On the formation of one-armed spirals in locally isothermal disks

(and the vertical shear instability in protoplanetary disks)

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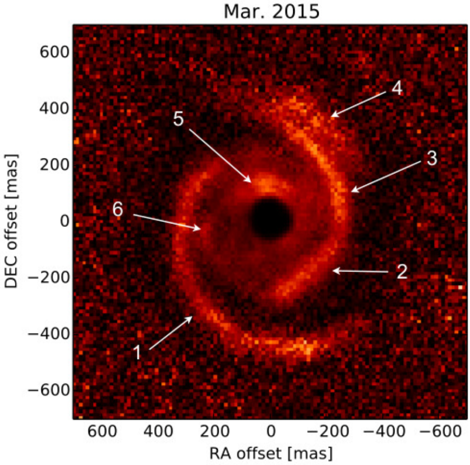
Outline

- Talk — one-armed spiral instability
- Poster plug — vertical shear instability

Large-scale structures in circumstellar disks

MWC 758



(Benisty et al., 2015)

Large-scale structures in circumstellar disks

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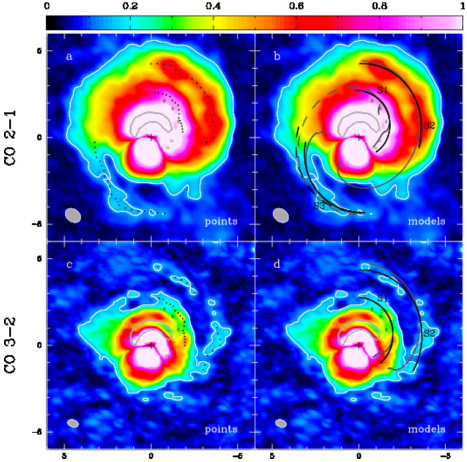
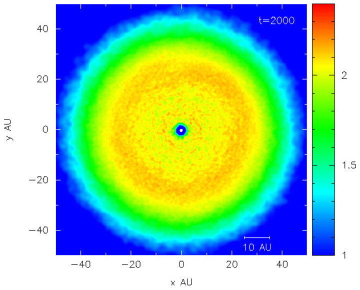


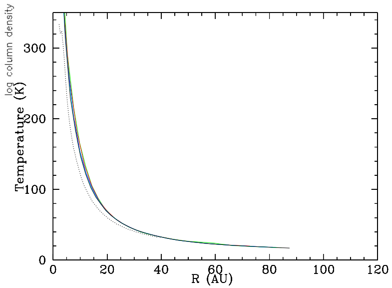
(Christiaens et al., 2014)

Spiral scales: S2 from $\sim$ 500AU to $\sim$ 600AU

Modeling hydrodynamics at large distances

- Can we make simplifications?
- Example: irradiated disks





Radiation hydrodynamic simulations from Stamatellos & Whitworth (2008)

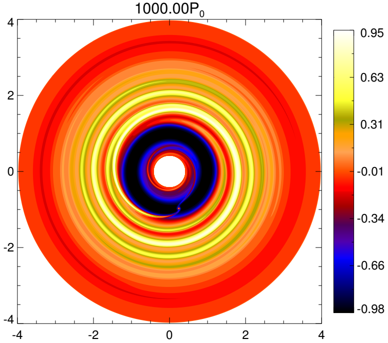
- Temperature does not change much as it is essentially set externally

The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$T = T(\boldsymbol{r}).$$

- Significant simplification: no energy equation to consider
  $\rightarrow$ efficient numerical simulations
- Example: long term disk-planet simulations



The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$\boxed{T = T(\boldsymbol{r}).}$$

- Significant simplification: no energy equation to consider
  $\rightarrow$ efficient numerical simulations
- What happens to waves in disks which are thermodynamically forced?

Angular momentum of non-axisymmetric perturbations

- Split the system into equilibrium and deviations

$$\Sigma \to \Sigma_{\mathrm{ref}}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\rightarrow$ time evolution of deviations or perturbations

Angular momentum of non-axisymmetric perturbations

- Split the system into equilibrium and deviations

$$\Sigma \rightarrow \Sigma_{\mathrm{ref}}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\rightarrow$ time evolution of deviations or perturbations
- Angular momentum conservation for the perturbations

$$\frac{\partial J_{\mathrm{pert}}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\mathrm{pert}} = T_{\mathrm{ext,pert}}.$$

- J_{pert}: angular momentun density of *the perturbations*
- $\boldsymbol{F}_{\mathrm{pert}}$: angular momentum flux
- $T_{\mathrm{ext,pert}}$: everything else (external torque)

External torque in linear theory

$$T_{\mathrm{ext,pert}} = \begin{cases} 0 & \text{barotropic or adiabatic flow} \\ -\frac{m}{2}\,\mathrm{Im}\left(\delta\Sigma\xi_r^*\frac{dc_s^2}{dr}\right) & \text{fixed } T(r) \text{ in 2D} \\ \frac{m}{2}\,\mathrm{Im}\left[\rho\left(\nabla\cdot\boldsymbol{\xi}\right)\boldsymbol{\xi}^*\cdot\nabla c_s^2\right] & \text{fixed } T(r,z) \text{ in 3D} \end{cases}$$

- Barotropic: $p(\rho)$, adiabatic: $\Delta S = 0$
- Locally isothermal: sound-speed $c_s(\boldsymbol{r})$ fixed
- $\boldsymbol{\xi}$: Lagrangian displacement, m: azimuthal wavenumber

External torque in linear theory

With thermal relaxation (Newtonian cooling) towards a fixed $c_s^2(R, z)$ structure:

$$
\begin{aligned}
T_{\mathrm{ext,pert}} =& \frac{m}{2}\,\mathrm{Im}[\cdots] \\
[\cdots] =& \frac{c_s^2}{\rho}|\delta\rho|^2 + c_s^2(\chi - 1)\rho|\nabla\cdot\boldsymbol{\xi}|^2 - \nabla c_s^2 \cdot \left(\delta\rho\boldsymbol{\xi}^* + \frac{\chi - 1}{\gamma - 1}\delta\rho^*\boldsymbol{\xi}\right) \\
& - \frac{\chi - 1}{\gamma - 1}\left(\left|\xi_R\right|^2 \partial_R\rho\partial_R c_s^2 + \left|\xi_z\right|^2 \partial_z\rho\partial_z c_s^2\right) \\
& + \frac{\gamma - \chi}{\gamma - 1}\xi_R^*\xi_z\partial_R\rho\partial_z c_s^2 - \partial_z\rho\partial_R c_s^2\left(\xi_R^*\xi_z + \frac{\chi - 1}{\gamma - 1}\xi_R\xi_z^*\right),
\end{aligned}
$$

where

$$\chi = \frac{1 - \mathrm{i}\bar{\omega}\gamma t_{\mathrm{cool}}}{1 - \mathrm{i}\bar{\omega} t_{\mathrm{cool}}},$$

and

$$\bar{\omega} = \omega - m\Omega.$$

External torque in linear theory

$$\frac{\partial J_{\mathrm{pert}}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\mathrm{pert}} = T_{\mathrm{ext,pert}}.$$

$$T_{\mathrm{ext,pert}} \neq 0$$

$\Rightarrow$ angular momentum exchange between waves and the background disk

- i.e. Σ_{ref} and $\delta\Sigma$ can interact (cf. classic DWT)

Can $T_{\mathrm{ext,pert}}$ make perturbations grow?

Ignoring fluxes,

$$\frac{\partial J_{\mathrm{pert}}}{\partial t} \sim T_{\mathrm{ext,pert}}.$$

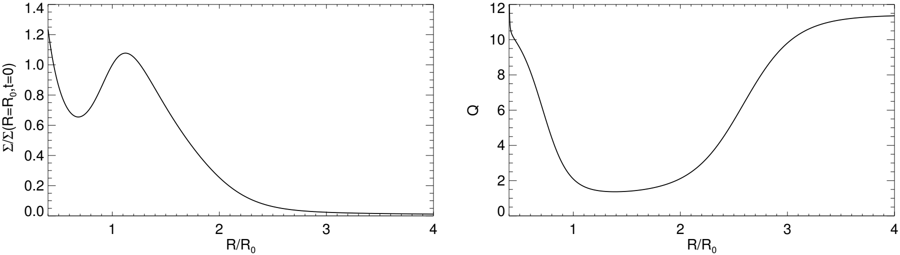
- May have an unstable situation if $T_{\mathrm{ext,pert}} \times J_{\mathrm{pert}} > 0$

Low-frequency trailing waves in a disk with temperature decreasing outwards

- E.g. eccentric disturbance (both torque and angular mometum are negative)

Numerical demonstration

2D, self-gravitating disk with radial structure



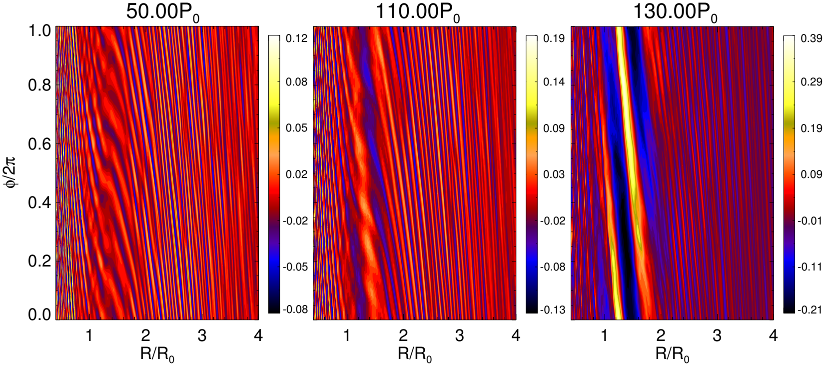
- Put a standard (WKB), one-armed low-frequency density wave in a global disk with $c_s^2 \propto R^{-q}$

$$\text{Growth rate} \sim \frac{T_{\mathrm{ext,pert}}}{J_{\mathrm{pert}}} \sim \frac{q\Omega}{2}\frac{H}{R}k_R H \sim \frac{q\Omega}{2Q}\frac{H}{R}$$

Numerical demonstration

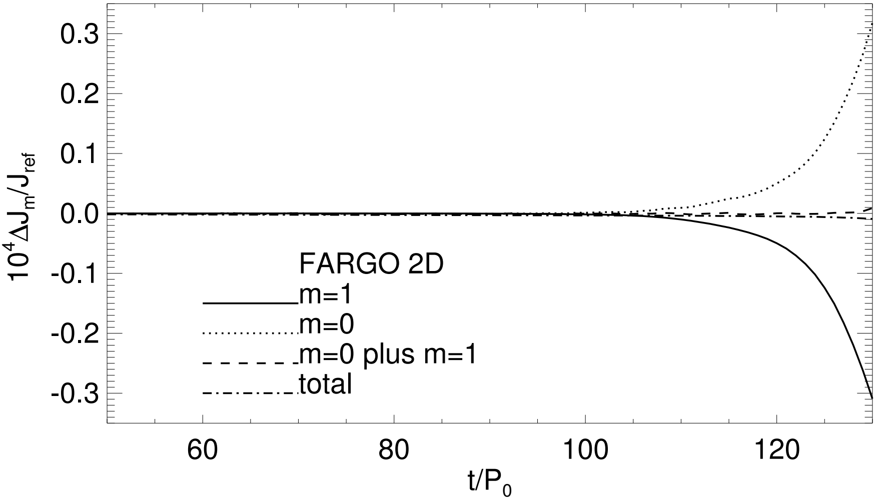
2D, self-gravitating disk with radial structure

FARGO simulations:



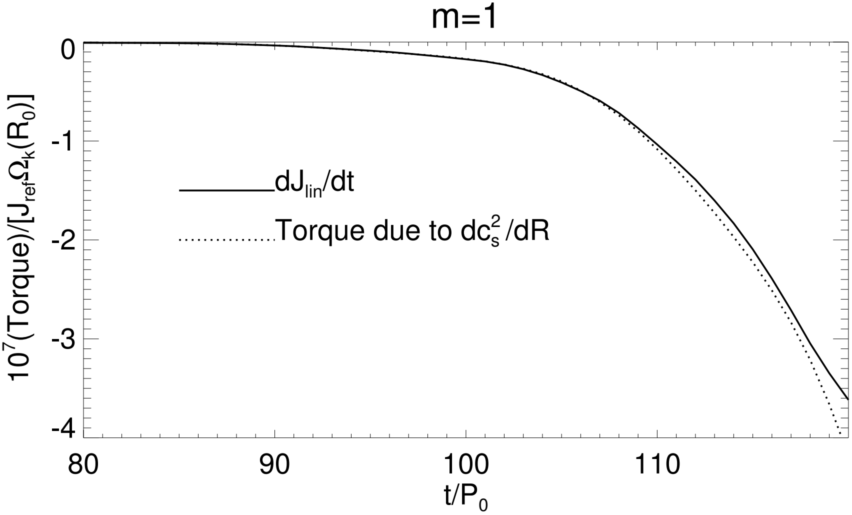
$T_{\mathrm{ext,pert}}$ in action

Angular momentum exchange between the background disk and the spiral:

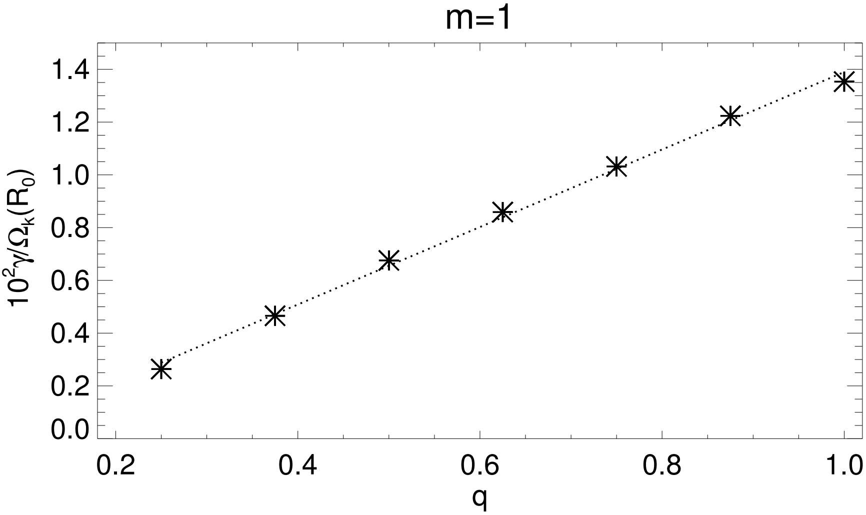


$T_{\mathrm{ext,pert}}$ in action

Extracting angular momentum from the spiral:



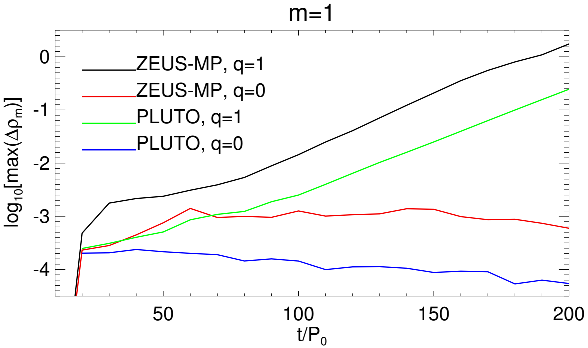
Dependence on the imposed temperature gradient



- Fixed sound-speed profile $c_s^2 \propto r^{-q}$.

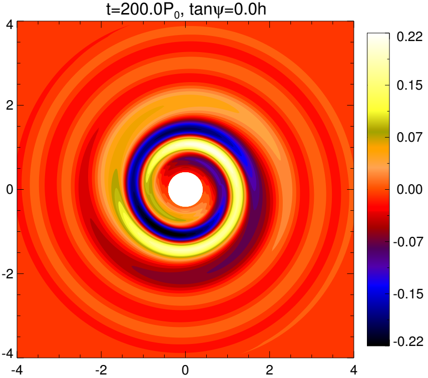
Three-dimensional simulations

Repeat experiment in 3D

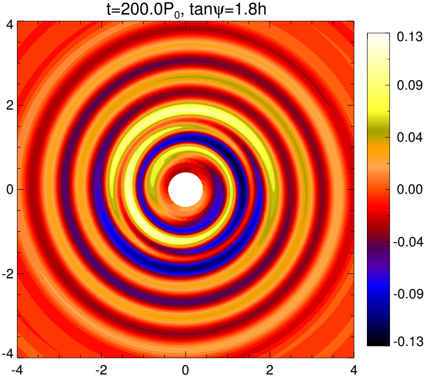


- ZEUS: finite difference, discretized Poisson
- PLUTO: Godunov, Poisson through spherical harmonics
- *No growth without imposed temperature gradient*

Three-dimensional simulations



Midplane

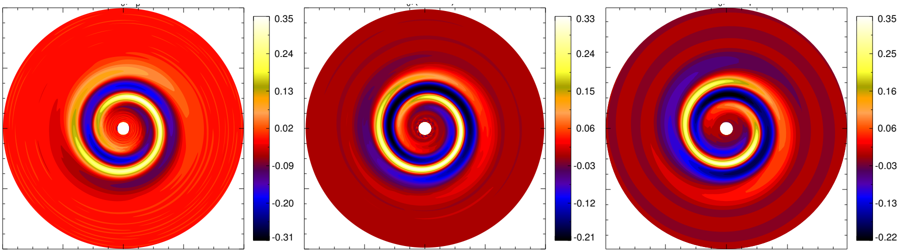


Atmosphere

- Spiral-induced spiral

Summary

(Lin, 2015)



FARGO 2D ZEUS 3D PLUTO 3D

- Forcing a temperature gradient may lead to an unstable exchange of angular momentum between non-axisymmetric waves and the background disk
- Possible application to outer parts of protoplanetary disks where heating is dominated by external irradiation
- Non-ideal thermal evolution imply non-trivial wave angular mometum conservation

Vertical shear instability in protoplanetary disks

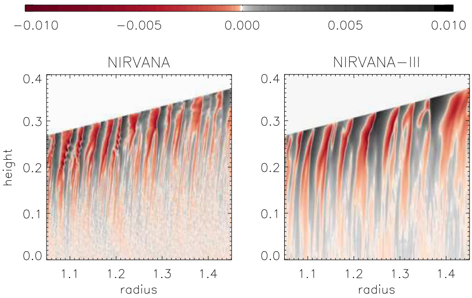
If

$$T = T(R) \propto c_s^2(R)$$

Then

$$\boxed{\frac{\partial \Omega}{\partial z} \neq 0.}$$

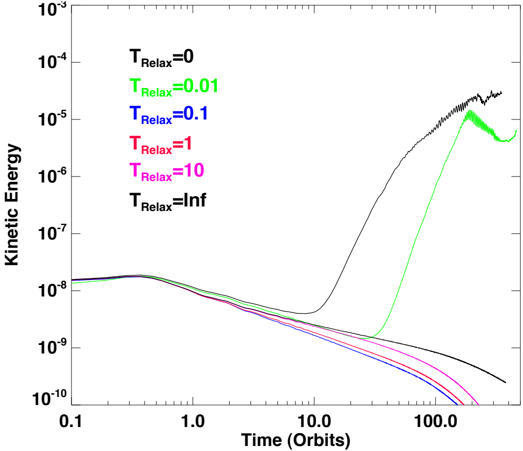
- This *vertical shear* may render the disk unstable



Axisymmetric simulations by Nelson et al. (2013)

Vertical shear instability requires fast thermal relaxation

Otherwise buoyancy prevents vertical motion



Simulations from Nelson et al. (2013) with

$$\frac{\partial T}{\partial t} = -\frac{T - T_{\mathrm{init}}}{T_{\mathrm{Relax}} P_{\mathrm{orb}}}.$$

What is ‘rapid cooling’ for the VSI?

$$\underbrace{r\partial_z\Omega}_{\text{destabilizing vert. shear}} \quad \text{v.s.} \quad \underbrace{N_z}_{\text{stabilizing vert. buoyancy}}$$

- Vertical shear is *weak*, $\boxed{r\partial_z ln\Omega \sim O(h) \ll 1}$
- Vertical buoyancy is *strong*, $\boxed{N_z/\Omega \sim O(1)}$, *but cooling can reduce it*

What is ‘rapid cooling’ for the VSI?

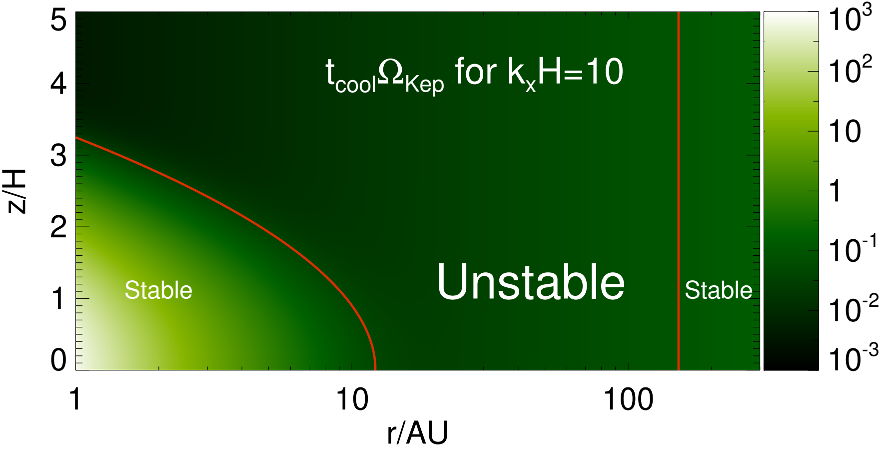
Lin-Youdin criterion

$$t_{\rm cool}\Omega_{\rm Kep} < \frac{h|q|}{\gamma - 1} \ll 1,$$

(Lin & Youdin, 2015)

- $h = H/R$
- $T \propto R^q$
- Note $\max(t_{\rm cool}) \longleftrightarrow \max(L_x)$

Where is VSI most effective in a PPD?



[Also talk to: A. Youdin, S. Richard, W. Kley, H. Latter, A. Barker]

References

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