Vortices in planetary migration

Min-Kai Lin
John Papaloizou

DAMTP
University of Cambridge

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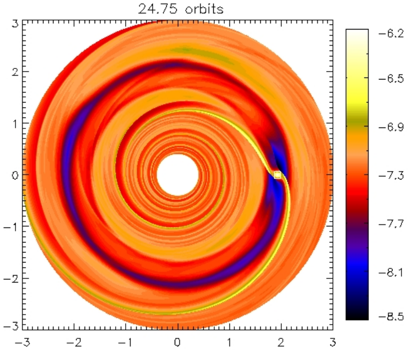
Introduction

- 374 exo-planets discovered (2 October 2009).
- First 'hot Jupiter' around 51 Pegasi, orbital period 4 days (Mayor & Queloz 1995). Fomalhaut b with semi-major axis 115AU.
- Formation difficult in situ, so invoke *migration*: interaction of planet with gaseous disc (Goldreich & Tremaine 1979; Lin & Papaloizou 1986).



Numerics

Standard numerical setup for disc-planet interaction. 2D disc in polar co-ordinates centered on primary but non-rotating. Units $G = M_* = 1$.

- Hydrodynamic equations with local isothermal equation of state:

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0,$$

$$\frac{\partial v_r}{\partial t} + \mathbf{v} \cdot \nabla v_r - \frac{v_\phi^2}{r} = -\frac{1}{\Sigma}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r} + \frac{f_r}{\Sigma},$$

$$\frac{\partial v_\phi}{\partial t} + \mathbf{v} \cdot \nabla v_\phi + \frac{v_\phi v_r}{r} = -\frac{1}{\Sigma r}\frac{\partial P}{\partial \phi} - \frac{1}{r}\frac{\partial \Phi}{\partial \phi} + \frac{f_\phi}{\Sigma},$$

$$P = c_s^2(r)\Sigma.$$

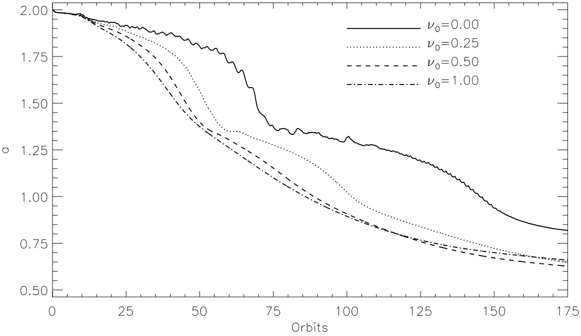
  Viscous forces $f \propto \nu = \nu_0 \times 10^{-5}$, temperature $c_s^2 = h^2/r$, $h = H/r$. Φ is total potential including primary, planet (softening $\epsilon = 0.6H$), indirect terms but no self-gravity.

- Method: FARGO code (Masset 2000), finite difference for hydrodynamics, RK5 for planet motion.

Type III in action

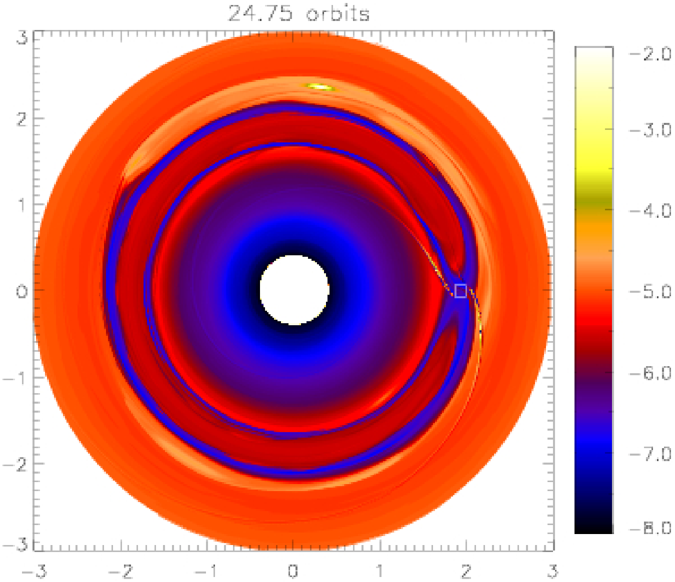
Discs: uniform density $\Sigma = 7 \times 10^{-4}$, aspect ratio $h = 0.05$ and different uniform kinematic viscosities.

Planet: Saturn mass $M_p = 2.8 \times 10^{-4}$ initially at $r = 2$.



What's going on at low viscosities?

Inviscid case: evolution of Σ/ω:



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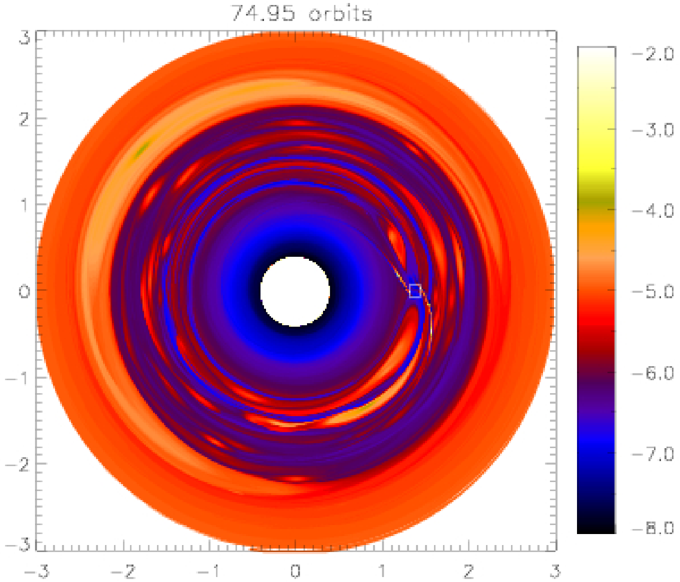

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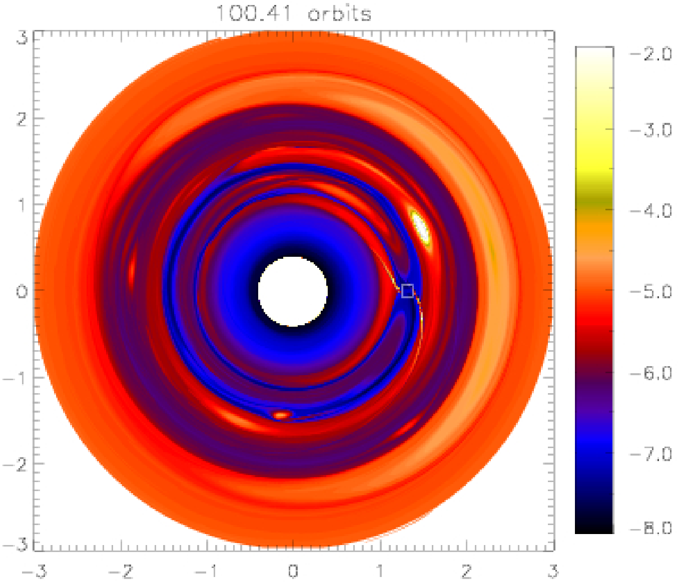
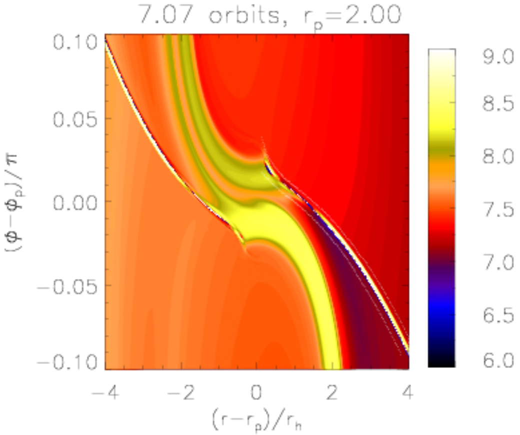


Vortensity rings: formation via shocks



Vortensity generated as fluid elements U-turn during its horse-shoe orbit.

Predicting the vortensity jump

We need:

- Vortensity jump across isothermal shock:

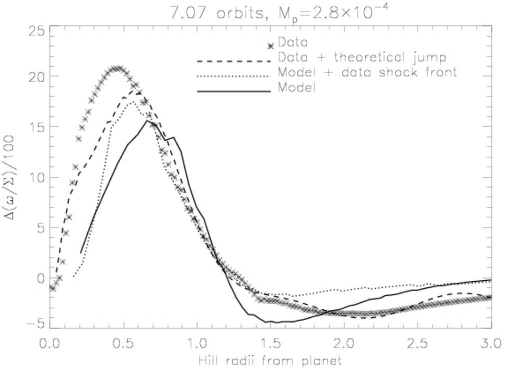
$$\left[\frac{\omega}{\Sigma}\right] = -\frac{(M^2-1)^2}{\Sigma M^4}\frac{\partial v_\perp}{\partial S} - \left(\frac{M^2-1}{\Sigma M^2 v_\perp}\right)\frac{\partial c_s^2}{\partial S}.$$

  RHS is pre-shock. $M = v_\perp/c_s$, S is distance along shock (increasing radius). Additional baroclinic term compared to Li et al. (2005) but has negligible effect ($c_s^2 \propto 1/r$).
- Flow field: shearing-box geometry, velocity field from zero-pressure momentum equations, density field from vortensity conservation following a particle.
- Shock location : generalised Papaloizou et al. (2004)

$$\frac{dy_s}{dx} = \frac{\hat{v}_y^2 - 1}{\hat{v}_x\hat{v}_y - \sqrt{\hat{v}_x^2 + \hat{v}_y^2 - 1}}.$$

$\hat{v} \equiv v/c_s$.

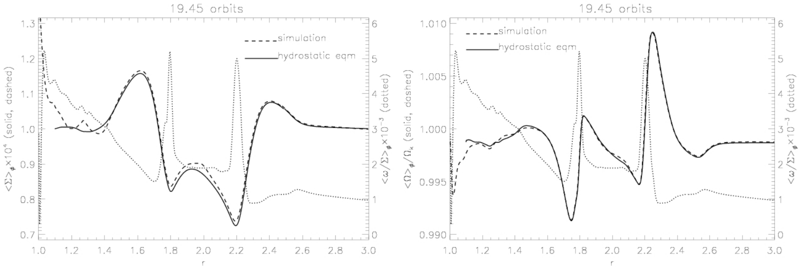
Theoretical jumps



- Vortensity generation near shock tip (horse-shoe orbits) , vortensity destruction further away (circulating region). Variation in flow properties on scales of $r_h \simeq H$.
- Variation in disc profiles on scale-heights enables shear instability $\Rightarrow$ vortices in non-linear stage (Lovelace et al. 1999, Li et al. 2001).

Ring stability

Idea: linear stability analysis of inviscid disc but use simulation vortensity profile as basic state: axisymmetric, $v_r = 0$.



Σ Ω/Ω_k

- In principle can predict gap structure via shock modelling / vortensity generation. Important to check axisymmetric hydrostatic basic state, otherwise linear analysis becomes very difficult.

Linear theory

- Governing equation for isothermal perturbations $\propto \exp i(\sigma t + m\phi)$:

$$\frac{d}{dr}\left(\frac{\Sigma}{\kappa^2 - \bar{\sigma}^2}\frac{dW}{dr}\right) + \left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{\kappa^2}{r\eta(\kappa^2 - \bar{\sigma}^2)}\right] - \frac{r\Sigma}{h^2} - \frac{m^2\Sigma}{r^2(\kappa^2 - \bar{\sigma}^2)}\right\} W = 0$$

$W = \delta\Sigma/\Sigma$; $\kappa^2 = 2\Sigma\eta\Omega$; $\bar{\sigma} = \sigma + m\Omega(r)$.

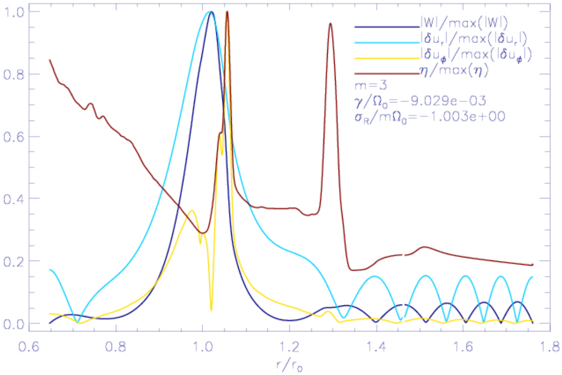
- Self-excited modes in inviscid disc with sharp vortensity profiles.

- Simplified equation for "co-rotational modes" ($\kappa^2 \gg |\bar{\sigma}^2|$, $m = O(1)$):

$$\frac{d}{dr}\left(\frac{rc^2\Sigma}{\kappa^2}\frac{dW}{dr}\right) + \left\{\frac{m}{\bar{\sigma}}\frac{d}{dr}\left[\frac{c^2}{\eta}\right] - r\Sigma\right\} W = 0.$$

Should have $(c^2/\eta)' \to 0$ as $\bar{\sigma} \to 0$ to stay regular.

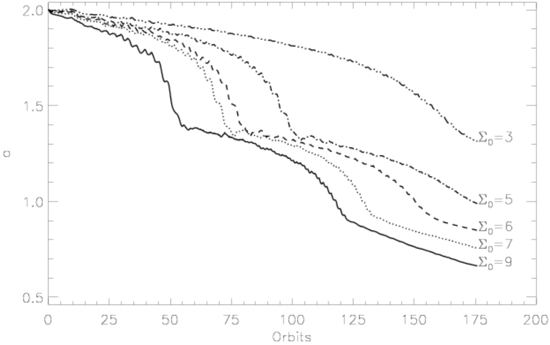
Example: $m = 3,\ h = 0.05$



- Disturbance focused around vortensity minimum (gap edge), exponential decays either side joined by vortensity term at co-rotation r_0. More extreme minimum $\Rightarrow$ more localised.
- Waves beyond the Lindblad resonances ($\kappa^2 - \bar{\sigma}^2 = 0$) but amplitude not large compared to co-rotation.

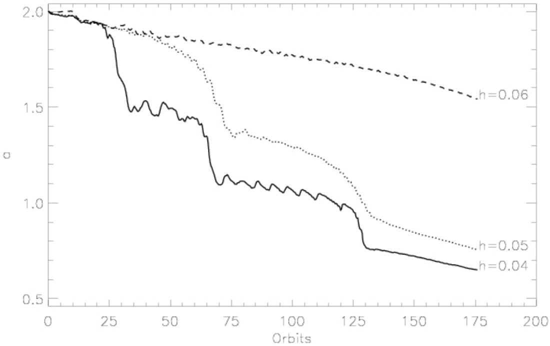
Implications of linear theory

Growth rate indepdent of density scale. Higher density just means less time needed for vortex to grow sufficiently large for interaction.



Implications of linear theory

$c_s^2 = T \propto h^2$. Lower temperature $\Rightarrow$ stronger shocks $\Rightarrow$ profile more unstable $\Rightarrow$ shorter time-scale to vortex-planet interaction.



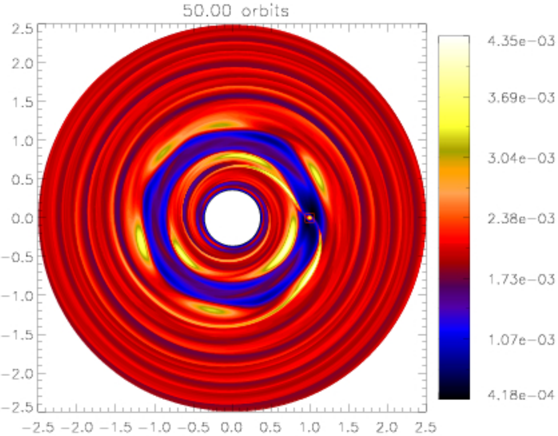
Require disc profile to be sufficiently extreme and have enough mass to trigger vortex-planet interaction, but the extent of migration during one episode is the same.

Summary

- Migration in low viscosity/inviscid discs is non-smooth due to shear instabilities associated with gap edge (vortensity minima).
- Provided an over-all picture of vortex-planet interaction: formation of unstable basic state via shocks, linear stability analysis and hydrodynamic simulations.
- Instability encourages type III by increasing co-orbital mass deficit. Vortex-planet interaction when $\delta m/M_p \sim 4$—5. Associated disruption of co-orbital vortensity structure.
- Vortex-induced migration stalls in uniform density discs but can act as trigger in $\Sigma \propto r^{-p}$ discs.

Future work: self-gravity

Thanks



(FARGO with Li et al.'s Poisson solver.)