Image Processing

Geometric Transform

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Objectives

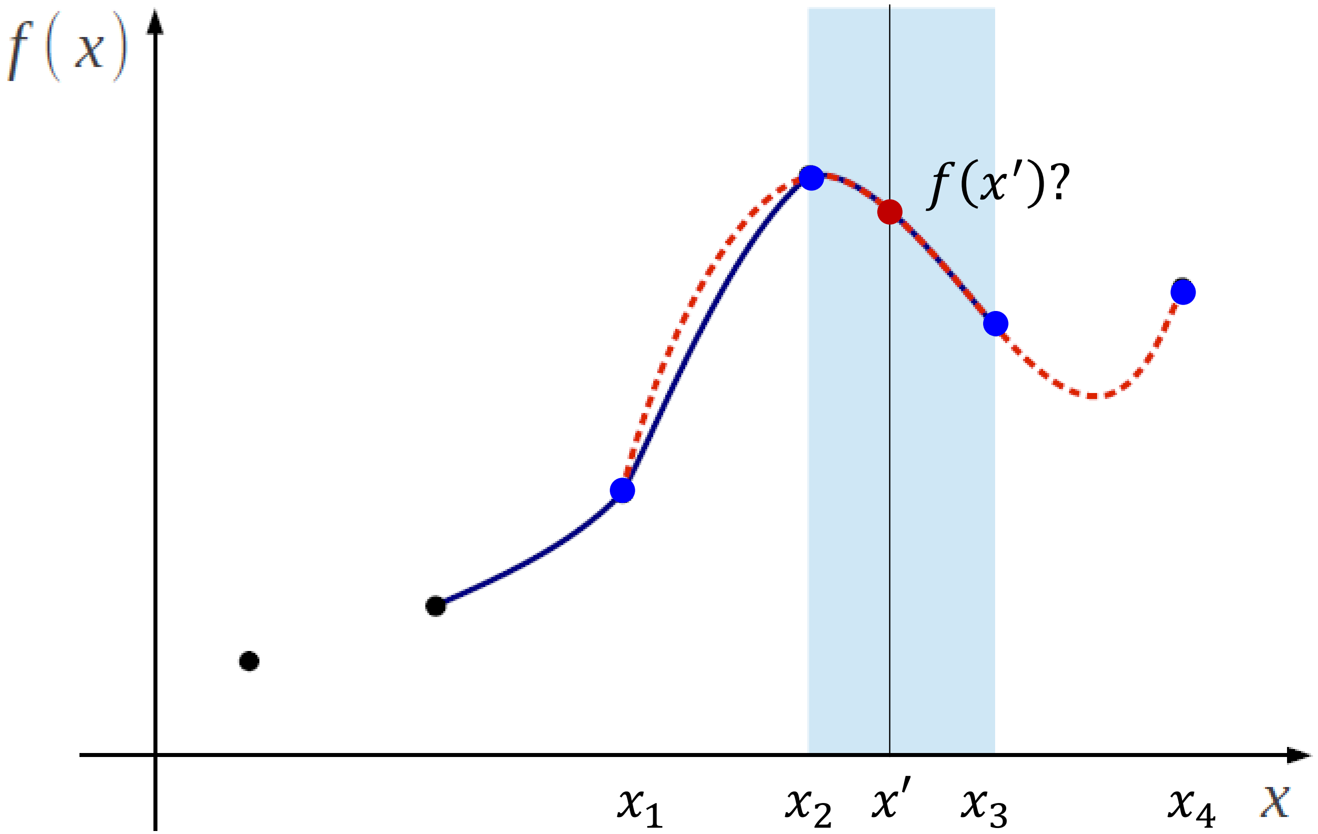
- Interpolation
  - 1D signal interpolation
  - 2D image interpolation

- Parametric transform
  - Rotation
  - Scaling
  - Translation
  - Affine

Cubic Interpolation

- Cubic interpolation

$$f(x') = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$$



Cubic Interpolation

$$R_3(u) = \begin{cases} 1.5|u|^3 - 2.5|u|^2 + 1 & if\ |u| \le 1 \\ -0.5|u|^3 + 2.5|u|^2 - 4|u| + 2 & if\ 1 < |u| \le 2 \end{cases}$$

$$f(x') = R_3(-1-\lambda)f(x_1) + R_3(-\lambda)f(x_2) + R_3(1-\lambda)f(x_3) + R_4(2-\lambda)f(x_4),$$

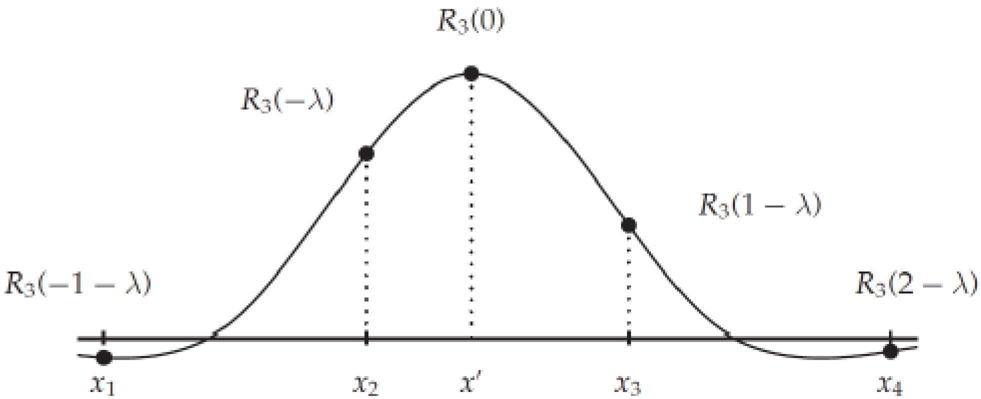
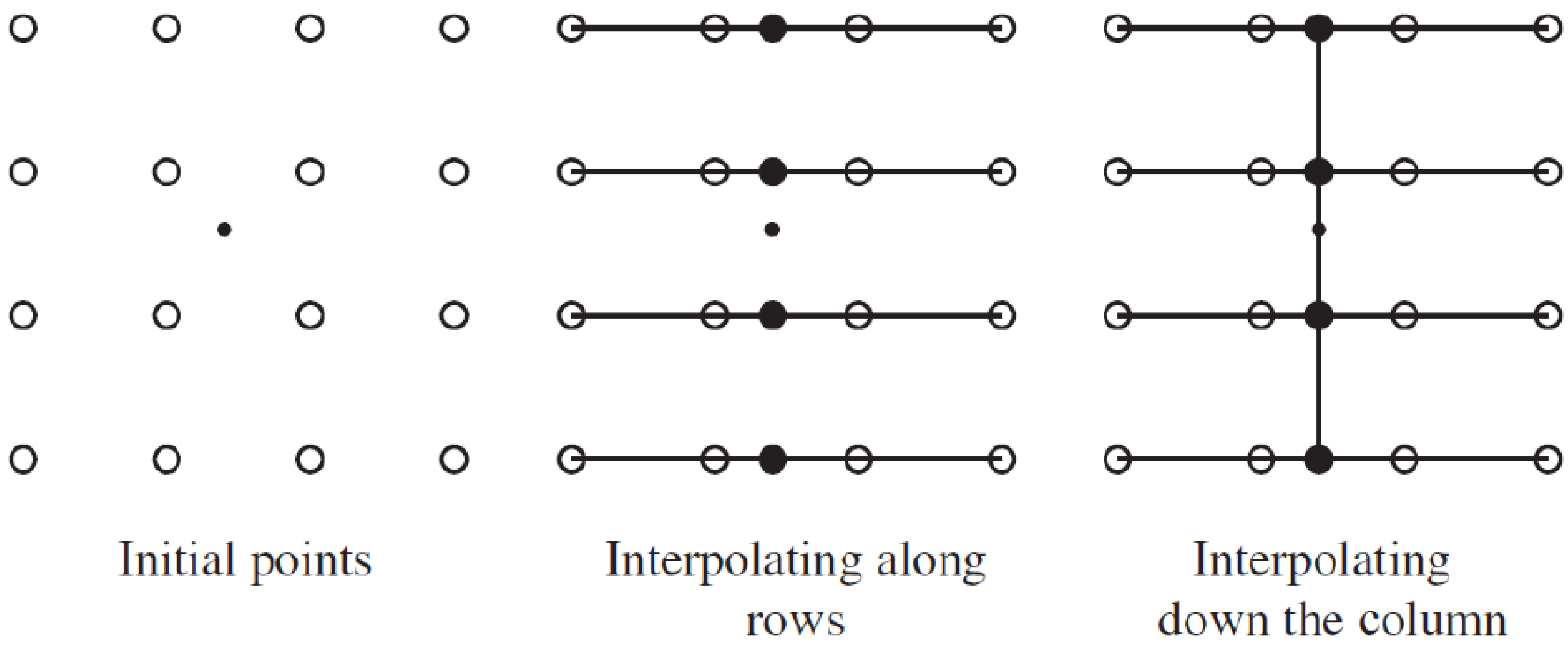


FIGURE 6.14 *Using $R_3(u)$ for interpolation.*

$$x' - x_2 = \lambda$$
$$x_3 - x' = 1 - \lambda$$

Bicubic Interpolation

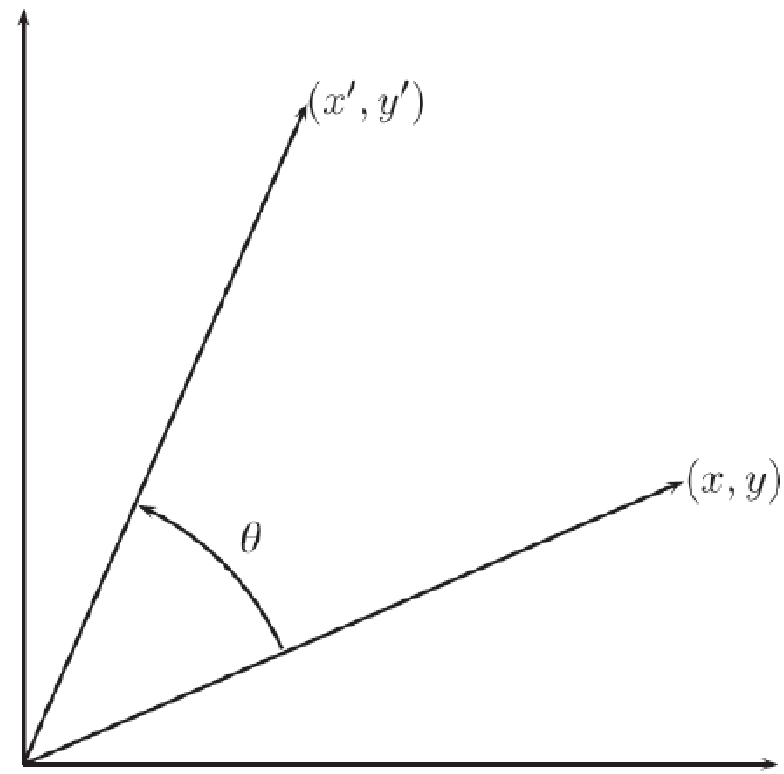


Rotation

- In a continuous signal, the rotation can be done using the following equation

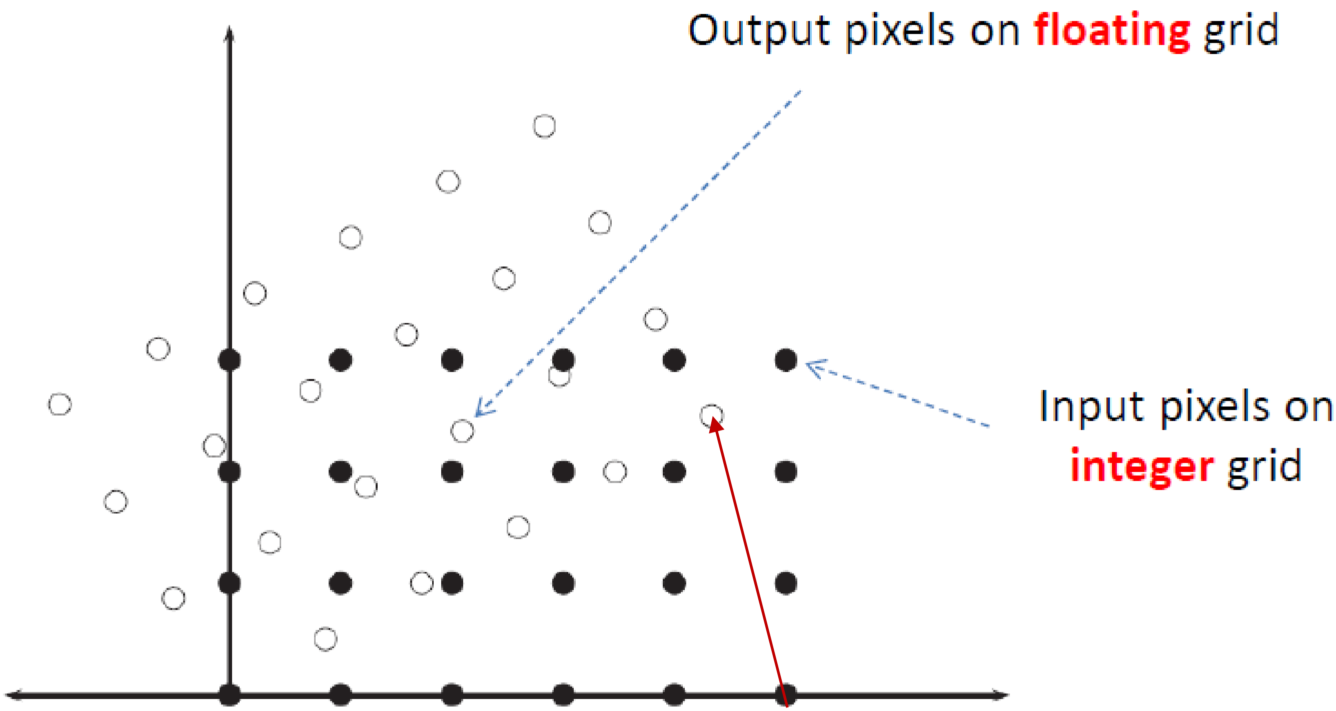
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$



Rotation

- Discrete signal?
  - Rotation does not always produce an integer pixel location
  - So, the interpolation (or re-sampling) is needed

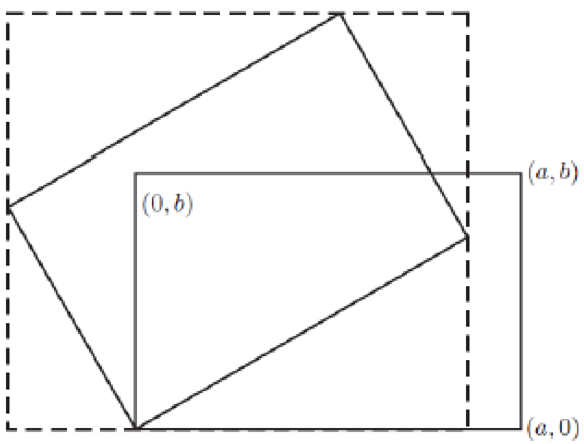


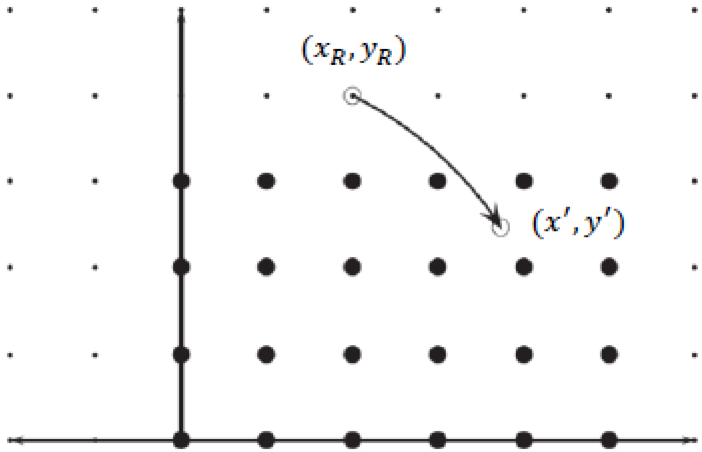
Rotation

- Define an enlarged, rotated image I_R whose size is larger than that of an original image
- Assume $(x_R, y_R) \in I_R$. Then, compute the transform $(x_R, y_R) \rightarrow (x', y')$ Note that (x', y') is a floating pixel

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_R \\ y_R \end{bmatrix}$$

- Compute $f(x', y')$ using the interpolation technique
- Then $f(x_R, y_R) = f(x', y')$





Rotation in MATLAB

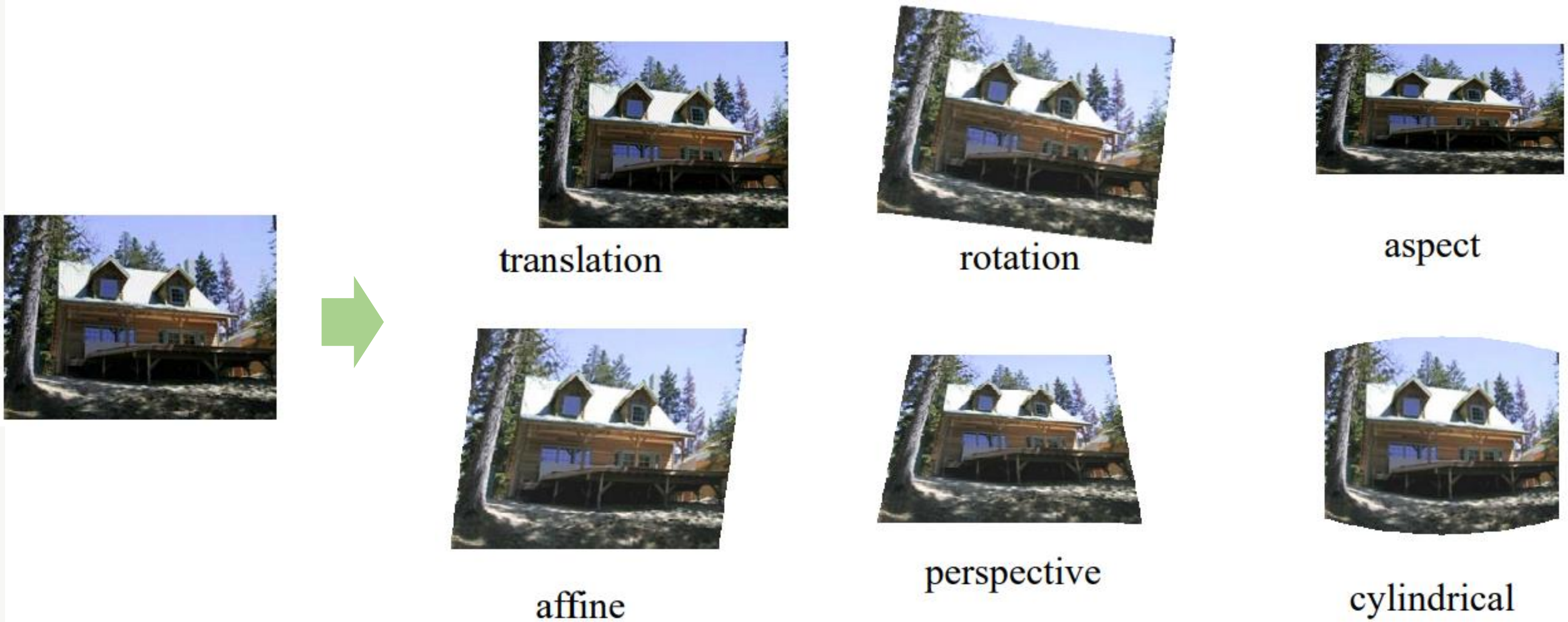
```
>> cr=imrotate(c,60);
>> imshow(cr)
>> crc=imrotate(c,60,'bicubic');
>> imshow(crc)
```

(a)

(b)

Image Warping

- Parametric (global) warping



Parametric Warping

- 2×2 Matrices
  - What types of transformation can be represented with a 2×2 matrix?

Scale matrix: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{Sx} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax \\ by \end{bmatrix}$$

Rotation matrix: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{Rx} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cos\theta x - sin\theta y \\ cos\theta x + sin\theta y \end{bmatrix}$$

Shear matrix: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{Hx} = \begin{bmatrix} 1 & h_x \\ h_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + h_x y \\ x h_y + y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ax + by \\ dx + ey \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{Mx}$$

Parametric Warping

- 2×2 Matrices
  - Scaling + Shear + Rotate

$$\mathbf{x}' = \mathbf{R}\left(\mathbf{H}\left((\mathbf{S}\mathbf{x})\right)\right) = \mathbf{M}\mathbf{x}$$

If we have to transform many points, it is easier to do one matrix multiplication