The stability of self-gravitating gaps

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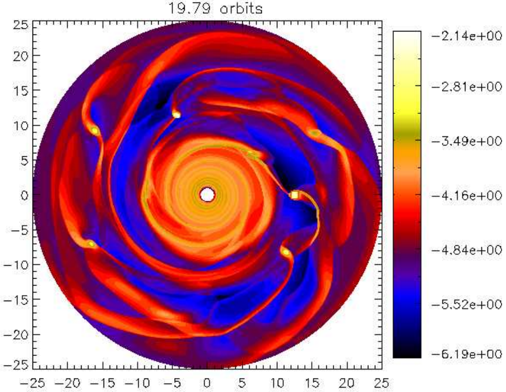
Outline

- Introduction
- Numerical simulations
- Vortices & spirals
- Disk-planet interactions
- Conclusion & future work

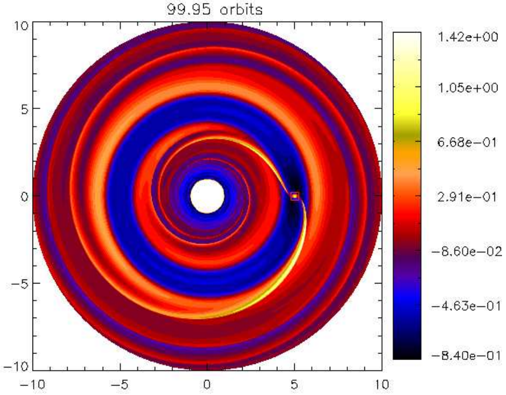
Disk stability

- Disks are ubiquitous in astrophysics.
- Instability is important: accretion, star formation, direct or indirect planet formation.
- Unstable because: magnetic fields, thermodynamics, **disk structure, self-gravity**.



Gaps in protoplanetary disks

- 490+ exo-planets discovered (October 2010).
- Planets form in disks. Sufficiently massive planets opens a gap (Papaloizou & Lin, 1984; Lin & Papaloizou, 1986).
- Dynamical instability at gap edges because of steep gradients.



Self-gravity

- Total gravitational potential: star, planet, disk.
- Usually ignore disk potential because $M_d \sim 0.01 M_*$.
- Massive disks needed for giant planet formation via GI, type III migration.
- How does SG affect gap stability?

Consider a series of disk-planet simulations with increasing M_d:

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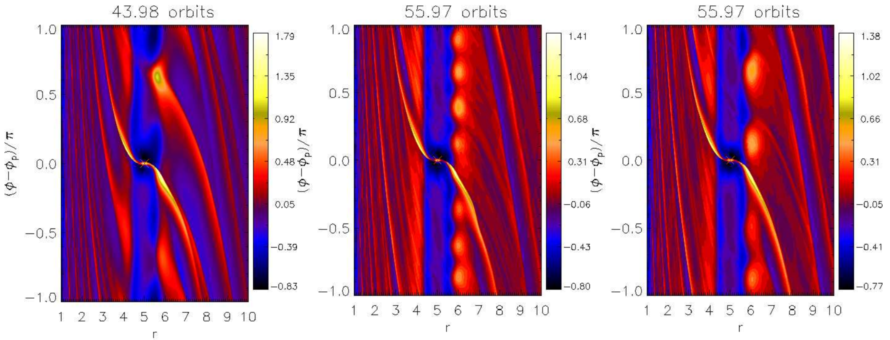
Consider a series of disk-planet simulations with increasing M_d:



$M_d = 0.063 M_*$ $M_d = 0.032 M_*$ $M_d = 0.011 M_*$

Model equations

2D disc in polar co-ordinates centered on primary but non-rotating.

- Hydrodynamic equations with local isothermal equation of state:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\mathbf{u}\Sigma) = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\Sigma}\nabla P - \nabla \Phi + \frac{\mathbf{f}}{\Sigma}.$$

- Viscous forces $f \propto \nu$, pressure $P = c_s^2 \Sigma$ with $c_s^2 = h^2 GM_* / r$ and potential Φ include star potential, indirect potentials and disk potential Φ_d:

$$\Phi_d = -\int \frac{G\Sigma(r', \phi')}{\sqrt{r^2 + r'^2 - 2rr' \cos(\phi - \phi') + \epsilon_g^2}} r' dr' d\phi'.$$

Linearized equations

- Simplify: $P = P(\Sigma)$, ignore indirect and planet potentials. Ignore viscosity.
- Perturb the system, e.g. $\Sigma \to \Sigma + \delta\Sigma(r) \exp i(\sigma t + m\phi)$. Linearize to get ODE in the form

$$\begin{aligned} L(S) &= \delta\Sigma \\ S &= c_s^2 \delta\Sigma/\Sigma + \delta\Phi \\ \delta\Phi &= -G \int K_m(r,\xi)\delta\Sigma(\xi)\xi d\xi. \end{aligned}$$

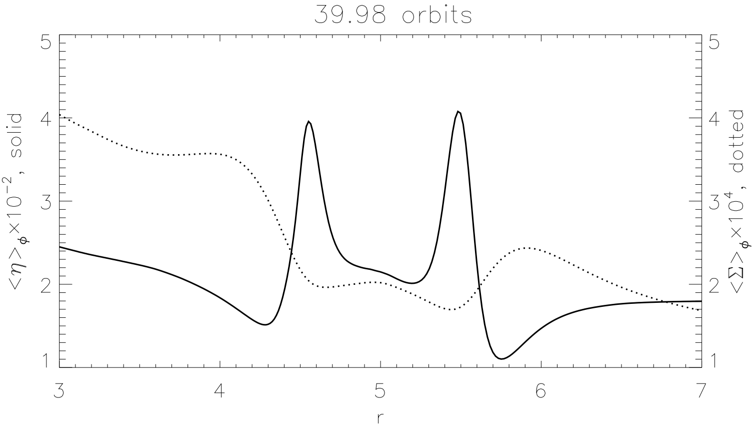
- L is a linear integro-differential operator

$$L(S) = \frac{mS}{r\bar{\sigma}(1-\bar{\nu}^2)} \frac{d}{dr}\left(\frac{1}{\eta}\right) + \cdots ,$$

$\bar{\sigma} = \sigma + m\Omega$, $\bar{\nu} = \bar{\sigma}/\kappa$ and

$\eta = \kappa^2/2\Omega\Sigma$ is the *vortensity*.

Vortensity profile of gaps



- Disturbances associated with vortensity extrema. Take $\bar{\sigma} \simeq 0$ at $r = r_c$ such that $\eta'(r_c) \simeq 0$.
- Consider $\int rS^* L(S) dr = \int rS^* \delta\Sigma dr$. Assume on LHS: vortensity gradient term dominates and spatially localised near r_c.

Vortensity profile of gaps

- $\int rS^* L(S)dr = \int rS^* \delta\Sigma dr$ has dimensions of energy. Assumptions lead to:

$$\int \frac{m|S|^2}{\bar{\sigma}} \frac{d}{dr}\left(\frac{1}{\eta}\right) dr \sim \int c_s^2 \frac{|\delta\Sigma|^2}{\Sigma} dr - G \int\int r\xi K_m(r,\xi)\delta\Sigma^*(r)\delta\Sigma(\xi) dr d\xi$$

- Sign of $(1/\eta)''/\Omega'$ at r_c determine sign of LHS. $\Omega' < 0$.
- Insignificant SG: RHS > 0 so r_c is $\min(\eta) \rightarrow$ VORTICES.
- Significant SG: RHS < 0 so r_c is $\max(\eta) \rightarrow$ SPIRALS.

Note: Toomre $Q = \kappa c_s/\pi G\Sigma$. $\max(Q)$ coincides with $\max(\eta)$.

A nice result

Theorem

The perturbative effect of self-gravity through the linear response, $\delta\Phi$, is to stabilize vortex modes and de-stablize spiral modes.

Proof.

- Consider marginally stable mode with real $\sigma = -m\Omega(r_c)$. $\eta'(r_c) = 0$.
- Change self-gravity via $G \to G + \delta G$.
- Perturb eigensolution: $\sigma \to \sigma + \delta\sigma$ with $\delta\sigma = \delta\sigma_R + i\gamma$; $S \to S + \delta S$; $\delta\Sigma \to \delta\Sigma + \delta\Sigma_1$. γ is assumed small negative.

Can show:

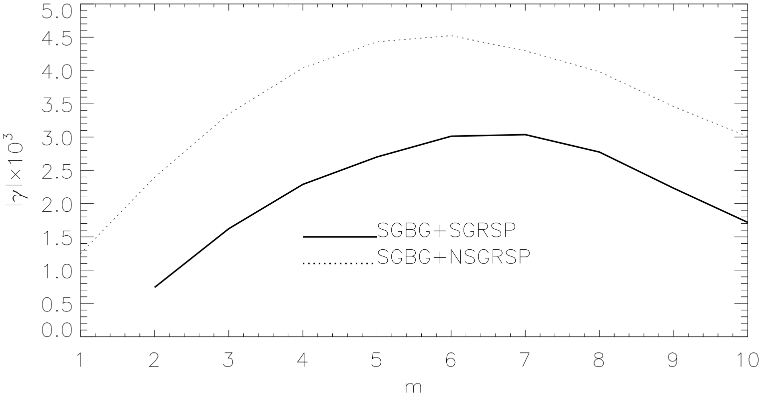
$$\boxed{\gamma = \beta \left.\frac{d^2\eta}{dr^2}\right|_{r_c} \times \delta G,}$$

with $\beta > 0$ for Ω decreasing outwards. Vortex modes have $\eta''(r_c) > 0$. Need $\delta G < 0$ to de-stabilize them, i.e. increasing SG stabilizes them. □

Stabilization of vortex modes

- Solve the linear problem numerically, with local isothermal equation of state.
- Also solved with $\delta\Phi = 0$.

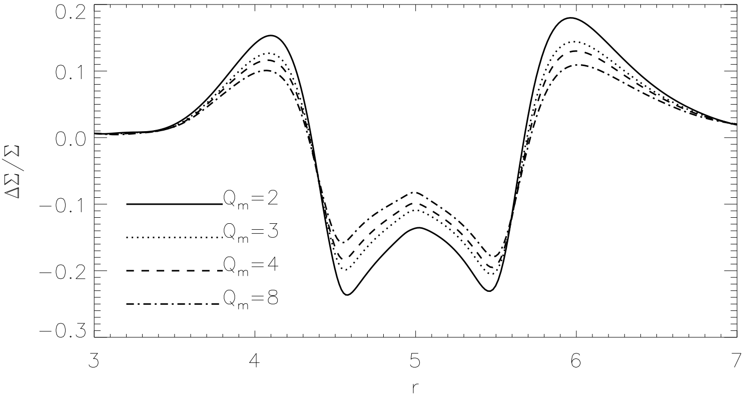
Growth rate $|\gamma|$ as a function of azimuthal wave-number m:



Solid: with SG in response. Dotted: no SG in response.

Effect of SG via the background

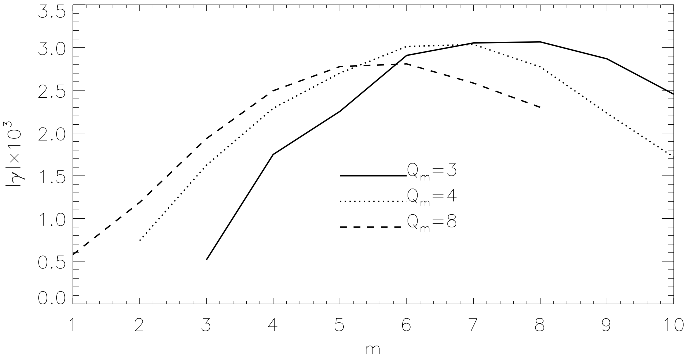
Altering SG affect the gap profile set up by simulation:



$Q_m \propto 1/M_d$. Deeper gaps with increasing SG → more unstable. Effect *diminishes* with lowering m.

Getting more vortices

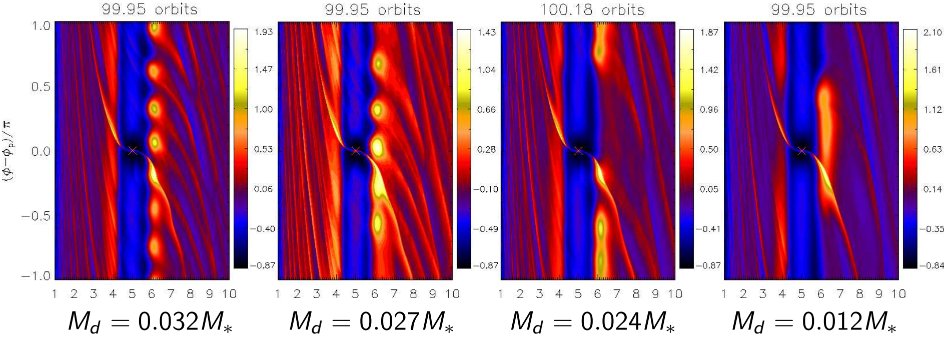
Now include SG all the way in linear problem.



Recall $Q_m \propto 1/M_d$.

- Higher m preferred with increasing SG.
- Loss of low m.

Effect of SG on vortex evolution



- Multiple-vortices configuration is sustained longer with increasing SG.

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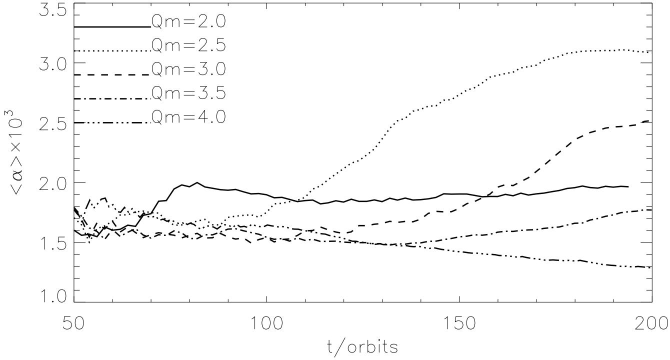
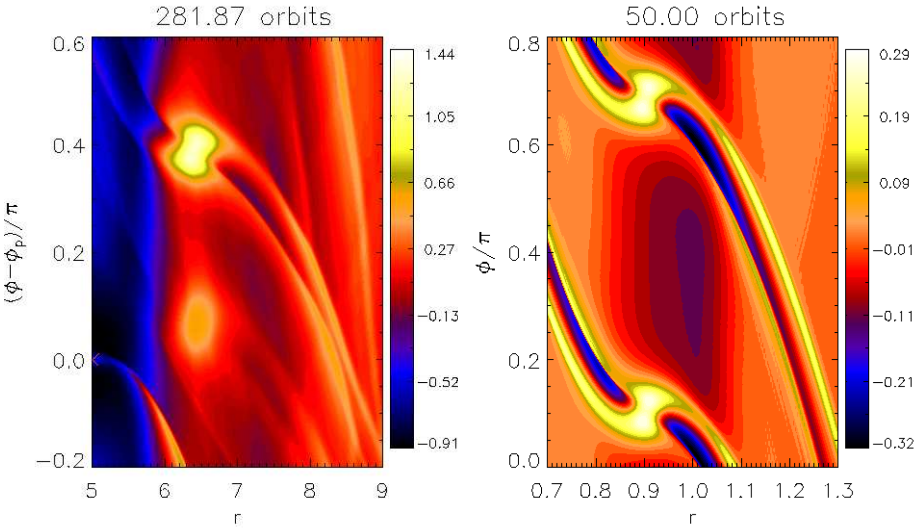


- Multiple-vortices configuration is sustained longer with increasing SG.
- Get α growth for intermediate range of disk mass.

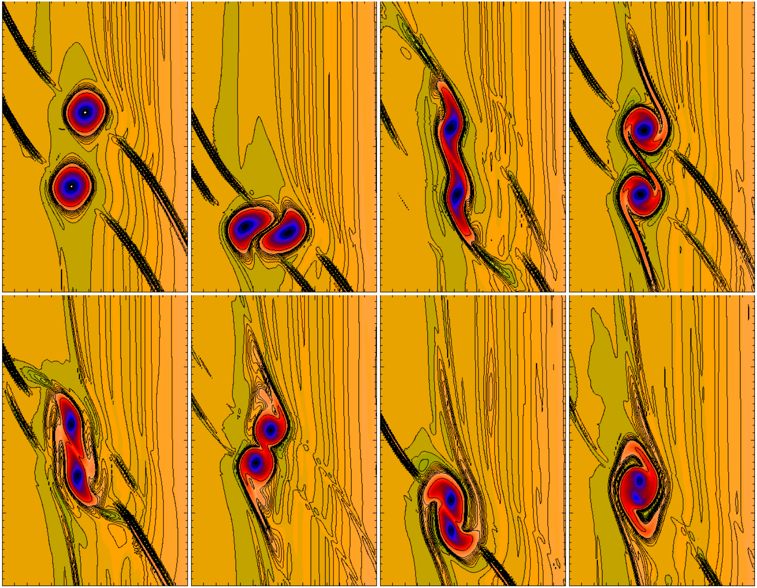
Vortices as co-orbital planets



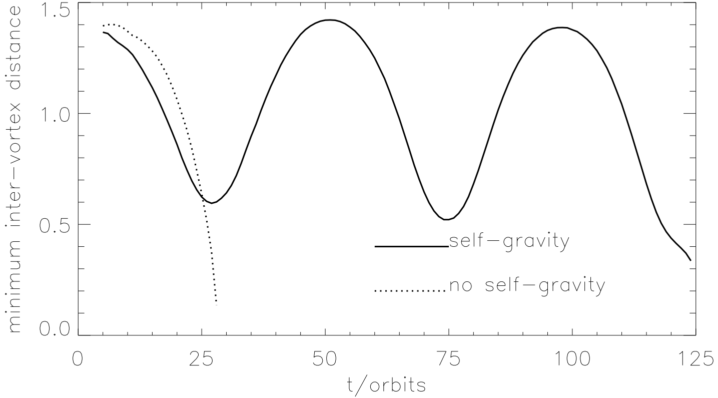
Gap vortex-pair

Kida vortex pair

Vortices as co-orbital planets



Vortices as co-orbital planets



SG on/off

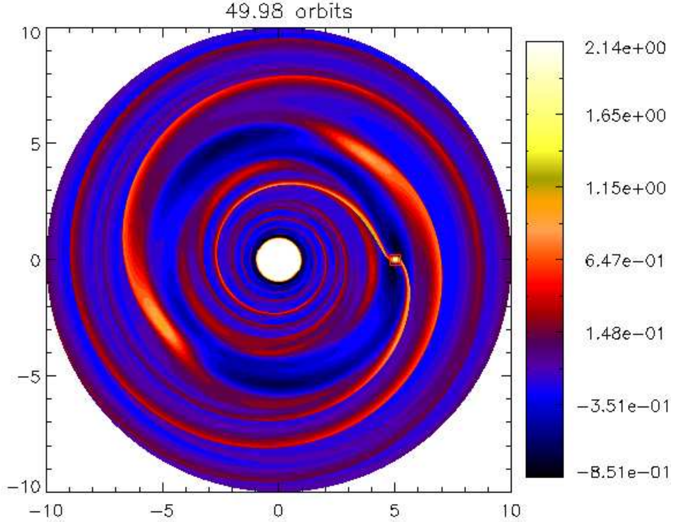
- SG imply minimum inter-vortex distance. If still larger than critical → no merging.

Application to vortex-induced migration

Repeat **?**'s fiducial case with SG.

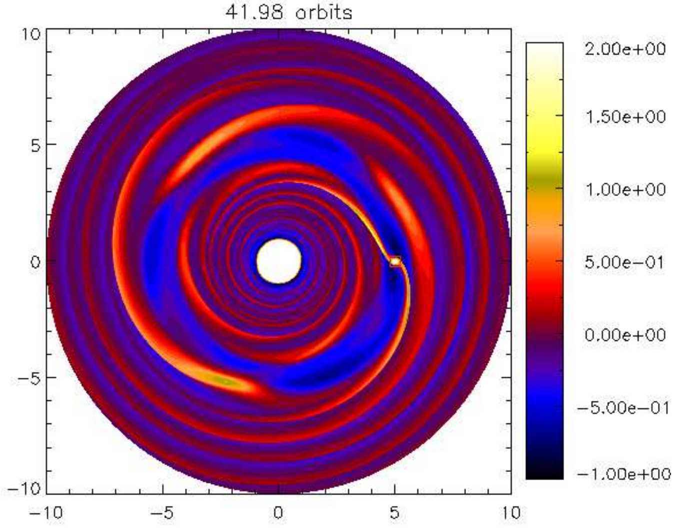
Strong self-gravity

A case with $M_d = 0.06M_*$. Co-rotation radius at $r \simeq 5.5$, local vortensity *maximum*. Confirms Meschiari & Laughlin (2008).



Strong self-gravity

An example with $M_d = 0.08M_*$.



Requirement for edge modes

- Recall ODE: $L(S) = \delta\Sigma$, $S = c_s^2\delta\Sigma/\Sigma + \delta\Phi$.
- Only keep vortensity gradient term in L. Get...

$$\lambda\mathcal{H}(r) = \int \mathcal{R}_m(r,\xi)\mathcal{H}(\xi)d\xi.$$

with $\lambda = 1$. $S \to \mathcal{H}$ (new eigenfunction); $K_m \to \mathcal{R}_m$ (new kernel).

- But there is a $\max(\lambda)$ for these equations. So if

$\max(\lambda) < 1$ then no mode.

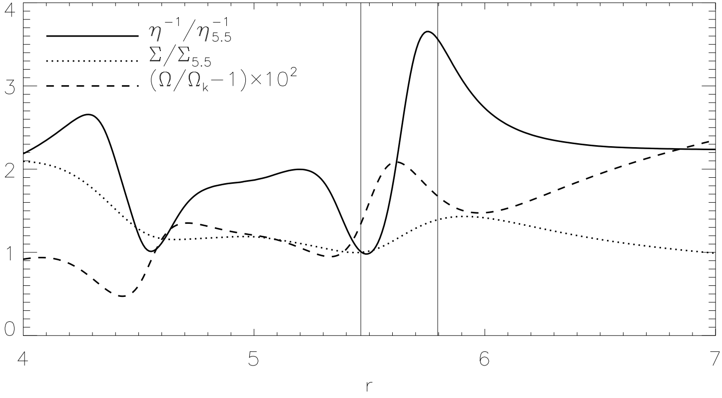
- Can show $\max(\lambda) < \Lambda$ and estimate Λ:

$$\Lambda \sim \frac{2GK_0(m\epsilon_g/r_c)}{r_c}\left|\frac{1}{\Omega'}\left(\frac{2\Omega\Sigma}{\kappa^2}\right)''\right| L_c.$$

K_0: Bessel function; L_c length-scale of edge.
Approximations: thin edge, zero pressure.

Linear calculations of edge modes

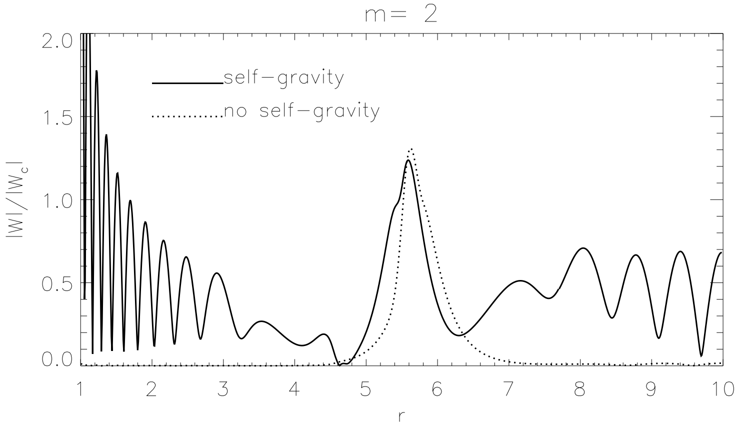
Solve the ODE with and without SG in response. Background has SG.



Solid line is $1/\eta$! NSG $r_c = 5.80$ and SG $r_c = 5.46$.

Linear calculations of edge modes

Here are the eigenfunctions:

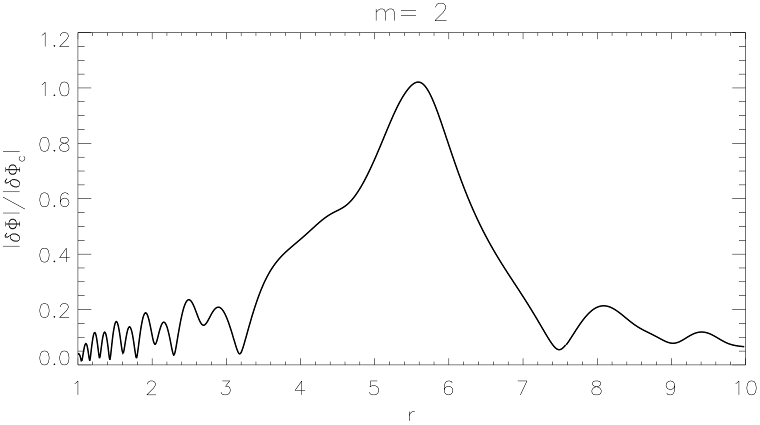


Analogy with disk-planet system

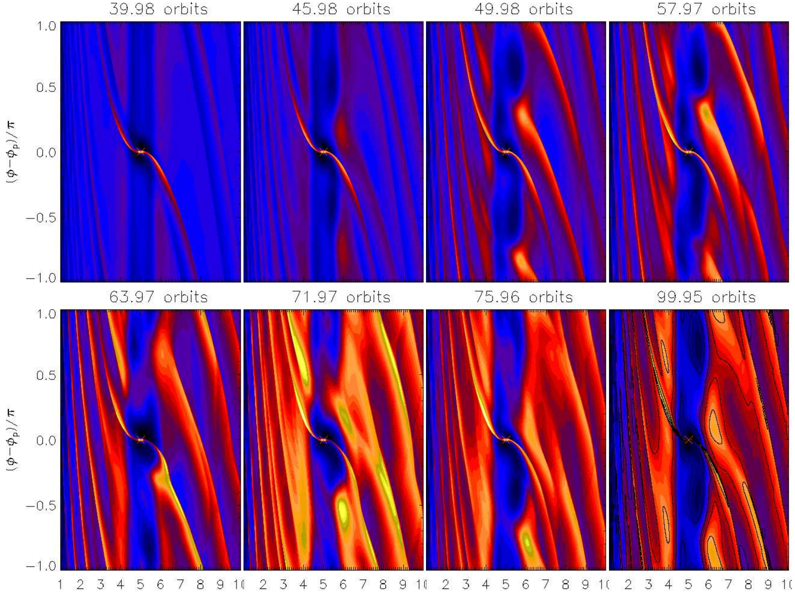
- Edge disturbance acts as external forcing on smooth part of the disk.
- Edge disturbance is like a planet. Does its potential vary on global scale?

Analogy with disk-planet system

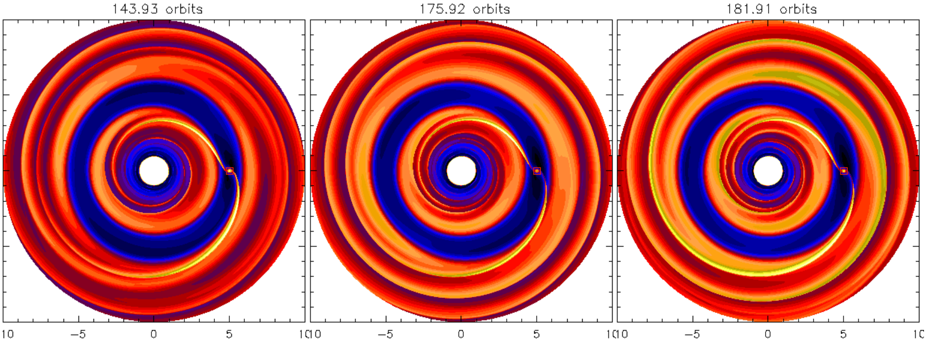
- Edge disturbance acts as external forcing on smooth part of the disk.
- Edge disturbance is like a planet. Does its potential vary on global scale? Well, sort of.



Back to hydrodynamics



Eccentric gap



Viscosity

- Need low viscosity to get vortices ($\nu < 10^{-6}$ or $\alpha < 10^{-4}$).
- Standard viscosity $\alpha = 10^{-3}$ can't kill edge modes.

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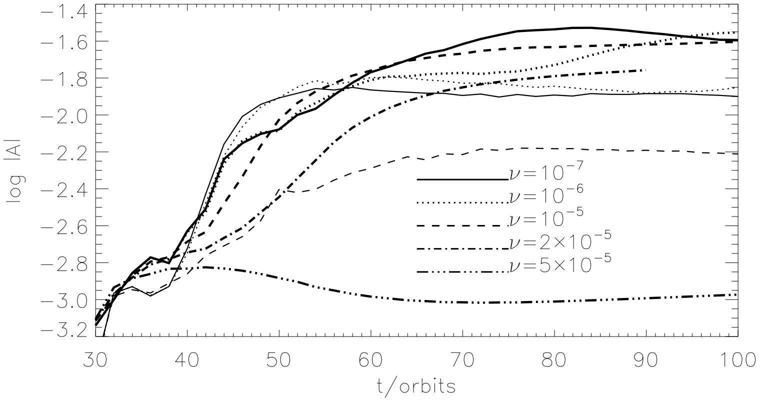
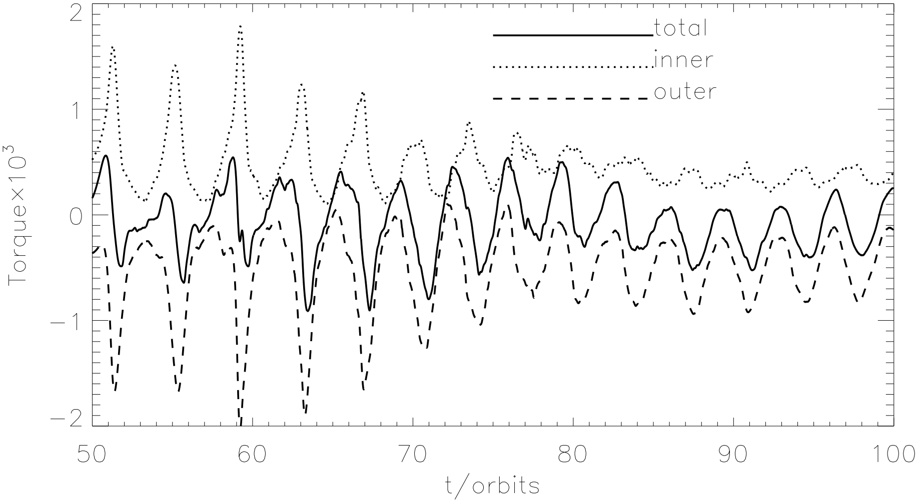


- Lower viscosity → higher *m* (sharper profiles).
- Eventually surpressed but because no local vortensity maximum.

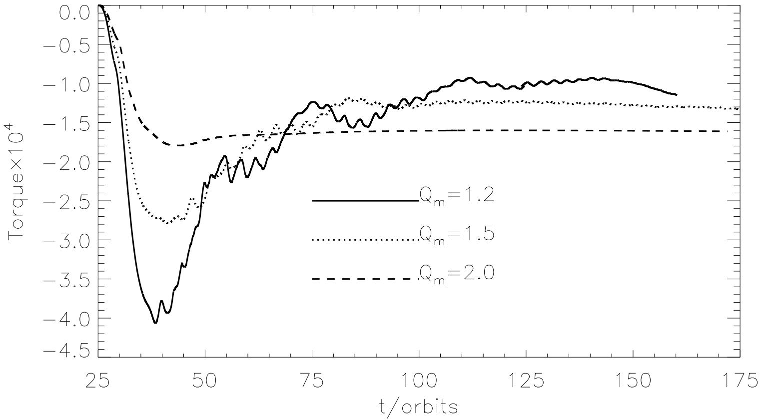
Application to disk-planet interaction

Disk-planet torque for fiducial case:



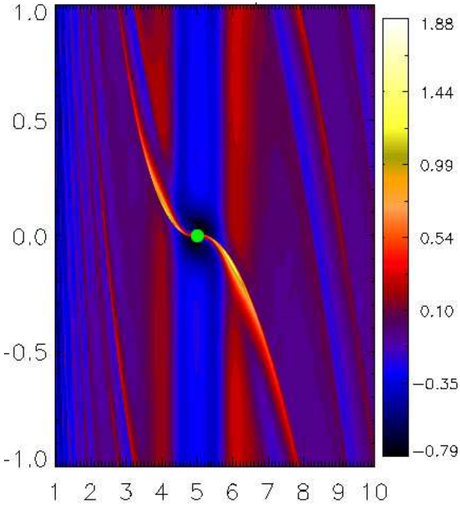
Application to disk-planet interaction

Time-averaged disk-on-planet torques:



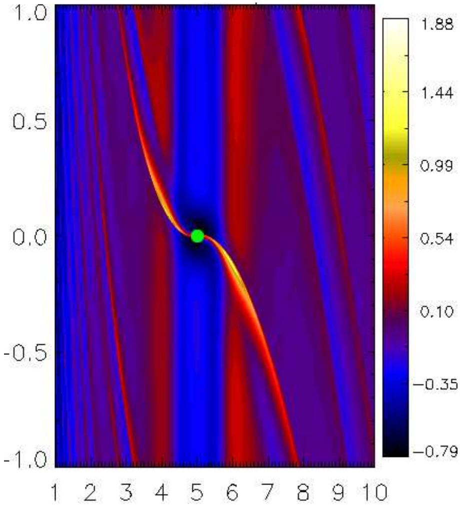
If unstable → average torque more positive with increasing disk mass.

Diluting the outer torque

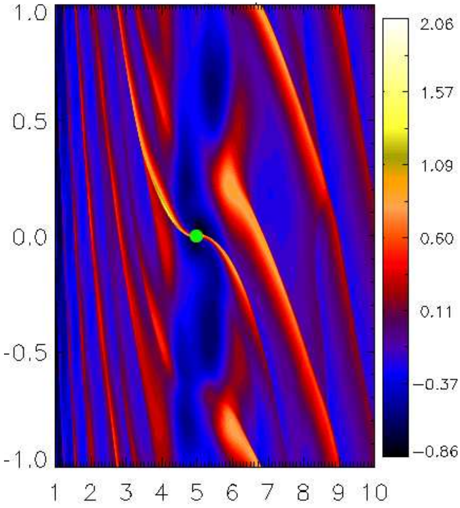


Stable disk.

Diluting the outer torque



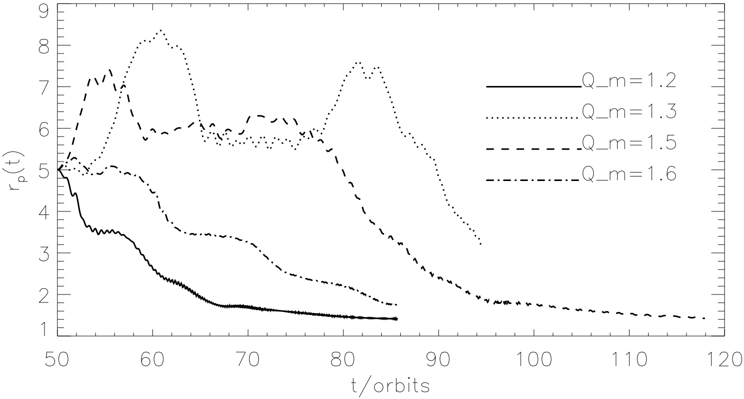
Stable disk.



Unstable disk.

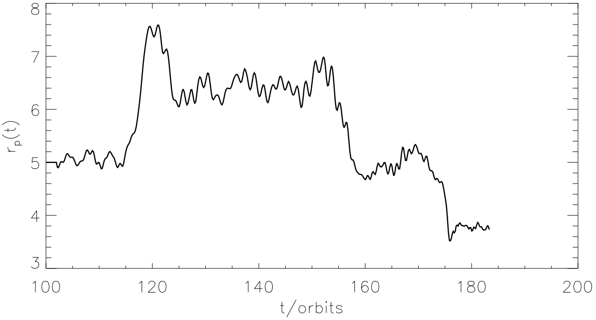
Migration in massive disks

- Numerically difficult: lots of parameters.
- Need better set up.



Summary

- Self-gravity affects onset of instability at gap edge, higher m modes preferred as SG becomes important.
- Self-gravity delays vortex merging. Final configuration without SG is a single large vortex. With SG, final vortex is compact and has local scale.
- When self-gravity is strong, get different type of mode altogehter: $m = 2$ global spirals. Sometimes have strong affects on planet migration.



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