Image Processing & Vision

Lecture 01: Images & Transformations

Hak Gu Kim

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Graduate School of Advanced Imaging Science, Multimedia & Film (GSAIM)

Chung-Ang University (CAU)

06 Mar. 2023

Topics

- **Introduction**
  - About Me & IRIS LAB
  - Course Logistics
  - A Brief Introduction of Image Processing & Vision

- **Images & Transformation**
  - Images
  - Vectors & Matrices
  - Transformations

Topics

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  - Images
  - Vectors & Matrices
  - Transformations

About Me

Hak Gu Kim

EXPERIENCE

- **Assistant Professor** in GSAIM, **Chung-Ang University** (Sept. 2021 – present)
- **Postdoctoral Researcher** in School of Computer and Communication Sciences, **EPFL** (Aug. 2021)
- **Visiting Postdoctoral Researcher** in Institute of Electrical Engineering, **EPFL** (Aug. 2020)

EDUCATION

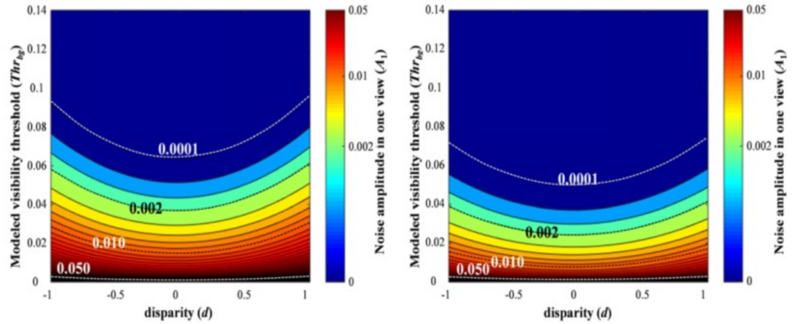
- **Ph.D.** in School of Electrical Engineering, **KAIST** (Feb. 2019)
- **M.S.** in Department of Electronic Engineering, **Inha University** (Feb. 2014)
- **B.S.** in Department of Electronic Engineering, **Inha University** (Feb. 2012)

RESEARCH ACHIEVEMENT

- **15 SCI/E Journal Papers (8 SCI/E Journals: 1st author)**
  - IEEE Trans. Image Process. / IEEE Trans. Circuits Syst. Video Technol. / Optics Express / Medical Physics ...
- **32 International Conferences (6 BK21+ CS Outstanding Conferences)**
  - 2 AAAI / 2 CVPR / 1 ECCV / 1 ACM VRST / ACM MMW / IEEE ICASSP / IEEE ICIP / IEEE EMBC …
- **Awards & Honors**
  - 2019: Postdoctoral Fellowship Scholarship (The 4-th Industrial Revolution), NRF of Korea
  - 2019: Best Paper Award Finalists (Top 2.1% of the accepted papers), IEEE ICIP
  - 2018: Robert F. Wagner All Conference Best Student Paper Final Lists Award, SPIE MI
  - 2018: Best Student Paper Award (Silver prize), IEEE/IEIE ICCE-Asia

My Research Experience and Key Achievements

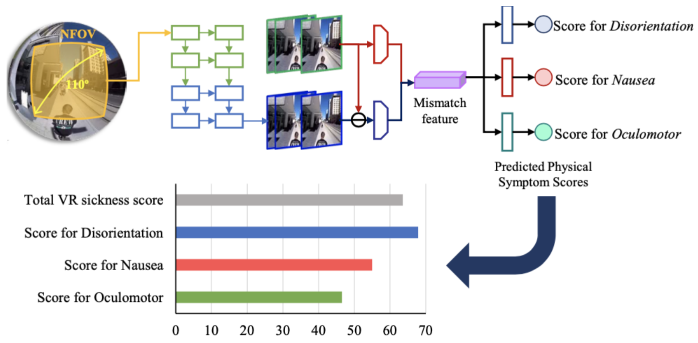
2012 – 2017: M.S. and Early Ph.D.



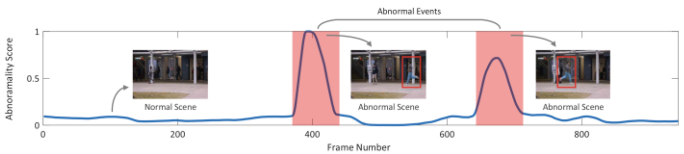
Human Perception in 3D Viewing
[IEEE TCSVT 2016, OE 2016]

Image and Video Processing
[JVCIR 2015, ICIP 2015, IEEE TCSVT 2017]

2017 – Present: Ph.D. and Postdoc

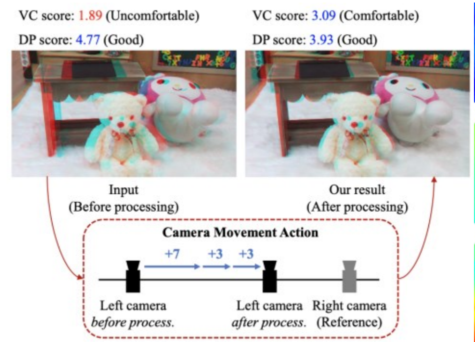


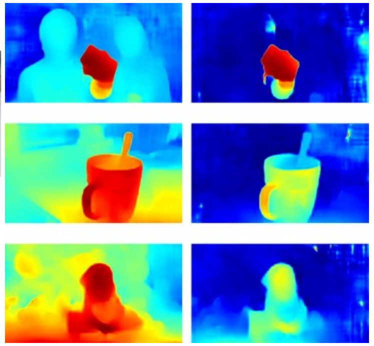
Brain-Inspired Neural Network
[ACM VRST 2017, ICIP 2017, ICASSP 2018, IEEE TIP 2019, IEEE TCSVT 2020, AAAI 2021]

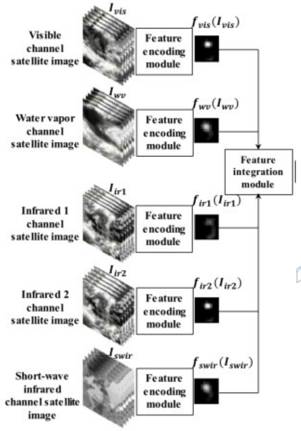


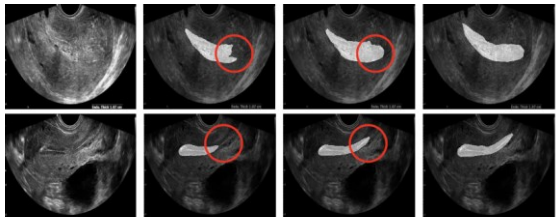
Attention-Aware Processing
[ICASSP 2018, IEEE TIP 2020]

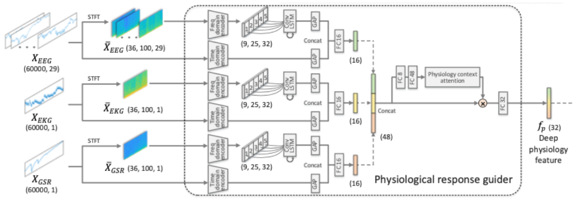
2020 – Present: Postdoc and Prof.











Learning with Domain Knowledge
[IEEE TRGS 2020, ECCV 2020, CVPR 2020, IEEE TCSVT 2021, AAAI 2021, CVPR 2021]

Immersive Reality & Intelligent Systems Lab (IRIS LAB)

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Graduate School of Advanced Imaging Science, Multimedia & Film (GSAIM), Chung-Ang Univ.

IRIS LAB
Immersive Reality & Intelligent Systems Lab.

Advisor

Prof. Hak Gu Kim

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- Office: Rm 818, 305 Bldg., CAU
- Email: hakgukim@cau.ac.kr
- Phone: 02-820-5972
- Web: www.irislab.cau.ac.kr

Introduction to IRIS LAB@CAU

CAU IRIS LAB

- **IRIS@CAU: Immersive Reality & Intelligent Systems (IRIS)**
  : Convergence of AI & VR/Metaverse
- **Major – VR/Game/Metaverse**
  : AI/ML-based 3D VR & Virtual human (Digital twin), AI in Metaverse

Main Research

- **Immersive Content Analysis**
  : Convergence of AI & VR/Metaverse
- **Attention-aware Processing**
  : Human vision-based AI modeling
- **Domain Knowledge Learning**
  : Interaction between human and AI

Recent Publications

Journals (2020, 2021)

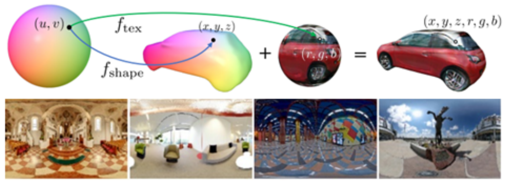
- **1 IEEE TIP** (JCR Top 5.7%, IF: 10.856)
- **4 IEEE TCSVT** (JCR Top 15.7%, IF: 4.685)

Conferences (2020, 2021)

- **2 CVPR** (*Top-tier AI & CV conf.*)
- **2 AAAI** (*Top-tier AI & CV conf.*)
- **1 ECCV** (*Top-tier AI & CV conf.*)

Immersive Content Analysis

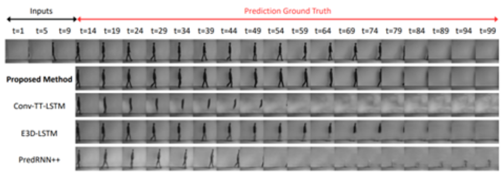
Stereoscopic 3D (S3D) depth editing



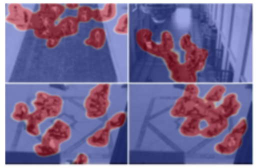
3D/VR/ 360° Content Representations

- ❖ AI-based S3D Content Editing
- ❖ AI-based 360° Image & Video Analysis for VR/Metaverse Content Creation

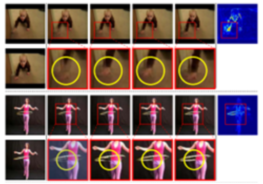
Attention-Aware Processing



Long-term video prediction



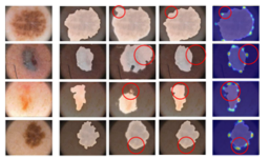
Anomaly detection



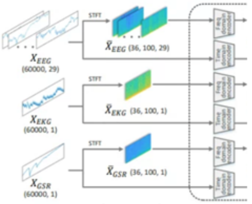
Video interpolation

- ❖ Human Visual Perception-based Video Understanding and Analysis
- ❖ AI-based Expression & Action Analysis

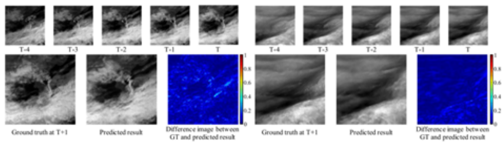
Domain Knowledge Learning



Medical image analysis



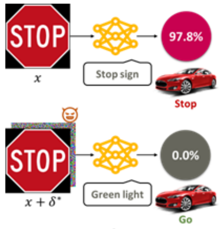
Physiological signal encoding



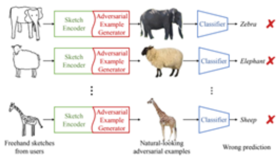
Weather forecasting

- ❖ Multi-Modal Learning (video & audio)
- ❖ Interactive Learning between Human Expert and AI Agent

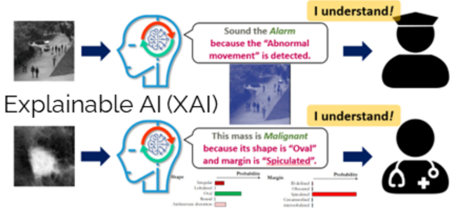
Adversarial Attack & Defense



Adversarial attack



Natural-looking adversarial examples



Explainable AI (XAI)

- ❖ Robustness of Deep Neural Network for Safe AI
- ❖ Explainable AI for reliable AI

Basic Course Information

- **Time & Location**
  - Time: Mon. 14:30 – 17:20
  - Location: Rm #706, 810 Art Center
  - Type: Online (Odd weeks) & Offline (Even weeks)

- **Instructor:** Hak Gu Kim
  - E-mail: hakgukim@cau.ac.kr
  - Webpage: www.irislab.cau.ac.kr
  - Office: Rm #818, 305 Building, CAU, Seoul
  - Office hour: Wed. 13:00 – 15:00

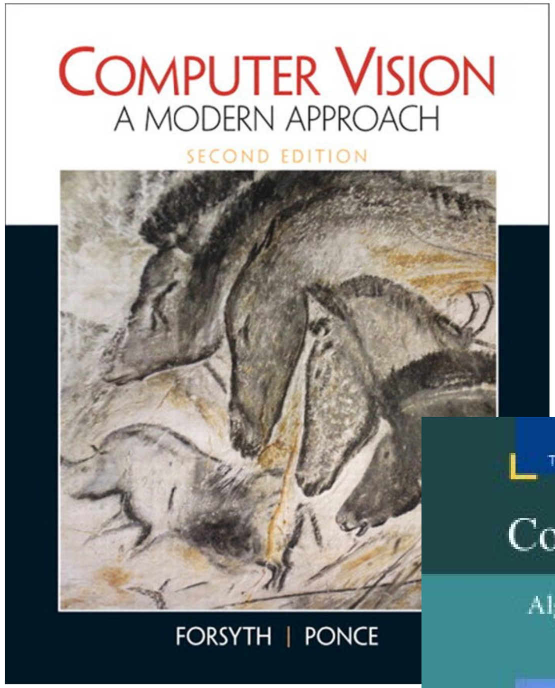
Basic Course Information: Textbook

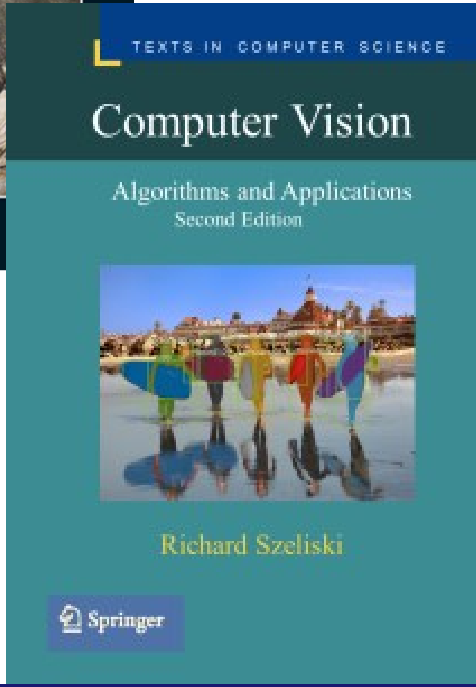
- Following are recommendation but not required

— *Computer Vision: A Modern Approach*
(David A. Forsyth and Jean Ponce)

— *Computer Vision: Algorithms and Applications*
(Richard Szeliski)

- https://szeliski.org/Book/





Grading Policy

- **Homework: 60%**
  - Homework 1 – 6 (60% = 10% x 6), Once in 2 weeks
  - Python Programming for Image Processing & Vision Practice

- **Final Exam: 30%**
  - Summary Report

- **Attendance: 10%**
  - Late policy – For each missed class, 0.5% point will be deducted

Learning Objectives

- Study Basic Image Processing & Computer Vision

— Understand the basics of image processing and computer vision

— Study a wide range of image processing and computer vision applications

- Practical Understanding of Image Processing & Computer Vision Tasks

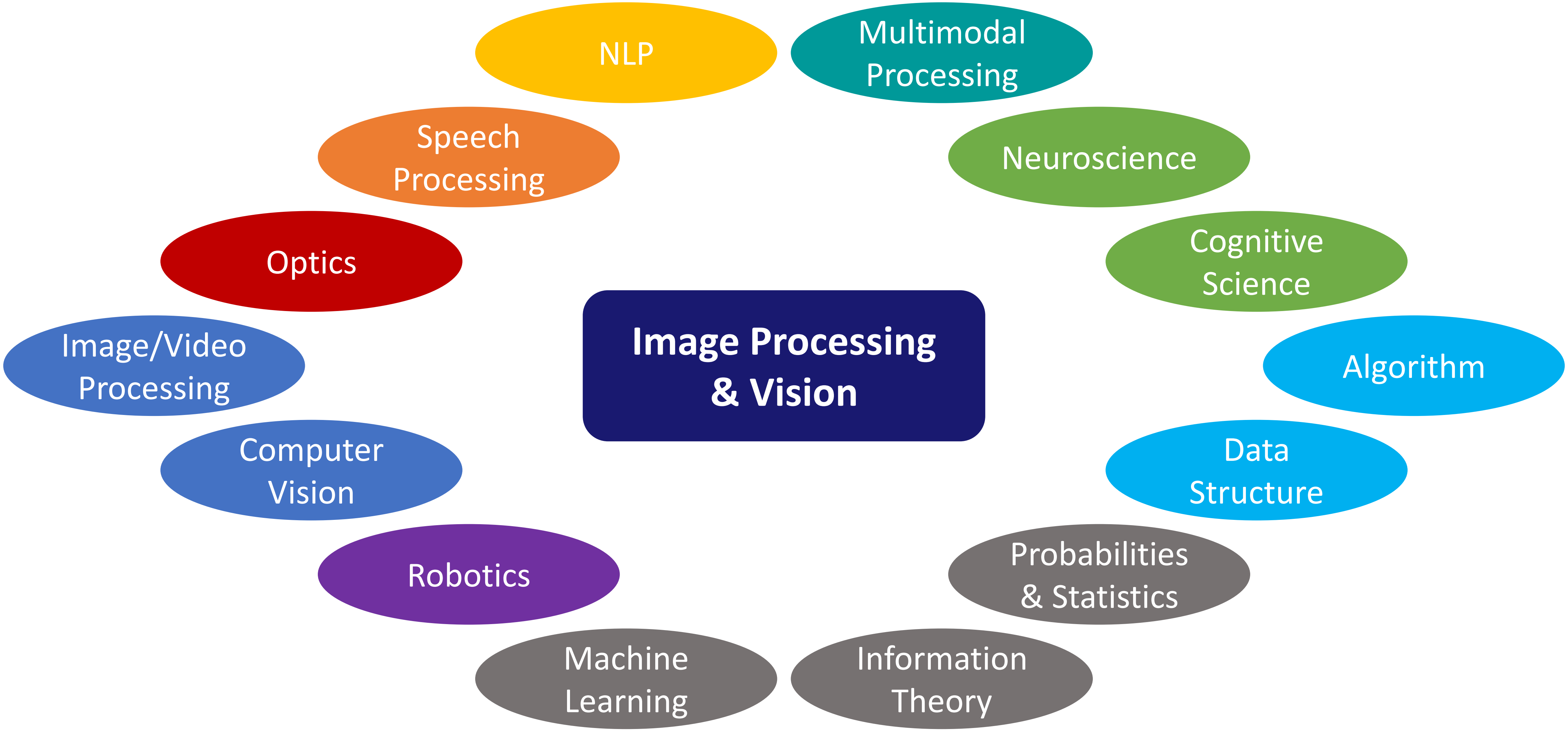
— Learn how fundamental theory is applied to image processing and computer vision applications

— Learn the link between image signal processing theory and implementation

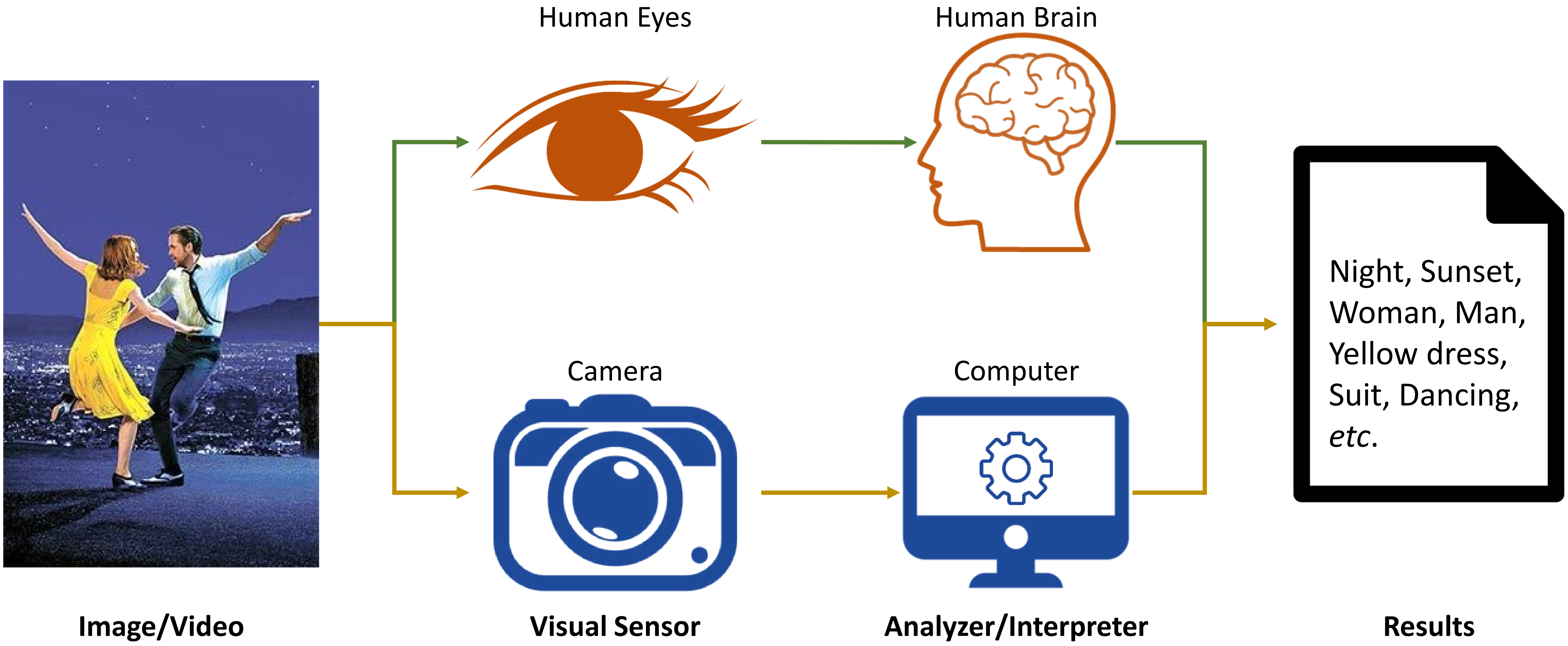
Roadmap: Image Processing & Vision

Pixels	Segments	Images	Videos	Advanced Topic
- Filtering	- Segmentation	- Recognition	- Motion	- Neural Networks
- Edges	- Clustering	- Detection	- Tracking	- CNNs for CV
- Features	- Resizing			

Brief Introduction



Human Vision vs. Computer Vision



The Goal of Computer Vision

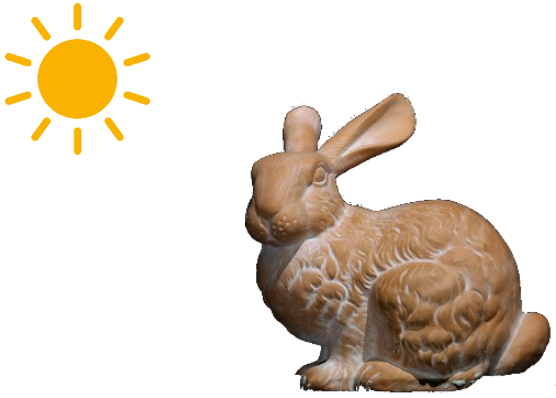
- To Bridge the Gap Between "Pixels" and "Meaning"

What we see

161	165	167	168	169	172	172	170
165	170	173	172	172	174	175	173
165	171	174	171	168	169	171	172
167	173	181	175	169	168	172	174
174	179	181	175	169	168	172	174
176	180	179	173	168	164	166	165
173	175	172	166	163	164	166	165
172	172	168	163	160	163	164	162

What a computer sees

Image Processing: **Digital Photography Pipeline**



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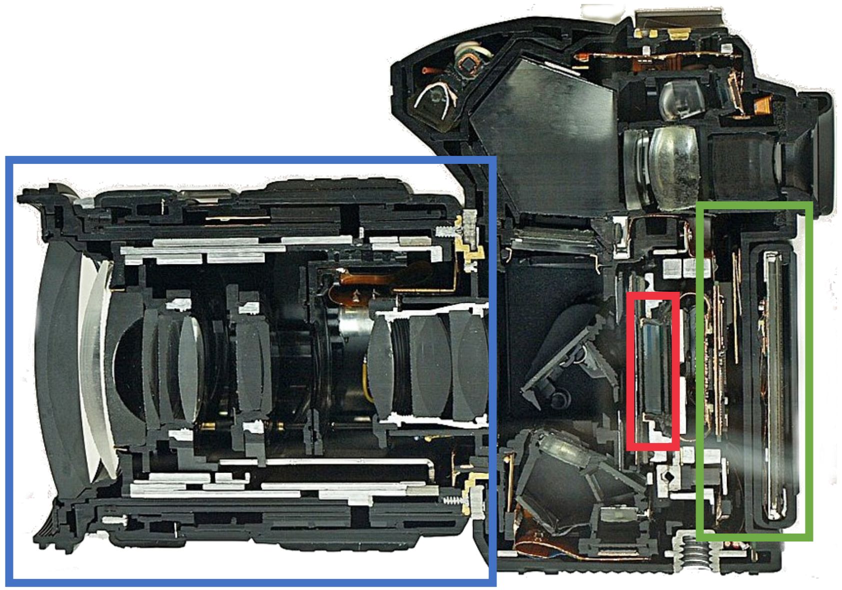
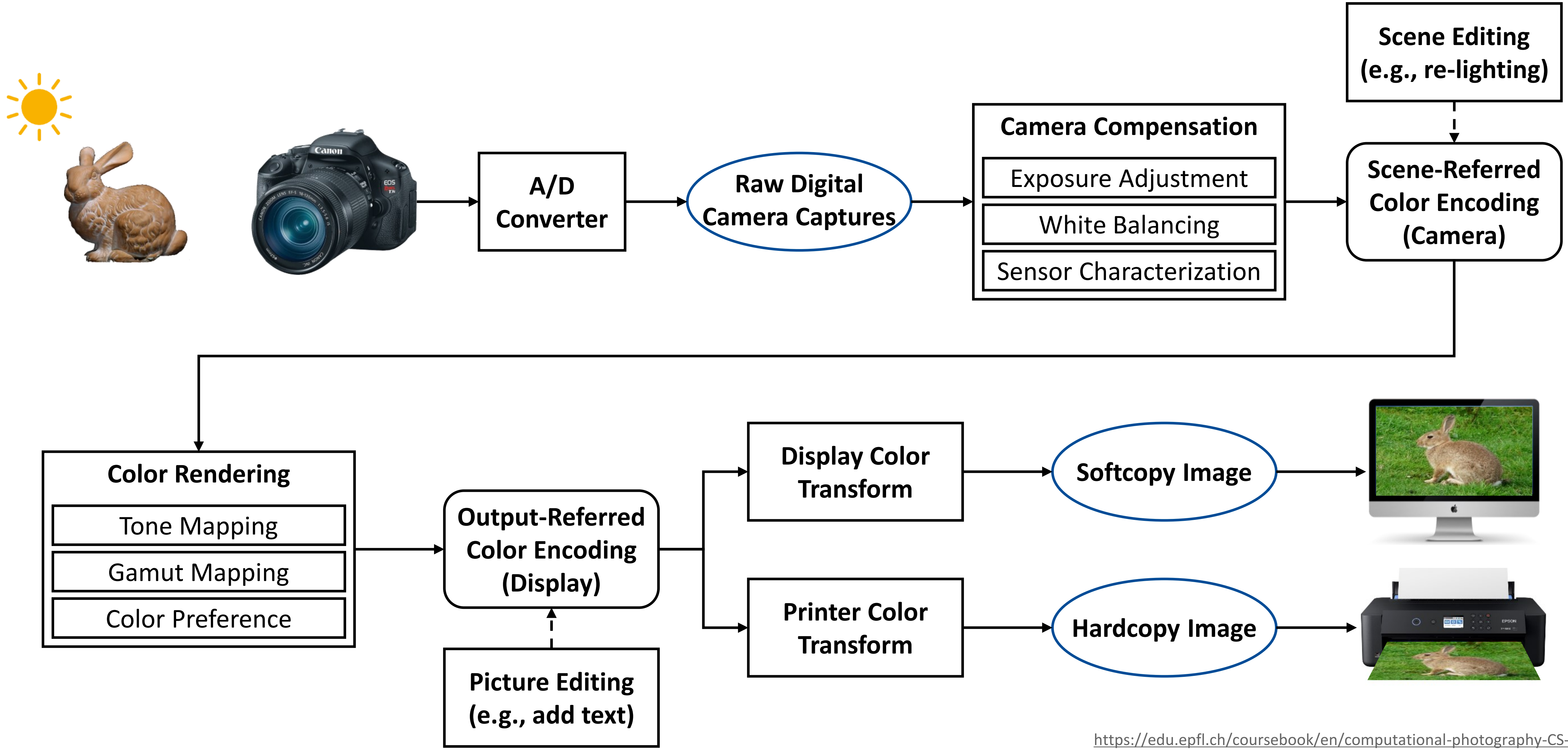


Physics & Optics → **Sensors** → **In-camera Image Processing**

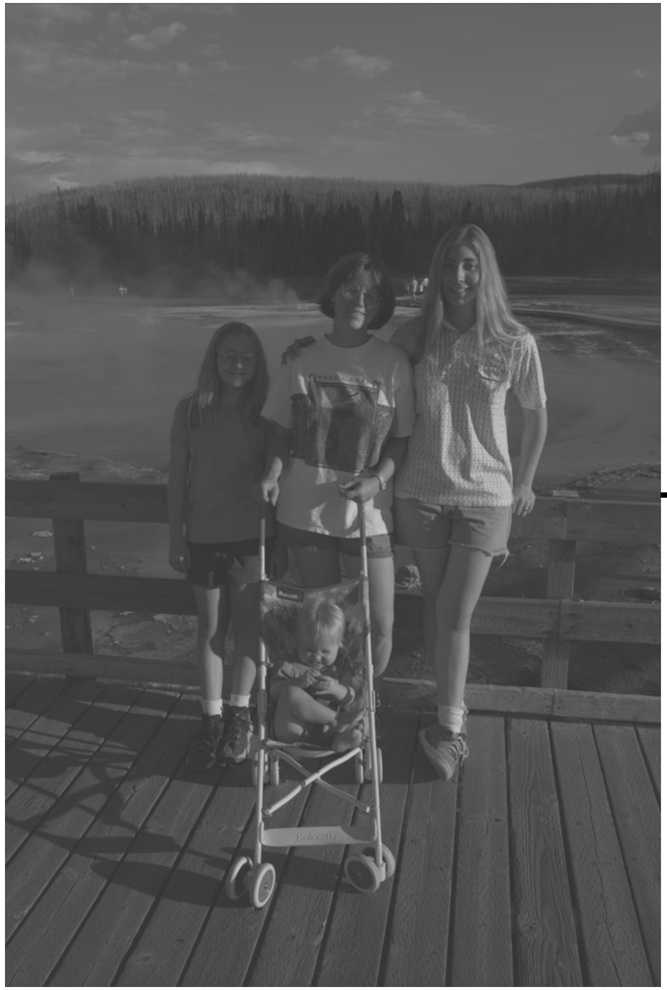
http://graphics.cs.cmu.edu/courses/15-463/

Image Processing: **From Raw To Rendered**

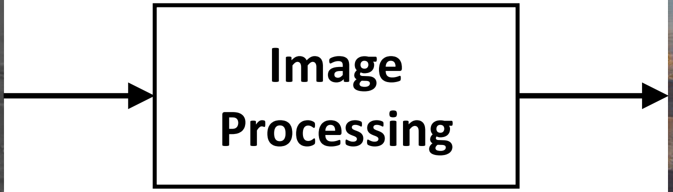


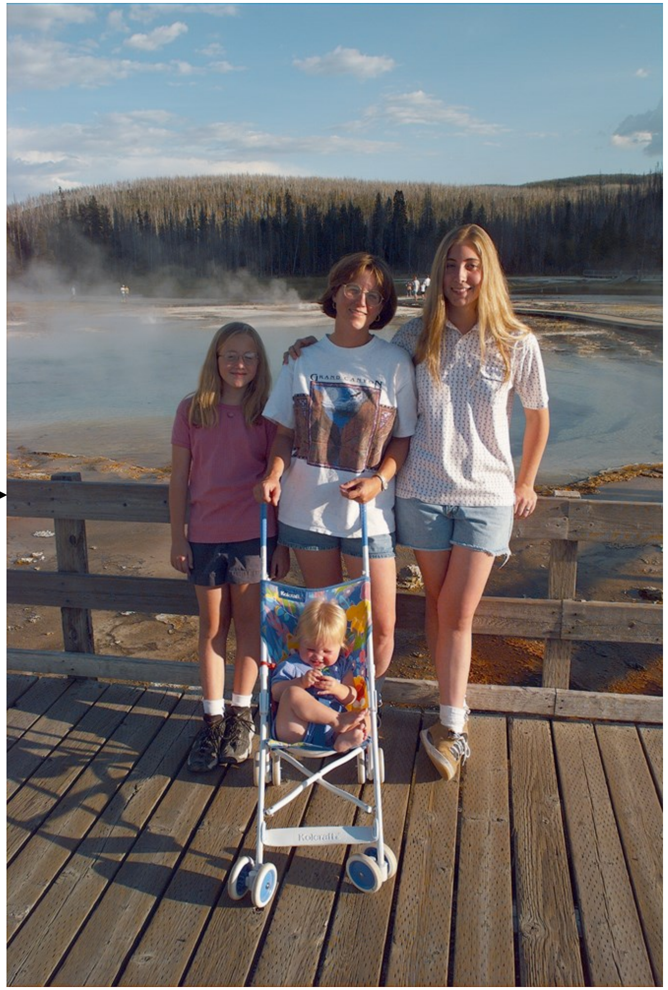
https://edu.epfl.ch/coursebook/en/computational-photography-CS-413

Image Processing: **Raw Image vs. Rendered Image**



Raw image (Captured)





Rendered image (Displayed)

Computer Vision

- The Classic Problems of Computational Vision
— Reconstruction in 3D space
— Recognition
— Reorganization

Computer Vision: **Applications**

- Reconstruction in 3D space:

— Large-scale 3D reconstruction

Building Rome in a Day, **ICCV, 2019**
Atlas: End-to-End 3D Scene Reconstruction from Posed Images, **ECCV, 2020**

Computer Vision: **Applications**

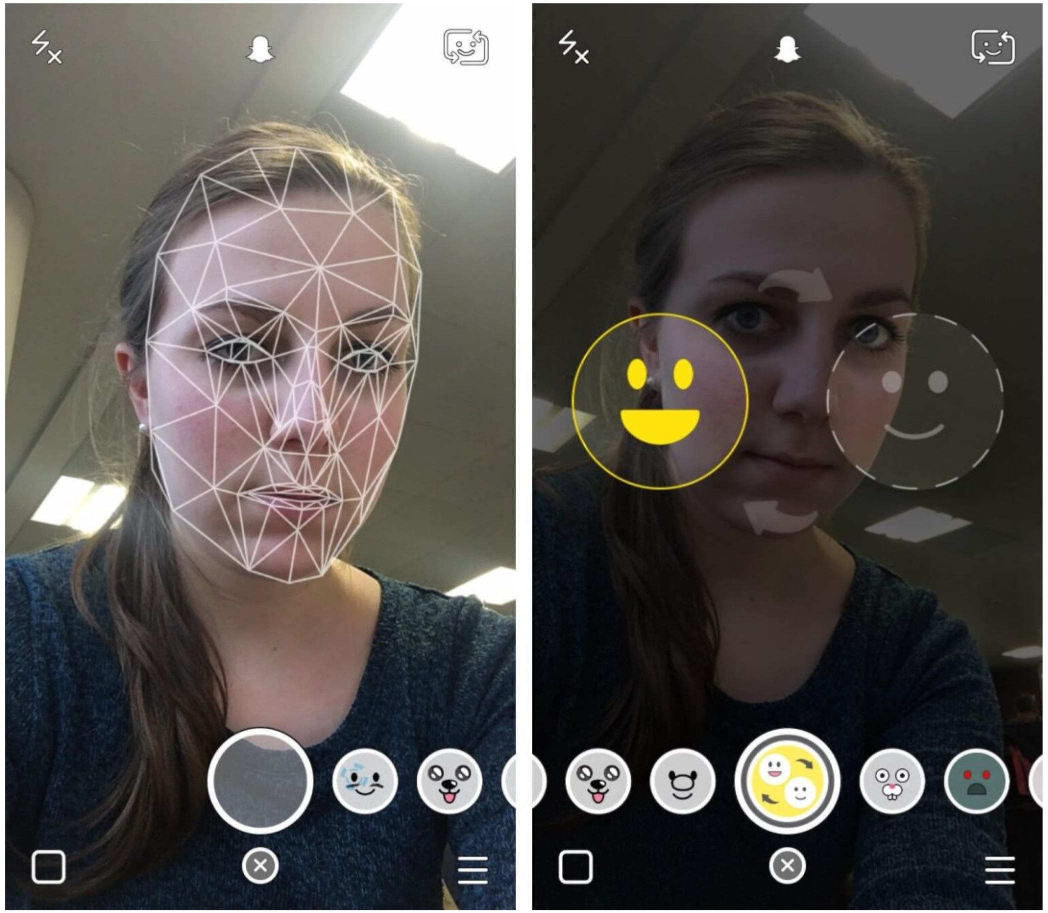
- Reconstruction in 3D space:

— Human shape capture and reconstruction

Computer Vision: **Applications**

- Recognition:

— Face Detection & Recognition

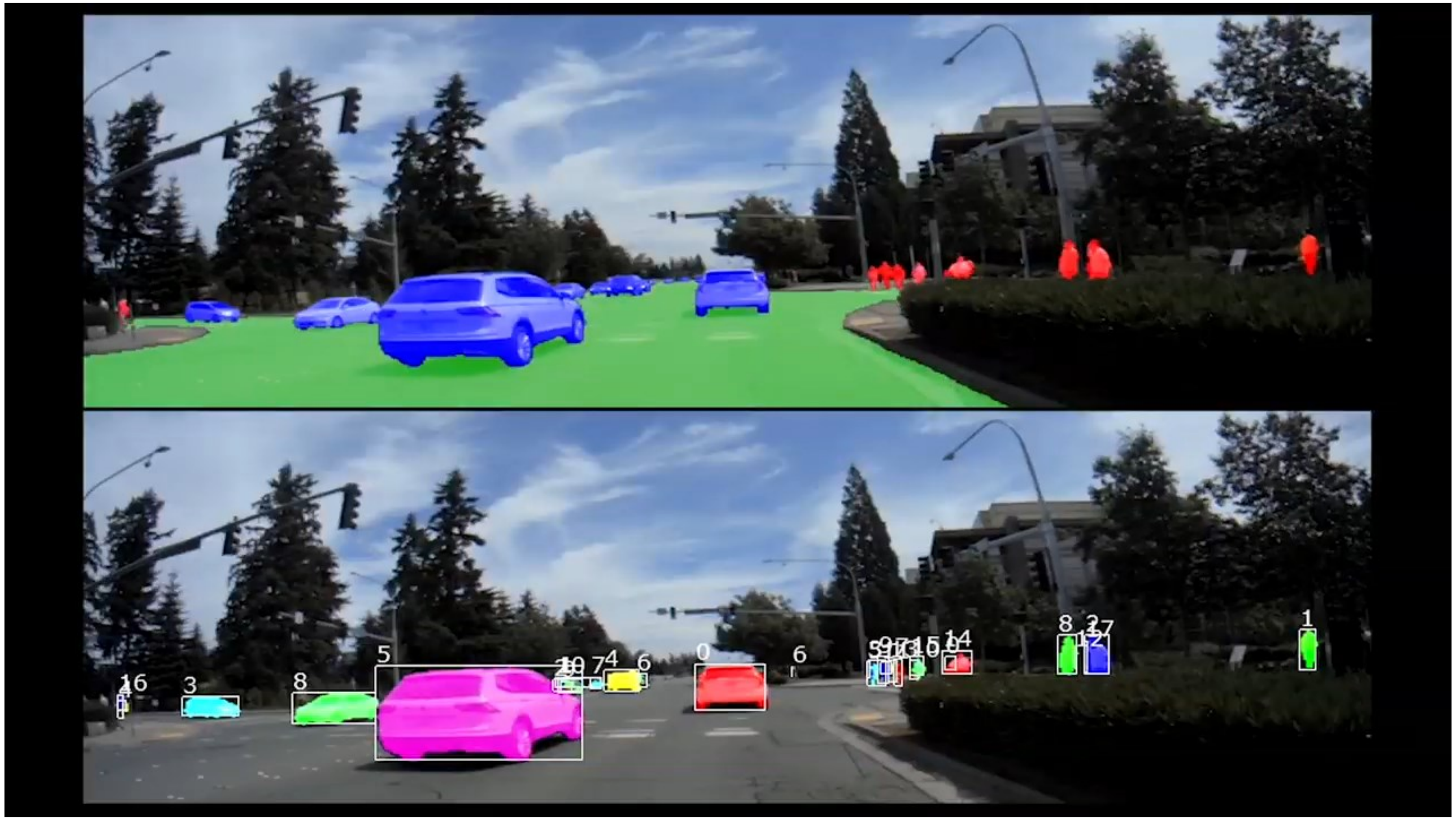


Computer Vision: **Applications**

- Recognition:

— Object Recognition in Autonomous Cars

https://www.youtube.com/watch?v=HS1wV9NMLr8



Computer Vision: **Reorganization**

- Reorganization:

— Semantic Segmentation

SceneCut: Joint Geometric and Object Segmentation for Indoor Scenes, **ICRA, 2018**
https://viso.ai/deep-learning/image-segmentation-using-deep-learning/

Image Processing & Vision: **Applications**

- NASA's Mars Exploration Rover Spirit
  - Panorama image stitching
  - 3D terrain modeling
  - Obstacle detection
  - Position tracking

Matthies et al., Computer Vision on Mars, 2007

Applications: Image Processing & Vision

- STAIR: STanford Artificial Intelligence Robot
  - Fetch or deliver items around the home or office
  - Tidy up a room, including picking up and throwing away trash, and using the dishwasher
  - Prepare meals using a normal kitchen
  - Use tools to assemble a bookshelf

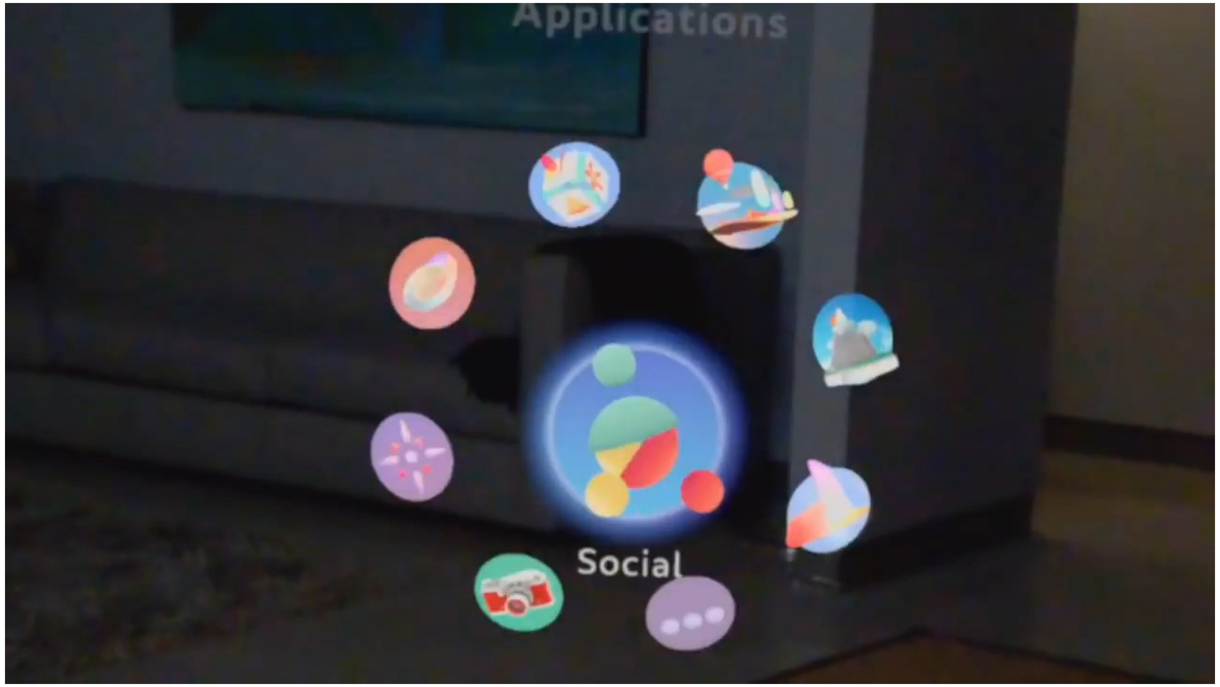
http://stair.stanford.edu/index.php

Learning grasp strategies with partial shape information, **AAAI, 2008**

Applications: Image Processing & Vision

- AR/VR/MR/XR/Metaverse
  - MS HoloLens
  - Oculus
  - Magic Leap

HoloLens 2 AR Headset: On Stage Live Demonstration



Introducing Avatar Chat on Magic Leap One | Feature Trailer

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- **Images & Transformation**
  - Images
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Images

- Types of Images

Binary Image

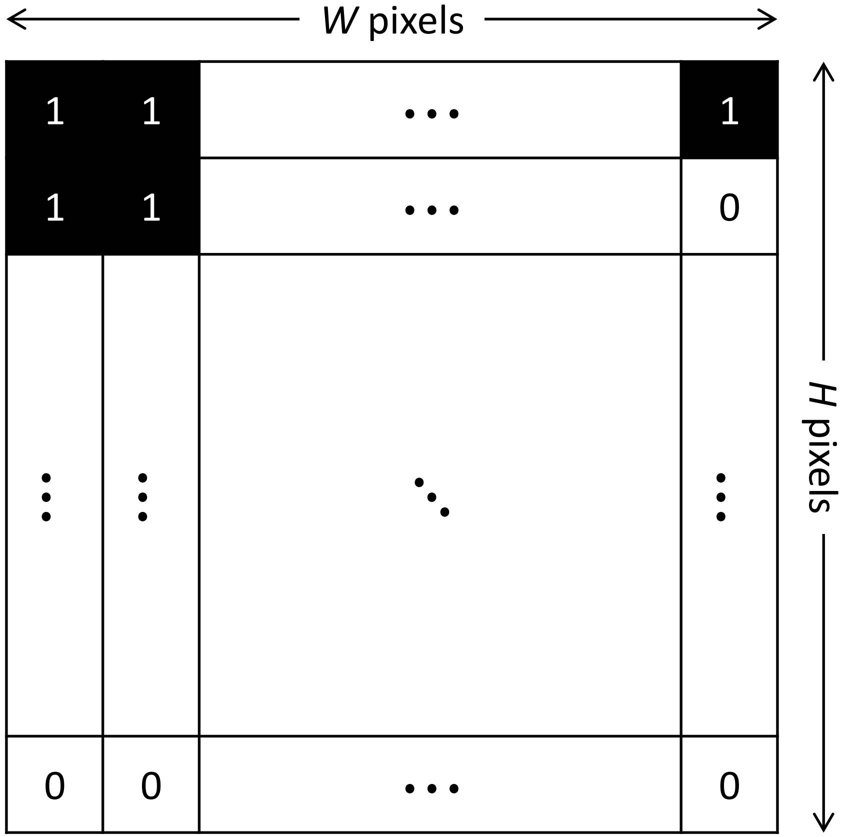
Grayscale Image

Color Image

Images: **Binary Image**

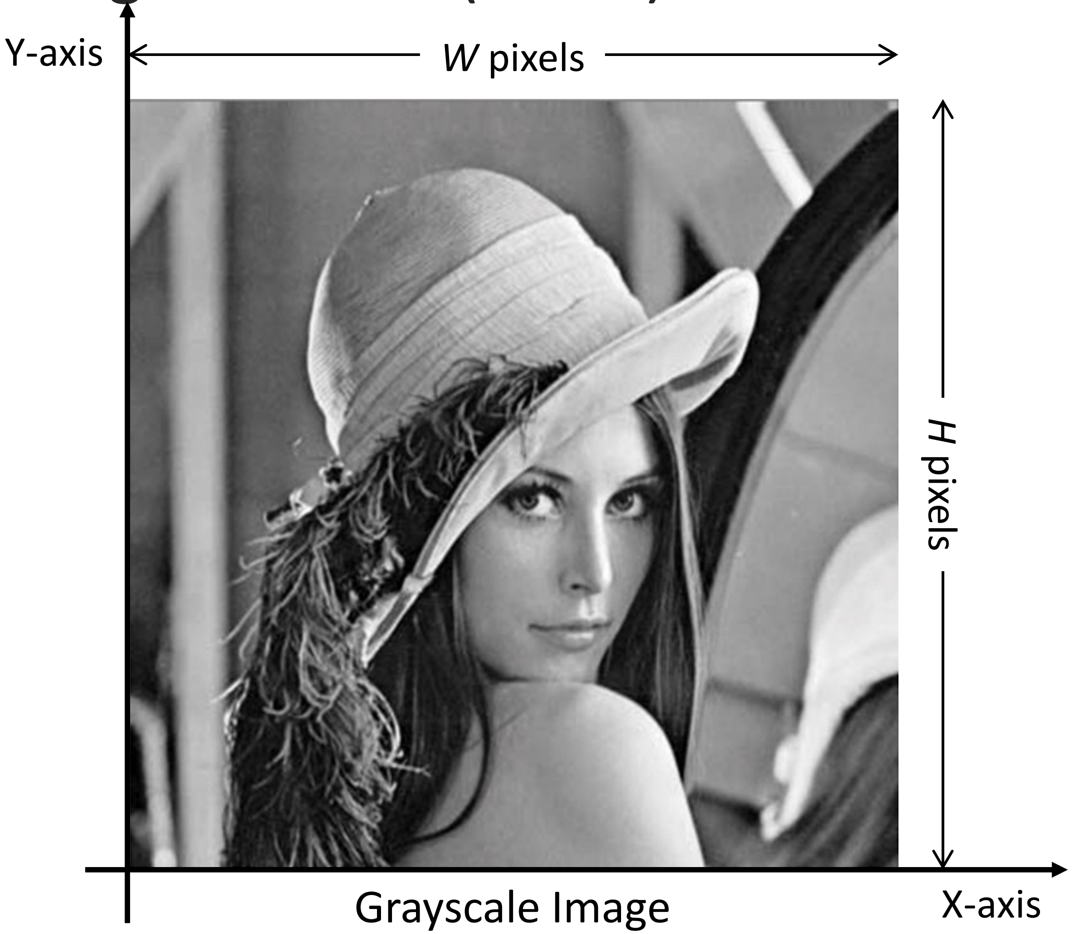
- Binary Image Representation
  - Range: 0 (black) or 1 (white)

Binary Image

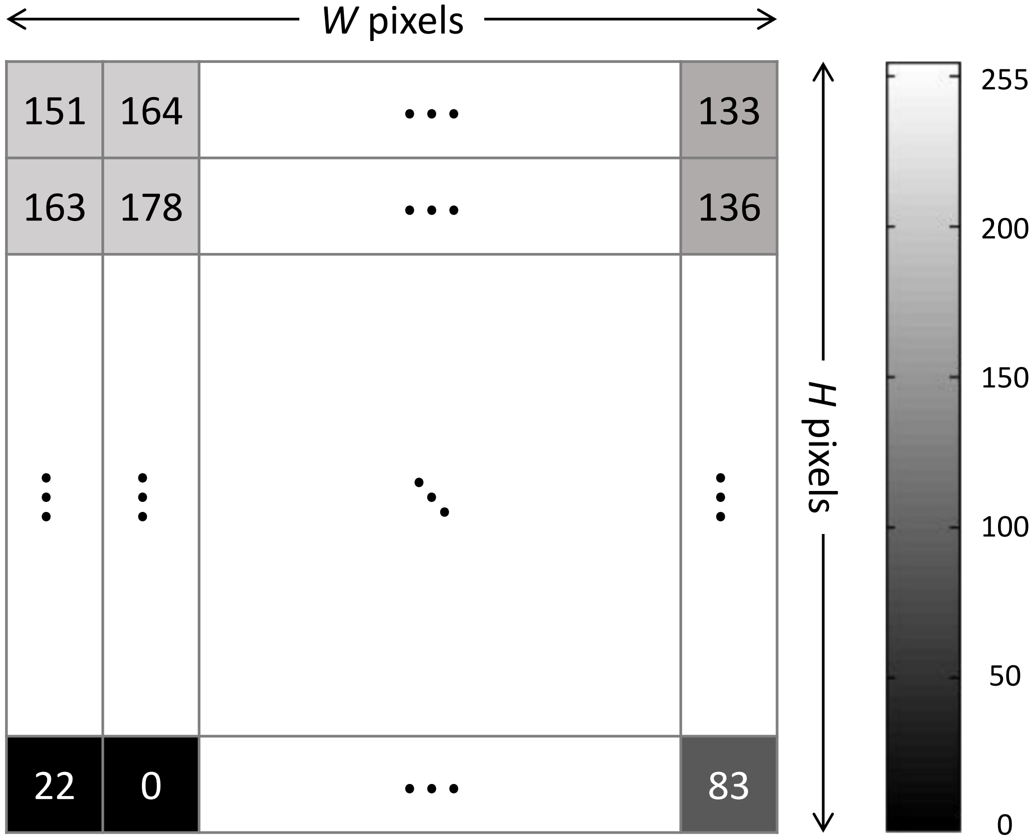


Images: **Grayscale Image**

- Grayscale Image Representation
  - Range: 0 – 255 (8 bits)



Grayscale Image



Images: **Color Image**

- Color Image Representation

— Range: 0 – 255 (8bits) & 3 Channels (R/G/B)

Color Image

R channel

224	230	⋯	198
226	234	⋯	196
⋮	⋮	⋱	⋮
29	24	⋯	153

159	159	⋯	108
158	153	⋯	108
⋮	⋮	⋱	⋮
19	14	⋯	79

G channel

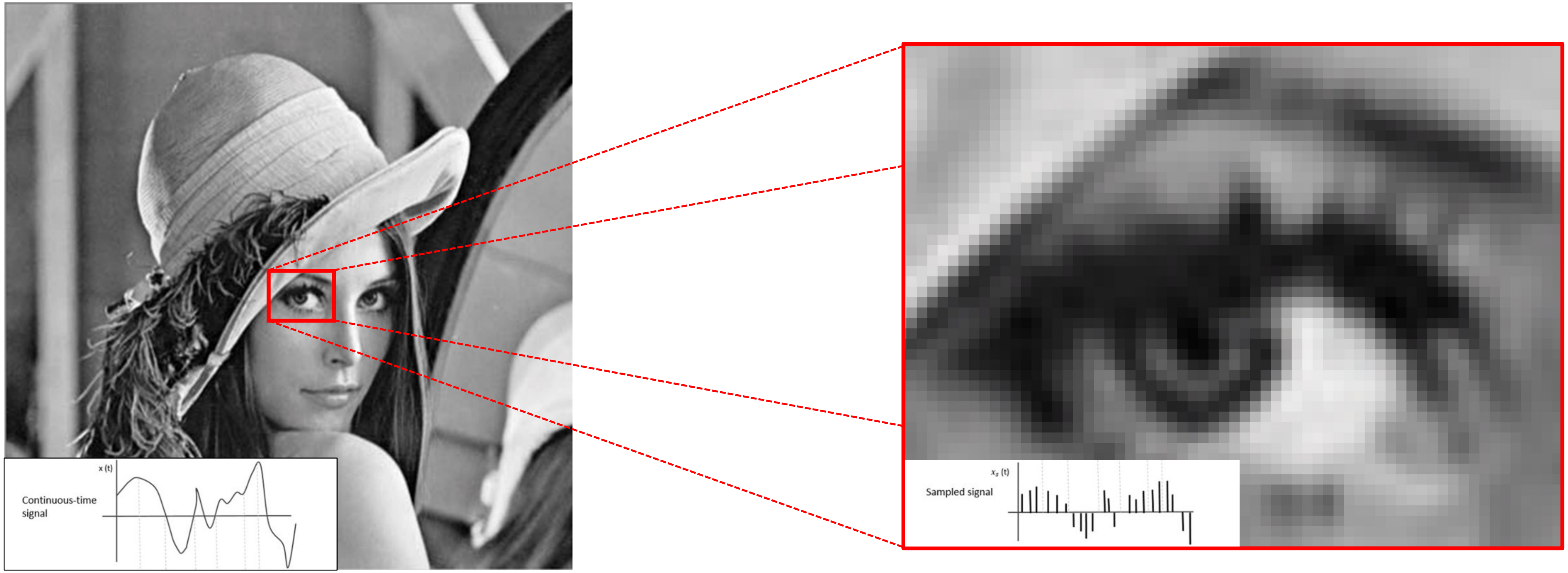
127	125	⋯	65
126	123	⋯	67
⋮	⋮	⋱	⋮
16	9	⋯	50

B channel

Conversion from Continuous to Discrete Signals

① **Sampling:** The process of extracting the samples from a continuous signal

② **Quantization:** The process of mapping input values from a large set (a continuous set) to output values in a smaller set (countable)

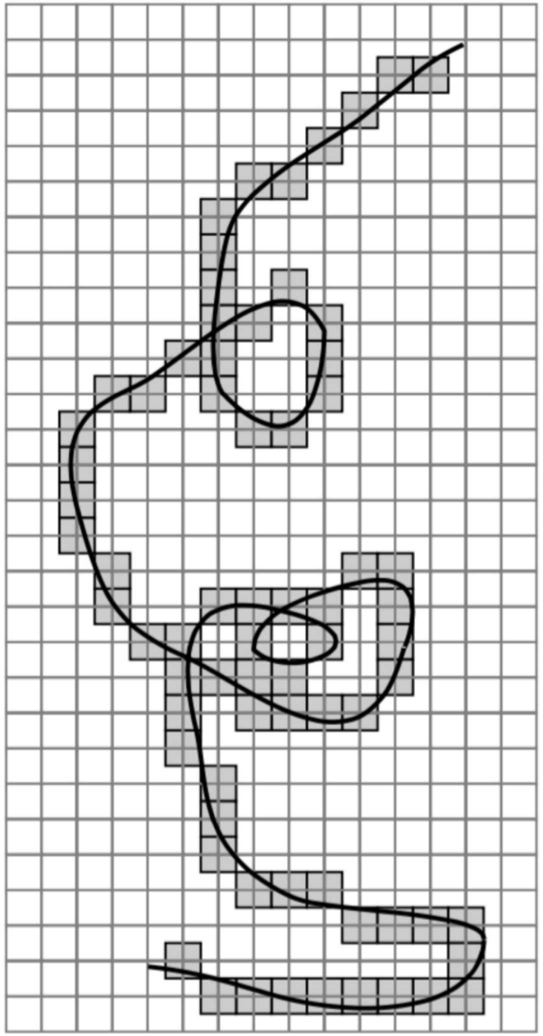


Continuous signal (Assumption)

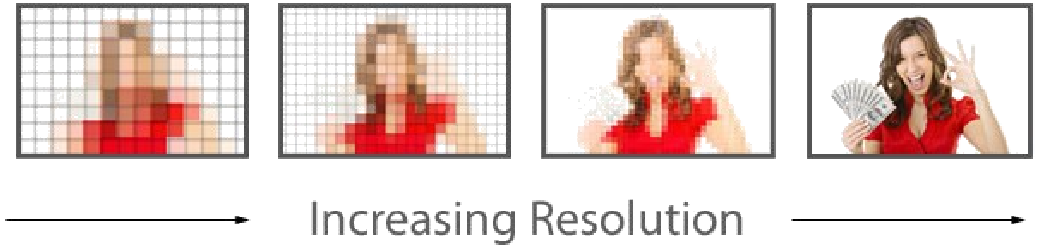
Digitalized image

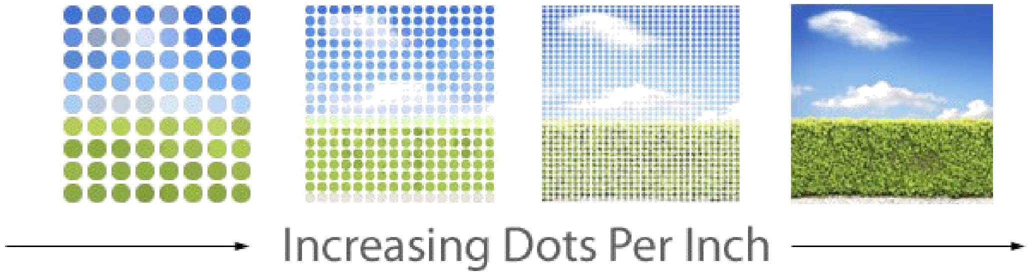
Sampling

- Errors due to Sampling



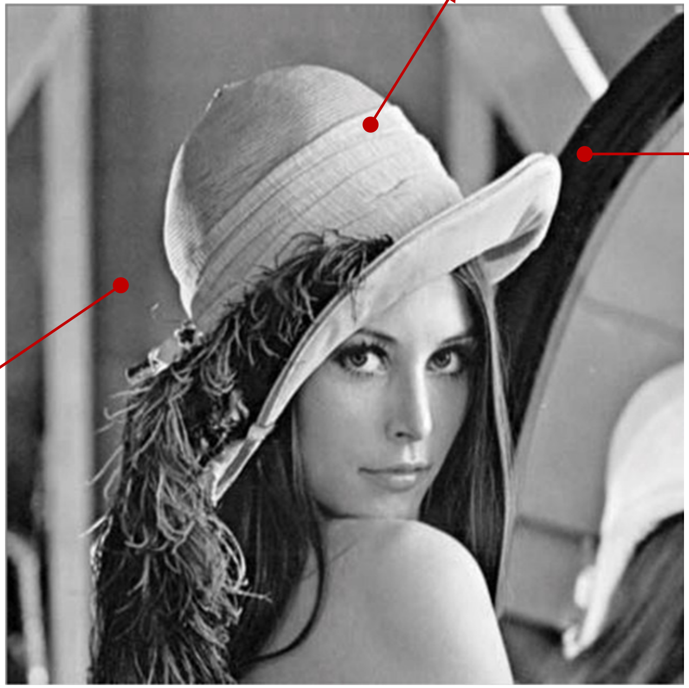
- **Spatial Resolution**





Quantization

- Discrete Number of Pixels in Digital Image
  - Pixel value in grayscale: [0, 255]
  - Pixel value in color image: [R; G; B]



Grayscale Image

Color Image

Vectors

- A Column Vector, $\mathbf{v} \in \mathbb{R}^{n\times 1}$
  - $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$
- A Row Vector, $\mathbf{v}^T \in \mathbb{R}^{1\times n}$
  - $\mathbf{v}^T = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}^T = [v_1 \quad v_2 \quad \cdots \quad v_n]$

Dot Product: Definition

- Dot Product

— The sum of the products of the corresponding entries of the two sequences of numbers

$$\mathbf{v} \cdot \mathbf{w} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$

- Algebraic Rules
  - $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$: Symmetry
  - $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$: Additivity
  - $c(\mathbf{v} \cdot \mathbf{w}) = c(\mathbf{v}) \cdot \mathbf{w} = \mathbf{v} \cdot (c\mathbf{w})$: Homogeneity
  - $\mathbf{v} \cdot \mathbf{v} \geq 0$: Positivity
  - $\mathbf{v} \cdot \mathbf{v} = 0$ if and only if $\mathbf{v} = 0$: Definiteness

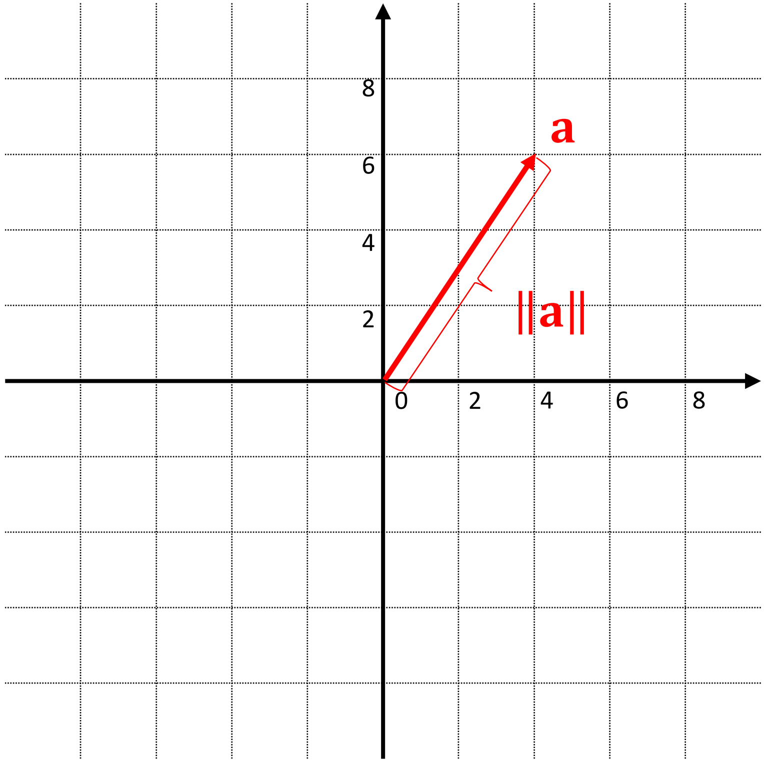
Dot Product: Length

- Length of Vector
  - A norm $\|\mathbf{v}\|$ is defined as a real-valued size measuring function on a vector $\mathbf{v}$

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2} = \sqrt{\sum_{i=1}^{n} v_i^2}$$

$$\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v} = v_1^2 + v_2^2 + \cdots + v_n^2$$

- Examples of Length
  - $\mathbf{a} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} \dashrightarrow \|\mathbf{a}\| = \sqrt{4^2 + 6^2} = \sqrt{52}$
  - $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \dashrightarrow \|\mathbf{b}\| = \sqrt{1^2 + 5^2 + 3^2} = \sqrt{35}$



Matrix: Definition

- Matrix

— A matrix is a rectangular, 2D array of values

— A matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ is an array of numbers with size m by n:

  - If $m = n$, $\mathbf{A}$ is a square matrix.

Matrix Operation: **Addition & Multiplication**

- Matrix Addition
  - $\mathbf{S} = \mathbf{A} + \mathbf{B}$

$$\begin{bmatrix} s_{1,1} & \cdots & s_{1,n} \\ \vdots & \ddots & \vdots \\ s_{m,1} & \cdots & s_{m,n} \end{bmatrix} = \begin{bmatrix} a_{1,1} + b_{1,1} & \cdots & a_{1,n} + b_{1,n} \\ \vdots & \ddots & \vdots \\ a_{m,1} + b_{m,1} & \cdots & a_{m,n} + b_{m,n} \end{bmatrix}$$

- Scalar Multiplication
  - $\mathbf{P} = c\mathbf{A}$

$$\begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,n} \end{bmatrix} = \begin{bmatrix} c \cdot a_{1,1} & \cdots & c \cdot a_{1,n} \\ \vdots & \ddots & \vdots \\ c \cdot a_{m,1} & \cdots & c \cdot a_{m,n} \end{bmatrix}$$

Matrix Operation: **Addition & Multiplication**

- Algebraic Rules of Matrix Addition and Scalar Multiplication
  - $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
  - $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
  - $\mathbf{A} + \mathbf{0} = \mathbf{A}$
  - $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$
  - $a(\mathbf{A} + \mathbf{B}) = a\mathbf{A} + a\mathbf{B}$
  - $a(b\mathbf{A}) = (ab)\mathbf{A}$
  - $(a + b)\mathbf{A} = a\mathbf{A} + b\mathbf{A}$
  - $1\mathbf{A} = \mathbf{A}$

Matrix Operation: **Transpose**

- Transpose

— The transpose of a matrix $\mathbf{A}$, $\mathbf{A}^T$, interchanges the rows and columns of $\mathbf{A}$

- It does this by exchanging elements across the matrix's main diagonal: $(\mathbf{A}^T)_{i,j} = (\mathbf{A})_{j,i}$
- The main diagonal doesn't change, or is invariant: $(\mathbf{A}^T)_{i,i} = (\mathbf{A})_{i,i}$

$$\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 0 & 2 \\ 6 & 3 \end{bmatrix} \qquad \mathbf{A}^T = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 2 & 3 \end{bmatrix}$$

- Algebraic Rules
  - $(\mathbf{A}^T)^T = \mathbf{A}$
  - $(a\mathbf{A}^T) = a\mathbf{A}^T$
  - $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$

Vector Representation: **Block Matrix**

- Block Matrix
  - A matrix can be represented by submatrices, rather than by individual elements. This is know as a block matrix

$$\mathbf{B} = \begin{bmatrix} 2 & 3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

  - A matrix can be represented by a set of row or column matrices

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$$

Vector Representation: **Matrix Product**

- Matrix Product
  - Multiplying a matrix by a compatible vector will transform the vector
  - Multiplying matrices together will create a single matrix that performs their combined transformations
    - $\mathbf{C} = \mathbf{AB}$

$$c_{i,j} = \sum_{k=1}^{n} a_{i,k} b_{k,j}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \begin{bmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \end{bmatrix} [\mathbf{b}_1 \quad \mathbf{b}_2] = \begin{bmatrix} \mathbf{a}_1 \cdot \mathbf{b}_1 & \mathbf{a}_1 \cdot \mathbf{b}_2 \\ \mathbf{a}_2 \cdot \mathbf{b}_1 & \mathbf{a}_2 \cdot \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1^T \mathbf{b}_1 & \mathbf{a}_1^T \mathbf{b}_2 \\ \mathbf{a}_2^T \mathbf{b}_1 & \mathbf{a}_2^T \mathbf{b}_2 \end{bmatrix}$$

Vector Representation: **Matrix Product**

- Algebraic Rules for Matrix Multiplication
  - $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$
  - $a(\mathbf{BC}) = (a\mathbf{B})\mathbf{C}$
  - $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
  - $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$
  - $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$

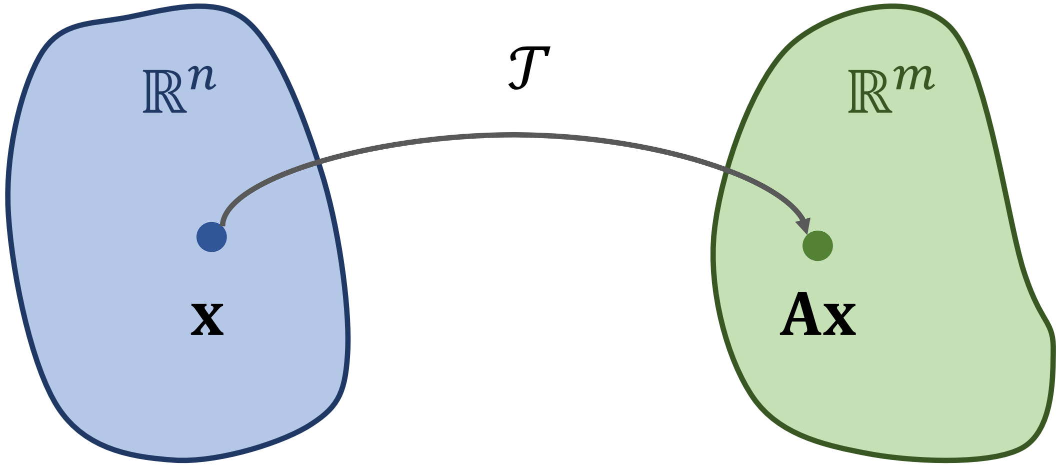
Linear Transformation

- Matrix-Vector Product as Linear Transformation

— Given $\mathbf{A}_{m\times n} = [\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_n]$, $\mathcal{T}: \mathbb{R}^n \to \mathbb{R}^m$:

$$\mathcal{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} = [\mathbf{v}_1\ \mathbf{v}_2\ \cdots\ \mathbf{v}_n]\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_n\mathbf{v}_n \in \mathbb{R}^m$$

- Example
  - $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $\mathcal{T}: \mathbb{R}^2 \to \mathbb{R}^2$, $\mathcal{T}(\mathbf{x}) = \mathbf{A}\mathbf{x}$
  - $\mathcal{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ 3x_1 + 4x_2 \end{bmatrix}$
  - $\mathcal{T}(x_1, x_2) = (2x_1 - x_2, 3x_1 + 4x_2,)$

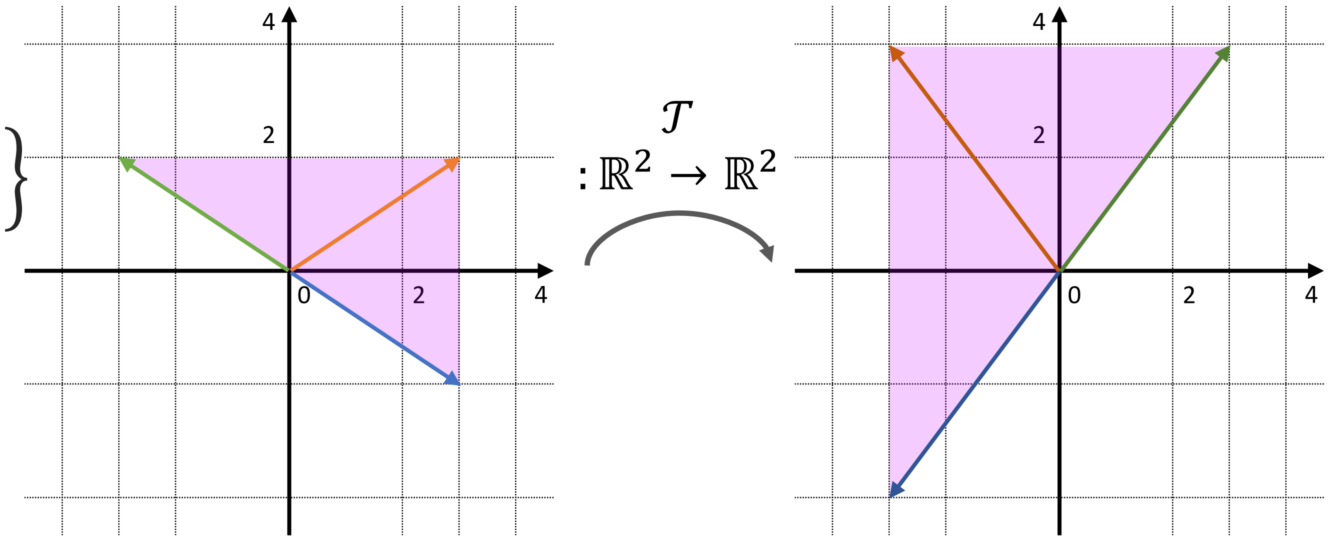


Linear Transformation: **Scaling and Reflection**

- $\mathcal{T}: \mathbb{R}^n \rightarrow \mathbb{R}^m,\ \mathcal{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} = [\mathcal{T}(\mathbf{e}_1)\ \mathcal{T}(\mathbf{e}_2)\ \cdots\ \mathcal{T}(\mathbf{e}_n)]$

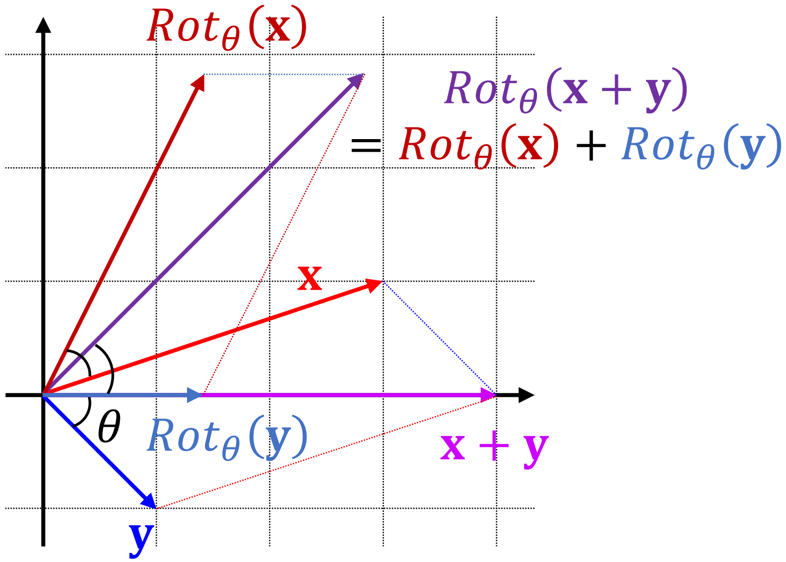
- Example: Reflect around y-axis & Stretch $\times$ 2 in y direction
  - $\mathcal{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ 2y \end{bmatrix} \rightarrow \mathbf{A} = \left[\mathcal{T}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)\ \ \mathcal{T}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right] = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow \mathcal{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$
  - $S = \left\{\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right\}$
  - $\mathcal{T}(S) = \left\{\mathcal{T}\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right), \mathcal{T}\left(\begin{bmatrix} -3 \\ 2 \end{bmatrix}\right), \mathcal{T}\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)\right\}$

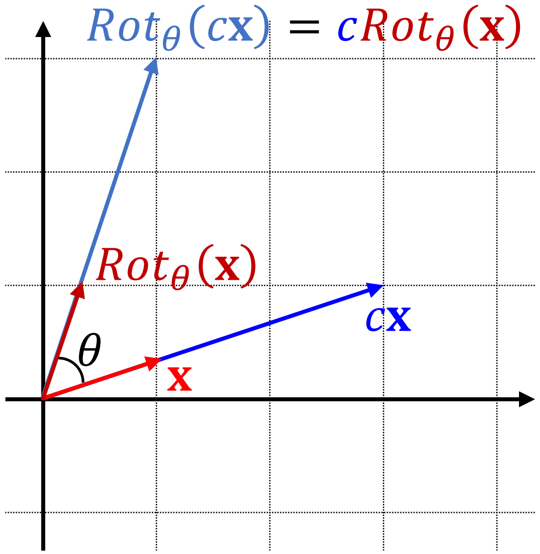
    $= \left\{\begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \end{bmatrix}\right\}$



Linear Transformation: **Rotation in $\mathbb{R}^2$**

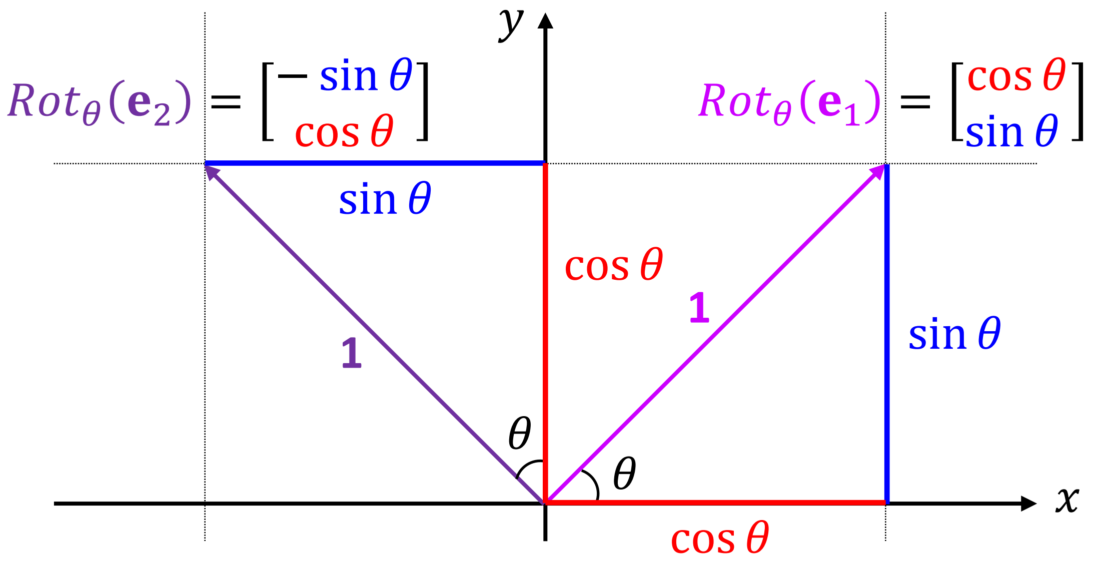
- $Rot_\theta(\mathbf{x})$: Counter clockwise θ degree rotation of $\mathbf{x}$
  - $Rot_\theta(\mathbf{x}+\mathbf{y}) = Rot_\theta(\mathbf{x}) + Rot_\theta(\mathbf{y})$
  - $Rot_\theta(c\mathbf{x}) = cRot_\theta(\mathbf{x})$





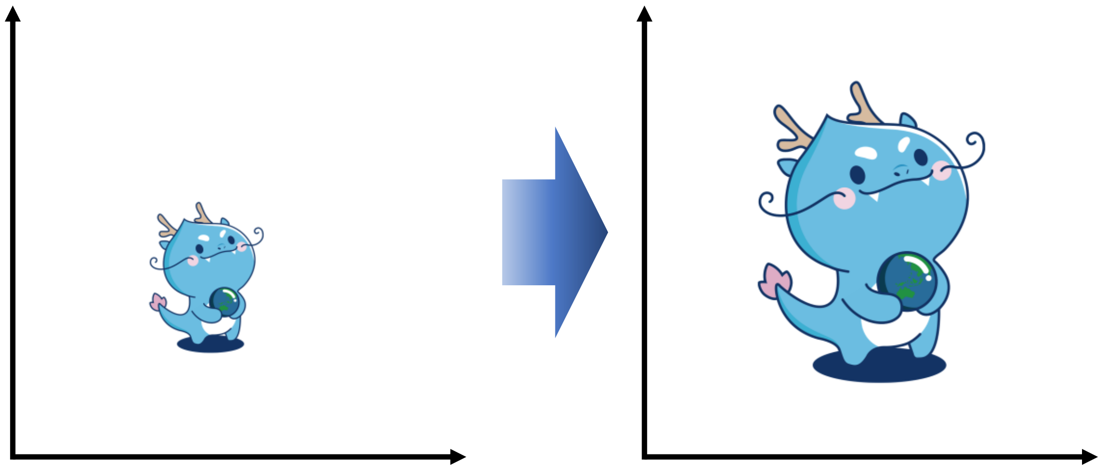
Linear Transformation: **Rotation in** $\mathbb{R}^2$

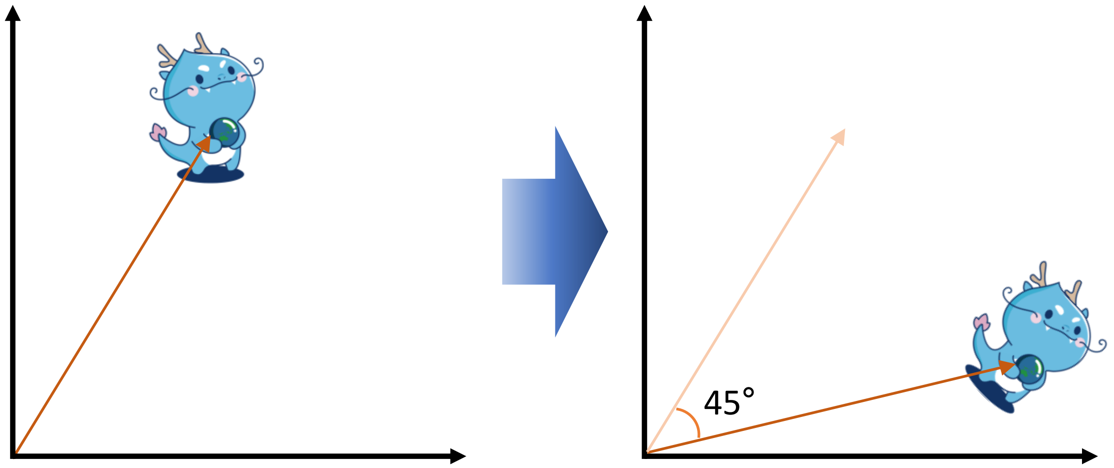
- $Rot_\theta(\mathbf{x})$: Counter clockwise θ degree rotation of $\mathbf{x}$
  - $Rot_\theta : \mathbb{R}^2 \to \mathbb{R}^2,\ Rot_\theta(\mathbf{x}) = \mathbf{A}\mathbf{x},\ \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\mathbf{e}_1 \quad \mathbf{e}_2]$
  - $Rot_\theta(\mathbf{x}) = \mathbf{A}\mathbf{x} = [Rot_\theta(\mathbf{e}_1) \quad Rot_\theta(\mathbf{e}_2)]\mathbf{x} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\mathbf{x}$



Homogeneous System

- Linear Transformation: Matrix-Vector Product (=Matrix Multiplication)
  - $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$
- Scaling
  - $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$
- Rotation
  - $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta\, x - \sin\theta\, y \\ \sin\theta\, y + \cos\theta\, x \end{bmatrix}$





Homogeneous Coordinates

- Homogeneous Transformation Matrix

— In homogeneous coordinates, the multiplication works out so the rightmost column of the matrix is a vector that gets added

— A homogeneous transformation matrix will have a bottom row of $[0\ 0\ 1]$, so that the result has a "1" at the bottom too

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{bmatrix}$$

Image Transformation: **Translation**

- Translation: For a single point $\mathbf{P} = (x, y)$, it is the same as adding a translation vector $\mathbf{t} = \left(t_x, t_y\right)$ to the point $\mathbf{P}$
  - All points are shifted equally in space, the size and shape of the object will not change

$$\mathbf{P}' = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{P} = \mathbf{TP} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

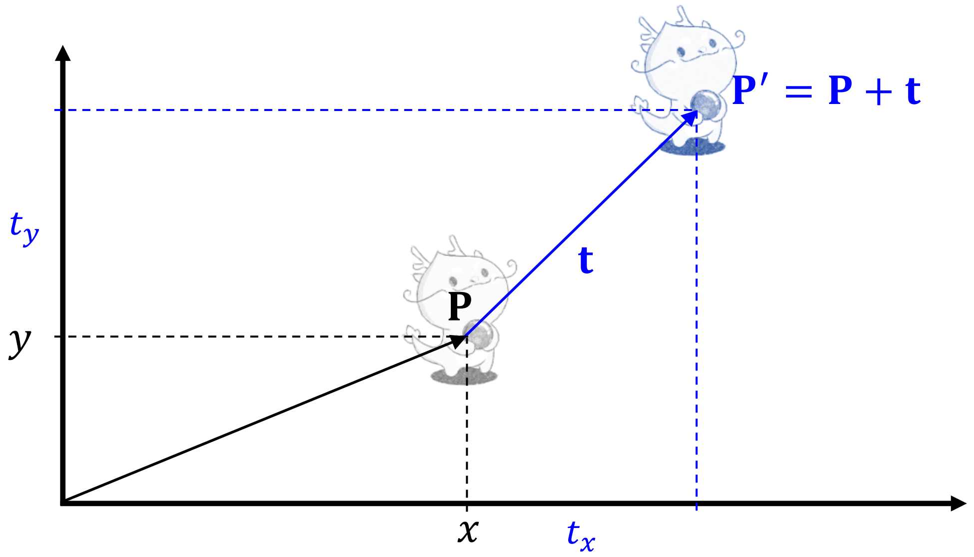


Image Transformation: **Scaling**

- Scaling: It is like a scalar multiplication but not quite the same. In scaling of image transformations, we consider the positive factor as a scale factor

$$\mathbf{P}' = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{P} = \mathbf{S}_{xy}\mathbf{P} = \begin{bmatrix} s_x x \\ s_y y \\ 1 \end{bmatrix}$$

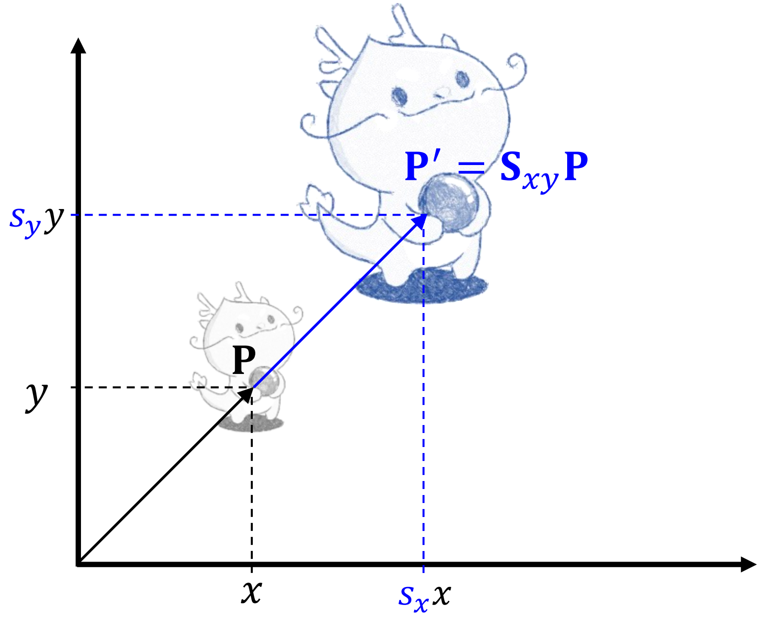


Image Transformation: **Rotation**

- Rotation: The points are rotated counterclockwise through θ with respect to the positive x axis about the origin of a 2D Cartesian coordinate system

— Considering the rotation of a vector, its direction is rigidly changed around an axis without changing its length

$$\mathbf{P}' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{P} = \mathbf{R}_\theta \mathbf{P} = \begin{bmatrix} \cos\theta\, x - \sin\theta\, y \\ \sin\theta\, y + \cos\theta\, x \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

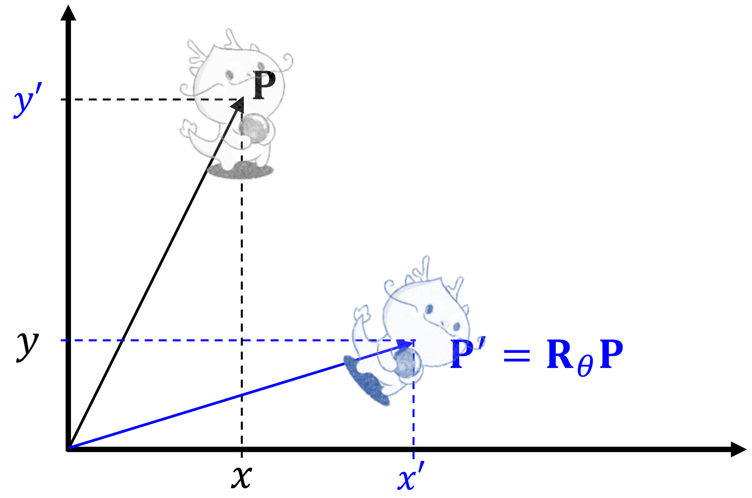


Image Transformation: **Composition of Transformations**

- Generalized Transformation Matrix:

— For the final world transformation, we will concatenate a sequence of these translation, rotation, and scaling transformations together.

— The concatenation of transformations is not commutative

$$\mathbf{P}' = \mathbf{T}\mathbf{R}_\theta\mathbf{S}_{xy}\mathbf{P} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \boldsymbol{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{RS} & \boldsymbol{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Summary

- Images
  - Types: Binary / Grayscale / Color
  - Digital images: By sampling and quantization from continuous signals

- Review of 2D Image Transformation
  - Transformation matrices
    - Translation
    - Scaling
    - Rotation
  - Homogeneous systems