On Maximizing NFT Profit through Social Influence

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ABSTRACT

Recently, the rise of non-fungible tokens (NFTs) has been one of the most significant developments for the art and technology in the Metaverse. Different from necessities, scarcity and rarity have huge impact on user valuations of NFTs. Moreover, the unprecedented NFT breeding mechanism creates new challenges for viral marketing, as the generated NFT offspring also yield huge profits. However, previous profit maximization studies on tangible products do not take scarcity, rarity, and the breeding mechanism into account. In this paper, we make the first attempt to formulate *NFT Profit Maximization (NPM)* by finding NFT airdrops and determining NFT quantities. We prove the hardness of NPM and design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*. QOOA leverages Quantity-Sensitive Profit to prune inferior airdrops and upper bounds the search for possible NFT quantities by deriving Valuation-based Quantity Inequality. To increase profit from NFT offspring, QOOA identifies Rare Trait Collectors and encourages them to purchase multiple NFTs with rare traits in order to generate valuable offspring. We compare the performance of QOOA with the state-of-the-art approaches with real NFT datasets. The experimental results demonstrate that QOOA effectively outperforms state-of-the-art approaches in large-scale social networks.

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1 INTRODUCTION

Recently, the rise of NFTs has been one of the most significant developments in the art and technology in the Metaverse. NFTs first gained popularity in 2017 with the introduction of CryptoKitties, which uses blockchain technologies to enables users to buy and sell virtual cats as unique digital assets. Since then, the market for NFTs has exploded, with NFTs fetching millions of dollars in high-profile sales. For instance, the digital artist Beeple sold an NFT of his artwork at auction for a record-breaking $69 million. Other notable NFT trades include the sale of Twitter CEO Jack Dorsey's first tweet for $2.9 million and a LeBron James highlight video for $208,000.

NFTs are digital assets that are verified with blockchain to guarantee their authenticity and ownership. Different from traditional viral marketing, the marketing of NFTs has the following new features. 1) Most NFT marketplaces sell NFTs via auction, with the highest bidders winning the NFTs. The profit is thus dominated by the *transaction prices*, which are the offers of the highest bidders based on their valuations, and the *reserve price*, which is the minimum transaction price accepted by the seller, in addition to the number of influenced users. For example, Nakamigos #3648 attracts more than 5200 views.[1] However, as the quantity is one, Nakamigos #3648 is sold to the best offer at a transaction price of $28,000. 2) Unlike necessities, the rarity of NFTs leads to high assessments. For instance, CryptoPunk #2924, one of 24 punk apes and the 38th rarest out of a total of 10,000 created by Larva Labs in 2017, is recently sold for $4.5 million.[2] 3) Most importantly, *NFT breeding* enables the combination of multiple NFTs to produce brand-new and unique NFT offspring. The feature offers the opportunity to produce an NFT that is rarer and more valuable than its parents. For example, in December 2022, Nike minted its NFT sneakers, named CryptoKicks, which can be redeemed as physical sneakers. CryptoKicks holders will be able to breed their CryptoKicks, which will be generated according to trait data and/or genetic code from the parents.[3] Similarly, Roaring Leaders, CryptoKitties, Axie Infinity, and STEPN also enable users to breed NFT offspring.[4] Among them, STEPN is the first move-to-earn, enabling users to earn tokens by walking while wearing an NFT sneaker. Rarer NFT sneakers usually accelerate the earning process significantly. With the tokens, users can breed rare NFT offspring for sale and/or rental to earn more tokens. Compared with traditional necessities and assets, promoting NFTs indeed introduces new research challenges in viral marketing.

In order to promote new NFT projects, NFT marketplaces, such as OpenSea and Blur, usually *airdrop* NFTs to influential users, i.e., providing free NFTs for users to drive public awareness.[5] Consider Figure 1 as an illustrative example to promote a new NFT project with three NFTs n_1, n_2, and n_3 in a social network, where nodes represent users, edges represent users' friendships with activation probabilities, and users' preferences for n_1, n_2, and n_3 are listed in order in square brackets near the nodes. The traits and (potential) assessments under different quantities of n_1, n_2, n_3, and the new offspring are presented below the social network. Note that the valuation of a user for an NFT is assumed to be calculated by multiplying her preference and the assessment of the NFT. Assume that the airdrop quota and the reserve price of each NFT are one and 2.5, respectively. Influence maximization approaches, aiming to find users to maximize the number of influenced users, intend to airdrop all NFTs to u_1. When the quantity of each NFT is one, the expected profit is about $0.09 + 0.05 + 0.06 = 0.2$, coming from n_1, n_2, and n_3

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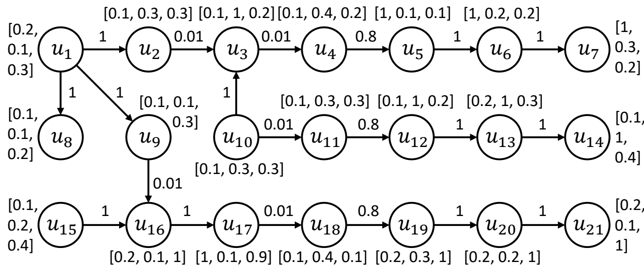
[1] Details on Nakamigos #3648 can be found at https://opensea.io/assets/ethereum/0xd774557b647330c91bf44cfeab205095f7e6c367/3648.

[2] More information about CryptoPunk #2924 can be found at https://opensea.io/assets/ethereum/0xb47e3cd837ddf8e4c57f05d70ab865de6e193bbb/2924.

[3] The patent of CryptoKicks can be found at https://patents.google.com/patent/US10505726B1/en.

[4] Roaring Leaders: https://roaringleaders.io/; CryptoKitties: https://www.cryptokitties.co/; Axie Infinity: https://axieinfinity.com/; STEPN: https://stepn.com/.

[5] Airdrop information of OpenSea and Blur at https://partners.opensea.io/drops and https://blur.io/airdrop, respectively.



		NFT project			New offspring
NFT		n_1	n_2	n_3	
Traits		Star, rainbow	Star	Moon	Moon, rainbow
(Potential) Assessment under different quantities	1	9	5	6	9
	2	4.5	2.5	3	4.5
	3	2.25	1.25	1.5	2.25

Figure 1: An illustrative example.

sold to u_{17}, u_3 and u_{16}, respectively, without any NFT offspring generated.[6] Similarly, classical profit maximization approaches identify u_4, u_{11}, and u_{18} for n_1, n_2, and n_3, respectively, as the airdrops to maximize the total profit without considering NFT quantities. The airdrops lead to an expected profit of $7.2 + 4 + 4.8 = 16$ when the quantity of each NFT is one. In this case, one of u_5, u_6, and u_7 wins n_1, one of u_{12}, u_{13}, and u_{14} wins n_2, one of u_{19}, u_{20}, and u_{21} wins n_3, and still no one can breed NFT offspring. When the quantity of each NFT increases in both cases of airdrops, the valuations of influenced users drop greatly, with many falling below the reserve prices, resulting in a smaller profit than when the quantity is one. In contrast to the above two types of approaches that fail to determine NFT quantities, by considering the NFT quantity and the breeding mechanism, airdropping n_1, n_2, and n_3 to u_{15}, u_{10}, and u_{15} respectively, and setting the quantities of n_1, n_2, and n_3 as 1, 1, and 2, respectively, can earn an expected profit of $9+5+(3+2.7)+3.44 = 23.14$. In this case, u_{17} and u_3 purchase n_1 and n_2, respectively, while both u_{16} and u_{17} purchase n_3. As u_{17} holds n_1 and n_3, she will generate one NFT offspring with a potential profit of $\frac{2}{3} \cdot \frac{1}{3} \cdot 4.5 + \frac{2}{3} \cdot \frac{2}{3} \cdot 2.5 + \frac{1}{3} \cdot \frac{2}{3} \cdot 1.5 + \frac{1}{3} \cdot \frac{1}{3} \cdot 9 = 3.44$ according to the rarity of each trait owned by n_1 and n_3 and the NFT assessment under the corresponding quantity (including the generated offspring).

Observed from the above scenario, several new challenges arise to maximize the profit of NFTs. 1) As the profit of NFTs is generated from the transaction prices, i.e., the valuations of users willing to purchase the NFTs, an important NFT airdrop is to reach the users with high valuations. Since the quantities of NFTs are limited, maximizing the influence spread does not necessarily maximize the profit This is because only a small number of influenced users pay for the NFTs. Previous works [] aim to maximize the influence spread but neglect user valuations, thus failing to ensure the maximization of profit. 2) As the scarcity of an NFT significantly affects users' valuations, limiting an NFT to be unique usually boosts users' valuations. However, it is important to note that the profit can only be earned from a single user in such cases. By contrast, supplying an NFT with a large quantity may allow more users to purchase the NFT but reduce users' valuations as well as transaction prices. Previous works [] do not take into account the impact of scarcity and rarity on valuations and cannot determine appropriate quantities of NFTs. 3) Due to the NFT breeding mechanism, it is encouraged for users to simultaneously own more than one NFT. This is because NFT offspring can generate additional profit for the users. As the assessments of NFT offspring are determined by the traits of their parent NFTs (i.e., offspring inherit traits from the parents, e.g., both parents with a happy face are likely to generate offspring with a happy face), it is preferable to facilitate two NFTs with rare traits purchased by the same user to breed rare offspring. On the other hand, the excessive breeding of NFTs may reduce the scarcity and rarity as well as the assessments of the NFT offspring. Therefore, it is crucial to encourage suitable NFTs as parents to generate appropriate offspring to maximize profit. Previous works [] consider multiple items without additional profits and cannot find appropriate NFTs to adopt simultaneously.

In this paper, we formulate a new problem, named *NFT Profit Maximization (NPM)*. Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T, a social network, a set of budgets $B = \{b_1, \ldots, b_{|N|}\}$, and a maximum quantity $q_{\max}$, NPM aims to find a set of NFT airdrops and determine the quantity of each NFT, such that the profit earned from the NFTs and their offspring is maximized, where the number of airdrops for each NFT n_k is subject to b_k and the quantity of each NFT is at most $q_{\max}$.[7] We first prove that NPM is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$. To solve NPM, we design an approximation algorithm, named *Quantity and Offspring-Oriented Airdrops (QOOA)*. 1) In order to achieve high transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* as the possible profit earned by the influence of a set of users under the quantity constraint if they are selected for airdrops. Specifically, QOOA first identifies the prospective purchasers whose valuations are not less than the reserve price. QOOA then evaluates QSP by estimating the likelihood of the set of users influencing the prospective purchasers with the highest valuations. 2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find the upper bound of profit that can be generated under a specific quantity. VQI captures the relationships between the reserve price and user valuations for varying quantities, and then infers the maximum quantity that users will accept to purchase the NFT. 3) For increasing the profit earned from NFT offspring, QOOA identifies the *Rare Trait Collector (RTC)* by evaluating the rarity of NFTs a user purchases. RTCs are encouraged to purchase additional NFTs with rare traits by tailoring the airdrops, so that more rare offspring can be generated and, consequently, higher profits can be achieved. We evaluate the performance of QOOA on real NFT projects, e.g., XXX. The contributions of this work include:

- To the best of our knowledge, NPM is the first attempt to study the profit maximization problem for NFTs, taking into

[6] Since the probability of u_5, u_6, and u_7 being influenced by u_1 is too small (i.e., 0.0008), we neglect it here for simplicity.

[7] In practice, the quantity of NFTs is usually not unlimited since most NFTs are regarded as artwork.

consideration NFT breeding, NFT scarcity, and trait rarity. We prove the hardness and inapproximability of NPM.

- We design an approximation algorithm QOOA with the notions of QSP, VQI, and RTCs.
- Via real NFT projects, experiments demonstrate that QOOA achieves more than XX time of profits over the state-of-the-art approaches.

2 RELATED WORK

Profit/revenue maximization. Previous works on profit/revenue maximization mainly focus on maximizing the difference between the influence spread and the cost of the seed group [7, 9]. Tang et al. [16] further introduce the benefit of users and aim at the total benefits instead of the influence spread. Different from maximizing the profit from activated users, Gao et al. [4] focus on the benefits related to interactions among activated nodes. Han et al. [5], on the other hand, study the problem from the host's perspective to maximize the revenue of all advertisers, where the revenue comes from the cost-per-engagement amount and the influence spread of an advertiser. However, all the above works do not study the profit/revenue/benefit that varies according to the quantity of the product, and cannot identify a proper quantity to maximize the profit/revenue/benefit. Furthermore, they do not include additional profits of simultaneously adopting multiple products, unable to deal with the NFT breeding mechanism embedded in NPM.

Viral marketing on multiple products. Prior research has investigated the problem of maximizing the influence (or profit/revenue) of a company's multiple products. While the studies [2, 18] assume that each product is independent, other studies [1, 8, 13, 17] consider the adoptions of products are dependent, such as the complementary and substitutable relationships between products. However, these works only model the positive or negative impact of adopting a product on another one, without capturing the effect of co-purchasing multiple products on the influence (or profit/revenue), i.e., the breeding mechanism. Moreover, all the above works neither value the products according to scarcity and rarity of the products, nor determine the quantities of the products. Therefore, they cannot be directly applied to NPM.

3 PROBLEM FORMULATION

In this section, we first describe the diffusion process of NFTs by incorporating NFT characteristics into existing diffusion models. Next, we define the profit function and the profit maximization problem of NFTs. Finally, we analyze the hardness of the proposed problem. Table 1 summarizes the notations in this paper.

3.1 Definition

In this work, we aim to maximize the profit from an NFT project $N = \{n_1, \ldots, n_k, \ldots, n_{|N|}\}$ with traits $T = \{t_1, \ldots, t_l, \ldots, t_{|T|}\}$, where each NFT n_k has a reserve price p_k and is associated with a set of traits $T_k \subseteq T$, e.g., with a happy face, with a bicycle helmet, and so on. In general, the assessment of an NFT can be estimated according to scarcity and rarity. According to [14], scarcity plays a crucial role in assessing non-fungible objects, such that the assessment of an NFT is likely to boost if it is scarce. In addition to scarcity, the rarity of the traits of an NFT is also crucial. Following [6, 15], the rarity of a trait t_l is inversely proportional to its occurrence in N, i.e.,

Table 1: Notation table.

Notation	Description
N; n_k	NFT project; an NFT
T; t_l	Universe set of traits; a trait
p_k; T_k	Reserve price of n_k; set of traits of n_k
$a_{i,j}$	Activation probability of u_i to u_j
S; (u_i, n_k); S_k	Set of NFT airdrops; an NFT airdrop; set of NFT airdrops for n_k
Q; q_k	Set of NFT quantities; quantity of n_k
$v_{u_i,n_k}(q_k)$	u_i's valuation of n_k when the quantity of n_k is q_k
β_i	Breeding probability of u_i forming NFT offspring
$o_{l,m}$	NFT offspring generated from n_l and n_m
$A(n_k, q_k)$; $A(o_{l,m}, S, Q)$	Assessment of n_k under q_k; assessment of offspring $o_{l,m}$ given S and Q
$f(S, Q)$	Profit of S and Q
$TP(S_k, q_k)$; $OS(S, Q)$	Total transaction prices of n_k generated by S_k subject to q_k; Total assessments of NFT offspring under the influence of S subject to Q
B; b_k	Set of budgets; budget for n_k

$$\gamma(t_l) = \frac{|N|}{Occ(t_l, N)}, \tag{1}$$

where $Occ(t_l, N)$ is the number of NFTs in N with t_l. Then, the overall rarity of an NFT n_k is the sum of the rarity of its traits, i.e.,

$$\Gamma(n_k) = \sum_{t_l \in T_k} \gamma(t_l). \tag{2}$$

Let $Q = \{q_1, \ldots, q_{|N|}\}$ denote the quantity set of the NFT project N, where q_k is the quantity of $n_k \in N$. According to [6], the assessment of an NFT can be estimated as[8]

$$A(n_k, q_k) = e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Gamma(n_k)}, \tag{3}$$

where η_0, η_1, and η_2 are weight parameters. In particular, n_k is assessed at $e^{\eta_1 \frac{1}{q_k}}$ and $e^{\eta_2 \Gamma(n_k)}$ according to scarcity (i.e., the NFT quantity) and rarity, respectively, while e^{η_0} represents the assessment related to the NFT project, e.g., the fame of the creator.

Consider a social network $G = (V, E)$, where V is the node set representing users, and E is the edge set standing for friendships. Each user $u_i \in V$ has a preference for an NFT n_k, denoted as $w_{u_i,n_k} \in [0, 1]$, which can be learned from [] according to the purchase history of the user and the traits of the NFT. Following [], a user u_i's valuation on an NFT n_k is derived according to u_i's preference for n_k (i.e., w_{u_i,n_k}) and the assessment of n_k (i.e., Equation (3)).

$$v_{u_i,n_k}(q_k) = w_{u_i,n_k} \cdot A(n_k, q_k). \tag{4}$$

Each edge $e_{i,j} \in E$ indicates that $u_i \in V$ has an activation probability of $a_{i,j}$ to influence $u_j \in V$.

According to [], the diffusion process of NFTs consists of two stages: the airdrop stage and the public stage. Let $S = \{(u_i, n_k), \ldots\}$ and $Q = \{q_1, \ldots, q_{|N|}\}$ denote a set of NFT airdrops and the quantity set of the NFT project N, respectively, where (u_i, n_k) represents an NFT airdrop that provides a free NFT $n_k \in N$ for a user $u_i \in V$, and q_k is the quantity of $n_k \in N$. The airdrop stage aims to propagate the influence of S, while the public stage determines who wins the NFTs

[8] $A(n_k, q_k)$ is an objective assessment that does not include user preferences.

according to the influence of S, user valuations, NFT quantities Q, and reserve prices. The details of each stage are described as follows.

Airdrop stage. The influence propagation of NFTs typically follows existing diffusion models, such as []. Initially, all users are inactive in all NFTs, except for the users in the set of NFT airdrops S who are active in their respective NFT. Following existing diffusion models, a user who is active in an NFT n_k will influence her inactive friend u_j to become active in n_k. Once u_j is successfully influenced by her active friend, she too becomes active in n_k.[9] The influence thus propagates until no users can be newly influenced.

Plubic stage. After the NFT airdrops influence the social network, the marketer allows active users to bid on NFTs and determines the winners according to user valuations, NFT quantities, and reserve prices of NFTs. Following most NFT marketplaces, such as OurSong and OpenSea, each n_k is sold to the highest bidders. Specifically, the users who are active in n_k and have the q_k highest valuations win the NFT n_k, where the valuations should be no smaller than the reserve price p_k. For a user u_i winning an NFT n_k, the *transaction price* is equal to her valuation $v_{u_i,n_k}(q_k)$, where $v_{u_i,n_k}(q_k) \geq p_k$. Note that n_k may not be sold out if there are less than q_k users with valuations at least p_k.

Due to the characteristics of NFTs, NFT marketplaces earn profit not only from the current NFTs N but also from NFT offspring. Following Roaring Leader, Axie Infinity, STEPN, and CryptoKitties, when a user u_i owns two different NFTs, she will form an NFT offspring with a breeding probability of β_i. The NFT offspring is determined by its parent NFTs' traits, which further dominate the assessment of this NFT offspring. Specifically, for parent NFTs n_k and n_m, let $o_{k,m}$ and $T_{k,m}$ denote the generated NFT offspring and its trait set, respectively.[10] Since the diffusion of S and Q will reach different users and then affect the breeding of NFT offspring, the assessment of $o_{k,m}$ is derived according to Equation (3) as follows.

$$A(o_{k,m}, S, Q) = e^{\eta_0+\eta_1 \frac{1}{q_{k,m}(S,Q)}+\eta_2 \sum_{t_l \in T_{k,m}} \frac{|N \cup O(S,Q)|}{Occ\left(t_l, N\cup O(S,Q)\right)}}, \quad (5)$$

where $q_{k,m}(S,Q)$ is the quantity of $o_{k,m}$ given S and Q, and $O(S,Q)$ is the set of NFT offspring given S and Q. Similar to Equation (3), $o_{k,m}$ is assessed at $e^{\eta_1 \frac{1}{q_{k,m}(S,Q)}}$ and $e^{\eta_2 \sum_{t_l \in T_{k,m}} \frac{|N\cup O(S,Q)|}{Occ\left(t_l, N\cup O(S,Q)\right)}}$ according to scarcity and rarity, respectively, while e^{η_0} indicates the assessment related to the NFT project. As NFT marketplaces earn profit from the service fees based on the transaction price, e.g., 2.5% on OpenSea, we aim to maximize the profit by maximizing the transaction prices of current NFTs and the assessments of NFT offspring (which determine the subsequent transaction prices). Therefore, the profit function is defined as follows.

Definition 3.1 (Profit Function). Given an NFT project N with traits T and a social network G, the profit of S for the NFT project N with quantities Q consists of the transaction prices of NFTs in N and the assessments of NFT offspring generated from N as follows.

$$f(S,Q) = \sum_{k=1}^{|N|} TP(S_k, q_k) + OS(S,Q), \quad (6)$$

where

$$TP(S_k, q_k) = \sum_{u_i \in V(S_k,q_k)} v_{u_i,n_k}(q_k) \quad (7)$$

is the transaction prices of n_k, and

$$OS(S,Q) = \sum_{u_i \in \mathcal{V}(S,Q)} \sum_{n_l,n_m \in W(u_i,S,Q), l<m} \beta_i \cdot \mathbb{E}[A(o_{l,m}, S, Q)] \quad (8)$$

is the assessments of NFT offspring generated under the influence of S with quantities Q. In Equation (7), $S_k = \{(u, n_k) : (u, n_k) \in S\} \subseteq S$ consists of NFT airdrops in S that provide a free NFT n_k for some user u, and $V(S_k, q_k)$ is the set of users winning the NFT n_k under the influence of S_k with the quantity q_k. In Equation (8), under the influence of S with quantities Q, $\mathcal{V}(S,Q)$ is the set of users winning at least two NFTs, $W(u_i, S, Q)$ is the set of NFTs won by u_i, and $\mathbb{E}[A(o_{l,m}, S, Q)]$ is the expected assessment of the offspring NFT $o_{l,m}$ generated from parents n_l and n_m.

Consequently, we formally formulate *NFT Profit Maximization (NPM)* as follows.

Definition 3.2 (NFT Profit Maximization (NPM)). Given an NFT project N with traits T, a social network G, a set of budgets $B = \{b_1, \ldots, b_{|N|}\}$ for N, and a maximum quantity $q_{\max}$, NPM aims to find a set of NFT airdrops S and a set of NFT quantities Q for N, such that the profit $f(S,Q)$ is maximized, under the budget constraint that $\forall k, |S_k| \leq b_k$ and the quantity constraint that $\forall k, q_k \leq q_{\max}$.

3.2 Hardness Analysis

Theorem 3.1. *NPM is NP-hard and cannot be approximated within a factor of* $1 - \frac{1}{e} + \varepsilon$ *for any* ε *unless* $P = NP$.

Proof. We prove this theorem with a reduction from the Influence maximization problem for the independent cascade model (INFMAX-IC), which is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$ [10]. Given a social network G, and two integers k, l with $k \leq l$, the decision version of INFMAX-IC is to decide whether there exists a seed set S of size k such that $\sigma(S) = l$, where $\sigma(S)$ is the expected number of active nodes at the end of process, given that S is the initial active set. Given an instance (G, k, l) of INFMAX-IC, we construct an instance of NPM as follows. i) The social network of NPM instance is a copy of G. ii) The NFT project $N = \{n_1\}$, reserve price $p_1 = 1$, seed budget $b_1 = k$ and valuation $v_{u,n_1}(q_1) = 1$ for each user $u \in V$ and any quantity q_1 of NFT n_1.

To complete the proof, we show that there exists a seed set S of size k such that $\sigma(S) = l$ in INFMAX-IC if and only if there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S,Q) = l - k$ in NPM. We first prove the necessary condition. Suppose that there exists a seed set S of size k such that $\sigma(S) = l$ in INFMAX-IC. Then the corresponding user set S in the constructed NPM instance together with the set of NFT quantity $Q = \{q_1 = \infty\}$ is a solution such that $f(S,Q) = l - k$. We then prove

[9] For the promotional relationship between different NFTs, i.e., a user influenced by an NFT n_k is more likely to be influenced by another NFT n_m ($m \neq k$), diffusion models for multiple correlated items [] can be adopted for the proposed problem.

[10] Basically, offspring have a high probability of inheriting traits of their parents. Nevertheless, different NFT breeding mechanisms have diverse ways of determining the traits of NFT offspring. For example, Cryptokitties consider one primary gene for appearance and three hidden genes for passing on to offspring, and further allow genes to mutate (see https://guide.cryptokitties.co/guide/cat-features/genes for more details).

the sufficient condition. Suppose that there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S, Q) = l - k$ in NPM. Then the corresponding set S in INFMAX-IC is a solution such that $\sigma(S) = l$. Therefore, NPM is NP-hard and cannot be approximated within a factor of $1 - \frac{1}{e} + \varepsilon$ for any ε unless $P = NP$. The theorem follows. □

4 APPROXIMATION ALGORITHM

4.1 Algorithm Overview

To efficiently solve NPM, we design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, including the following novel ideas. 1) To achieve high transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to evaluate the possible profit of a set of users if they are selected for airdrops. Let *prospective purchasers* represent the users with valuations no smaller than the reserve price. QOOA first finds the likelihood of each user in the social network influencing the prospective purchasers. For an NFT quantity q, QSP carefully evaluates the total profit of a set of users by deriving the likelihood of them influencing the top-q prospective purchasers and the valuations of the top-q prospective purchasers. Equipped with QSP, QOOA is able to efficiently filter out inappropriate sets of users for airdrops.

2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find the upper bound of the profit subject to the quantity for an NFT. Specifically, as the highest bidders win the NFT, the influenced users with the highest valuations are vital since they affect the transaction prices as well as the profit. According to Equations (3), (4), and (5), when a larger quantity is available, user valuations tend to decrease as the NFT becomes less scarce, while the profit may rise due to additional transaction prices generated by additional buyers. However, if user valuations fall behind the reserve price, no additional transaction price will be generated. Hence, VQI captures the relationship between the reverse price and user valuations under varying quantities. QOOA is thus able to find the upper bound of the profit subject to each specific quantity to facilitate the identification of the proper quantity.

3) To increase the profit earned from NFT offspring, QOOA aims to encourage users to purchase multiple NFTs with rare traits. First, QOOA identifies *Rare Trait Collectors (RTCs)* as the users who have purchased at least one NFT with the rarest traits. Then, for each NFT with the rarest traits, QOOA attempts to find alternative airdrops that encourage RTCs, rather than other users, to purchase it. This is because joint purchases of the NFTs with rare traits can generate additional valuable NFT offspring.

In summary, QOOA consists of two steps: Quantity-driven Airdrop Selection (QAS) and Offspring Profit Enhancement (OPE). In QAS, for each NFT, QOOA evaluates QSP of users to find proper airdrops to maximize the transaction prices. It iteratively evaluates the profits with increasing quantities until no more profit can be generated by providing an additional quantity, based on the upper bound of the profit derived by VQI. After finding the best NFT airdrops and quantities identified in QAS, QOOA leverages OPE to improve profit from offspring, by encouraging RTCs to jointly purchase multiple NFTs with rare traits. The pseudo-code of QOOA is presented in Algorithm 1.

Algorithm 1: QOOA

Input: NFT project N with T, social network G, budget B
Output: NFT airdrops S and quantities Q

/* QAS phase */
for *each* $n_k \in N$ **do**
  $q_k \leftarrow 1$
  $q_k^* \leftarrow null$; $S_k^* \leftarrow null$; $TP_k^* \leftarrow 0$
  while $UB(n_k, q_k) > TP_k^*$ **do**
    $S_k \leftarrow \emptyset$
    $TP_k \leftarrow \sum_{u_j \in V(S_k, q_k, p_k)} v_{u_j, n_k}(q_k)$
    while $|S_k| < b_k$ **do**
      $U \leftarrow \emptyset$
      for *each* $(u_i, n_k) \notin S_k$ **do**
        if $QSP(S_k \cup \{(u_i, n_k)\}, n_k, q_k) \geq TP_k$ **then**
          $U \leftarrow U \cup \{u_i\}$
      $(u_i^*, n_k) \leftarrow \operatorname{argmax}_{u_i \in U} Gain\big((u_i, n_k), S_k\big)$
      if $Gain\big((u_i^*, n_k), S_k\big) > 0$ **then**
        $S_k \leftarrow S_k \cup \{(u_i^*, n_k)\}$
        $TP_k \leftarrow TP_k + Gain\big((u_i^*, n_k), S_k\big)$
      else
        break
    if $TP_k > TP_k^*$ **then**
      $q_k^* \leftarrow q_k$; $S_k^* \leftarrow S_k$; $TP_k^* \leftarrow TP_k$
$S \leftarrow \bigcup_{n_k \in N} S_k^*$; $Q \leftarrow \{q_1^*, \ldots, q_{|N|}^*\}$
/* OPE phase */
$N^{\mathrm{T}} \leftarrow$treasures of NFTs; $RTC \leftarrow RTC(S, Q)$
for *each* $u_c \in RTC$ **do**
  for *each* $n_k \in N^{\mathrm{T}} \setminus W(u_c, S, Q)$ **do**
    $(u_i^*, n_k) \leftarrow \operatorname{argmin}_{(u_i, n_k) \in S_k^*} MTI(u_i, n_k, q_k^*)$
    $(u_j^*, n_k) \leftarrow \operatorname{argmax}_{(u_j, n_k) \notin S_k^*} MTI(u_j, n_k, q_k^*)$
    if $MTI(u_i^*, n_k, q_k^*) < MTI(u_j^*, n_k, q_k^*)$ **then**
      $S' \leftarrow S \setminus \{(u_i^*, n_k)\} \cup \{(u_j^*, n_k)\}$
      if $f(S', Q) > f(S, Q)$ **then**
        $S \leftarrow S'$
return S, Q

4.2 Algorithm Description

Quantity-driven Airdrop Selection (QAS). QAS aims to maximize the transaction prices by finding appropriate airdrops and NFT quantity. Let $S_k^{q_k}$ denote the set of airdrops for NFT n_k identified by QAS under q_k. Specifically, for each NFT n_k, QAS starts from $q_k = 1$ and finds S_k^1 that maximizes the total transaction prices $TP(S_k^1, 1)$. At each iteration, before increasing q_k from x to $x + 1$ (where x is a natural number), QAS derives the upper bound of the total transaction prices for $q_k = x + 1$, and examines whether the upper bound is greater than the total transaction prices generated by $S_k^{q_k}$ under $q_k = 1, \ldots, x$. Finally, QAS identifies the best quantity and the corresponding airdrops that maximize the total transaction prices.

In order to efficiently extract appropriate users for airdrops, QAS first identifies the *prospective purchasers*, who have valuations no less than the reserve price, since only influencing prospective purchasers can lead to profit. As user valuations decrease when a larger quantity is provided, QAS evaluates user valuations subject to the minimum NFT quantity (i.e., $q_k = 1$) to identify the prospective purchasers. This approach includes all users that have chances to purchase the NFT subject to larger quantities. Specifically, for NFT n_k with the reserve price p_k, the set of prospective purchasers is $V_{\text{pp}}(n_k) = \{u : v_{u,n_k}(1) \geq p_k\}$.[11] For each prospective purchaser, QAS searches for its reverse reachable sets in different deterministic realized graphs of G. Then, QAS derives the likelihood of any user $u_i \in V$ influencing a prospective purchaser $u_j \in V_{\text{pp}}(n_k)$ according to the frequency of u_i occurring in u_j's reverse reachable sets, denoted as $feq(u_i, u_j)$. Accordingly, for a set of users $U = \{u_1, \ldots, u_{|U|}\}$, QAS obtains U's Influence on Prospective Purchasers (IPP) as a set, denoted as $IPP(U, n_k)$, as follows.

$$IPP(U, n_k) = \{\big(u, \sum_{u_i \in U} feq(u_i, u)\big) : u \in V_{\text{pp}}(n_k), \sum_{u_i \in U} feq(u_i, u) > 0\}. \tag{9}$$

Equipped with IPP, QAS evaluates *Quantity-Sensitive Profit (QSP)* of a set of users subject to a specific quantity. For NFT n_k with $q_k = x$, QSP of a set of users U is the sum of weighted transaction prices of the prospective purchasers with the top-x valuations influenced by U. Let $IPP(U, n_k)[x]$ denote the set of the prospective purchasers with the top-x valuations in $IPP(U, n_k)$. QSP of U under $q_k = x$ is

$$QSP(U, n_k, x) = \sum_{u_j \in IPP(U, n_k)[x]} \big(v_{u_j, n_k}(x) \sum_{u_i \in U} feq(u_i, u_j)\big), \tag{10}$$

where the sum of likelihood of each $u_i \in U$ influencing the prospective purchaser u_j stands for the weight $\sum_{u_i \in U} feq(u_i, u_j)$ on u_j's transaction price $v_{u_j, n_k}(x)$.

Afterward, to maximize the total transaction prices for NFT n_k with $q_k = x$, QAS iteratively selects the best airdrop and adds it to the current set S_k^x of airdrops. To efficiently prune inferior airdrops, when considering (u_i, n_k), QAS first evaluates QSP of $S_k^x \cup \{(u_i, n_k)\}$ and compares it with the total transaction prices obtained by S_k^x. If $QSP\big(S_k^x \cup \{(u_i, n_k)\}, n_k, x\big)$ is less than $TP(S_k^x, x)$, i.e., adding (u_i, n_k) to S_k^x does not yield greater total transaction prices, it is unnecessary to consider (u_i, n_k), since QSP is the upper bound of the total transaction prices. Otherwise, QAS evaluates the marginal gain of total transaction prices of adding (u_i, n_k) to S_k^x, i.e.,

$$Gain\big((u_i, n_k), S_k^x\big) = TP(S_k^x \cup \{(u_i, n_k)\}, x) - TP(S_k^x, x). \tag{11}$$

Then, when $\left|S_k^x\right| < b_k$, QAS adds (u_i, n_k) with the largest marginal gain to S_k^x if $Gain\big((u_i, n_k), S_k^x\big)$ is positive, because $TP(S_k^x, x)$ is not monotonically increasing (proved in Lemma 4.1 later) and adding (u_i, n_k) with a negative marginal gain to S_k^x will decrease the total transaction prices.

After $q_k = x$ is examined, QAS continues to find $S_k^{q_k}$ for $q_k = x+1$. Before the search for $q_k = x + 1$, QAS derives *Valuation-based Quantity Inequality (VQI)* by capturing the relationship between the reserve price p_k and user valuations to find the upper bound of the total transaction prices for $q_k = x + 1$. Let $W(n_k, y)$ denote the y-th largest user preference for n_k. The corresponding user valuation is eligible for a transaction price only if it is larger than the reserve price p_k, i.e.,

$$\begin{aligned} & W(n_k, y) \cdot A(n_k, q_k) \\ = \; & W(n_k, y) \cdot e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Gamma(n_k)} \\ = \; & W(n_k, y) \cdot e^{\eta_0 + \eta_2 \Gamma(n_k)} \cdot e^{\frac{\eta_1}{q_k}} \\ = \; & W(n_k, y) \cdot \eta_3 \cdot e^{\frac{\eta_1}{q_k}} \qquad (\text{Let } \eta_3 = e^{\eta_0 + \eta_2 \Gamma(n_k)}) \\ \geq \; & p_k. \end{aligned}$$

Accordingly, VQI infers the maximum quantity that ensures the user valuation $W(n_k, y) \cdot A(n_k, q_k)$ eligible for a transaction price

$$q_k \leq \frac{\eta_1}{\ln(\frac{p_k}{W(n_k, y) \cdot \eta_3})} \tag{12}$$

when $W(n_k, y) \cdot \eta_3 < p_k$. If $W(n_k, y) \cdot \eta_3 \geq p_k$, there is no constraint on the quantity since the corresponding user valuation must be larger than p_k for any quantity.

Next, QAS derives the upper bound of $TP(S_k^{x+1}, x + 1)$ under $q_k = x + 1$ by summing up the transaction prices of the top-$(x + 1)$ user valuations, because the best airdrops subject to $q_k = x + 1$ is to influence all the users with the top-$(x + 1)$ valuations. According to Equation (12), the transaction price of the y-th largest user valuation is $W(n_k, y) \cdot A(n_k, x + 1)$ if $x + 1 \leq \frac{\eta_1}{\ln(\frac{p_k}{W(n_k, y) \cdot \eta_3})}$; otherwise, the transaction price is 0. Hence, the upper bound of $TP(S_k^{x+1}, x + 1)$ under $q_k = x + 1$ is

$$UB(n_k, x + 1) = \sum_{y=1}^{y'} W(n_k, y) \cdot A(n_k, x + 1), \tag{13}$$

where y' is the minimum value making Equation (12) not hold. Consequently, if there exists $x' \leq x$ leading to $UB(n_k, x + 1) \leq TP(S_k^{x'}, x')$, i.e., $TP(S_k^{x+1}, x + 1)$ for all possible S_k^{x+1} will not be greater than the total transaction prices generated by the identified airdrops under some quantity examined so far, QAS terminates the searches for more quantities. QAS then assigns $q_k = x'$ and $S_k = S_k^{x'}$ if $TP(S_k^{x'}, x')$ is the largest among all examined quantities $1, \ldots, x$.

Offspring Profit Enhancement(OPE). To further increase the profit, OPE encourages generating rare NFT offspring. OPE first ranks the rarity of traits to identify the rarest traits. Then, OPE regards the NFTs with the rarest traits as *treasures*, and users purchasing at least one treasure are recognized as *Rare Trait Collectors (RTCs)*. The goal of OPE is to encourage RTCs to purchase more treasures by adjusting the airdrops of the treasures.

Specifically, let $T^{\text{R}} \subseteq T$ denote the set of rarest traits according to the trait rarity.[12] Based on T^{R}, OPE defines the *treasures* of NFTs as $N^{\text{T}} = \{n_k : T_k \cap T^{\text{R}} \neq \emptyset\}$, where each $n_k \in N^{\text{T}}$ is associated with a treasure value $\theta_k = \left|T_k \cap T^{\text{R}}\right|$.[13] Then, OPE introduces *Rare Trait Collectors (RTCs)*, who purchase at least one treasure. It evaluates each RTC u_c with the *Collector Index (CI)*, which captures the

[11] As the reserve price p_k of an NFT n_k is given as a fixed value, we omit p_k from $V_{\text{pp}}(n_k)$.

[12] Following [15], the top 10% rarest ones are usually discussed.

[13] For an NFT that is not a treasure, its treasure value is 0.

probability of u_c to breed offspring and the treasure values of the NFTs u_c purchases, i.e., $CI(u_c, S, Q) = \beta_c \cdot \sum_{n_k \in W(u_c,S,Q)} \theta_k$, where β_c is the breeding probability of u_c, and $W(u_c, S, Q)$ is the set of NFT u_c purchases under the influence of S with Q. OPE prioritizes RTCs with a high CI since they have more chances to generate rare NFT offspring.

For each RTC u_c, OPE explores the treasures not purchased by u_c, starting from the NFT with the highest treasure value, and tailors the airdrops to persuade u_i to purchase them. For each treasure n_k, OPE first examines whether u_c's valuation on n_k exceeds the reserve price p_k. If $v_{u_c,n_k}(q_k) < p_k$, OPE does not adjust the airdrops of n_k for u_c since u_c will not be able to purchase n_k. Otherwise, OPE evaluates each user with the likelihood of influencing RTCs and CI of the influenced RTCs. Similar to IPP (Equation (9)), OPE discovers the Influence on Top-q_k Purchasers (ITP-q_k) of a user u_i's as follows.

$$ITP(u_i, n_k, q_k) = \{\big(u, feq(u_i, u)\big) : v_{u,n_k}(q_k) \geq p_k, feq(u_i, u) > 0, \\ |ITP(u_i, n_k, q_k)| = q_k\}, \quad (14)$$

such that any $\big(u', feq(u_i, u')\big) \notin ITP(u_i, n_k, q_k)$ does not satisfy $feq(u_i, u') > 0$ and $v_{u',n_k}(q_k) > v_{u,n_k}(q_k)$, where $\big(u, feq(u_i, u)\big) \in ITP(u_i, n_k, q_k)$. Note that different from IPP, ITP-q_k takes into account q_k since q_k has been determined in QAS. OPE can evaluate the *Multi-Treasure Influence (MTI)* of a user u_i more accurately by summing up the weighted CI of RTCs in the top-q_k influenced purchasers.

$$MTI(u_i, n_k, q_k) \quad (15)$$
$$= \sum_{\big(u_c, feq(u_i,u_c)\big) \in ITP(u_i,n_k,q_k), u_c \in RTC(S,Q)} CI(u_c, S, Q) \cdot feq(u_i, u_c),$$

where $RTC(S, Q)$ is the set of RTCs under the influence of S with Q, and $feq(u_i, u_c)$ stands for the weight on u_c's CI. For 1) each user u_i selected for airdrops but having the least MTI and 2) another user u_j not selected for airdrops and having the largest MTI, if u_i is less likely to persuade RTCs to purchase n_k than u_j, i.e., $MTI(u_i, n_k, q_k) < MTI(u_j, n_k, q_k)$, OPE attempts to replace the airdrop from u_i to u_j in order to facilitate NFT offspring with rare traits. It updates S as $S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}$ if the above replacement increases the profit, i.e., $f(S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}, Q) > f(S, Q)$.

4.3 Theoretical Results

For each $k \in \{1, 2, \ldots, |N|\}$, let $Act(S_k)$ denote the set of users being active in NFT n_k. Please note that $V(S_k, q_k) \subseteq Act(S_k)$ and $|V(S_k, q_k)| \leq q_k$ for each $k \in \{1, 2, \ldots, |N|\}$. Moreover, $S = S_1 \cup S_2 \cup \cdots \cup S_{|N|}$, and $S_k \cap S_l = \emptyset$ for any $k, l \in \{1, 2, \ldots, |N|\}$ with $k \neq l$. For each $k \in \{1, 2, \ldots, |N|\}$, define

$$r(S_k, q_k) = \sum_{u_i \in Act(S_k), v_{u_i,n_k}(q_k) \geq p_k} v_{u_i,n_k}(q_k)$$

to be the function for the total valuations of being active in NFT n_k with transaction price constraint (i.e., $v_{u_i,n_k}(q_k) \geq p_k$). Since $V(S_k, q_k) \subseteq Act(S_k)$, it follows

$$TP(S_k, q_k) \leq r(S_k, q_k) \quad (16)$$

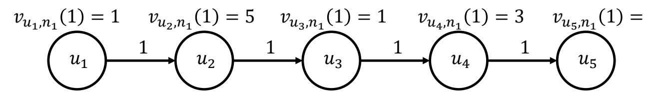


Figure 2: An example for Lemma 4.1.

for each $k \in \{1, 2, \ldots, |N|\}$. Moreover, by Equations (3) and (4), we have $v_{u_i,n_k}(q_k) \geq v_{u_i,n_k}(q'_k)$ and thus

$$r(S_k, q_k) \geq r(S_k, q'_k) \quad (17)$$

for $q_k \leq q'_k$. Please note that in the influence propagation of NFTs, for each NFT n_k, its total transaction price function $TP(S_k, q_k)$ is independent of each other.

In the following, we first check whether the total transaction price function $TP(S_k, q_k)$ is submodular.

Definition 4.1 (Submodular function [12]). Given a ground set U, a set function $\rho : 2^U \mapsto \mathbb{R}$ is submodular if for any subsets X, Y with $X \subseteq Y$ and any element $u \in U \setminus Y$,

$$\rho(Y \cup \{u\}) - \rho(Y) \leq \rho(X \cup \{u\}) - \rho(X). \quad (18)$$

Unfortunately, $TP(S_k, q_k)$ is non-monotonically increasing and far from submodular.

Lemma 4.1. *For each $k \in \{1, 2, \ldots, |N|\}$, the total transaction price function $TP(S_k, q_k)$ is neither monotonically increasing nor submodular.*

PROOF. We prove this result by giving an instance as follows. Let the NFT project $N = \{n_1\}$, seed budget $b_1 = 3$, NFT quantity $q_1 = 1$, reserve price $p_1 = 1$ and the social network G be a path graph P_5 of five vertices, i.e., there are five users $u_1, u_2, \ldots, u_5$, and four edges $e_{i,i+1}$, $1 \leq i \leq 4$, where each edge $e_{i,i+1}$ has the activation probability of $a_{i,i+1} = 1$, as shown in Figure 2. The valuations of all users u_i for NFT n_1 are $v_{u_1,n_1}(q_1) = 1$, $v_{u_2,n_1}(q_1) = 5$, $v_{u_3,n_1}(q_1) = 1$, $v_{u_4,n_1}(q_1) = 3$, and $v_{u_5,n_1}(q_1) = 1$, respectively. In this instance, when (u_3, n_1) is chosen as the only seed, users u_4 and u_5 are activated, and thus u_4 wins the NFT n_1, i.e., $TP(\{(u_3, n_1)\}, q_1) = 3$. However, if we choose (u_3, n_1) and (u_4, n_1) as the seeds, only user u_5 is activated, and thus $TP(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 1$, which implies that the revenue function $TP(S_1, q_1)$ is non-monotonically increasing.

To prove the non-submodularity, we consider two seed groups $\{(u_1, n_1), (u_3, n_1)\}$ and $\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}$. The former seed group results in $f(\{(u_1, n_1), (u_3, n_1)\}, q_1) = 5$ because u_2, u_4 and u_5 are activated, and u_2 wins NFT n_1, and the latter results in $TP(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) = 5$ because u_2 and u_5 are activated, and u_2 wins NFT n_1. Therefore,

$$TP(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) - TP(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 4$$

and

$$TP(\{(u_1, n_1), (u_3, n_1)\}, q_1) - TP(\{(u_3, n_1)\}, q_1) = 2,$$

which implies that the total transaction price function $TP(S_1, q_1)$ is non-submodular. The lemma follows. □

Different from the total transaction price function $TP(S_k, q_k)$, the total valuation function $r(S_k, q_k)$ is submodular.

Lemma 4.2. *For each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing but submodular in the seed selection.*

PROOF. Since the profit of all users being seeds is zero, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing for each $k \in \{1, 2, \ldots, |N|\}$. To prove the submodularity, we show that each function $r(S_k, q_k)$ satisfies (18). Following the proof in [10] where the submodularity of the influence function on the IC model can be reduced to that in a deterministic graph realized by flipping the coin for each edge of $G = (V, E)$, we show the submodularity of $r(S_k, q_k)$ in a deterministic graph $G' = (V, E') \subseteq G$ realized by flipping the coin for each edge of G with the activation probability. The process of influence propagation on G can be regarded as the influence propagation process upon the deterministic graph G'. Thus, for any activated user, its influence propagation is a connected subgraph of G' rooted by some seed user u, i.e., $(u, n_k) \in S_k$. For any seed group S_k, the function $r(S_k, q_k)$ is the total valuations (excluding the valuations violating the transaction price constraint $v_{u_i,n_k}(q_k) \geq p_k$) of the union of the connected subgraphs rooted by all seed users in S_k. Therefore, for each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is a coverage function, which is submodular [11]. The lemma follows. □

Let (S^{opt}, Q^{opt}) denote the optimal solution of NPM, where $S^{opt} = S_1^{opt} \cup S_2^{opt} \cup \cdots \cup S_{|N|}^{opt}$ and $Q^{opt} = \{q_1^{opt}, q_2^{opt}, \ldots, q_{|N|}^{opt}\}$. To derive the approximation ratio, we consider a problem similar to NPM, named NPM-QO, where the quantity constraint is removed. In NPM-QO, all active users with valuations satisfying the reserve price constraint $v_{u_i,n_k}(q_k) \geq p_k$, without the restriction that the q_k highest valuations win the NFT n_k. The user u_i's valuation on NFT n_k is $v_{u_i,n_k}(1)$, and $\beta_i = 0$ for each $i \in \{1, 2, \ldots, |V|\}$. Analogously, let $\hat{S}^{opt}$ denote the optimal solution of NPM-QO, where $\hat{S}^{opt} = \hat{S}_1^{opt} \cup \hat{S}_2^{opt} \cup \cdots \cup \hat{S}_{|N|}^{opt}$ satisfies $\hat{S}_k^{opt} = \arg\max_{S_k} r(S_k, 1)$ for each $k \in \{1, 2, \ldots, |N|\}$.

Lemma 4.3. *For each $k \in \{1, 2, \ldots, |N|\}$, $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot TP(S_k^{opt}, q_k^{opt})$ holds, where $\hat{S}_k$ is the solution found by the continuous greedy algorithn [3] to solve NPM-QO for NFT n_k.*

PROOF. By Lemma 4.2, the total valuation function $r(S_k, 1)$ is submodular for each $k \in \{1, 2, \ldots, |N|\}$. According to [3], the seed set $\hat{S}_k$ found by the continuous greedy algorithm is a $\frac{1}{e}$-approximation solution of NPM-QO for NFT n_k, i.e., $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot r(\hat{S}_k^{opt}, 1)$. To complete the proof, for each $k \in \{1, 2, \ldots, |N|\}$, we show $r(\hat{S}_k^{opt}, 1) \geq TP(S_k^{opt}, q_k^{opt})$ as follows.

$$r(\hat{S}_k^{opt}, 1) \geq r(S_k^{opt}, 1) \geq r(S_k^{opt}, q_k^{opt}) \geq TP(S_k^{opt}, q_k^{opt}),$$

where the first, second, and last inequalities are obtained due to the optimality of $\hat{S}_k^{opt}$ to NPM-QO for NFT n_k, Equation (17), and Equation (16), respectively. The lemma follows. □

Next, we show the relation between the total transaction prices of N and the assessments of NFT offspring generated from N. Let $A_{\max}$ denote the maximum assessment of an offspring, i.e.,

$$A_{\max} = e^{\eta_0 + \eta_1 \frac{1}{1} + \eta_2 |T| \frac{2^{|T|}}{2}} = e^{\eta_0 + \eta_1 + \eta_2 |T| 2^{|T|-1}}, \tag{19}$$

where we assume that the quantity of this offspring is 1 (leading to $e^{\eta_1 \frac{1}{1}}$), it has $|T|$ traits, each trait is only owned by it and one of its parents, and at most $2^{|T|}$ offspring are generated (leading to $e^{\eta_2 |T| \frac{2^{|T|}}{2}}$). For any (S, Q) in NPM, we have

$$\begin{aligned}
&OS(S, Q) \\
&\leq \sum_{k<m} q_{k,m}(S, Q) \cdot A_{\max} \\
&\leq \frac{q_{\max} \frac{|N|(|N|-1)}{2}}{|N|} \cdot \frac{A_{\max}}{p_{\min}} \cdot p_{\min} \cdot |N| \\
&\leq c \sum_{k=1}^{|N|} p_k d_k \qquad (\text{Let } c = \frac{q_{\max}(|N|-1)}{2} \cdot \frac{A_{\max}}{p_{\min}}) \\
&\leq c \sum_{k=1}^{|N|} TP(S_k, q_k),
\end{aligned}$$

where $p_{\min}$ is the minimum reserve price, and d_k is the actual number of the NFT n_k sold. Note that $\sum_{k<m} q_{k,m}(S, Q) \leq q_{\max} \frac{|N|(|N|-1)}{2}$ holds because $q_{\max} \frac{|N|(|N|-1)}{2}$ is the maximum number of offspring that can be generated assuming that $q_{\max}$ quantities of each NFT are sold and exactly $q_{\max}$ users own all $|N|$ NFTs, and $p_{\min} |N| \leq \sum_{k=1}^{|N|} p_k d_k$ holds since $p_{\min} |N|$ is the minimum transaction prices assuming each NFT is sold at $p_{\min}$ with only one quantity.[14] Then, for any (S, Q) in NPM,

$$\begin{aligned}
f(S, Q) &= \sum_{k=1}^{|N|} TP(S_k, q_k) + OS(S, Q) \\
&\leq (1 + c) \cdot \sum_{k=1}^{|N|} TP(S_k, q_k).
\end{aligned} \tag{20}$$

For each $k \in \{1, 2, \ldots, |N|\}$, let (S_k^{alg}, q_k^{alg}) denote the algorithm solution obtained by QOOA for NFT n_k. Then (S^{alg}, Q^{alg}) is the algorithm solution obtained by QOOA, where $S^{alg} = S_1^{alg} \cup S_2^{alg} \cup \cdots \cup S_{|N|}^{alg}$, $Q^{alg} = \{q_1^{alg}, q_2^{alg}, \ldots, q_{|N|}^{alg}\}$, and $f(S^{alg}, Q^{alg}) = \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) + OS(S^{alg}, Q^{alg})$.

Theorem 4.4. *QOOA is a $\frac{1}{ea(1+c)}$-approximation algorithm in $O\big(q_{\max} F |N| (b_{\max} |V| + |N|)\big)$ time for NPM, where $c = \frac{q_{\max}(|N|-1)}{2} \cdot \frac{A_{\max}}{p_{\min}}$, $a = \max_{1 \leq k \leq |N|} \frac{r(\hat{S}_k, 1)}{TP(S_k^{alg}, q_k^{alg})}$ is a data-dependent ratio, $p_{\min}$ is the minimum reserve price, F is the time to evaluate f depending on the evaluation error $\epsilon > 0$, and $b_{\max}$ is the maximum budget in B.*

[14] To ensure that an NFT is marketable in the real world, its reserve price is typically determined according to the valuations of its intended audience.

PROOF. To prove this result, we show $f(S^{alg}, Q^{alg}) \geq \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt})$. For each $k \in \{1, 2, \ldots, |N|\}$, we have

$$\begin{aligned} TP(S_k^{alg}, q_k^{alg}) =& \frac{TP(S_k^{alg}, q_k^{alg})}{r(\hat{S}_k, 1)} \cdot r(\hat{S}_k, 1) \\ \geq& \frac{1}{e} \cdot \frac{TP(S_k^{alg}, q_k^{alg})}{r(\hat{S}_k, 1)} \cdot TP(S_k^{opt}, q_k^{opt}) \\ \geq& \frac{1}{ea} \cdot TP(S_k^{opt}, q_k^{opt}), \end{aligned} \tag{21}$$

where the first and second inequalities are obtained by Lemma 4.3 and the setting of a, respectively. Therefore,

$$\begin{aligned} f(S^{alg}, Q^{alg}) =& \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) + OS(S^{alg}, Q^{alg}) \\ \geq& \sum_{k=1}^{|N|} TP(S_k^{alg}, q_k^{alg}) \\ \geq& \frac{1}{ea} \cdot \sum_{k=1}^{|N|} TP(S_k^{opt}, q_k^{opt}) \\ \geq& \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt}), \end{aligned}$$

where the first inequality is obtained because $OS(S^{alg}, Q^{alg})$ is non-negative by definition, and the second and third inequalities are obtained by Equation (21) and Equation (20), respectively.

Next, we analyze the time complexity. Let F be the time to evaluate the profit function f by Monte Carlo sample depending on the evaluation error $\epsilon > 0$. In the QAS phase, for an NFT n_k with budget b_k, it takes $O(F|V|b_k)$ time to find the airdrops under a specific quantity. As the maximum quantity is $q_{\max}$, it requires $O(q_{\max} b_k F|V|)$ time to identify the most appropriate quantity and the corresponding airdrops for each NFT n_k. Therefore, QAS takes $O(q_{\max} b_{\max} F|V||N|)$ time to find an initial solution of NPM, where $b_{\max}$ is the maximum budget in B. Next, in the OPE phase, it examines $O(|N|)$ replacements of airdrops for an RTC to encourage her to purchase additional treasures, where each examination costs $O(F)$ time. As there are $O(q_{\max}|N|)$ RTCs, i.e., all NFTs (with $q_{\max}$ quantities) are treasures and sold to distinct users, OPE takes $O(q_{\max} F|N|^2)$ time. Consequently, the time complexity of QOOA is $O\big(q_{\max} F|N|(b_{\max}|V| + |N|)\big)$. The theorem follows. □

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