Strange aspects of locally isothermal astrophysical disks and the stability of magnetized massive disks

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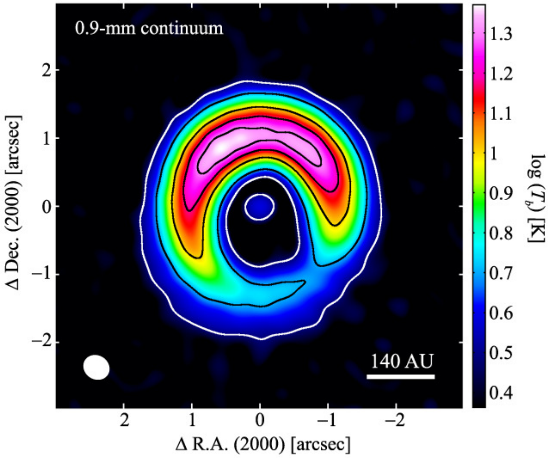
Interests

- Astrophysical fluid dynamics
- Disk-planet interactions
- Self-gravitating disks
- Disk instabilities, large-scale structures
- Magneto-hydrodynamics (new)
- Numerical simulations
- Linear/analytical hydrodynamics/methods

Large-scale structures in circumstellar disks

Example: transition disk around HD 142527



(Fukagawa et al., 2013)

Note the scale: $O(10^2)$AU

Large-scale structures in circumstellar disks

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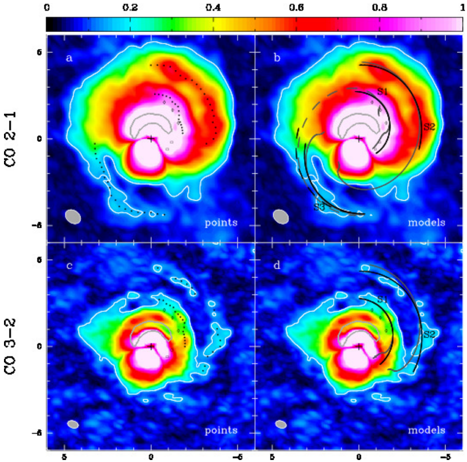
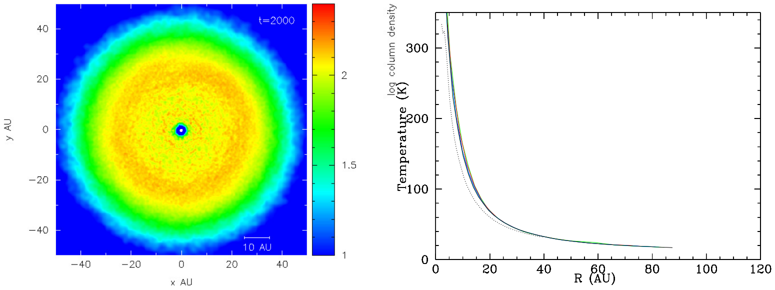


(Christiaens et al., 2014)

Spiral scales: S2 from $\sim$ 500AU to $\sim$ 600AU

Modeling hydrodynamics at large distances

- Can we make simplifications?
- Example: irradiated disks



Radiation hydrodynamic simulations from Stamatellos & Whitworth (2008)

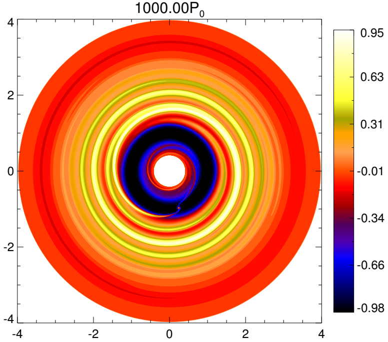
- Temperature does not change much as it is essentially set externally

The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$T = T(\boldsymbol{r}).$$

- Tremendous simplification: no energy equation to consider
  $\rightarrow$ cheaper numerical simulations
- Example: long term disk-planet simulations



The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$\boxed{T = T(\boldsymbol{r}).}$$

- Tremendous simplification: no energy equation to consider
  $\rightarrow$ cheaper numerical simulations
- What are the fundamental consequences?

Angular momentum conservation

Essential to all rotating disk problems:

$$\frac{\partial J}{\partial t} + \nabla \cdot \boldsymbol{F} = T_{\mathrm{ext}}.$$

- J: angular momentum density
- $\boldsymbol{F}$: angular momentum flux
- T_{ext}: external torques

Linear stability 101

- Split the system into equilibrium and deviations

$$\Sigma \to \Sigma_{\mathrm{ref}}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\rightarrow$ time evolution of deviations or perturbations

Linear stability 101

- Split the system into equilibrium and deviations

$$\Sigma \to \Sigma_{\rm ref}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\to$ time evolution of deviations or perturbations

Linearized equations $\to$ angular momentum conservation for the perturbations

$$\boxed{\frac{\partial J_{\rm pert}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\rm pert} = T_{\rm ext,pert}.}$$

- 'pert' quanties associated with perturbations
- Definition not obvious

External torque in linear theory

$$T_{\mathrm{ext,pert}} = \begin{cases} 0 & \text{barotropic or adiabatic flow} \\ -\frac{m}{2}\,\mathrm{Im}\left(\delta\Sigma\xi_r^* \frac{dc_s^2}{dr}\right) & \text{locally isothermal in 2D} \\ \frac{m}{2}\,\mathrm{Im}\left[\rho\left(\nabla\cdot\boldsymbol{\xi}\right)\boldsymbol{\xi}^*\cdot\nabla c_s^2\right] & \text{locally isothermal in 3D} \end{cases}$$

- Barotropic: $p(\rho)$, adiabatic: $\Delta S = 0$
- Locally isothermal: sound-speed $c_s(\boldsymbol{r})$ fixed
- $\boldsymbol{\xi}$: Lagrangian displacement, m: azimuthal wavenumber

$$T_{\mathrm{ext,pert}} \neq 0$$

angular momentum exchange between perturbations and the background disk

Can $T_{\text{ext,pert}}$ make perturbations grow?

Ignoring angular momentum fluxes,

$$\frac{\partial J_{\text{pert}}}{\partial t} \sim T_{\text{ext,pert}}.$$

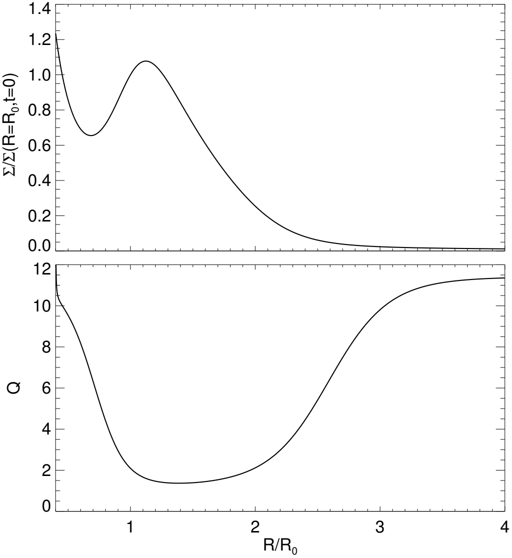
- May have an unstable situation if $T_{\text{ext,pert}}$ is the same sign as J_{pert}
- Possible for

  low-frequency disturbances in a disk with temperature decreasing outwards

  (both torque and angular mometum are negative)
- Low-frequency: $\delta\Sigma \sim e^{i\omega t}$ and $|\omega| \ll m\Omega$.

Numerical demonstration

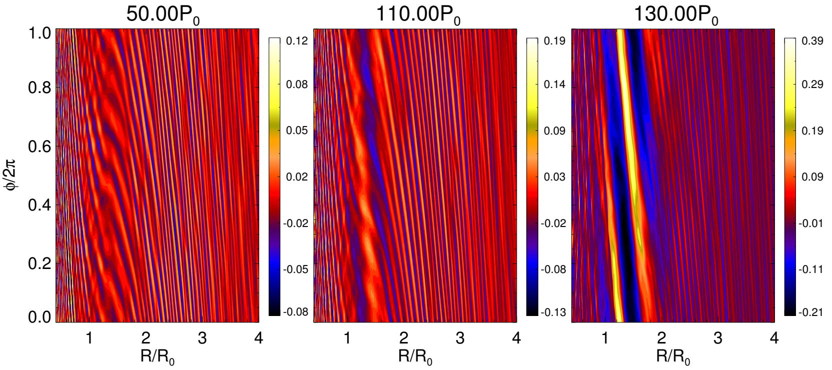
2D, self-gravitating disk with radial structure



Numerical demonstration

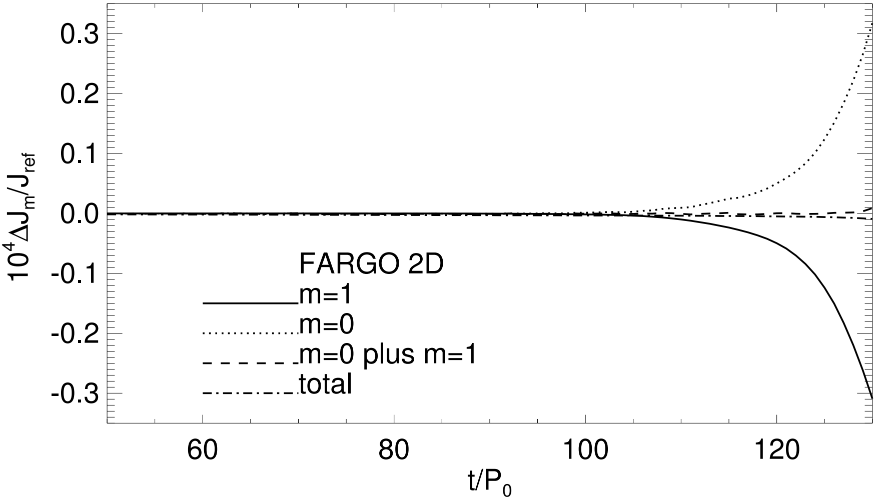
2D, self-gravitating disk with radial structure

FARGO simulations:



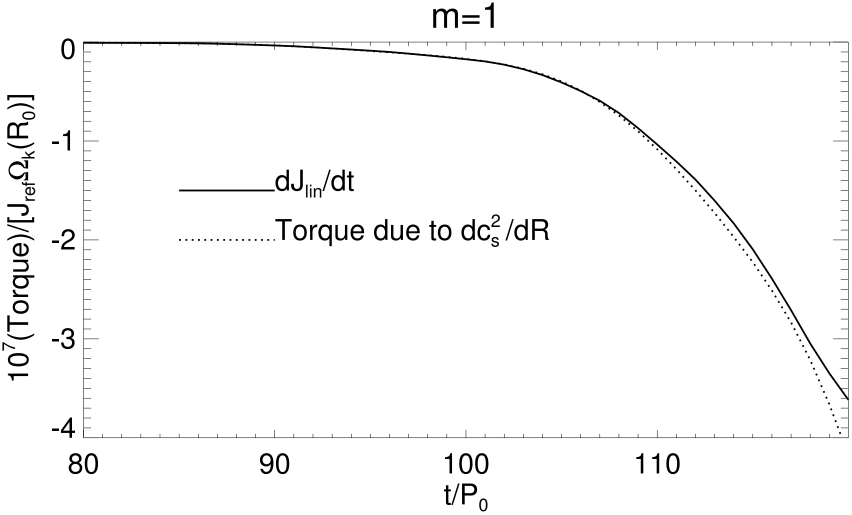
$T_{\mathrm{ext,pert}}$ in action

Angular momentum exchange between the background disk and the spiral:

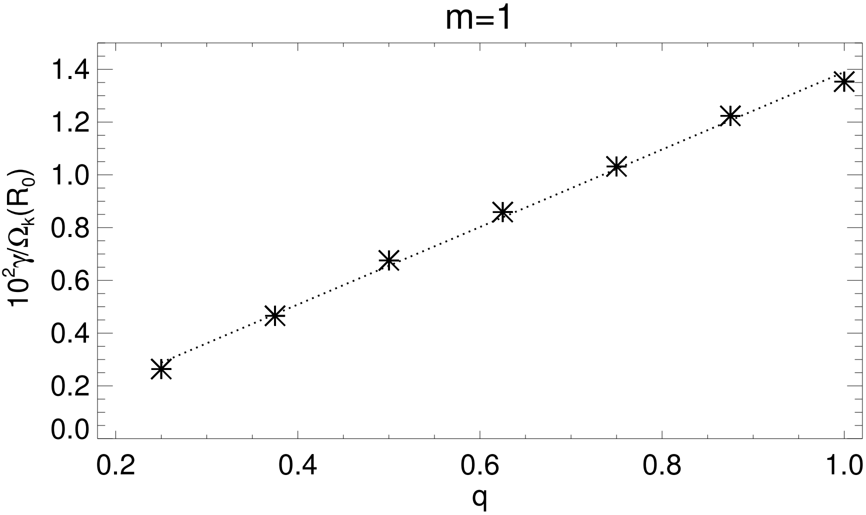


$T_{\mathrm{ext,pert}}$ in action

Extracting angular momentum from the spiral:



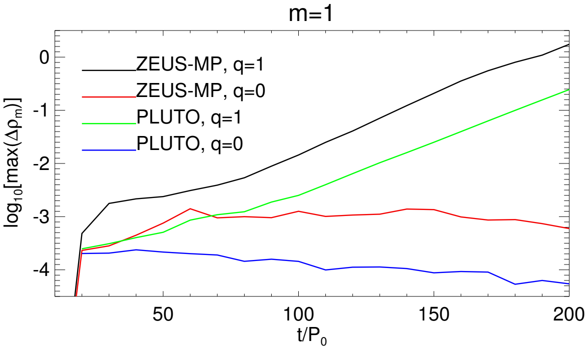
Dependence on the imposed temperature gradient



- Fixed sound-speed profile $c_s^2 \propto r^{-q}$.

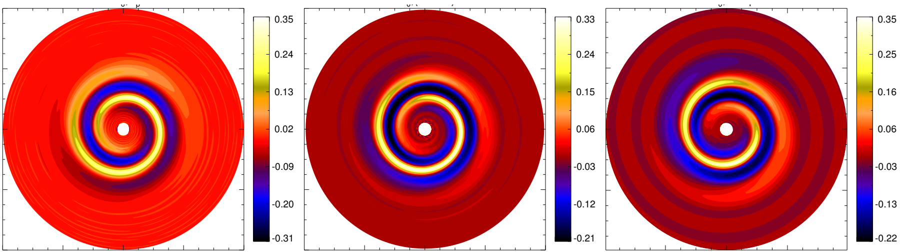
Three-dimensional simulations

Repeat experiment in 3D



- ZEUS: finite difference, discretized Poisson
- PLUTO: Godunov, Poisson through spherical harmonics
- ZEUS results off-set because of numerical issues at boundary
- *No growth without imposed temperature gradient*

Summary



FARGO 2D ZEUS 3D PLUTO 3D

- Locally isothermal disks are weird : forcing a temperature gradient permit disturbances to exchange angular momentum with the disk
- Application to protoplanetary disks uncertain
- Be star disks?
- Need to develop a more rigorous theory

Three-dimensional locally isothermal disks are baroclinic

If

$$T = T(R) \propto c_s^2$$

Then

$$R\frac{\partial \Omega^2}{\partial z} = -\frac{\partial \ln \rho}{\partial z}\frac{dc_s^2}{dR} \neq 0$$

- This *vertical shear* may render the disk unstable

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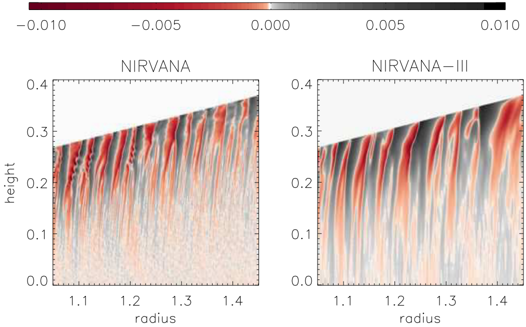
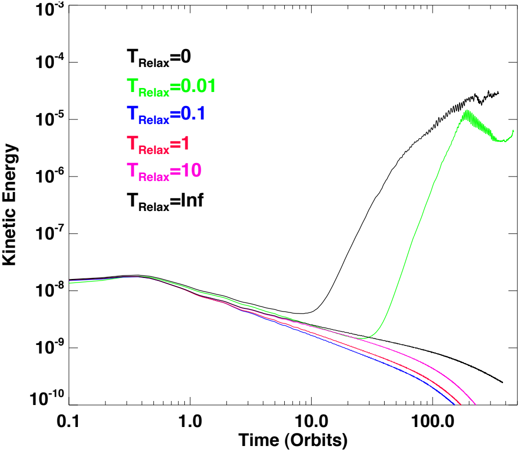


Axisymmetric simulations by Nelson et al. (2013)

Vertical shear instability requires fast thermal relaxation

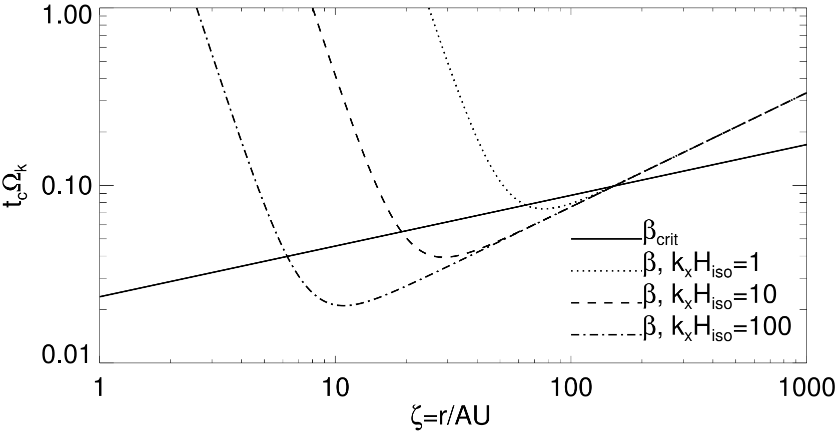
Otherwise buoyancy prevents vertical motion



Simulations from Nelson et al. (2013) with

$$\frac{\partial T}{\partial t} = -\frac{T - T_{\rm init}}{T_{\rm Relax} P_{\rm orb}}.$$

What is 'fast' for VSI?

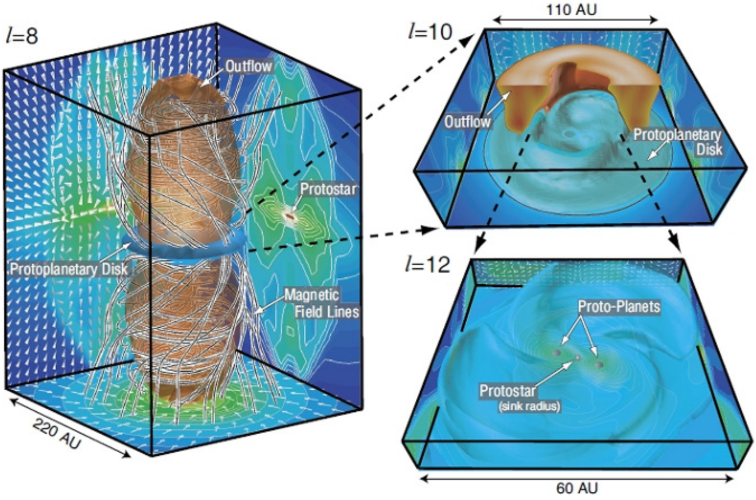


(Lin & Youdin, in prep.)

- $T_{\mathrm{Relax}} = \beta\Omega^{-1}$
- Applied to Minimum Mass Solar Nebulae

Magnetized massive disks

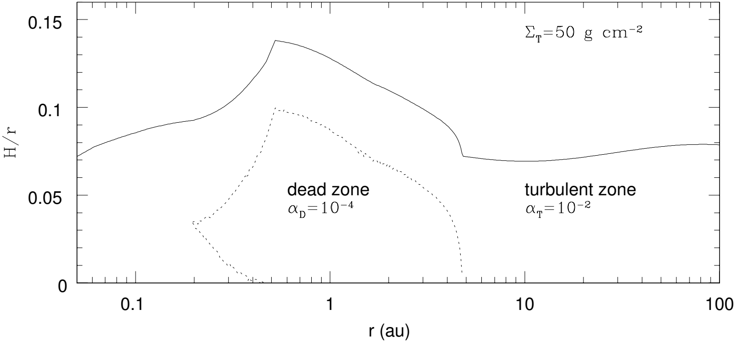
Example 1: protostellar disk formation



Non-ideal MHD plus self-gravity simulation from Inutsuka et al. (2010)

Magnetized massive disks

Example 2: 'dead zones' and layered accretion in protoplanetary disks



(Terquem, 2008)

- Can you have a self-gravitating midplane plus MHD turbulent upper/lower layers?

Previous work

- Self-gravitating

  $\rightarrow$ gravitational instability (GI)
- Magnetized

  $\rightarrow$ magneto-rotational instbaility (MRI)

- Fromang et al. (2004)
  $\rightarrow$ latest dedicated simulations

Also

- Lizano et al. (2010)
  $\rightarrow$ flat disk model (no MRI): effect of field on GI

MRI plus GI from scratch

Linear model:

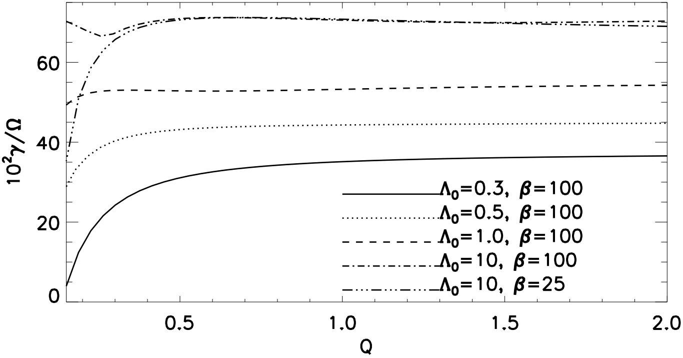
- Axisymmetric shearing box
- Self-gravitating
- Magnetized, initially uniform (both B_z and B_y allowed)
- Isothermal or polytropic
- Ohmic resistivity, can be non-uniform

Questions for adding SG to a magnetized disk:

- Are MRI growth rates affected?
- Is 'layered' structure possible (GI at the midplane, MRI at top and bottom)?
- Can SG enhance density perturbations from MRI?

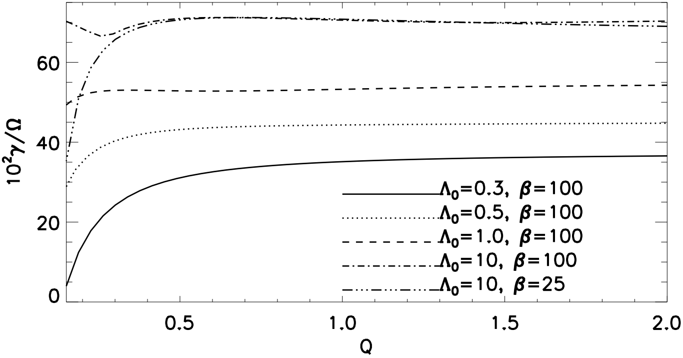
(Lin, 2014)

Upper limit on the field strength for MRI in a massive disk



- $Q = \Omega^2/4\pi G\rho_0$ small $Q \rightarrow$ strong self-gravity
- Plasma $\beta = c_{s0}^2/v_{A0}^2$ small $\beta \rightarrow$ strong field
- Elsasser number $\Lambda_0 = v_{A0}^2/\eta_0\Omega$ small $\Lambda_0 \rightarrow$ strong resistivity

Upper limit on the field strength for MRI in a massive disk

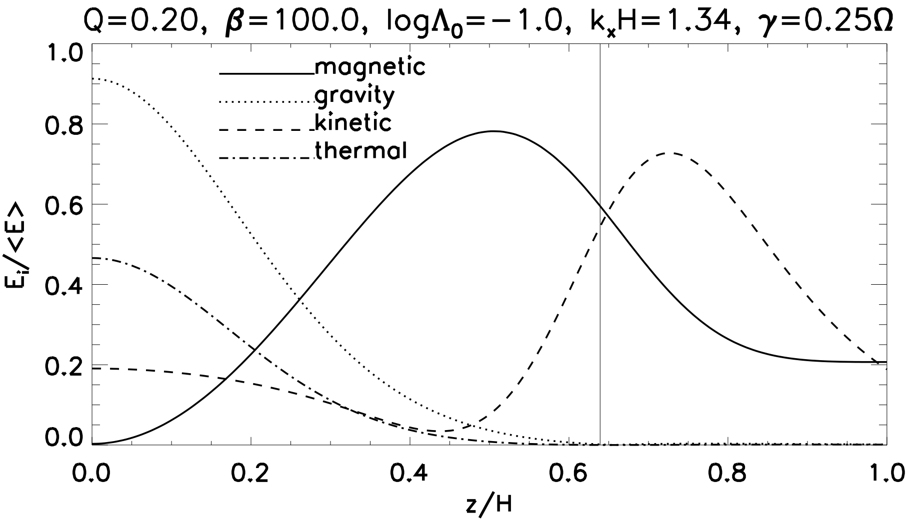


Ideal MHD, polytropic disk ($P \propto \rho^2$), vertical field, need

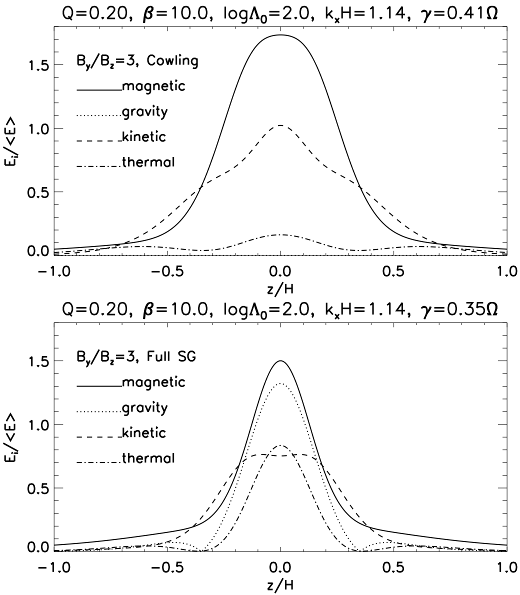
$$\frac{B_z}{c_{s0}\Omega}\sqrt{\frac{\pi G}{\mu_0}} \lesssim \frac{\sqrt{15}}{16}$$

to get MRI in strongly self-gravitating disks.

Is there a layered MRI-GI mode?



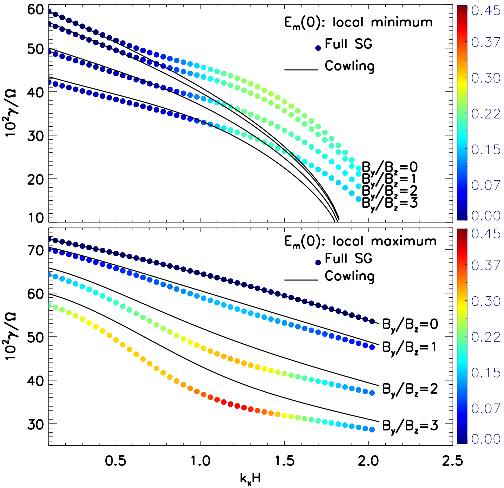
Enhancing MRI density perturbations



- No self-gravity
- Small thermal/density perturbation

- With self-gravity
- Large thermal/density perturbation

How SG affects MRI depends on symmetry



gravity-dominated, **magnetic**-dominated

- MRI can be symmetric or anti-symmetric across $z = 0$
- GI can only be symmetric

Summary and future directions

- MRI-GI interaction requires them to have similar scales
  $\rightarrow$ need weak MRI so its vertical lengthscale $\sim H$ (cf. Fromang et al., 2004)
- Next step: non-axisymmetric perturbations

Eventual goal:

- Full MHD simulations of self-gravitating disks
- Questions:
  angular momentum transport, effect of MRI turbulence on disk fragmentation

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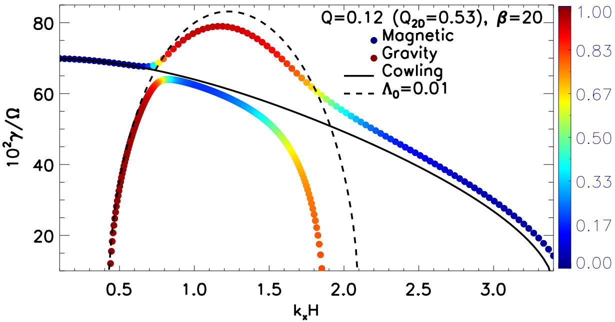
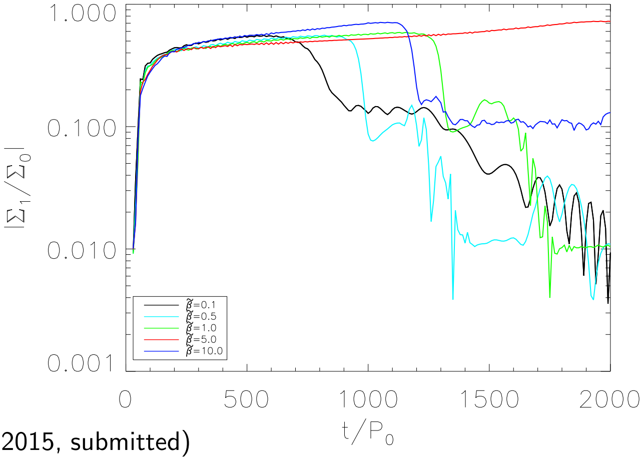
For fun: 'avoided crossing' between MRI and GI?



The vortex instability in non-isothermal disks

(2014 CITA summer student program)



(Les & Lin, 2015, submitted)

- $\beta \ll 1$: fast cooling (isothermal), $\beta \gg 1$: slow cooling (adiabatic)
- There is an optimal cooling rate to maximize vortex lifetime

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