On the formation of one-armed spirals in locally isothermal disks

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Introduction

Large-scale spiral structures have been observed in the outer parts of transition disks. These could be due to unseen planets, but theoretical alternatives should be explored. We describe a new but simple mechanism that can lead to the growth of one-armed spirals, or eccentric modes, in irradiated astrophysical disks, which may be applicable to the outer parts of protoplanetary disks. We explain this new instability with linear density wave theory, confirm it through 2D and 3D hydrodynamic simulations, and find that this mechanism can produce long-lived spiral structures.

The locally isothermal disk

In the outer parts of protoplanetary disks, thermal timescales can be short due to stellar irradiation. Theoretical modeling then simplifies significantly by adopting the *locally isothermal disk*, in which the disk temperature is prescribed as a function of position, e.g.,

$$T(r) = T_0\left(\frac{r}{r_0}\right)^{-q} \quad \text{or} \quad c_s^2(r) = c_{s0}^2\left(\frac{r}{r_0}\right)^{-q}.$$

In fact, locally isothermal disks are widely used in numerical simulations. Because no energy equation is explicitly solved, this reduces computational costs tremendously. However, this leads to new effects when considering how small disturbances in the disk evolve.

Non-conservation of angular momentum

- Consider linear perturbations to the disk, e.g.

$$\boldsymbol{\Sigma}(r,\phi,t) \to \boldsymbol{\Sigma}(r) + \delta\boldsymbol{\Sigma}(r)\exp\left[\mathrm{i}\left(m\phi - \sigma t\right)\right]$$

  etc., σ: frequency, m: azimuthal wavenumber.
- Non-axisymmetric disturbances ($m \neq 0$) in a rotating disk satisfy angular momentum conservation,

$$\frac{\partial J_{\rm pert}}{\partial t} + \nabla\cdot F_{\rm pert} = \begin{cases} 0 & \text{barotropic/adiabatic} \\ -\frac{m}{2}\,\mathrm{Im}\left(\delta\Sigma\xi_r^*\frac{dc_s^2}{dr}\right) & \text{locally isothermal 2D} \\ \frac{m}{2}\,\mathrm{Im}\left[\rho\left(\nabla\cdot\xi\right)\xi^*\cdot\nabla c_s^2\right] & \text{locally isothermal 3D.}\end{cases}$$

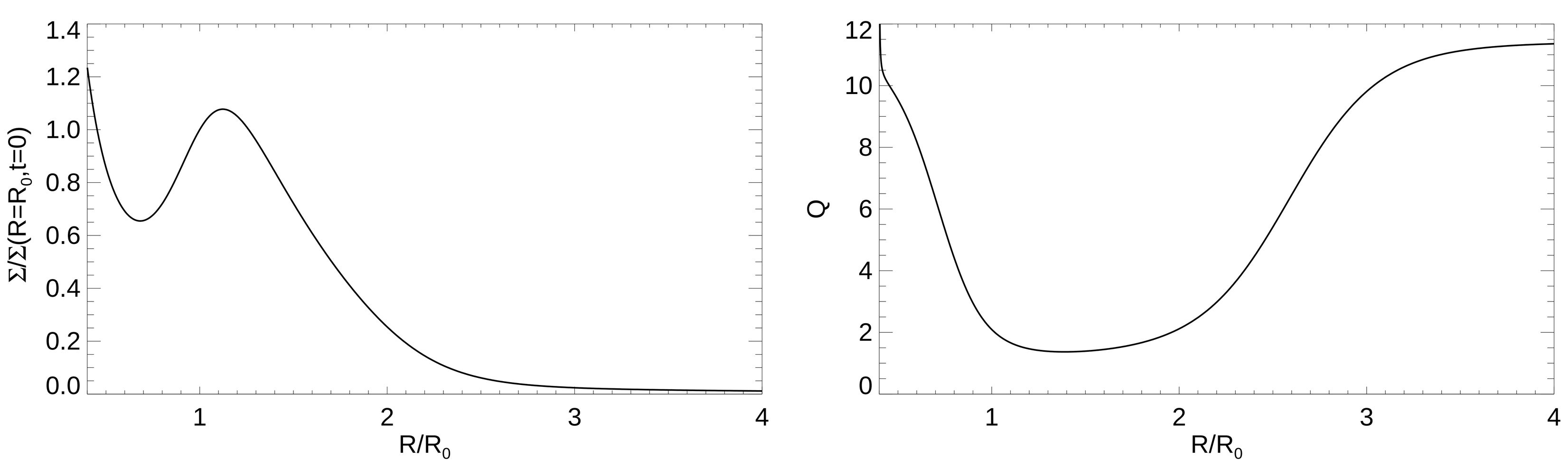
- ξ: Lagrangian displacement.
- Locally isothermal disk: $J_{\rm pert}$ can change due to temperature gradient.
- **SMALL DISTURBANCES DO NOT CONSERVE THEIR ANGULAR MOMENTUM IN LOCALLY ISOTHERMAL DISKS!**
- Exchange of angular momentum between small disturbances and the background disk $\longrightarrow$ **POSSIBLE INSTABILITY**!
- Estimate of growth rate for low frequency disturbances:

$$\gamma \sim \frac{1}{2}q\left(\frac{H}{r}\right)\left(\frac{k_rH}{m}\right)\Omega.$$

- H: disk scale height, Ω: disk rotation, k_r: radial wavenumber.

Numerical demonstration

Example: radially structured, self-gravitating disk, fixed temperature profile:



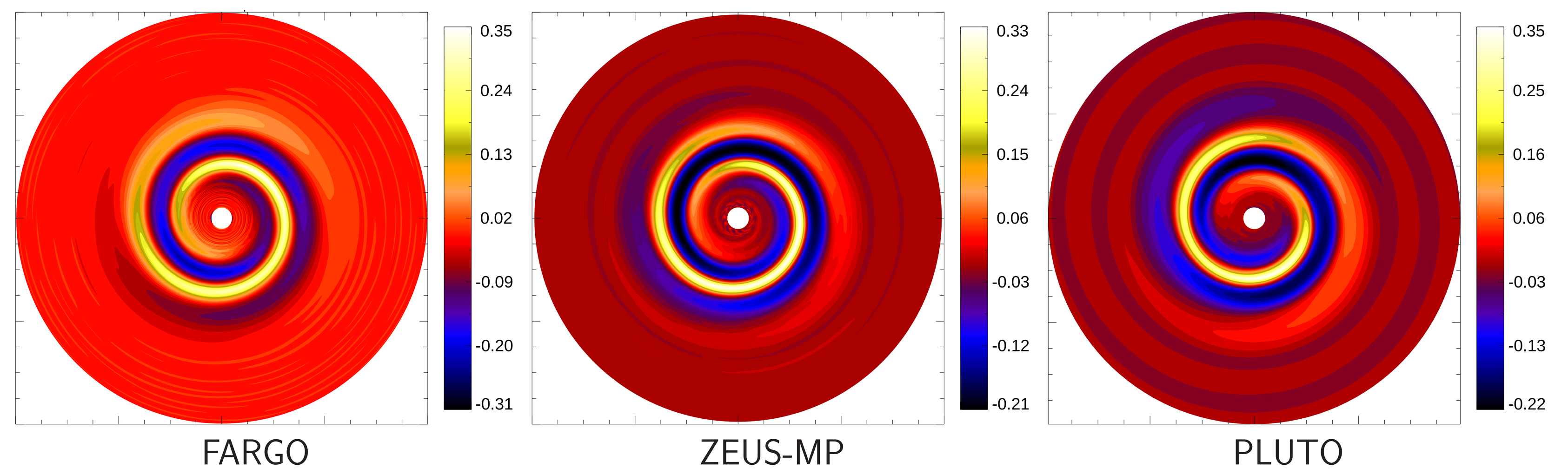
Surface density   Toomre parameter

- Radial structure 'traps' low frequency $m = 1$ modes ($|\sigma| \ll \Omega$).
- Temperature gradient then destabilizes them.

Codes:
- **FARGO**: 2D cylindrical, finite difference, Poisson via FFT.
- **ZEUS-MP**: 3D spherical, finite difference, Poisson via linear solver.
- **PLUTO**: 3D spherical, Godunov, Poisson via spherical harmonics.

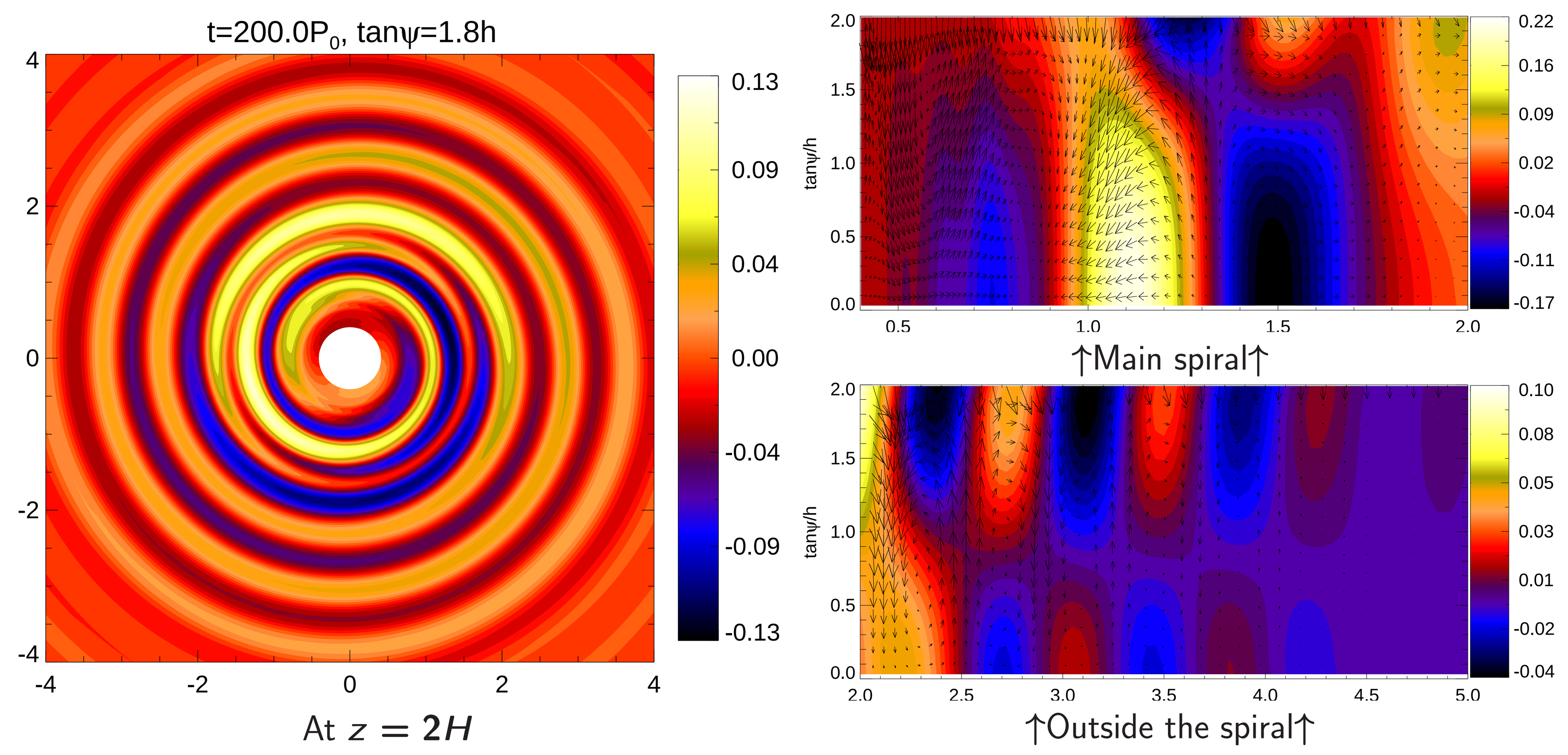
Growth of one-armed spirals

Non-axisymmetric density perturbation at the midplane ($z = 0$):



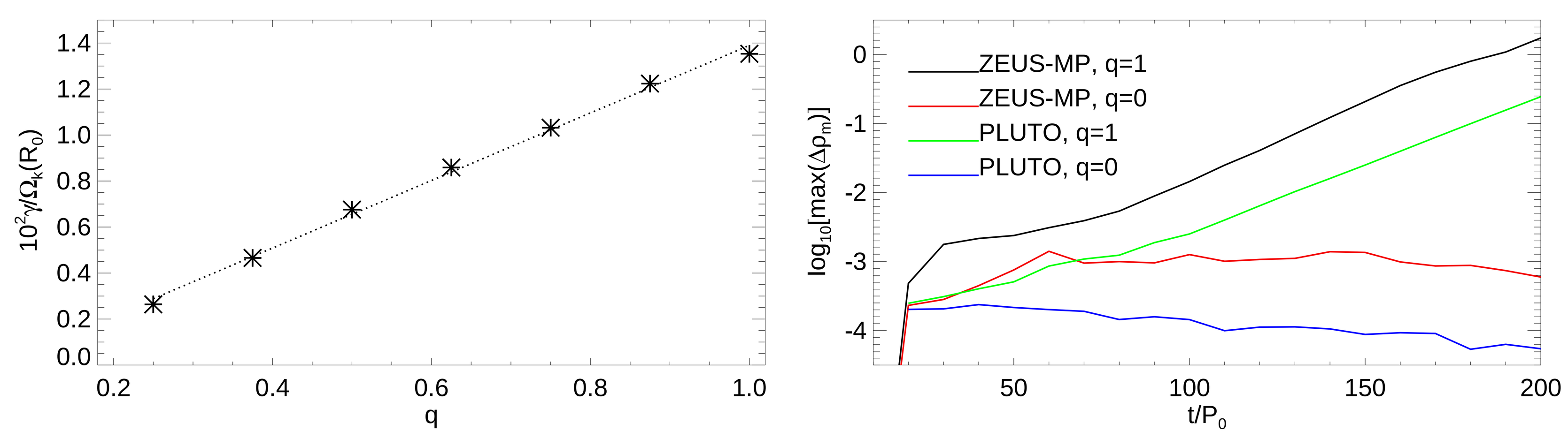
- *Not* a gravitational instability (GI) in the traditional sense.
- Does *not* require motion of the central star or very massive disks (cf. 'classic' GI that produce one-armed spirals).

Three-dimensional structure



- Spiral not radially confined away from the midplane $\longrightarrow$ 'rings' near surface.

Dependence on the temperature profile

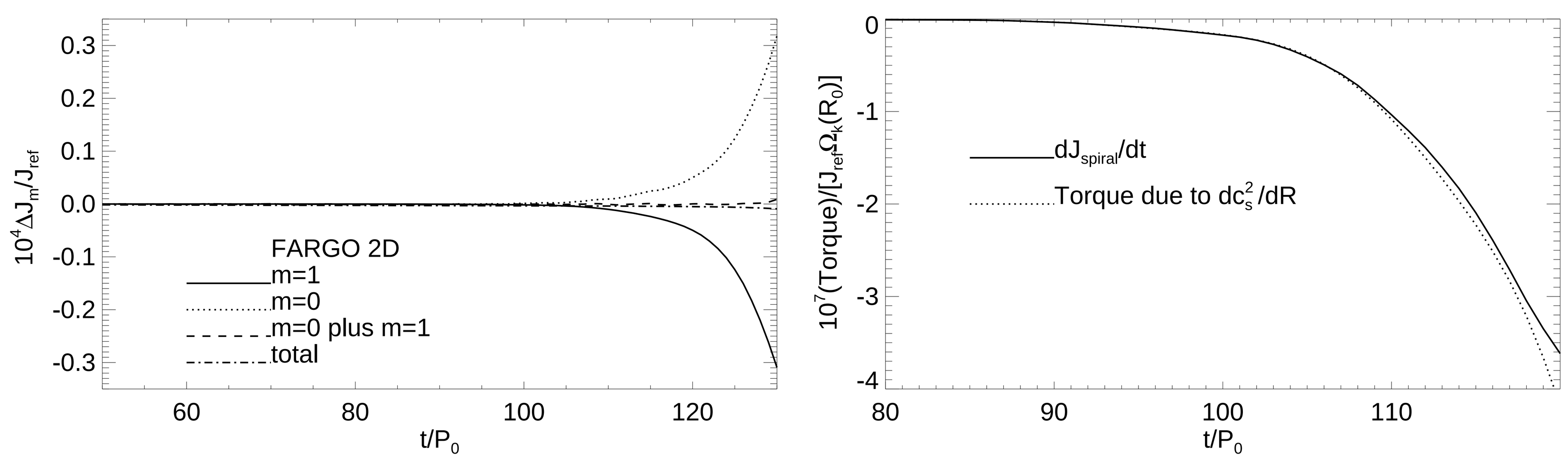


Growth rates   Mode amplitudes

- No growth in strictly isothermal (barotropic) disks.

Internal angular momentum exchange

Total angular momentum is conserved, but angular momentum is exchanged between different disk symmetries



Angular momentum components   Torque on the one-armed spiral

- One-armed spiral ($m = 1$) grows by losing angular momentum to the background disk ($m = 0$) through the imposed temperature gradient.

Summary

Astrophysical disks with short thermal timescales allow small, non-axisymmetric disturbances to exchange angular momentum with the background disk in the presence of a temperature gradient. This can lead to the growth of large-scale, long-lived spiral structures in the disk, which may be applicable to the irradiated outer parts of protoplanetary disks, where large-scale spirals are indeed observed.

- **Lin, M.-K., 2015, MNRAS, 448, 3806**