## Optimum Global Thresholding

For each region, the following property holds.

$$q_{1}(k) = \sum_{i=0}^{k} p(i)$$

$$q_{2}(k) = \sum_{i=k+1}^{L-1} p(i)$$

$$m_{1}(k) = \frac{\sum_{i=0}^{k} ip(i)}{\sum_{k=0}^{k} p(i)} = \frac{1}{q_{1}(k)} \sum_{i=0}^{k} ip(i)$$

$$m_{2}(k) = \frac{\sum_{i=k+1}^{L-1} ip(i)}{\sum_{i=k+1}^{L-1} p(i)} = \frac{1}{q_{2}(k)} \sum_{i=k+1}^{L-1} ip(i)$$

$$\sigma_{1}^{2}(k) = \frac{1}{q_{1}(k)} \sum_{i=0}^{k} [i - m_{1}(k)]^{2} p(i)$$

$$\sigma_{2}^{2}(k) = \frac{1}{q_{2}(k)} \sum_{i=k+1}^{L-1} [i - m_{2}(k)]^{2} p(i)$$

$$= \frac{1}{q_{1}(k)} \sum_{i=0}^{k} i^{2} p_{i} - m_{1}^{2}(k)$$

$$= \frac{1}{q_{2}(k)} \sum_{i=k+1}^{L-1} i^{2} p(i) - m_{2}^{2}(k)$$

For an entire image,

$$m_G = \sum_{i=0}^{L-1} i p(i),$$
  $\sigma_G^2 = \sum_{i=0}^{L-1} [i - m_G]^2 p(i)$   $q_1(k) = q_1(k)$ 

$$q_1(k) + q_2(k) = 1$$
  
 $q_1(k)m_1(k) + q_2(k)m_2(k) = m_G$