response to elevated sovereign risk. Our unique contribution is to bring two strands of literature (sovereign debt literature and intangible investment in business cycles literature) together and

quantify the aggregate losses due to sovereign risk through the intangible investment channel.

Our model implications for the TFP losses of sovereign debt crises relates to an extensive literature that studies intangible assets and productivity. The positive effects of intangible assets on firm productivity and economic performance are well established in the literature. This branch of literature traces back to Griliches (1958). Research on intangible investment and firm productivity has bloomed ever since (Griliches (1979), Geroski (1989), Hall et al. (2010), Gunn and Johri (2011), Johri and Karimzada (2021), among others). Several recent studies show that low firm-level incentives to invest in intangibles would result in TFP and output losses. The lack of incentive can be caused by either distortions (Ranasinghe (2014)), monetary policy (Moran and Queralto (2018)), equity financing shocks (Bianchi et al. (2019)), or financial crises (Queralto (2020)). Although traditional literature shows that intangible investment is countercyclical due to the lower opportunity cost of long-term innovative investments in recessions than in booms (Bean (1990), Aghion and Saint-Paul (1998)), more recent literature, such as Aghion et al. (2012), show that this traditional view is only true for firms that are not financially constrained. Several contributions also highlight the key role of credit constraints in intangible investment more generally (Brown et al. (2012), Hall et al. (2016), Peia and Romelli (2020), Xue et al. (2021)).

Road map. This paper proceeds as follows. Section 2 shows the data and key empirical findings. Section 3 presents our model with sovereign default risk, financial intermediaries, and firm investment in both tangible and intangible assets. Section 4 calibrates the model, measures the aggregate losses of sovereign risk, and highlights the role of intangible investment by comparing our benchmark model with several reference models. Section 5 concludes.

2   Empirical facts

This section documents the empirical results about the impact of sovereign risk on firm tangible and intangible investment. Section 2.1 describes the construction of the variables of interest and provides summary statistics. Section 2.2 shows the firm-level

responses in terms of both tangible and intangible investment during sovereign debt crises. Section 2.3 provides additional empirical evidence.

2.1 Data description

Firm-level variables. Our main sample uses annual manufacturing firm-level data from the Orbis dataset, covering the period 2006-2016. The dataset covers a large majority of Italian firms including both private and public firms, and includes rich balance-sheet information. The core variables in our analysis are firm investment (investment in intangible fixed assets and tangible fixed assets), key balance sheet indicators (total assets, short-term debt, and long-term debt), and additional firm-level variables. In Appendix A, we provide details on variables and sample selection.

Intangible fixed assets in the Orbis dataset are defined as all balance sheet intangible assets such as formation expenses, research expenses, goodwill, development expenses, and all other expenses with a long-term effect. Following literature, we measure investment using log difference of the fixed assets.[2]

In our baseline regression, leverage is defined as the ratio of total debt to total assets, where total debt is the sum of short-term debt and long-term debt. We label leverage calculated according to this method as "total leverage". We also use short-term leverage (the ratio of short-term debt to total assets) and net leverage (the ratio of net debt to total assets, where net debt is the sum of short-term loans and long-term debt minus net current assets) as robustness checks. As standard in literature, we define size, liquidity, sales growth, and the net current asset ratio. Table 1 reports a set of summary statistics for the main variables.

Sovereign debt crisis. The Italian economy was hit by the global financial crisis in 2008-2009 and recovered slightly in 2010. Then the economy experienced a second deep recession featuring a sovereign debt crisis in 2011-2013, when real GDP further declined by a further 4.4%. Figure 1 plots monthly sovereign spreads for Italy, which

---

[2]Investment is highly skewed, suggesting a log-linear rather than a simple linear regression specification. One possible concern about this log-difference measure for intangible investment is the loss of some observations if firms have zero intangible fixed assets in some certain years. To ensure that this does not change our results, we also consider the extensive margin of intangible investment in Section 2.3.

Table 1: Summary statistics

	Obs.	Mean	St.Dev.	Min.	Max.	P25	Median	P75
Net leverage	389560	0.053	0.37	-0.937	3.963	-0.205	0.054	0.309
Short-term leverage	389560	0.131	0.148	0	0.796	0	0.078	0.229
Total leverage	389560	0.202	0.195	0	0.983	0	0.165	0.349
Size	389560	10.123	1.203	4.565	13.323	9.283	10.145	11.011
Liquidity	389560	0.075	0.111	0	0.848	0.004	0.025	0.1
Investment in tangibles (log-diff)	389560	-0.124	0.746	-2.769	3.623	-0.462	-0.197	0.046
Investment in intangibles (log-diff)	389560	-0.04	0.397	-1.877	2.484	-0.204	-0.086	0.041

Notes: Statistics are calculated using the manufacturing firm-level data from the Orbis dataset, covering the period 2006-2016. We restrict the sample to observations with available intangible investment and tangible investment, to guarantee that the firm-level investment responses are comparable across different assets. The detailed sample selection can be found in Appendix A.

measures the severity of Italian sovereign debt crisis. The sovereign spread is defined as the gap between Italian and German 30-year government bond yields.[3] During 2011-2013, the spread spiked to about 4.5%. Sovereign debt credit ratings can also indicate the severity of a sovereign debt crisis. In 2011, the Standard & Poor's (S&P) revised their credit rating for Italian bonds from A+ to A, and further downgraded the rating to BBB+ in 2012, BBB in 2013, and BBB- in 2014.
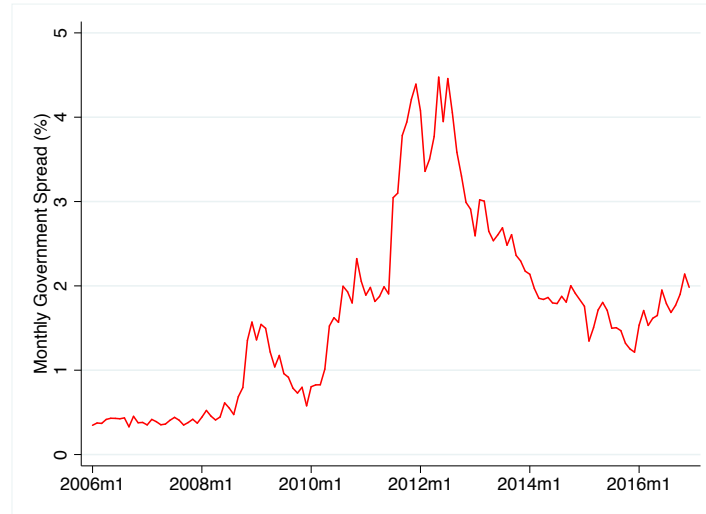

Figure 1: Sovereign spreads

Notes: Italian sovereign spread (monthly data). The spread is defined as the gap between 30-year Italian and German sovereign yields. Data is obtained from GFDFinaeon.

---

[3]Similar patterns hold for sovereign spreads data with different maturities.

2.2 Firm-level responses during a sovereign debt crisis

We focus on how firm investments react differently during a sovereign debt crisis. We first examine and estimate how the heterogeneous effects of sovereign risk on investment (including intangible and tangible investment) depend on firm characteristics, controlling for sector-year fixed effects. To obtain the average effect of sovereign risk, we relax the sector-year fixed effects and include more aggregate controls in a second estimation. This specification allows us to include government spreads in the regression. After examining the responses of intangible investment and tangible investment, we test for asset reallocation pattern during the sovereign debt crisis.

2.2.1 Heterogeneous effects

To estimate the responses of firm-level investment to sovereign risk and how the responses depend on firm-level characteristics, we estimate variants of our baseline empirical specification:

$$\Delta \log(assets_{i,t+1}) = \beta(x_i \times sp_t) + Controls + \delta_i + \eta_{st} + \epsilon_{it}, \tag{1}$$

where *assets* represent two types of assets: intangible assets and tangible assets. $\Delta \log(assets_{i,t+1}) \in \{\Delta \log(intangibles_{i,t+1}), \Delta \log(tangibles_{i,t+1})\}$ denotes intangible investment or tangible investment of firm i at time t, which are defined as the log-difference of intangible assets $[\log(intangibles_{i,t+1}) - \log(intangibles_{it})]$ or the log-difference of tangible assets $[\log(tangibles_{i,t+1}) - \log(tangibles_{it})]$. sp_t is the sovereign spread at year t, which is the average of the monthly data. x_i represents firm-level characteristics and we focus on two key characteristics: size and leverage. Thus, $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$ is a firm's size or total leverage in 2006.[4] *Controls* contains the interaction term of x_i and GDP growth (ΔGDP_t) to control for the general economic condition, and a vector of firm-level variables at time $t-1$, which includes size, total leverage, liquidity, sales growth, the ratio of liabilities to total assets, and the ratio of net current assets to total assets. δ_i controls for firm fixed effects, which capture permanent differences in investment behavior across firms. η_{st} is a sector-year

---

[4]We use firm characteristics in the first year of the sample to guarantee the variables are predetermined. We also use other measures of leverage including net leverage and short leverage for robustness checks.

fixed effect, which captures differences in sectoral exposure to aggregate shocks. ϵ_{it} is a residual. We cluster the standard errors at the firm level. Our main coefficient of interest is β, which depicts how firms invest in response to the sovereign spread, conditional on firm characteristics.

Panel (a) of Table 2 reports the results from estimating this baseline specification (1) for intangible investment. We have standardized size and total leverage, so their units are in standard deviations relative to the mean. Column (1) shows the results when only focusing on firm heterogeneity in size. Column (2) reports the results when focusing on firm heterogeneity in leverage. Column (3) adds both interactions with size and total leverage.[5] The results show that a firm of one standard deviation larger size than average has approximately a 2.0 *higher* semi-elasticity of intangible investment to sovereign spread, and a firm of one standard deviation higher leverage than average has approximately a 0.65 *lower* semi-elasticity.

Panel (b) of Table 2 reports the results of estimating Eq. (1) for tangible investment. Column (3) shows that a firm of one standard deviation larger size than average has approximately a 0.65 *higher* semi-elasticity of tangible investment to sovereign spread, and a firm of one standard deviation higher leverage than average has approximately a 0.13 *higher* semi-elasticity. Comparing Panel (a) and (b), we find that firms at each leverage level have opposite responses in terms of intangible and tangible investment: a high-leverage firm would invest less in intangible assets but invest more in tangible assets, compared to other firms, during a sovereign debt crisis.

As the sector-year fixed effects absorb the average effect of sovereign risk, we can only obtain the heterogeneous effects. We now relax the sector-year fixed effects and instead include aggregate controls, which allows us to compare the heterogeneous responses to the average effects.

2.2.2   Average effects

To assess the economic significance of our estimated interaction coefficients β, we now relax the sector-year fixed effects to obtain the average effects of sovereign risk

---

[5]The correlation between size and total leverage is not high (0.1474) in our baseline sample.

Table 2: Heterogeneous responses of firm investment

(a) Intangible investment

	$\Delta \log(intangibles_{i,t+1})$		
	(1)	(2)	(3)
$size_{i,2006} \times sp_t$	1.905***		1.998***
	(0.189)		(0.191)
$totallev_{i,2006} \times sp_t$		-0.426***	-0.652***
		(0.165)	(0.166)
Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes
Observations	303,935	303,935	303,935
R-squared	0.026	0.025	0.026
Number of id	59,706	59,706	59,706

Notes: Results from estimating Eq. (1) for intangible investment. We have normalized $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$ to standardized size and leverage, so their units are in standard deviation relative to the mean. Robust standard errors (in parentheses) are clustered by firms. *** p<0.01, ** p<0.05, * p<0.1.

(b) Tangible investment

	$\Delta \log(tangibles_{i,t+1})$		
	(1)	(2)	(3)
$size_{i,2006} \times sp_t$	0.671***		0.652***
	(0.091)		(0.092)
$totallev_{i,2006} \times sp_t$		0.202***	0.129*
		(0.074)	(0.074)
Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes
Observations	303,935	303,935	303,935
R-squared	0.068	0.068	0.068
Number of id	59,706	59,706	59,706

Notes: Results from estimating Eq. (1) for tangible investment. We have normalized $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$ to standardized within-firm size and total leverage, so their units are in standard deviation relative to the mean. Robust standard errors (in parentheses) are clustered by firms. *** p<0.01, ** p<0.05, * p<0.1.

by estimating:

$$\Delta \log(assets_{i,t+1}) = \beta_0 sp_t + \beta_1(x_i \times sp_t) + Controls + AggControls + \delta_i + \epsilon_{it}, \quad (2)$$

where $\Delta \log(assets_{i,t+1}) \in \{\Delta \log(intangibles_{i,t+1}), \Delta \log(tangibles_{i,t+1})\}$ denotes intangible or tangible investment of firm i at time t, sp_t is the sovereign spread at time t, and $x_i \in \{size_{i,2006}, totallev_{i,2006}\}$ is firm's size or total leverage in the year 2006. *Controls* is a vector of firm-level variables at time $t-1$ as defined in Eq. (1). We also include a vector of aggregate controls *AggControls*, which includes GDP growth (ΔGDP_t) and its interactions with firm characteristics ($x_i \times \Delta GDP_t$).[6] δ_i is firm fixed effects. Our coefficients of interest are β_0 and β_1. β_0 shows the average effects of the sovereign spread on firm investment, and β_1 measures how the semi-elasticity of investment with respect to the spread depends on firm size or total leverage.

Table 3 reports the results from estimating Eq. (2). The coefficient of sp_t for intangible (tangible) investment is around 0.1-0.3 (0.6-0.7) in absolute value. Thus, the interaction coefficients imply an economically meaningful degree of heterogeneity. The coefficients for the interaction terms are similar to the ones in Table 2. We make several observations when there is an increase in sovereign spread. First, on

---

[6]Appendix B.3 provides results when we add more aggregate controls, such as world GDP growth, trade openness, etc, and the results remain robust.

average, firms invest less in both intangible assets and tangible assets. Second, firms are heterogeneous in their investment responses. Especially, compared with other firms, firms with higher leverage invest more in tangible assets and less in intangible assets, indicating a higher degree of asset reallocation. In the next section, we use the tangibles-to-intangibles ratio as the dependent variable to examine the asset reallocation.

Table 3: Average responses of firm investment

(a) Intangible investment

	$\Delta \log(intangibles_{it+1})$			
	(1)	(2)	(3)	(4)
sp_t	-0.117	-0.376**	-0.109	-0.375**
	(0.163)	(0.164)	(0.163)	(0.164)
$size_{i,2006} \times sp_t$		1.984***		2.073***
		(0.187)		(0.189)
$totallev_{i,2006} \times sp_t$			-0.376**	-0.617***
			(0.164)	(0.166)
Firm controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Observations	303,935	303,935	303,935	303,935
R-squared	0.012	0.013	0.012	0.013
Number of id	59,706	59,706	59,706	59,706

Notes: Results from estimating Eq. (2) for intangible investment. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

(b) Tangible investment

	$\Delta \log(tangibles_{it+1})$			
	(1)	(2)	(3)	(4)
sp_t	-0.689***	-0.777***	-0.693***	-0.778***
	(0.074)	(0.078)	(0.074)	(0.078)
$size_{i,2006} \times sp_t$		0.683***		0.663***
		(0.089)		(0.089)
$totallev_{i,2006} \times sp_t$			0.219***	0.142*
			(0.073)	(0.074)
Firm controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Observations	303,935	303,935	303,935	303,935
R-squared	0.042	0.042	0.042	0.043
Number of id	59,706	59,706	59,706	59,706

Notes: Results from estimating Eq. (2) for tangible investment. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

2.2.3 Reallocation towards tangibles

When firms cut more intangible investment than tangible investment, their asset allocation changes. To better visualize the reallocation pattern indicated by the previous results, we replace the dependent variable in Eq. (1) and (2) with tangibles-to-intangibles ratio. The estimation results are shown in Table 4. The positive sign for sp_t shows that on average, firms increase their tangibles-to-intangibles ratios when the sovereign spread goes up. This asset reallocation pattern depends on firm characteristics: the negative sign for the $size \times spread$ interaction term and the positive sign for the $leverage \times spread$ show that small firms and high-leverage firms reallocate more towards tangible assets.

Table 4: Responses of tangibles-to-intangibles ratio

	Tangibles-to-intangibles ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
sp_t				1.686***	1.453***	1.678***
				(0.150)	(0.140)	(0.151)
$size_{i,2006} \times sp_t$	-1.009***		-1.064***	-0.986***		-1.042***
	(0.166)		(0.169)	(0.165)		(0.167)
$totallev_{i,2006} \times sp_t$		0.195	0.323**		0.199	0.327**
		(0.143)	(0.145)		(0.143)	(0.145)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No
Observations	380,769	380,769	380,769	380,769	380,769	380,769
R-squared	0.145	0.144	0.145	0.141	0.141	0.141
Number of id	73,697	73,697	73,697	73,697	73,697	73,697

Notes: Results from estimating Eq. (1) and (2) for tangibles-to-intangibles ratio. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

2.3 Additional empirical results

This section presents several key robustness checks, including firm-level borrowing responses, using alternative measures for investment, sovereign risk, and firm-level leverage. More robustness checks, including accounting for depreciation, adding region-year fixed effects, adding more aggregate controls, clustering at sector level, winsorizing the sample with alternative criteria, deflating with alternative price indices, using alternative maturities for government spreads, etc, can be found in Appendix B. None of these checks materially changes our baseline conclusions.

Firm-level borrowing. Deteriorated bank balance sheets have negative impacts on credit supply for firms (Bofondi et al. (2018), Acharya et al. (2018), Bottero et al. (2020)). Using our baseline sample, we find consistent results that firm borrowing is negatively affected by the debt crisis and the responses are heterogeneous. Table 5 reports the results when we replace the dependent variable in Eq. (2) with firm borrowing. The negative sign for sp_t shows that firms on average borrow less during the sovereign debt crisis. The borrowing responses are heterogeneous across firms. Firms that are less financially constrained, large firms and low-leverage firms, are less affected. In contrast, small firms and high-leverage firms are associated with major

declines in borrowing. The responses in firm debt provide additional evidence for the role of financial frictions in generating the (heterogeneous) investment responses across firms.

Table 5: Responses of firm borrowing

	Firm borrowing			
	(1)	(2)	(3)	(4)
sp_t	-2.060***	-2.193***	-1.932***	-2.064***
	(0.164)	(0.182)	(0.200)	(0.213)
$size_{i,2006} \times sp_t$	0.486**		0.513**	
		(0.205)		(0.205)
$totallev_{i,2006} \times sp_t$			-0.413**	-0.442**
			(0.189)	(0.190)
Firm controls	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Observations	186,751	186,751	186,751	186,751
R-squared	0.155	0.156	0.156	0.156
Number of id	44,582	44,582	44,582	44,582

Notes: Results from estimating Eq. (1) and (2) using the log-difference of total debt as dependent variable, where total debt is the sum of short-term debt and long-term debt. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Extensive margin of intangible investment. In the baseline regressions, we use the log-difference of assets to measure investment. Although being widely used, one potential concern is that the log-difference measure omits observations with zero assets by construction, which could be more pronounced for intangible assets. To deal with this concern, we further characterize the effects of sovereign risk on the extensive margin of intangible investment and also construct an alternative measure for investment. Here we focus on the extensive margin of intangible assets[7], i.e., whether to continue holding any intangible assets. Denote $\mathbb{1}(intangibles_{i,t+1})$ as an indicator that equals 1 if firm i continues to hold any intangible fixed assets in period $t + 1$, and equals 0 if firm i stops having any intangible fixed assets. Table 6 reports the results when we substitute the dependent variables with $\mathbb{1}(intangibles_{i,t+1})$ in Eq. (1) and (2). Table 6 shows that large firms and low-leverage firms are more likely to continue to hold intangible assets, consistent with the intensive margin results of adjusting intangible assets shown in Section 2.2.1 and 2.2.2.

---

[7]For tangible assets, using the log-difference to measure investment is less of a concern (if any), because we exclude firms with negative or zero total assets in the baseline sample, and the remaining firms have at least positive tangible assets.

Table 6: Extensive margin of intangibles

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
				$\mathbb{1}(intangibles_{i,t+1})$			
sp_t				-1.789***	-1.866***	-1.788***	-1.866***
				(0.041)	(0.043)	(0.041)	(0.043)
$size_{i,2006} \times sp_t$	0.607***		0.621***		0.630***		0.643***
	(0.050)		(0.050)		(0.050)		(0.050)
$totallev_{i,2006} \times sp_t$		-0.017	-0.093**			-0.002	-0.083*
		(0.043)	(0.043)			(0.043)	(0.043)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	383,121	383,121	383,121	383,121	383,121	383,121	383,121
R-squared	0.029	0.028	0.029	0.017	0.018	0.017	0.018
Number of id	71,339	71,339	71,339	71,339	71,339	71,339	71,339

Notes: Results from estimating Eq. (1) and (2) with the dependent variable as an indicator that equals 1 if continuing to hold any intangible assets and equals 0 if ceasing to hold any intangible assets. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Alternative measure for investment. To account for asset changes at both the intensive and extensive margin, we borrow a growth measure from the job creation literature (Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1998), Huber, Oberhofer, and Pfaffermayr (2013), among others) that accounts for both the intensive and extensive margin. We analogously define a measure of firm investment.

For intangible assets, firms in each year can be classified into three groups:

$$\begin{cases} \text{exiting firms} & G_x = \{i | k_{it} \neq 0, k_{i,t+1} = 0\} \\ \text{continuing firms} & G_c = \{i | k_{it} \neq 0, k_{i,t+1} \neq 0\} \\ \text{entering firms} & G_n = \{i | k_{it} = 0, k_{i,t+1} \neq 0\} \end{cases}$$

where k_{it} denotes intangible fixed assets of firm i at period t. Here "exiting" and "entering" only indicate whether firm i continues to hold intangible fixed assets. Then, investment in intangible assets (which is also the growth rate between two averages) can be defined as:

$$g(intangibles_{i,t+1}) = \frac{k_{i,t+1} - k_{it}}{0.5(k_{i,t+1} + k_{it})} = \begin{cases} -2 & i \in G_x \\ \dfrac{k_{i,t+1}/k_{it} - 1}{0.5(k_{i,t+1}/k_{it} + 1)} & i \in G_c \\ 2 & i \in G_n \end{cases} \tag{3}$$

14

We refer to this measure of investment as DHS (abbreviation for Davis, Haltiwanger, and Schuh (1998)) investment. The main advantage of DHS investment is that it can accommodate both entry (into the asset market, i.e., beginning to hold assets) and exit (from the asset market, i.e., no longer holding assets). It is a second-order approximation of the log-difference growth rate around 0 and it is bounded in the range [–2,2]. We estimate both empirical specifications, Eq. (1) and (2), using DHS investment as the dependent variable. Table 7 shows that, consistent with our baseline regression results, small firms and high-leverage firms decrease their intangible investment more than other firms during a sovereign debt crisis.

Table 7: Alternative measure for intangible investment: DHS investment

	$g(\text{intangibles}_{i,t+1})$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sp_t				-2.672***	-2.964***	-2.671***	-2.975***
				(0.150)	(0.152)	(0.150)	(0.152)
$size_{i,2006} \times sp_t$	2.469***		2.589***		2.580***		2.697***
	(0.174)		(0.175)		(0.171)		(0.173)
$totallev_{i,2006} \times sp_t$		-0.465***	-0.786***			-0.415***	-0.758***
		(0.154)	(0.155)			(0.154)	(0.155)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-year FE	Yes	Yes	Yes	No	No	No	No
Observations	392,201	392,201	392,201	392,201	392,201	392,201	392,201
R-squared	0.032	0.031	0.032	0.013	0.014	0.013	0.015
Number of id	71,523	71,523	71,523	71,523	71,523	71,523	71,523

Notes: Results from estimating Eq. (1) and (2) with the dependent variable as DHS investment. Robust standard errors are in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Alternative measure for sovereign risk. Another way to measure the severity of a sovereign debt crisis is to use credit ratings for sovereign bonds from a credit rating agency. We apply the credit ratings from Standard & Poor's (S&P) to replace baseline sovereign spread. The credit rating from S&P ranges from AAA (prime grade) to D (default). For instance, A+ implies an upper medium grade and B+ suggests highly speculative. As shown in Table 8, the S&P credit rating for Italian sovereign bonds was downgraded from A+ to A in 2011, and further downgraded to BBB+ in 2012, BBB in 2013, and BBB- in 2014.

These letter credit ratings are converted to discrete numerical scale between 100

(riskless) and 0 (likely to default) by Trading Economics.[8] The values corresponding to Italian sovereign credit ratings from 2006 to 2016 are: A+ (80), A (75), BBB+ (65), BBB (60), BBB- (55).

Table 8: Credit rating for Italy by S&P

Year (end of year)	S&P	Assigned number
2006	A+	80
2007	A+	80
2008	A+	80
2009	A+	80
2010	A+	80
2011	A	75
2012	BBB+	65
2013	BBB	60
2014	BBB-	55
2015	BBB-	55
2016	BBB-	55

We estimate the baseline empirical specifications using the credit rating score rather than the debt spread. Table 9 shows the results. Column (1) shows the heterogeneous effects of credit rating on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period $t + 1$, and 0 if firm i stops holding any intangible fixed assets. Column (4) and (5) are the results for tangible investment. In general, during the sovereign debt crisis (lower credit rating for sovereign debt), small firms and high-leverage firms decrease their intangible investment more than other firms, consistent with our baseline results.

Alternative measure for leverage. In the baseline regressions, leverage is defined as the ratio of total debt to total assets (we refer to this ratio as "total leverage"), where total debt includes both short-term loans and long-term debt. Here we use net leverage and short leverage as alternative measures of leverage. Short-term leverage measures firm short-run ability to meet its financial obligations and net leverage, defined by total leverage minus the ratio of net current asset to total asset, measures the firms' insolvency. Table 10 and Table 11 show that the baseline results are robust

---

[8]Refer to https://tradingeconomics.com/italy/rating for full correspondence table. AAA corresponds to 100, AA+ corresponds to 95, and each downgrade corresponds to a reduction by 5 in the score.

Table 9: Alternative measure for sovereign risk using credit rating

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
$rating_t$		0.612***	0.205***		0.264***
		(0.017)	(0.004)		(0.009)
$size_{i,2006} \times rating_t$	-0.151***	-0.145***	-0.075***	-0.020*	-0.015
	(0.019)	(0.019)	(0.005)	(0.010)	(0.010)
$totallev_{i,2006} \times rating_t$	0.068***	0.059***	0.001	0.001	0.002
	(0.017)	(0.017)	(0.004)	(0.008)	(0.008)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.019	0.020	0.068	0.047
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effects of sovereign credit rating on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, and 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). The credit rating is scaled down by 100. For example, we use 0.65 as the credit rating in 2012. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

to these alternative definitions.

Table 10: Alternative measure for leverage: net leverage

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.402**	-1.879***		-0.767***
		(0.163)	(0.043)		(0.077)
$size_{i,2006} \times sp_t$	1.886***	1.961***	0.628***	0.674***	0.682***
	(0.189)	(0.187)	(0.050)	(0.091)	(0.089)
$netlev_{i,2006} \times sp_t$	-0.523***	-0.433**	-0.089*	0.150*	0.184**
	(0.179)	(0.177)	(0.048)	(0.081)	(0.079)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,715	303,715	382,908	303,715	303,715
R-squared	0.026	0.013	0.019	0.068	0.042
Number of id	59,583	59,583	71,209	59,583	59,583

Notes: Estimation results here use net leverage instead of total leverage. Column (1) shows the heterogeneous effects of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, and 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Appendix B provides some further robustness checks, none of which materially change our conclusions. Appendix B.1 provides results when we allow tangible and intangible assets to depreciate at different rates. Appendix B.2 additionally controls for region-year fixed effects or province-year fixed effects, which capture possible geographical differences across sample firms. Appendix B.3 includes more aggregate controls in the empirical specifications. Our results are also robust to alternative group dummies (Appendix B.4 and B.5). Appendix B.6 clusters the standard errors at the

Table 11: Alternative measure for leverage: short-term leverage

Dependent variable	$\Delta \log(intangibles_{it})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{it})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.403**	-1.861***		-0.797***
		(0.164)	(0.043)		(0.078)
$size_{i,2006} \times sp_t$	1.916***	1.991***	0.629***	0.644***	0.655***
	(0.189)	(0.186)	(0.049)	(0.091)	(0.088)
$shortlev_{i,2006} \times sp_t$	-0.493***	-0.465***	-0.067	0.122	0.133*
	(0.165)	(0.165)	(0.043)	(0.077)	(0.076)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	304,704	304,704	383,904	304,704	304,704
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	59,699	59,699	71,295	59,699	59,699

Notes: Estimation results here use short-term leverage instead of total leverage. Column (1) shows the heterogeneous effects of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, and 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

sector level. Appendix B.7 winsorizes the variables of interest at the top and bottom 0.5% (baseline uses 1%). Appendix B.8 shows robustness to deflating intangible and tangible fixed assets by the Producer Price Index, which partially rules out the possibility that the different responses of intangibles and tangibles are driven by the price effects. Our results are also robust to replacing baseline sovereign spreads with an aggregate-level firm spread (Appendix B.9), and sovereign spreads with different maturities (Appendix B.10).

3 Model

Our empirical findings show that firm-level intangible and tangible asset investment are negatively affected by sovereign risk. Firms decrease both types of investment, but decrease intangible investment by more, especially for the small and high-leverage firms. As documented in literature, lower intangible investment may hurt future productivity. To quantify the impact of sovereign risk and evaluate aggregate implications, we develop a sovereign default model incorporating firm intangible investment.

The economy is composed of a central government, heterogeneous firms, financial intermediaries, and households. The government borrows by issuing long-term bonds to the financial intermediaries. Government can default on its debt. The probability of a government default evolves over time according to a reduced-form

stochastic process. Government issues bonds and collects tax revenues from final goods firms to finance the lump-sum transfers to the households and service the outstanding government debt.

There are two types of firms: final goods firms and intermediate goods firms. Final goods firms are competitive, and they convert intermediate goods to final goods. Intermediate goods firms operate under monopolistic competition, and they use both tangible capital and intangible capital to produce differentiated goods. They borrow from the financial intermediaries to finance a fraction of investment costs, and the borrowing interest rate is firm-specific, depending on the firm's tangible collateral. The intermediate goods firms are exogenously heterogeneous in productivity and financing need.

Households are composed of consumers and bankers, and they own the intermediate goods firms. Households decide on their consumption and how much to save with the financial intermediaries. The financial intermediaries are run by bankers who use repayments of prior loans and the savings of households to lend to the intermediate goods firms and the government.

The timing within the period is as follows. First, the aggregate shock on government default risk and the idiosyncratic shocks for intermediate goods firms are realized. Given the aggregate shock that exogenously determines the default risk, the government chooses borrowing and lump-sum transfers. The intermediate goods firms choose investment on intangible capital and tangible capital, and how much to borrow. The households save by depositing with the financial intermediaries. At the end of the period, production takes place. The financial intermediaries receive payments from firms and the government, and repay household deposits.

We start by describing the problems of each type of agent: the government, final goods firms, intermediate goods firms, consumers, and financial intermediaries. We then define the equilibrium for this economy.

3.1 The government

The government provides transfers to households. It finances the transfers T_t by issuing long-term bonds to the financial intermediaries and levying a tax on aggregate

final goods Y_t at rate τ. In every period, a fraction ϑ of debt matures and the remaining fraction remains outstanding. The government can default on its debt in every period by writing off a fraction $f \in [0,1]$ of its outstanding obligations.

Following Bocola (2016), we assume an exogenous process for government default risk. Assume that in every period the economy is hit by a shock ε_d that follows a standard logistic distribution. The default process follows:

$$d_{t+1} = \begin{cases} 1 & \text{if } \varepsilon_{d,t+1} < s_t \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where s is an AR(1) process:[9]

$$\log(s_t) = (1 - \rho_s)\log(s^*) + \rho_s \log(s_{t-1}) + \sigma_s \varepsilon_{st}, \quad \varepsilon_{st} \sim N(0,1). \tag{5}$$

The probability of default is then given by:

$$p_t^d \equiv \text{Prob}(d_{t+1} = 1 | s_t) = \frac{\exp(s_t)}{1 + \exp(s_t)}. \tag{6}$$

Every period, the government maximizes transfers T_t by choosing a new stock of bonds B_{t+1}, subject to its budget constraint:

$$q_t[B_{t+1} - (1-\vartheta)(1-d_t f)B_t] + \tau Y_t = \vartheta(1-d_t f)B_t + \frac{\phi_b}{2}[B_{t+1} - (1-\vartheta)(1-d_t f)B_t]^2 + T_t, \tag{7}$$

where q_t is the government bond price and B_t is the stock of bonds at time t. When the government defaults, $d_t = 1$ and a fraction $f \in [0,1]$ of its outstanding obligations is written off. ϕ_b parametrizes the bond adjustment cost.[10]

---

[9]As explained in Bocola (2016), this default process is consistent with literature showing that self-fulfilling beliefs were key drivers of sovereign risk during the European debt crisis. It also allows us to isolate the economic mechanisms underlying the propagation of sovereign default risk.

[10]This is a parsimonious way to pin down government bonds. Alternatively, one could set a reduced-form fiscal rule, and then government bonds would balance the budget constraint as in Bocola (2016).

3.2 Final goods firms

The final good Y_t is produced from a fixed variety of intermediate goods $i \in [0, 1]$ using the technology:

$$Y_t \leq [\int (y_{it})^\eta di]^{\frac{1}{\eta}}, \tag{8}$$

where the elasticity of demand is $\frac{1}{1-\eta} > 1$. We normalize the price of final goods to one, so total taxes paid to the government is τY_t. The price of intermediate good i is p_{it}. The final goods producers choose quantities of intermediate goods $\{y_{it}\}$ to solve:

$$\max_{\{y_{it}\}} (1 - \tau)Y_t - \int p_{it}y_{it}di, \tag{9}$$

subject to (8). Thus, the demand function y_{it} for intermediate good i is solved as:

$$y_{it} = (\frac{1 - \tau}{p_{it}})^{\frac{1}{1-\eta}} Y_t. \tag{10}$$

Demand function (10) shows that the demand for good i is negatively correlated with price p_{it}, and positively correlated with total output Y_t.

3.3 Intermediate goods firms

There is a unit measure of intermediate goods firms producing differentiated goods. We abstract from firm entry and exit for simplicity. Each firm i produces output y_{it} with tangible capital $k_{T,it}$ and intangible capital $k_{I,it}$:

$$y_{it} = z_{it}k_{T,it}^{\alpha_T}k_{I,it}^{\alpha_I} \tag{11}$$

where z_{it} is the idiosyncratic productivity shock. z_{it} evolves following $\log(z_{it}) = \rho_z \log(z_{it-1}) + \sigma_z \varepsilon_{it}$, where ε_{it} follows a standard normal random process. $k_{T,it}$ is the stock of tangible capital, and $k_{I,it}$ is the stock of intangible capital. α_T and α_I are the income shares of tangible capital and intangible capital, respectively. $\alpha_T + \alpha_I \leq 1$ so the production function is non-increasing returns to scale technology. TFP is calculated from the Solow residual, which is the portion of an economy's output

growth that cannot be attributed to the accumulation of tangible capital.[11] Tangible capital depreciates every period at the rate δ_T, and there is an adjustment cost for changing the capital stock. Thus, the investment in tangible capital at period t is given by:

$$i_{T,it} = k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it}), \tag{12}$$

where $\Theta(k_{T,it+1}, k_{T,it}) = \frac{\theta_T}{2}(\frac{k_{T,it+1}}{k_{T,it}} - 1 + \delta_T)^2 k_{T,it}$ is the convex adjustment cost for tangible capital. Similarly, the investment in intangible capital at period t is given by:

$$i_{I,it} = k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it}), \tag{13}$$

where δ_I is the depreciation rate for intangible capital, and $\Theta(k_{I,t+1}, k_{I,t}) = \frac{\theta_I}{2}(\frac{k_{I,t+1}}{k_{I,t}} - 1 + \delta_I)^2 k_{I,t}$ is the adjustment cost for investing in intangible capital.

At the beginning of each period, firm i's idiosyncratic productivity z_{it} is realized. Then firm i chooses next period tangible capital $k_{T,it+1}$ and intangible capital $k_{I,it+1}$. We assume that firms need to borrow a fraction of their investment before production, and the financing needs λ_i are firm-specific and time-invariant. Heterogeneity in λ_i captures the heterogeneous borrowing requirement. The financial intermediaries provide loans b_{it} to firm i at a firm-specific interest rate R_{it}, and the working capital requirement for firm i is:

$$b_{it} = \lambda_i(i_{T,it} + i_{I,it}). \tag{14}$$

We denote the time-invariant distribution of λ_i by Λ. The firm-specific interest rate R_{it} depends on the tangible capital of firm i. The financial intermediaries' problem in Section 3.4 will introduce the functional form of the firm-specific interest rate.

At the end of the period, production takes place. Firm i decides on the price p_{it} for its production y_{it}, taking the demand function (10) as given, and repays its debt $R_{it}b_{it}$. The dividend of firm i at period t is:

$$\begin{aligned}
D_{it} = {} & p_{it}z_{it}k_{T,it}^{\alpha_T}k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T)k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})] \\
& - [k_{I,it+1} - (1 - \delta_I)k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it}b_{it}.
\end{aligned} \tag{15}$$

---

[11]Literature on TFP estimation commonly measures TFP by subtracting growth rates of labor and physical capital inputs from the growth rate of value added. For example, Olley and Pakes (1996) constructs the capital measure applying a perpetual inventory method to buildings and equipment. The capital measure in Levinsohn and Petrin (2003) is the sum of the real peso value of depreciated buildings, machinery and vehicles.

Taking aggregate demand Y_t and the interest rate R_{it} as given, the intermediate goods firm i chooses intangible and tangible investment to maximize the present value of the dividends $\sum_{t=0}^{\infty} \beta^t D_{it}$, subject to the demand function (10) and the working capital constraint (14). Appendix C.1 provides the analytical optimality conditions for a simplified case with no capital adjustment costs.

3.4 Households

The representative household is composed of consumers and bankers. The household's preferences over consumption C_t are given by:

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t C_t \right], \tag{16}$$

where $\beta \in (0,1)$ is the discount factor and C_t is consumption in period t. Each period, the household sends bankers to operate the financial intermediaries, and provides them with net worth N_t. At the end of the period, the bankers bring back their returns from operations F_t to the household. The household can save using one-period deposits M_t with the financial intermediary at the price q_t^m.

In each period, the household also receives dividends D_t from the intermediate goods firms and a lump-sum transfer T_t from the government. The budget constraint of the household is:

$$C_t + q_t^m M_t + N_t = M_{t-1} + D_t + F_t + T_t. \tag{17}$$

The household maximizes (16) subject to (17). The optimality condition indicates that the price of deposits is given by $q_t^m = \beta$, which is constant over time.

Financial intermediaries. The bankers run the financial intermediaries. The financial intermediaries use their net worth N_t and the deposits of the household M_t to purchase government bonds and issue loans to firms. The financial intermediaries are competitive.

The net worth N_t the household provides to the bankers consists of a constant

transfer $\bar{n}$ and the value of government bonds that did not mature:

$$N_t = \bar{n} + (1 - d_t f)(1 - \vartheta)q_t B_t. \tag{18}$$

The evolution of government default risk drives the dynamics of the government bond price q_t, as well as actual default behavior d_t, both of which change the value of the net worth for the financial intermediaries.

At the beginning of the period, the *budget constraint* of the financial intermediaries is:

$$q_t B_{t+1} + \int b_{it} di \leq \underbrace{\bar{n} + (1 - d_t f)(1 - \vartheta)q_t B_t}_{N_t} + q_t^m M_t. \tag{19}$$

Aside from the budget constraint, the financial intermediaries are also subject to a *deposit constraint* that limits the amount of deposits the financial intermediaries can get from households:

$$q_t^m M_t \leq q_t B_{t+1} + \int \theta_{it} b_{it} di. \tag{20}$$

Here we assume that government bonds can be fully pledged, while loans to firms can only be partially pledged. θ_{it} denotes the fraction of firm i's debt can be pledged. This fraction θ_{it} depends on firm i's tangible capital share, as intangible capital typically cannot be used as collateral. We assume the firm-specific fraction θ_{it} that can be pledged is given by:

$$\theta_{it} = \frac{k_{T,it}}{\bar{k}} < 1, \tag{21}$$

where $\bar{k}$ is constant [12] and $k_{T,it}$ is the stock of tangible capital of firm i at time t. $\theta_{it} < 1$ reflects that firms' loans can't be fully pledged.

Combining the budget constraint (19) and the deposit constraint (20) gives that the unpledged amount of firm debt is bounded by the financial intermediaries' net worth, and we label this as the *leverage constraint*:

$$\int (1 - \theta_{it}) b_{it} di \leq N_t. \tag{22}$$

At the end of the period, the financial intermediaries receive payments from firms

---

[12]In the quantitative part, we set $\bar{k}$ to the maximum capital level that firms can choose in order to guarantee $\theta_{it} < 1$.

and the government, and repay household deposits. The return for the financial intermediaries equals:

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t. \quad (23)$$

The financial intermediary chooses $\{M_t, B_{t+1}, b_{it}\}$ to maximize the expected return $\mathbb{E}_t[\beta F_{t+1}]$ subject to (19) and (22). The optimality conditions give the following pricing conditions for government bonds and firm loans:

$$q_t = \mathbb{E}_t \beta[(1 - d_{t+1}f)(\vartheta + q_{t+1}(1 - \vartheta)], \quad (24)$$

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}, \quad (25)$$

where ζ_t is the Lagrange multiplier on the leverage constraint (22). The price for firm i's loans (25) implies that firm i will pay a premium $\frac{(1 - \theta_{it})\zeta_t}{\beta}$ over the risk-free rate when the leverage constraint of the financial intermediaries binds. When firms determine their investment in tangible and intangible capital, they are aware of how their capital decisions affect their financing costs through θ_{it}.

3.5 Equilibrium

We now define the equilibrium for this economy. Define $S = [s, B, d, \Lambda]$ as the state variables, where s is the government default risk process, B denotes the initial level of government debt, $d(s)$ is the default event determined by the exogenous default risk process, and $\Lambda(z, \lambda, k_T, k_I)$ is the distribution of the intermediate firms. We omit the time subscript t and use x' to denote a variable x in the next period.

Given an aggregate state S, the equilibrium consists of: (i) intermediate goods firms' policies for tangible capital $k_T'(z, \lambda, k_T, k_I; S)$, intangible capital $k_I'(z, \lambda, k_T, k_I; S)$ and borrowing $b(z, \lambda, k_T, k_I; S)$, and final goods firms' output $Y(S)$; (ii) policies for aggregate tangible capital $K_T(S)$, aggregate intangible capital $K_I(S)$, and consumption $C(S)$; (iii) price functions for firm borrowing rates $R(k_T, S)$, a government bond price function $q(s)$, and a constant deposit price q^m; and (iv) the distribution of firms over idiosyncratic productivity and capitals $\Lambda(z, \lambda, k_T, k_I)$ such that: (a) policy

functions of intermediate and final goods firms satisfy their optimization problem; (b) intermediate firms' borrowing rates satisfy (25) and the leverage constraint (22) holds; (c) the distribution of firms is consistent with the idiosyncratic shocks; (d) policies for households satisfy their optimal conditions; (e) next period government bonds satisfy the government budget constraint; (f) the government bond price satisfies (24); and (g) the markets for capital, goods, and bonds clear.

Next, we analyze key conditions that explain how an increase in sovereign risk affects private loan interest rates and firm investment choices. Recall that the financial intermediaries hold government bonds and face a leverage constraint which could be binding. When the sovereign spread increases, the value of the government bonds on the financial intermediaries' balance sheets falls, which leads to a lower net worth N_t.

Recall that the loan interest rate for firm i is given by

$$R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}, \text{where } \theta_{it} = \frac{k_{T,it}}{\bar{k}} < 1. \tag{26}$$

The Lagrange multiplier $\zeta_t > 0$ when the leverage constraint (22) binds. A decline in financial intermediaries' net worth N_t reduces credit supply and further tighten the leverage constraint, leading to an increase in ζ_t and thus the loan interest rate R_{it}. The firm specific interest rate R_{it} also depends on its tangible assets through θ_{it}. Thus, firms internalize the impact of tangible capital on interest rates when choosing tangible and intangible investment. Tangible investment helps offset tightening financial conditions by decreasing a firm's interest rate, while intangible investment does not ($\partial R_{it}/\partial k_{T,it} < 0$ and $\partial R_{it}/\partial k_{I,it} = 0$ when the leverage constraint binds).

Eq. (27) shows the first order condition for firm i's next period tangible capital $k_{T,it+1}$.[13] The left-hand-side is the marginal cost of increasing tangible capital, and the right-hand-side is the marginal benefit of tangible capital. Unlike intangible capital, investing in tangible capital has an extra benefit of decreasing the firm's interest rate,

---

[13]To simplify notation, Eq. (27) is for the case when the adjustment costs equal zero. This simplification is only for analytical purposes. We solve for the general case quantitatively.

as shown in the bracket.

$$
\begin{aligned}
[1 + (R_{it} - 1)\lambda_i] =& \beta[\eta(1 - \tau)Y_{t+1}^{1-\eta}y_{it+1}^{\eta-1}(\alpha_T z_{it+1}k_{T,it+1}^{\alpha_T-1}k_{I,it+1}^{\alpha_I}) \\
& + (1 - \delta_T)[1 + (R_{it+1} - 1)\lambda_i] \\
& \underbrace{- \left[k_{T,it+2} - (1 - \delta_T)k_{T,it+1} + k_{I,it+2} - (1 - \delta_I)k_{I,it+1}\right]\lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}}}_{\text{extra benefit of tangible capital investment}}]
\end{aligned}
$$

$$(27)$$

During sovereign debt crises, the net worth of the financial intermediaries shrinks, thus tightening the leverage constraint. When ζ_t increases, the impact of tangible capital on private sector interest rates is larger, i.e., a higher absolute value of $\partial R_i / \partial k_{T,i}$. For firms who largely rely on external financing (high-λ firms), the marginal benefit of investing in tangibles is larger. This explains our empirical finding that high-leverage firms reallocate more resources from intangible capital to tangible capital during the Italian sovereign debt crisis. The movements in the loan interest rate affect firms' input choices, which then affect aggregate TFP and output.

4   Quantitative Analysis

We now fit the model to Italian data. This section proceeds in four steps. Section 4.1 parametrizes the model. Section 4.2 studies the impulse responses of firm investment, output, and TFP to an increase in government default risk, as well as the model counterpart of firm heterogeneous responses. Section 4.3 reports the results of our quantitative experiment, in which we use the model to measure the output and TFP losses due to Italian debt crisis. Section 4.4 highlights the role of endogenous intangible investment by comparing our benchmark model to two reference models.

4.1   Parameterization

The model is at an annual frequency. There are two groups of parameters. The parameters in the first group are fixed exogenouly and are taken directly from the literature or from our empirical exercise, and those in the second group are jointly

chosen to match a set of moments relating to the Italian economy and its constituent firms. Table 12 lists all the parameter values.

Fixed parameters. The fixed parameters are $\{\alpha_T, \eta, \delta_T, \delta_I, \tau, \rho_z, \sigma_z, \beta, \vartheta, \rho_s, f\}$. The parameters $\{\alpha_T, \eta\}$ affect the shape of the production function of intermediate and final goods firms. We set α_T to 0.36 following Pérez-Orive (2016). η is set to be 0.75, which is the conventional value in the literature. The depreciation rate for intangible assets δ_I is 24.3% and the depreciation rate for tangible assets δ_T is 10.1%, according to our estimation for Italian depreciation rates in 2006 using the EU KLEMS database. The tax rate τ is 0.24, which is the corporate tax rate in Italy. The persistence and standard deviation of the firm productivity shock are set to be 0.9516 and 0.0033, following Lopez and Olivella (2018). The discount factor β is set to match an annual risk-free rate of 2%. The fraction of bonds maturing ϑ is set to be 0.05. The parameters governing the persistence of the sovereign risk process ρ_s are taken from Bocola (2016). The haircut fraction f is consistent with empirical evidence in Cruces and Trebesch (2013).

Fitted parameters. The remaining parameters in the model include parameters for the income share of intangible capital α_I, parameters governing investment adjustment costs $\{\theta_T, \theta_I\}$, parameters for the working capital requirement $\{\lambda_l, \lambda_h\}$, a constant transfer to the financial intermediaries $\bar{n}$, a parameter ϕ_B measuring government bond adjustment cost, and parameters for the sovereign risk process $\{\sigma_s, s^*\}$.

To set these parameters, we target 9 sample moments that reflect the behaviors of firms, banks, and government. Firm statistics include the volatility of tangible capital relative to the volatility of real sales, the counterpart statistic for intangible capital, the average leverage for firms within the low-leverage and high-leverage groups in 2006, and the average asset tangibility (ratio of tangible assets to total assets). The bank statistics is the ratio of credit to non-financial corporations to government credit. The government statistics include the ratio of government bonds to tax revenues in 2006, and the average spread and the volatility of the spread during the sample period.

We solve the model using global methods. Given the model policy functions, we perform simulations to obtain the model-implied counterparts of our targets. We jointly choose the fitted parameters to match these 9 sample moments by minimizing

the sum of the distance between the moments in the model and their corresponding counterparts in the data.

Although we choose all parameters jointly to match the moments, we can provide a heuristic description of how the moments inform specific parameters. First, the income share of intangible capital and the capital adjustment costs mostly affect firm tangibility and capital volatility. Second, the leverage statistics mainly pin down the working capital parameters $\{\lambda_l, \lambda_h\}$. Third, there is a tight relationship between $\bar{n}$—the lowest lending ability of financial intermediaries—and the ratio of credit to non-financial corporations to government credit. Fourth, the ratio of government bonds to tax revenue disciplines the government adjustment cost parameter ϕ_B. Finally, the mean and volatility of the spread primarily inform the sovereign risk process parameters $\{\sigma_s, s^*\}$. Table 13 reports the moments in the data and in the model. The model generates similar statistics to the ones in the data.

Table 12: Parameters

Parameter	Description	Value	Target/Source
Fixed parameters			
α_T	Income share of tangible capital	0.36	Pérez-Orive (2016)
η	Markup parameter	0.75	Conventional value
δ_T	Depreciation of tangible capital	0.101	Our estimation
δ_I	Depreciation of intangible capital	0.243	Our estimation
τ	Tax rate	0.24	Corporate tax rate
ρ_z	Persistence of firm productivity shock	0.9516	Lopez and Olivella (2018)
σ_z	Volatility of firm productivity shock	0.0033	Lopez and Olivella (2018)
β	Discount factor	0.98	Annual risk-free rate of 2%
ϑ	Fraction of bonds maturing	0.05	Conventional value
ρ_s	Sovereign risk process	0.95	Bocola (2016)
f	Haircut fraction	0.37	Cruces and Trebesch (2013)
Fitted parameters			
α_I	Income share of intangible capital	0.11	Firm tangibility
θ_T	Adjustment cost of tangible investment	2.41	Vol(tangible capital)/Vol(sales)
θ_I	Adjustment cost of intangible investment	0.051	Vol(intangible capital)/Vol(sales)
$[\lambda_l, \lambda_h]$	Working capital requirements	[0.111,1.445]	Average leverage of firms
$\bar{n}$	Constant transfer	0.03	Credit to firms/Credit to government
ϕ_B	Bond adjustment cost	44.7	Average government bonds/Tax revenue
σ_s	Sovereign risk process	0.272	Volatility of spread
s^*	Sovereign risk process	-3.46	Average spread

4.2 Effects of elevated sovereign risk

During sovereign debt crises, facing higher borrowing costs, firms reduce their investment. To offset tightening financial conditions, firms also reallocate their investment

Table 13: Moments in the data and model

	Data	Model
std(tangible capital)/std(sales)	1.606	1.603
std(intangible capital)/std(sales)	3.026	3.260
mean(leverage) for low-leverage firms	0.020	0.020
mean(leverage) for high-leverage firms	0.338	0.338
government bonds/tax revenue	2.595	2.602
credit to firms/credit to government	0.644	0.697
mean(firm tangibility)	0.865	0.856
mean(spread)	0.017	0.017
std(spread)	0.011	0.011

Notes: See Appendix A.3 for the construction of moments in
the data.

from intangible capital to tangible capital. That is to say, although they reduce both their tangible and intangible investment, they cut their intangible investment more.

To see this in the model, we plot the firms' impulse response functions (IRFs) to a positive spread shock, s, so that the government spread increases by one standard deviation. We simulate 40,000 paths for the model for 200 periods. From periods 1 to 100, the aggregate s shock follows its underlying Markov chain. In period 101, there is a positive shock to s so that the government spread increases by one standard deviation. From period 101 on, the s shocks follow the conditional Markov process. The impulse responses plot the average, across the 40,000 paths, of the variables for the last 100 periods.

Figure 2 shows these impulse responses for the firms when there is a one standard deviation increase in sovereign spreads (Panel (a)). When sovereign spreads increase, the balance sheets of the financial intermediaries deteriorate. With lower net worth, the financial intermediaries' leverage constraint binds, increasing the interest rates offered to firms (Panel (b)). Face a higher borrowing cost, the firms lower both their tangible assets and intangible assets (Panel (c)). However, firms reduce their intangible investment by more because intangible assets can't be used as collateral. Tangible assets, as collateral, can help lower their loan interest rate. Panel (d) confirms this asset reallocation pattern where the ratio of tangible assets to intangible assets increases following the shock. Since capital decreases, firms' output decreases in response (Panel (e)). Because firms decrease their intangible investment, their TFP decreases (Panel (f)). Note that the only shock here is the s shock. Thus, the model

endogenously generates the output decline and TFP decline when the sovereign spread increases.
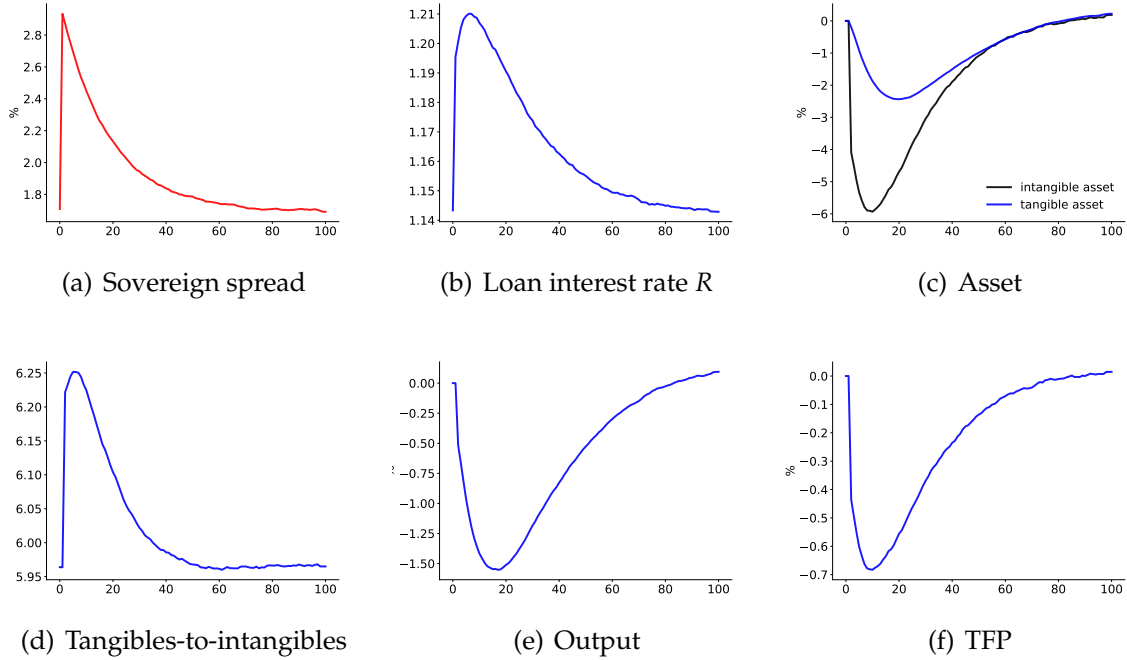

Figure 2: IRFs to a one standard deviation increase in sovereign spreads

Notes: Impulse response functions to a positive s shock (so that the sovereign spread increases by one standard deviation). Before the shock, the aggregate s follows its underlying Markov chain. In period 1, there is a positive shock to s so that the government spread increases by 1 standard deviation. After period 1, the s shocks follow the conditional Markov process. The impulse responses plot the average across the simulations.

The model can also replicate the observed heterogeneous asset reallocation pattern, as in the empirical section. We use the calibrated model to mimic the Italian economy and generate model-simulated data. Consistent with the sample length in our empirical section, we focus on the Italian economy from 2006 to 2016. We feed the model with a sequence of s_t shocks and z_t shocks[14] such that the model replicates the observed path of Italian sovereign risk and real GDP. Then we simulate the model to generate a panel sample of heterogeneous firms. Using the model-simulated sample, we run the same regressions as those in Table 4, where the dependent variable is tangibles-to-intangibles ratio. We investigate how the sovereign debt crisis affects

---

[14]This exogenous productivity shock summarizes the shocks that are not directly induced by the sovereign debt crisis, e.g., aggregate demand declining due to the global recession.

this ratio and how firm characteristics (size and leverage) affect the magnitude of the reallocation.

Table 14 compares the estimated coefficients using the Italian data with those using the model-simulated data. The coefficients from the Italian data are taken from column (6) in Table 4. The model generates a similar asset reallocation pattern as in the data: firms increase their tangibles-to-intangibles ratio during the sovereign debt crisis. Moreover, small firms and high-leverage firms reallocate more towards tangible investment compared with other firms. However, the model fails to match the magnitude of the regression coefficients.

Table 14: Regression results: data and model

	Data	Model
sp_t	1.678	0.179
$size_{i,2006} \times sp_t$	-1.042	-0.070
$totallev_{i,2006} \times sp_t$	0.327	0.132

Notes: Regression coefficients for the data and the model. The coefficients from the data are taken from column (6) in Table 4. The model regression specification mimics the data regression as much as possible. The sample time length is consistent with data regression.

4.3 Output and TFP losses from the Italian debt crisis

In this section, we quantify the output and TFP losses from the Italian sovereign debt crisis using our model with intangible investment. First, we feed the model with a sequence of s_t shocks and z_t shocks such that the model replicates the observed path of Italian sovereign risk and real GDP. Then we construct a scenario in which the Italian economy does not experience a sovereign debt crisis. We then compare the result of our benchmark model and that of the counterfactual model with no debt crisis. The differences between the paths of key variables in the benchmark model and those in the *no-debt-crisis* counterfactual model isolate the impact of the sovereign crisis on the Italian economy.

Figure 3 reports the time paths for sovereign spreads, GDP, and TFP during 2006-2016. The black dotted lines plot the paths in the data, the red solid lines plot the result of the benchmark model, and the blue dashed lines plot the result of the counterfactual scenario where there was no debt crisis. The unit of sovereign spreads

in Panel (a) is percentage points. Panel (b) and Panel (c) plot the percentage changes of GDP and TFP from the 2006 level.

By construction, the benchmark model (red solid lines) matches the sovereign spread and GDP in the data (black dotted lines). In general, the model needs a negative z shock and a positive s shock to reproduce the dynamics of sovereign spreads and GDP observed in the data. The sovereign spread increases from 0.4% in 2006 to 3.7% in 2012. Real GDP decreases by 4.7% from 2006 to 2009, recovers slightly in 2010, and then decreases another 4.3% during 2011-2013.

The blue dashed lines show the corresponding result in the *no-debt-crisis* counterfactual case, where we adjust the series of s_t shocks to fix the sovereign spread at their 2006 level throughout the simulation. Thus, the sovereign spread is constant in this counterfactual case. In this scenario, there is no increase in sovereign default risk and there is no transmission of sovereign risk to the financial intermediaries or the firms.
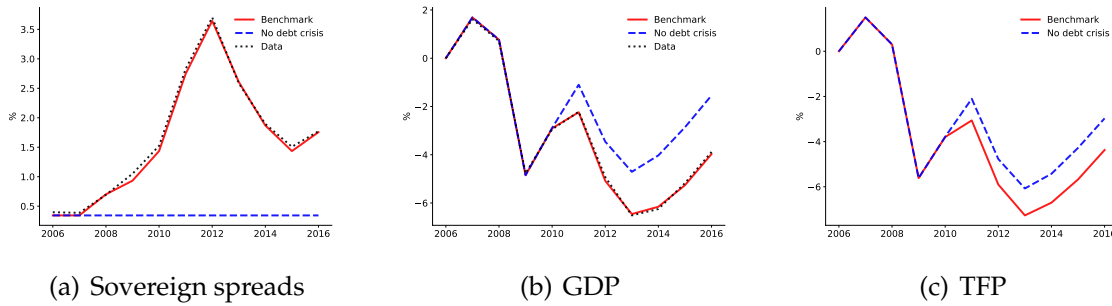

(a) Sovereign spreads   (b) GDP   (c) TFP

Figure 3: Paths for spreads, GDP and TFP

Notes: Paths for Italian sovereign spreads, GDP and TFP during 2006-2016. The black dotted lines plot for the data, the red solid lines plot for the benchmark model result, and the blue dashed lines plot the results from the counterfactual scenario where there was no debt crisis.

Output and TFP are still below the trend after the year 2013, indicating a prolonged recession. To quantify the losses, Table 15 reports output and TFP during and after the sovereign debt crisis. During 2011-2016, on average, output is 4.86% below trend, while it would have been 2.95% below trend without the sovereign debt crisis. As for TFP, on average, TFP is 5.5% below trend, while it would have been 4.27% without debt crisis. Therefore, our model predicts that sovereign risk was responsible for about 40% of the output losses and 22% of the TFP losses from 2011 to 2016.

33

Table 15: Output and TFP losses

	2011	2012	2013	2014	2015	2016	Avg. 11-16
output	-2.24	-5.08	-6.46	-6.16	-5.26	-3.97	-4.86
output-no debt crisis	-1.10	-3.46	-4.71	-4.03	-2.86	-1.54	-2.95
TFP	-3.06	-5.90	-7.27	-6.70	-5.68	-4.37	-5.50
TFP-no debt crisis	-2.10	-4.77	-6.07	-5.42	-4.28	-2.97	-4.27

Notes: "output" and "TFP" are the dynamics for the benchmark model. "output-no debt crisis" and "TFP-no debt crisis" are the dynamics for the no-debt-crisis counterfactual case. Output and TFP are reported as the percentage deviation from their 2006 value.

4.4 Role of intangible assets

To highlight the role of intangible assets, we compare the benchmark model with those without intangible investment. We use two ways to shut down intangible investment and call them reference models. In the first reference model, we eliminate intangible assets completely. We refer to this model as the *no-intangible-asset* model. In the second reference model, we fix intangible assets for each firm (and no depreciation) to the median level of the invariant distribution from the benchmark model and call this the *fixed-intangible-asset* model. Then we compare the benchmark model to these reference models to explore the role of intangible assets.

For the no-intangible-asset model, we discard intangible assets and keep the implied labor share constant, thus the income share of tangible capital α_T is now the sum of the original shares of tangible and intangible capital (i.e., α_T=0.47). We then choose $\{\theta_T, \lambda_l, \lambda_h, \bar{n}, \phi_B\}$ to match the relative volatility of tangible capital, the average leverage for firms within the low-leverage and high-leverage groups, the ratio of government bonds to tax revenues, and the ratio of credit to non-financial corporations to government credit.

For the fixed-intangible-asset model, we fix each firm's intangible assets to the median level of the invariant distribution from the benchmark model and assume no depreciation of intangible assets. The capital shares are the same as in the benchmark model (α_T=0.36, α_I=0.11). We then choose $\{\theta_T, \lambda_l, \lambda_h, \bar{n}, \phi_B\}$ to match the same set of moments as in the no-intangible-asset model. Table 16 reports the parameters in each reference model after recalibration. The bottom panel shows that the benchmark model and the reference models generate similar moments as in the data. The middle

panel lists the parameters that are different from the benchmark.

Table 16: Parameters and moments in different models

	benchmark	no-intangible-asset	fixed-intangible-asset
Parameters same with benchmark			
Markup parameter η	0.75	0.75	0.75
Depreciation of tangible capital δ_T	0.101	0.101	0.101
Tax rate τ	0.24	0.24	0.24
Persistence of firm productivity shock ρ_z	0.9516	0.9516	0.9516
Volatility of firm productivity shock σ_z	0.0033	0.0033	0.0033
Discount factor β	0.98	0.98	0.98
Fraction of bonds maturing ϑ	0.05	0.05	0.05
Sovereign risk process ρ_s	0.95	0.95	0.95
Haircut fraction f	0.37	0.37	0.37
Sovereign risk process σ_s	0.272	0.255	0.255
Sovereign risk process s^*	-3.46	-3.35	-3.35
Parameters changed from benchmark			
Income share of tangible capital α_T	0.36	0.47	0.36
Income share of intangible capital α_I	0.11	-	0.11
Depreciation of intangible capital δ_I	0.243	-	0
Adjustment cost of tangible investment θ_T	2.41	6.37	3.84
Adjustment cost of intangible investment θ_I	0.051	-	-
Working capital requirements $[\lambda_l, \lambda_h]$	[0.111,1.445]	[0.130,1.990]	[0.147,2.170]
Constant transfer $\bar{n}$	0.030	0.036	0.055
Bond adjustment cost ϕ_B	44.7	39.6	43.1
Model moments			
std(tangible capital)/std(sales)	1.603	1.524	1.610
std(intangible capital)/std(sales)	3.260	-	0
mean(leverage) for low-leverage firms	0.020	0.021	0.020
mean(leverage) for high-leverage firms	0.338	0.338	0.340
government bonds/tax revenue	2.602	2.521	2.608
credit to firms/credit to government	0.697	0.695	0.693
mean(firm tangibility)	0.856	-	0.836
mean(spread)	0.017	0.017	0.017
std(spread)	0.011	0.011	0.011

Notes: This table presents the parameters in the benchmark, no-intangible-asset model, and fixed-intangible-asset model. In both reference models, we redo the calibration so that the model moments are approximately identical to those generated by the benchmark model.

With the recalibrated reference models, we compare the IRFs in Figure 4. The IRFs plot the responses following the same positive s shock so that the sovereign spread increases by one standard deviation (Panel (a)). The red solid lines are the responses for our benchmark model, the blue dotted lines are for the no-intangible-asset model, and the black dashed lines are for the fixed-intangible-asset model.

Panel (b) of Figure 4 plots the responses of tangible assets. The benchmark model (red solid line) generates a larger decline than the reference models. This is because the decline in intangible assets (as shown in Panel (c)) reduces measured TFP, thus

lowering the marginal product of tangible assets in the benchmark model.

Panel (d) plots the tangibles-to-intangibles ratio. In our benchmark model, firms that face tightening financing conditions reallocate assets toward tangible assets, thus increasing the tangibles-to-intangibles ratio. However, in the fixed-intangible-asset model, the decline in tangible assets directly leads to a decline in the tangibles-to-intangibles ratio.

Panel (e) plots the IRFs of output. The benchmark model shows the largest decline. With one standard deviation increase in spread, output declines by 1.52% in the benchmark model, 0.54% in the no-intangible-asset model, and 0.47% in the fixed-intangible-asset model. The reduction in intangible assets lowers measured TFP (as shown in Panel (f)) and thus leads to a larger decline in output. In contrast, the responses of TFP in the reference models are muted.

In sum, our benchmark model generates the key empirical implications for intangible assets, asset reallocation, and measured TFP as in the data, while the reference models are silent on those empirical patterns. Also, the TFP losses and output losses would be underestimated if we ignore the responses in intangibles, highlighting the role of intangible assets when quantifying the impact of sovereign risk.

5 Conclusion

Sovereign debt crises have adverse effects on firm investment. Empirical evidence shows that firms reduce investment in both intangible assets and tangible assets during a crisis. Furthermore, although firms reduce tangible investment, they reallocate their investment away from intangible assets and towards tangible assets. This asset reallocation pattern is more pronounced in small firms and high-leverage firms.

We build a sovereign default model incorporating firm intangible investment to explain these empirical findings and measure the aggregate output and TFP costs of sovereign risk. Firms internalize that tangibles can be used as collateral and thus can help offset tightening financial constraints. When sovereign risk is transmitted to firms through the financial intermediaries, firms lower their investment, especially investment in intangible assets. Quantitatively, sovereign risk explains a large fraction
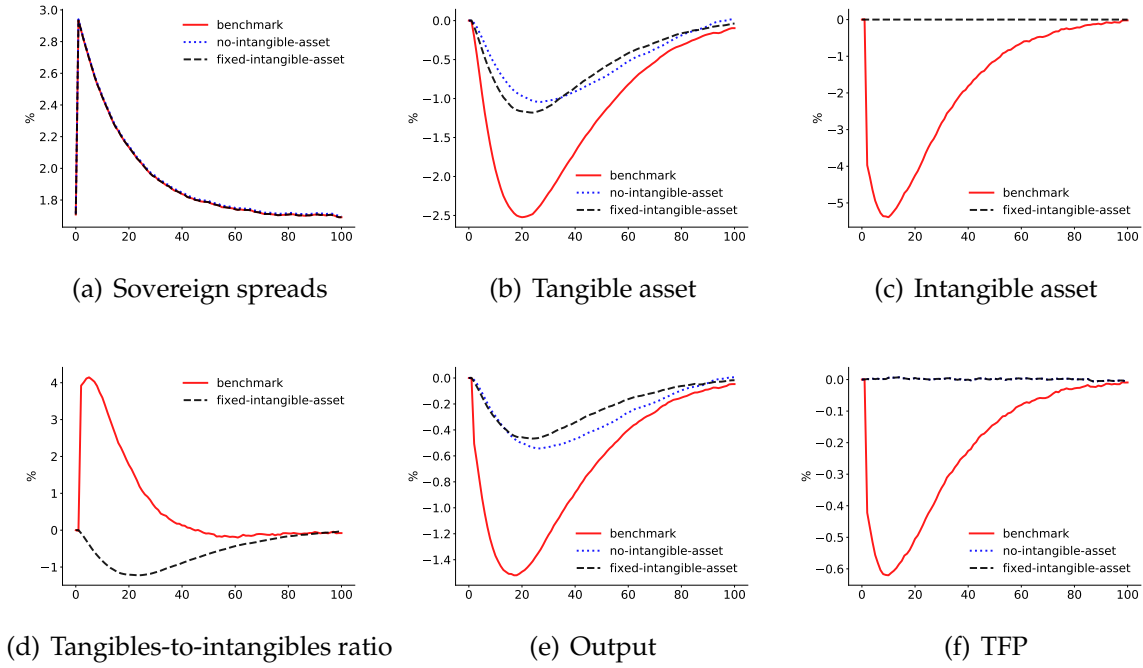
(a) Sovereign spreads   (b) Tangible asset   (c) Intangible asset

(d) Tangibles-to-intangibles ratio   (e) Output   (f) TFP

Figure 4: IRFs in benchmark model and reference models following recalibration

Notes: Impulse response functions to a positive s shock (so that the sovereign spread increases by one standard deviation) in the benchmark model (red solid lines), no-intangible-asset model (blue dotted lines), and fixed-intangible-asset model (black dashed lines). The parameters for the reference models are recalibrated so that the reference models also match the data moments. Before the shock, s follows its underlying Markov chain. In period 1, there is a positive shock to s so that the government spread increases by 1 standard deviation. After period 1, the s shocks follow the conditional Markov process. The impulse responses plot the average across the simulations.

of the output and TFP losses during the Italian debt crisis.

We focus on firm investment and our approach could be generalized along other dimensions. Sovereign risk could impact firms though different channels. For example, sovereign debt crises may affect the entry and exit decisions of firms, or their import and export decisions. We believe using firm-level data to estimate the impact of sovereign risk and explore other potential mechanisms is a compelling future research opportunity.

Due to data limitations, we do not observe substantial details on the nature of firms' intangible asset holdings. It would also be interesting to decompose intangible assets and explore if different types of intangible assets play different roles in explaining firm choices and outcomes. Understanding the heterogeneous investment behaviors of firms during crises, especially in terms of investment—which has beneficial long-run effects—provides key information for policy makers. We leave these applications to future research.

References

Acharya, V. V., T. Eisert, C. Eufinger, and C. Hirsch (2018). Real effects of the sovereign debt crisis in europe: Evidence from syndicated loans. *The Review of Financial Studies 31*(8), 2855–2896.

Aghion, P., P. Askenazy, N. Berman, G. Cette, and L. Eymard (2012). Credit constraints and the cyclicality of R&D investment: Evidence from france. *Journal of the European Economic Association 10*(5), 1001–1024.

Aghion, P. and G. Saint-Paul (1998). Virtues of bad times interaction between productivity growth and economic fluctuations. *Macroeconomic Dynamics 2*(3), 322–344.

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*(1), 64–83.

Alessandria, G., Y. Bai, and M. Deng (2020). Migration and sovereign default risk. *Journal of Monetary Economics 113*, 1–22.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 98*(3), 690–712.

Arellano, C., Y. Bai, and L. Bocola (2019). Sovereign default risk and firm heterogeneity.

Arellano, C., Y. Bai, and G. Mihalache (2018). Default risk, sectoral reallocation, and persistent recessions. *Journal of International Economics 112*, 182–199.

Bean, C. R. (1990). Endogenous growth and the procyclical behaviour of productivity. *European Economic Review 34*(2-3), 355–363.

Bernstein, J. I. and T. P. Mamuneas (2006). R&D depreciation, stocks, user costs and productivity growth for US R&D intensive industries. *Structural Change and Economic Dynamics 17*(1), 70–98.

Bianchi, F., H. Kung, and G. Morales (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics 101*, 47–63.

Bocola, L. (2016). The pass-through of sovereign risk. *Journal of Political Economy 124*(4), 879–926.

Bofondi, M., L. Carpinelli, and E. Sette (2018). Credit supply during a sovereign debt crisis. *Journal of the European Economic Association 16*(3), 696–729.

Bottero, M., S. Lenzu, and F. Mezzanotti (2020). Sovereign debt exposure and the bank lending channel: impact on credit supply and the real economy. *Journal of International Economics 126*, 103328.

Brown, J. R., G. Martinsson, and B. C. Petersen (2012). Do financing constraints matter for R&D? *European Economic Review 56*(8), 1512–1529.

Corrado, C., C. Hulten, and D. Sichel (2009). Intangible capital and US economic growth. *Review of Income and Wealth 55*(3), 661–685.

Cruces, J. J. and C. Trebesch (2013). Sovereign defaults: The price of haircuts. *American Economic Journal: Macroeconomics 5*(3), 85–117.

Davis, S. J. and J. Haltiwanger (1992). Gross job creation, gross job destruction, and employment reallocation. *The Quarterly Journal of Economics 107*(3), 819–863.

Davis, S. J., J. C. Haltiwanger, and S. Schuh (1998). Job creation and destruction. *MIT Press Books 1*.

Demmou, L., G. Franco, and I. Stefanescu (2020). Productivity and finance: the intangible assets channel-a firm level analysis. *OECD Economic Department Working Papers* (1596), 0_1–60.

Deng, M. (2019). Inequality, taxation, and sovereign default risk. *Available at SSRN 3545501*.

D'Erasmo, P., H. Moscoso Boedo, M. P. Olivero, and M. Sangiácomo (2020). Relationship networks in banking around a sovereign default and currency crisis. *IMF Economic Review 68*, 584–642.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Fakos, A., P. Sakellaris, and T. Tavares (2022). Investment slumps during financial crises: The real effects of credit supply. *Journal of Financial Economics 145*(1), 29–44.

Garcia-Macia, D. (2017). *The financing of ideas and the great deviation*. International Monetary Fund.

Gennaioli, N., A. Martin, and S. Rossi (2014). Sovereign default, domestic banks, and financial institutions. *The Journal of Finance 69*(2), 819–866.

Geroski, P. A. (1989). Entry, innovation and productivity growth. *The Review of Economics and Statistics*, 572–578.

Griliches, Z. (1958). Research costs and social returns: Hybrid corn and related innovations. *Journal of Political Economy 66*(5), 419–431.

Griliches, Z. (1979). Issues in assessing the contribution of research and development to productivity growth. *The Bell Journal of Economics*, 92–116.

Gunn, C. M. and A. Johri (2011). News and knowledge capital. *Review of Economic Dynamics 14*(1), 92–101.

Hall, B. H., J. Mairesse, and P. Mohnen (2010). Measuring the returns to R&D. In *Handbook of the Economics of Innovation*, Volume 2, pp. 1033–1082. Elsevier.

Hall, B. H., P. Moncada-Paternò-Castello, S. Montresor, and A. Vezzani (2016). Financing constraints, R&D investments and innovative performances: new empirical evidence at the firm level for Europe.

Huber, P., H. Oberhofer, and M. Pfaffermayr (2013). Who creates jobs? Estimating job creation rates at the firm level. *University of Salzburg, Economics and Finance Working Paper* (2013-05).

Johri, A. and M. Karimzada (2021). Learning efficiency shocks, knowledge capital and the business cycle: A bayesian evaluation. *Canadian Journal of Economics/Revue canadienne d'économique 54*(3), 1314–1360.

Kalemli-Özcan, L. Laeven, and D. Moreno (2018). Debt overhang, rollover risk, and corporate investment: Evidence from the European crisis. Technical report, National Bureau of Economic Research.

Levinsohn, J. and A. Petrin (2003). Estimating production functions using inputs to control for unobservables. *The review of economic studies 70*(2), 317–341.

Lopez, J. I. and V. Olivella (2018). The importance of intangible capital for the transmission of financial shocks. *Review of Economic Dynamics 30*, 223–238.

Mendoza, E. G. and V. Z. Yue (2012). A general equilibrium model of sovereign default and business cycles. *The Quarterly Journal of Economics 127*(2), 889–946.

Moran, P. and A. Queralto (2018). Innovation, productivity, and monetary policy. *Journal of Monetary Economics 93*, 24–41.

Moretti, M. (2020). The asymmetric pass-through of sovereign risk. Technical report, Working Paper.

Olley, G. S. and A. Pakes (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica 64*(6), 1263–1297.

Pakes, A., M. A. Schankerman, et al. (1978). The rate of obsolescence of knowledge, research gestation labs, and the private rate of return to research resources. Technical report.

Peia, O. and D. Romelli (2020). Did financial frictions stifle R&D investment in Europe during the great recession? *Journal of International Money and Finance*, 102263.

Perez, D. J. et al. (2015). Sovereign debt, domestic banks and the provision of public liquidity.

Pérez-Orive, A. (2016). Credit constraints, firms' precautionary investment, and the business cycle. *Journal of Monetary Economics 78*, 112–131.

Queralto, A. (2020). A model of slow recoveries from financial crises. *Journal of Monetary Economics 114*(C), 1–25.

Ranasinghe, A. (2014). Impact of policy distortions on firm-level innovation, productivity dynamics and TFP. *Journal of Economic Dynamics and Control* (46), 114–129.

Sosa-Padilla, C. (2018). Sovereign defaults and banking crises. *Journal of Monetary Economics 99*, 88–105.

Stehrer, R., A. Bykova, K. Jäger, O. Reiter, and M. Schwarzhappel (2019). Industry level growth and productivity data with special focus on intangible assets. Report on methodologies and data construction for the EU KLEMS release 2019. *The Vienna Institute for International Economic Studies*.

Xue, J., C. K. Yip, and J. Zheng (2021). Innovation capability, credit constraint and the cyclicality of R&D investment. *Economics Letters 199*, 109705.

ONLINE APPENDIX TO "SOVEREIGN RISK AND INTANGIBLE INVESTMENT"

BY MINJIE DENG, CHANG LIU

A Data

A.1 Variables

1. *Investment*

Our baseline measure of investment in period t is defined as the log difference of intangible fixed assets between period $t + 1$ and period t. That is, $\Delta \log(intangibles_{i,t+1})$ in Eq. (1) denotes the investment in intangibles of firm i at the end of period t. Similarly, tangible investment is defined as the log-difference of tangible fixed assets.

Intangible and tangible fixed assets are scaled by the price of intangibles and tangibles every year. Figure A.1 shows the asset components for tangible assets and intangible assets in the EU KLEMS database. The EU KLEMS database reports the price for each asset type. We construct the aggregate price for intangible assets as the weighted average price of each component in the right square, weighted by the share of each asset.[15] An identical construction is carried out for the price of tangibles.

2. *Net leverage*

Net leverage is measured as the ratio of firm i's net debt to total assets, where net debt is the sum of short-term loans and long term debt net of net current assets.

3. *Short leverage*

Short leverage is defined as the ratio of firm i's short-term loans to total assets.

4. *Total leverage*

Total leverage is defined as the ratio of firm i's total debt to total assets, where total debt is the sum of short-term loans and long-term debt.

---

[15]For example, Software and Database ("Soft_DB") accounts for 15% of intangible assets and R&D ("RD") accounts for 40% of intangible assets.
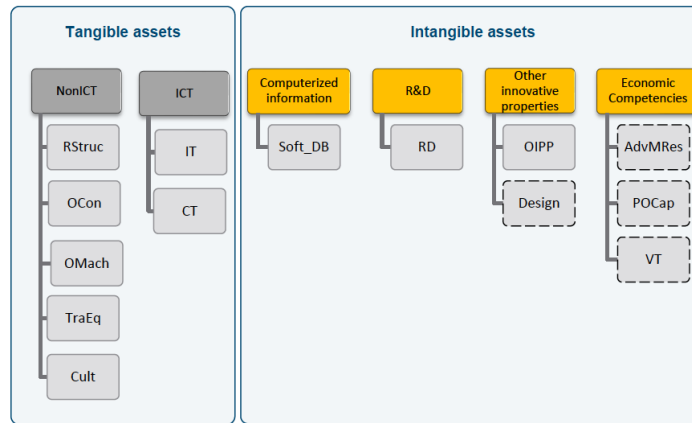
Figure A.1: Aggregates of capital services

Notes: Dashed lines indicate asset types outside the boundaries of National Accounts. Source: Report on methodologies and data construction for the EU KLEMS Release 2019 (Stehrer et al. (2019)).

5. *Size*

Size is measured as the log of total assets.

6. *Liquidity*

Liquidity is measured as the ratio of cash and cash equivalents to total assets.

7. *Sales growth*

Sales growth is defined as the log difference of sales, i.e. sales growth$_{it} = log(sales_{it}) - log(sales_{it-1})$.

8. *Liability ratio*

The liability ratio is defined as the ratio of total liabilities to total assets, where total liabilities is the difference between total assets and shareholders funds.

9. *Net current assets ratio*

Net current assets ratio is measured as the ratio of net current assets to total assets.

A.2 Sample selection

Our main sample excludes (in order of operation):

1. Firms not in the manufacturing sector.

2. Firms with negative or zero total assets.

3. Firms with negative intangible fixed assets or tangible fixed assets.

4. Firms which have missing values for total assets, intangible fixed assets, or tangible fixed assets over the sample period.

5. Firms that were not observed in 2006.

After applying the sample selection operations, we winsorize the variables mentioned above at the top and bottom 1% of the distribution.

A.3 Data moments

Standard deviation ratio   For tangible $k_{T,it}$, intangible assets $k_{I,it}$, and sales (all deflated by PPI), we detrend the data series for each firm i assuming a log-linear trend. The standard deviation of tangible assets is calculated as the average standard deviation of detrended tangible assets (cyclical component) across firms. A similar calculation yields the standard deviation of intangible assets and sales. The ratio of the standard deviation of tangible assets to that of sales is 1.606, while the counterpart for intangible assets is 3.026.

Leverage   Firms in our sample are divided into high-/low-leverage groups by each firm's total leverage in the base year of 2006. The mean leverage of the high-leverage group is 0.338 and that of the low-leverage group is 0.020.

Government bonds/tax revenue   The ratio of government bonds to tax revenue is calculated by the ratio of general government debt to general government revenue, which is 2.595. Data is from the OECD.Stat database.

Credit to firms/credit to government   The average ratio of total credit to the private non-financial sector to total credit to the government sector is 0.644 during the sample period, according to Bank for International Settlements statistics.

Firm tangibility   We construct the tangibility measure for firm i at period t as:

$$tangibility_{it} = \frac{k^d_{T,it}}{k^d_{T,it} + k^d_{I,it}}.$$

Since our Italian data features an unbalanced sample, we first take the average tangibility across sample periods to derive the average tangibility of firm i. Aggregate tangibility is then defined as the mean of tangibility measures across firms.

Moments of spread   The spread is defined as the gap between 30-year Italian and German sovereign yields. The mean and standard deviation of spread are calculated based on Italian 30-year government bond spread during the sample period. The average spread is 0.017 and the standard deviation of spread is 0.011.

B Robustness

This section shows additional robustness checks of our main empirical findings in Section 2.

B.1 Depreciation

One difference between intangible assets and tangible assets is that they depreciate at different speeds. Relatively little is known about depreciation rates for intangibles. Corrado et al. (2009) estimates the depreciation rate of R&D capital in the U.S. to be 20%. The U.S. Bureau of Economic Analysis (BEA) places its central estimate of the depreciation rate for R&D at 15%. Pakes et al. (1978) get an average depreciation rate of 25%, using data for several European countries.

Fortunately, the EU KLEMS database provides depreciation rates for each asset type, which allows us to construct depreciation rates for intangible and tangible assets in Italy—at the aggregate level. The depreciation rate for intangible assets is the weighted average of the depreciation rates of: computer software and databases, research and development, and other intellectual property patent (IPP) assets, with the weight being the asset share. For tangible assets, the depreciation rate is the weighted average of the depreciation rates of: computing equipment, communications equipment, transport equipment, other machinery and equipment, total non-residential investment, residential structures and cultivated assets, with the weight being the asset share. The calculated depreciation rate for intangible assets is 24.3% and the depreciation rate for tangible assets is 10.1% in 2006. Our estimates are in line with the rates reported in the existing literature.

Alternatively, we can construct intangible investment as [16]:

$$\Delta \log(intangibles_{dep,it}) = [\log(y_{i,t+1}) - \log((1 - dep_{intangible}) * y_{it}] * (1 - dep_{intangible})$$

where y_{it} denotes the intangible fixed assets of firm i at period t. $dep_{intangible} = 0.243$ is the weighted average depreciation rate for intangible investment. Corresponding

---

[16] intangible investment$_{it}/y_{it}$ $= [y_{i,t+1} - (1 - dep_{intangible}) * y_{it}]/y_{it}$ $\approx [\log(y_{i,t+1}) - \log((1 - dep_{intangible}) * y_{it}] * (1 - dep_{intangible})$

tangible investment can be constructed similarly, with a depreciation rate of 0.101. Table B.1 presents the estimation results for investment with depreciation.

Table B.1: Results for investment with depreciation

Dependent variable	$\Delta \log(intangibles_{dep,it+1})$		$\mathbb{1}(intangibles_{dep,it+1})$	$\Delta \log(tangibles_{dep,it+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.284**	-1.866***		-0.699***
		(0.124)	(0.043)		(0.070)
$size_{i,2006} \times sp_t$	1.513***	1.569***	0.643***	0.586***	0.596***
	(0.145)	(0.143)	(0.050)	(0.082)	(0.080)
$totallev_{i,2006} \times sp_t$	-0.494***	-0.467***	-0.083*	0.116*	0.128*
	(0.126)	(0.126)	(0.043)	(0.067)	(0.066)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.018	0.068	0.043
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1) using $\Delta \log(intangibles_{dep,it})$ as the dependent variable. Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table B.2: Intangible investment with alternative depreciation rate

Dependent variable	$\Delta \log(intangibles_{CHS2009_{dep,it+1}})$		$\Delta \log(intangibles_{BM2006_{dep,it+1}})$		$\Delta \log(intangibles_{PS1978_{dep,it+1}})$	
	(1) heterogeneity	(2) average	(3) heterogeneity	(4) average	(5) heterogeneity	(6) average
sp_t		-0.300**		-0.307**		-0.281**
		(0.131)		(0.135)		(0.123)
$size_{i,2006} \times sp_t$	1.599***	1.658***	1.639***	1.700***	1.499***	1.555***
	(0.153)	(0.151)	(0.157)	(0.155)	(0.143)	(0.142)
$totallev_{i,2006} \times sp_t$	-0.522***	-0.494***	-0.535***	-0.506***	-0.489***	-0.463***
	(0.133)	(0.133)	(0.136)	(0.136)	(0.125)	(0.124)
Firm controls	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	303,935	303,935	303,935	303,935
R-squared	0.026	0.013	0.026	0.013	0.026	0.013
Number of id	59,706	59,706	59,706	59,706	59,706	59,706

Notes: Column (1) and (2) are results using an intangible depreciation rate of 0.2 (Corrado et al. (2009)). Column (3) and (4) are results using an intangible depreciation rate of 0.18 (Bernstein and Mamuneas (2006)). Column (5) and (6) are results using an intangible depreciation rate of 0.25 (Pakes et al. (1978)). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table B.2 displays the results using $\Delta \log(intangibles_{dep,it+1})$ with alternative depreciation rates as dependent variables. The baseline results are robust to all choices of depreciation rate.

B.2 Region-year fixed effects

We further add region-year fixed effects and province-year fixed effects into our baseline regressions, controlling for any geographical differences. For example, firms

near the border may be highly exposed to foreign trade, which is possibly less affected by the Italian sovereign debt crisis. Table B.3 and Table B.4 show that our baseline results are robust to including region-year fixed effects or province-year fixed effects.

Table B.3: Region-year FE

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		7.988	3.050		-0.147
		(5.932)	(1.921)		(2.702)
$size_{i,2006} \times sp_t$	1.986***	2.058***	0.619***	0.646***	0.674***
	(0.194)	(0.191)	(0.050)	(0.093)	(0.091)
$totallev_{i,2006} \times sp_t$	-0.659***	-0.649***	-0.116***	0.110	0.104
	(0.169)	(0.168)	(0.043)	(0.075)	(0.075)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Region_year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.026	0.025	0.029	0.069	0.068
Number of id	58,918	58,918	70,408	58,918	58,918

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1) with additional region-year fixed effects. Column (2) is the corresponding estimation of Eq. (2) with region-year fixed effects. The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table B.4: Province-year FE

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		8.997	-3.493		-1.324
		(11.169)	(3.070)		(6.614)
$size_{i,2006} \times sp_t$	1.977***	2.046***	0.621***	0.653***	0.683***
	(0.194)	(0.191)	(0.050)	(0.093)	(0.091)
$totallev_{i,2006} \times sp_t$	-0.655***	-0.647***	-0.110**	0.121	0.116
	(0.169)	(0.169)	(0.044)	(0.075)	(0.075)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Province-year FE	Yes	Yes	Yes	Yes	Yes
Observations	299,742	299,742	377,900	299,742	299,742
R-squared	0.029	0.028	0.032	0.071	0.070
Number of id	58,918	58,918	70,408	58,918	58,918

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1) with additional province-year fixed effects. Column (2) is the corresponding estimation of Eq. (2) with region-year fixed effects. The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.3 More aggregate controls

The baseline results hold when we add more aggregate controls, including GDP, world GDP growth, and trade openness.

Table B.5: Adding more aggregate controls

Dependent variable	$\Delta \log(intangibles_{i,t+1})$	$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$
sp_t	4.101***	-1.160***	1.386***
	(0.241)	(0.062)	(0.114)
$size_{i,2006} \times sp_t$	2.054***	0.634***	0.652***
	(0.189)	(0.049)	(0.090)
$totallev_{i,2006} \times sp_t$	-0.641***	-0.094**	0.129*
	(0.166)	(0.043)	(0.074)
Firm controls	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Observations	303,935	383,121	303,935
R-squared	0.022	0.028	0.050
Number of id	59,706	71,339	59,706

Notes: Column (1) shows the estimation of Eq. (2) with additional aggregate controls. The dependent variable in Column (2) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (3) is the tangible investment counterpart to Column (1). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.4 Group dummies

Instead of using continuous standardized size and total leverage in 2006, we construct dummies for size and total leverage. Eq. (1) and (2) are modified as:

$$\Delta \log(assets_{i,t+1}) = \beta(dx_i \times sp_t) + Controls + \delta_i + \eta_{st} + \epsilon_{it}, \tag{B.1}$$

$$\Delta \log(assets_{i,t+1}) = \beta_0 sp_t + \beta_1(dx_i \times sp_t) + Controls + AggControls + \delta_i + \epsilon_{it}, \tag{B.2}$$

where $dx_i \in \{dsize_{i,2006}, dtotallev_{i,2006}\}$ are the dummies for firm size or total leverage in the year 2006. $dsize_{i,2006}$ is 1 if the size of firm i is larger than the median in 2006, 0 otherwise. $dtotallev_{i,2006}$ is 1 if the total leverage of firm i is higher than the median in 2006, 0 otherwise. The baseline results hold for these alternative indicators for firm size and total leverage.

Table B.6: Group dummies

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{it+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-1.329***	-2.267***		-1.294***
		(0.291)	(0.082)		(0.149)
$dsize_{i,2006} \times sp_t$	2.697***	2.852***	0.873***	0.706***	0.744***
	(0.332)	(0.329)	(0.086)	(0.157)	(0.154)
$dtotallev_{i,2006} \times sp_t$	-0.812**	-0.748**	-0.026	0.341**	0.365**
	(0.330)	(0.329)	(0.082)	(0.149)	(0.148)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	305,109	305,109	384,564	305,109	305,109
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	60,048	60,048	71,745	60,048	60,048

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (B.1). Column (2) is the corresponding estimation of Eq. (B.2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.5 Group dummies based on sector median

There are concerns for Appendix B.4 because firm distribution in size or total leverage may be highly skewed in some specific sectors. Therefore, we use sector median as the standard to construct dummy indicators for size and total leverage. For Eq. (B.1) and (B.2), $dsize_{i,2006}$ is 1 if the size of firm i is larger than the sector median in 2006, and 0 otherwise. $dtotallev_{i,2006}$ is 1 if the total leverage of firm i is higher than the sector median in 2006, and 0 otherwise. The baseline results are also robust to this redefinition of the group dummies.

Table B.7: Group dummies based on sector median

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-1.324***	-2.253***		-1.267***
		(0.289)	(0.082)		(0.147)
$dsize_{i,2006} \times sp_t$	2.832***	2.782***	0.846***	0.745***	0.724***
	(0.330)	(0.329)	(0.086)	(0.154)	(0.154)
$dtotallev_{i,2006} \times sp_t$	-0.710**	-0.668**	-0.019	0.308**	0.332**
	(0.329)	(0.329)	(0.083)	(0.149)	(0.148)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	304,954	304,954	384,402	304,954	304,954
R-squared	0.025	0.013	0.018	0.068	0.042
Number of id	60,022	60,022	71,720	60,022	60,022

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (B.1). Column (2) is the corresponding estimation of Eq. (B.2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.6 Clustering at sector-level

The baseline results are robust to clustering the standard errors at sector level.

Table B.8: Clustering at sector-level

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.375**	-1.866***		-0.778***
		(0.169)	(0.072)		(0.118)
$size_{i,2006} \times sp_t$	1.998***	2.073***	0.643***	0.652***	0.663***
	(0.218)	(0.193)	(0.050)	(0.112)	(0.114)
$totallev_{i,2006} \times sp_t$	-0.652***	-0.617**	-0.083**	0.129*	0.142**
	(0.232)	(0.236)	(0.036)	(0.069)	(0.069)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.018	0.068	0.043
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.7 Winsorizing at 0.5%

The baseline sample winsorizes the variables of interest at the top and bottom 1%. This section shows the baseline estimation results are robust if we instead winsorize the variables of interest at the top and bottom 0.5%.

Table B.9: 0.5% winsorizing

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.749***	-1.882***		-0.972***
		(0.172)	(0.042)		(0.082)
$size_{i,2006} \times sp_t$	1.983***	2.091***	0.607***	0.670***	0.681***
	(0.193)	(0.190)	(0.046)	(0.093)	(0.090)
$totallev_{i,2006} \times sp_t$	-0.625***	-0.589***	-0.109**	0.018	0.024
	(0.176)	(0.175)	(0.043)	(0.079)	(0.078)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	341,283	341,283	406,802	341,283	341,283
R-squared	0.026	0.014	0.019	0.066	0.044
Number of id	63,631	63,631	73,770	63,631	63,631

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm i continues to hold intangible fixed assets in period t, 0 if firm i ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.8 Deflating intangible and tangible fixed assets with PPI

The baseline estimation deflates intangible (tangible) fixed assets by the price of intangible (tangible) assets. We replace the price of investment with the PPI and the results remain robust.

Table B.10: PPI deflation

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-0.772***	-1.866***		-0.719***
		(0.164)	(0.043)		(0.078)
$size_{i,2006} \times sp_t$	1.981***	2.052***	0.643***	0.651***	0.660***
	(0.191)	(0.189)	(0.050)	(0.092)	(0.089)
$totallev_{i,2006} \times sp_t$	-0.640***	-0.603***	-0.083*	0.133*	0.142*
	(0.166)	(0.166)	(0.043)	(0.074)	(0.074)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,941	303,941	383,121	303,941	303,941
R-squared	0.025	0.012	0.018	0.065	0.042
Number of id	59,705	59,705	71,339	59,705	59,705

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.9 Alternative measure of debt crisis severity: firm spreads

Figure B.2 plots the Italian firm-level spread which is defined as the gap between the interest rate for loans (other than bank overdrafts) to non-financial corporations and the risk-free interest rate. The nominal risk-free rate is given by the Eurosystem main refinancing operations interest rate. Data was accessed via the Bank of Italy Statistical Database. During the Italian sovereign debt crisis, the interest rate spread for firms also increased. We replace the sovereign spread with the firm spread and the baseline results do not vary too much.
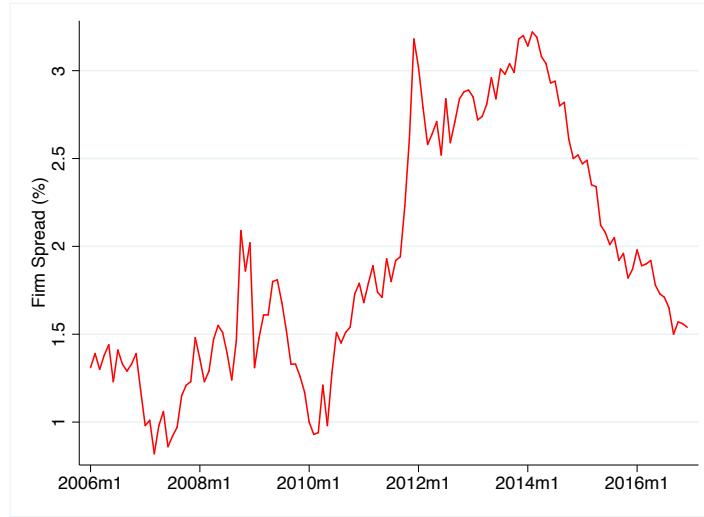
Figure B.2: Italy, firm spreads

Notes: A measure of average interest rate spreads for firms. The series is given by the spread over the risk-free rate of the interest rate for Italian non-financial corporations on non-overdraft loans (total maturity). The nominal risk-free rate is given by the Eurosystem main refinancing operations interest rate. Data source: Bank of Italy Statistical Database.

Table B.11: Alternative measure of debt crisis severity: firm spreads

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		-2.552***	-0.912***		-0.523***
		(0.159)	(0.041)		(0.079)
$size_{i,2006} \times sp_t$	0.950***	0.730***	0.102**	0.186**	0.016
	(0.186)	(0.181)	(0.046)	(0.092)	(0.090)
$totallev_{i,2006} \times sp_t$	-0.351**	-0.279*	-0.050	0.127*	0.143**
	(0.160)	(0.158)	(0.040)	(0.073)	(0.073)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.014	0.013	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

B.10 Alternative measure of debt crisis severity: spreads using yields of government bonds with different maturities

We replace the baseline 30-year government bond spread with 1-year/5-year/10-year government bond spreads, and our baseline results are robust to alternative spreads.

Table B.12: Results using 1-year government bond yield spreads

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		2.021***	-1.696***		0.220**
		(0.200)	(0.053)		(0.095)
$size_{i,2006} \times sp_t$	1.964***	2.189***	0.636***	0.758***	0.853***
	(0.230)	(0.229)	(0.061)	(0.112)	(0.110)
$totallev_{i,2006} \times sp_t$	-0.591***	-0.608***	-0.115**	0.140	0.140
	(0.201)	(0.202)	(0.053)	(0.091)	(0.090)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.025	0.014	0.015	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table B.13: Results using 5-year government bond yield spreads

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		0.684***	-1.178***		-0.240***
		(0.134)	(0.036)		(0.064)
$size_{i,2006} \times sp_t$	1.440***	1.544***	0.393***	0.523***	0.546***
	(0.155)	(0.153)	(0.041)	(0.075)	(0.073)
$totallev_{i,2006} \times sp_t$	-0.419***	-0.418***	-0.084**	0.128**	0.139**
	(0.135)	(0.135)	(0.035)	(0.060)	(0.060)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.015	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table B.14: Results using 10-year government bond yield spreads

Dependent variable	$\Delta \log(intangibles_{i,t+1})$		$\mathbb{1}(intangibles_{i,t+1})$	$\Delta \log(tangibles_{i,t+1})$	
	(1) heterogeneity	(2) average	(3) extensive	(4) heterogeneity	(5) average
sp_t		0.228	-1.405***		-0.401***
		(0.146)	(0.039)		(0.070)
$size_{i,2006} \times sp_t$	1.644***	1.723***	0.451***	0.570***	0.574***
	(0.170)	(0.169)	(0.045)	(0.082)	(0.080)
$totallev_{i,2006} \times sp_t$	-0.497***	-0.484***	-0.088**	0.142**	0.155**
	(0.148)	(0.148)	(0.039)	(0.066)	(0.065)
Firm controls	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Observations	303,935	303,935	383,121	303,935	303,935
R-squared	0.026	0.013	0.016	0.068	0.042
Number of id	59,706	59,706	71,339	59,706	59,706

Notes: Column (1) shows the heterogeneous effect of spreads on intangible investment by estimating Eq. (1). Column (2) is the corresponding estimation of Eq. (2). The dependent variable in Column (3) is a dummy variable which equals 1 if firm *i* continues to hold intangible fixed assets in period *t*, 0 if firm *i* ceases to hold intangible fixed assets. Column (4) and (5) are the tangible investment counterparts to Column (1) and (2). Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

C Proofs

C.1 Intermediate Goods Firms

Each intermediate goods firm i takes the aggregate demand Y_t and interest rate R_{it} as given, and maximizes the present value of dividends, which is subject to the demand function (10) and the working capital constraint (14).

$$\max \sum_{t=0}^{\infty} \beta^t D_{it},$$

where

$$D_{it} = p_{it} z_{it} k_{T,it}^{\alpha_T} k_{I,it}^{\alpha_I} - [k_{T,it+1} - (1 - \delta_T) k_{T,it} + \Theta(k_{T,it+1}, k_{T,it})]$$
$$- [k_{I,it+1} - (1 - \delta_I) k_{I,it} + \Theta(k_{I,it+1}, k_{I,it})] + b_{it} - R_{it} b_{it},$$

subject to

$$y_{it} = (\frac{1 - \tau}{p_{it}})^{\frac{1}{1-\eta}} Y_t,$$

$$b_{it} = \lambda_i (i_{T,it} + i_{I,it}).$$

When there's no adjustment cost ($\theta_T = \theta_I = 0$)[17], the first order conditions (FOCs) are given by:

$$[k_{T,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = \beta[\eta(1 - \tau) Y_{t+1}^{1-\eta} y_{it+1}^{\eta-1} (\alpha_T z_{it+1} k_{T,it+1}^{\alpha_T - 1} k_{I,it+1}^{\alpha_I})$$
$$+ (1 - \delta_T)[1 + (R_{it+1} - 1)\lambda_i]$$
$$- \left[k_{T,it+2} - (1 - \delta_T) k_{T,it+1} + k_{I,it+2} - (1 - \delta_I) k_{I,it+1} \right] \lambda_i \frac{\partial R_{it+1}}{\partial k_{T,it+1}}]$$

$$[k_{I,it+1}] \quad [1 + (R_{it} - 1)\lambda_i] = \beta[\eta(1 - \tau) Y_{t+1}^{1-\eta} y_{it+1}^{\eta-1} (\alpha_I z_{it+1} k_{T,it+1}^{\alpha_T} k_{I,it+1}^{\alpha_I - 1})$$
$$+ (1 - \delta_I)[1 + (R_{it+1} - 1)\lambda_i]]$$

---

[17]This simplification is only for analytical purposes. We solved for the general case in the quantitative section.

57

C.2 Households

The problem of the households is to maximize their preferences (16), subject to the budget constraint (17). The FOCs are given as:

$$[C_t] \quad \beta^t = \xi_t \tag{C.3}$$

$$[M_t] \quad \xi_{t+1} = q_t^m \xi_t \tag{C.4}$$

where ξ_t is the Lagrange multiplier on the budget constraint (17). Then we obtain:

$$q_t^m = \beta \tag{C.5}$$

Notice the price of deposits is constant over time.

C.3 Financial intermediaries

The financial intermediary's problem is:

$$\max_{\{M_t, B_{t+1}, b_{it}\}} E_t[\beta F_{t+1}] \tag{C.6}$$

where

$$F_{t+1} = (1 - d_{t+1}f)[\vartheta B_{t+1} + q_{t+1}(1 - \vartheta)B_{t+1}] + \int R_{it}b_{it}di - M_t.$$

subject to

$$(\mu_t) \quad q_t B_{t+1} + \int b_{it}di \leq N_t + q_t^m M_t, \tag{C.7}$$

$$(\zeta_t) \quad \int (1 - \theta_{it})b_{it}di \leq N_t. \tag{C.8}$$

The FOCs are:

$$[b_{it}] \quad \beta R_{it} - \mu_t - (1 - \theta_{it})\zeta_t = 0 \tag{C.9}$$

$$[B_{t+1}] \quad \beta E_t[(1 - d_{t+1}f)(\vartheta + q_{t+1}(1 - \vartheta))] - \mu_t q_t = 0 \tag{C.10}$$

$$[M_t] \quad -\beta + \mu_t q_t^m = 0 \tag{C.11}$$

Using the FOC from the households' problem $q_t^m = \beta$, we can get the pricing conditions for government bonds (24) and firm loans (25).

D   Numerical Solution

The aggregate state of the economy includes the aggregate shock to the government default risk process s and the initial level of government debt B. The government chooses borrowing $B'(s, B, d)$ to maximize lump-sum transfers, subject to the budget constraint and the bond price. The exogenous default risk process determines the default event $d(s)$ and government bond price $q(s)$.

For any state (s, B, d), net worth is given by $N(s, B, d) = \bar{n} + (1 - df)(1 - \vartheta)q(s)B$. Net worth affects the firm loan interest rate $R_{it} = \frac{1 + (1 - \theta_{it})\zeta_t}{\beta}$, where ζ_t is the Lagrange multiplier for the leverage constraint $\int (1 - \theta_{it})b_{it}di \leq N_t$. When the leverage constraint does not bind, $\zeta_t = 0$, otherwise, $\zeta_t > 0$.

We now describe the computational algorithm in detail.

1. Create grid points for the default risk process s, government bonds B, and an indicator d to denote whether the government is in default or not.

2. Create grid points for the productivity shock z, financing needs λ, tangible capital k_T, intangible capital k_I, and the Lagrange multiplier $\zeta \in [0, \zeta_{max}]$.

3. Guess the initial government bond price $q(s)$.

4. Update the bond price using Eq. (24). Iterate until the bond price converges. Since government default process does not depend on firms, we iterate it separately from the rest of the loops.

5. Guess the initial value function of firm $V_0(z, \lambda, k_T, k_I, \zeta)$ and aggregate output $Y_0(\Lambda(z, \lambda, k_T, k_I), \zeta)$.

6. Update the value function and policy functions $k_T'(z, \lambda, k_T, k_I, \zeta)$, $k_I'(z, \lambda, k_T, k_I, \zeta)$ and $b(z, \lambda, k_T, k_I, \zeta)$.

(1) In particular, we parameterize the forecasting function for ζ using the following linear functional form:

$$\zeta' = \phi_0 + \phi_1\zeta$$

(2) Set the initial guess for ϕ_0 and ϕ_1: ϕ_0^0 and ϕ_1^0.

(3) Using the guess for ϕ_0 and ϕ_1, solve the optimization problem for the firms.

(4) Using the optimal decision rules obtained in the previous step, implement the simulation. Using the simulated paths to run OLS regression to obtain the new set of coefficients for the forecasting function ϕ_0^1 and ϕ_1^1.

(5) Compare ϕ_0^0 and ϕ_0^1, and ϕ_1^0 and ϕ_1^1. If the distance of coefficients are all less than the tolerance level, done. Otherwise, update both coefficients and go back to (3).

7. Compute the stationary distribution.

8. Update aggregate output $Y_{upd}(\Lambda(z,\lambda,k_T,k_I),\zeta)$ and an aggregate term that summarizes firm loan demand b: $X(\Lambda(z,\lambda,k_T,k_I),\zeta) \equiv \int (1-\theta_{it})b_{it}di$.

9. Given shocks and government policies $\{s,B,d\}$, compute net worth $N(s,B,d)$.

10. Compute the equilibrium Lagrange multiplier $\zeta(s,B,d)$: if $X(\Lambda(z,\lambda,k_T,k_I),0) \leq N(s,B,d)$, then $\zeta = 0$, otherwise, ζ is chosen such that $X(\Lambda(z,\lambda,k_T,k_I),\zeta) = N(s,B,d)$.

11. If the distance of firm value function and its previous value and aggregate output and its previous value are all less than the tolerance level, done. Otherwise, update both and go back to 6.