Vertical shear instability in protoplanetary disks

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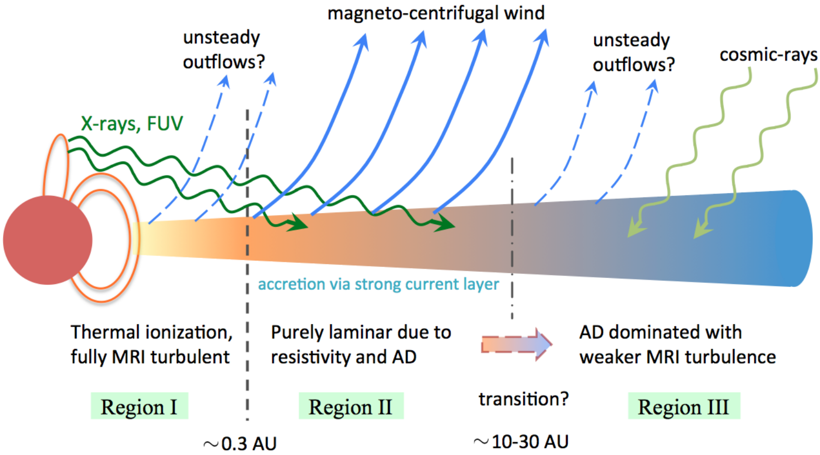
June 18 2015

Transport in protoplanetary disks

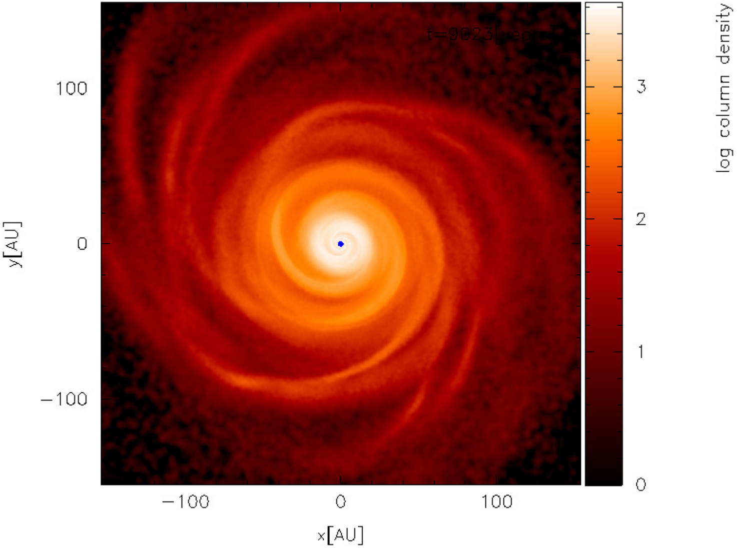
- Magnetic



(Bai, 2013)

Transport in protoplanetary disks

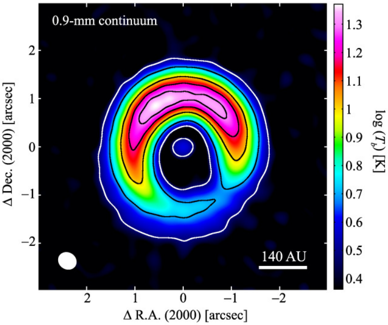
- Self-gravity



(Forgan et al., 2011)

Hydrodynamic instabilities

- Vortices



(Fukagawa et al., 2013)

- 'Rossby wave instability' (Lovelace et al., 1999; Li et al., 2000): Kelvin-Helmholtz in a disk
- 'Convective overstability' (Klahr & Hubbard, 2014; Lyra, 2014): growing epicycles
- 'Baroclinic instability' (Lesur & Papaloizou, 2009): non-linear amplification of vortices

Baroclinic disks and vertical shear

- Astrophysical disks are generally *baroclinic* ($\nabla P \times \nabla \rho \neq 0$) so that

$$\frac{\partial \Omega}{\partial z} \neq 0.$$

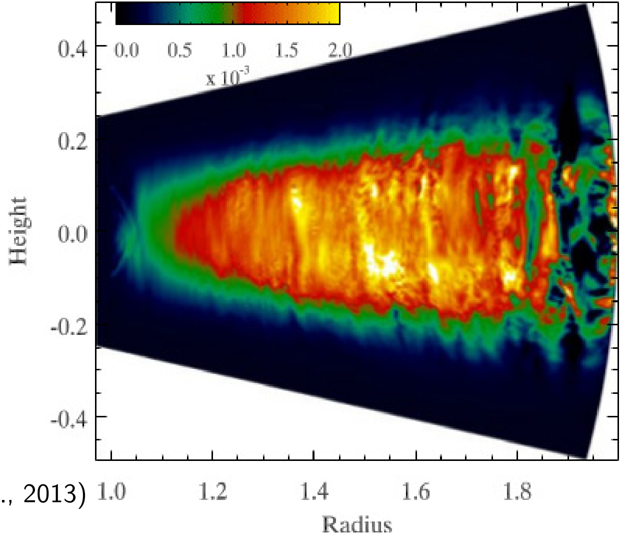
- Example: vertically isothermal thin-disk with radial temperature gradient $T \propto r^q$,

$$r \frac{\partial \Omega}{\partial z} \simeq \frac{1}{2} h q \Omega_{\mathrm{Kep}} \left(\frac{z}{H} \right)$$

  $h = H/r$: disk aspect-ratio; Ω_{Kep}: Keplerian rotation

- Shear flow $\Rightarrow$ free energy $\Rightarrow$ instability?

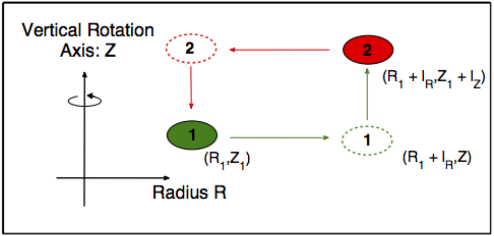
The 'vertical shear instability'



(Nelson et al., 2013)

Hydrodynamic turbulence.

Basic physics of the VSI



(Umurhan et al., 2013)

- Change in kinetic energy:

$$\Delta E \sim l_r^2 \left(\Omega^2 + \frac{l_z}{l_r} \cdot r \frac{\partial \Omega^2}{\partial z} \right).$$

- Vertical shear is weak, **BUT**

$$\Delta E < 0 \quad \text{is possible if} \quad |l_z| \gg |l_r|,$$
$\Rightarrow$ **INSTABILITY!**

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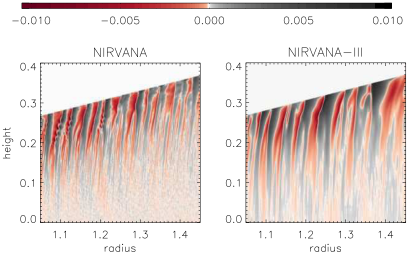

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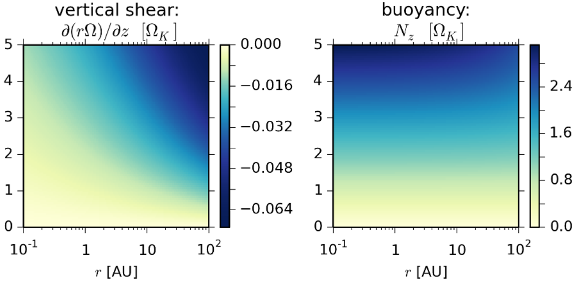
Role of buoyancy

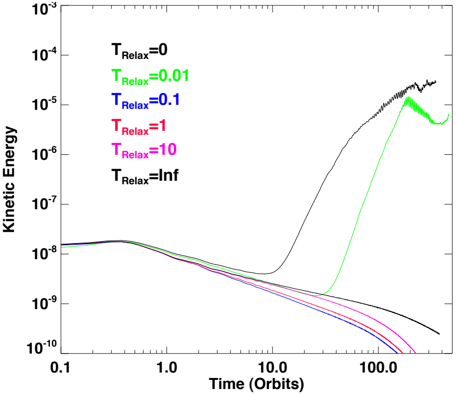
Vertical motion associated with VSI is opposed by buoyancy forces

$$\underbrace{r\partial_z\Omega}_{\text{destabilizing vert. shear}} \quad \text{v.s.} \quad \underbrace{N_z}_{\text{stabilizing vert. buoyancy}}$$



- Vertical shear is *weak*, $r\partial_z ln\Omega \sim O(h) \ll 1$
- Vertical buoyancy is *strong*, $N_z/\Omega \sim O(1)$

The need for rapid cooling



(Nelson et al., 2013)

- Buoyancy ineffective if cooling times are short $\rightarrow$ VSI can operate
- Stoll & Kley (2014): VSI in radiation-hydrodynamic simulations only when external heating included

Is there a quantitative thermodynamic criteria?

Previous stability analyses and our contribution

- Vertically and radially local analyses: (Urpin & Brandenburg, 1998; Urpin, 2003)
- Vertically global, radially local analyses, no buoyancy: (Nelson et al., 2013; Barker & Latter, 2015)

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Lin & Youdin (2015)

- Vertically global, radially local, including energy equation (i.e. with buoyancy)
- Both constant cooling and realistic cooling functions

Linear theory: simplified model

- Axisymmetric perturbations in a vertically isothermal disk
- Wave-ansatz radial dependence with wavenumber k_x, low-frequency limit $|\omega| \ll \Omega_{\rm Kep}$
- Vertically constant cooling time, $t_c \equiv \beta\Omega_{\rm Kep}^{-1}$

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Reduction to single ODE

$$0 = \delta v_z''(z) - zA\delta v_z'(z) + \left(B - Cz^2\right)\delta v_z(z).$$

- Hermite differential equation after transformation
- Dispersion relation $\omega = \omega(k_x; \beta)$ for the frequency

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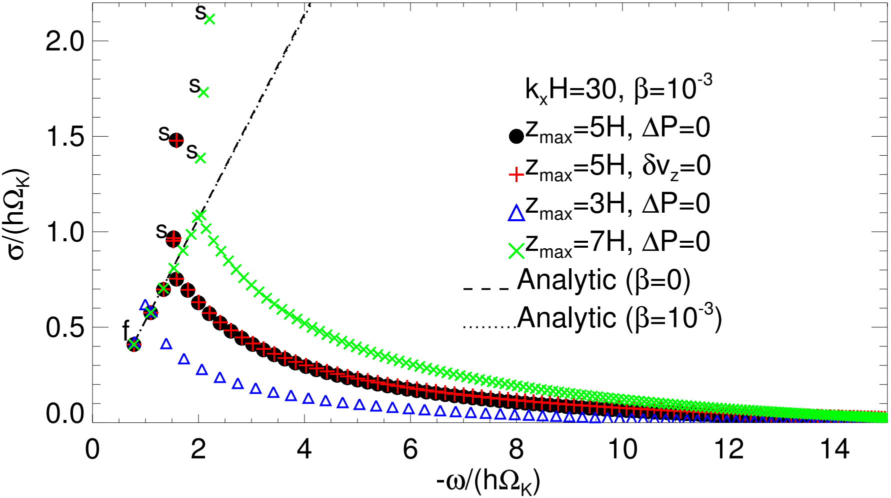
Seek $\mathrm{Im}(\omega) = 0$ for large k_x to find VSI requires

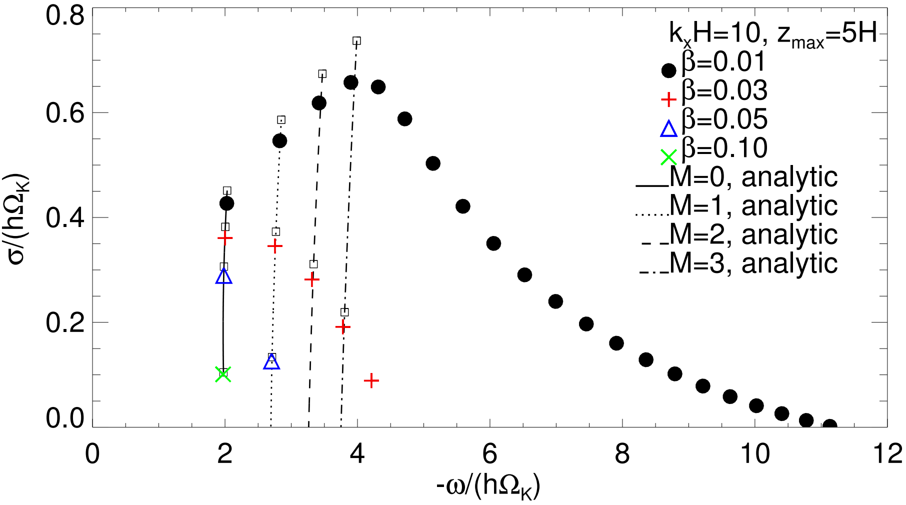
$$t_c\Omega_{\mathrm{Kep}} < \frac{h|q|}{\gamma - 1} \equiv \beta_{\mathrm{crit}}.$$

- $h|q|$: vertical shear
- $\gamma - 1$: vertical buoyancy
- $\beta_{\mathrm{crit}} \ll 1$, i.e. rapid cooling required

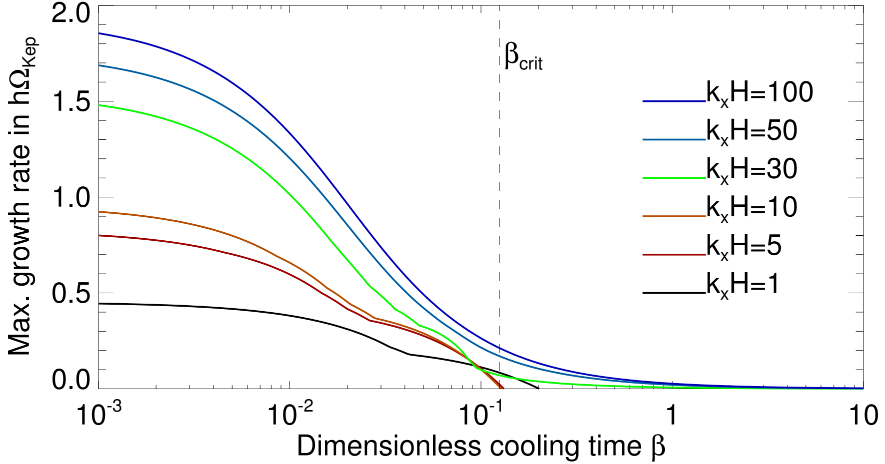
Linear theory: numerical treatment



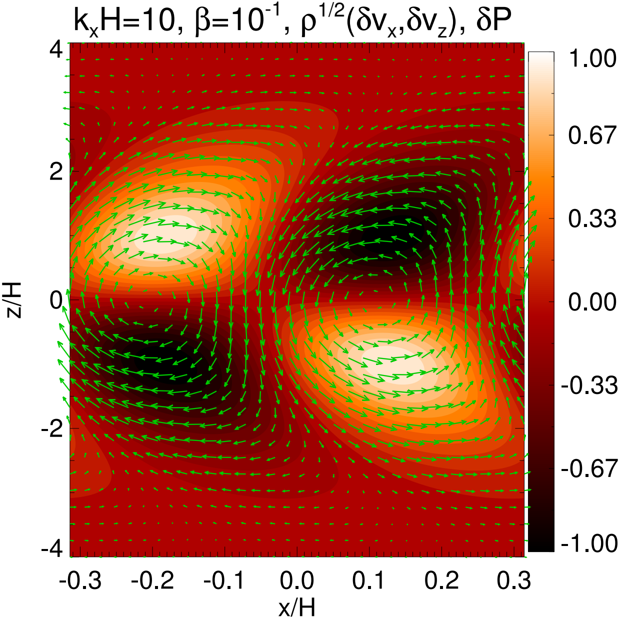
Effect of increasing the cooling time



Testing the critical cooling timescale

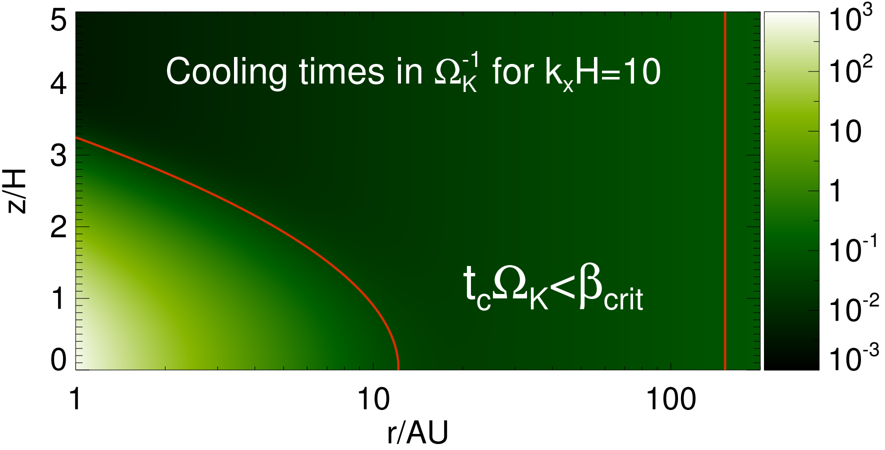


Visualization



Application to protoplanetary disks

Estimate cooling times in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010) based on dust opacity:



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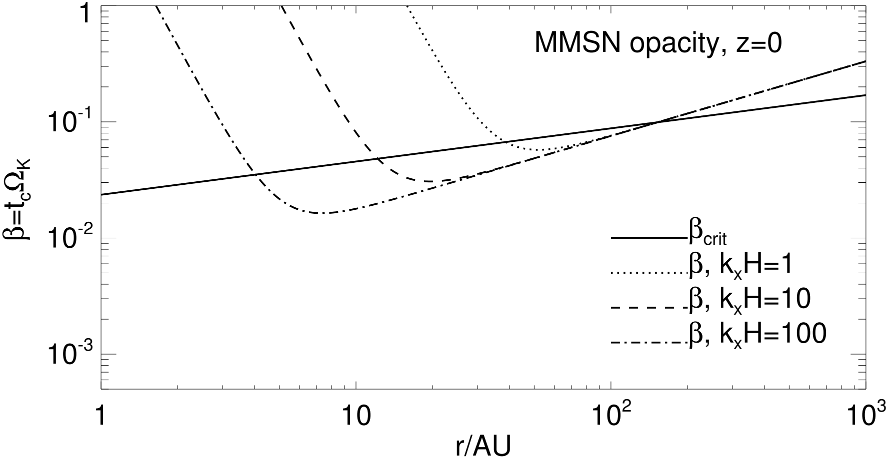

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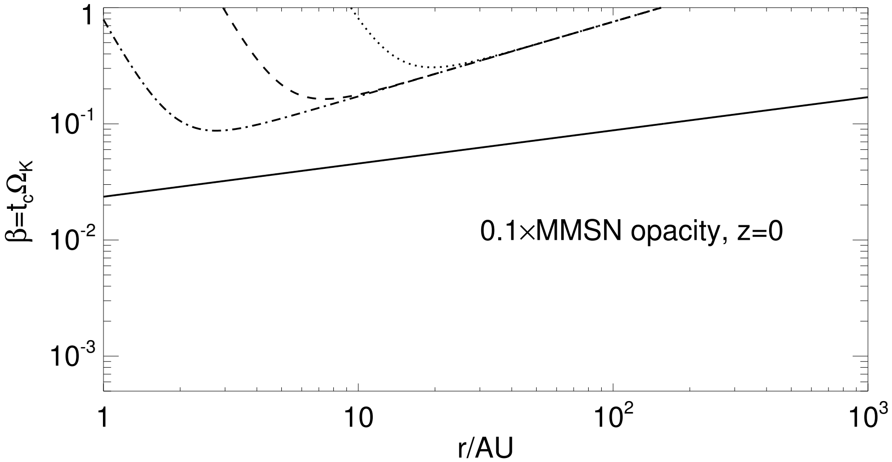
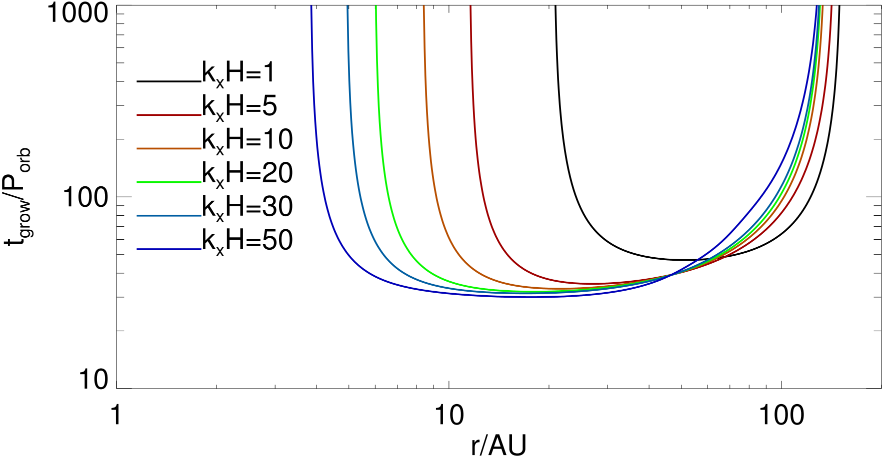


VSI in the Solar Nebula



Conclusions

- Astrophysical disks generally possess vertical shear
- Unstable if buoyancy ineffecitive: $N_z = 0$ and/or $\boxed{t_c\Omega_{\mathrm{Kep}} < \beta_{\mathrm{crit}} \ll 1}$
- Fast cooling needed because vertical shear is weak but buoyancy is strong
- Stringent thermodynamic requirement satisfied at 10s of AU in typical PPDs

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