Supervised classification - improving capacity learning

0. Import library

Import library

In [1]:

```
# Import libraries
import warnings

warnings.filterwarnings(action='ignore')

# math library
import numpy as np

# visualization library
%matplotlib inline
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('png2x','pdf')
import matplotlib.pyplot as plt

# machine learning library
from sklearn.linear_model import LogisticRegression

# 3d visualization
from mpl_toolkits.mplot3d import axes3d
from matplotlib import cm

# computational time
import time

import math
```

1. Load and plot the dataset (dataset-noise-02.txt)

The data features for each data i are $x_i=(x_{i(1)},x_{i(2)})$.

The data label/target, y_i, indicates two classes with value 0 or 1.

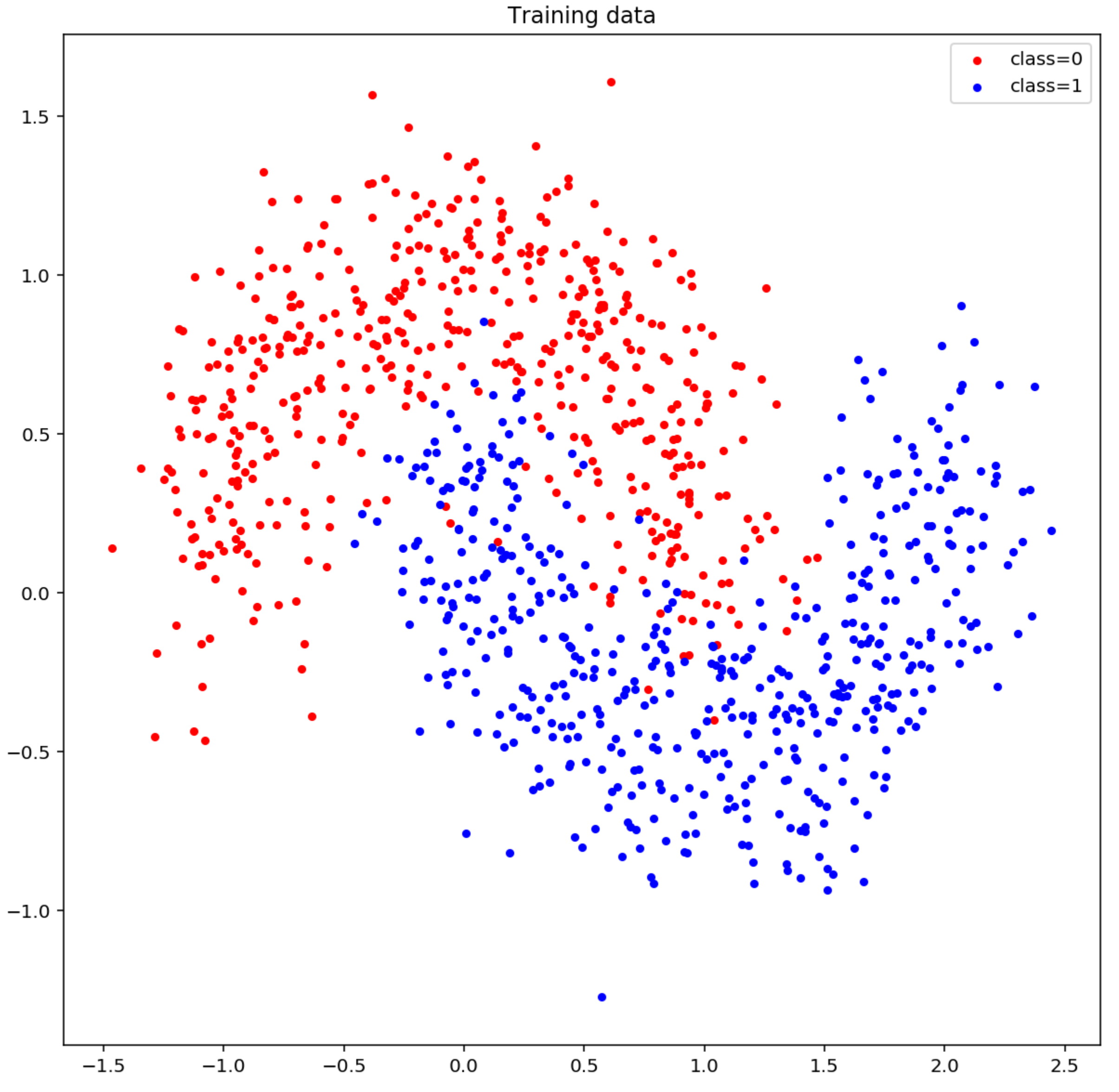
Plot the data points.

You may use matplotlib function `scatter(x,y)`.

In [3]:

```
# import data with numpy
data = np.loadtxt('dataset-b.txt', delimiter=',')

# number of training data
n = data.shape[0]
print('Number of the data = {}'.format(n))
print('Shape of the data = {}'.format(data.shape))
print('Data type of the data = {}'.format(data.dtype))

# plot
x1 = data[:,0].reshape(data.shape[0], 1) # feature 1
x2 = data[:,1].reshape(data.shape[0], 1) # feature 2
idx = data[:,2].reshape(data.shape[0], 1) # label

idx_class0 = np.array([[x1[i], x2[i]] for i in range(len(idx)) if idx[i] == 0]) # index of class0
idx_class1 = np.array([[x1[i], x2[i]] for i in range(len(idx)) if idx[i] == 1]) # index of class1

plt.figure(1,figsize=(10,10))
plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```

```
Number of the data = 1000
Shape of the data = (1000, 3)
Data type of the data = float64
```



2. Define a logistic regression loss function and its gradient

In [5]:

```
def sigmoid(z):

    return 1 / (1 + math.e ** -z)

# predictive function definition
def f_pred(X,w):

    f = sigmoid(np.dot(X, w))

    return f

# Test predicitive function

def loss_logreg(z_pred,z):
    n = len(z)
    loss = - (1/n) * np.sum(z * np.log(z_pred) + (1-z) * np.log(1-z_pred))
    return loss

def grad_loss(z_pred,z,X):

    grad = np.dot(X.T, z_pred - z) / len(X) * 2

    return grad

# gradient descent function definition
def grad_desc(X, z, w_init,tau=0.01, max_iter=500):

    L_iters = np.zeros([max_iter]) # record the loss values
    w = w_init # initialization

    for i in range(max_iter): # loop over the iterations

        z_pred = f_pred(X,w) # linear predicition function
        grad_f = grad_loss(z_pred, z, X) # gradient of the loss
        w = w - tau * grad_f
        L_iters[i] =  loss_logreg(z_pred, z)  # save the current loss value


    return w, L_iters
```

3. define a prediction function and run a gradient descent algorithm

The logistic regression/classification predictive function is defined as:
$$ \begin{aligned} p_w(x) &= \sigma(X w) \end{aligned} $$

The prediction function can be defined in terms of the following feature functions f_i as follows:
$$ X = \left[\begin{array}{cccccc} f_0(x_1) & f_1(x_1) & f_2(x_1) & f_3(x_1) & f_4(x_1) & f_5(x_1) & f_6(x_1) & f_7(x_1) & f_8(x_1) & f_9(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2) & f_3(x_2) & f_4(x_2) & f_5(x_2) & f_6(x_2) & f_7(x_2) & f_8(x_2) & f_9(x_2) \\ \vdots\\ f_0(x_n) & f_1(x_n) & f_2(x_n) & f_3(x_n) & f_4(x_n) & f_5(x_n) & f_6(x_n) & f_7(x_n) & f_8(x_n) & f_9(x_n) \\ \end{array} \right] \quad \textrm{ and } \quad w = \left[\begin{array}{cccc} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{array} \right] \quad $$

where $x_i = (x_i(1), x_i(2))$ and you can define a feature function f_i as you want.

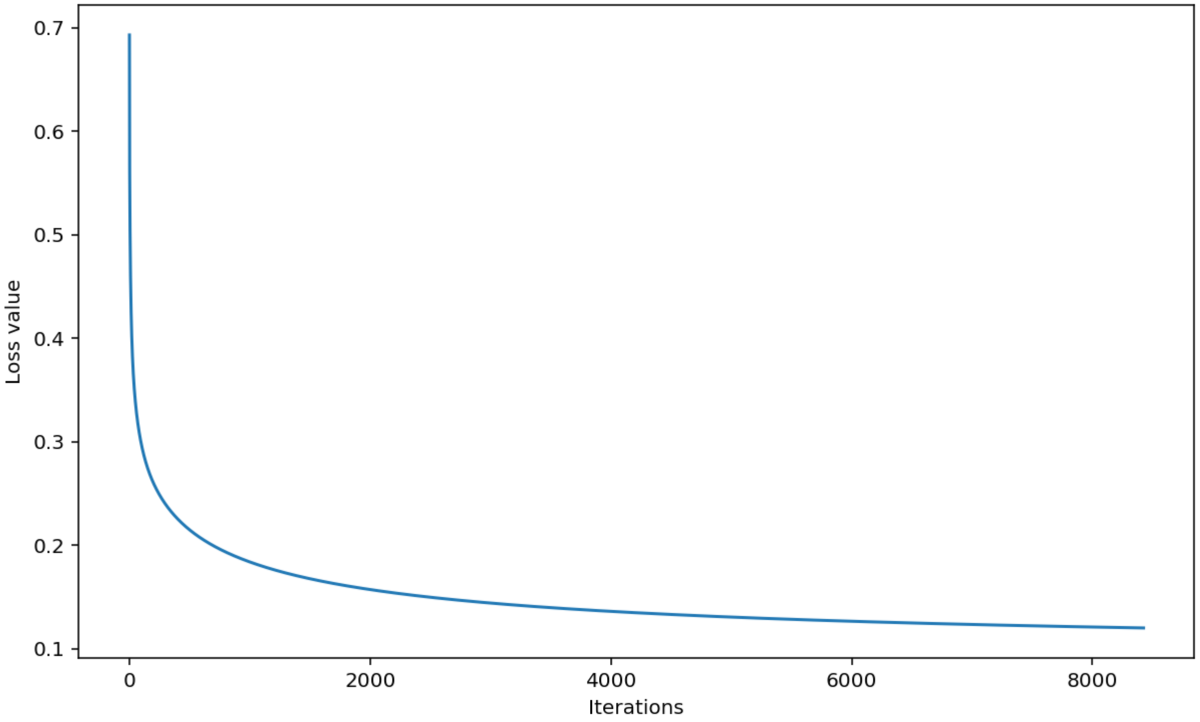
You can use at most 10 feature functions f_i, $i = 0, 1, 2, \cdots, 9$ in such a way that the classification accuracy is maximized. You are allowed to use less than 10 feature functions.

Implement the logistic regression function with gradient descent using a vectorization scheme.

In [25]:

```
x1 = data[:,0].reshape(data.shape[0], 1)
x2 = data[:,1].reshape(data.shape[0], 1)
y = data[:,2].reshape(data.shape[0], 1)

# construct data matrix

n = data.shape[0]
X = np.ones([n,10])
for k in range(X.shape[1]):
    if k % 3 == 0:
        X[:,k] = [(x1[i] * x2[i]) ** (int(k/3)) for i in range(len(X[:,1]))]
    elif k %3 == 1:
        X[:,k] = [x1[i] ** (int(k/3) + 1) for i in range(len(X[:,1]))]
    else:
        X[:,k] = [x2[i] ** (int(k/3) + 1) for i in range(len(X[:,1]))]

# parameters vector
w = np.array([1,1,1,1,1,1,1,1,1,1])[:,None] # [:,None] adds a singleton dimension

# run gradient descent algorithm
start = time.time()
w_init = np.array([0 for i in range(10)])[:,None]
tau = 1e-1; max_iter = 30000
w, L_iters = grad_desc(X, y, w_init, tau, max_iter)
print('Time=',time.time() - start)
print(L_iters[-1])
print(w)

# plot
plt.figure(3, figsize=(10,6))
plt.plot([i for i in range(len(L_iters))], L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```

```
Time= 5.108136177062988
nan
[[  4.7288973 ]
 [ -0.21921241]
 [ -6.49392092]
 [ -2.77997786]
 [-15.38729076]
 [  0.29552354]
 [  3.01822954]
 [  9.74601719]
 [ -4.29789142]
 [  2.88687976]]
```



0.7
0.6
0.5
0.4
0.3
0.2
0.1
Loss value
0
2000
4000
6000
8000
Iterations

/

4. Plot the decisoin boundary

In [26]:

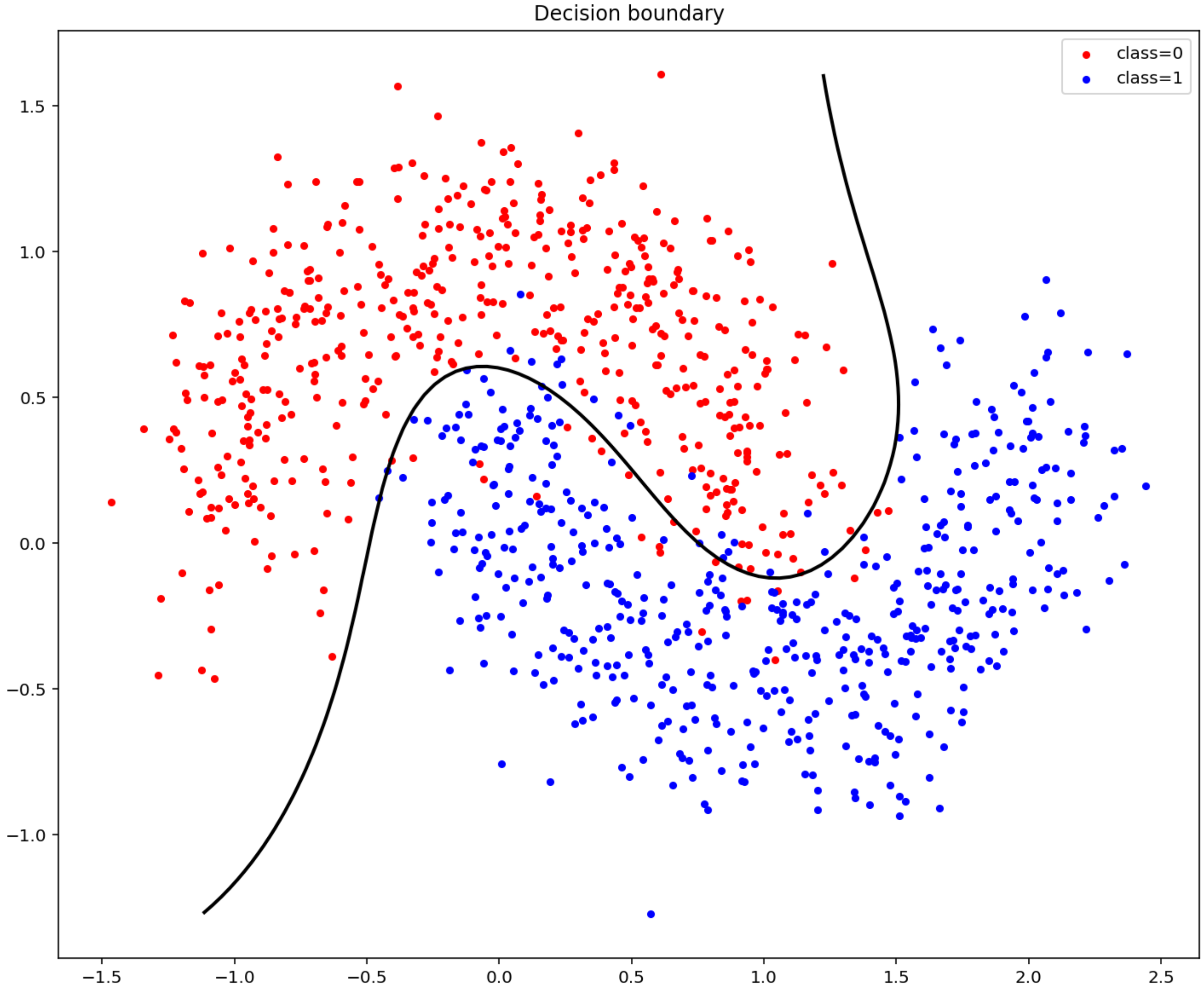
```
def function_f(x, y, k):
    sum = 0
    if k % 3 == 0:
        sum = (x * y) ** (int(k/3))
    elif k %3 == 1:
        sum = x ** (int(k/3) + 1)
    else:
        sum = y ** (int(k/3) + 1)
    return sum
```

In [27]:

```
# compute values p(x) for multiple data points x
x1_min, x1_max = min(x1), max(x1) # min and max of grade 1
x2_min, x2_max = min(x2), max(x2) # min and max of grade 2
x1_coordinate = np.linspace(x1_min, x1_max)
x2_coordinate = np.linspace(x2_min, x2_max)

xx1, xx2 = np.meshgrid(x1_coordinate, x2_coordinate) # create meshgrid
X2 = np.zeros((len(xx1), len(xx2)))

for i in range(len(x1_coordinate)):
    for j in range(len(x2_coordinate)):
        sum = 0
        for k in range(10):
            sum += w[k] * function_f(x1_coordinate[i], x2_coordinate[j], k)
        X2[i,j] = sigmoid(sum)
```

In [28]:

```
# compute values p(x) for multiple data points x
x1_min, x1_max = min(x1), max(x1) # min and max of grade 1
x2_min, x2_max = min(x2), max(x2) # min and max of grade 2
x1_coordinate = np.linspace(x1_min, x1_max, 100)
x2_coordinate = np.linspace(x2_min, x2_max, 100)

xx1, xx2 = np.meshgrid(x1_coordinate, x2_coordinate) # create meshgrid
X2 = np.zeros((len(xx1), len(xx2)))

for i in range(len(x1_coordinate)):
    for j in range(len(x2_coordinate)):
        sum = 0
        for k in range(10):
            sum += w[k] * function_f(x1_coordinate[i], x2_coordinate[j], k)
        X2[i,j] = sum

# plot
plt.figure(4,figsize=(12,10))


plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, np.transpose(X2), [0], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary')
plt.show()
```



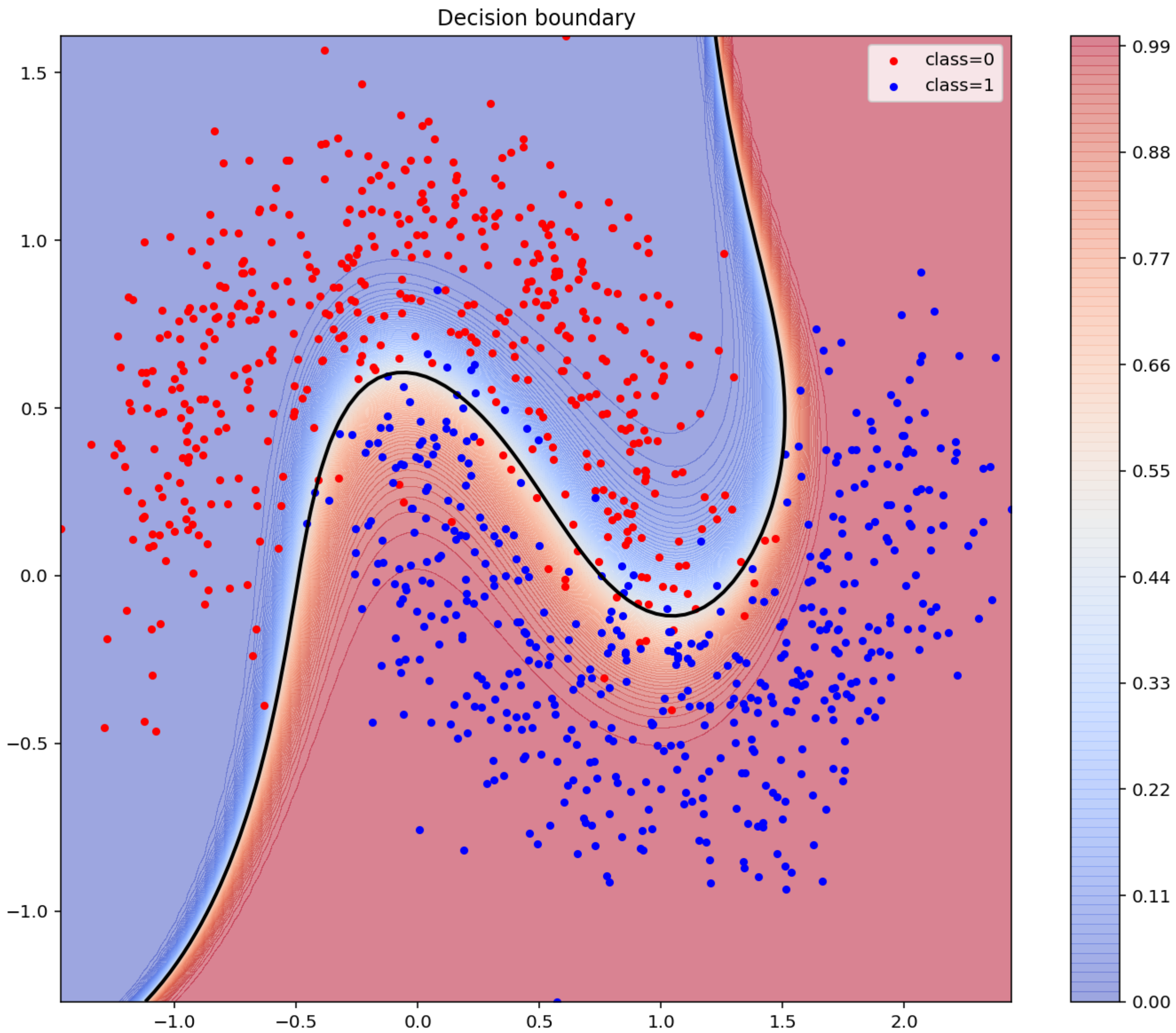
Decision boundary
class=0
class=1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5

/

5. Plot the probability map

In [29]:

```
# compute values p(x) for multiple data points x
x1_min, x1_max = min(x1), max(x1) # min and max of grade 1
x2_min, x2_max = min(x2), max(x2) # min and max of grade 2
x1_coordinate = np.linspace(x1_min, x1_max, 100)
x2_coordinate = np.linspace(x2_min, x2_max, 100)

xx1, xx2 = np.meshgrid(x1_coordinate, x2_coordinate) # create meshgrid
X2f = np.zeros((len(xx1), len(xx2)))

for i in range(len(x1_coordinate)):
    for j in range(len(x2_coordinate)):
        sum = 0
        for k in range(10):
            sum += w[k] * function_f(x1_coordinate[i], x2_coordinate[j], k)
        X2f[i,j] = sigmoid(sum)

# plot
fig = plt.figure(4,figsize=(12,10))
ax = fig.add_subplot(111)
ax.set_xlim(x1_min, x1_max)
ax.set_ylim(x2_min, x2_max)
ax = plt.contourf(xx1, xx2, np.transpose(X2f), cmap=cm.coolwarm, alpha=0.5, levels = np.arange(0, 1.01, 0.01))
cbar = plt.colorbar( )
cbar.update_ticks()

plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, np.transpose(X2), [0], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary')
plt.show()
```


Decision boundary
class=0
class=1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
0.99
0.88
0.77
0.66
0.55
0.44
0.33
0.22
0.11
0.00

/

6. Compute the classification accuracy

The accuracy is computed by:

$$ \textrm{accuracy} = \frac{\textrm{number of correctly classified data}}{\textrm{total number of data}} $$

In [30]:

```
def check_acc(y_pred, y):
    sum = 0
    for i in range(len(y)):
        if y[i] == 0 and y_pred[i] < 0.5:
            sum += 1
        if y[i] == 1 and y_pred[i] >= 0.5:
            sum += 1
    return sum / len(y)
```

In [31]:

```
# compute the accuracy of the classifier
n = data.shape[0]
p = f_pred(X, w)

print('total number of data', n)
print('total number of correctly classified data = ', check_acc(p, y) * len(y))
print('accuracy(%) = ', check_acc(p, y))
```

```
total number of data 1000
total number of correctly classified data =  957.0
accuracy(%) =  0.957
```

Output using the dataset (dataset-noise-02.txt)

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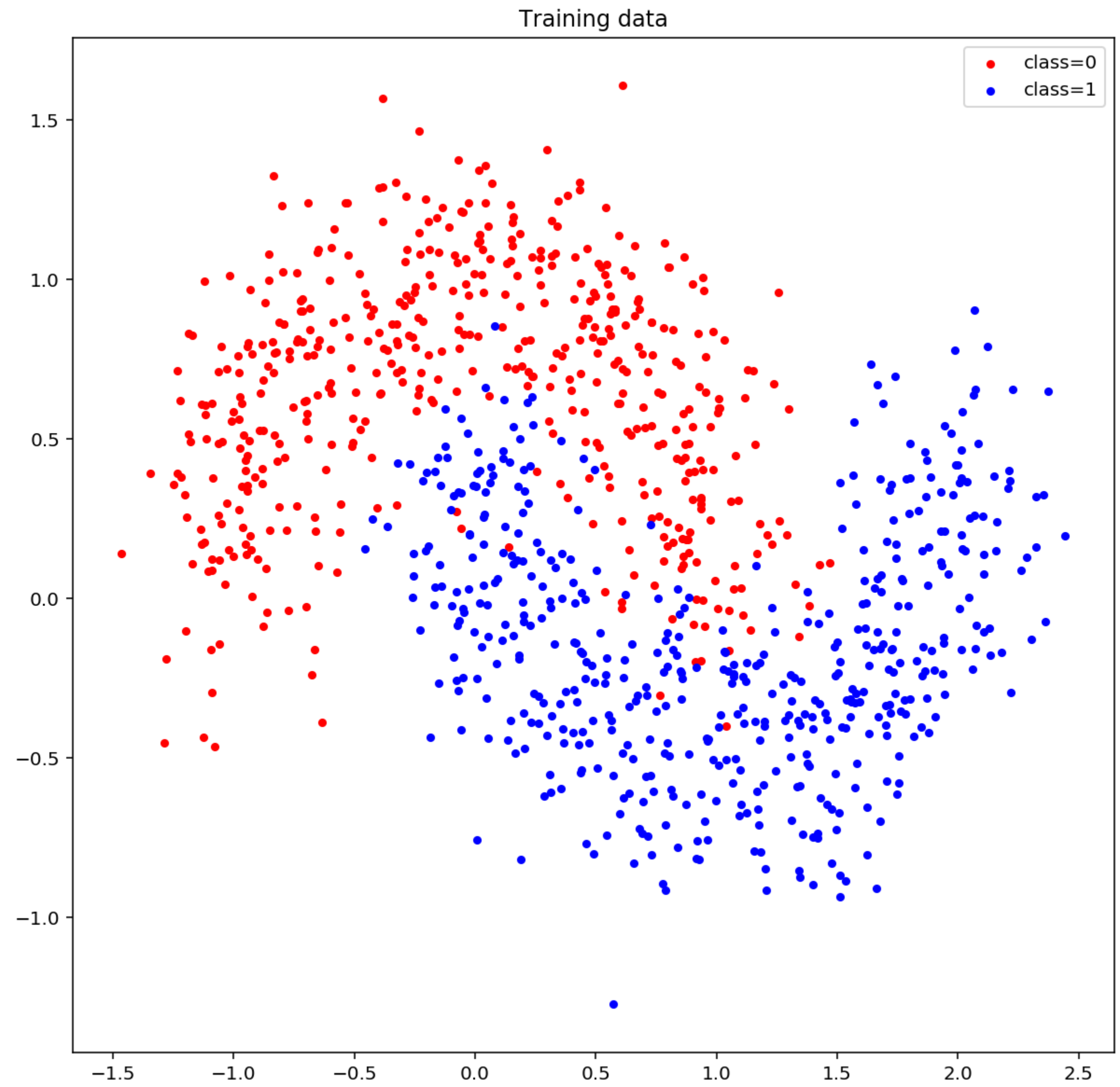
1. Visualize the data [1pt]

In [32]:

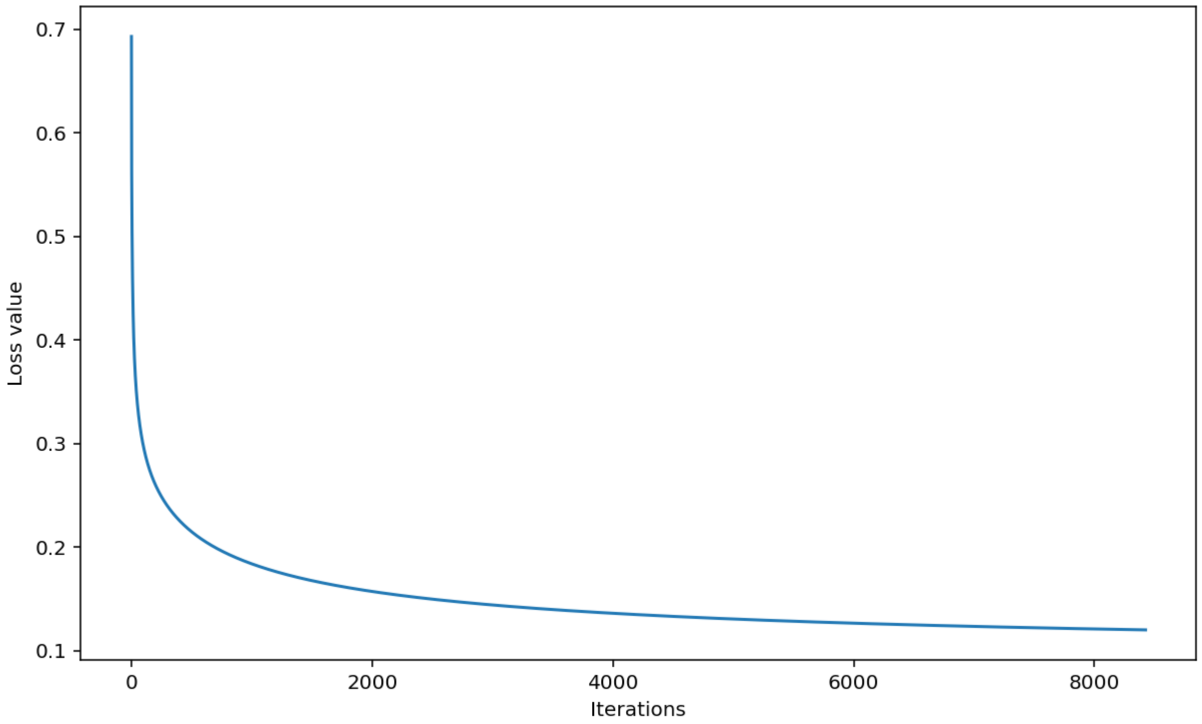
```
plt.figure(1,figsize=(10,10))
plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



2. Plot the loss curve obtained by the gradient descent until the convergence [2pt]

In [33]:

```
plt.figure(3, figsize=(10,6))
plt.plot([i for i in range(len(L_iters))], L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```



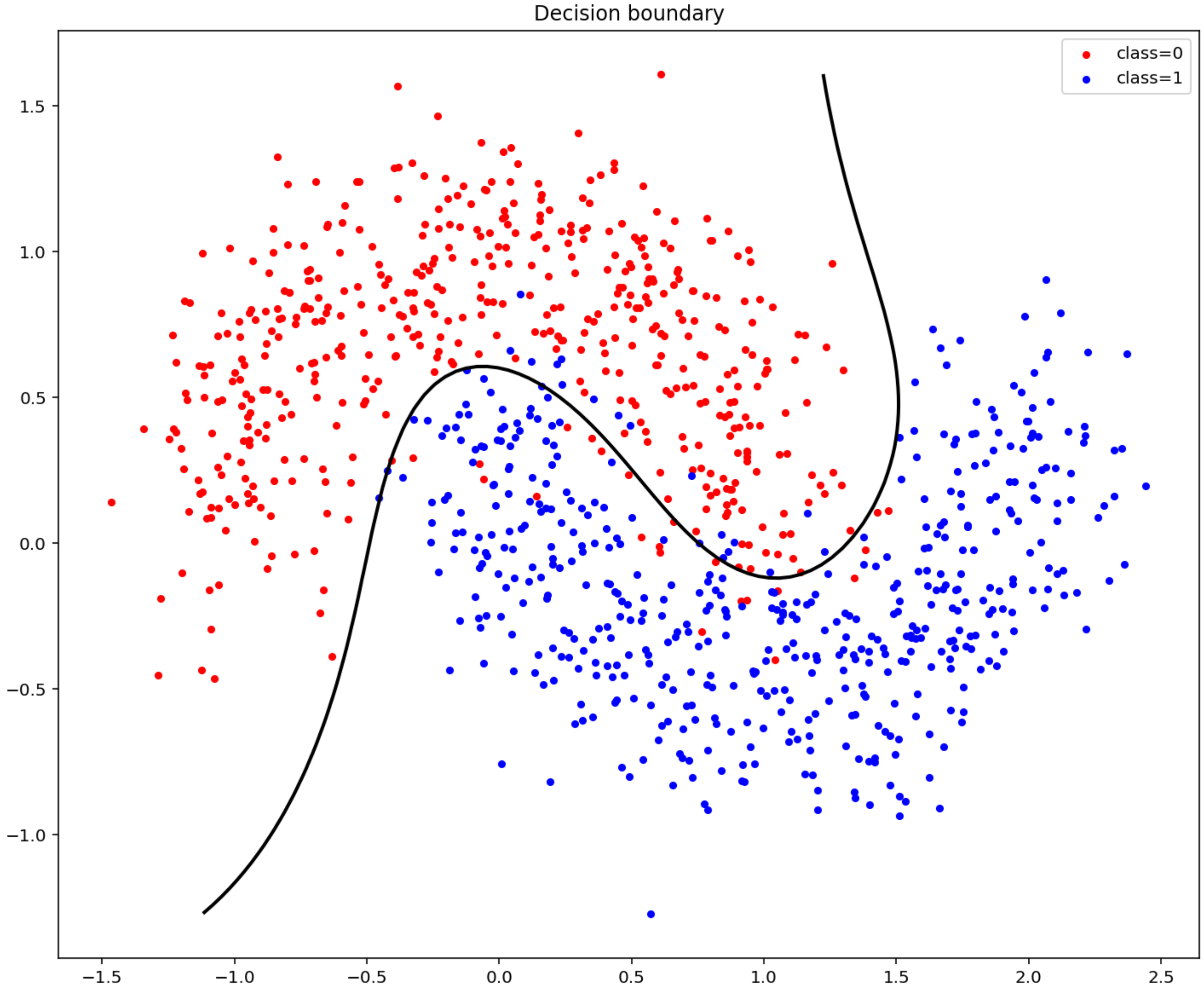
3. Plot the decisoin boundary of the obtained classifier [2pt]

In [34]:

```
plt.figure(4,figsize=(12,10))



plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, np.transpose(X2), [0], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary')
plt.show()
```

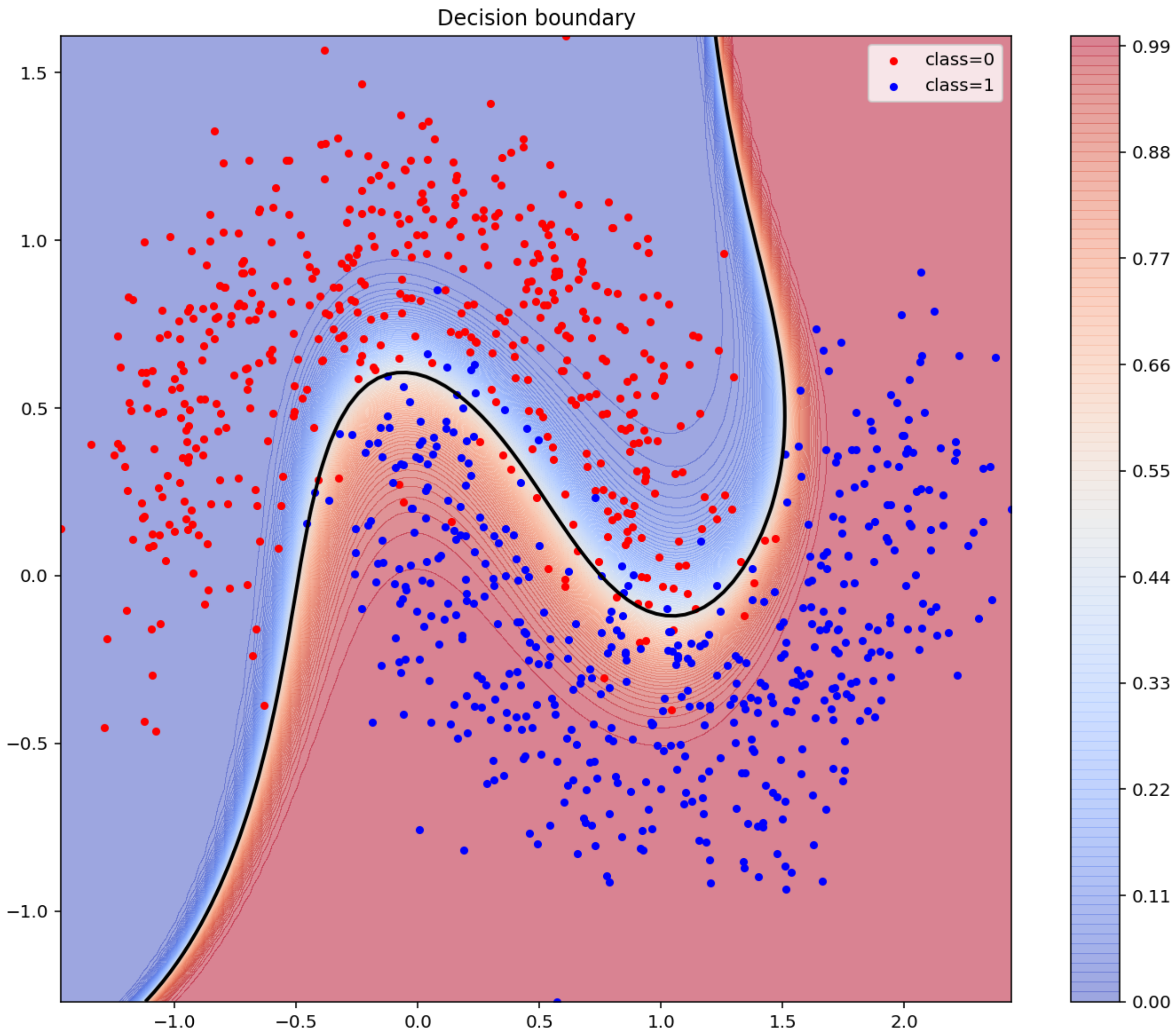


4. Plot the probability map of the obtained classifier [2pt]

In [35]:

```
fig = plt.figure(4,figsize=(12,10))
ax = fig.add_subplot(111)
ax.set_xlim(x1_min, x1_max)
ax.set_ylim(x2_min, x2_max)
ax = plt.contourf(xx1, xx2, np.transpose(X2f), cmap=cm.coolwarm, alpha=0.5, levels = np.arange(0, 1.01, 0.01))
cbar = plt.colorbar( )
cbar.update_ticks()

plt.scatter(idx_class0[:,0], idx_class0[:,1] , s=50, c='r', marker='.', label='class=0')
plt.scatter(idx_class1[:,0], idx_class1[:,1], s=50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, np.transpose(X2), [0], linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary')
plt.show()
```



Decision boundary
class=0
class=1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
0.99
0.88
0.77
0.66
0.55
0.44
0.33
0.22
0.11
0.00

/

5. Compute the classification accuracy [1pt]

In [36]:

```
print('total number of data', n)
print('total number of correctly classified data = ', check_acc(p, y) * len(y))
print('accuracy(%) = ', check_acc(p, y))
```

```
total number of data 1000
total number of correctly classified data =  957.0
accuracy(%) =  0.957
```