Three-layer magnetoconvection

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Introduction

We study the evolution of a three-layered fluid permeated by a vertical magnetic field, where the middle layer is convectively stable but the upper and lower layers are unstable to convection. Such configuration is believed to exist in A-type stars that have multiple convection zones near its surface. The lack of solid boundaries allow fluid elements to overshoot from unstable to stable regions, affecting transport and mixing processes in stellar interiors. Here we focus on the effect of magnetic field strength on the convection and on the interaction between the layers.

Model

We consider a slab of fluid composed of three layers of equal thickness and permeated by a vertical magnetic field. The governing equations, in dimensionless form, are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla(P + FB^2/2) + \theta(m+1)\rho\hat{\mathbf{z}} + \nabla\cdot(F\mathbf{BB} + \rho\sigma\kappa\tau)$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T + (\gamma-1)T\nabla\cdot\mathbf{u} = \frac{\gamma\kappa}{\rho}\nabla^2 T + \kappa(\gamma-1)(\sigma\tau^2/2 + F\zeta_0 J^2/\rho)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\wedge(\mathbf{u}\wedge\mathbf{B} - \zeta_0\kappa\nabla\wedge\mathbf{B}),$$

together with $\nabla\cdot\mathbf{B} = 0$ and $P = \rho T$. Lengths are scaled by the depth d of each layer; density and temperature by ρ_0 and T_0, (values at $z = 0$, the top); time by $d/\sqrt{R_* T_0}$ where R_* is the gas constant; and magnetic field by B_0, the magnitude of the initial uniform field. Here $F = B_0^2/R_* T_0\rho_0\mu_0$, $\kappa = K/d\rho_0 c_p\sqrt{R_* T_0}$ is the dimensionless thermal diffusivity, τ the stress tensor and $\zeta_0 = \eta c_p\rho_0/K$ where η is the magnetic diffusivity. The Chandrasekhar number $Q = F/\zeta_0\sigma\kappa^2$, where σ is the Prandtl number, is a measure of field strength relative to diffusion.
In this work we discuss three cases, namely $Q = 100, 500, 1000$. Other parameters are fixed: $\gamma = \frac{5}{3}$; $\sigma = 1.0$; $\zeta_0 = 0.2$; $\theta = 10$. The three-layered structure is constructed by choosing thermal conductivity of the form:

$$K = \frac{K_1}{2}\left[1 + \frac{K_2+K_3}{K_1} - \tanh\left(\frac{z-1}{\Delta}\right) + \frac{K_3}{K_1}\tanh\left(\frac{z-2}{\Delta}\right) - \frac{K_2}{K_1}\tanh\left(\frac{z-2}{\Delta}\right)\tanh\left(\frac{z-1}{\Delta}\right)\right],$$

and $\Delta = 0.1$. The middle layer has polytropic index $m_2 = 4$ and the top and bottom polytropic index $m_1 = m_2 = 1$, making the latter two convectively unstable. The basic state is subject to random velocity perturbations and evolved, subject to

$$T = 1,\ u_z = \frac{\partial u_x}{\partial z} = B_x = B_y = \frac{\partial B_z}{\partial z} = 0 \text{ at } z = 0,$$

$$\frac{\partial T}{\partial z} = \theta,\ u_z = \frac{\partial u_x}{\partial z} = B_x = B_y = \frac{\partial B_z}{\partial z} = 0 \text{ at } z = 3,$$

and the domain taken to be periodic in x and y with periods $x_m = y_m = 8$.

Results

We discuss a comparison of the 3 cases below but we will begin with a discussion of the intermediate case. Fig. 1 show horizontal slices through each layer for $Q = 500$. In the top layer, B_z is concentrated in circular and triangular cells, corresponding to up-flow and down-flow regions in ρu_z. These regions are are separated by rings of low B_z, and correspond to regions of low ρu_z. The field structure is transported into the stable layer as shown by the slice at $z = 1.5$. Field lines may act as connection between the layers and provide channels to guide particles from the unstable to stable region. However, increasing field strength generally reduces motion, so in terms of over-shooting there is a competition between the two effects. In the mid-layer, there is no correlation between ρu_z and B_z. The hexagonal structure in ρu_z is absent at $z = 1.5$, but has some similarity to that in $z = 2.25$. Increasing the field reduces over-shooting from the top relative to that from the bottom.

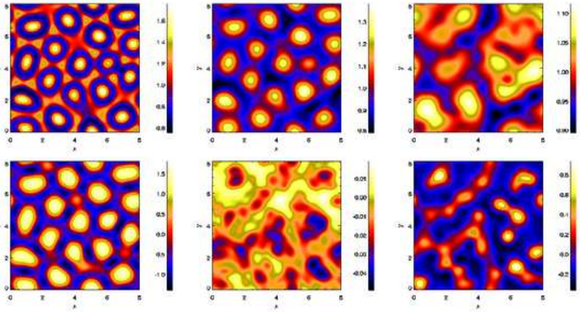
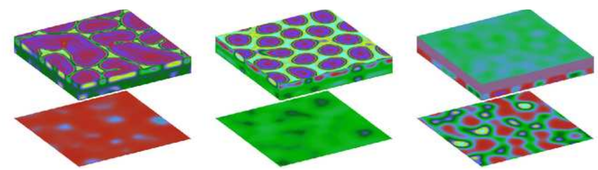


Fig. 1: *Vertical component of magnetic field (top panel) and vertical component of momentum (lower panel) in the horizontal plane at $z = 0.75, 1.5, 2.25$ (left to right); for $Q = 500$ and $t = 37.12$.*

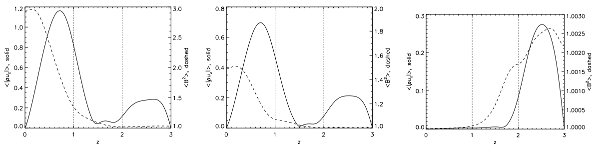
The effect of increasing Q from $100 \rightarrow 1000$ is shown in Fig. 2. We see convection patterns switch from the top layer to bottom. When $Q = 100$, large convection cells dominate the top and correlate to up-flow/down-flow regions while the bottom has relatively uniform B_z. When $Q = 500$, the stronger field reduces horizontal scales to produce hexagonal-type cells that dominate the top layer. The $Q = 1000$ case shows an almost inverted distribution of structure. The bottom has magnetic structure with a horizontal scale comparable to that for the top of the $Q = 500$ case, although the distribution is less ordered. Convection is predominantly in the bottom layer, the strong field renders the top almost featureless with a nearly uniform B_z and little vertical motion.



$Q = 100$ $\quad$ $Q = 500$ $\quad$ $Q = 1000$

Fig. 2: *Relative distribution of vertical component of magnetic field (near the top and bottom) and vertical component of momentum (sides).*

Fig. 3 demonstrates the effect of magnetic field strength on the interaction between the three layers. There is, on average non-zero ρu_z in the middle stable layer, so motion here is due to over-shooting plumes from either convection zone. The distribution of $\langle|\rho u_z|\rangle$ and $\langle B^2\rangle$ is qualitatively similar for $Q = 100$ and $Q = 500$, but smaller amplitudes in the latter. In particular the top is more suppressed in $Q = 500$ than $Q = 100$. There is relatively more magnetic energy pumped downwards into the middle for $Q = 500$. There is more connection, but since the vigour of motion is suppressed (compared to $Q = 100$) there is no increased over-shooting from the top layer. The situation is very different for $Q = 1000$, magnetic disturbance and motion is predominantly in the lower convection zone and relatively more magnetic energy is transported *upwards* into the stable layer. As convection in the top layer is suppressed, over-shooting from the bottom layer will dominate.



$Q = 100$ $\quad$ $Q = 500$ $\quad$ $Q = 1000$

Fig. 3: *Interaction between the layers indicated by horizontal average of vertical momentum density (solid) and magnetic energy (dashed).*

Conclusions

We have presented a simple model for multiple convection zones in stellar interiors and focused on the effect of magnetic field strength. For $Q = 100$ and $Q = 500$, convection occurs in both unstable layers with the top having more vigorous motion, and more overshooting occurs top→middle. When $Q = 1000$ motion and the top is suppressed and overshooting occurs bottom→middle. We have shown that communication between unstable layers can be significantly affected by a magnetic field permeating all layers. More details of this work can be found in *Lin, Silvers & Proctor, 2008, Physics Letters A, 373, 69-75.*