Hydrodynamic activity in protoplanetary disks

Min-Kai Lin

Steward Theory Fellow
University of Arizona

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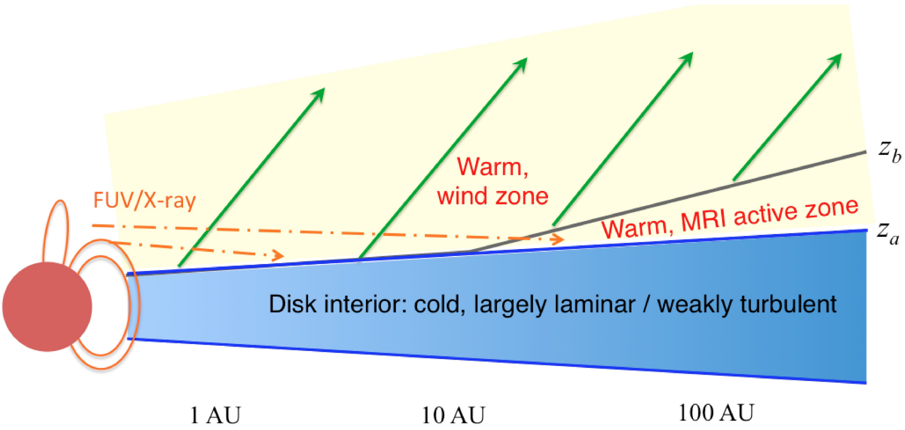
Interests

- Astrophysical fluid dynamics of accretion/protoplanetary disks
- Disk-planet interactions, orbital migration
- Self-gravitating disks
- Hydrodynamic instabilities
- Magneto-hydrodynamics
- Non-linear numerical simulations
- Analytical/semi-analytical methods

Role of magnetic fields in protoplanetary disks



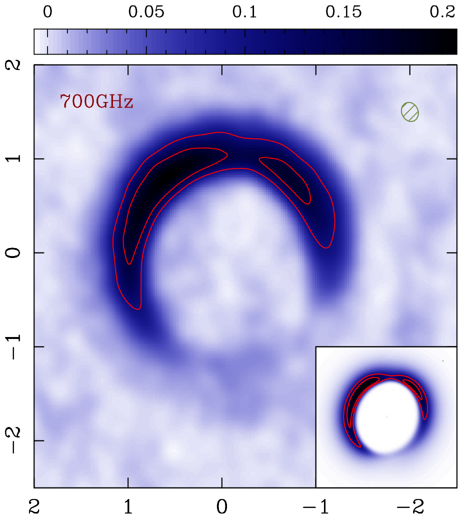
(Bai, 2016)

- Purely hydrodynamical effects are important in the planet-forming disk bulk

Hydrodynamical processes in protoplanetary disks

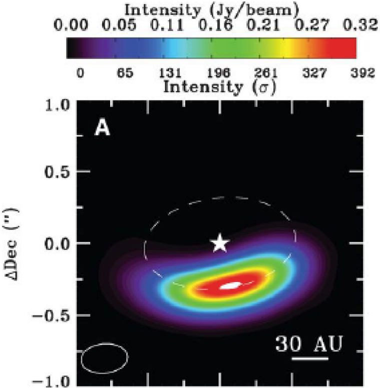
- Vortex formation and evolution
  - Vortices as dust traps
  - Explaining asymmetric transition disks
  - Assisting planetesimal formation
  - 3D effects in massive disks (Lin & TBC, in prep.)
- Vertical shear instability
  - Hydrodynamic turbulence
  - Angular momentum transport
  - Does it occur in realistic PPDs? (Lin & Youdin, 2015)
- Generalized gravitational instabilities
  - 'Gravito-turbulence' and transport
  - Fragmentation to form giant planets/sub-stellar companions
  - Going beyond Toomre and Lin-Shu analyses (Lin & Kratter, re-submitted)

Transition disk asymmetries: vortices?



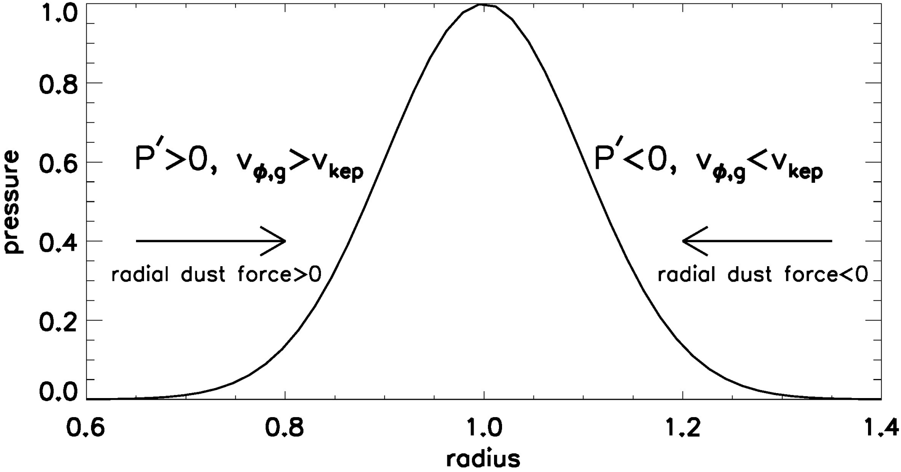
(HD142527, Casassus et al., 2015)

Transition disk asymmetries: vortices?



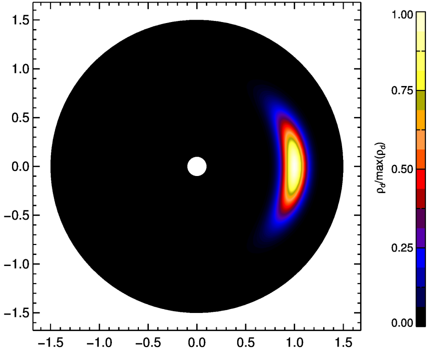
(Oph IRS 48, van der Marel et al., 2013)

Dust trapping at pressure maxima



- Drag forces cause dust to accumulate at pressure bumps

Dust distribution in disk vortices



$$\rho_d(a) \propto \exp\left(-\frac{a^2}{2H_v^2}\right),$$

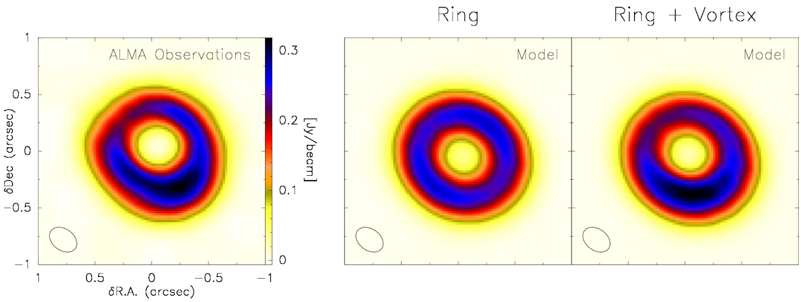
- a: distance from the vortex center

(Lyra & Lin, 2013)

$$H_v(\chi, \delta, \mathrm{St}) = \frac{H_g}{f(\chi)}\sqrt{\frac{\delta}{\delta + \mathrm{St}}}.$$

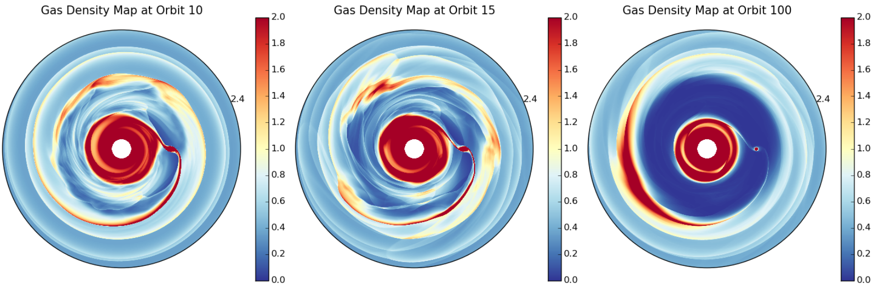
- χ: vortex aspect-ratio
- δ: turbulence in the vortex
- St: Stokes number (dust-gas friction)
- H_g: gas scale height

Application to observations



(SAO 206462, Pérez et al., 2014)
$\chi_{\rm obs} \sim 7$, model+ data $\rightarrow v_{\rm turb} \sim 0.22c_s$.

Gap edges as sites for vortex formation



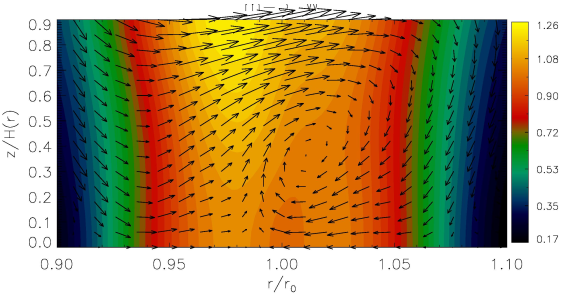
(2D simulations by M. Hammer, 'Vortices and orbital migration')

- Disk-planet interaction → gaps
- Surface density maxima at gap edges, or PV minima due to strong shear
- Rossby wave instability → edges 'roll up' into vortices (Li et al., 2001)

Gap edges as sites for vortex formation

Rossby vortices are vertically global

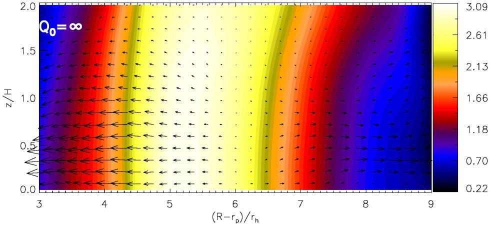
$\Delta P/\rho$



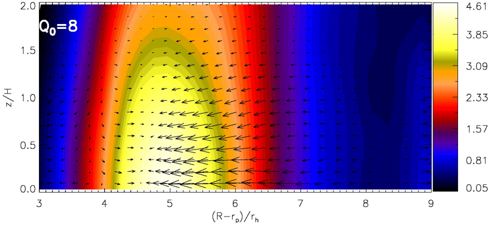
- 3D linear theory (Lin, 2013b)

Rossby vortices are vertically global

$\Delta\rho/\rho$



↑ NO self-gravity

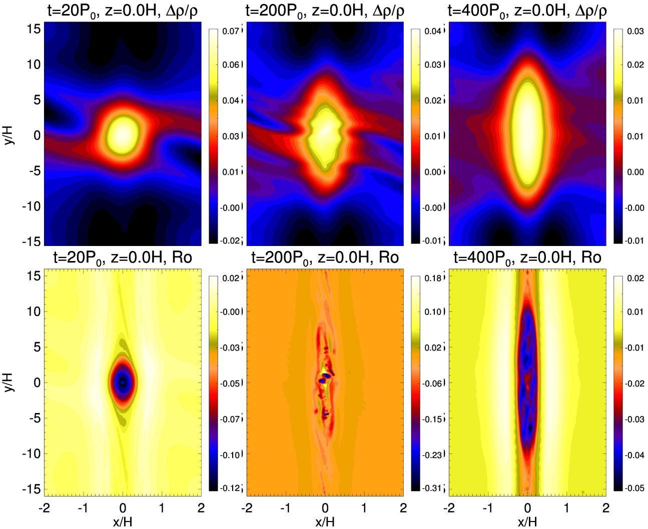


↑ WITH self-gravity

- Global 3D Zeus simulations (Lin, 2012)

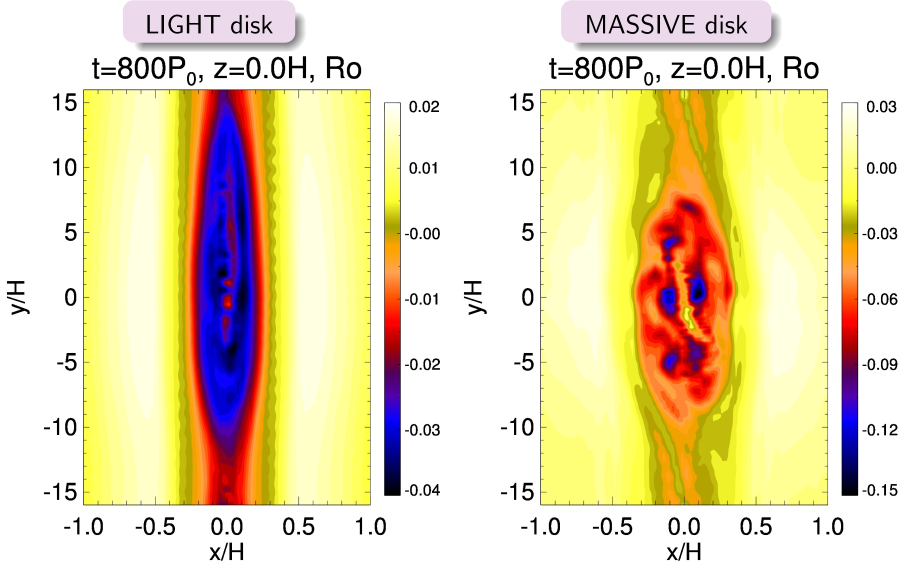
Elliptic instability of 3D vortices

- A 3D instability that weakens/destroys vortices (Lesur & Papaloizou, 2009)



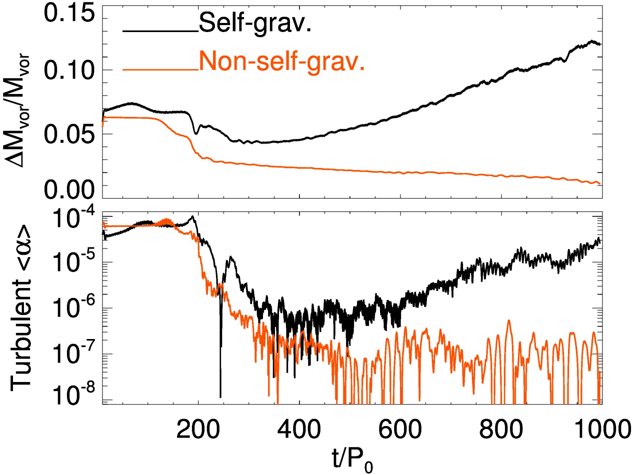
- Athena simulations (Lin & TBC, in prep.)

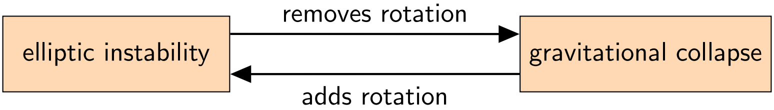
Elliptic instability & self-gravity



$(Q = 100)$ $(Q = 3)$

Elliptic instability & self-gravity





Astrophysical disks have vertical shear

- Accretion disks are generally baroclinic, $\nabla P \times \nabla \rho \neq 0$

$$\Rightarrow \boxed{\frac{\partial \Omega}{\partial z} \neq 0}$$

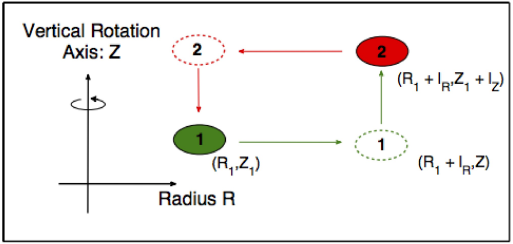
- Vertically isothermal thin-disk with $T \propto r^q$,

$$r\frac{\partial \Omega}{\partial z} \simeq \left(\frac{qz}{2H}\right) \times \frac{H}{r}\Omega_{\mathrm{Kep}}$$

- $O(H/r)$ effect

Vertical shear instability

$\partial_z \Omega \neq 0 \Rightarrow$free energy$\rightarrow$instability?



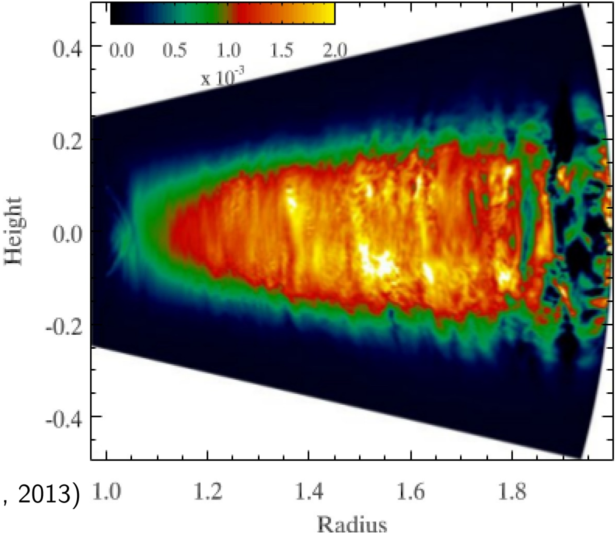
(Umurhan et al., 2013)

- Change in kinetic energy:

$$\Delta E \sim l_r^2 \left(\Omega^2 + \frac{l_z}{l_r} \cdot r \frac{\partial \Omega^2}{\partial z} \right).$$

$\Delta E < 0 \quad \text{if} \quad |l_z| \gg |l_r| \Rightarrow$ **INSTABILITY**

VSI: non-linear outcome

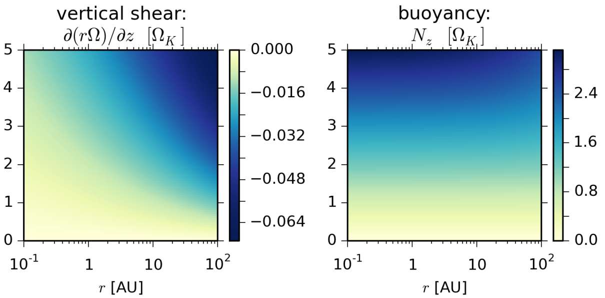


(Nelson et al., 2013)

Hydrodynamic turbulence

VSI needs to fight buoyancy in real disks

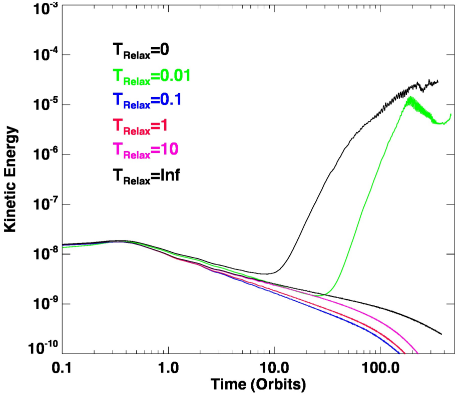
$$\underbrace{r\partial_z\Omega}_{\text{destabilizing vert. shear}} \quad \text{v.s.} \quad \underbrace{N_z}_{\text{stabilizing vert. buoyancy}}$$



- Vertical shear is weak, $r\partial_z \ln\Omega \sim O(h) \ll 1$ (so need $l_z/l_r \gg 1$)
- Vertical buoyancy is strong, $N_z/\Omega \sim O(1)$

Ultra-fast cooling can overcome buoyancy forces

- Cooling parameterization: $\partial_t T = -(T - T_0)/t_{\rm cool}$



(Nelson et al., 2013)

Can we quantify this requirement?

Lin & Youdin (2015): linear theory with finite cooling

From single ODE, reduced model for $T \propto r^q$ disk, find that VSI requires:

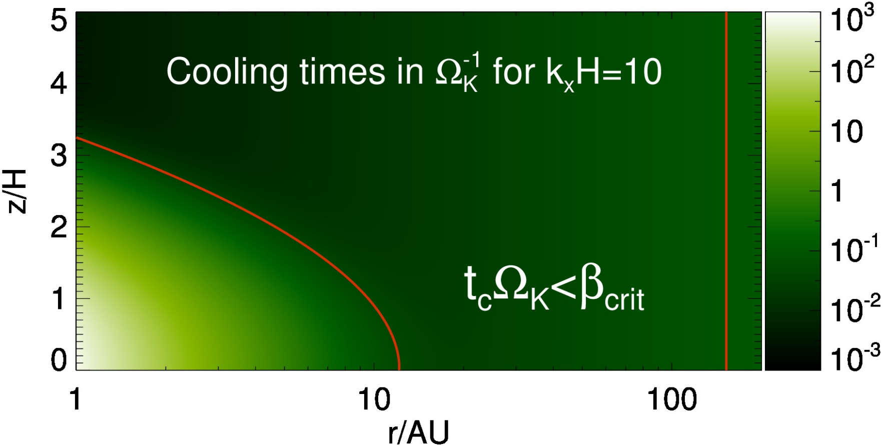
$$t_{\mathrm{cool}}\Omega_{\mathrm{K}} < \frac{h|q|}{\gamma - 1}$$

- $h|q|$: vertical shear ($h \equiv H/r$) — destabilizing
- $\gamma - 1$: vertical buoyancy — stabilizing
- $t_{\mathrm{cool}}\Omega_{\mathrm{K}} \ll 1$ required, i.e. rapid cooling

Vertical shear instability in the Solar Nebula

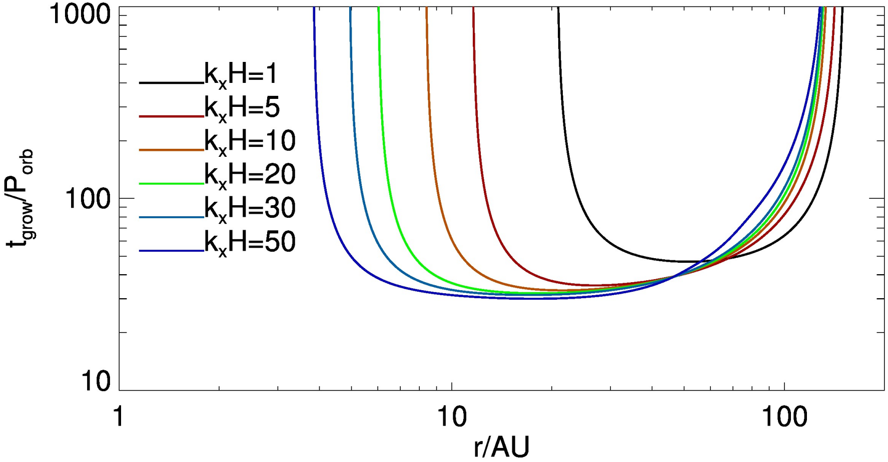
Cooling via dust-opacity ($\propto T^2$) in the Minimum Mass Solar Nebula (Chiang & Youdin, 2010)

$$\beta_{\mathrm{crit}} \equiv \frac{h|q|}{\gamma - 1}$$



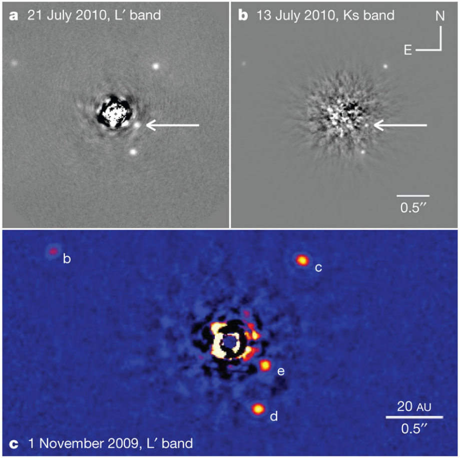
Typical growth times in the Solar Nebula

- Solve the full linearized fluid equations in the radially local approximation, with radiative diffusion/optically-thin cooling



- VSI is most active in the outer disk 10—100AU
- Forced to develop on smaller scales towards inner disk

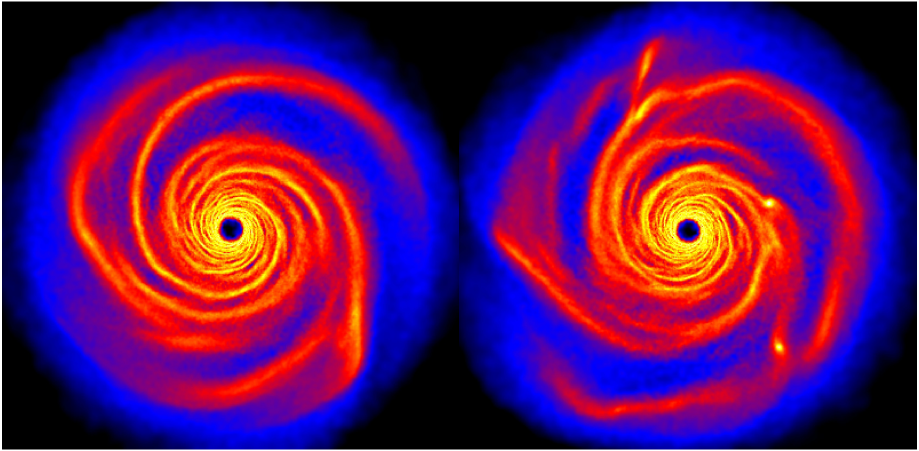
Wide orbit planets



(Marois et al., 2010)

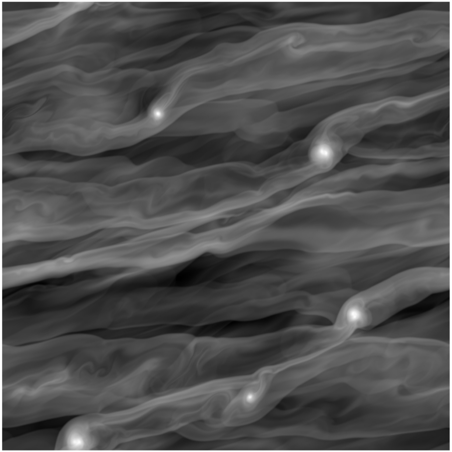
Disk instability theory

- Young, massive protoplanetary disks fragment under its own gravity



(Rice et al., 2005)

When do (simulated) protostellar disks fragment?



(Rice et al., 2011)

- Massive disk

$$Q \equiv \frac{c_s\Omega}{\pi G\Sigma} \lesssim 2$$

- Fast cooling

$$t_{\mathrm{cool}}\Omega \lesssim 3$$

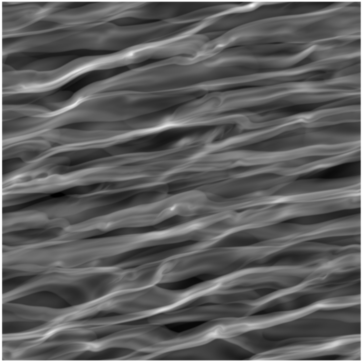
The cooling criterion is empirical.

And when it does not fragment

Gravito-turbulence

$$\begin{tikzcd} \text{stable} \arrow[r, shift left, "\text{cooling, } \beta"] & \text{unstable} \arrow[l, shift left, "\text{heating, } \alpha"] \end{tikzcd}$$

- Cooling: $\beta \equiv t_{\mathrm{cool}}\Omega$
- Heating: α viscosity
- Equilibrium $\Rightarrow \alpha = \alpha(\beta)$
- Need $\beta > \beta_c$ or $\alpha < \alpha_c$
  But why?



(Rice et al., 2011)

When do realistic protostellar disks fragment?

Work out $\Sigma(R)$, $T(R)$..etc., then ask

1. Is Toomre $Q \sim 1$?
2. Is $t_{\mathrm{cool}}\Omega \sim 1$ or $\alpha \sim 0.1$?

Possible issues:

- Need to choose critical values
- Numerical uncertainties in critical values (e.g. resolution)

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Motivation 1:

Assess disk fragmentation without input from hydrodynamic simulations

Beyond classical gravitational instability

Modern simulations

- Cooling physics, e.g.

$$\frac{\partial E}{\partial t} = -\frac{E}{t_{\mathrm{cool}}}$$

- Turbulent/viscous, e.g.

$$\nu = \alpha \frac{c_s^2}{\Omega}$$

Analytic toolbox

Lin-Shu dispersion relation, Toomre Q

$$\omega^2 = \kappa^2 - 2\pi G\Sigma|k| + c_s^2 k^2$$

$$Q \equiv \frac{c_s \kappa}{\pi G \Sigma}$$

- Isothermal/adiabatic (no cooling)
- Laminar (inviscid)

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Motivation 2:

Generalize analytic treatment of GI to include cooling, irradiation and viscosity

$$\omega = \omega(k; Q, t_{\mathrm{cool}}, \alpha)$$

Quantifying cooling

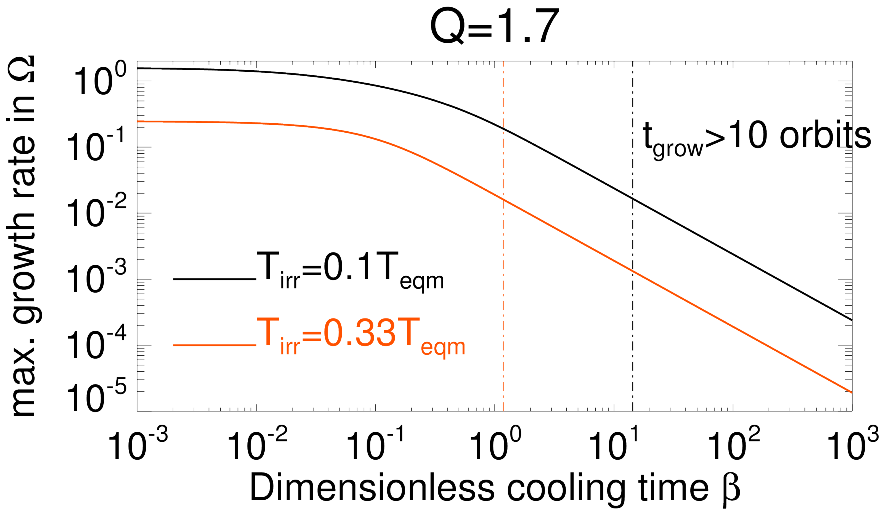
Dispersion relation with cooling

$$\underbrace{s^2}_{\text{growth}} = \underbrace{2\pi G\Sigma|k|}_{+\text{gravity}} \underbrace{-\Omega^2}_{-\text{rotation}} \underbrace{-\left(\frac{T_{\rm irr}/T + \gamma t_{\rm cool}s}{1 + t_{\rm cool}s}\right)c_s^2 k^2}_{-\text{modified pressure}}$$

- $T_{\rm irr}$: irradiation or floor temperature
- Can be formally unstable for any $t_{\rm cool} < \infty$
- $T_{\rm irr} = 0 \sim$ pressureless disk

Just a fancy way to compare compressional heating v.s. thermal losses

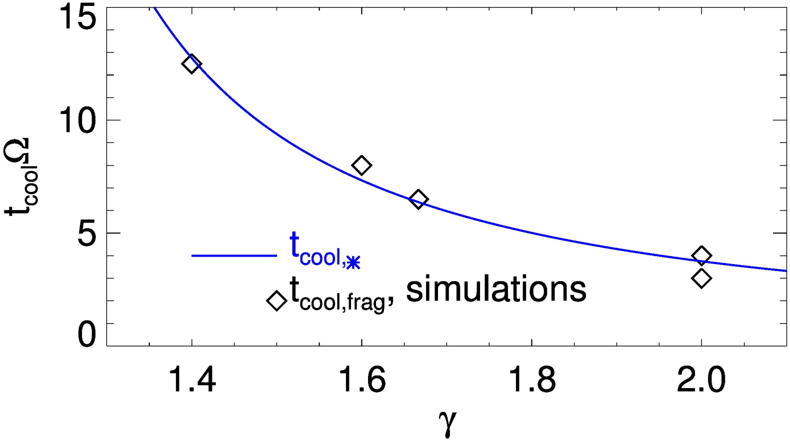
Cooling-driven gravitational instability



Understanding (some) simulations

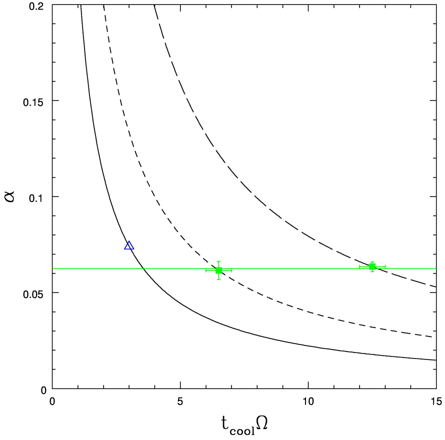
Cooling timescale to remove pressure over a lengthscale $\sim H$

$$t_{\mathrm{cool},*} = \left(\sqrt{\gamma} - 1\right)^{-3/2} \Omega^{-1}$$



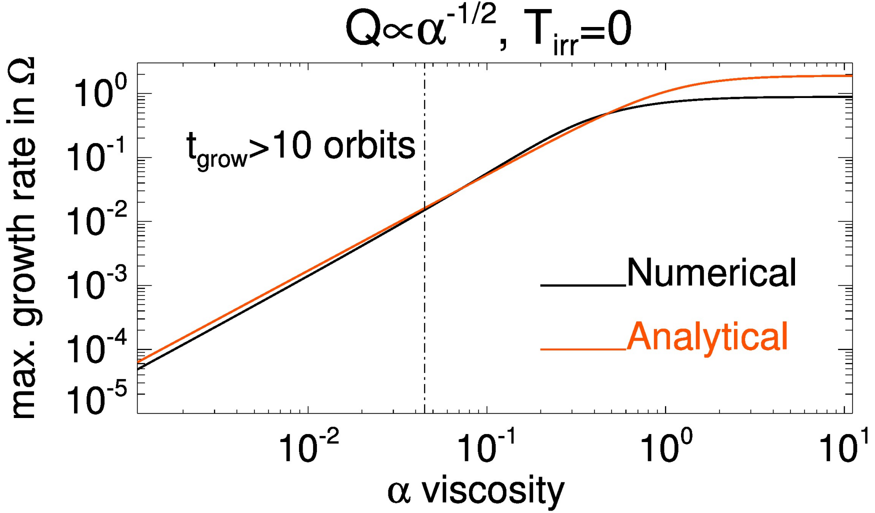
Simulations: Gammie (2001); Rice et al. (2005, 2011); Paardekooper (2012)

Maximum stress/viscosity in simulations



- Rice et al. (2005) report a $\alpha_{\max}$ before fragmentation (also supported by Clarke et al. (2007))

Viscous gravitational instability

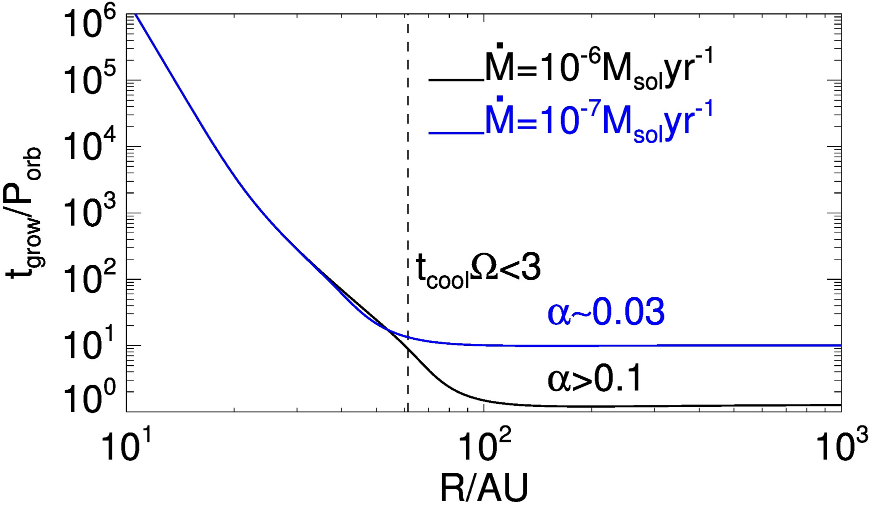


- Viscosity/friction removes rotational stabilization (cf. inwards migration of particles due to gas-dust drag)

(Lynden-Bell & Pringle, 1974; Willerding, 1992; Gammie, 1996)

Application to protoplanetary disks

- Input physical disk model into stability calculation — get growth timescales



- High $\dot{M}$ disk fragments $\gtrsim$ 60AU, growth times $\sim$ one orbit

Summary

Vortices in massive 3D disks

- Self-gravity helps vortex survival, but with a turbulent core?

Vertical shear instability

- Feeds off free energy in $\partial_z \Omega \neq 0$
- Require large vertical motions to tap into weak vertical shear
- Enabled by ultra-fast cooling
- VSI possible in the outer disk between 10—100AU

Generalized gravitational instability

- Cooling: reduces thermal support
- Viscosity: reduces rotational support
- Fragmentation: inability to maintain steady gravito-turbulence because cooling and/or turbulent stresses (viscosity)

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