Hydrodynamic instabilities in irradiated protoplanetary disks

Min-Kai Lin
minkailin@email.arizona.edu

Steward Theory Fellow
University of Arizona

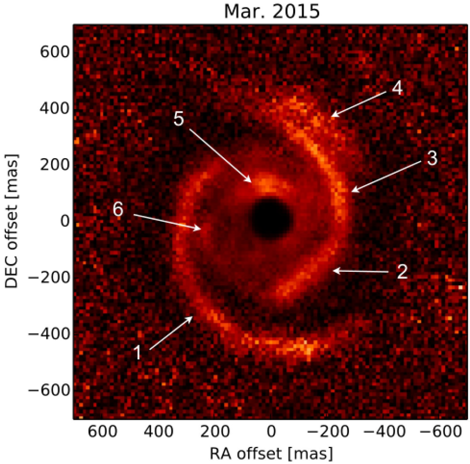
April 8 2016

Outline

- One-arm/eccentric instability : new result from a classic disk model
- Vertical shear instability : the details

Large-scale structures in circumstellar disks

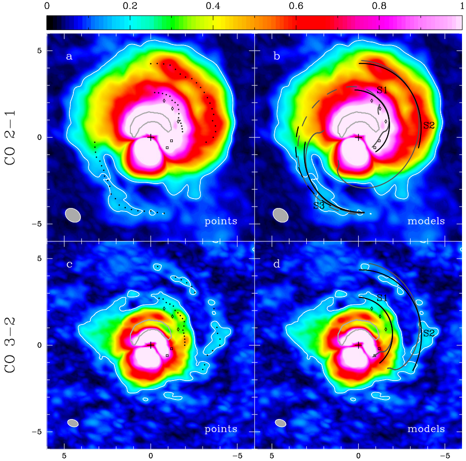
MWC 758



(Benisty et al., 2015)

Large-scale structures in circumstellar disks

HD 142527

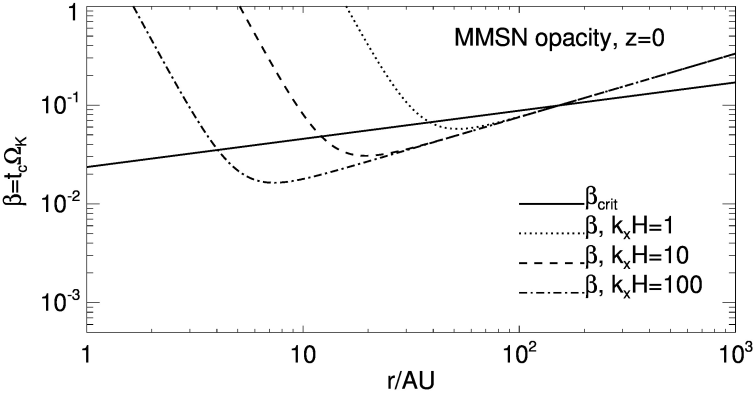


(Christiaens et al., 2014)

Spiral scales: S2 from ~ 500AU to ~ 600AU

Modeling hydrodynamics at large distances

- Can we make simplifications?
- Example: irradiated disks

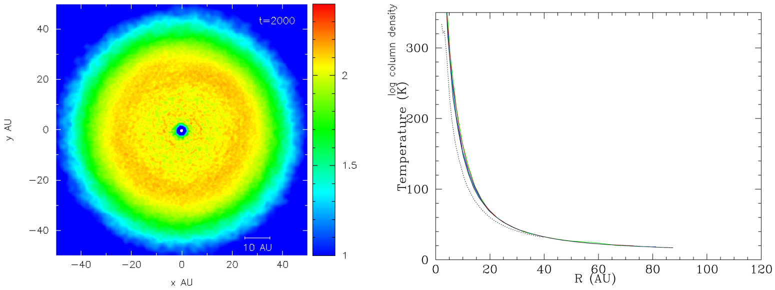


(Lin & Youdin, 2015)

Cooling time $t_{\rm cool}$: timescale for temperature flucuations to decay

Modeling hydrodynamics at large distances

- Can we make simplifications?
- Example: irradiated disks



Radiation hydrodynamic simulations from Stamatellos & Whitworth (2008)

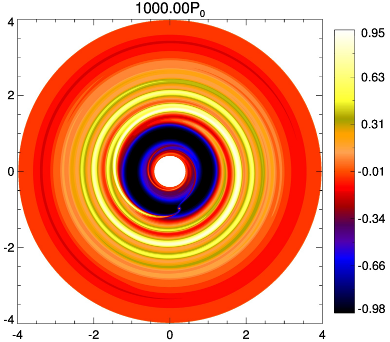
- Temperature does not change much as it is essentially set externally

The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$\boxed{T = T(\boldsymbol{r}).}$$

- Significant simplification: no energy equation to consider
  $\rightarrow$ efficient numerical simulations
- Example: long term disk-planet simulations



The locally isothermal disk

- Take this to the *idealized limit* of prescribing the temperature distribution:

$$T = T(\boldsymbol{r}).$$

- Significant simplification: no energy equation to consider
  $\rightarrow$ efficient numerical simulations
- What happens to waves in disks which are thermodynamically forced?

Angular momentum of non-axisymmetric perturbations

- Split the system into equilibrium and deviations

$$\Sigma \to \Sigma_{\mathrm{ref}}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\rightarrow$ time evolution of deviations or perturbations

Angular momentum of non-axisymmetric perturbations

- Split the system into equilibrium and deviations

$$\Sigma \to \Sigma_{\mathrm{ref}}(\boldsymbol{r}) + \delta\Sigma(\boldsymbol{r}, t)$$

- Linearized equations $\to$ time evolution of deviations or perturbations
- Angular momentum conservation for the perturbations

$$\boxed{\frac{\partial J_{\mathrm{pert}}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\mathrm{pert}} = T_{\mathrm{ext,pert}}.}$$

- J_{pert}: angular momentun density of *the perturbations*
- $\boldsymbol{F}_{\mathrm{pert}}$: angular momentum flux
- $T_{\mathrm{ext,pert}}$: everything else (external torque)

External torque in linear theory

$$T_{\mathrm{ext,pert}} = \begin{cases} 0 & \text{barotropic or adiabatic flow} \\ -\frac{m}{2}\,\mathrm{Im}\left(\delta\Sigma\xi_r^* \frac{dc_s^2}{dr}\right) & \text{fixed } T(r) \text{ in 2D} \\ \frac{m}{2}\,\mathrm{Im}\left[\rho\left(\nabla\cdot\boldsymbol{\xi}\right)\boldsymbol{\xi}^*\cdot\nabla c_s^2\right] & \text{fixed } T(r,z) \text{ in 3D} \end{cases}$$

- Barotropic: $p(\rho)$, adiabatic: $\Delta S = 0$
- Locally isothermal: sound-speed $c_s(\boldsymbol{r})$ fixed
- $\boldsymbol{\xi}$: Lagrangian displacement, m: azimuthal wavenumber

External torque in linear theory

With thermal relaxation (Newtonian cooling) towards a fixed $c_s^2(R, z)$ structure:

$$\begin{aligned}
T_{\mathrm{ext,pert}} =& \frac{m}{2}\,\mathrm{Im}[\cdots] \\
[\cdots] =& \frac{c_s^2}{\rho}|\delta\rho|^2 + c_s^2(\chi-1)\rho|\nabla\cdot\boldsymbol{\xi}|^2 - \nabla c_s^2 \cdot \left(\delta\rho\boldsymbol{\xi}^* + \frac{\chi-1}{\gamma-1}\delta\rho^*\boldsymbol{\xi}\right) \\
& - \frac{\chi-1}{\gamma-1}\left(|\xi_R|^2\,\partial_R\rho\partial_R c_s^2 + |\xi_z|^2\,\partial_z\rho\partial_z c_s^2\right) \\
& + \frac{\gamma-\chi}{\gamma-1}\xi_R^*\xi_z\partial_R\rho\partial_z c_s^2 - \partial_z\rho\partial_R c_s^2\left(\xi_R^*\xi_z + \frac{\chi-1}{\gamma-1}\xi_R\xi_z^*\right),
\end{aligned}$$

where

$$\chi = \frac{1 - \mathrm{i}\bar{\omega}\gamma t_{\mathrm{cool}}}{1 - \mathrm{i}\bar{\omega} t_{\mathrm{cool}}},$$

and

$$\bar{\omega} = \omega - m\Omega.$$

External torque in linear theory

$$\frac{\partial J_{\mathrm{pert}}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\mathrm{pert}} = T_{\mathrm{ext,pert}}.$$

$$T_{\mathrm{ext,pert}} \neq 0$$

$\Rightarrow$ angular momentum exchange between waves and the background disk

- i.e. Σ_{ref} and $\delta\Sigma$ can interact (cf. classic DWT)

Can $T_{\rm ext,pert}$ make perturbations grow?

Ignoring fluxes,

$$\frac{\partial J_{\rm pert}}{\partial t} \sim T_{\rm ext,pert}.$$

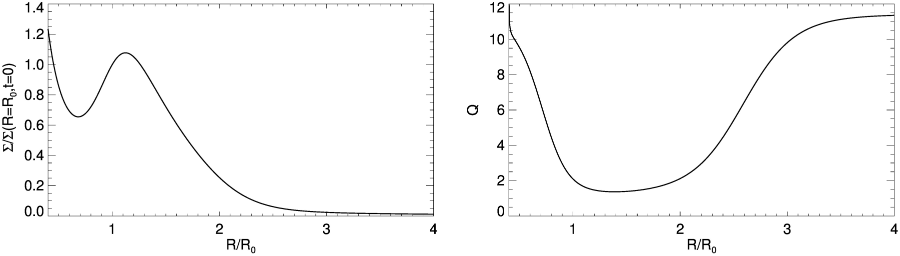
- May have an unstable situation if $T_{\rm ext,pert} \times J_{\rm pert} > 0$

Low-frequency trailing waves in a disk with temperature decreasing outwards

- E.g. eccentric disturbance (both torque and angular mometum are negative)

Numerical demonstration

2D, self-gravitating disk with radial structure



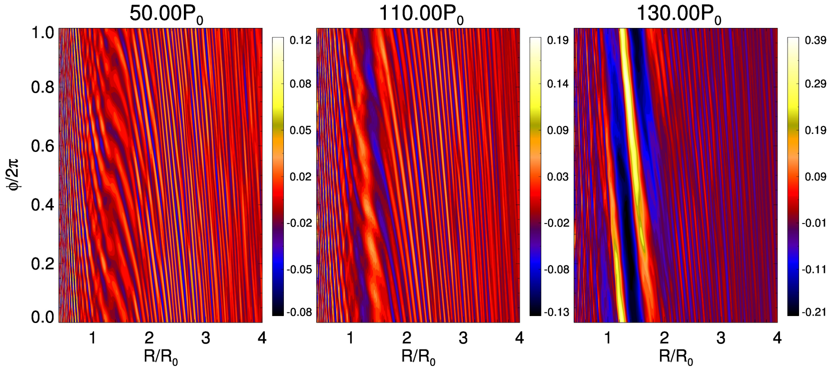
- Put a standard (WKB), one-armed low-frequency density wave in a global disk with $c_s^2 \propto R^{-q}$

$$\text{Growth rate} \sim \frac{T_{\mathrm{ext,pert}}}{J_{\mathrm{pert}}} \sim \frac{q\Omega}{2}\frac{H}{R}k_R H \sim \frac{q\Omega}{2Q}\frac{H}{R}$$

Numerical demonstration

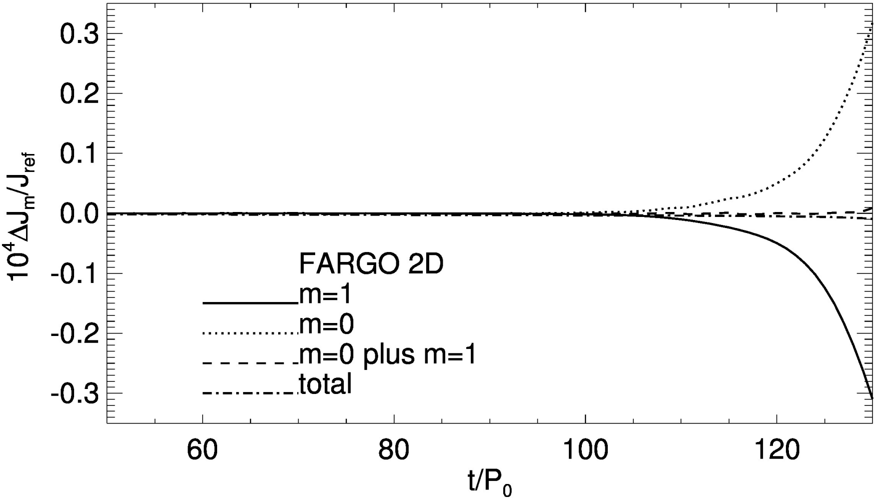
2D, self-gravitating disk with radial structure

FARGO simulations:



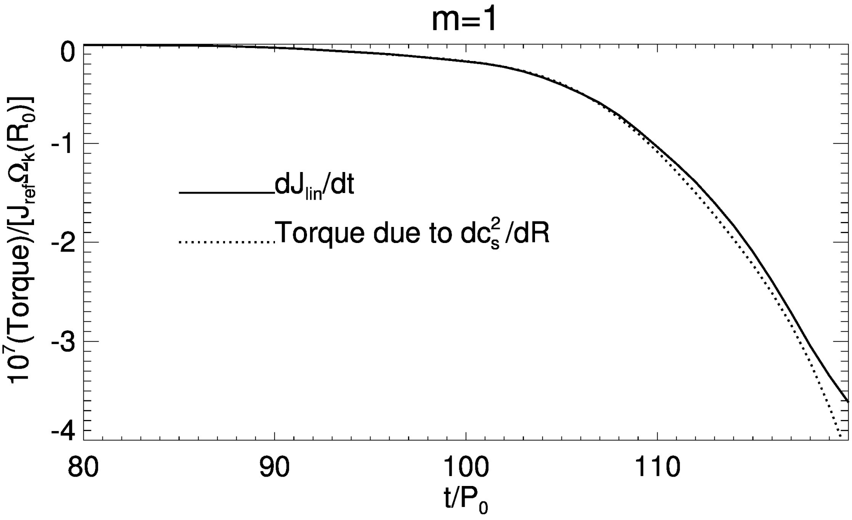
$T_{\mathrm{ext,pert}}$ in action

Angular momentum exchange between the background disk and the spiral:

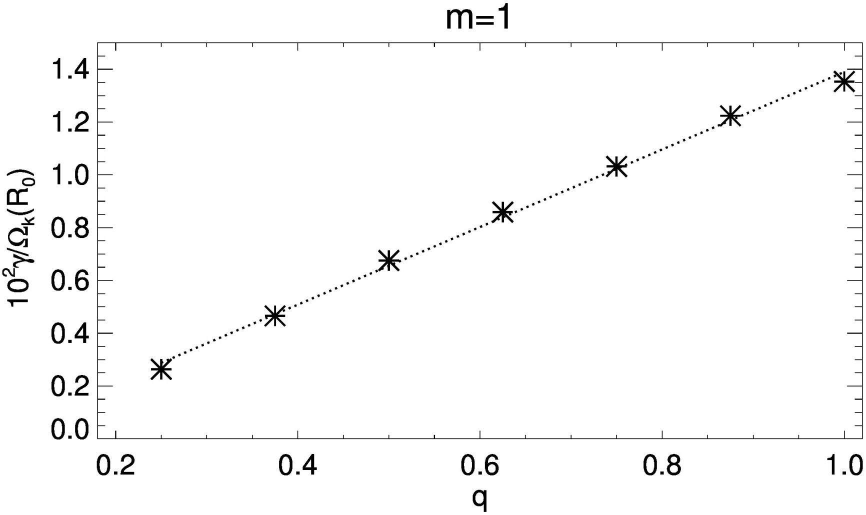


$T_{\mathrm{ext,pert}}$ in action

Extracting angular momentum from the spiral:



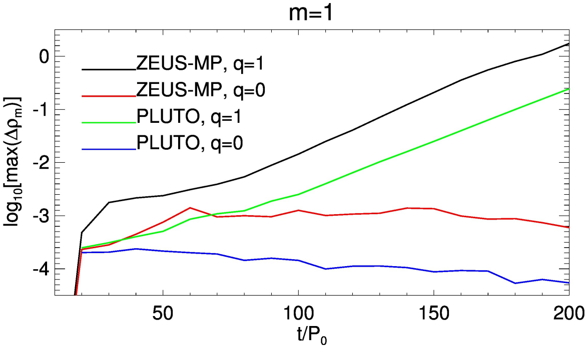
Dependence on the imposed temperature gradient



- Fixed sound-speed profile $c_s^2 \propto r^{-q}$.

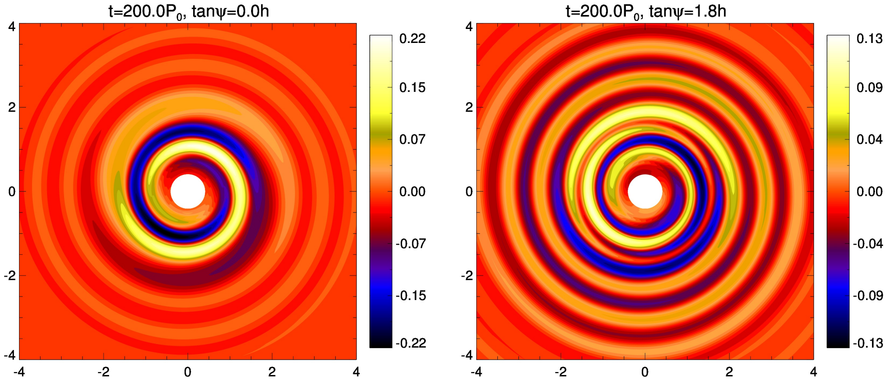
Three-dimensional simulations

Repeat experiment in 3D



- ZEUS: finite difference, discretized Poisson
- PLUTO: Godunov, Poisson through spherical harmonics
- *No growth without imposed temperature gradient*

Three-dimensional simulations



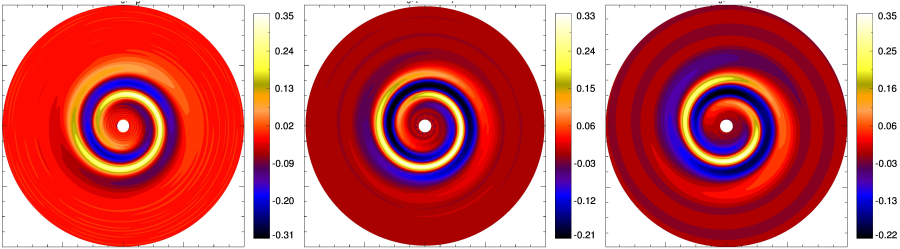
Midplane

Atmosphere

- Spiral-induced spiral

Summary

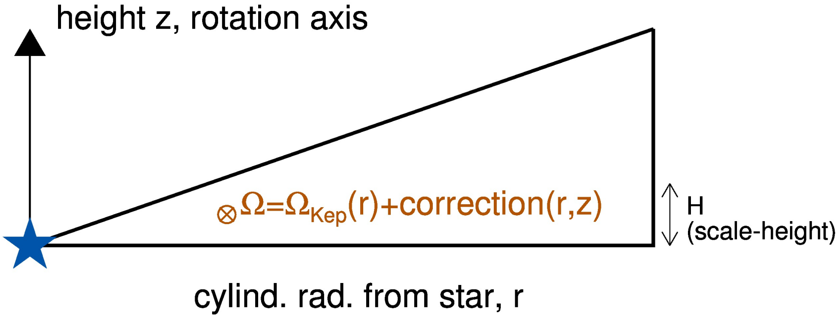
(Lin, 2015)



FARGO 2D | ZEUS 3D | PLUTO 3D

- Non-ideal thermal evolution
  $\rightarrow$ non-trivial wave angular mometum conservation
- Possible application to irradiated parts of protoplanetary disks
- Need to extend to finite thermal timescales

Astrophysical disks have vertical shear



$$\frac{\partial \Omega}{\partial z} \neq 0$$

(Because $\nabla P \times \nabla \rho \neq 0$)

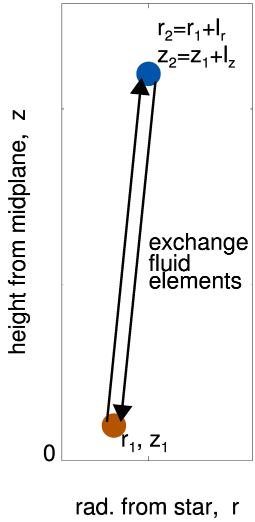
- Vertically isothermal thin-disk with $T \propto r^q$,

$$r\frac{\partial \Omega}{\partial z} \simeq \left(\frac{qz}{2H}\right) \times \frac{H}{r}\Omega_{\mathrm{Kep}}$$

- $H/r \sim 0.05$ in PPDs

Vertical shear instability

$\partial_z \Omega \neq 0 \Rightarrow$free energy$\rightarrow$instability?



- Change in kinetic energy:

$$\Delta E \sim l_r^2 \left(\Omega^2 + \frac{l_z}{l_r} \cdot r \frac{\partial \Omega^2}{\partial z} \right)$$

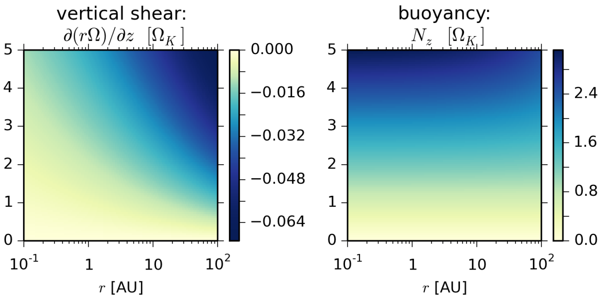
- Vertical shear is weak, **BUT**

$$\Delta E < 0 \quad \text{if} \quad |l_z| \gg |l_r|$$
$\Rightarrow$ **INSTABILITY**

- Energy released for vertically elongated disturbances.

VSI needs to fight buoyancy in real disks

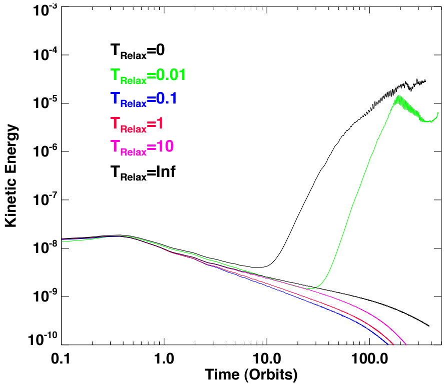
$$\underbrace{r\partial_z\Omega}_{\text{destabilizing vert. shear}} \quad \text{v.s.} \quad \underbrace{N_z}_{\text{stabilizing vert. buoyancy}}$$



- Vertical shear is *weak*, $\boxed{r\partial_z \ln\Omega \sim O(h) \ll 1}$ (so need $l_z/l_r \gg 1$)
- Vertical buoyancy is *strong*, $\boxed{N_z/\Omega \sim O(1)}$

Rapid cooling can enable VSI

- Astrophysical disks generally have $\partial_z \Omega \neq 0$ — neccessary for VSI, but also need rapid cooling .



(Nelson et al., 2013)

Linear theory: previous analyses and our contribution

- Vertically and radially local, with energy equation
  (Urpin & Brandenburg, 1998; Urpin, 2003, G. Mohandas)
- Vertically global, radially local, no buoyancy
  (Nelson et al., 2013; McNally & Pessah, 2014; Barker & Latter, 2015)
- Vertically and radially global, no buoyancy
  (Barker & Latter, 2015; Umurhan et al., 2015)

Lin & Youdin (2015)

- Vertically global, radially local, including energy equation (i.e. with buoyancy)
- Both constant cooling and realistic cooling functions

Isothermal limit (instantaneous cooling)

Linearized fluid equations $\rightarrow$ Hermite ODE

$$0 = W'' + \left[\ln \rho' - \frac{\mathrm{i}K}{(1-\nu^2)\,\Omega_\mathrm{K}^2 h}\frac{d\Omega^2}{dz}\right] W' + \nu^2\left(1 + \frac{K^2}{1-\nu^2}\right) W.$$

K: dimensionless wavenumber; ν: dimensionless frequency

- Formal[1] limit on the growth rate of low-frequency modes

$$\boxed{\sigma < \max\left|r\frac{d\Omega}{dz}\right|}$$

(Unbound if approximating vertical shear $\propto z$)

- General frequency waves in a thin-disk without a surface

$$\nu^4 - \left(L + 1 + K^2\right)\nu^2 + L\left(1 + \mathrm{i}hqK\right) = 0, \quad L = 1, 2\cdots$$

VSI is the low-frequency (inertial) branch.

[1] Via Cauchy-Schwarz inequality...etc.

Linear theory with finite cooling

- Parameterized cooling: $t_{\rm cool}\Omega_{\rm K} \equiv \beta = \text{const.}$

$$
\begin{aligned}
\mathrm{i}\upsilon\frac{Q}{c_s^2} &= \mathrm{i}k_x\delta v_x + \frac{d\delta v_z}{dz} + \frac{\partial \ln\rho}{\partial z}\delta v_z + \frac{\partial \ln\rho}{\partial r}\delta v_x, \\
\mathrm{i}\upsilon\delta v_x &= \mathrm{i}k_x W - 2\Omega\delta v_y - \frac{1}{\rho}\frac{\partial P}{\partial r}\frac{Q}{c_s^2} \\
\mathrm{i}\upsilon\delta v_y &= r_0\frac{\partial\Omega}{\partial z}\delta v_z + \frac{\kappa^2}{2\Omega}\delta v_x, \\
\mathrm{i}\upsilon\delta v_z &= \frac{dW}{dz} + \frac{\partial\ln\rho}{\partial z}\left(W - Q\right), \\
\mathrm{i}\upsilon W &= c_s^2\frac{\partial\ln\rho}{\partial z}\delta v_z + c_s^2\frac{\gamma}{\Gamma}\left(\mathrm{i}k_x\delta v_x + \frac{d\delta v_z}{dz}\right) \\
&\quad + \frac{1}{t_c}\left(W - \frac{Q}{\Gamma}\right) + \frac{1}{\rho}\frac{\partial P}{\partial r}\delta v_x.
\end{aligned}
$$

- $\upsilon = \omega + \mathrm{i}\sigma$: complex frequency; k_x: radial wavenumber

Linear theory with finite cooling

- Parameterized cooling: $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \equiv \beta = \mathrm{const.}$

Single ODE reduced model (low-freq., thin-disk, no explicit $\partial_r P$)

$$0 = \delta v_z''(z) - zA\delta v_z'(z) + \left(B - Cz^2\right)\delta v_z(z).$$

Linear theory with finite cooling

- Parameterized cooling: $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \equiv \beta = \mathrm{const.}$

Single ODE reduced model (low-freq., thin-disk, no explicit $\partial_r P$)

$$\boxed{0 = \delta v_z''(z) - zA\delta v_z'(z) + \left(B - Cz^2\right)\delta v_z(z).}$$

- Transformation $\rightarrow$ Hermite ODE (as before)
- A, B, C complex because $\partial_z \Omega \neq 0$
- Important: reduced model is only valid for rapid cooling (OK for VSI)

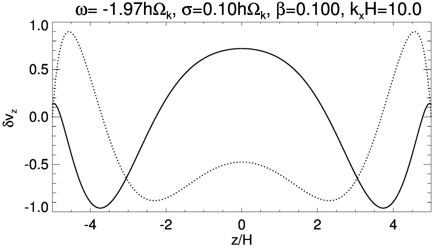
Linear theory with finite cooling

- Parameterized cooling: $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \equiv \beta = \mathrm{const.}$

Single ODE reduced model (low-freq., thin-disk, no explicit $\partial_r P$)

$$0 = \delta v_z''(z) - zA\delta v_z'(z) + \left(B - Cz^2\right)\delta v_z(z).$$

- Finite K.E. density as $|z| \to \infty \Rightarrow$ dispersion relation $\omega = \omega(k_x; \beta, M)$
- Mode number $M = 0, 1, 2 \cdots$
- Fundamental mode $M = 0$ has special importance



Critical cooling time

- Assume $\beta = \beta_{\rm c}$ at marginal stability ($\sigma = 0$) and large k_x

Find

$$\frac{\partial \beta_{\rm c}}{\partial M} < 0$$

(if the disk is sufficiently thin). Then $M = 0$ has the longest critical cooling time.

The fundamental mode is the most difficult to stabilize with increasing $t_{\rm cool}$.

Critical cooling time

- Assume $\beta = \beta_c$ at marginal stability ($\sigma = 0$) and large k_x

Find

$$\frac{\partial \beta_c}{\partial M} < 0$$

(if the disk is sufficiently thin). Then $M = 0$ has the longest critical cooling time.

The fundamental mode is the most difficult to stabilize with increasing $t_{\rm cool}$.

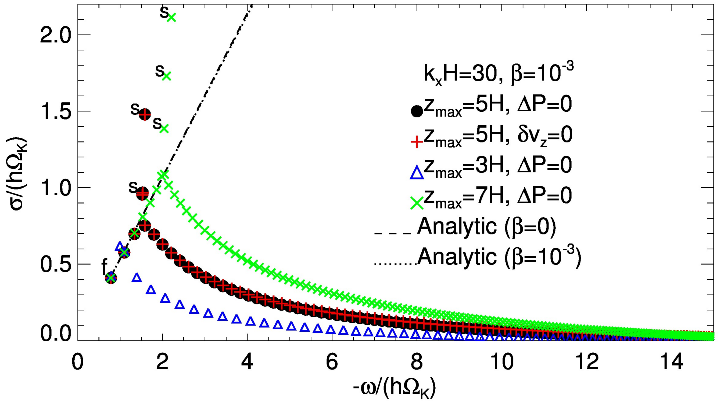
So condition for VSI is

$$\boxed{t_{\rm cool}\Omega_{\rm K} < \beta_c(M=0) = \frac{h|q|}{\gamma - 1}}$$

- $h|q|$: vertical shear (destabilizing)
- $\gamma - 1$: vertical buoyancy (stabilizing)

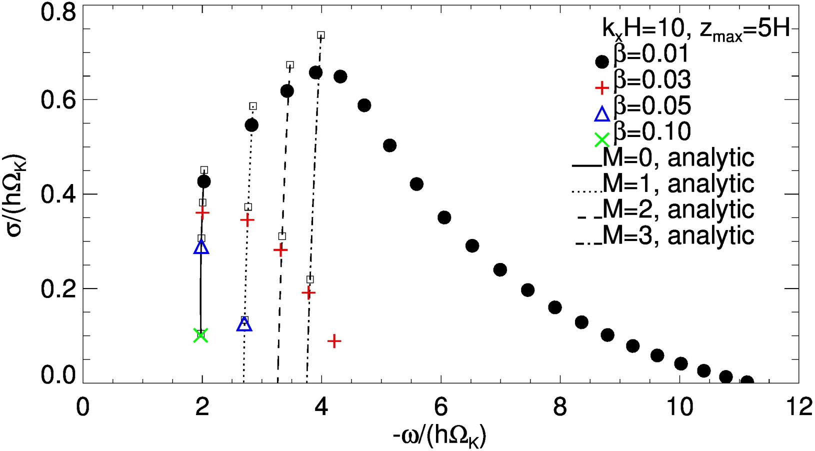
Numerical calculations

- Solve linearized equation in the radially local approx.
- Relax all other assumptions in reduced model

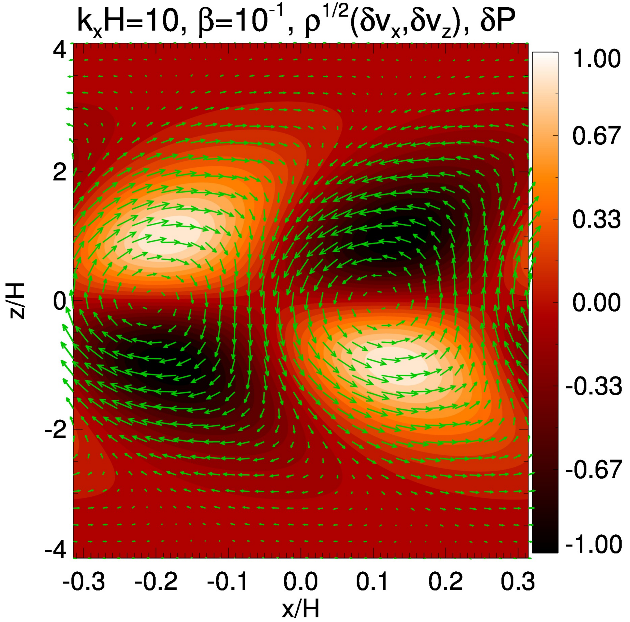


- Theory describes the lowest order modes inc. fundamental mode
- 'Surface modes' are entirely due to disk surface (imposed or physical)

Effect of increasing the cooling time

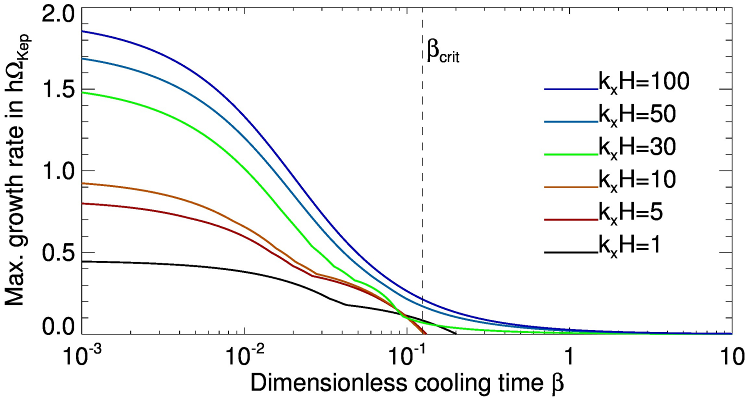


2D visualization



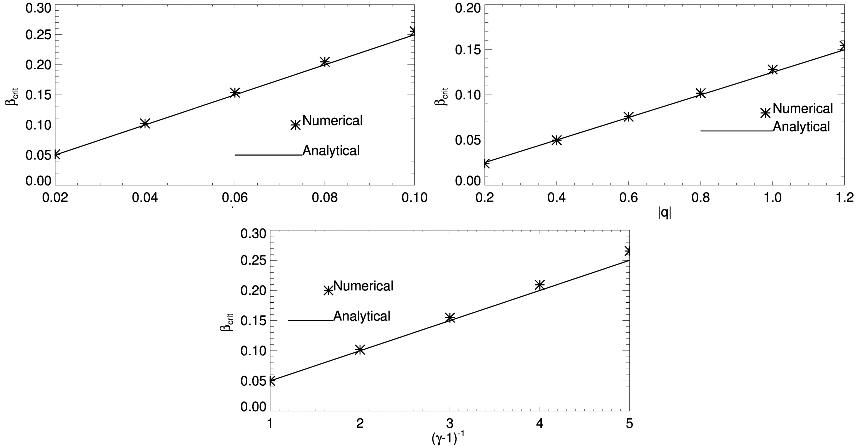
Testing the critical cooling timescale

$$t_{\rm cool}\Omega_{\rm K} < \frac{h|q|}{\gamma - 1}$$



Testing the critical cooling timescale

$$t_{\mathrm{cool}}\Omega_{\mathrm{K}} < \frac{h|q|}{\gamma - 1}$$



Application to protoplanetary disks

Estimate cooling times in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010) based on dust opacity ($\propto T^2$):

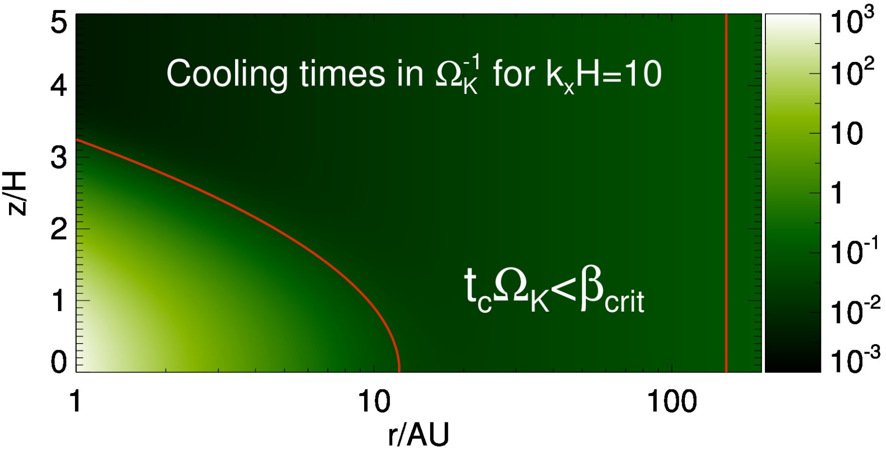
$$t_{\rm cool}\Omega_{\rm K} \equiv$$

$$\beta(z; r, K) = 3.9\times10^{-3}\frac{r_{\rm AU}^{9/14}}{\kappa_{\rm d}}\left[1 + \frac{1.9\times10^7\kappa_{\rm d}^2}{r_{\rm AU}^{33/7}K^2}\exp\left(-\frac{z^2}{2H^2}\right)\right]$$

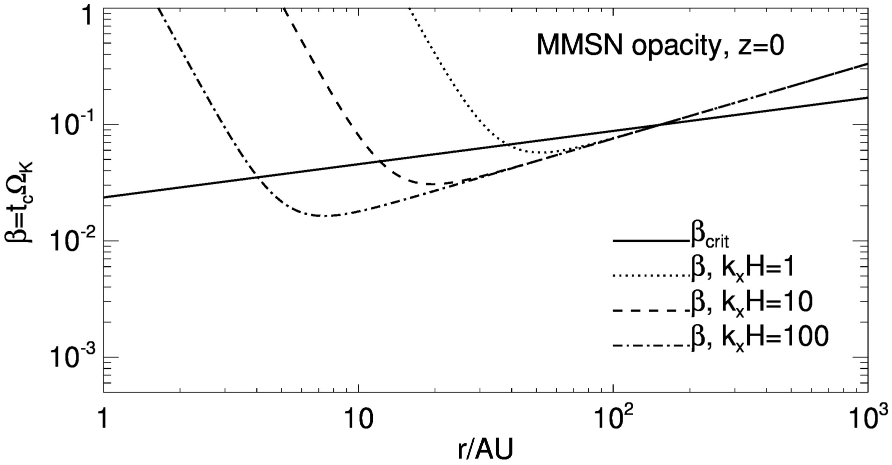
- $\kappa_{\rm d}$: opacity scale relative to MMSN
- Optically thin/Newtonian cooling for very small scales, fast for large $\kappa_{\rm d}$
- Radiative diffusion for longer scales, fast for small $\kappa_{\rm d}$
- Vert. dependence through ρ

Application to protoplanetary disks

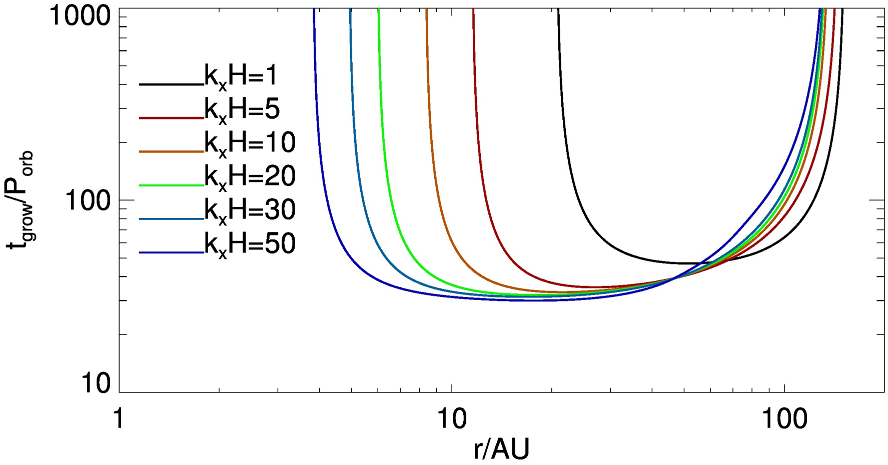
Estimate cooling times in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010) based on dust opacity ($\propto T^2$):



β versus $\beta_{\rm crit}$



VSI in the solar nebula



Summary & future work

- VSI operates in the MMSN outer disk, 10—100AU
- VSI in dusty disks
- VSI due to infall?
- Radially global analysis

References

Barker A. J., Latter H. N., 2015, MNRAS, 450, 21

Benisty et al. 2015, A&A, 578, L6

Chiang E., Youdin A. N., 2010, Annual Review of Earth and Planetary Sciences, 38, 493

Christiaens V., Casassus S., Perez S., van der Plas G., Ménard F., 2014, ApJL, 785, L12

Lin M.-K., 2015, MNRAS, 448, 3806

Lin M.-K., Youdin A. N., 2015, ApJ, 811, 17

McNally C. P., Pessah M. E., 2014, ArXiv e-prints

Nelson R. P., Gressel O., Umurhan O. M., 2013, MNRAS, 435, 2610

Stamatellos D., Whitworth A. P., 2008, A&A, 480, 879

Umurhan O. M., Nelson R. P., Gressel O., 2015, ArXiv e-prints

Urpin V., 2003, A&A, 404, 397

Urpin V., Brandenburg A., 1998, MNRAS, 294, 399