

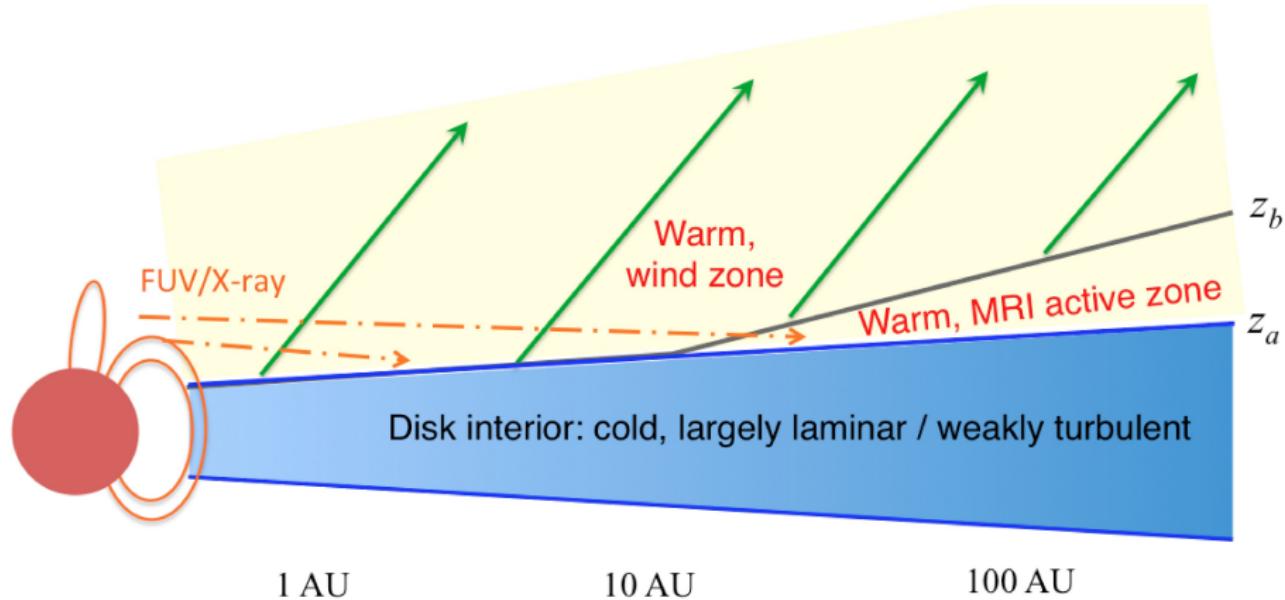
Hydrodynamic activity in protoplanetary discs

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Current picture of magnetic accretion

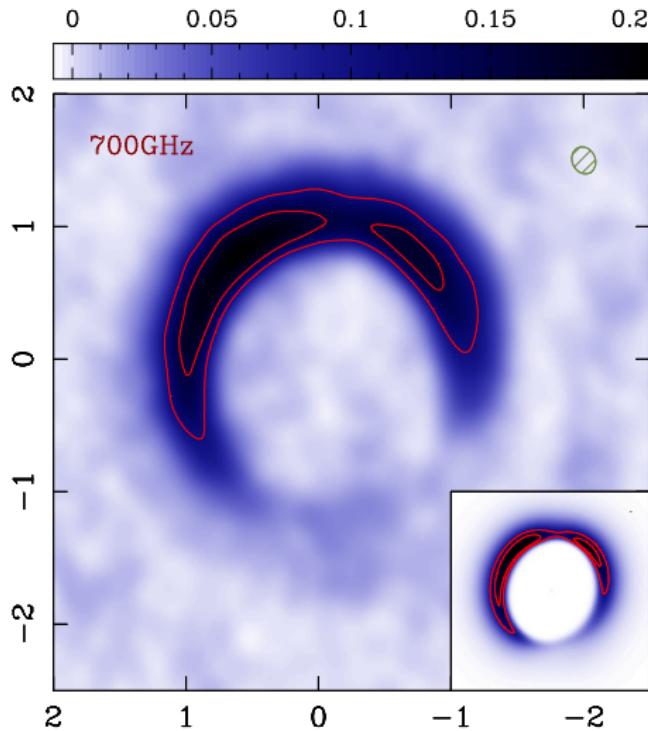


(Bai, 2016)

Outline

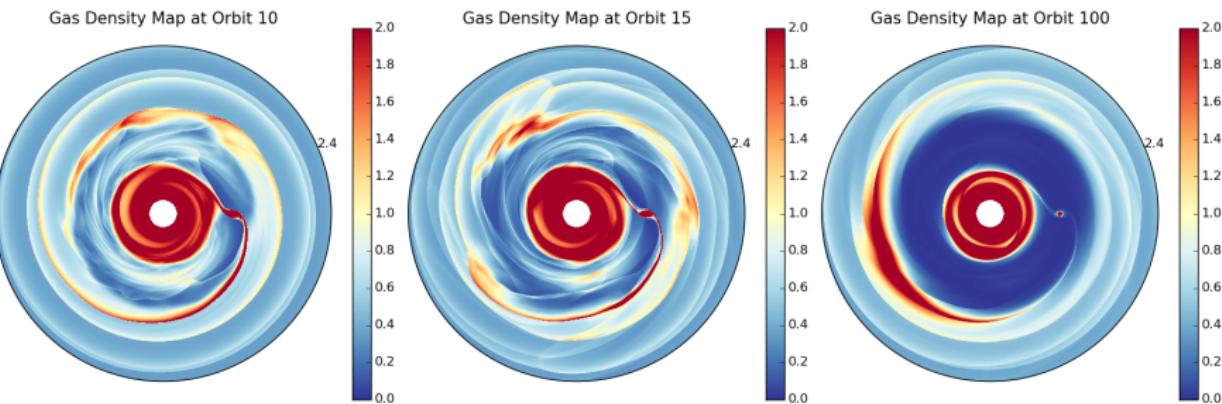
- Vortex evolution in 3D massive discs
(Lin & TBC, in prep.)
- Vertical shear instability
(Lin & Youdin, 2015)
- Generalized gravitational instabilities
(Lin & Kratter, submitted)

Transition disc asymmetries



(HD142527, Casassus et al., 2015)

Gap edges as sites for vortex formation



(2D simulations by M. Hammer, 'Vortices and orbital migration')

- Disc-planet interaction → gaps
- Surface density maxima at gap edges, or PV minima due to strong shear
- Rossby wave instability → 'roll up' into vortices
(Li et al., 2001)

Gap edges as sites for vortex formation



Rossby wave instability in 3D

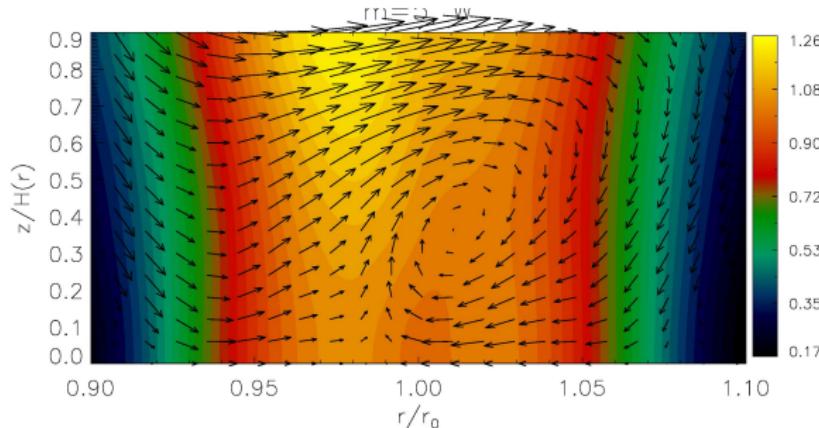
3D linear theory

Meheut et al. (2012); Lin (2012, 2013)

3D simulations

Meheut et al. (2010); Richard & Barge (2013)

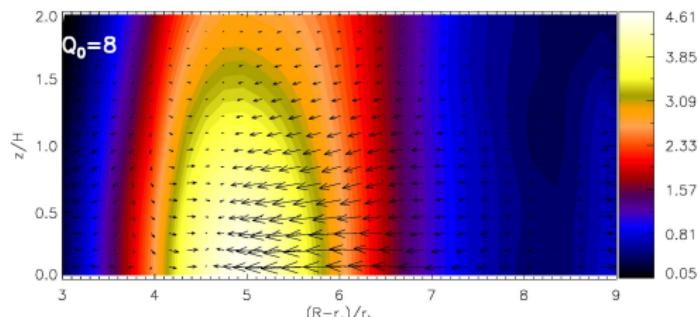
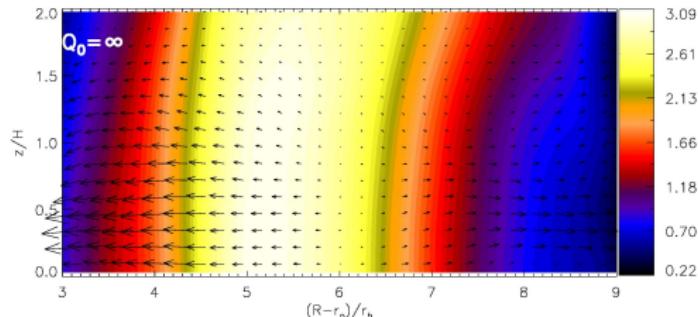
- Instability is physically 2D
- Increased vertical/structure flow with stiffer equation of state, or when buoyancy forces are present



$$(\gamma = 2.5, P \propto \rho^{3/2}, \text{Lin, 2013})$$

Rossby vortices are vertically global

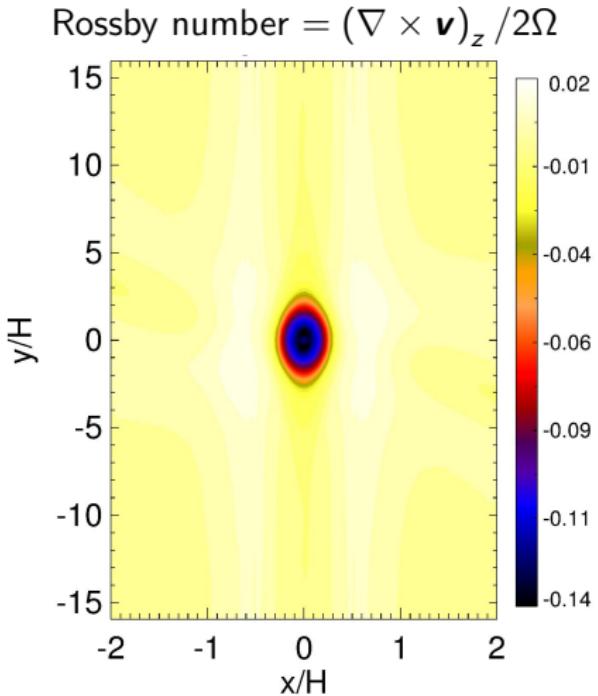
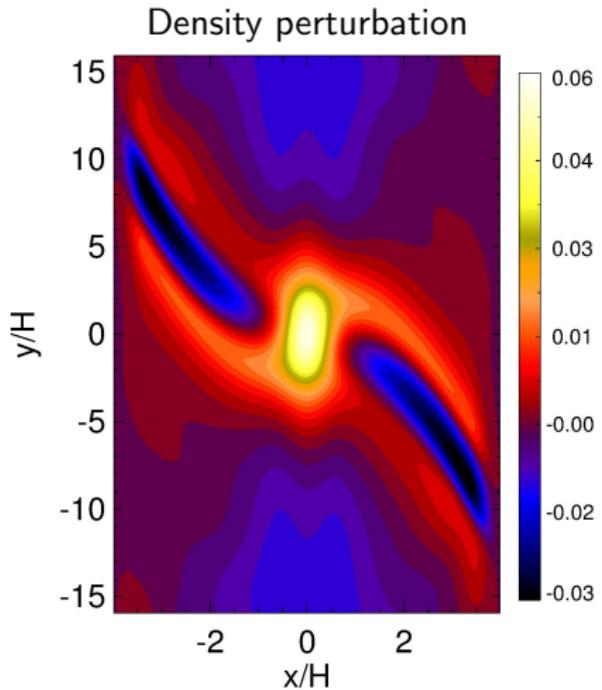
$$\Delta\rho/\rho$$



- $Q_{\text{planet}} \simeq 15$ initially
- SG \rightarrow more stratified
- $|v_z/c_s| \sim \text{few \%}$

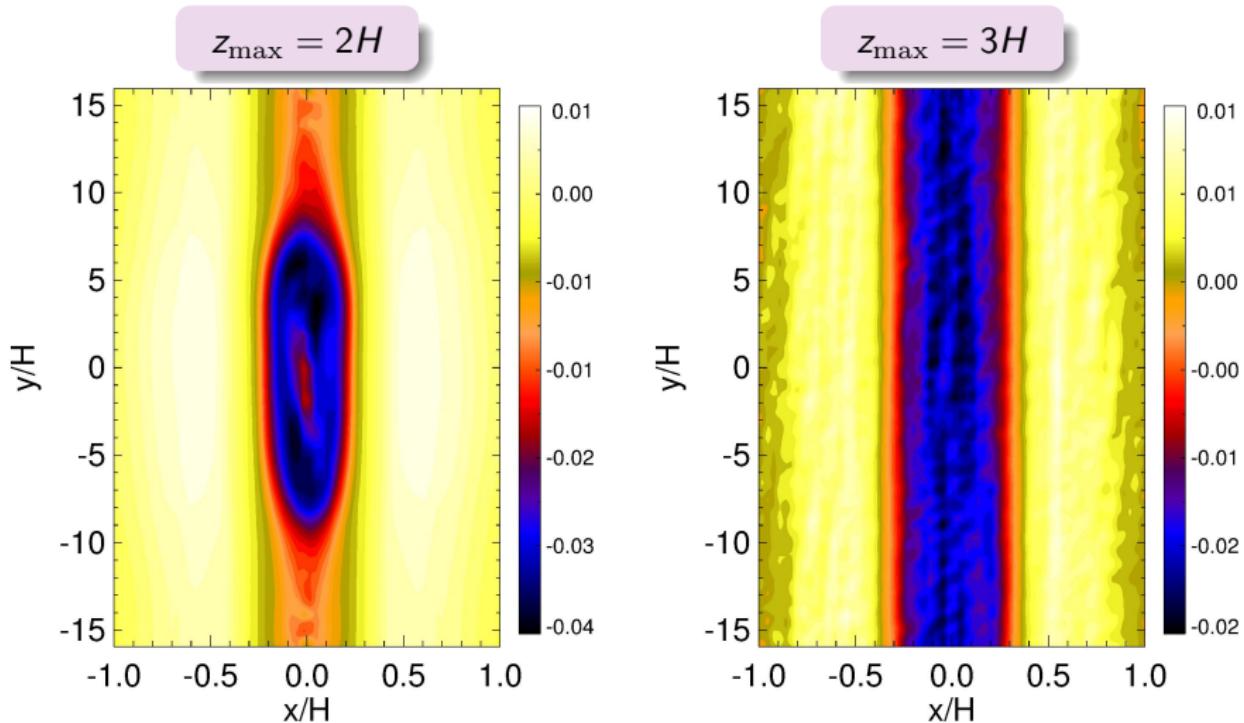
So vertical boundary conditions matter

- Athena isothermal shearing box initialized with a vortex



So vertical boundary conditions matter

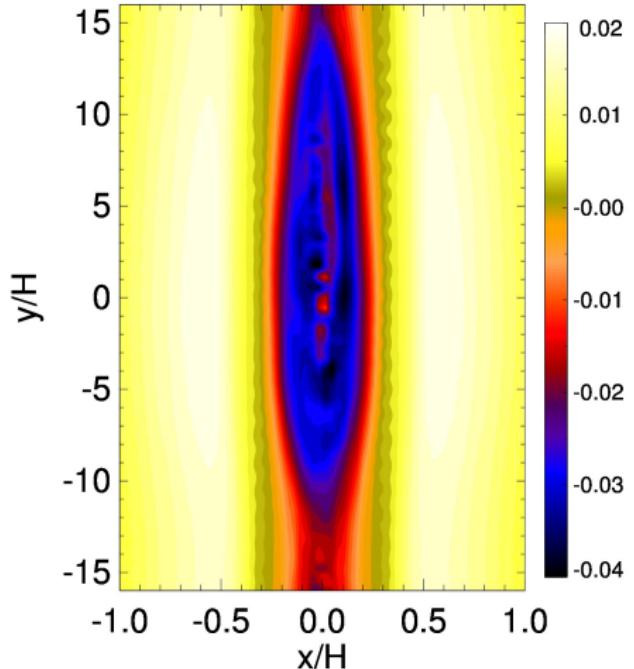
- Short vortices survive, tall vortices die
(‘elliptic instability’, Lesur & Papaloizou, 2009; Lithwick, 2009)



Elliptic instability & self-gravity

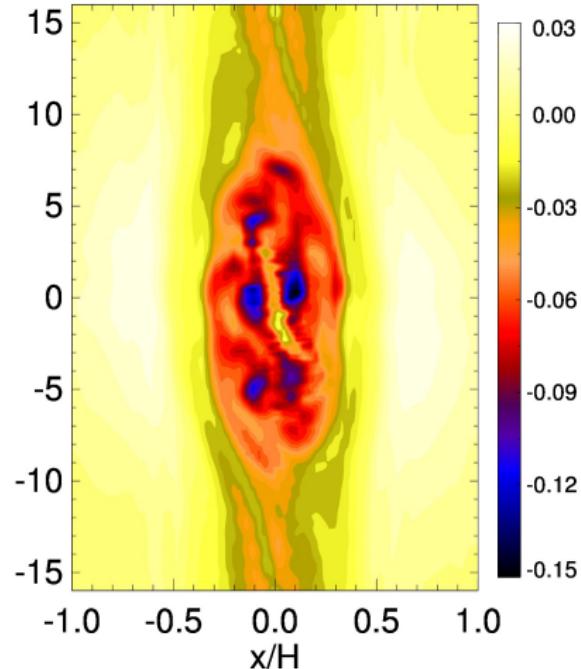
$Q = 100$

$t=800P_0$, $z=0.0H$, Ro

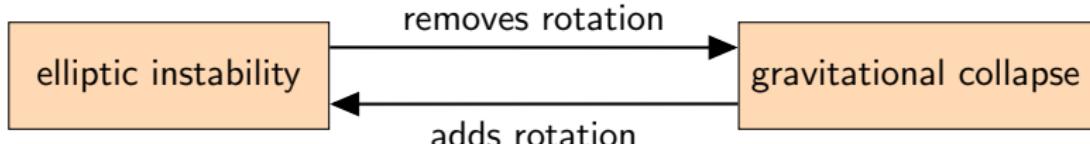
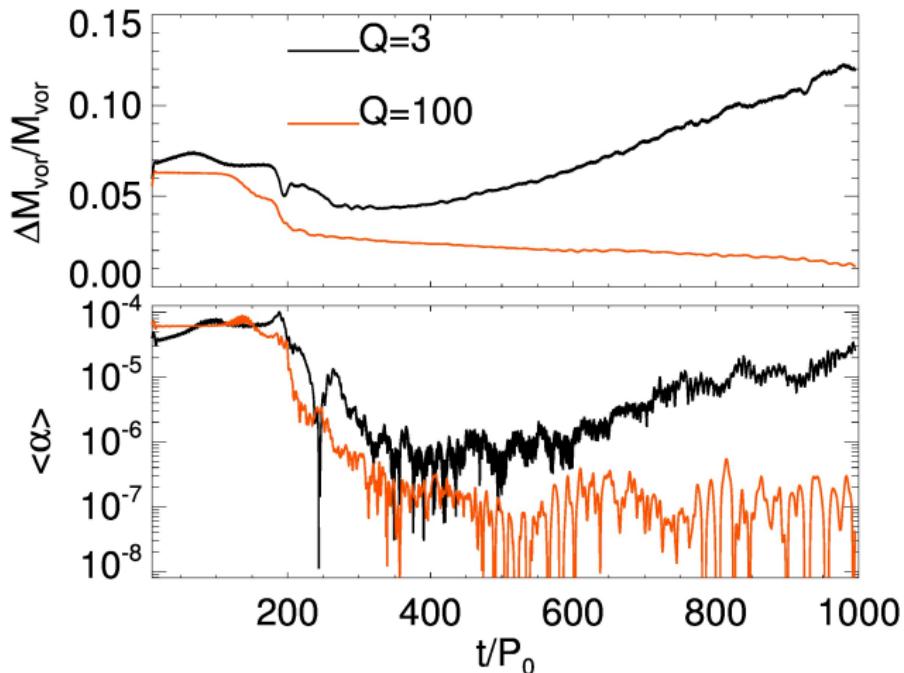


$Q = 3$

$t=800P_0$, $z=0.0H$, Ro



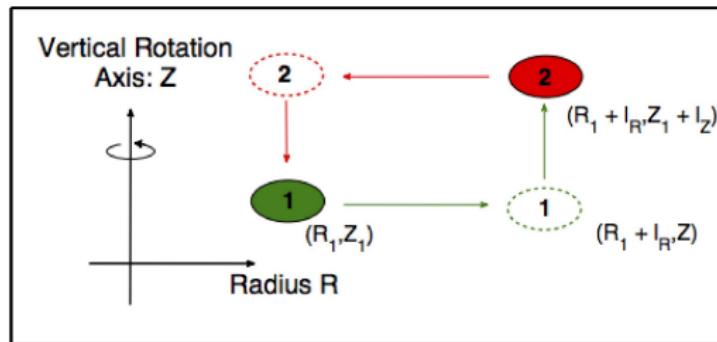
Elliptic instability & self-gravity



Vertical shear instability

- Accretion discs are generally baroclinic, $\nabla P \times \nabla \rho \neq 0 \Rightarrow$

$\partial_z \Omega \neq 0 \Rightarrow$ free energy \rightarrow instability?



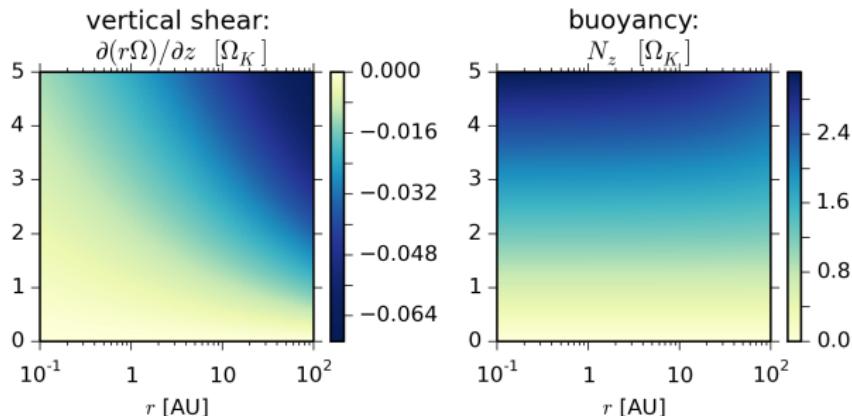
(Umurhan et al., 2013)

- Change in kinetic energy:

$$\Delta E \sim l_r^2 \left(\Omega^2 + \frac{l_z}{l_r} \cdot r \frac{\partial \Omega^2}{\partial z} \right).$$

$\Delta E < 0 \quad \text{if} \quad |l_z| \gg |l_r| \Rightarrow \text{INSTABILITY}$

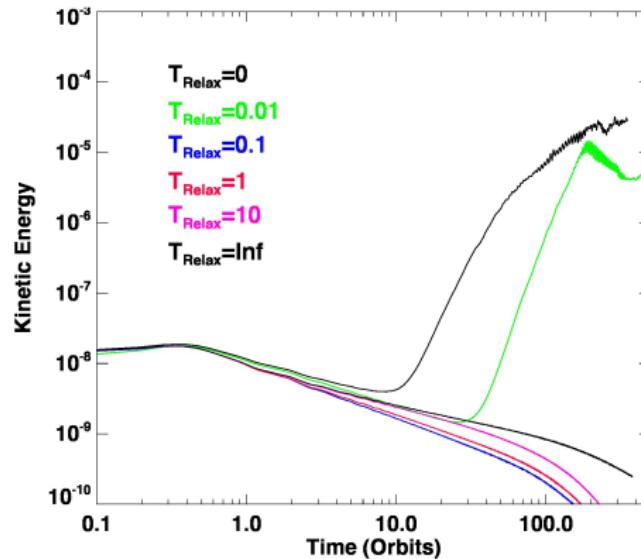
VSI needs to fight buoyancy in real discs



- Vertical shear is weak, $r\partial_z\ln\Omega \sim O(h) \ll 1$ (so need $l_z/l_r \gg 1$)
- Vertical buoyancy is strong, $N_z/\Omega \sim O(1)$

Thermodynamic condition for the VSI

- Can overcome buoyancy forces by cooling the disc rapidly
- Parameterization: $\partial_t T = -(T - T_0)/t_{\text{cool}}$



(Nelson et al., 2013)

Can we quantify this requirement?

Lin & Youdin (2015): linear theory with finite cooling

From single ODE, reduced model for $T \propto r^q$ disc, find that VSI requires:

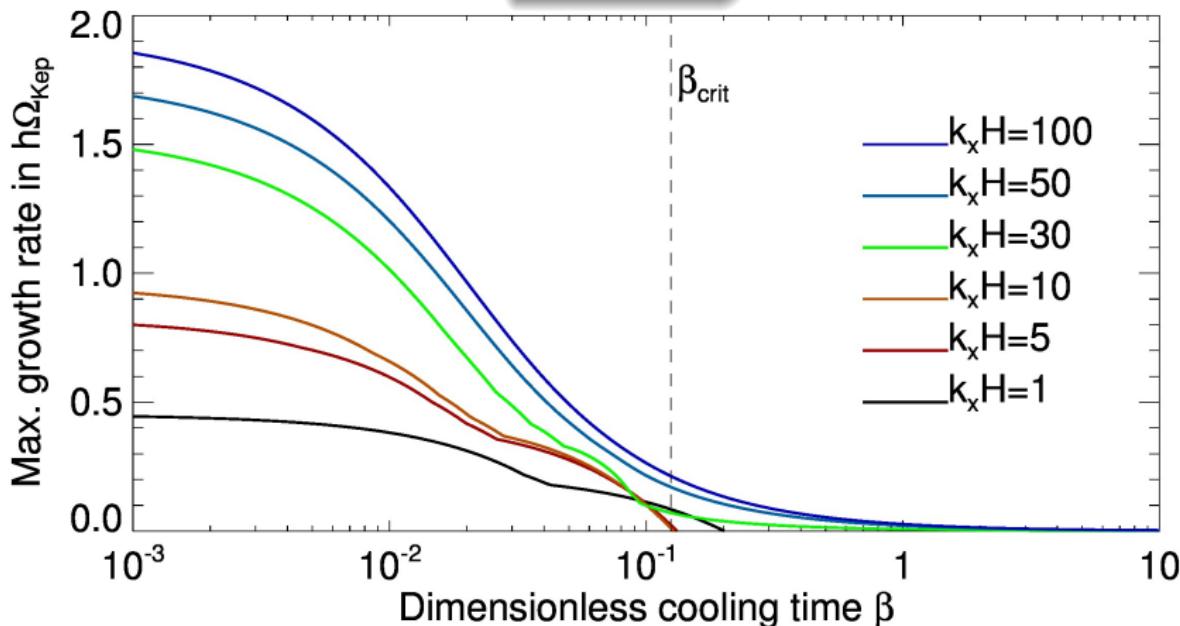
$$t_{\text{cool}}\Omega_K < \frac{h|q|}{\gamma - 1}$$

- $h|q|$: vertical shear ($h \equiv H/r$) — destabilizing
- $\gamma - 1$: vertical buoyancy — stabilizing
- $t_{\text{cool}}\Omega_K \ll 1$ required, i.e. rapid cooling

Full linear theory

- Solve linearized fluid equations in the radially local approximation
 - $\beta = t_{\text{cool}} \Omega_K$

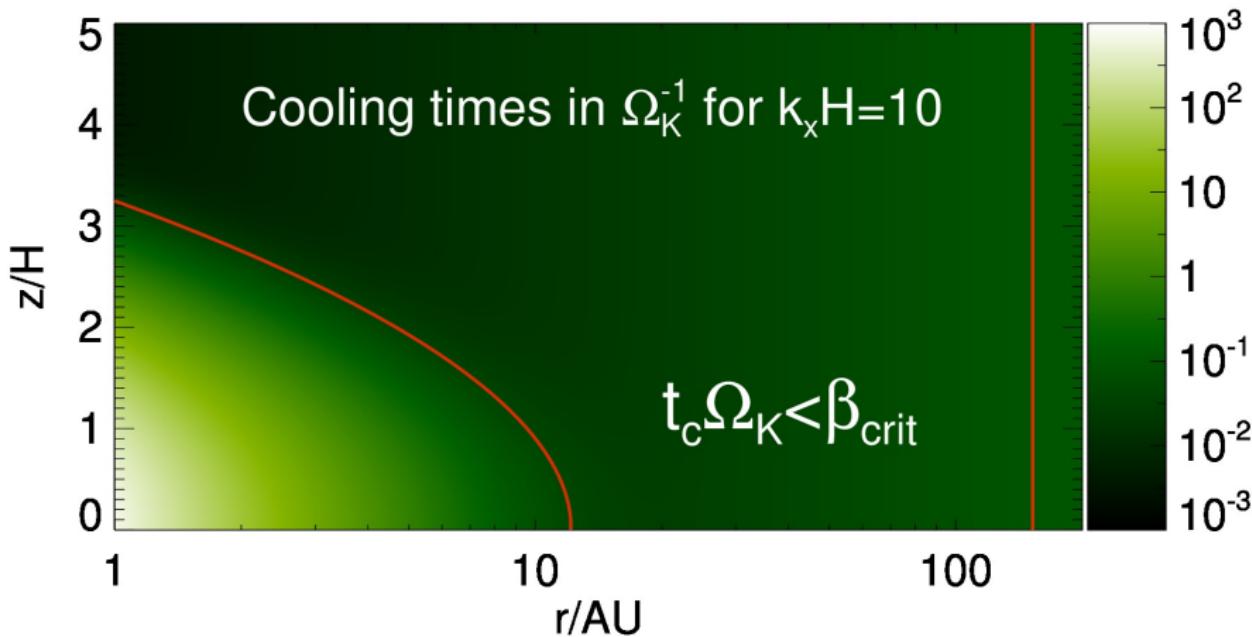
$$\beta_{\text{crit}} \equiv \frac{h|q|}{\gamma - 1}$$



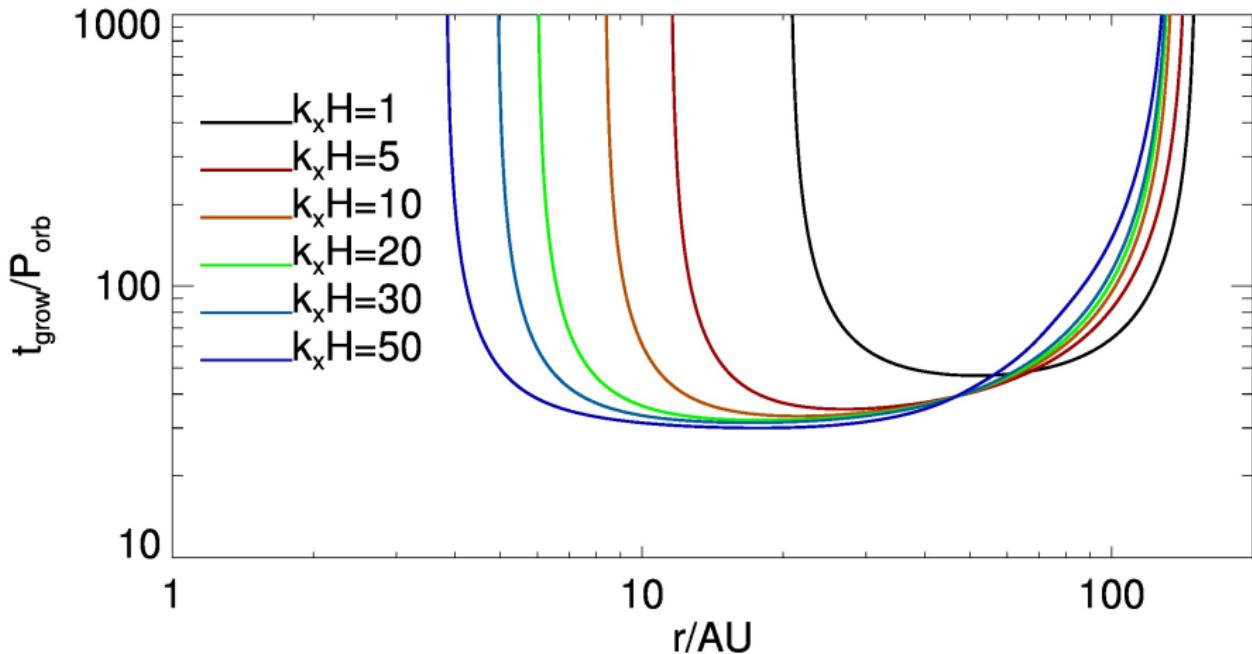
Vertical shear instability in the Solar Nebula

Cooling via dust-opacity ($\propto T^2$) in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010):

$$t_{\text{cool}} \Omega_K =$$

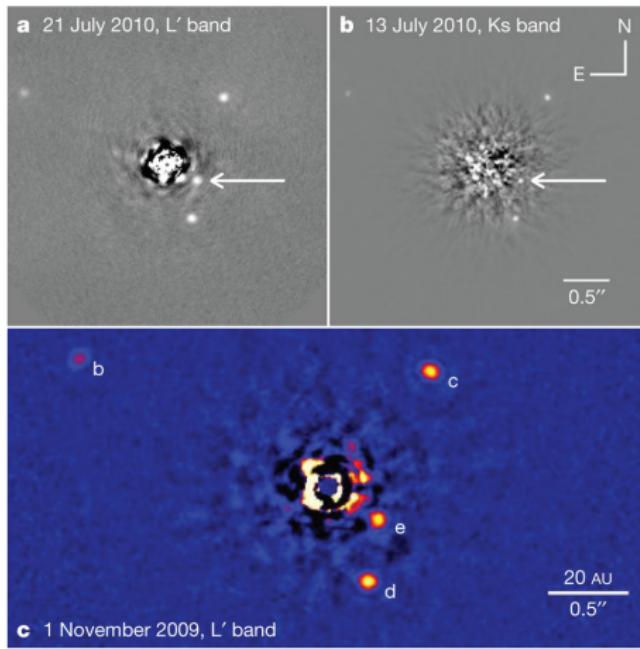


Typical growth times in the Solar Nebula



- VSI is most active in the outer disc 10—100AU
- Forced to develop on smaller scales towards inner disc

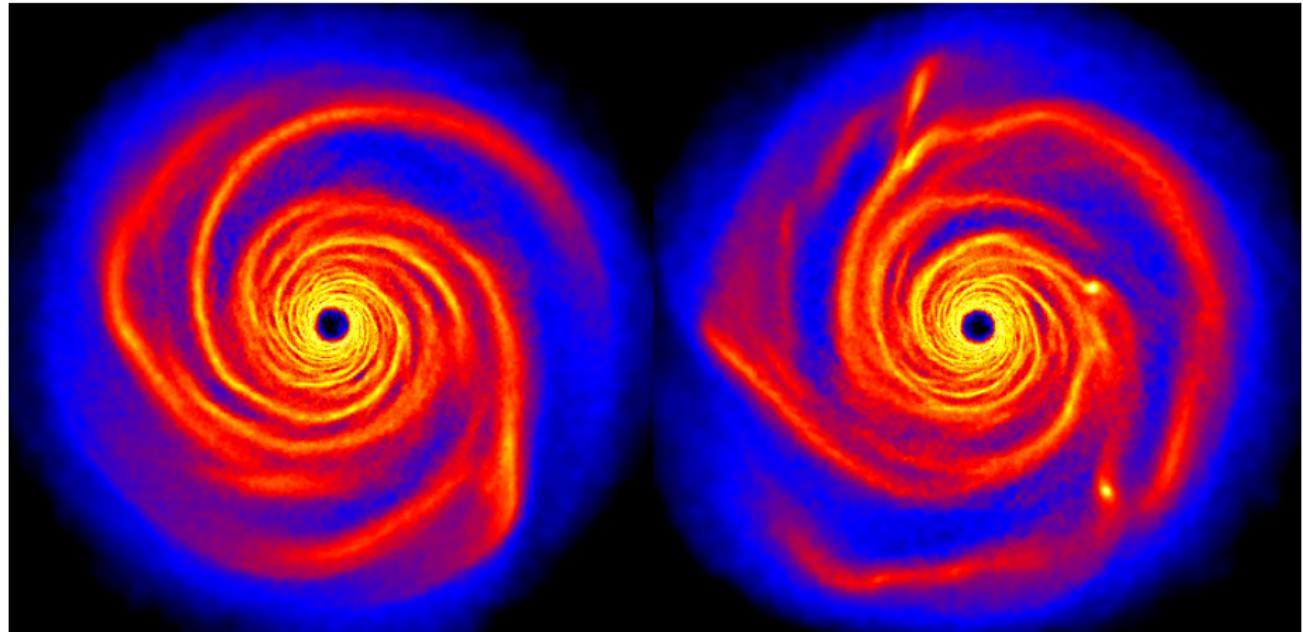
Wide orbit planets



(Marois et al., 2010)

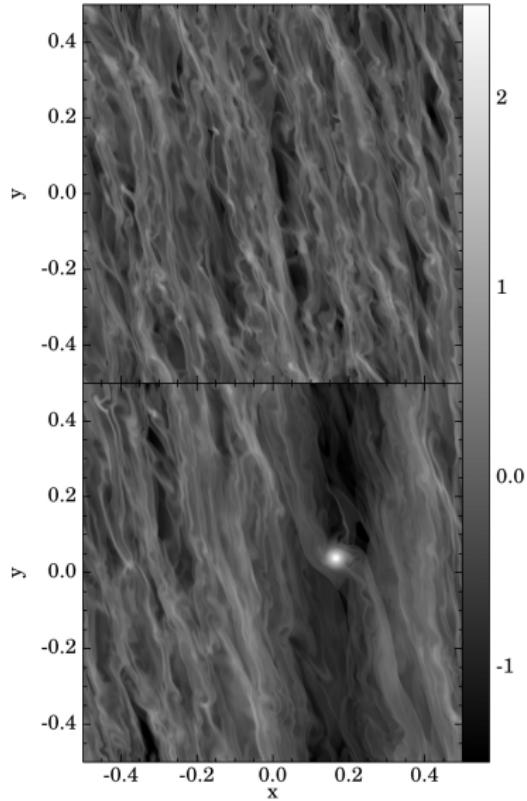
Disc instability theory

- Young, massive protoplanetary discs fragment under its own gravity



(Rice et al., 2005)

When do (simulated) protostellar discs fragment?



• Massive disc

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 2$$

• Fast cooling

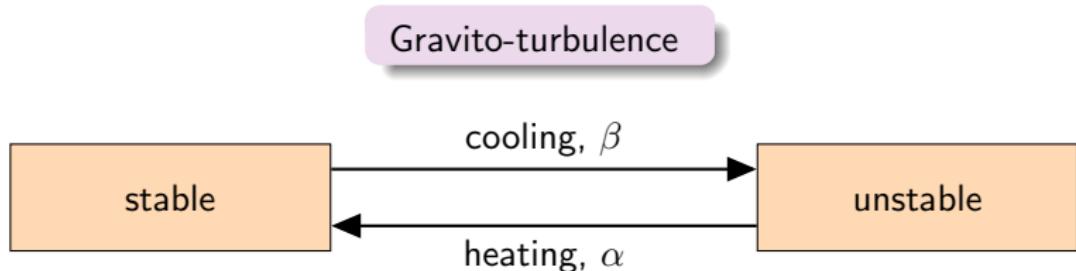
$$t_{\text{cool}} \Omega \lesssim 3$$

The cooling criterion is empirical.

N.B. local simulations only

(Paardekooper, 2012)

And when it does not fragment



- Cooling: $\beta \equiv t_{\text{cool}}\Omega$
- Heating: α viscosity
- Equilibrium $\Rightarrow \alpha = \alpha(\beta)$
- Fragments if $\beta < \beta_c$ or $\alpha > \alpha_c$

When do realistic protostellar discs fragment?

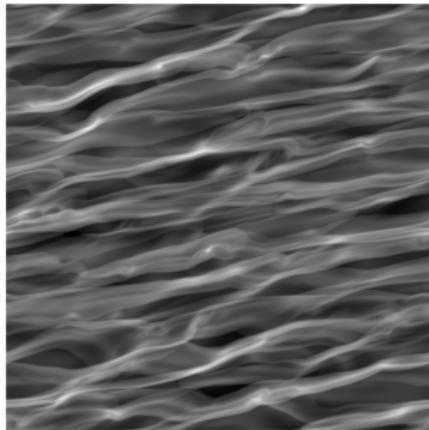
Work out $\Sigma(R)$, $T(R)$..etc.,

- ① Is Toomre $Q \sim 1$?
- ② Is $t_{\text{cool}}\Omega \sim 1$ or $\alpha \sim 0.1$?

Possible issues:

- Need to choose critical values
- Complex physics to get Σ , T were not included in the numerical experiments that established those critical values
- Numerical uncertainties in critical values (e.g. resolution)

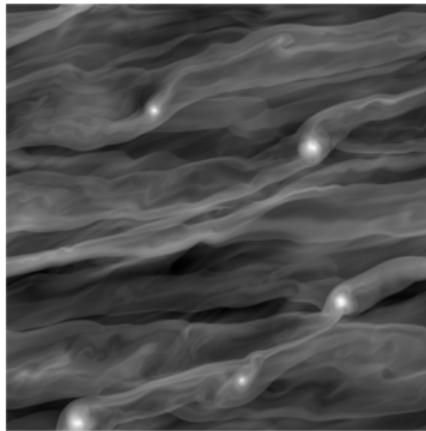
An alternative approach



(Rice et al., 2011)

- Write down a classic, viscous disc model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze its stability properties

An alternative approach



(Rice et al., 2011)

- Write down a classic, viscous disc model to describe quasi-steady, gravito-turbulent state, include cooling physics
- Analyze its stability properties
- Unstable → fragmentation

Local model

- 2D/3D shearing box with self-gravity
- Cooling physics and irradiation

$$\frac{\partial E}{\partial t} = -\Lambda$$

- Navier-Stokes viscosity for turbulent transport

$$\frac{\partial \mathbf{v}}{\partial t} = +\frac{1}{\rho} \nabla \cdot \mathbf{T}$$

(and viscous heating in energy equation)

Quantifying cooling

Classic dispersion relation without cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G \Sigma |k|}_{\text{+gravity}} - \underbrace{\Omega^2}_{\text{-rotation}} - \underbrace{\gamma c_s^2 k^2}_{\text{-pressure}}$$

Quantifying cooling

New result with cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G \Sigma |k|}_{\text{+gravity}} - \underbrace{\Omega^2}_{\text{-rotation}} - \underbrace{\left(\frac{T_{\text{irr}}/T + \gamma t_{\text{cool}} s}{1 + t_{\text{cool}} s} \right) c_s^2 k^2}_{\text{-modified pressure}}$$

$$\frac{\partial E}{\partial t} = - \frac{\mathcal{R}_* \rho}{\mu(\gamma - 1)} \frac{(T - T_{\text{irr}})}{t_{\text{cool}}}$$

Note if $T_{\text{irr}} = 0$:

$$\frac{\partial E}{\partial t} = - \frac{E}{t_{\text{cool}}}$$

(Gammie, 2001)

Quantifying cooling

New result with cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G \Sigma |k|}_{\text{+gravity}} - \underbrace{\Omega^2}_{\text{-rotation}} - \underbrace{\left(\frac{T_{\text{irr}}/T + \gamma t_{\text{cool}} s}{1 + t_{\text{cool}} s} \right) c_s^2 k^2}_{\text{-modified pressure}}$$

- Dispersion relation changes from $s^2 \rightarrow s^3$
- Can be formally unstable for any $t_{\text{cool}} < \infty$
- $T_{\text{irr}} = 0 \sim \text{pressureless disc}$

Just a fancy way to compare compressional heating v.s. thermal losses

A little magic

Cooling timescale to remove pressure over a lengthscale $\sim H$

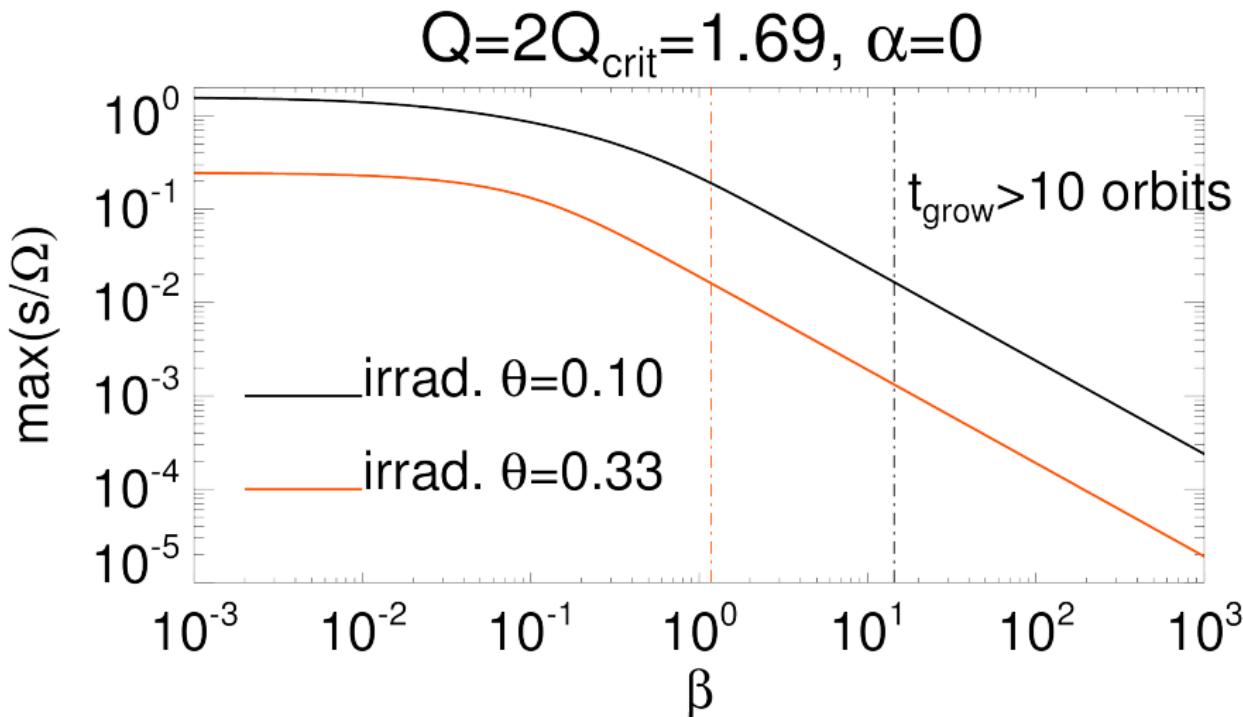
$$t_{\text{cool},*} = (\sqrt{\gamma} - 1)^{-3/2} \Omega^{-1}$$

$$\alpha_* = \frac{4}{9} \frac{\sqrt{\sqrt{\gamma} - 1}}{\gamma(\sqrt{\gamma} + 1)}$$

γ	α_*	$t_{\text{cool},*} \Omega$	Sim., $t_{\text{cool,frag}} \Omega$	Reference
7/5	0.062	12.75	12—13	Rice et al. (2005)
1.6	0.063	7.33	8	Rice et al. (2011)
5/3	0.063	6.37	6—7	Rice et al. (2005)
2	0.059	3.75	3	Gammie (2001)

(Lin & Kratter, submitted)

Irradiation is important



Viscosity-enabled gravitational instability

Classic instability condition

$$2\pi G \Sigma |k| - \Omega^2 - \gamma c_s^2 k^2 > 0$$

+ gravity - rotation - pressure > 0

Viscosity-enabled gravitational instability

Add viscosity:

$$2\pi G \Sigma |k| - c_s^2 k^2 > 0$$

$$+ \text{ gravity} \quad - \text{ pressure} > 0$$

(Lynden-Bell & Pringle, 1974; Willerding, 1992; Gammie, 1996)

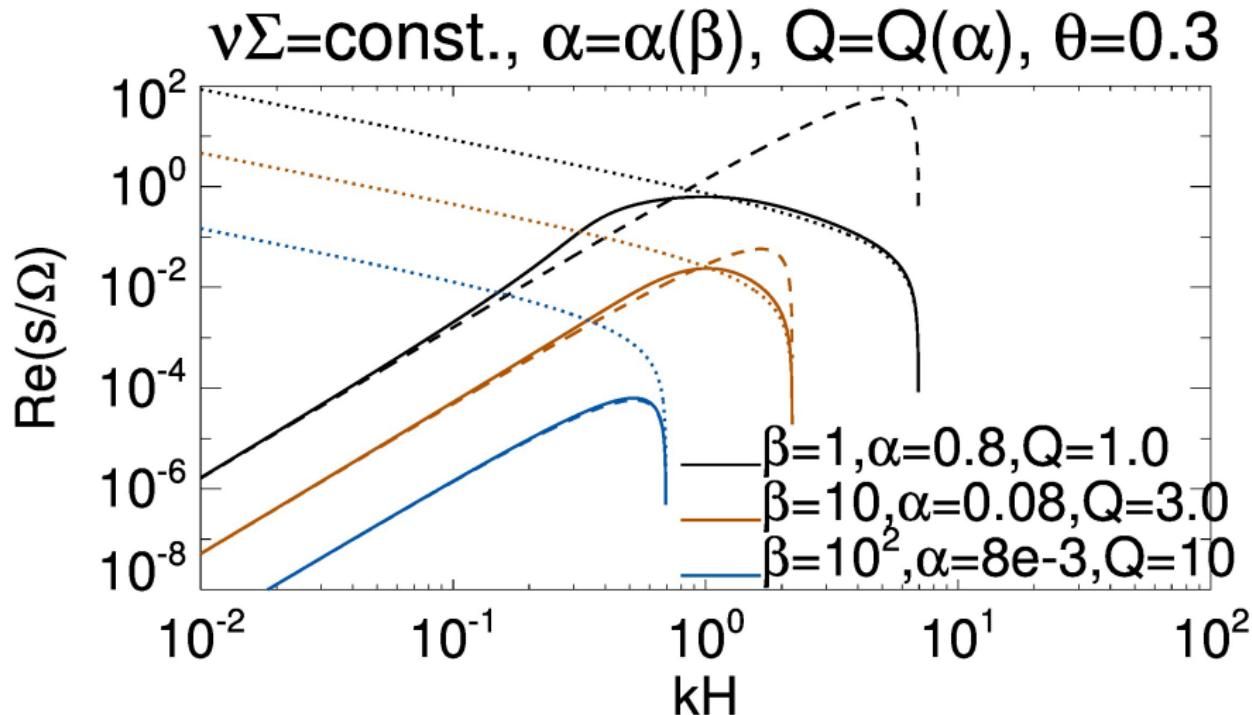
- Viscosity or frictional forces remove rotational stabilization (cf. inwards migration of particles due to gas-dust drag)
- Rice et al. (2005) report a $\alpha_{\max} \sim 0.1$ before fragmentation, also supported by Clarke et al. (2007)

Viscous GI growth rate:

$$s = \frac{27\alpha}{16Q^4} \Omega$$

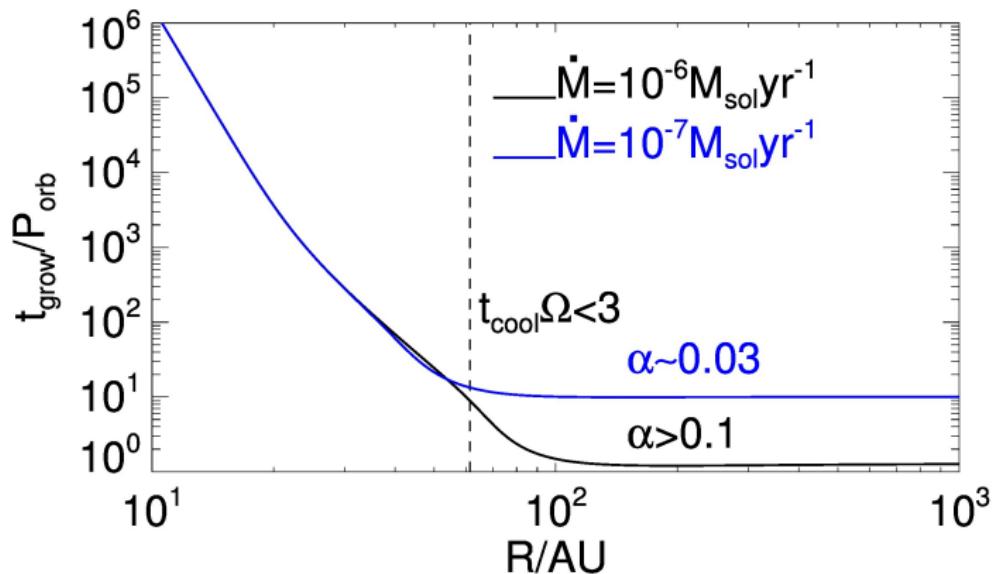
(isothermal/irradiated limit)

Viscous gravitational instability example



Application to protoplanetary discs

- Input physical disc model into stability calculation — get *growth timescales*



Beyond $\sim 60 \text{ AU}$:

- Cooling criterion \Rightarrow both disc fragments
- Viscosity criterion \Rightarrow high \dot{M} disc fragments, growth times \sim one orbit

Summary

Elliptical vortices in massive discs

- Self-gravity helps vortex survival, but with a turbulent core?

Vertical shear instability

- Feeds off free energy in $\partial_z \Omega \neq 0$
- Require ultra-fast cooling in convectively stable, thin discs
- VSI possible in the outer disc between 10—100AU

Generalized gravitational instability

- Cooling: reduces thermal support
- Viscosity: reduces rotational support
- Fragmentation: inability to maintain steady gravito-turbulence because cooling and/or turbulent stresses (viscosity)

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