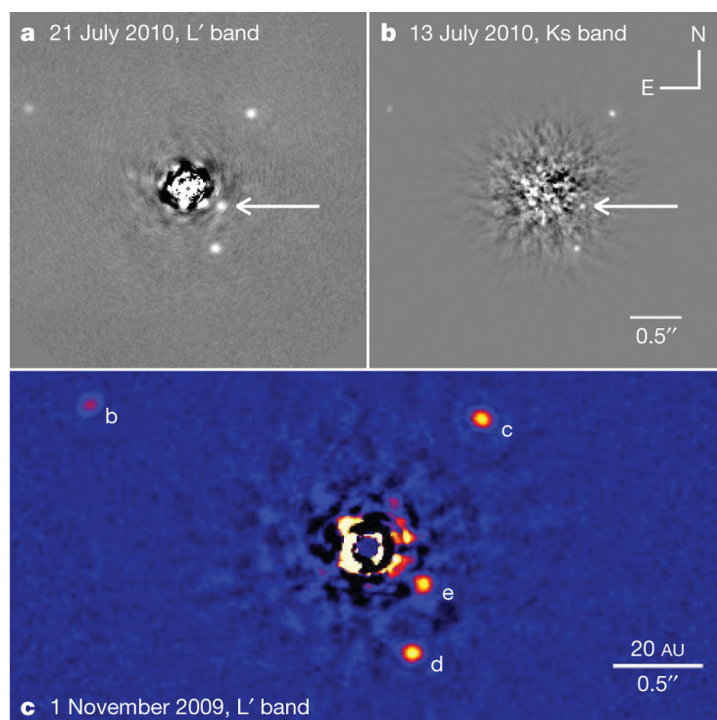


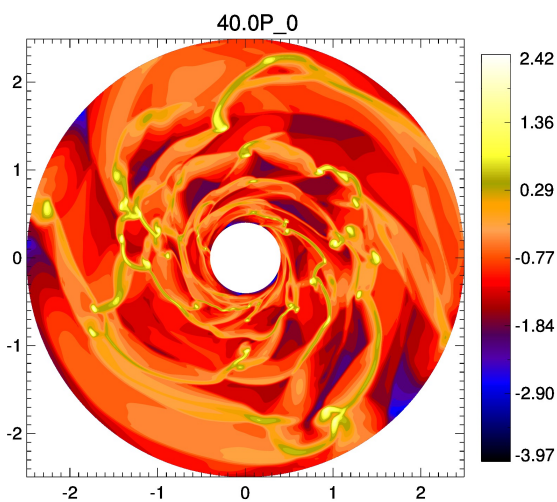
Directly imaged wide orbit planets/brown dwarfs



(Marois et al., 2010)

Disk instability theory

- Young, massive protoplanetary disks can fragment under its own gravity



(Lin, Fargo sims., log density)

Fragmentation conditions

- Massive disk

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 2 \text{ or } M_{\text{disk}} \gtrsim 0.1 M_*$$

- Fast cooling

$$t_{\text{cool}} \Omega \lesssim 3$$

The cooling criterion is empirical!

When do realistic protostellar disks fragment?

Work out $\Sigma(R)$, $T(R)$..etc., then ask

- ① Where/when is Toomre $Q \lesssim 2$?
- ② Where/when is $t_{\text{cool}}\Omega \lesssim 3$?

WARNING

Critical cooling depends on the numerical simulation!

(resolution, 2D/3D, local/global, particle-based or grid-based simulations)

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Motivation 1:

Assess disk fragmentation without input from hydrodynamic simulations

Beyond classical gravitational instability

Modern simulations (c. 2010)

- Cooling physics, e.g.

$$\frac{\partial E}{\partial t} = - \frac{E}{t_{\text{cool}}}$$

- Turbulent/viscous, e.g.

$$\nu = \alpha \frac{c_s^2}{\Omega}$$

Analytic toolbox (c. 1960)

Lin-Shu dispersion relation, Toomre Q

$$\omega^2 = \kappa^2 - 2\pi G \Sigma |k| + c_s^2 k^2$$

$$Q \equiv \frac{c_s \kappa}{\pi G \Sigma}$$

- Isothermal/adiabatic (no cooling)
- Laminar (inviscid)

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Motivation 2:

Generalize analytic treatment of GI to include cooling, irradiation and viscosity

$$\omega = \omega(k; Q, t_{\text{cool}}, \alpha)$$

Quantifying cooling

Dispersion relation with cooling

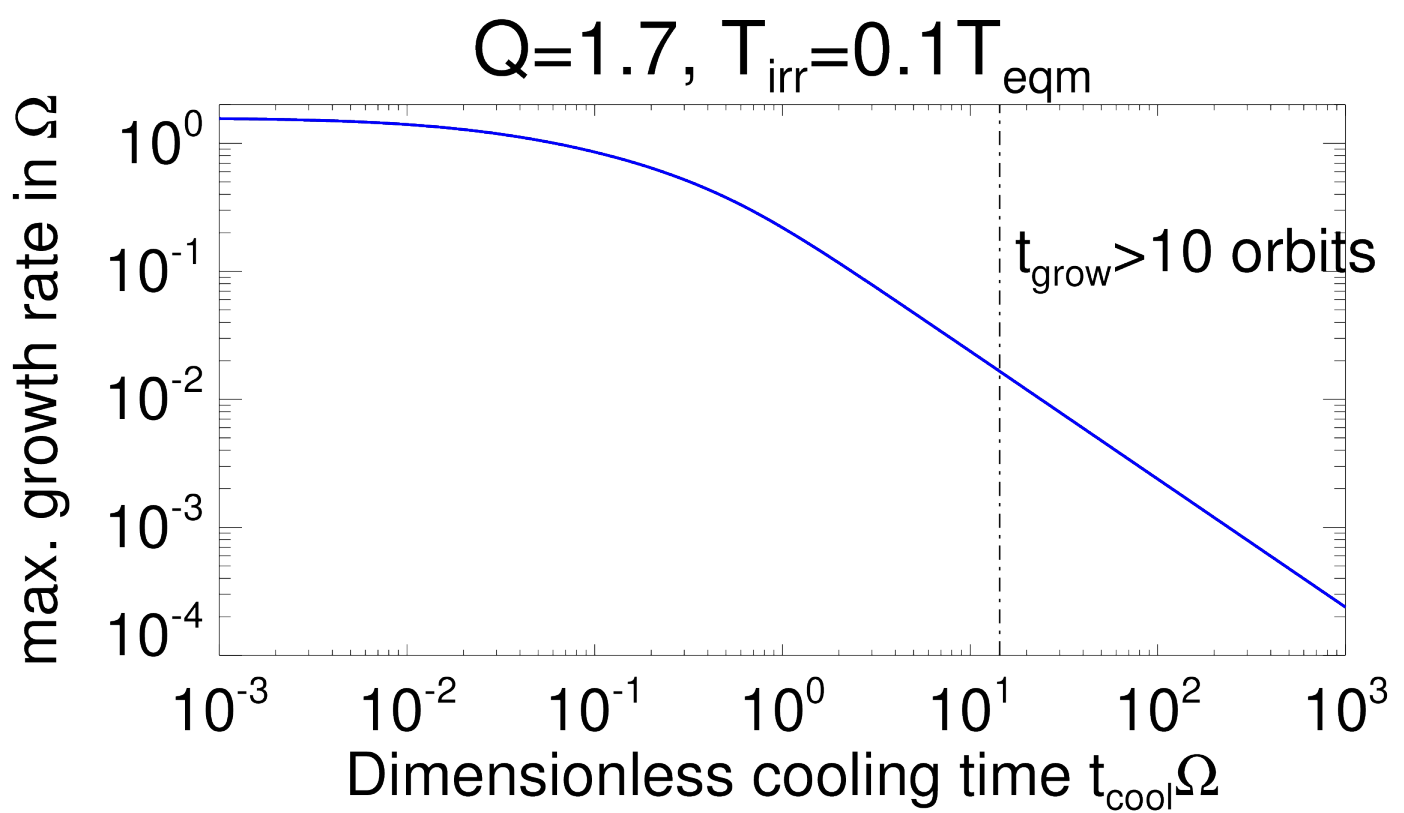
$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G \Sigma |k|}_{\text{+gravity}} \underbrace{-\Omega^2}_{\text{-rotation}} \underbrace{- \left(\frac{T_{\text{irr}}/T + \gamma t_{\text{cool}} s}{1 + t_{\text{cool}} s} \right) c_s^2 k^2}_{\text{-modified pressure}}$$

(Lin & Kratter, 2016, arXiv:1603.01613)

- T_{irr} : irradiation or floor temperature
- Can be unstable even for $Q > 1$ (cf. $Q < 1$ for classic GI)

Cooling changes the fundamental nature of disk GI

Cooling-driven gravitational instability

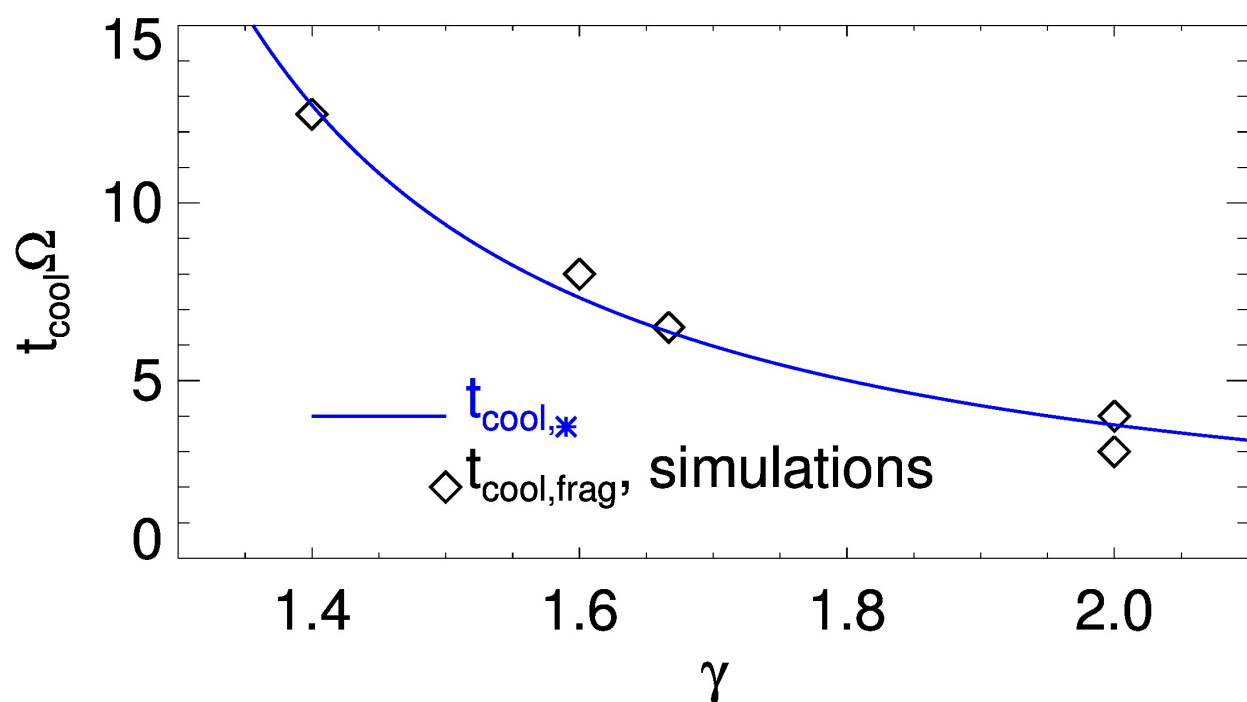


(Lin & Kratter, 2016, arXiv:1603.01613)

Understanding simulations

Cooling timescale to remove pressure over a lengthscale $\sim H$

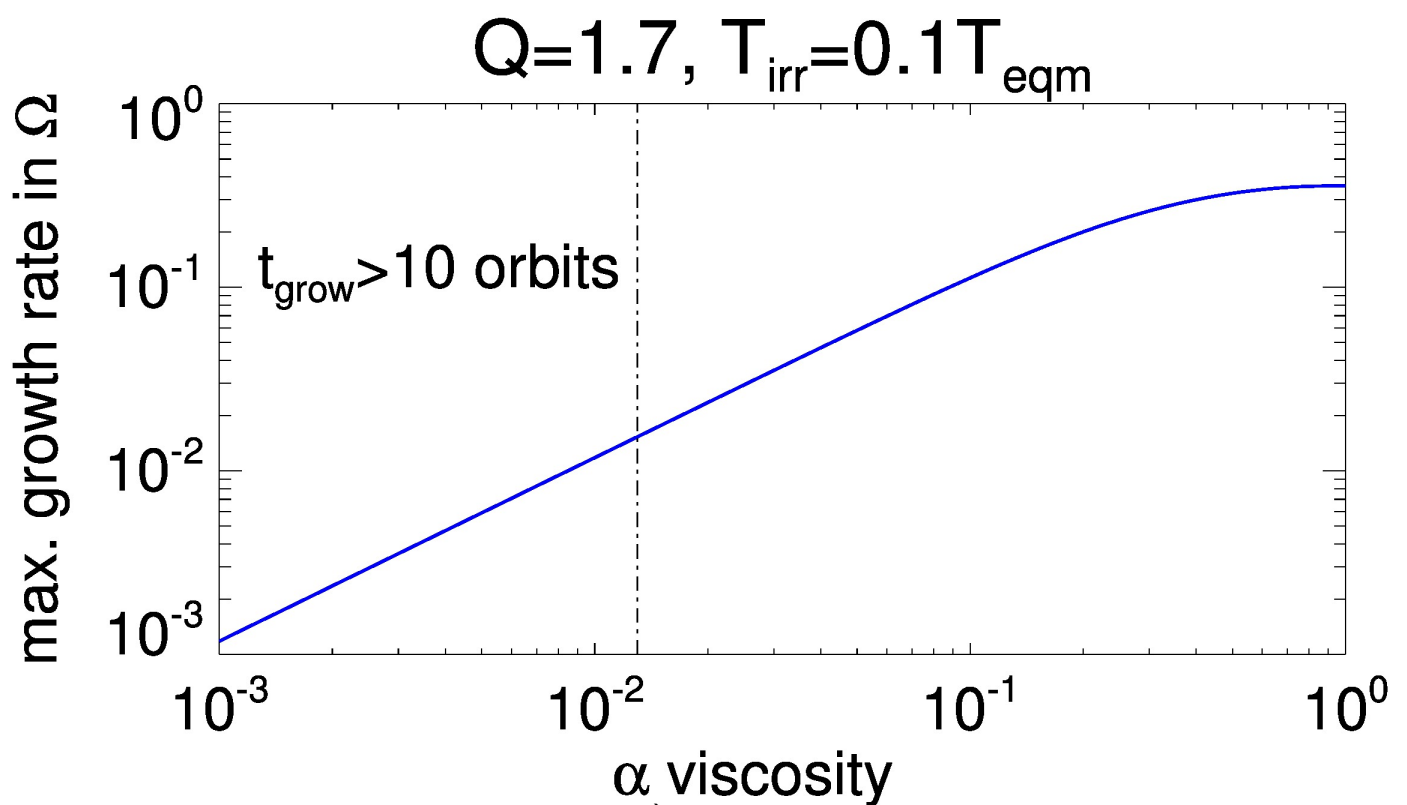
$$t_{\text{cool},*} = (\sqrt{\gamma} - 1)^{-3/2} \Omega^{-1} \quad (\text{Lin \& Kratter, 2016, arXiv:1603.01613})$$



Simulations: Gammie (2001); Rice et al. (2005, 2011); Paardekooper (2012)

Viscous gravitational instability

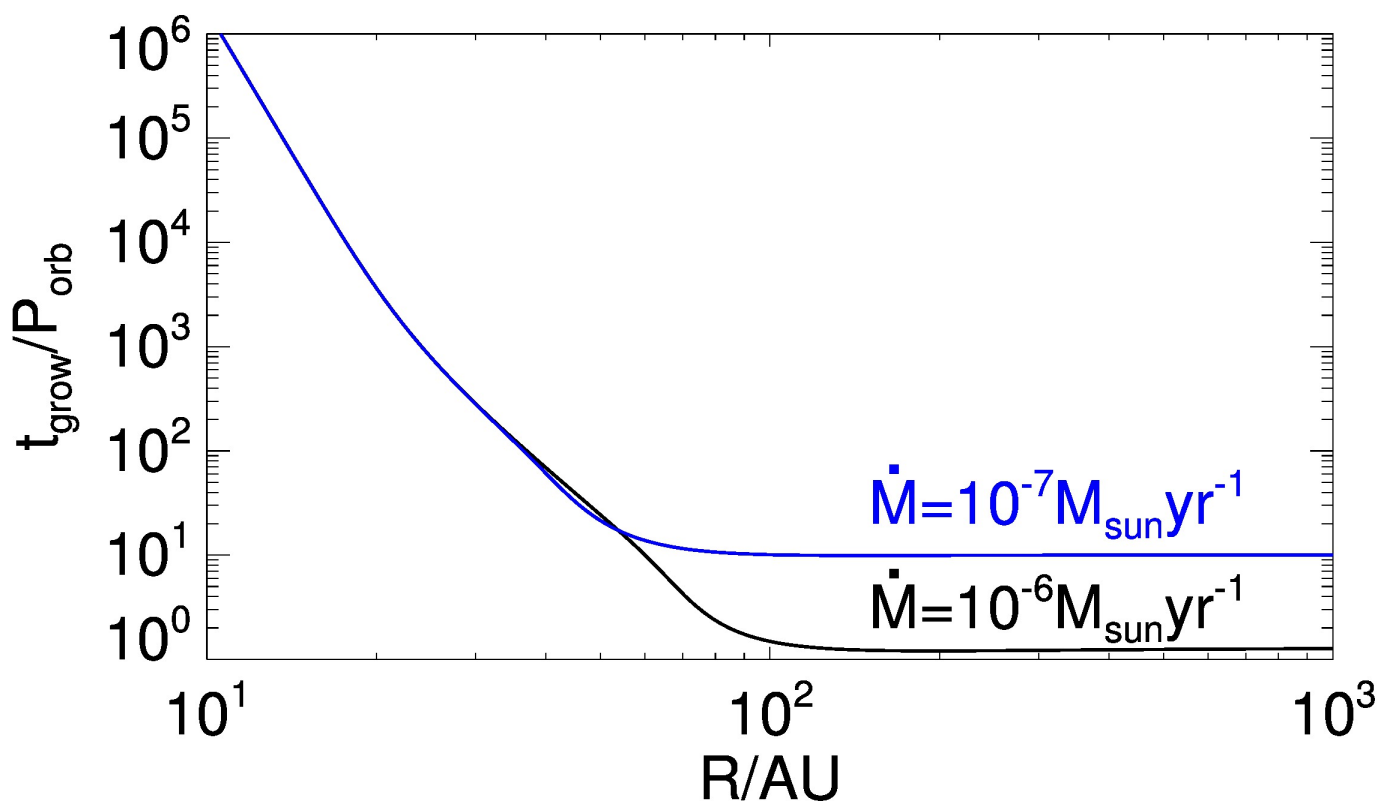
- Viscosity/friction can remove rotational stabilization (Lynden-Bell & Pringle, 1974)



(Lin & Kratter, 2016, arXiv:1603.01613)

Putting it all together: application to protoplanetary disks

- Input physical disk model with cooling and viscosity — get growth timescales



(Lin & Kratter, 2016, arXiv:1603.01613)

- High \dot{M} disk fragments $\gtrsim 60 \text{ AU}$, growth times \sim one orbit

What's next for disk GI theory?

- Global effects with cooling and viscosity
 - ▶ Mass infall
 - ▶ Disks with radial structure
 - ▶ Large-scale spiral instabilities
- Magnetic effects : good or bad for stability?
 - ▶ Extend Lin (2014) to include cooling/viscosity