On NFT Profit Maximization with Social Influence

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ABSTRACT

Recently, the rise of non-fungible tokens (NFTs) has been one of the most significant developments for art and technology in the Metaverse. Different from necessities, scarcity and rarity have a huge impact on user valuations of NFTs. Moreover, the unprecedented NFT breeding mechanism creates new challenges for viral marketing, as the generated NFT offspring also yield huge profits. However, previous profit maximization studies on tangible products do not take scarcity, rarity, and the breeding mechanism into account. In this paper, we make the first attempt to formulate *NFT Profit Maximization (NPM)* by finding NFT airdrops and determining NFT quantities. We prove the hardness of NPM and design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*. QOOA leverages Quantity-Sensitive Profit to prune inferior airdrops and upper bounds the search for possible NFT quantities by deriving Valuation-based Quantity Inequality. To increase profit from NFT offspring, QOOA identifies Rare Trait Collectors and encourages them to purchase multiple NFTs with rare traits in order to generate valuable offspring. We compare the performance of QOOA with the state-of-the-art approaches with real NFT datasets. The experimental results demonstrate that QOOA effectively achieves up to 4 times the profit of state-of-the-art approaches in large-scale social networks.

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1 INTRODUCTION

Recently, Non-Fungible Tokens (NFTs) have emerged as a significant development at the intersection of art and technology within the Metaverse. NFTs can be traced back to 2017 when CryptoKitties, an innovative application utilizing blockchain technologies, introduced the concept of acquiring and trading virtual cats as distinctive digital assets. Since then, the market for NFTs has experienced an unprecedented surge, witnessing remarkable transactions commanding multimillion-dollar sums. Notably, the digital artist Beeple made headlines by auctioning an NFT of his artwork for a groundbreaking $69 million. Additional noteworthy NFT transactions include the acquisition of Twitter CEO Jack Dorsey's inaugural tweet for $2.9 million and a LeBron James highlight video for $208,000.

NFTs, as digital assets verified by blockchain technology to ensure their authenticity and ownership, exhibit distinctive marketing features compared to conventional viral marketing. These new features include: 1) *Auction-Based Sales*: Most NFT marketplaces sell NFTs to the highest bidder via auction mechanisms. This places significant emphasis on the *transaction prices* determined by the highest bidders based on their valuations, as well as the *reserve price* established by the seller as the lowest acceptable transaction price. The profitability is substantially determined by the impact of these prices, rather than the number of influenced users. For instance, Nakamigos #3648 received over 5200 views, but ultimately sold for $28,000 to the highest bidder due to its limited quantity.[1] 2) *Rarity and High Valuations*: Unlike everyday necessities, the rarity of NFTs contributes to their elevated value. As an example, CryptoPunk #2924, which belongs to a collection of 10000 unique punk apes created by Larva Labs in 2017, recently fetched an impressive $4.5 million due to its ranking as the 38th rarest out of the entire series.[2] 3) *NFT Breeding*: A distinguishing characteristic of NFTs is the concept of NFT breeding, which permits the combination of a pair of NFTs to produce entirely new and unique offspring. This feature enables the creation of NFTs that are more scarce and valuable than their parent NFTs. For instance, in December 2022, Nike introduced NFT sneakers, known as CryptoKicks, that contain genotype information, e.g., attributes, colors, styles, backgrounds, etc. CryptoKicks holders can breed their NFTs, i.e., creating offspring inheriting traits from their parents based on genotype information, and redeem them for physical sneakers.[3] Similar breeding mechanisms are employed by NFT collections such as Heterosis, STEPN, Roaring Leaders, CryptoKitties, and Axie Infinity.[4] For example, offspring's quality typically depends on their parents in STEPN, e.g., there is no chance for two "common" parents to produce "epic" offspring. Within the STEPN marketplace, offspring categorized as "common" are priced at a minimum of 126 GMT, whereas offspring classified as "epic" can fetch a minimum price of 7777 GMT.[5] Hence, selecting suitable parents to breed offspring is crucial for maximizing profits. Compared to conventional necessities and assets, the promotion of NFTs presents distinct research challenges in viral marketing.

In order to promote new NFT projects, NFT marketplaces such as OpenSea and Blur frequently employ an *airdrop* strategy in which influential users are provided with free NFTs to increase public awareness.[6] Figure 1 illustrates an example for the promotion of a new NFT project consisting of three NFTs denoted n_1, n_2 and n_3 within a social network, where nodes represent users, edges represent friendships between users associated with activation probabilities,

[1] Details on Nakamigos #3648 can be found at https://opensea.io/assets/ethereum/0xd774557b647330c91bf44cfeab205095f7e6c367/3648.

[2] More information about CryptoPunk #2924 can be found at https://opensea.io/assets/ethereum/0xb47e3cd837ddf8e4c57f05d70ab865de6e193bbb/2924.

[3] News can be found at https://dappradar.com/blog/nike-x-rtfkt-unveiled-cryptokicks-through-gamified-mechanics.

[4] Heterosis: https://og.art/collections/heterosis/; STEPN: https://stepn.com/; Roaring Leaders: https://roaringleaders.io/; CryptoKitties: https://www.cryptokitties.co/; Axie Infinity: https://axieinfinity.com/.

[5] The marketplace of STEPN: https://m.stepn.com/. The current price of STEPN is $0.26 per GMT on 2 June 2023.

[6] Airdrop information of OpenSea and Blur at https://partners.opensea.io/drops and https://blur.io/airdrop, respectively.

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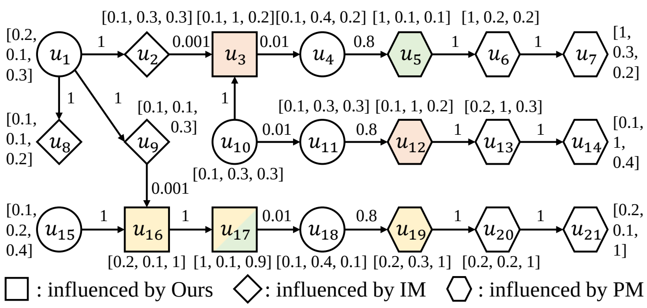
		NFT project			New offspring
NFT		n_1	n_2	n_3	
Traits		Star, rainbow	Star	Moon	Moon, rainbow
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



(b) A social network.

Figure 1: An illustrative example.

and the users' preferences for n_1, n_2, and n_3 are indicated in square brackets adjacent to the nodes, as shown in Figure 1(b). The traits and (potential) assessments under different quantities of n_1, n_2, and n_3, as well as the offspring, are presented in Figure 1(a). For example, in the NFT project, the traits "moon" and "rainbow" are rarer than the other trait "star" since they are possessed by only one NFT. Here a user's valuation of an NFT is calculated by multiplying her preference for the NFT with its assessment in this example. Assume that the airdrop quota for each NFT is one and the reserve price of each NFT is 2.6. To maximize the profit, airdropping n_1, n_2, and n_3 to u_{15}, u_{10}, and u_{15}, respectively, with the quantities set as 1, 1, and 2 is effective. It yields an expected profit of 28.72, where the influenced users are represented by square shapes and the NFTs they purchase are indicated by colors. Since u_{17} possesses both n_1 and n_3, she has the opportunity to generate an NFT offspring with a profit of 4.32.

By contrast, previous influence maximization (IM) approaches seek to identify users to maximize the number of influenced individuals and will airdrop all NFTs to u_1, due to its expected influence spread of 3, where the influenced users are represented by diamond shapes. However, the influenced users by u_1 do not have high valuations of NFTs. Hence, the expected profit is almost 0 when the quantity of each NFT is one. On the other hand, classical profit maximization approaches will designate u_4, u_{11}, and u_{18} as the airdrop recipients for n_1, n_2, and n_3, respectively, thereby maximizing the total expected profit of 60.57 without considering NFT quantities, where the influenced users are represented by hexagonal shapes. Nevertheless, when the quantity of each NFT is subject to one, the expected profit is reduced to 20.19, the NFTs they purchase are indicated by colors. In addition, for both cases, no user can breed NFT offspring to generate additional profits. On the other hand, when the quantity of each NFT increases, the valuations of influenced users decline significantly, with many falling below the reserve prices. The profit obtained is thus smaller compared to the case where the quantity is one. As a result, both approaches of maximizing influence and profit fall short in maximizing the profit of NFTs, when considering the features of NFTs, including breeding, scarcity, and trait rarity, and the need to determine NFT quantities.

The aforementioned scenario highlights several new challenges that arise in the pursuit of maximizing NFT profits: 1) *Maximizing transaction prices*: The profit generated from NFTs relies on the transaction prices, which correspond to the valuations of users willing to purchase the NFTs. A crucial aspect of NFT airdrops is targeting users with high valuations. Merely maximizing the spread of influence among users does not guarantee maximum profit since only a small fraction of influenced users actually make purchases. Previous works [1, 2, 5, 14, 26, 29, 36–38] aim to maximize the influence spread but neglect user valuations, thus failing to ensure the maximization of profit. 2) *Balancing scarcity and quantity*: The scarcity of an NFT significantly impacts users' valuations. Limiting an NFT to being unique can boost valuations, but it restricts profit generation to a single user. Conversely, supplying a large quantity of an NFT allows more users to purchase it but may reduce valuations and transaction prices. Previous works [3, 7–9, 11, 13, 15, 22, 33, 35] fail to account for the impact of scarcity and rarity on valuations and cannot determine appropriate quantities of NFTs. 3) *Leveraging NFT breeding*: The NFT breeding mechanism encourages users to own multiple NFTs simultaneously, as offspring can generate an additional profit. The traits of parent NFTs determine the assessments of offspring (e.g., CryptoKicks parents decorated with angel wings are likely to produce offspring decorated with angel wings). It is advantageous to facilitate the breeding of two NFTs with rare traits owned by the same user to produce rare offspring. However, excessive breeding may diminish scarcity, rarity, and the assessments of NFT offspring. Thus, it is crucial to encourage the adoption of suitable NFTs as parents to generate offspring that maximize profit. Previous works [1, 2, 5, 14, 26, 29, 36–38] consider multiple items without additional profits and cannot find appropriate airdrops to encourage simultaneous ownership.

In this paper, we formulate a new problem, named *NFT Profit Maximization (NPM)*. Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T and their reserve prices $P = \{p_1, \ldots, p_{|N|}\}$, a social network, a set of budgets $B = \{b_1, \ldots, b_{|N|}\}$, and quantity limits $L = \{l_1, \ldots, l_{|N|}\}$, NPM aims to determine the set of NFT airdrops and the quantity of each NFT to maximize the profit earned from the NFTs and their offspring. The number of airdrops for each NFT n_k is subject to the budget constraint $b_k \in B$, and the quantity of each NFT n_k is limited to l_k. We first prove that NPM is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$. To solve NPM, we design an approximation algorithm, named *Quantity and Offspring-Oriented Airdrops (QOOA)*. 1) To maximize transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to estimate the potential profit earned from the influence of a set of users, considering the quantity constraint. It identifies prospective purchasers with valuations higher than the reserve price and evaluates the likelihood of the set of users influencing the prospective purchasers with the highest valuations. 2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find an upper bound on the profit generated under a specific quantity constraint. VQI captures the relationships between the reserve price and user valuations for different quantities and infers the maximum quantity that users are willing to purchase. 3) To increase the profit from NFT offspring, QOOA identifies the *Rare Trait Collector (RTC)*, which evaluates the rarity of NFTs purchased by users. By tailoring the airdrops, RTCs are encouraged to acquire additional NFTs with rare traits, increasing the chances of generating rare offspring and maximizing profits. We evaluate the performance of QOOA on real NFT projects, e.g., Defimons Characters, Lascaux, and Timpers Pixelworks. The contributions of this work include:

- To the best of our knowledge, NPM is the first attempt to study the profit maximization problem for NFTs, taking into consideration NFT breeding, NFT scarcity, and trait rarity. We prove the hardness and inapproximability of NPM.
- We design an approximation algorithm QOOA with the notions of QSP, VQI, and RTCs.
- Via real NFT projects, experiments demonstrate that QOOA achieves up to 4 times the profits over the state-of-the-art approaches.

2 RELATED WORK

Profit/revenue maximization. Previous works on profit/revenue maximization in social networks have primarily focused on maximizing the difference between the influence spread and the cost of the seed group [9, 13, 15]. Li et al. [22] introduce the concept of user benefits and aim to maximize the total benefits rather than just the influence spread. Several studies [3, 33, 35] further incorporate the concept of benefits into profit maximization, i.e., maximizing the difference between the benefits of influence and the cost of seeding. Unlike the above works maximizing the profit from nodes, the works [7, 8] examine the benefits related to interactions among activated nodes. Han et al. [11] consider the perspective of the host and maximize the revenue of all advertisers, incorporating the cost-per-engagement amount and the influence spread. However, these works do not address the issue of profit/revenue/benefit that varies based on the quantity of the product. They cannot determine the optimal quantity to maximize profit/revenue/benefit. Additionally, they do not account for the potential additional profits that can be obtained by simultaneously adopting multiple products, thus making them unsuitable to handle the NFT breeding mechanism embedded in the NPM.

Viral marketing on multiple products. Existing research on viral marketing has explored the problem of maximizing the influence or profit/revenue of multiple products within a company. Some studies [5, 38] assume that each product is independent and focus on optimizing the adoption of each product individually. On the other hand, other studies [1, 2, 14, 26, 29, 36, 37] consider the interdependencies between product adoptions, such as complementary or substitutable relationships. However, these works primarily focus on modeling the positive or negative impact of adopting one product on another, without explicitly considering the effect of co-purchasing multiple products on influence or profit/revenue. They do not capture the breeding mechanism inherent in NFT projects, where owning multiple NFTs can lead to additional profits through NFT offspring generation. Additionally, these studies do not consider the valuation of products based on their scarcity and rarity, nor do they address the determination of optimal quantities for the products. As a result, previous viral marketing on multiple products cannot be directly applied to NPM.

Table 1: Notation table.

Notation	Description
N; n_k	NFT project; an NFT
T; t_h; T_k	Universe set of traits; a trait; trait set of n_k
P; p_k	Set of reserve prices of N; reserve price of n_k;
Q; q_k	Set of NFT quantities; quantity of n_k
G; u_i	Social network; a user
$a_{i,j}$	Activation probability of u_i to u_j
$v_{u_i,n_k}(q_k)$;	u_i's valuation of n_k when the quantity of n_k is q_k;
w_{u_i,n_k}	u_i's preference for n_k
β_i	Breeding probability of u_i forming NFT offspring
S; (u_i, n_k); S_k	Set of NFT airdrops; an NFT airdrop; set of NFT airdrops for n_k
$o_{h,m}$	NFT offspring generated from n_h and n_m
$A(n_k, q_k)$; $A(o_{h,m}, S, Q)$	Assessment of n_k under q_k; assessment of offspring $o_{h,m}$ given S and Q
$f(S, Q)$	Profit of S and Q
$TP(S_k, q_k)$; $OS(S, Q)$	Total transaction prices of n_k generated by S_k subject to q_k; Total assessments of NFT offspring under the influence of S subject to Q
B; b_k	Set of budgets; budget for n_k
L; l_k	Set of quantity limits; quantity limit for n_k

3 PROBLEM FORMULATION

In this section, we first describe the diffusion process of NFTs by incorporating NFT characteristics into existing diffusion models. Next, we define the profit function and the profit maximization problem of NFTs. Finally, we analyze the hardness of the proposed problem. Table 1 summarizes the notations in this paper.

3.1 Definition

In this work, we aim to maximize the profit from an NFT project $N = \{n_1, \ldots, n_k, \ldots, n_{|N|}\}$ with traits $T = \{t_1, \ldots, t_h, \ldots, t_{|T|}\}$ and their reserve prices $P = \{p_1, \ldots, p_k, \ldots, p_{|N|}\}$, where each NFT n_k is associated with a reserve price p_k and a set of traits $T_k \subseteq T$, e.g., with a happy face, with a bicycle helmet, and so on. In general, the assessment of an NFT can be estimated according to scarcity and rarity. According to [27], scarcity plays a crucial role in assessing non-fungible objects, such that the assessment of an NFT is likely to boost if it is scarce. In addition to scarcity, the rarity of the traits of an NFT is also crucial. Following [12, 28], the rarity of a trait t_h is inversely proportional to its occurrence in N, i.e.,

$$\gamma(t_h) = \frac{|N|}{Occ(t_h, N)}, \tag{1}$$

where $Occ(t_h, N)$ is the number of NFTs in N with t_h. Following [12, 28], the overall rarity of an NFT n_k is the sum of the rarity of its traits, i.e.,

$$\Gamma(n_k) = \sum_{t_h \in T_k} \gamma(t_h). \tag{2}$$

Let $Q = \{q_1, \dots, q_{|N|}\}$ denote the quantity set of the NFT project N, where q_k is the quantity of $n_k \in N$. According to [12, 32], the assessment of an NFT can be estimated as[7]

$$A(n_k, q_k) = e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Gamma(n_k)}, \tag{3}$$

where η_0, η_1, and η_2 are weight parameters. In particular, n_k is assessed at $e^{\eta_1 \frac{1}{q_k}}$ and $e^{\eta_2 \Gamma(n_k)}$ according to scarcity (i.e., the NFT quantity) and rarity, respectively, while e^{η_0} represents the assessment related to the NFT project, e.g., the fame of the creator.

Consider a social network $G = (V, E)$, where V is the node set representing users, and E is the edge set standing for friendships. Each user $u_i \in V$ has a preference for an NFT n_k, denoted as $w_{u_i,n_k} \in [0, 1]$, which can be derived from learning models, such as HG-GNN [31] and DGNN [23], according to the purchase history of the user and the traits of the NFT. Following [17, 18], a user u_i's valuation on an NFT n_k is derived according to u_i's preference for n_k (i.e., w_{u_i,n_k}) and the assessment of n_k (i.e., Equation (3)).

$$v_{u_i,n_k}(q_k) = w_{u_i,n_k} \cdot A(n_k, q_k). \tag{4}$$

Each edge $e_{i,j} \in E$ indicates that $u_i \in V$ has an activation probability of $a_{i,j}$ to influence $u_j \in V$.

Following most NFT marketplaces, such as OpenSea, the diffusion process of NFTs consists of two stages: the airdrop stage and the public stage.[8] Let $S = \{(u_i, n_k), \dots\}$ and $Q = \{q_1, \dots, q_{|N|}\}$ denote a set of NFT airdrops and the quantity set of the NFT project N, respectively, where (u_i, n_k) represents an NFT airdrop that provides a free NFT $n_k \in N$ for a user $u_i \in V$, and q_k is the quantity of $n_k \in N$. The airdrop stage aims to propagate the influence of S, while the public stage determines who wins the NFTs according to the influence of S, user valuations, NFT quantities Q, and reserve prices P. The details of each stage are described as follows.

Airdrop stage. The influence propagation of NFTs typically follows existing diffusion models, such as the PTC, LT, and IC models [4, 16]. Initially, all users are inactive in all NFTs, except for the users in the set of NFT airdrops S who are active in their respective NFT. Following existing diffusion models, a user who is active in an NFT n_k will influence her inactive friend u_j to become active in n_k. Once u_j is successfully influenced by her active friend, she too becomes active in n_k.[9] The influence thus propagates until no users can be newly influenced.

Plubic stage. After the NFT airdrops influence the social network, the marketer allows active users to bid on NFTs and determines the winners according to user valuations, quantities, and reserve prices of NFTs. Following most NFT marketplaces, such as OurSong and OpenSea, each n_k is sold to the highest bidders. Specifically, the users who are active in n_k and have the q_k highest valuations win the NFT n_k, where the valuations are no smaller than the reserve price p_k. Following most NFT marketplaces, e.g., OpenSea, for a user u_i winning an NFT n_k, the *transaction price* is equal to her offer according to her valuation $v_{u_i,n_k}(q_k)$, where $v_{u_i,n_k}(q_k) \geq p_k$. Note that n_k may not be sold out if there are fewer than q_k users with valuations at least p_k.

The unique breeding enables NFT marketplaces to earn profit not only from the current NFTs N but also from NFT offspring. Following CryptoKicks, Heterosis, STEPN, Roaring Leader, Axie Infinity, and CryptoKitties, when a user u_i owns two different NFTs, she will form an NFT offspring with a breeding probability of β_i, which can be derived from users' engagement of breeding activities.[10] The NFT offspring is determined by its parent NFTs' traits, which further dominate the assessment of this NFT offspring.[11] Specifically, for parent NFTs n_k and n_m, let $o_{k,m}$ and $T_{k,m}$ denote the generated NFT offspring and its trait set, respectively. Since the diffusion of S and Q will reach different users and then affect the breeding of NFT offspring, following [12, 28, 32], the assessment of $o_{k,m}$ is derived according to Equation (3) as follows.

$$A(o_{k,m}, S, Q) = e^{\eta_0 + \eta_1 \frac{1}{q_{k,m}(S,Q)} + \eta_2 \sum_{t_h \in T_{k,m}} \frac{|N \cup O(S,Q)|}{Occ\left(t_h, N \cup O(S,Q)\right)}}, \tag{5}$$

where $q_{k,m}(S, Q)$ is the quantity of $o_{k,m}$ given S and Q, and $O(S, Q)$ is the set of NFT offspring given S and Q. Similar to Equation (3), $o_{k,m}$ is assessed at $e^{\eta_1 \frac{1}{q_{k,m}(S,Q)}}$ and $e^{\eta_2 \sum_{t_h \in T_{k,m}} \frac{|N \cup O(S,Q)|}{Occ\left(t_h, N \cup O(S,Q)\right)}}$ according to scarcity and rarity, respectively, while e^{η_0} indicates the assessment related to the NFT project. As NFT marketplaces earn profit from the service fees based on the transaction price, e.g., 2.5% on OpenSea, it is important to maximize the profit by maximizing the transaction prices of current NFTs and the assessments of NFT offspring (which determine the subsequent transaction prices). Therefore, the profit function is defined as follows.

Definition 3.1 (Profit Function). Given an NFT project N with traits T and their reserve prices P and a social network G, the profit of S for the NFT project N with quantities Q consists of the transaction prices of NFTs in N and the assessments of NFT offspring generated from N as follows.

$$f(S, Q) = \sum_{k=1}^{|N|} TP(S_k, q_k) + OS(S, Q), \tag{6}$$

where

$$TP(S_k, q_k) = \sum_{u_i \in V(S_k, q_k)} v_{u_i,n_k}(q_k) \tag{7}$$

is the total transaction price of n_k, and

$$OS(S, Q) = \sum_{u_i \in \mathcal{V}(S,Q)} \sum_{n_h, n_m \in W(u_i,S,Q), h<m} \beta_i \cdot \mathbb{E}[A(o_{h,m}, S, Q)] \tag{8}$$

is the assessments of NFT offspring generated under the influence of S with quantities Q. In Equation (7), $S_k = \{(u, n_k) : (u, n_k) \in S\} \subseteq S$

[7] $A(n_k, q_k)$ is an objective assessment that does not include user preferences.

[8] https://support.opensea.io/hc/en-us/articles/13592013208851-Part-2-Prepare-your-drop-schedule.

[9] For the promotional relationship between different NFTs, i.e., a user influenced by an NFT n_k is more likely to be influenced by another NFT n_m ($m \neq k$), diffusion models for multiple correlated items [1, 14, 26, 36] can be adopted for the proposed problem.

[10] For most NFT breeding mechanisms, any two NFTs can generate offspring. However, to maintain economic equilibrium, there may be additional limitations in place. For example, CryptoKitties has a breeding cooldown period, and STEPN stipulates a maximum of 7 shoe-mint-events per NFT sneaker.

[11] Basically, offspring have a high probability of inheriting traits from their parents. Nevertheless, different NFT breeding mechanisms have diverse ways of determining the traits of NFT offspring. For example, Cryptokitties consider one primary gene for appearance and three hidden genes for passing on to offspring, and further allow genes to mutate (see https://guide.cryptokitties.co/guide/cat-features/genes for more details).

consists of NFT airdrops in S that provide a free NFT n_k for some user u, and $V(S_k, q_k)$ is the set of users winning the NFT n_k under the influence of S_k with the quantity q_k. In Equation (8), under the influence of S with quantities Q, $\mathcal{V}(S,Q)$ is the set of users winning at least two NFTs, $W(u_i, S, Q)$ is the set of NFTs won by u_i, and $\mathbb{E}[A(o_{h,m}, S, Q)]$ is the expected assessment of the offspring NFT $o_{h,m}$ generated from parents n_h and n_m.

Consequently, we formally formulate *NFT Profit Maximization (NPM)* as follows.

Definition 3.2 (NFT Profit Maximization (NPM)). Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T and their reserve prices P, a social network G, a set of budgets $B = \{b_1, \ldots, b_{|N|}\}$ for N, and a set of quantity limits $L = \{l_1, \ldots, l_{|N|}\}$, NPM aims to find a set of NFT airdrops S and a set of NFT quantities Q for N, such that the profit $f(S,Q)$ is maximized, under the budget constraint $\forall k, |S_k| \leq b_k$ and the quantity constraint $\forall k, q_k \leq l_k$.

3.2 Hardness Analysis

Theorem 3.1. *NPM is NP-hard and cannot be approximated within a factor of* $1 - \frac{1}{e} + \varepsilon$ *for any* ε *unless* $P = NP$.

Proof. We prove this theorem with a reduction from the Influence maximization problem for the independent cascade model (INFMAX-IC), which is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$ [16]. Given a social network G, and two integers k, h with $k \leq h$, the decision version of INFMAX-IC is to decide whether there exists a seed set S of size k such that $\sigma(S) = h$, where $\sigma(S)$ is the expected number of active nodes at the end of process, given that S is the initial active set. Given an instance (G, k, h) of INFMAX-IC, we construct an instance of NPM as follows. i) The social network of NPM instance is a copy of G. ii) The NFT project $N = \{n_1\}$, reserve price $p_1 = 1$, seed budget $b_1 = k$ and valuation $v_{u,n_1}(q_1) = 1$ for each user $u \in V$ and any quantity q_1 of NFT n_1.

To complete the proof, we show that there exists a seed set S of size k such that $\sigma(S) = h$ in INFMAX-IC if and only if there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S,Q) = h - k$ in NPM. We first prove the necessary condition. Suppose that there exists a seed set S of size k such that $\sigma(S) = h$ in INFMAX-IC. Then the corresponding user set S in the constructed NPM instance together with the set of NFT quantity $Q = \{q_1 = \infty\}$ is a solution such that $f(S,Q) = h - k$. We then prove the sufficient condition. Suppose that there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S,Q) = h - k$ in NPM. Then the corresponding set S in INFMAX-IC is a solution such that $\sigma(S) = h$. Therefore, NPM is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$. The theorem follows. □

4 APPROXIMATION ALGORITHM

4.1 Algorithm Overview

To efficiently solve NPM, we design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, including the following novel ideas. 1) To achieve high transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to evaluate the possible profit of a set of users if they are selected for airdrops. Let *prospective purchasers* represent the users with valuations no smaller than the reserve price for a specific quantity. QOOA first finds the likelihood of each user in the social network influencing the prospective purchasers. For an NFT quantity q, QSP carefully evaluates the total profit of a set of users by deriving the likelihood of them influencing the top-q prospective purchasers and the valuations of the top-q prospective purchasers. Equipped with QSP, QOOA is able to efficiently filter out inappropriate sets of users for airdrops.

2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find the upper bound of the profit subject to the quantity for an NFT. Specifically, as the highest bidders win the NFT, the influenced users with the highest valuations are vital since they affect the transaction prices as well as the profit. According to Equations (3), (4), and (5), when a larger quantity is available, user valuations tend to decrease as the NFT becomes less scarce, while the profit may rise due to additional transaction prices generated by additional buyers. However, if user valuations fall behind the reserve price, no additional transaction price will be generated. Hence, VQI captures the relationship between the reverse price and user valuations under varying quantities. QOOA is thus able to find the upper bound of the profit subject to each specific quantity to facilitate the identification of the proper quantity.

3) To increase the profit earned from NFT offspring, QOOA aims to encourage users to purchase multiple NFTs with rare traits. QOOA identifies *Rare Trait Collectors (RTCs)* as the users who have purchased at least one NFT with the rarest traits. Then, for each NFT with traits of the greatest rarity, QOOA attempts to find alternative airdrops that encourage RTCs, rather than other users, to purchase it. This is because joint purchases of NFTs with rare traits can generate additional valuable NFT offspring.

In summary, QOOA consists of two steps: Quantity-driven Airdrop Selection (QAS) and Offspring Profit Enhancement (OPE). In QAS, for each NFT, QOOA evaluates QSP of users to find proper airdrops to maximize the total transaction price. It iteratively evaluates the profits with increasing quantities until no more profit can be generated by providing an additional quantity, based on the upper bound of the profit derived by VQI. After finding the best NFT airdrops and quantities identified in QAS, QOOA leverages OPE to improve profit from offspring by encouraging RTCs to jointly purchase multiple NFTs with rare traits. The pseudo-code of QOOA is presented in Algorithm 1.

4.2 Algorithm Description

Quantity-driven Airdrop Selection (QAS). QAS aims to maximize the total transaction price by finding appropriate airdrops and NFT quantities. Let $S_k^{q_k}$ denote the set of airdrops for NFT n_k identified by QAS under q_k. Specifically, for each NFT n_k, QAS starts from $q_k = 1$ and finds S_k^1 that maximizes the total transaction price $TP(S_k^1, 1)$. At each iteration, before increasing q_k from x to $x+1$, QAS derives the upper bound of the total transaction price for $q_k = x+1$, and examines whether the upper bound is greater than the total transaction price generated by $S_k^{q_k}$ under $q_k = 1, \ldots, x$, to facilitate efficient pruning. Finally, QAS identifies the best quantities and the corresponding airdrops that maximize the total transaction price.

Algorithm 1: QOOA

Input: NFT project N with traits T and reserve prices P, social network G, budgets B, quantity limits L
Output: NFT airdrops S and quantities Q
/* QAS phase */
for *each* $n_k \in N$ **do**
  $q_k^* \leftarrow null$; $S_k^* \leftarrow null$; $TP_k^* \leftarrow 0$
  for $q_k = 1, \ldots, l_k$ **do**
    if $UB(n_k, q_k) \leq TP_k^*$ **then**
      continue
    $S_k \leftarrow \emptyset$; $TP_k \leftarrow \sum_{u_j \in V(S_k, q_k, p_k)} v_{u_j,n_k}(q_k)$
    while $|S_k| < b_k$ **do**
      $U \leftarrow \emptyset$
      for *each* $(u_i, n_k) \notin S_k$ **do**
        if $QSP(S_k \cup \{(u_i, n_k)\}, n_k, q_k) \geq TP_k$ **then**
          $U \leftarrow U \cup \{u_i\}$
      $(u_i^*, n_k) \leftarrow \operatorname{argmax}_{u_i \in U} Gain\big((u_i, n_k), S_k\big)$
      if $Gain\big((u_i^*, n_k), S_k\big) > 0$ **then**
        $S_k \leftarrow S_k \cup \{(u_i^*, n_k)\}$
        $TP_k \leftarrow TP_k + Gain\big((u_i^*, n_k), S_k\big)$
      else
        break
    if $TP_k > TP_k^*$ **then**
      $q_k^* \leftarrow q_k$; $S_k^* \leftarrow S_k$; $TP_k^* \leftarrow TP_k$
$S \leftarrow \bigcup_{n_k \in N} S_k^*$; $Q \leftarrow \{q_1^*, \ldots, q_{|N|}^*\}$
/* OPE phase */
$N^{\mathrm{T}} \leftarrow$treasures of NFTs
for *each* $n_k \in N^{\mathrm{T}}$ **do**
  $S_k' \leftarrow S_k^*$; $\bar{S}_k' \leftarrow \emptyset$
  while $S_k' \neq \emptyset$ **do**
    $(u_i^*, n_k) \leftarrow \operatorname{argmin}_{(u_i,n_k) \in S_k'} MTI(u_i, n_k, q_k^*)$
    $(u_j^*, n_k) \leftarrow \operatorname{argmax}_{(u_j,n_k) \notin S_k' \cup \bar{S}_k'} MTI(u_j, n_k, q_k^*)$
    if $MTI(u_i^*, n_k, q_k^*) < MTI(u_j^*, n_k, q_k^*)$ **then**
      $S_k' \leftarrow S_k' \setminus \{(u_i^*, n_k)\}$; $\bar{S}_k' \leftarrow \bar{S}_k' \cup \{(u_j^*, n_k)\}$
      $S' \leftarrow S \setminus \{(u_i^*, n_k)\} \cup \{(u_j^*, n_k)\}$
      if $f(S', Q) > f(S, Q)$ **then**
        $S \leftarrow S'$
    else
      break
return S, Q

In order to efficiently extract appropriate users for airdrops, QAS first identifies the *prospective purchasers*, who have valuations no smaller than the reserve price, since only influencing prospective purchasers can lead to profit. Specifically, for NFT n_k under a specific quantity q_k with the reserve price p_k, the set of prospective purchasers is $V_{\mathrm{pp}}(n_k, q_k) = \{u : v_{u,n_k}(q_k) \geq p_k\}$. For each prospective purchaser, QAS searches for its reverse reachable sets in different deterministic realized graphs of G. Then, QAS derives the likelihood of any user $u_i \in V$ influencing a prospective purchaser $u_j \in V_{\mathrm{pp}}(n_k, q_k)$ according to the frequency of u_i occurring in u_j's reverse reachable sets, denoted as $feq(u_i, u_j)$. Accordingly, for a user u_i, QAS obtains its Influence on Prospective Purchasers (IPP) as a set, denoted as $IPP(u_i, n_k, q_k)$, as follows.

$$IPP(u_i, n_k, q_k) = \big\{\big(u, feq(u_i, u)\big) : u \in V_{\mathrm{pp}}(n_k, q_k), feq(u_i, u) > 0\big\}. \tag{9}$$

Equipped with IPP, QAS evaluates the *Quantity-Sensitive Profit (QSP)* of a set of users subject to a specific quantity. For NFT n_k with $q_k = x$, QSP of a set of users $U = \{u_1, \ldots, u_{|U|}\}$ is the sum of weighted transaction prices of the prospective purchasers with the top-x valuations influenced by U. Let $Top(U, n_k, x)$ denote the set of the prospective purchasers with the top-x valuations in the union of all IPP of $u_i \in U$. QSP of U under $q_k = x$ is

$$QSP(U, n_k, x) = \sum_{u_j \in Top(U, n_k, x)} \Big(v_{u_j,n_k}(x) \sum_{u_i \in U} feq(u_i, u_j)\Big), \tag{10}$$

where the sum of likelihood of each $u_i \in U$ influencing the prospective purchaser u_j, i.e., $\sum_{u_i \in U} feq(u_i, u_j)$, stands for the weight on u_j's transaction price $v_{u_j,n_k}(x)$, because the chance that u_j pays $v_{u_j,n_k}(x)$ for NFT n_k is positively correlated with the likelihood of u_j being influenced.

Afterward, to maximize the total transaction price for NFT n_k with $q_k = x$, QAS iteratively selects the best airdrop and adds it to the current set S_k^x of airdrops. To efficiently prune inferior airdrops, when considering (u_i, n_k), QAS first evaluates QSP of $S_k^x \cup \{(u_i, n_k)\}$ and compares it with the total transaction price obtained by S_k^x. If $QSP\big(S_k^x \cup \{(u_i, n_k)\}, n_k, x\big)$ is smaller than $TP(S_k^x, x)$, i.e., adding (u_i, n_k) to S_k^x does not yield a greater total transaction price, it is unnecessary to consider (u_i, n_k), since QSP is the upper bound of the total transaction price. Otherwise, QAS evaluates the marginal gain of a total transaction price of adding (u_i, n_k) to S_k^x, i.e.,

$$Gain\big((u_i, n_k), S_k^x\big) = TP(S_k^x \cup \{(u_i, n_k)\}, x) - TP(S_k^x, x). \tag{11}$$

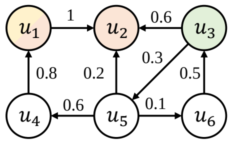
Then, when $\big|S_k^x\big| < b_k$, i.e., the airdrop budget is sufficient, QAS adds (u_i, n_k) with the largest marginal gain to S_k^x if $Gain\big((u_i, n_k), S_k^x\big)$ is positive, because $TP(S_k^x, x)$ is not monotonically increasing (proved in Lemma 4.1 later), and adding (u_i, n_k) with a negative marginal gain to S_k^x will decrease the total transaction price.

After $q_k = x$ is examined, QAS continues to find $S_k^{q_k}$ for $q_k = x+1$. Before the search for $q_k = x + 1$, QAS derives *Valuation-based Quantity Inequality (VQI)* by capturing the relationship between the reserve price p_k and user valuations to find the upper bound of the total transaction price for $q_k = x + 1$. Let $W(n_k, y)$ denote the y-th largest user preference for n_k. The corresponding user valuation is eligible for a transaction price only if it is no smaller than the reserve price p_k, i.e.,

$$\begin{aligned}
& W(n_k, y) \cdot A(n_k, q_k) \\
= \;& W(n_k, y) \cdot e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Gamma(n_k)} \\
= \;& W(n_k, y) \cdot e^{\eta_0 + \eta_2 \Gamma(n_k)} \cdot e^{\frac{\eta_1}{q_k}} \\
= \;& W(n_k, y) \cdot \eta_3 \cdot e^{\frac{\eta_1}{q_k}} \qquad (\text{Let } \eta_3 = e^{\eta_0 + \eta_2 \Gamma(n_k)}) \\
\geq \;& p_k.
\end{aligned}$$

Accordingly, VQI infers the maximum quantity that ensures the user valuation $W(n_k, y) \cdot A(n_k, q_k)$ eligible for a transaction price

$$q_k \leq \frac{\eta_1}{\ln\big(\frac{p_k}{W(n_k, y) \cdot \eta_3}\big)} \tag{12}$$



Preference	u_1	u_2	u_3	u_4	u_5	u_6
n_1	0.6	0.5	0.8	0.3	0.7	0.2
n_2	0.8	0.8	0.7	0.6	0.7	0.9
n_3	0.9	0.6	0.9	0.7	0.5	0.4

Figure 2: An example for QOOA.

when $W(n_k, y) \cdot \eta_3 < p_k$. If $W(n_k, y) \cdot \eta_3 \geq p_k$, there is no constraint on the quantity since $e^{\frac{\eta_1}{q_k}} > 1$ for any q_k and the corresponding user valuation will be larger than p_k for any quantity.

Next, QAS derives the upper bound of $TP(S_k^{x+1}, x+1)$ under $q_k = x+1$ by summing up the transaction prices of the top-$(x+1)$ user valuations, because the best airdrops subject to $q_k = x+1$ are to influence all users with the top-$(x+1)$ valuations. According to Equation (12), the transaction price of the y-th largest user valuation is $W(n_k, y) \cdot A(n_k, x+1)$ if $x+1 \leq \frac{\eta_1}{\ln(\frac{p_k}{W(n_k,y)\cdot\eta_3})}$; otherwise, the transaction price is 0. Hence, the upper bound of $TP(S_k^{x+1}, x+1)$ under $q_k = x+1$ is

$$UB(n_k, x+1) = \sum_{y=1}^{\min\{y', x+1\}} W(n_k, y) \cdot A(n_k, x+1), \tag{13}$$

where y' is the minimum value making Equation (12) not hold. Consequently, if there exists $x' \leq x$ leading to $UB(n_k, x+1) \leq TP(S_k^{x'}, x')$, i.e., $TP(S_k^{x+1}, x+1)$ for every possible S_k^{x+1} will not be greater than the total transaction price generated by the identified airdrops under some quantity examined so far, QAS skips the search for $q_k = x+1$. After $q_k = l_k$ is examined, QAS assigns $q_k = x'$ and $S_k = S_k^{x'}$ if $TP(S_k^{x'}, x')$ is the largest among all examined quantities.

Example 4.1. Consider a new NFT project $N = \{n_1, n_2, n_3\}$ in Figure 1(a) to be promoted in a social network in Figure 2. Assume that $B = \{2, 2, 2\}$, $L = \{2, 2, 2\}$, $P = \{1.8, 1.8, 1.8\}$, $\eta_0 = 0$, $\eta_1 = 1.5$, and $\eta_2 = 0.2$. QAS starts from n_1 subject to $q_1 = 1$. At first, QAS examines all users and chooses u_1 for an airdrop to maximize the total transaction price, i.e., $S_1^1 = \{(u_1, n_1)\}$ and $TP(S_1^1, 1) = 5.51$. Then, QAS derives QSP as follows.

$$QSP(\{u_1, u_2\}, n_1, 1) = 4.32, \quad QSP(\{u_1, u_3\}, n_1, 1) = 4.32,$$
$$QSP(\{u_1, u_4\}, n_1, 1) = 7.73, \quad QSP(\{u_1, u_5\}, n_1, 1) = 7.44,$$
$$QSP(\{u_1, u_6\}, n_1, 1) = 8.64.$$

Since both $QSP(\{u_1, u_2\}, n_1, 1)$ and $QSP(\{u_1, u_3\}, n_1, 1)$ are smaller than $TP(S_1^1, 1) = 5.51$, QOOA chooses the second user for an airdrop only from u_4, u_5, and u_6. As $TP(\{u_1, u_6\}, 1) = 7.1$ is the greatest, QAS updates $S_1^1 = \{u_1, u_6\}$ and finishes the search for $q_1 = 1$ due to $|S_1^1| = b_1 = 2$.

Before the search for $q_1 = 2$ begins, QAS identifies $W(n_1, 1) = 0.8$ and $W(n_1, 2) = 0.7$. Since $W(n_1, 1)\eta_3 = 0.8 \times e^{0.2\times4.5} = 1.97 > p_1 = 1.8$, $W(n_1, 1)A(n_1, 2)$ is eligible for a transaction price. As $W(n_1, 2)\eta_3 = 0.7 \times e^{0.2\times4.5} = 1.72 < p_1 = 1.8$, QAS infer the maximum quantity $\frac{1.5}{\ln(\frac{1.8}{1.72})} = 33.74 > q_1 = 2$. Hence, $W(n_1, 2)A(n_1, 2)$ is also eligible for a transaction price. As a result, QAS derives $UB(n_1, 2) = W(n_1, 1)A(n_1, 2) + W(n_1, 2)A(n_1, 2) = 7.81$. Since $UB(n_1, 2) = 7.81 > TP(\{u_1, u_6\}, 1) = 7.1$, QAS will continue the search under $q_1 = 2$. ■

Offspring Profit Enhancement(OPE). To further increase the profit, OPE encourages generating rare NFT offspring. OPE identifies the NFTs with the traits of the greatest rarity as *treasures*, and users purchasing at least one treasure are recognized as *Rare Trait Collectors (RTCs)*. The goal of OPE is to encourage RTCs to purchase more treasures by tailoring the airdrops of the treasures.

Specifically, let $T^{\mathrm{R}} \subseteq T$ denote the set of rarest traits according to the trait rarity (Equation (1)).[12] Based on T^{R}, OPE defines the *treasures* of NFTs (with at least one trait in T^{R}) as $N^{\mathrm{T}} = \{n_k : T_k \cap T^{\mathrm{R}} \neq \emptyset\}$, where each $n_k \in N^{\mathrm{T}}$ is associated with a treasure value $\theta_k = |T_k \cap T^{\mathrm{R}}|$ indicating the number of rare traits in n_k.[13] Then, OPE introduces *Rare Trait Collectors (RTCs)*, who purchase at least one treasure. It evaluates each RTC u_c with the *Collector Index (CI)*, which represents the necessity of encouraging u_c to engage in multiple treasure purchases according to the probability of u_c to breed offspring and the treasure values of the NFTs that u_c purchases, i.e., $CI(u_c, S, Q) = \beta_c \cdot \sum_{n_k \in W(u_c, S, Q)} \theta_k$, where β_c is the breeding probability of u_c, and $W(u_c, S, Q)$ is the set of NFT that u_c purchases under the influence of S with Q. OPE prioritizes RTCs with a high CI since they have more chances to generate rare NFT offspring.

For each treasure $n_k \in N^{\mathrm{T}}$, OPE tailors the airdrops of n_k in order to persuade RTCs to purchase it, accompanying the purchases of other treasures. OPE first examines whether each RTC u_c's valuation on n_k exceeds the reserve price p_k. If $v_{u_c, n_k}(q_k) < p_k$ for all n_c, OPE does not adjust the airdrops of n_k since no RTC will be able to purchase n_k. Otherwise, OPE evaluates each user $u_i \in V$ with CI of RTCs that can be influenced by u_i, weighted by the likelihood of the RTCs being influenced. Recall IPP in Equation (9) and the set of prospective purchasers with the top-x valuations in the IPP of u_i, i.e., $Top(\{u_i\}, n_k, x)$. Similar to QSP, OPE evaluates the *Multi-Treasure Influence (MTI)* of a user u_i by summing up the weighted CI of RTCs in $Top(\{u_i\}, n_k, x)$.

$$\begin{aligned} & MTI(u_i, n_k, q_k) \\ = & \sum_{u_c \in RTC(S,Q) \cup Top(\{u_i\}, n_k, x)} CI(u_c, S, Q) \cdot feq(u_i, u_c), \end{aligned} \tag{14}$$

where $RTC(S, Q)$ is the set of RTCs under the influence of S with Q, and $feq(u_i, u_c)$ is the likelihood of u_i influencing u_c recorded in u_i's IPP, i.e., $(u_c, feq(u_i, u_c)) \in IPP(u_i, n_k, x)$, representing the weight on u_c's CI. For 1) each user u_i selected for airdrops but having the least MTI (to encourage RTCs in multiple treasure purchases) and 2) another user u_j not selected for airdrops and having the largest MTI (for facilitating multiple treasure purchases of RTCs), if u_i is less likely to persuade RTCs to purchase n_k (compared with u_j), i.e., $MTI(u_i, n_k, q_k) < MTI(u_j, n_k, q_k)$, OPE attempts to replace the airdrop from u_i to u_j in order to facilitate NFT offspring with rare traits. It updates S as $S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}$ if the above replacement improves the profit, i.e., $f(S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}, Q) > f(S, Q)$.

Example 4.2. Following Example 4.1, QOOA obtains $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_6, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 25.93$. Since "rainbow" and "moon" are the rarest traits,

[12] Following [28], the top 10% rarest ones are usually discussed.

[13] For an NFT that is not a treasure, its treasure value is 0.

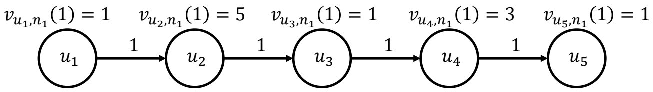


Figure 3: An example for Lemma 4.1.

OPE identifies the treasures $N^{\mathrm{T}} = \{n_1, n_3\}$. For n_3, OPE examines MTI as follows.

$$\begin{aligned} &\text{Airdrops:} && MTI(u_4, n_3, 1) = 0.51, MTI(u_6, n_3, 1) = 0.26 \\ &\text{Not Airdrops:} && MTI(u_3, n_3, 1) = 0.37, MTI(u_5, n_3, 1) = 0.36, \\ & && MTI(u_1, n_3, 1) = 0, MTI(u_2, n_3, 1) = 0. \end{aligned}$$

Accordingly, OPE attempts to replace u_3 with u_6 for airdrops, due to $MTI(u_3, n_3, 1) = 0.37 > MTI(u_6, n_3, 1) = 0.26$. Since $f(S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}, Q) = 26.97 > f(S, Q) = 25.93$, OPE updates $S = S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}$. As $MTI(u_5, n_3, 1) = 0.36 < MTI(u_4, n_3, 1) = 0.51$, OPE terminates the enhancement for n_3. Consequently, the solution is $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_3, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 26.97$. ■

4.3 Theoretical Results

For each $k \in \{1, 2, \ldots, |N|\}$, let $Act(S_k)$ denote the set of users being active in NFT n_k. Please note that $V(S_k, q_k) \subseteq Act(S_k)$ and $|V(S_k, q_k)| \leq q_k$ for each $k \in \{1, 2, \ldots, |N|\}$. Moreover, $S = S_1 \cup S_2 \cup \cdots \cup S_{|N|}$, and $S_k \cap S_h = \emptyset$ for any $k, h \in \{1, 2, \ldots, |N|\}$ with $k \neq h$. For each $k \in \{1, 2, \ldots, |N|\}$, let

$$r(S_k, q_k) = \sum_{u_i \in Act(S_k), v_{u_i,n_k}(q_k) \geq p_k} v_{u_i,n_k}(q_k)$$

represent the total valuation function of users being active in NFT n_k following the transaction price constraint (i.e., $v_{u_i,n_k}(q_k) \geq p_k$). Since $V(S_k, q_k) \subseteq Act(S_k)$,

$$TP(S_k, q_k) \leq r(S_k, q_k) \tag{15}$$

for each $k \in \{1, 2, \ldots, |N|\}$. Moreover, by Equations (3) and (4), for every $q_k \leq q'_k$, we have $v_{u_i,n_k}(q_k) \geq v_{u_i,n_k}(q'_k)$ and thus

$$r(S_k, q_k) \geq r(S_k, q'_k). \tag{16}$$

Note that in the influence propagation of NFTs, for each NFT n_k, its total transaction price function $TP(S_k, q_k)$ is independent to each other.

In the following, we first examine whether the total transaction price function $TP(S_k, q_k)$ is submodular.

Definition 4.3 (Submodular function [20]). Given a ground set U, a set function $\rho : 2^U \mapsto \mathbb{R}$ is submodular if for any subsets X, Y with $X \subseteq Y$ and any element $u \in U \setminus Y$,

$$\rho(Y \cup \{u\}) - \rho(Y) \leq \rho(X \cup \{u\}) - \rho(X). \tag{17}$$

Unfortunately, $TP(S_k, q_k)$ is non-monotonically increasing and far from submodular.

Lemma 4.1. *For each $k \in \{1, 2, \ldots, |N|\}$, the total transaction price function $TP(S_k, q_k)$ is neither monotonically increasing nor submodular.*

PROOF. We prove this result by giving an instance as follows. Let the NFT project $N = \{n_1\}$, seed budget $b_1 = 3$, NFT quantity $q_1 = 1$, reserve price $p_1 = 1$ and the social network G be a path graph P_5 of five vertices, i.e., there are five users $u_1, u_2, \ldots, u_5$, and four edges $e_{i,i+1}$, $1 \leq i \leq 4$, where each edge $e_{i,i+1}$ has the activation probability of $a_{i,i+1} = 1$, as shown in Figure 3. The valuations of all users u_i for NFT n_1 are $v_{u_1,n_1}(q_1) = 1$, $v_{u_2,n_1}(q_1) = 5$, $v_{u_3,n_1}(q_1) = 1$, $v_{u_4,n_1}(q_1) = 3$, and $v_{u_5,n_1}(q_1) = 1$, respectively. In this instance, when (u_3, n_1) is chosen as the only seed, users u_4 and u_5 are activated, and thus u_4 wins the NFT n_1, i.e., $TP(\{(u_3, n_1)\}, q_1) = 3$. However, if we choose (u_3, n_1) and (u_4, n_1) as the seeds, only user u_5 is activated, and thus $TP(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 1$, which implies that the revenue function $TP(S_1, q_1)$ is non-monotonically increasing.

To prove the non-submodularity, we consider two seed groups $\{(u_1, n_1), (u_3, n_1)\}$ and $\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}$. The former seed group results in $f(\{(u_1, n_1), (u_3, n_1)\}, q_1) = 5$ because u_2, u_4 and u_5 are activated, and u_2 wins NFT n_1, and the latter results in $TP(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) = 5$ because u_2 and u_5 are activated, and u_2 wins NFT n_1. Therefore,

$$TP(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) - TP(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 4$$

and

$$TP(\{(u_1, n_1), (u_3, n_1)\}, q_1) - TP(\{(u_3, n_1)\}, q_1) = 2,$$

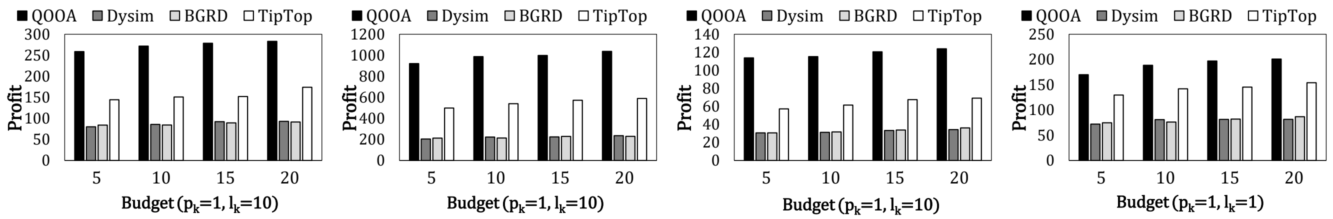
which implies that the total transaction price function $TP(S_1, q_1)$ is non-submodular. The lemma follows. □

Nevertheless, different from the total transaction price function $TP(S_k, q_k)$, the total valuation function $r(S_k, q_k)$ is submodular.

Lemma 4.2. *For each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing but submodular in the seed selection.*

PROOF. Since the profit of all users being seeds is zero, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing for each $k \in \{1, 2, \ldots, |N|\}$. To prove the submodularity, we show that each function $r(S_k, q_k)$ satisfies (17). Following the proof in [16] where the submodularity of the influence function on the IC model can be reduced to that in a deterministic graph realized by flipping the coin for each edge of $G = (V, E)$, we show the submodularity of $r(S_k, q_k)$ in a deterministic graph $G' = (V, E') \subseteq G$ realized by flipping the coin for each edge of G with the activation probability. The process of influence propagation on G can be regarded as the influence propagation process upon the deterministic graph G'. Thus, for any activated user, its influence propagation is a connected subgraph of G' rooted by some seed user u, i.e., $(u, n_k) \in S_k$. For any seed group S_k, the function $r(S_k, q_k)$ is the total valuation (excluding the valuations violating the transaction price constraint $v_{u_i,n_k}(q_k) \geq p_k$) of the union of the connected subgraphs rooted by all seed users in S_k. Therefore, for each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is a coverage function, which is submodular [19]. The lemma follows. □

Let (S^{opt}, Q^{opt}) denote the optimal solution of NPM, where $S^{opt} = \bigcup_{k \in \{1,2,\ldots,|N|\}} S_k^{opt}$ and $Q^{opt} = \{q_1^{opt}, q_2^{opt}, \ldots, q_{|N|}^{opt}\}$. To derive the approximation ratio, we first consider a problem similar to NPM, named NPM-QO, where the quantity constraint is removed. In NPM-QO, all active users with valuations satisfying the reserve price constraint $v_{u_i,n_k}(q_k) \geq p_k$, without the restriction that the q_k highest valuations win the NFT n_k. The user u_i's valuation on NFT n_k is $v_{u_i,n_k}(1)$, and $\beta_i = 0$ for each $i \in \{1, 2, \ldots, |V|\}$. Similarly,



(a) Defimons Characters (*Gowalla*). (b) Laxcaux (*Epinion*). (c) Timpers Pixelworks (*Pokec*). (d) Defimons Characters (*Gowalla*).

Figure 4: Comparison on profits for different approaches.

let $\hat{S}^{opt}$ denote the optimal solution of NPM-QO, where $\hat{S}^{opt} = \bigcup_{k\in\{1,2,\ldots,|N|\}} \hat{S}_k^{opt}$ satisfies $\hat{S}_k^{opt} = \arg\max_{S_k} r(S_k, 1)$ for each $k \in \{1, 2, \ldots, |N|\}$.

Lemma 4.3. *For each $k \in \{1, 2, \ldots, |N|\}$, $r(\hat{S}_k, 1) \geq \frac{1}{e}\cdot TP(S_k^{opt}, q_k^{opt})$ holds, where $\hat{S}_k$ is the solution found by the continuous greedy algorithn [6] to solve NPM-QO for NFT n_k.*

Proof. By Lemma 4.2, the total valuation function $r(S_k, 1)$ is submodular for each $k \in \{1, 2, \ldots, |N|\}$. According to [6], the seed set $\hat{S}_k$ found by the continuous greedy algorithm is a $\frac{1}{e}$-approximation solution of NPM-QO for NFT n_k, i.e., $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot r(\hat{S}_k^{opt}, 1)$. To complete the proof, for each $k \in \{1, 2, \ldots, |N|\}$, we show $r(\hat{S}_k^{opt}, 1) \geq TP(S_k^{opt}, q_k^{opt})$ as follows,

$$r(\hat{S}_k^{opt}, 1) \geq r(S_k^{opt}, 1) \geq r(S_k^{opt}, q_k^{opt}) \geq TP(S_k^{opt}, q_k^{opt}),$$

where the first, second, and last inequalities are obtained due to the optimality of $\hat{S}_k^{opt}$ to NPM-QO for NFT n_k, Equation (16), and Equation (15), respectively. The lemma follows. □

Next, we show the relation between the total transaction price of N and the assessments of NFT offspring generated from N. Let $A_{\max}$ denote the maximum assessment of an offspring, i.e.,

$$A_{\max} = e^{\eta_0+\eta_1\frac{1}{1}+\eta_2|T|\frac{2^{|T|}}{2}} = e^{\eta_0+\eta_1+\eta_2|T|2^{|T|-1}}, \tag{18}$$

where we assume that the quantity of this offspring is 1 (leading to $e^{\eta_1\frac{1}{1}}$), it has $|T|$ traits, each trait is only owned by it and one of its parents, and at most $2^{|T|}$ offspring are generated (leading to $e^{\eta_2|T|\frac{2^{|T|}}{2}}$). For any (S, Q) in NPM, we have

$$\begin{aligned}
&OS(S,Q)\\
\leq &\sum_{k<m} q_{k,m}(S,Q)\cdot A_{\max}\\
\leq &\frac{q_{\max}\frac{|N|(|N|-1)}{2}}{|N|}\cdot\frac{A_{\max}}{p_{\min}}\cdot p_{\min}\cdot|N|\\
\leq &c\sum_{k=1}^{|N|} p_k d_k \qquad (\text{Let } c = \frac{q_{\max}(|N|-1)}{2}\cdot\frac{A_{\max}}{p_{\min}})\\
\leq &c\sum_{k=1}^{|N|} TP(S_k, q_k),
\end{aligned}$$

where $p_{\min}$ is the minimum reserve price, and d_k is the actual number of the NFT n_k sold. Note that $\sum_{k<m} q_{k,m}(S,Q) \leq q_{\max}\frac{|N|(|N|-1)}{2}$ holds because $q_{\max}\frac{|N|(|N|-1)}{2}$ is the maximum number of offspring that can be generated assuming that $q_{\max}$ quantities of each NFT are sold and exactly $q_{\max}$ users own all $|N|$ NFTs, and $p_{\min}|N| \leq \sum_{k=1}^{|N|} p_k d_k$ holds since $p_{\min}|N|$ is the minimum transaction prices assuming each NFT is sold at $p_{\min}$ with only one quantity.[14] Then, for any (S, Q) in NPM,

$$f(S,Q) = \sum_{k=1}^{|N|} TP(S_k, q_k) + OS(S,Q) \leq (1+c)\cdot\sum_{k=1}^{|N|} TP(S_k, q_k). \tag{19}$$

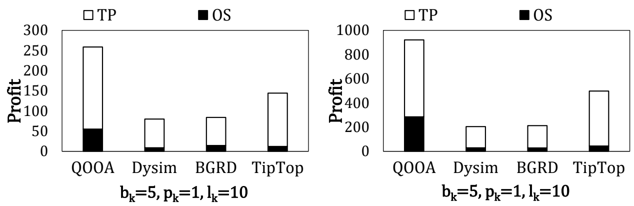
For each $k \in \{1, 2, \ldots, |N|\}$, let (S_k^{alg}, q_k^{alg}) denote the algorithm solution obtained by QOOA for NFT n_k. Then (S^{alg}, Q^{alg}) is the algorithm solution obtained by QOOA, where $S^{alg} = \bigcup_{k\in\{1,2,\ldots,|N|\}} S_k^{alg}$, $Q^{alg} = \{q_1^{alg}, q_2^{alg}, \ldots, q_{|N|}^{alg}\}$, and $f(S^{alg}, Q^{alg}) = \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) + OS(S^{alg}, Q^{alg})$.

Theorem 4.4. *QOOA is a $\frac{1}{ea(1+c)}$-approximation algorithm in $O(l_{\max} b_{\max} |V|\,|N|)$ time for NPM, where $a = \max_{1\leq k\leq|N|} \frac{r(\hat{S}_k,1)}{TP(S_k^{alg}, q_k^{alg})}$, $c = \frac{A_{\max} l_{\max}(|N|-1)}{2p_{\min}}$, $A_{\max}$ is the maximum assessment of offspring, $l_{\max} = \max_{l_k\in L} l_k$ is the maximum quantity limit, $p_{\min}$ is the minimum reserve price, and $b_{\max}$ is the maximum budget in B.*

Proof. To prove this result, we show $f(S^{alg}, Q^{alg}) \geq \frac{1}{ea(1+c)}\cdot f(S^{opt}, Q^{opt})$. For each $k \in \{1, 2, \ldots, |N|\}$, we have

$$\begin{aligned}
TP(S_k^{alg}, q_k^{alg}) = &\frac{TP(S_k^{alg}, q_k^{alg})}{r(\hat{S}_k, 1)}\cdot r(\hat{S}_k, 1)\\
\geq &\frac{1}{e}\cdot\frac{TP(S_k^{alg}, q_k^{alg})}{r(\hat{S}_k, 1)}\cdot TP(S_k^{opt}, q_k^{opt})\\
\geq &\frac{1}{ea}\cdot TP(S_k^{opt}, q_k^{opt}),
\end{aligned} \tag{20}$$

[14] To ensure that an NFT is marketable in the real world, its reserve price is typically determined according to the valuations of its intended audience.



(a) Defimons Characters (*Gowalla*). (b) Laxcaux (*Epinion*).

Figure 5: Comparison on profits from the original NFTs (i.e., TP) and the offspring (i.e., OS).

where the first and second inequalities are obtained by Lemma 4.3 and the setting of a, respectively. Therefore,

$$\begin{aligned} f(S^{alg}, Q^{alg}) &= \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) + OS(S^{alg}, Q^{alg}) \\ &\geq \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) \\ &\geq \frac{1}{ea} \cdot \sum_{k=1}^{|N|} TP(S_k^{opt}, q_k^{opt}) \\ &\geq \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt}), \end{aligned}$$

where the first inequality is obtained because $OS(S^{alg}, Q^{alg})$ is non-negative by definition, and the second and third inequalities are obtained by Equation (20) and Equation (19), respectively.

Next, we analyze the time complexity. In the QAS phase, for an NFT n_k with budget b_k, it takes $O(|V|\, b_k)$ time to find the airdrops under a specific quantity. As the maximum quantity limit is $l_{\max} = \max_{l_k \in L} l_k$, it requires $O(l_{\max} b_k\, |V|)$ time to identify the most appropriate quantity and the corresponding airdrops for each NFT n_k. Therefore, QAS takes $O(l_{\max} b_{\max}\, |V|\, |N|)$ time to find an initial solution of NPM, where $b_{\max}$ is the maximum budget in B. Next, in the OPE phase, it examines $O(b_k)$ replacements of airdrops for a treasure n_k to encourage RTCs to purchase additional treasures. As there are $O(|N|)$ treasures, OPE takes $O(b_{\max}\, |N|)$ time. Consequently, the time complexity of QOOA is $O(l_{\max} b_{\max}\, |V|\, |N|)$. The theorem follows. □

5 EXPERIMENTS

5.1 Experiment Setup

Datasets. We conduct experiments on three real NFT projects and three real social networks. The NFT projects include i) Defimons Characters:[15] It has 14 NFTs and 5 traits, with the rarity of NFTs ranging from 3.75 to 9. ii) Laxcaux:[16] It has 17 NFTs and 10 traits, with the rarity of NFTs ranging from 4.05 to 39.89. iii) Timpers Pixelworks:[17] It has 7 NFTs and 3 traits, with the rarity of NFTs ranging from 1.4 to 7. The social networks include *Gowalla* [24] (259K users and 900K friendships), *Epinions* [10] (114K users and 717K friendships), and Pokec [34] (1M users and 30M friendships). We follow [21, 25, 35] to set the user preferences and the activation probabilities. Following [30], the breeding probability β_i is set as the activity strength of each user u_i. Following most of the famous NFT projects, e.g., Trump Digital Trading Card,[18] the quantity limit l_k of n_k is set to 10 for all $n_k \in N$. Following OpenSea, the reserve price p_k for n_k is set to 1.[19]

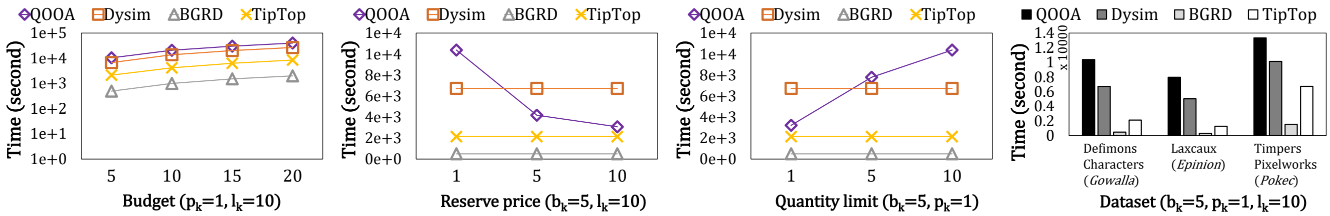
Baselines and metrics. We compare QOOA with three state-of-the-art approaches: Dysim [36], BGRD [1], and TipTop [22]. As TipTop only targets a single product, we apply it to an NFT at each time to find the airdrops. Since all baselines are not able to find the quantities of NFTs, we examine every quantity to find the one with the maximum profit. The performance metrics include profit $f(S, Q)$ and the execution time. We perform a series of sensitivity tests in terms of 1) the budgets B, 2) the quantity limits L, and 3) the reserve prices P. We conduct all experiments on an HP DL580 server with an Intel 2.10GHz CPU and 1TB RAM. Each simulation result is averaged over 100 samples.

5.2 Performance Comparison

Figures 4(a)-4(c) compare the profits under different budgets, where the x-axis specifies the budget b_k for each NFT n_k. For all datasets, QOOA achieves the greatest profit since it exploits QAS to maximize the total transaction price, instead of the influence spread or total valuation. Among the baselines, TipTop is superior since it takes into account user valuations. However, TipTop is unaware of NFT quantities and thus outperformed by QOOA. The profits generated by Dysim and BGRD are very similar since they both aim to maximize the influence spread. Besides, QOOA outperforms the baselines more considerably for Laxcaux (compared with the other NFT projects). This is because half traits in Laxcaux are highly rare (i.e., with the trait rarity of 17), and by leveraging OPE, QOOA is able to encourage RTCs to produce offspring assessed higher than Defimons Characters and Timpers Pixelworks. As the budgets increase, all approaches generate more profits. Nevertheless, the increasing trends of different approaches are diverse. The profit increments generated by Dysim and BGRD are insignificant, since they only enlarge the influence spread, not the profit. On the other hand, Figure 4(d) presents the profits under different budgets when the quantity limit $l_k = 1$ for each NFT n_k. Compared to Figure 4(a), the profits generated by all approaches are smaller. It manifests the importance of balancing scarcity and quantity, since the profits are not necessarily maximized even when the scarcity of NFTs boosts user valuations.

Figure 5 examines the profits generated by original NFTs (i.e., TP) and offspring (i.e., OS). Obviously, QOOA is most effective in generating offspring with high assessments because OPEencourages RTCs to engage in multiple NFT purchases. By contrast, the baselines do not account for the generation of offspring and solely focus on profiting from the transaction prices of the original NFTs. Moreover, as half of the traits in Laxcaux are rare, QOOA exhibits a higher likelihood of producing offspring with rare traits. This phenomenon is evident in Figure 5(b), where the profit generated by offspring accounts for nearly 30% of the total profit, surpassing the 21% observed in Figure 5(a).

[15]https://opensea.io/collection/defimons-characters.
[16]https://opensea.io/collection/lascauxfuture.
[17]https://opensea.io/collection/timperspixelworks.
[18]https://collecttrumpcards.com/.
[19]https://support.opensea.io/hc/en-us/articles/1500003246082-How-do-timed-auctions-work-.



(a) Defimons Characters (*Gowalla*). (b) Defimons Characters (*Gowalla*). (c) Defimons Characters (*Gowalla*). (d) All datasets.

Figure 6: Comparison on execution times for different approaches.

Figures 6(a), 6(b), and 6(c) compare the execution time under different budgets, reserve prices, and quantity limits, respectively. QOOA requires slightly more time than the baselines since it carefully modifies the airdrops to encourage RTCs to purchase multiple NFTs with rare traits. Additionally, QOOA explores various quantities to determine the best airdrops that maximize profits. In contrast, the baselines are quantity-agnostic and do not invest time in identifying airdrops with varying quantities. Among all baselines, BGRD requires the least time since it does not perform separate searches for airdrops corresponding to different NFTs. TipTop is more efficient than Dysim by leveraging reverse influence sampling to identify airdrops. In Figure 6(a), as more budgets are provided, all approaches require more time. Differently, in Figure 6(b), as the reserve price increases, the efficacy of VQI becomes more pronounced, allowing QOOA to prune unnecessary searches for different quantities. Hence, the execution time of QOOA decreases for higher reserve prices. By contrast, all baselines are not aware of reserve prices, and their execution times remain unchanged. Besides, in Figure 6(b), when the quantity limit is reduced from 10 to 5 and 1, the execution time of QOOA noticeably decreases, since fewer searches for different quantities are required. In contrast, the baselines, which do not identify airdrops for different quantities, exhibit execution times that are insensitive to the quantity limits. On the other hand, Figure 6(d) compares the execution time on different datasets. It is observed that the execution times of QOOA, Dysim, and TipTop are proportional to the sizes of the NFT project and the social network, while that of BGRD only correlated to the size of the social network. Note that despite the large scale of *Pokec*, QOOA remains efficient since it leverages QSP to prune inferior airdrops that are unlikely to yield higher profits.

6 CONCLUSION

To the best of our knowledge, we make the first attempt to investigate the profit maximization problem for NFTs. By incorporating key features of NFTs, including breeding, scarcity, and trait rarity, we formulate a new problem, named NPM, to find the airdrops and determine the quantities in viral marketing. We prove the hardness of NPMand design an approximation algorithm QOOA. QOOA effectively tackles NPM by initially identifying airdrops under varying quantities to maximize the profit. To enhance efficiency, QOOA evaluates QSP to discard inferior airdrops and derives VQI to find the upper bounds on profits for different quantities to trim off redundant searches. QOOA further increases the profit from offspring by encouraging RTCs to purchase multiple NFTs with rare traits. Experiments on real NFT projects and social networks demonstrate that QOOA can effectively achieve about twice the profits over the state-of-the-art approaches.

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