Non-ideal hydrodynamics in protoplanetary discs

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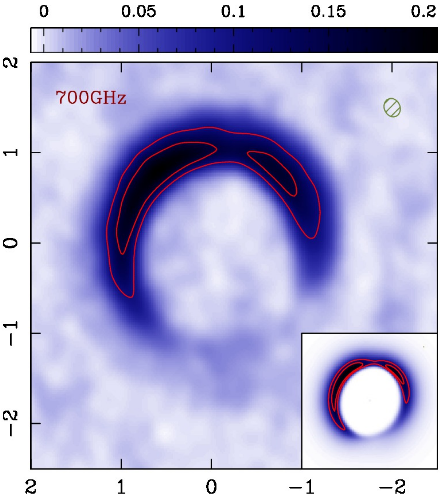
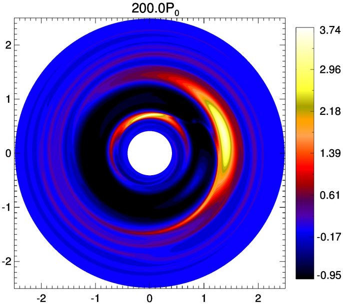
Example: disc asymmetries



(HD142527, Casassus et al., 2015)

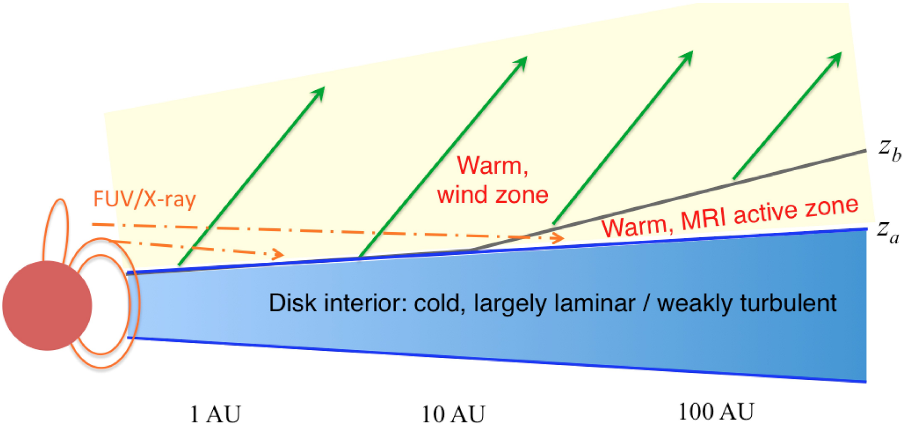


Vortices at planetary gap edges

'Non-ideal' in the sense that

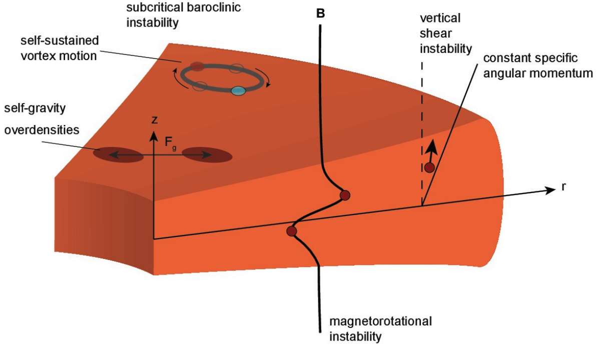
- Involves a structured disc
- No magnetic fields

Current picture of magnetic accretion



(Bai, 2016)

Fluid instabilities/turbulence in protoplanetary discs



(Armitage, 2015) Today:

- Vertical shear instability : thermodynamic requirements
- Gravitational instability : including viscosity and cooling

Post-talk:

- Thermodynamic-driving of eccentricity in discs

Astrophysical discs have vertical shear

- Accretion discs are generally <u>baroclinic</u>

$$\nabla P \times \nabla \rho \neq 0$$

$$\Rightarrow \boxed{\frac{\partial \Omega}{\partial z} \neq 0}$$

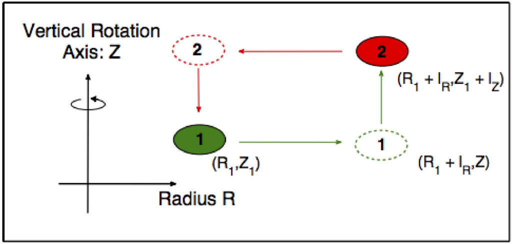
- Vertically isothermal thin-disc with $T \propto r^q$,

$$r\frac{\partial \Omega}{\partial z} \simeq \left(\frac{qz}{2H}\right) \times \frac{H}{r}\Omega_{\mathrm{Kep}}$$

- $O(H/r)$ effect

Vertical shear instability

$\partial_z \Omega \neq 0 \Rightarrow$free energy$\rightarrow$instability?



(Umurhan et al., 2013)

- Change in kinetic energy:

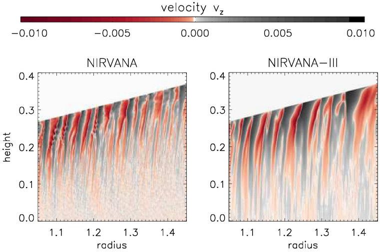
$$\Delta E \sim l_r^2 \left(\Omega^2 + \frac{l_z}{l_r} \cdot r \frac{\partial \Omega^2}{\partial z} \right).$$

- Vertical shear is weak, **BUT**

$\Delta E < 0$ if $|l_z| \gg |l_r| \Rightarrow$ **INSTABILITY**

Vertical shear instability

$\partial_z \Omega \neq 0 \Rightarrow$free energy$\rightarrow$instability?



(Nelson et al., 2013)

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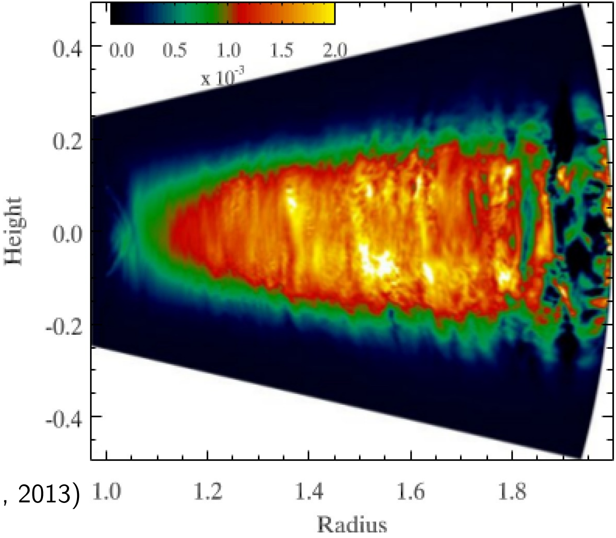
VSI: non-linear outcome

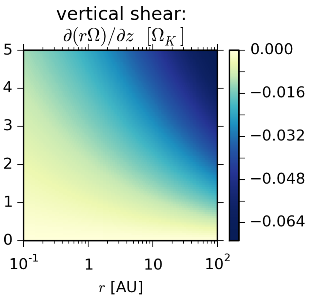


(Nelson et al., 2013)

Hydrodynamic turbulence

VSI needs to fight buoyancy in real discs

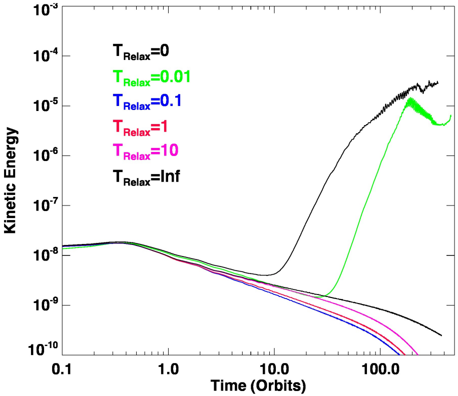
$$\underbrace{r\partial_z\Omega}_{\text{destabilizing vert. shear}} \quad \text{v.s.} \quad \underbrace{N_z}_{\text{stabilizing vert. buoyancy}}$$



- Vertical shear is weak, $r\partial_z ln\Omega \sim O(h) \ll 1$ (so need $l_z/l_r \gg 1$)
- Vertical buoyancy is strong, $N_z/\Omega \sim O(1)$

Thermodynamic condition for the VSI

- Can overcome buoyancy forces by cooling the disc rapidly
- Parameterization: $\partial_t T = -(T - T_0)/t_{\rm cool}$



(Nelson et al., 2013)

Can we quantify this requirement?

Lin & Youdin (2015): linear theory with finite cooling

From single ODE, reduced model for $T \propto r^q$ disc, find that VSI requires:

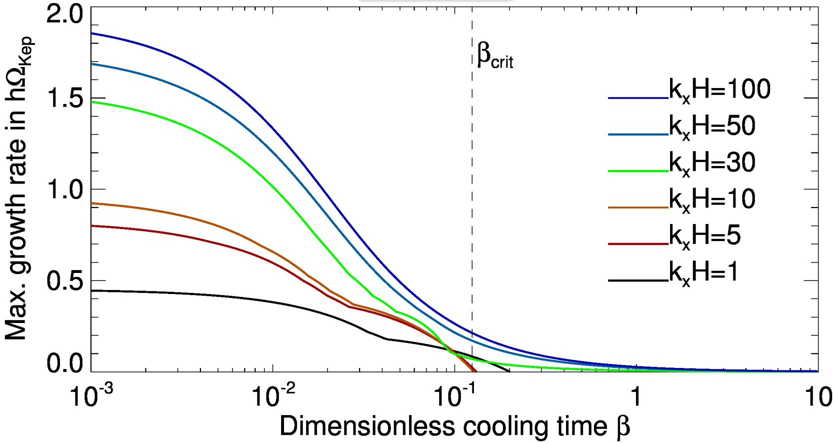
$$\boxed{t_{\mathrm{cool}}\Omega_{\mathrm{K}} < \frac{h|q|}{\gamma - 1}}$$

- $h|q|$: vertical shear ($h \equiv H/r$) — destabilizing
- $\gamma - 1$: vertical buoyancy — stabilizing
- $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \ll 1$ required, i.e. rapid cooling

Full linear theory

- Solve linearized fluid equations in the radially local approximation
- $\beta = t_{\rm cool}\Omega_{\rm K}$

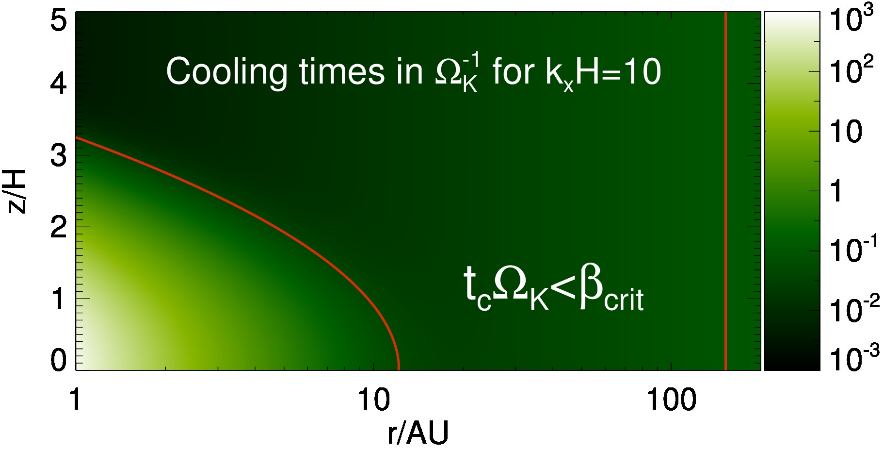
$$\beta_{\rm crit} \equiv \frac{h|q|}{\gamma - 1}$$



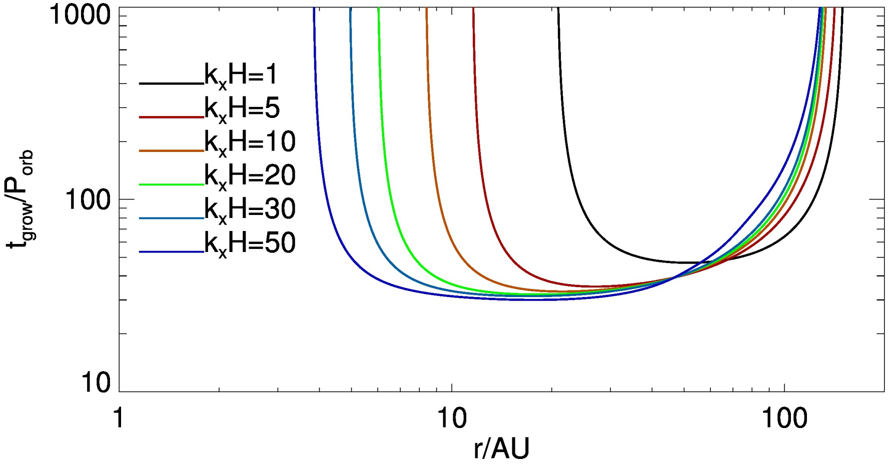
Vertical shear instability in the Solar Nebula

Cooling via dust-opacity ($\propto T^2$) in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010):

$$t_{\rm cool}\Omega_{\rm K} =$$

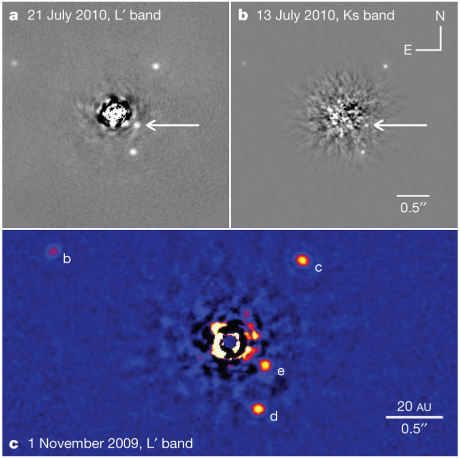


Typical growth times in the Solar Nebula



- VSI is most active in the outer disc 10—100AU
- Forced to develop on smaller scales towards inner disc

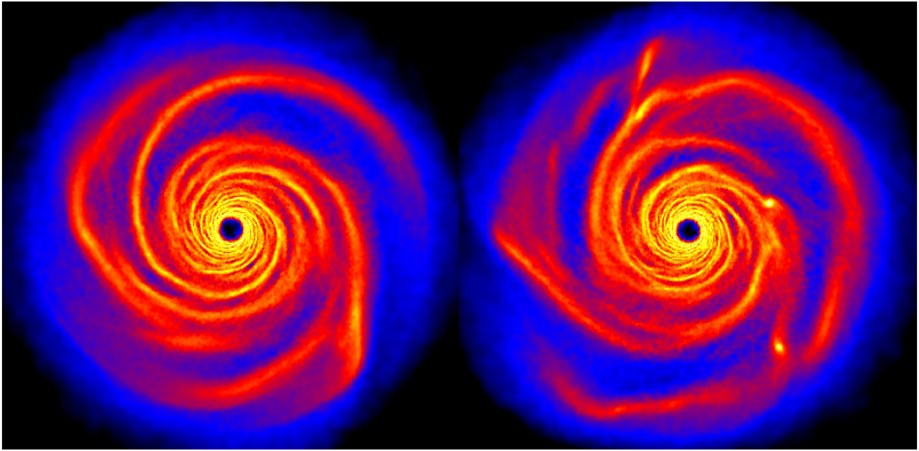
Wide orbit planets



(Marois et al., 2010)

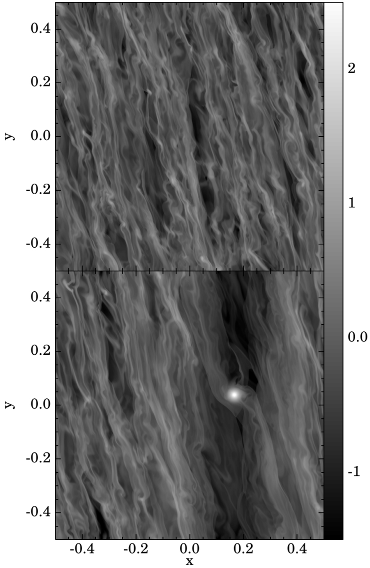
Disc instability theory

- Young, massive protoplanetary discs fragment under its own gravity



(Rice et al., 2005)

When do (simulated) protostellar discs fragment?



(Paardekooper, 2012)

- Massive disc

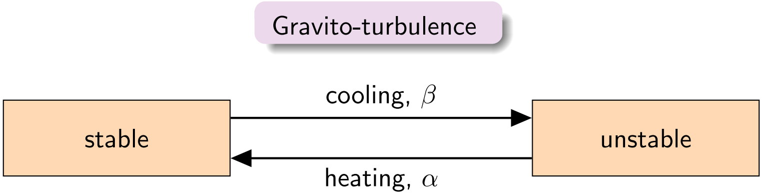
$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 2$$

- Fast cooling

$$t_{\mathrm{cool}} \Omega \lesssim 3$$

The cooling criterion is empirical.

And when it does not fragment



- Cooling: $\beta \equiv t_{\rm cool}\Omega$
- Heating: α viscosity
- Equilibrium $\Rightarrow \alpha = \alpha(\beta)$
- Fragments if $\beta < \beta_{\rm c}$ or $\alpha > \alpha_{\rm c}$

When do realistic protostellar discs fragment?

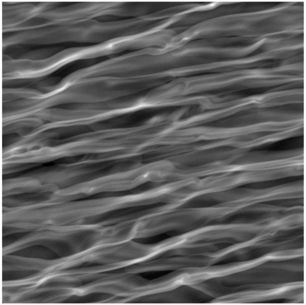
Work out $\Sigma(R)$, $T(R)$..etc.,

1. Is Toomre $Q \sim 1$?
2. Is $t_{\rm cool}\Omega \sim 1$ or $\alpha \sim 0.1$?

Possible issues:

- Need to choose critical values
- Complex physics to get Σ, T were not included in the numerical experiments that established those critical values
- Numerical uncertainties in critical values (e.g. resolution)

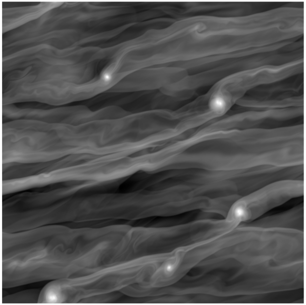
An alternative approach



(Rice et al., 2011)

- Write down a classic, viscous disc model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze its stability properties

An alternative approach



(Rice et al., 2011)

- Write down a classic, viscous disc model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze its stability properties
- Unstable $\rightarrow$ fragmentation

Quantifying cooling

Classic dispersion relation without cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G\Sigma|k|}_{+\text{gravity}} \quad \underbrace{-\Omega^2}_{-\text{rotation}} \quad \underbrace{-\gamma c_s^2 k^2}_{-\text{pressure}}$$

Quantifying cooling

New result **with cooling**

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G\Sigma|k|}_{+\text{gravity}} \quad \underbrace{-\Omega^2}_{-\text{rotation}} \quad \underbrace{-\left(\frac{T_{\rm irr}/T + \gamma t_{\rm cool} s}{1 + t_{\rm cool} s}\right) c_s^2 k^2}_{-\text{modified pressure}}$$

$$\frac{\partial E}{\partial t} = -\frac{\mathcal{R}_* \rho}{\mu(\gamma - 1)} \frac{(T - T_{\rm irr})}{t_{\rm cool}}$$

Note if $T_{\rm irr} = 0$:

$$\frac{\partial E}{\partial t} = -\frac{E}{t_{\rm cool}}$$

(Gammie, 2001)

Quantifying cooling

New result with cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G\Sigma|k|}_{+\text{gravity}} \underbrace{-\Omega^2}_{-\text{rotation}} \underbrace{-\left(\frac{T_{\mathrm{irr}}/T + \gamma t_{\mathrm{cool}} s}{1 + t_{\mathrm{cool}} s}\right) c_s^2 k^2}_{-\text{modified pressure}}$$

- Dispersion relation changes from $s^2 \to s^3$
- Can be formally unstable for any $t_{\mathrm{cool}} < \infty$
- $T_{\mathrm{irr}} = 0 \sim$ pressureless disc

Just a fancy way to compare compressional heating v.s. thermal losses

A little magic

Cooling timescale to remove pressure over a lengthscale $\sim H$

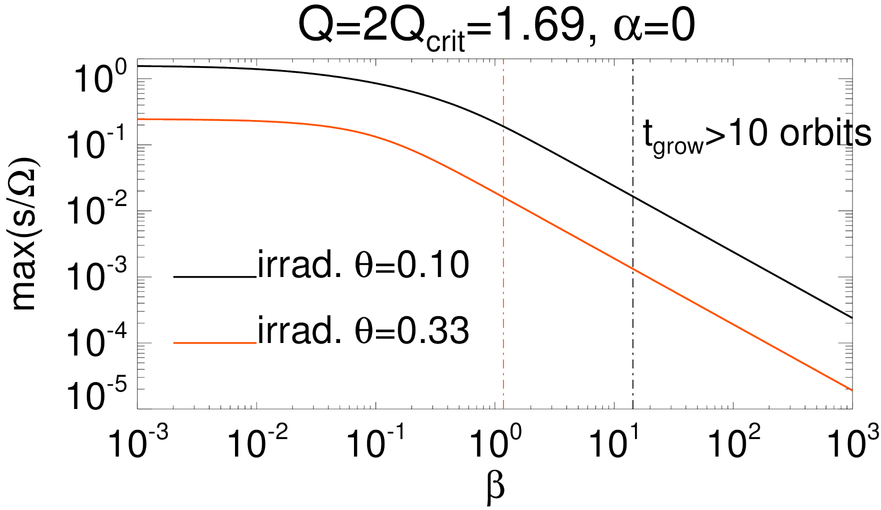
$$t_{\mathrm{cool},*} = \left(\sqrt{\gamma} - 1\right)^{-3/2} \Omega^{-1}$$

$$\alpha_* = \frac{4}{9} \frac{\sqrt{\sqrt{\gamma} - 1}}{\gamma(\sqrt{\gamma} + 1)}$$

γ	α_*	$t_{\mathrm{cool},*}\Omega$	Sim., $t_{\mathrm{cool,frag}}\Omega$	Reference
7/5	0.062	12.75	12—13	Rice et al. (2005)
1.6	0.063	7.33	8	Rice et al. (2011)
5/3	0.063	6.37	6—7	Rice et al. (2005)
2	0.059	3.75	3	Gammie (2001)

(Lin & Kratter, submitted)

Irradiation is important



Viscosity-enabled gravitational instability

Classic instability condition

$$2\pi G\Sigma|k| - \Omega^2 - \gamma c_s^2 k^2 > 0$$

$$+ \text{ gravity } - \text{ rotation } - \text{ pressure } > 0$$

Viscosity-enabled gravitational instability

Add viscosity:

$$2\pi G\Sigma|k| \qquad -c_s^2 k^2 > 0$$

$$+\ \text{gravity} \qquad\qquad -\ \text{pressure} > 0$$

(Lynden-Bell & Pringle, 1974; Willerding, 1992; Gammie, 1996)

- Viscosity or frictional forces remove rotational stabilization (cf. inwards migration of particles due to gas-dust drag)
- Rice et al. (2005) report a $\alpha_{\max} \sim 0.1$ before fragmentation, also supported by Clarke et al. (2007)

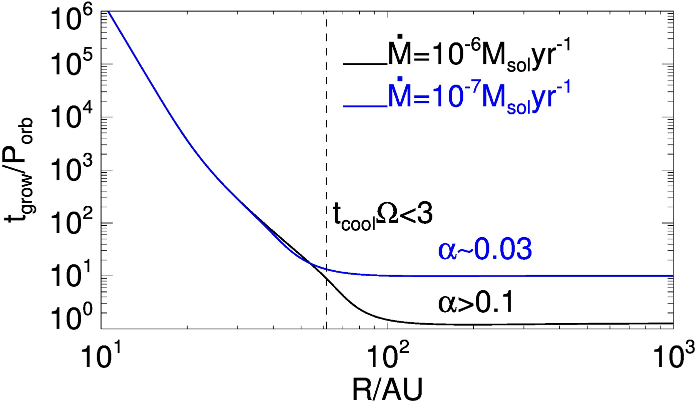
Viscous GI growth rate:

$$\boxed{s = \frac{27\alpha}{16Q^4}\Omega}$$

(isothermal/irradiated limit)

Application to protoplanetary discs

- Input physical disc model into stability calculation — get growth timescales



Beyond $\sim$ 60AU:

- ~~Cooling criterion $\Rightarrow$ both disc fragments~~
- Viscosity criterion $\Rightarrow$ high $\dot{M}$ disc fragments, growth times $\sim$ one orbit

Summary

Vertical shear instability

- Feeds off free energy in $\partial_z \Omega \neq 0$
- Require large vertical motions to tap into weak vertical shear
- Require ultra-fast cooling
- VSI possible in the outer disc between 10—100AU

Generalized gravitational instability

- Cooling: reduces thermal support
- Viscosity: reduces rotational support
- Fragmentation: inability to maintain steady gravito-turbulence because cooling and/or turbulent stresses (viscosity)

References

Armitage P. J., 2015, ArXiv e-prints

Bai X.-N., 2016, ArXiv e-prints

Casassus S., Wright C. M., Marino S., Maddison S. T., Wootten A., Roman P., Pérez S., Pinilla P., Wyatt M., Moral V., Ménard F., Christiaens V., Cieza L., van der Plas G., 2015, ApJ, 812, 126

Chiang E., Youdin A. N., 2010, Annual Review of Earth and Planetary Sciences, 38, 493

Clarke C. J., Harper-Clark E., Lodato G., 2007, MNRAS, 381, 1543

Gammie C. F., 1996, ApJ, 457, 355

Gammie C. F., 2001, ApJ, 553, 174

Lin M.-K., Youdin A. N., 2015, ApJ, 811, 17

Lynden-Bell D., Pringle J. E., 1974, MNRAS, 168, 603

Marois C., Zuckerman B., Konopacky Q. M., Macintosh B., Barman T., 2010, Nature, 468, 1080

Nelson R. P., Gressel O., Umurhan O. M., 2013, MNRAS, 435, 2610

Paardekooper S.-J., 2012, MNRAS, 421, 3286

Rice W. K. M., Armitage P. J., Mamatsashvili G. R., Lodato G., Clarke C. J., 2011, MNRAS, 418, 1356

Rice W. K. M., Lodato G., Armitage P. J., 2005, MNRAS, 364, L56

Umurhan O. M., Nelson R. P., Gressel O., 2013, in European Physical Journal Web of Conferences Vol. 46 of European Physical Journal Web of Conferences, Breathing Life Into Dead-Zones. p. 3003

Willerding E., 1992, Earth Moon and Planets, 56, 173