How to fragment protostellar disks with your bare hands

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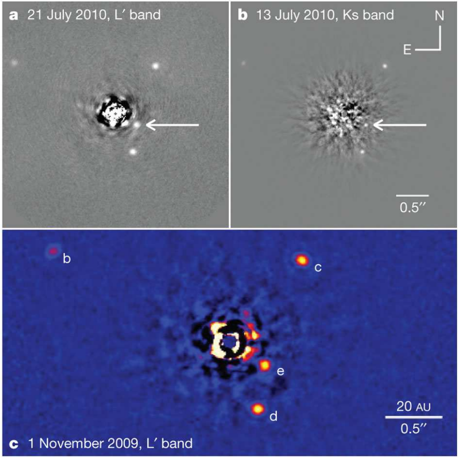
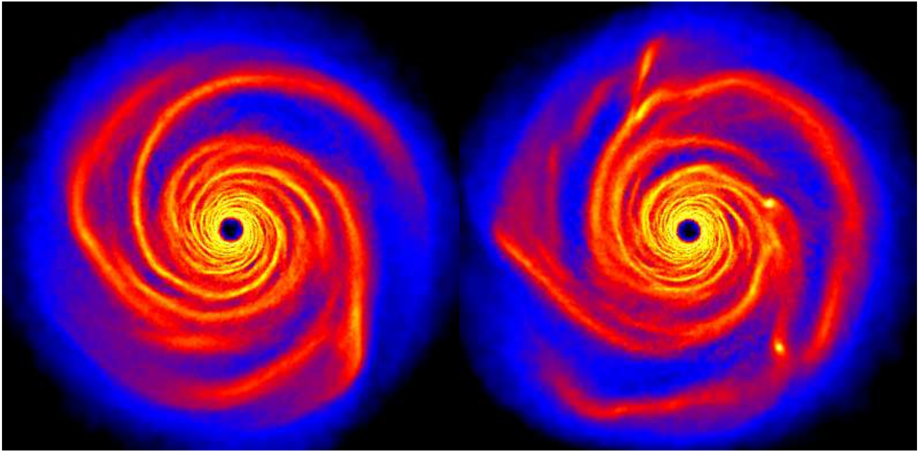
Wide orbit planets



(Marois et al., 2010)

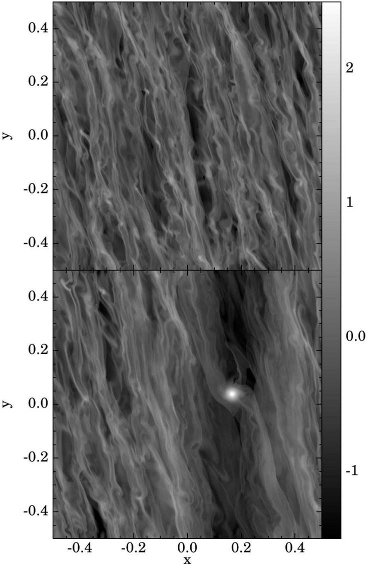
Disk instability theory

- Young, massive protoplanetary disks fragment under its own gravity



(Rice et al., 2005)

When do protostellar disks fragment on a computer?



- Massive disk

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 2 \text{ or } \frac{M_{\rm disk}}{M_*} \gtrsim 0.1$$

- Fast cooling

$$t_{\rm cool}\Omega \lesssim \text{"}3\text{"}$$

The cooling criterion is empirical.

(Paardekooper, 2012)

When does a real protostellar disk fragment?

If you don't want to run expensive simulations, then:

1. Work out disk structure: surface density Σ, temperature T
   (This might include physics such as: turbulence, stellar irradiation, radiative cooling...etc.)
2. Is Toomre $Q \sim 1$?
3. Is $t_{\mathrm{cool}}\Omega \sim 1$?

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Possible issues:

- Need to choose critical values (mass, cooling rate)
- Complex physics were not included in the numerical experiments that established those values

Experimental uncertainties

What is $t_{\mathrm{cool,crit}}$?

- Meru & Bate (2011): $t_{\mathrm{cool,crit}}$ increases with numerical resolution!
- Numerical details matter! (Lodato & Clarke, 2011; Meru & Bate, 2012; Rice et al., 2014; Young & Clarke, 2015)

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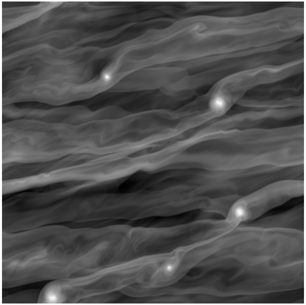
Does the concept of a $t_{\mathrm{cool,crit}}$ even make sense?

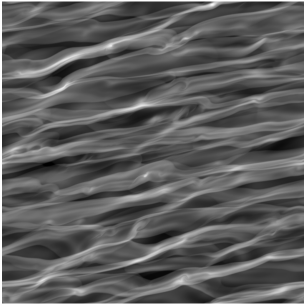
- Paardekooper (2012): just wait for it! Stochasitc in nautre
- Hopkins & Christiansen (2013): fragmentation is statistical

Conceptual approach



(Rice et al., 2011)

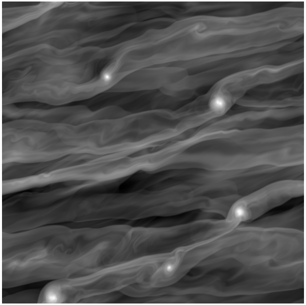
Conceptual approach



(Rice et al., 2011)

- Write down a classic, viscous disk model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze stability properties

Conceptual approach



(Rice et al., 2011)

- Write down a classic, viscous disk model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze stability properties
- Look for parameter regimes where model breaks down
  $\rightarrow$ fragmentation

Fragmentation by cooling

- Cooling removes pressure support against gravity, but how fast should it be?
- Look at dispersion relation for growth rate $s(k)$ and wavenumber k

Classic result without cooling

$$s^2 = 2\pi G\Sigma|k| - \Omega^2 \qquad - \gamma c_s^2 k^2$$

Growth = + gravity − rotation − pressure

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New result with cooling

$$s^2 = 2\pi G\Sigma|k| - \Omega^2 - \left(\frac{T_{\mathrm{irr}}/T + \gamma t_{\mathrm{cool}} s}{1 + t_{\mathrm{cool}} s}\right) c_s^2 k^2$$

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Growth = + gravity − rotation − pressure

- Dispersion relation changes from $s^2 \to s^3$
- Cooling changes the fundamental nature of the problem
- Condition for stability depends on irradiation temperature $T_{\rm irr}$
- Can be formally unstable for any $t_{\rm cool} < \infty$

Just another way to compare compressional heating v.s. thermal losses

A nice result

Special case:

$$s_{\rm max} \propto t_{\rm cool}^{-1/3}$$

Define $t_{\rm cool,*}$ as

the cooling timescale to remove pressure over a lengthscale $\sim$ disk thickness

A nice result

Special case:

$$s_{\max} \propto t_{\rm cool}^{-1/3}$$

Define $t_{\rm cool,*}$ as

$$t_{\rm cool,*} = \left(\sqrt{\gamma} - 1\right)^{-3/2} \Omega^{-1}$$

γ	$t_{\rm cool,*}\Omega$	Simulation, $t_{\rm cool,frag}\Omega$	Reference
7/5	12.75	12—13	Rice et al. (2005)
1.6	7.33	8	Rice et al. (2011)
5/3	6.37	6—7	Rice et al. (2005)
2	3.75	3	Gammie (2001)

Fragmentation by viscosity

Classic instability condition

$$2\pi G\Sigma|k| - \Omega^2 - \gamma c_s^2 k^2 > 0$$

$$+\ \text{gravity} - \text{rotation} - \text{pressure} > 0$$

Fragmentation by viscosity

Viscous instability condition

$$2\pi G\Sigma|k| \qquad -c_s^2 k^2 > 0$$

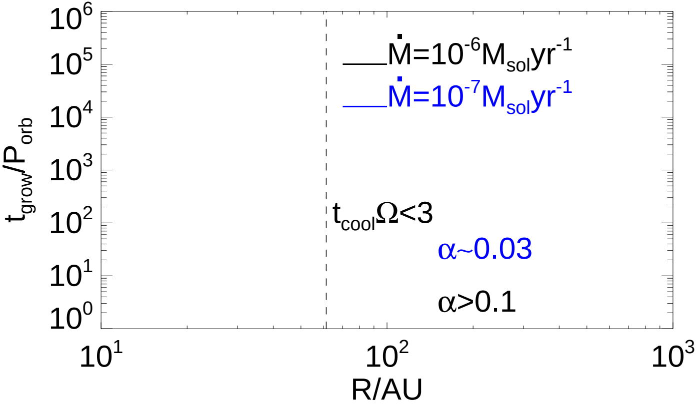
$$+\ \text{gravity} \qquad\qquad -\ \text{pressure} > 0$$

(Lynden-Bell & Pringle, 1974; Willerding, 1992; Gammie, 1996)

- Viscosity or frictional forces remove rotational stabilization (e.g. inwards migration of particles due to gas-dust drag)
- Model: use α-viscosity to mimic turbulence
- Simulations: Rice et al. (2005) report a $\alpha_{\max} \sim 0.1$ before fragmentation, also supported by Clarke et al. (2007)

Application to protoplanetary disks

- Input physical disk model into stability calculation — get growth timescales



Beyond $\sim$ 60AU:

- Cooling criterion $\Rightarrow$ both disk fragments
- Viscosity criterion $\Rightarrow$ high $\dot{M}$ disk fragments

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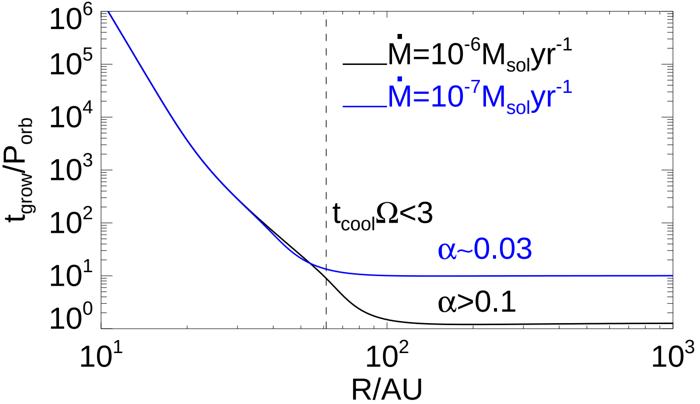

Beyond $\sim$ 60AU:

- ~~Cooling criterion $\Rightarrow$ both disk fragments~~
- Viscosity criterion $\Rightarrow$ high $\dot{M}$ disk fragments, growth times $\sim$ one orbit

Summary

- Analyze the stability properties of a model for non-fragmenting disks (2D/3D shearing box, viscosity, energy equation, optically-thin cooling or radiative diffusion, irradiation)
- Dynamical instability $\rightarrow$ fragmentation
- Application to physical disk models, determine *where and why* fragmentation occurs
- Minimal input from numerical simulations

Lin & Kratter, submitted

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