Harnessing Rarity, Scarcity, and Breeding for NFT Influence Maximization on Social Networks

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ABSTRACT

Non-fungible tokens (NFTs), touted as one of the most significant developments for art and technology in the Metaverse, have heavily relied on social networks for marketing. The scarcity, rarity, and unprecedented breeding mechanisms of NFTs have a huge impact on user valuations/assessments of NFTs, thus creating new challenges to viral marketing. In this paper, we make the first attempt to formulate the problem of *NFT Profit Maximization (NPM)*, which aims to maximize the sale profit from the marketplace's perspective, by selecting users for viral marketing (called NFT airdrops) and determining the NFT quantities for sale. We prove the hardness of NPM and design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, which leverages the proposed Quantity-Sensitive Profit to prune inferior airdrops and derives Valuation-based Quantity Inequality to bound the NFT quantities. To increase profit from NFT breeding, QOOA identifies and encourages the Rare Trait Collectors to purchase multiple NFTs with rare traits in order to generate valuable offspring. Experimental results demonstrate that QOOA effectively achieves up to 3.8 times the profit of state-of-the-art approaches in large-scale social networks.

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1 INTRODUCTION

The recent rise of Non-Fungible Tokens (NFTs) has been touted as a significant development intersecting the art and technology in the Metaverse. NFTs can be traced back to 2017 when CryptoKitties, an innovative application of Web3 technologies, demonstrated the idea of trading virtual cats as distinctive digital assets. Since then, the market for NFTs has experienced an unprecedented surge, witnessing remarkable growth of transactions commanding multimillion-dollar sums. Notably, a digital artist Beeple made headlines by auctioning an NFT of his artwork for a groundbreaking $69 million [3]. Other noteworthy NFT transactions include the acquisition of former Twitter CEO Jack Dorsey's inaugural tweet for $2.9 million [14] and a LeBron James highlight video for $208,000 [15].

NFTs, as digital assets verified by blockchain technology to ensure their authenticity and ownership, rely heavily on online social networks for promotion and marketing. Compared with conventional viral marketing, NFTs exhibit new and distinctive marketing features: 1) *Auction-Based Sales*: Most NFT marketplaces sell NFTs via auction mechanisms, which place significant emphasis on the *transaction prices* offered by the highest bidders based on their valuations (i.e., assessments and preferences) of the interested NFTs, as well as the *reserve price* (i.e., the lowest acceptable transaction price) set in the marketplaces. Unlike conventional viral marketing, the profitability is substantially determined by the highest bidding prices, rather than the number of influenced users.

2) *Rarity and Ownership-Driven Valuation*: Unlike everyday necessities, the rarity and notable ownership of NFTs contribute to their elevated value. As an example, CryptoPunk #2924, ranked as the 38th rarest in a collection of 10000 unique punk apes created by Larva Labs, recently fetched an impressive $4.5 million [10]. Moreover, ownership plays a significant role. CryptoPunk #9997 is an excellent example. Before being acquired by the renowned actor Shawn Yue, it was sold for around $159,000. Once in his possession, however, the subsequent sale price soared to $4.35 million [11].

3) *NFT Breeding*: The concept of NFT breeding, which permits a pair of NFTs to breed entirely new and unique offspring, is worth noting. New breeds of NFTs could be more scarce and valuable than their parent NFTs. As these offspring may be auctioned off in the future, they can bring additional profit to the marketplace. By contrast, traditional viral marketing is limited to promoting existing products and cannot leverage the perspective of potential offspring. For instance, in December 2022, Nike introduced CryptoKicks [18], NFT sneakers that contain genotype information, e.g., attributes, colors, styles, backgrounds, etc. CryptoKicks holders can use their NFTs to breed offspring inheriting traits from their parents based on genotype information, and *redeem* them for physical sneakers through Nike's Forging Mechanic.

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Breeding mechanisms have been employed by NFT collections such as Heterosis [13], STEPN [23], Roaring Leaders [21], CryptoKitties [7], and Axie Infinity [2]. Note that the traits of offspring are heavily influenced by the traits of their parent NFTs. In addition, it is a common practice to restrict the user *breeding quota*. For instance, in MODragon, a user is permitted to have at most five breeding pairs of NFTs concurrently [17]. These constraints are designed to foster sustainable growth, ensure a balanced ecosystem, and maintain the rarity of NFT offspring. Furthermore, holders can breed their NFTs with friends' NFTs if the friends' NFTs are available for siring. For example, holders of Roaring Leaders can be matched with friends for *social collaborative breeding*, facilitating the creation of diverse offspring from two holders' NFTs [21]. Owing to these unique features, the promotion of NFTs presents distinct challenges in viral marketing.

NFT marketplaces, such as OpenSea and Blur, host a variety of for-sale NFT projects. In each NFT transaction, the marketplace earns a commission, which is often a percentage of the transaction price, e.g., a 2.5% service fee on OpenSea. This transaction-based profit model motivates these platforms to actively promote the NFT projects they host. One effective promotional method they employ is the *airdrop* strategy for viral marketing on social networks.[1] Through airdrops, influential users are identified and given free NFTs, aiming to boost public awareness and potentially increase participation in those auctions, further maximizing the marketplace's profits by increasing transaction prices from the auctions of NFTs and the potential values of NFT offspring.

The unique characteristics of NFTs highlight several new challenges that arise in the pursuit of maximizing NFT profits: 1) *Maximizing transaction prices*: The profit generated from NFTs relies on the transaction prices, which correspond to the valuations of users willing to purchase the NFTs. A crucial aspect of NFT airdrops is to target users with high valuations. Merely maximizing the spread of influence among users does not guarantee maximum profit, since only a small fraction of influenced users actually make purchases, i.e., only these transactions count for profit. Previous works [26, 27, 35, 48, 61, 66, 80, 82, 84] aim to maximize the influence spread but neglect user valuations, thus failing to ensure the maximization of profit. 2) *Balancing scarcity and widespread adoption*: The scarcity of an NFT significantly impacts users' valuations. Limiting an NFT to being unique can boost valuations, but it restricts profit generation to a single user. Conversely, supplying a large quantity of an NFT allows more users to purchase it but may reduce valuations and transaction prices. Thus, it is critical to place the right amount of NFTs in the marketplace. Previous works [32, 37, 38, 42, 44, 47, 49, 58, 75, 79] fail to account for the impact of scarcity and rarity on valuations and cannot determine appropriate quantities of NFTs. 3) *Leveraging NFT breeding*: The NFT breeding mechanism incentivizes users to own multiple NFTs or to collaborate with friends to produce offspring, both of which have the potential to generate additional profits. The traits and ownership history of parent NFTs significantly impact the assessment of their offspring (e.g., Roaring Leaders parents with rainbow wings are likely to produce offspring with similar traits). Breeding NFTs with rare traits can lead to the creation of coveted offspring. However, excessive breeding may diminish scarcity, rarity, and the assessments of NFT offspring. Hence, the strategic selection of parent NFTs is crucial to producing unique and profitable offspring. Previous works [26, 27, 35, 48, 61, 66, 80, 82, 84] that consider acquisitions of multiple uncorrelated items without additional breeding profits cannot find appropriate airdrops that take breeding into account. An illustrative example, comparing the profit maximization of NFTs with traditional influence/profit maximization, is presented in Appendix A [81].

In this paper, we formulate a new problem, named *NFT Profit Maximization (NPM)*. Consider the social network in a marketplace. Given a project of NFTs with traits, their reserve prices and quantity limits, a user breeding quota, and a set of airdropping budgets, NPM aims to plan for an NFT launch by determining a set of NFT airdrops on the social network and the for-sale quantity of each NFT, to maximize the profit earned from the auctions of NFTs and the potential values of their offspring. The number of airdrops for each NFT is subject to the airdropping budget constraint, and the quantity of each NFT is restricted by the quantity limit. We prove that NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$, and V is the set of users.

To solve NPM, we design an approximation algorithm, named *Quantity and Offspring-Oriented Airdrops (QOOA)*, which incorporates several novel ideas: 1) To maximize transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to estimate the potential profit earned from purchases made by influenced users, by taking account of the quantity constraint. It also identifies prospective purchasers with valuations higher than the reserve price and evaluates the likelihood of the set of airdrops influencing the prospective purchasers with the highest valuations. 2) To balance scarcity and widespread adoption, we derive the *Valuation-based Quantity Inequality (VQI)* for QOOA to efficiently find an upper bound on the profit generated under a specific for-sale quantity. VQI captures the relationships between the reserve price and user valuations for different quantities and infers the quantity for the best profit. 3) To increase the profit from NFT offspring, QOOA identifies the *Rare Trait Collector (RTC)*, which recognizes users according to the trait rarity of the NFTs they may hold. QOOA favors RTCs who are likely to engage in the breeding of valuable offspring, because rarity can influence an NFT's assessment. With the user breeding quota, QOOA meticulously tailors airdrops to influence RTCs for potential breedings of NFTs with rare traits.

With these ideas, QOOA efficiently identifies users inclined to achieve high transaction prices with QSP and trims redundant searches for undesired quantities based on the profit upper bound inferred by VQI. Furthermore, QOOA enhances profit from generating valuable offspring by promoting RTCs and their friends to purchase NFTs with rare traits. We prove that QOOA is an approximation algorithm with a guaranteed performance bound and evaluate the performance on real NFT projects, e.g., Defimons Characters, Lascaux, and Timpers Pixelworks. The contributions of this work are summarized as follows.

[1] Airdrop information for OpenSea and Blur is at https://opensea.io/drops and https://blur.io/airdrop, respectively.

- To the best of our knowledge, NPM, by considering auction-oriented viral marketing, NFT scarcity, trait rarity, and NFT breeding, has not been studied previously. We prove the hardness and inapproximability of NPM.
- We design an approximation algorithm QOOA for NPM. By exploiting the proposed QSP and VQI, QOOA efficiently finds airdrops that maximize the profit (i.e., transaction prices of NFTs and potential values of their offspring) with a performance bound. Furthermore, QOOA identifies RTCs and tailors airdrops to enhance their potential for generating rare offspring.
- Experiments on data of real NFT projects demonstrate that QOOA achieves profits up to 3.8 times over the baselines.

2 RELATED WORK

Influence maximization. Existing research on viral marketing has explored the problem of maximizing the influence or profit/revenue of multiple products within a company. Previous studies [35, 84] assume that products are independent of each other and thus focus on optimizing the adoption of each product individually. Following studies [26, 27, 48, 61, 66, 80, 82] consider the interdependencies between product adoptions, such as complementary (positive impact) or substitutable (negative impact) relationships. Previous works on profit maximization, a variant of influence maximization, in social networks have primarily focused on maximizing the difference between the influence spread and the cost of the seed group [42, 47, 49]. Li et al. [58] introduce the concept of user benefits, aiming to maximize the total benefits, instead of the influence spread. Several studies [32, 75, 79] further incorporate the concept of benefits into profit maximization, i.e., maximizing the difference between the benefits of influence and the cost of seeding. Unlike the above works maximizing the profit from nodes, some works [37, 38] further examine the benefits related to interactions among activated nodes. Han et al. [44] consider the perspective of the host to maximize the revenue of all advertisers. However, the above works do not capture the breeding mechanisms in NFT projects, where owning multiple NFTs may lead to additional profits through NFT breeding. Additionally, these studies do not consider the valuation of products based on their scarcity and rarity, nor do they determine the optimal quantities for the products. As a result, they are inapplicable to NFTs considered in NPM.

NFTs in social networks. NFTs, being digital assets, are particularly well-suited for viral marketing via social networks. Recent studies [50, 72] demonstrate significant impacts of user engagement metrics on Twitter, such as counts of user membership lists, likes, and replies, on NFT trends and prices. Meanwhile, some research [34, 65] focuses on predicting NFT prices based on factors like images, texts, and rarity. Additionally, Casale-Brunet et al. [30] explore how users holding NFTs in leading projects play a pivotal role within large social network communities. However, while these studies investigate the correlation between social networks and the NFT market, they do not address the promotion of NFTs through viral marketing on social networks. Our work builds upon the insights gained from these studies to formulate NPM that closely reflects real-world dynamics in the NFT market.

Table 1: Notation and abbreviation table.

Notation	Description
N; n_k	NFT project; an NFT
T; t_d; T_k	Universal set of traits; a trait; trait set of n_k
P; p_k	Set of reserve prices of N; reserve price of n_k
Q; q_k	Set of NFT quantities; quantity of n_k
$\phi(t_d)$; $\Phi(n_k)$	Rarity of trait t_d; rarity of n_k
H_k; h_k	Ownership history of n_k; impact of H_k
$A(n_k, q_k)$	Assessment of n_k under q_k
G; u_i	Social network; a user
$a_{i,j}$	Activation probability of u_i to u_j
$v_{u_i,n_k}(q_k)$; w_{u_i,n_k}	u_i's valuation of n_k when the quantity of n_k is q_k; u_i's preference for n_k
β_i; γ_i	Breeding probability of u_i; siring probability of u_i
c_{BQ}	User breeding quota
S; (u_i, n_k)	Set of NFT airdrops; an NFT airdrop
S_k	Set of NFT airdrops for n_k
$o_{k,m}$	NFT offspring generated from n_k and n_m
$A(o_{k,m}, S, Q)$	Assessment of offspring $o_{k,m}$ given S and Q
$f(S, Q)$	Profit of S and Q
$TP(S_k, q_k)$; $OS(S, Q)$	Total transaction price of n_k generated by S_k subject to q_k; potential assessment of NFT offspring under the influence of S subject to Q
λ	Parameter to adjust the weighting of OS
ω	Estimated preference of social network users for the entire NFT project
$V(S_k, q_k)$	Set of influenced users holding n_k under the influence of S_k with the quantity q_k
B; b_k	Set of budgets; budget for n_k
L; l_k	Set of quantity limits; quantity limit for n_k
Abbreviation	**Description (where it is defined)**
NPM	NFT Profit Maximization (p. 6)
QOOA	Quantity and Offspring-Oriented Airdrops (p. 6)
QAS	Quantity-driven Airdrop Selection (p. 6)
OPE	Offspring Profit Enhancement (p. 8)
PEP	Potential Expected Profit (p. 7)
QSP	Quantity-Sensitive Profit (p. 7)
VQI	Valuation-based Quantity Inequality (p. 8)
RTC	Rare Trait Collectors (p. 8)
TI	Target Index (p. 9)
VOGI	Valuable Offspring Generation Influence (p. 9)

3 PROBLEM FORMULATION

We first introduce the background of NFTs. Then, we describe the diffusion process of NFT in social networks, followed by the problem formulation of NPM. Finally, we analyze the hardness result. Table 1 summarizes the notations and abbreviations in this paper.

3.1 NFT Terminology

Definition 3.1 (Airdrop). An airdrop, denoted as (u_i, n_k), designates the free distribution of a digital asset n_k, such as a token or a coin, to a user u_i. NFT airdrops aim to incentivize users to engage in viral marketing on social networks by giving complimentary gifts.

Definition 3.2 (Trait). A trait, denoted as t, refers to a distinguishable feature or characteristic associated with an NFT. Traits can include visual elements (e.g., sleepy eyes in Bored Ape Yacht Club [4]), metadata (e.g., the max forging status in Cryptokicks [22]), or other distinctive attributes (e.g., the solar oracle level in Quaere Genesis Pass [20]). Each NFT n_k has a unique combination of traits, denoted as T_k, rendering it a one-of-a-kind piece of artwork.

Definition 3.3 (NFT project). An NFT project N, also known as an NFT collection, refers to a curated assortment of digital assets released by a creator, comprising a limited number of individual NFTs. These collections typically maintain a consistent artistic style across multiple NFTs, while showcasing variations in traits such as appearance or background, collectively forming a universal set of traits T. NFT projects span various forms, including virtual real estate, music albums, and photography.

For example, in Cath Simard Editions [5], the universal set of traits includes {'5000 meters,' '4270 meters,' 'Cath Simard,' 'Peru,' 'Huayhuash,' '2019'}. There are two NFTs. One has five traits: {'5000 meters,' 'Cath Simard,' 'Peru,' 'Huayhuash,' '2019'}, while the other has four traits: {'4270 meters,' 'Cath Simard,' 'Peru,' 'Huayhuash'}.

3.2 Problem Definition

In this paper, we aim to maximize the profit from an NFT project $N = \{n_1, \dots, n_k, \dots, n_{|N|}\}$, where each NFT n_k is associated with a corresponding reserve price $p_k \in P = \{p_1, \dots, p_k, \dots, p_{|N|}\}$ and described by a set of traits $T_k \subseteq T = \{t_1, \dots, t_d, \dots, t_{|T|}\}$, and a history of ownership $H_k \subseteq V$ (where V is the set of users). The value of an NFT can be assessed based on its scarcity, rarity, and ownership [46, 52, 63, 65, 67, 74, 83]. 1) *Scarcity of NFTs:* Following [52, 63, 83], scarcity plays a crucial role in assessing non-fungible objects. Let $Q = \{q_1, \dots, q_{|N|}\}$ denote the quantity set of the NFT project N, where q_k is the quantity of $n_k \in N$. The value of an NFT n_k is likely to boost if it is scarce, i.e., q_k is small. 2) *Rarity of traits:* The rarity of the traits of an NFT is also crucial. Following [46, 65], the rarity of a trait t_d is inversely proportional to its occurrence in N, i.e.,

$$\phi(t_d) = \frac{|N|}{Occ(t_d, N)}, \tag{1}$$

where $Occ(t_d, N)$ is the number of NFTs in N with t_d. The overall rarity of an NFT n_k is thus the sum of the rarity of its traits, i.e.,

$$\Phi(n_k) = \sum_{t_d \in T_k} \phi(t_d). \tag{2}$$

3) *Ownership:* In addition to the intrinsic characteristics (i.e., scarcity and rarity), the ownership history of n_k, especially when held by notable users (e.g., celebrities) [62], usually increases its value.[2] Let h_k denote the impact imparted by the ownership history H_k. The value of h_k can be evaluated by the prominence of users in H_k. Generally, the popularity of celebrities on social networks can be measured by various approaches [31, 45]. For the NFT market, it can be further enhanced by the number of NFTs they hold and the frequency of their transactions [29, 69].

Following [46, 74], the value of n_k with a quantity q_k is assessed as follows.[3]

$$A(n_k, q_k) = e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Phi(n_k) + \eta_3 h_k}, \tag{3}$$

where η_0, η_1, η_2, and η_3 are weight parameters, which can be learned from previous NFT projects with ordinary least square regression in [46]. In particular, n_k is assessed at $e^{\eta_1 \frac{1}{q_k}}$ (assessed higher as n_k becomes scarcer), $e^{\eta_2 \Phi(n_k)}$ (assessed higher when n_k possesses more rare traits [46, 65]), and $e^{\eta_3 h_k}$ (assessed higher when n_k is held by more notable users [62], where h_k is the impact imparted by the ownership history H_k).[4] Meanwhile, for all $n_k \in N$, e^{η_0} represents the assessment related to the whole NFT project, e.g., the reputation of the creator [50].

Consider a social network $G = (V, E)$, where V is the node set representing users, and E is the edge set standing for friendships. Each user $u_i \in V$ has a personal preference for an NFT n_k, denoted as $w_{u_i,n_k} \in [0, 1]$, which can be derived from learning models, such as HG-GNN [68] and DGNN [59], according to the purchase history of the user and the traits of the NFT. Following [53, 54], a user u_i's valuation on an NFT n_k is derived according to u_i's personal preference for n_k (i.e., w_{u_i,n_k}) and the value assessment of n_k (i.e., Equation (3)).

$$v_{u_i,n_k}(q_k) = w_{u_i,n_k} \cdot A(n_k, q_k). \tag{4}$$

Each edge $e_{i,j} \in E$ indicates that $u_i \in V$ has an activation probability of $a_{i,j}$ to influence $u_j \in V$.

In most NFT marketplaces, e.g., OpenSea, the launching process of NFTs consists of two stages: the *airdrop* stage for viral marketing and the *public* stage for auction.[5] Let $S = \{(u_i, n_k), \dots\}$ and $Q = \{q_1, \dots, q_{|N|}\}$ denote a set of NFT airdrops and the quantity set of the NFT project N, respectively, where (u_i, n_k) represents an NFT airdrop that provides a free NFT $n_k \in N$ to a user $u_i \in V$, and q_k is the quantity of $n_k \in N$ for sale in the public stage. The airdrop stage aims to exploit the influence of S to propagate the NFT information over the social network, while the public stage starts the auctions which proceed under the influence of airdrops S, user valuations, NFT quantities Q, and reserve prices P, detailed as follows.

Airdrop stage. The influence propagation of NFTs typically follows existing diffusion models, such as the PTC, MF, TSC-HDM, LT, and IC models [33, 43, 51, 73]. Initially, all users are inactive in all NFTs, except for the users in the set of NFT airdrops S who are active for their respective NFT.[6] Following existing diffusion models, a user who is active in an NFT n_k may influence her inactive friend u_j to become active in n_k. Once u_j is successfully influenced by her active friend, she too becomes active in n_k.[7] The influence thus propagates until no more users can be influenced.

Public stage. After the airdrop stage, the active users bid on NFTs based on their valuations of NFTs. In most NFT marketplaces,

[2] As the user who breeds the offspring becomes its initial holder, ownership has a more pronounced impact on offspring assessment in our model.

[3] $A(n_k, q_k)$ is an objective assessment that is irrelevant to subjective user preferences.

[4] When an NFT n_k has not been held by any user, i.e., $H_k = \emptyset$, h_k is equal to 0.

[5] The airdrop guide of OpenSea: https://support.opensea.io/hc/en-us/articles/13592013208851-Part-2-Prepare-your-drop-schedule.

[6] A user active in an NFT n_k is said to be interested in n_k. Following most diffusion models [33, 43, 51, 73], user states are progressive; that is, active users do not revert to being inactive.

[7] For the promotional relationship between different NFTs, when a user influenced by an NFT n_k is more likely to be influenced by another NFT n_m ($m \neq k$), diffusion models for multiple correlated items [26, 48, 61, 80] can be adopted for the proposed problem.

such as OpenSea and Blur, each n_k is sold to its top q_k bidders.[8] Specifically, each of the active users of n_k who have the top q_k highest valuations acquires a single NFT n_k, where the valuations are no smaller than the reserve price p_k. For a user u_i winning an NFT n_k, the *transaction price* is equal to her offer made according to her valuation $v_{u_i,n_k}(q_k)$, where $v_{u_i,n_k}(q_k) \geq p_k$. Note that n_k may not be sold out if there are fewer than q_k users with valuations of at least p_k.

As NFT marketplaces earn profit from the service fees based on the transaction prices [40], it is natural to maximize the profit by maximizing the transaction prices of current NFTs and the assessments of NFT offspring (which in turn determine the subsequent transaction prices). Given that the breeding mechanism directly affects the total assessment of offspring, most NFT projects have designed their mechanisms with considerations in the following aspects:[9]

1) *Breeding Constraints:* An NFT breeding mechanism usually has some constraints to maintain the NFTs' rarity and regulate breeding frequency. Almost all NFT projects enforce a cooldown period, ensuring *non-concurrent breeding* [1, 6, 8, 13, 19, 24]. Besides, users are restricted by a breeding quota c_{BQ}, i.e., the maximum number of simultaneous breeding pairs owned by a user. For example, in MODragon, $c_{\text{BQ}} = 5$ [17].

2) *Genetic Mechanisms:* Generally, offspring inherit traits from their parent NFTs, but rarer traits have lower inheritance chances. Boosters can raise the inheritance probability of a specific trait by c_{BT}. For example, in Crypto Unicorns, using a berry (booster) associated with a trait increases its inheritance probability by 10% [6].

3) *Collaborative Breeding:* This mechanism, which facilitates community interaction, is prominently adopted by Roaring Leader [21], Axie Infinity [16], and CryptoKitties [9]. An NFT holder u_j has a siring probability, denoted as γ_j, to designate his NFTs as available for siring, indicating a willingness to collaborate with friends' NFTs in the breeding process.[10] Let u_j's friend u_i, desiring to breed her NFTs, have a breeding probability, denoted as β_i. Besides breeding with her own NFTs, u_i can choose to engage with u_j's siring-ready NFTs. It is noteworthy that the initiator of the breeding (i.e., u_i) is the legal owner of the offspring.

An illustrative example is presented in Appendix A [81]. Following famous NFT projects, such as Cryptokitties, Roaring Leader, Axie Infinity, and MODragon, we model the breeding mechanism to include non-concurrent breeding, the user breeding quota, inheritance, and collaborative breeding.[11] After acquiring NFTs during the public stage, users can decide whether to participate in t he breeding process.[12] For a user u_i who possesses NFT n_k, she may offer n_k as a siring-ready NFT to friends with a siring probability γ_i. Alternatively, with a breeding probability β_i, she may opt to breed n_k with another NFT n_m. This n_m can be either an NFT she owns or a siring-ready NFT held by one of her friends. To gain the maximum profit, u_i breeds her n_k with n_m if the generated offspring, denoted as $o_{k,m}$, has the highest assessment among her possible NFT breeding pairs that adhere to the user breeding quota c_{BQ} and non-concurrent breeding.[13] For the offspring $o_{k,m}$, let $T_{k,m}$ and $H_{k,m}$ denote its trait set and ownership history, respectively. Since the diffusion of S with Q reaches different users and then affects the breeding of NFT offspring, following [46, 65, 74], the assessment of $o_{k,m}$ is derived according to Equation (3) as follows.

$$A(o_{k,m}, S, Q) = e^{\eta_0 + \eta_1 \frac{1}{q_{k,m}(S,Q)} + \eta_2 \sum_{t_d \in T_{k,m}} \frac{|N \cup O(S,Q)|}{Occ\left(t_d, N \cup O(S,Q)\right)} + \eta_3 h_{k,m}},$$

where $q_{k,m}(S,Q)$ is the quantity of $o_{k,m}$ given S and Q, $O(S,Q)$ is the set of NFT offspring given S and Q, and $h_{k,m}$ is the impact of u_i since $H_{k,m} = \{u_i\}$.

As NFT marketplaces earn profits primarily from service fees of transactions [40], the profit function is defined as follows.

Definition 3.4 (Profit Function). Consider an NFT project N with traits T and their reserve prices P, a social network G, and a user breeding quota c_{BQ}. The profit of S for the NFT project N with quantities Q consists of the transaction prices of NFTs in N and the assessments of NFT offspring generated from N as follows.

$$f(S,Q) = TP(S,Q) + \lambda \cdot OS(S,Q), \quad (5)$$

where $TP(S,Q)$ is the total transaction price influenced by S under quantities Q, $OS(S,Q)$ is the total assessment of NFT offspring generated under the influence of S with quantities Q, and λ is a parameter to adjust the weighting of OS. Specifically, the total transaction price is defined as follows.

$$TP(S,Q) = \sum_{k=1}^{|N|} TP_k(S_k, q_k) = \sum_{k=1}^{|N|} \sum_{u_i \in V(S_k,q_k)} v_{u_i,n_k}(q_k), \quad (6)$$

where $TP_k(S_k, q_k)$ is the total transaction price of n_k influenced by S_k under the quantity q_k, $S_k = \{(u, n_k) : (u, n_k) \in S\} \subseteq S$ consists of NFT airdrops in S that provide a free NFT n_k to some users, and $V(S_k, q_k)$ is the set of influenced users holding the NFT n_k under the influence of S_k with the quantity q_k.[14] Moreover, $S = S_1 \cup S_2 \cup \cdots \cup S_{|N|}$, and $S_k \cap S_m = \emptyset$ for any $k, m \in \{1, 2, \ldots, |N|\}$ with $k \neq m$. On the other hand, the total assessment of offspring is defined as follows.

$$OS(S,Q) = \sum_{u_i \in \bigcup_{k=1}^{|N|} V(S_k,q_k)} \sum_{z=1}^{c_{\text{BQ}}} \omega \mathbb{E}[A(o_z^i, S, Q)], \quad (7)$$

where o_z^i is the z-th NFT offspring bred by u_i within the user breeding quota c_{BQ} under the influence of S with quantities Q, $\mathbb{E}[A(o_z^i, S, Q)]$ is its expected assessment depending on the breeding mechanism, and ω is the estimated preference of social network users for the entire NFT project (derived from the engagement rate in [39, 70]), as future winning bidders are uncertain. are uncertain.

[8] Following research on influence, revenue, and profit maximization [28, 51, 78], the seed users (i.e., those who receive free samples, referred to as airdrops in the context of NFTs) have already obtained the products. Hence, there is no need for them to participate in the bidding process to acquire the NFTs.

[9] Details on breeding are introduced in Appendix B [81].

[10] Offering NFTs for siring usually enhances one's reputation within the NFT community and may also earn siring fees.

[11] Our problem can accommodate the aforementioned considerations, such as boosters and non-collaborative breeding. Meanwhile, our proposed approach, QOOA, also supports these considerations, as detailed in Appendix C.2 [81].

[12] The likelihood for users to participate in breeding (i.e., the siring and breeding probabilities) can be derived based on their activity histories [67].

[13] If n_m is a siring-ready NFT and is chosen by multiple NFT holders for breeding, n_m breeds with the NFT holder who requests pairing first, as seen in Derby Stars [12].

[14] Since the set $V(S_k, q_k)$ depends on the influence diffusion of S_k and q_k, from this perspective, it can be considered as a random set. Nevertheless, for ease of illustration, we follow [76, 77] to adopt the concept of the live-edge graph, as $V(S_k, q_k)$ is deterministic in each live-edge graph.

Formally, we formulate the problem of *NFT Profit Maximization (NPM)* as follows, where an illustrative example of NPM is presented in Appendix A [81].

Definition 3.5 (NFT Profit Maximization (NPM)). Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T and their reserve prices P, a social network G, a user breeding quota c_{BQ}, a set of airdrop budgets $B = \{b_1, \ldots, b_{|N|}\}$ for N, and a set of quantity limits $L = \{l_1, \ldots, l_{|N|}\}$, NPM aims to plan for an NFT launch by finding a set of NFT airdrops S and a set of NFT quantities Q for N, such that the profit made from airdrops $f(S, Q)$ is maximized, under the budget constraint $\forall k, |S_k| \leq b_k$ and the quantity constraint $\forall k, q_k \leq l_k$.

Theorem 3.1. *NPM is NP-hard and cannot be approximated within a factor of* $|V|^{1/(\log\log|V|)^c}$ *assuming the exponential time hypothesis (ETH), where* $c > 0$ *is a constant independent of* $|V|$.

PROOF. Please refer to Appendix D.1 [81] for the details. □

4 APPROXIMATION ALGORITHM

4.1 Algorithm Overview

To efficiently solve NPM, we design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, based on the following new ideas. 1) To achieve high transaction prices, we propose the notion of *Quantity-Sensitive Profit (QSP)* to estimate the possible profit w.r.t. a set of users if they are selected for airdrops. Given an NFT and a specific quantity for it, let *prospective purchasers* be the users with valuations of the NFT no smaller than its reserve price. QOOA first finds the likelihood for each individual user in the social network to influence the prospective purchasers. QSP carefully upper bounds the total profit from a set of candidate airdrop users by deriving the likelihood for them to influence the prospective purchasers and evaluating their valuations. Equipped with QSP, QOOA is able to efficiently filter out unlikely users for airdrops.

2) To deal with the tradeoff between NFT quantities and user valuations, we derive the *Valuation-based Quantity Inequality (VQI)* in QOOA to efficiently find the upper bound of the profit subject to the quantity for an NFT. Specifically, the influenced users with the highest valuations (i.e., expected winners of the auction) are vital since their bids set the transaction prices and the profit. When a larger quantity is available, user valuations tend to drop as the NFT becomes less scarce, while the profit may rise due to additional transactions generated. However, if user valuations fall below the reserve price, no additional transaction can be achieved. Hence, VQI carefully examines the maximum number of users with valuations no smaller than the reverse price under various quantities, in order to establish corresponding upper bounds on the profit. By exploiting the upper bound subject to each specific quantity, QOOA efficiently prunes redundant searches for quantities producing low profits.

3) To increase the profit earned from NFT offspring, we target users with high impact (those who can elevate the offspring's assessment) and their friends (who may leverage collaborative breeding) to produce offspring with rare traits, while complying with the user breeding quota. QOOA identifies *Rare Trait Collectors (RTCs)* as users who may acquire at least one NFT with the rarest traits. Subsequently, for each such rare-trait NFT, QOOA finds alternative airdrops that specifically target RTCs with great breeding probabilities, significant impacts, and high valuations, in order to increase the breeding opportunity and assessments of offspring. For collaborative breeding, these airdrops prioritize the friends of RTCs if they exhibit great siring probabilities and high valuations. Accordingly, these alternative airdrops replace the original airdrops, leading to the breeding of more valuable NFT offspring.

In summary, QOOA consists of two steps: Quantity-driven Airdrop Selection (QAS) and Offspring Profit Enhancement (OPE). For each NFT, QAS evaluates QSP w.r.t. different sets of users to find airdrops that maximize the total transaction price. It iteratively evaluates the profits with increasing quantities until no more profit can be generated while efficiently trimming redundant searches, according to the upper bound of the profit derived by VQI. After finding the best NFT airdrops and quantities identified in QAS, QOOA leverages OPE to improve profit from breeding by encouraging RTCs and their friends to purchase multiple NFTs with rare traits. The pseudo-code of QOOA is presented in Algorithm 1.[15]

4.2 Algorithm Description

4.2.1 Quantity-driven Airdrop Selection (QAS).

QAS first maximizes the profit by finding appropriate airdrops and NFT quantities. Let $S_k^{q_k}$ denote the set of airdrops for NFT n_k identified by QAS under quantity q_k. Specifically, for each NFT n_k, QAS starts from $q_k = 1$ and finds S_k^1 that maximizes the total transaction price $TP_k(S_k^1, 1)$. Then, QAS iteratively increases q_k by 1 until q_k reaches the quantity limit l_k (Line 3). Finally, QAS identifies the best quantities and the corresponding airdrops that maximize TP_k (Line 21). To improve efficiency, QAS is equipped with two pruning strategies realized by *Quantity-Sensitive Profit (QSP)* (Lines 9-11) and *Valuation-based Quantity Inequality (VQI)* (Lines 4-5). Specifically, QSP is the upper bound of TP_k for a set of airdrops S_k under a specific quantity (proved in Lemma D.2 in Section 4.3), which helps eliminate ineffective airdrops that generate a smaller TP_k than the best airdrops identified so far. On the other hand, VQI infers the upper bound of TP_k under a specific q_k, irrespective of the airdrop set S_k, to facilitate effective pruning of redundant quantities.

QSP-based pruning. Specifically, for each quantity $q_k = x$ and NFT n_k, to maximize TP_k, QAS iteratively selects the best airdrop and adds it to the current set S_k^x of airdrops (Lines 12-17). To efficiently prune ineffective airdrops, when considering (u_i, n_k), QAS first evaluates QSP of $\hat{S}_k^x = S_k^x \cup \{(u_i, n_k)\}$ and compares it with TP_k of S_k^x (Line 10), as QSP serves as the upper bound of TP_k. If QSP of $\hat{S}_k^x$ is smaller than $TP_k(S_k^x, x)$, adding (u_i, n_k) to S_k^x does not yield a greater TP_k. Otherwise, QAS carefully evaluates the marginal gain of TP_k by adding (u_i, n_k) to S_k^x (Lines 14-15), i.e., $TP_k(\hat{S}_k^x, x) - TP_k(S_k^x, x)$. When the budget is sufficient, i.e., $\left|S_k^x\right| < b_k$, QAS adds (u_i, n_k) with the largest marginal gain to S_k^x if the gain is positive (Lines 16-17).[16]

The idea behind QSP w.r.t $\hat{S}_k^x$ is to extract the users generating the top-x expected profits of n_k under $q_k = x$, while being influenced

[15] The workflow of QOOA is illustrated in Appendix C.1 [81].

[16] Since $TP_k(S_k^x, x)$ is not monotonically increasing (proved in Appendix D.2 [81]), adding (u_i, n_k) with a negative marginal gain to S_k^x decreases the profit, because the total transaction price will be lowered.

Algorithm 1: QOOA

Input: NFT project N with traits T and reserve prices P, social network G, user breeding quota c_{BQ}, budgets B, quantity limits L

Output: NFT airdrops S and quantities Q

/* QAS phase */

for *each* $n_k \in N$ **do**
- $q_k^* \leftarrow null$; $S_k^* \leftarrow null$; $TP_k^* \leftarrow 0$
- **for** $q_k = 1, \ldots, l_k$ **do**
  - **if** $UB_k(q_k) \leq TP_k^*$ **then**
    - **continue**
  - $S_k \leftarrow \emptyset$; $TP_k \leftarrow \sum_{u_j \in V(S_k, q_k)} v_{u_j, n_k}(q_k)$
  - **while** $|S_k| < b_k$ **do**
    - $U \leftarrow \emptyset$
    - **for** *each* $(u_i, n_k) \notin S_k$ **do**
      - **if** $QSP_k(S_k \cup \{(u_i, n_k)\}, q_k) \geq TP_k$ **then**
        - $U \leftarrow U \cup \{u_i\}$
    - $u_i^* \leftarrow null$; $gain \leftarrow 0$
    - **for** $u_i \in U$ **do**
      - **if** $TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k) > gain$ **then**
        - $u_i^* \leftarrow u_i$; $gain \leftarrow TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k)$
    - **if** $gain > 0$ **then**
      - $S_k \leftarrow S_k \cup \{(u_i^*, n_k)\}$; $TP_k \leftarrow TP_k + gain$
    - **else**
      - **break**
  - **if** $TP_k > TP_k^*$ **then**
    - $q_k^* \leftarrow q_k$; $S_k^* \leftarrow S_k$; $TP_k^* \leftarrow TP_k$

$S \leftarrow \bigcup_{n_k \in N} S_k^*$; $Q \leftarrow \{q_1^*, \ldots, q_{|N|}^*\}$

/* OPE phase */

N^{T} ←treasures of NFTs; RTC ← RTCs according to N^{T}

Sort N^{T} according to the number of rarest traits possessed

for *each* $n_k \in N^{\text{T}}$ **do**
- $S_k' \leftarrow S_k^*$; $\bar{S}_k' \leftarrow \emptyset$
- **while** $S_k' \neq \emptyset$ **do**
  - $(\underline{u}, n_k) \leftarrow \text{argmin}_{(u, n_k) \in S_k'} VOGI_k(u, q_k^*)$
  - $(u^*, n_k) \leftarrow \text{argmax}_{u \in V, (u, n_k) \notin S_k' \cup \bar{S}_k'} VOGI_k(u, q_k^*)$
  - **if** $VOGI_k(\underline{u}, q_k^*) < VOGI_k(u^*, q_k^*)$ **then**
    - $S_k' \leftarrow S_k' \setminus \{(\underline{u}, n_k)\}$; $\bar{S}_k' \leftarrow \bar{S}_k' \cup \{(u^*, n_k)\}$
    - $S' \leftarrow (S \setminus \{(\underline{u}, n_k)\}) \cup \{(u^*, n_k)\}$
    - **if** $f(S', Q) > f(S, Q)$ **then**
      - $S \leftarrow S'$
  - **else**
    - **break**

return S, Q

by $\hat{S}_k^x$. The sum of these top-x expected profits serves as the QSP of $\hat{S}_k^x$. Thus, we define and identify the *prospective purchasers* as those who have the potential to generate profit.

Definition 4.1 (Prospective purchasers). Prospective purchasers of an NFT n_k are those with the valuations no smaller than the reserve price of n_k. Specifically, for NFT n_k under a specific quantity $q_k = x$ with the reserve price p_k, the set of prospective purchasers is defined as $V_k^{\text{pp}}(x) = \{u : v_{u,n_k}(x) \geq p_k\}$.

QAS evaluates each prospective purchaser u's *Potential Expected Profit (PEP)* of n_k under the influence of $\hat{S}_k^x$ with quantity $q_k = x$, serving as the upper bound of the expected profit on n_k when $\hat{S}_k^x$ successfully influences u. Afterward, QSP w.r.t $\hat{S}_k^x$ is derived by summing the top-x PEP under the influence of $\hat{S}_k^x$ to efficiently upper bound TP_k of $\hat{S}_k^x$ (proved later in Lemma D.2 in Section 4.3).

To find PEP of a prospective purchaser $u \in V_k^{\text{pp}}(x)$ under the influence of $\hat{S}_k^x$, QAS searches for reverse reachable sets in different deterministic realized graphs of G.[17] Then, it derives the likelihood for any user $u_i \in \hat{S}_k^x$ to influence u according to the average occurrence of u_i in u's reverse reachable sets, denoted as $oc(u_i, u)$. Consequently, u's PEP of n_k under the influence of $\hat{S}_k^x$ based on the likelihood for every user in $\hat{S}_k^x$ to influence u is formulated as follows.

$$PEP_k(u, \hat{S}_k^x, x) = v_{u,n_k}(x) \cdot \min\{1, \sum_{(u_i, n_k) \in \hat{S}_k^x} oc(u_i, u)\}, \quad (8)$$

where the sum of average occurrences of users in $\hat{S}_k^x$ upper bounds the probability of u being successfully influenced by $\hat{S}_k^x$ (with a maximum value of 1), since different users in $\hat{S}_k^x$ may concurrently occur in u's reverse reachable sets, leading to duplicate counts and thus overestimating the probability of successful influence by $\hat{S}_k^x$. Accordingly, PEP of u on NFT n_k can serve as the upper bound of u's expected profit on NFT n_k (proved in Appendix D.2 [81]).

Equipped with PEP, QAS evaluates the QSP of a set of users subject to a specific quantity, in order to derive the upper bound of TP_k for $\hat{S}_k^x = S_k^x \cup \{(u_i, n_k)\}$ to decide if (u_i, n_k) is an ineffective airdrop. For NFT n_k with $q_k = x$, QSP of $\hat{S}_k^x$ is the sum of PEP of the prospective purchasers with the top-x PEP as follows.

$$QSP_k(\hat{S}_k^x, x) = \sum_{u \in V_k^{\text{PEP}}(\hat{S}_k^x, x)} PEP_k(u, \hat{S}_k^x, x), \quad (9)$$

where $V_k^{\text{PEP}}(\hat{S}_k^x, x)$ is the set of prospective purchasers with the top-x PEP on NFT n_k under the influence of $\hat{S}_k^x$. Note that $TP_k(\hat{S}_k^x, x)$ is no greater than $QSP_k(\hat{S}_k^x, x)$ (proved later in Lemma D.2 in Section 4.3). QAS thus skips the evaluation of $TP_k(\hat{S}_k^x, x)$ when $QSP_k(\hat{S}_k^x, x)$ is no greater than $TP_k(S_k^x, x)$.

Example 4.2. Consider a new NFT project $N = \{n_1, n_2, n_3\}$ (extracted from CrypotoKitties [7]), as depicted in Figure 1(a), to be promoted in the social network in Figure 1(b). Assume that $B = \{2, 2, 2\}$, $L = \{2, 2, 2\}$, $P = \{1.8, 1.8, 1.8\}$, $\eta_0 = 0$, $\eta_1 = 1.5$, $\eta_2 = 0.2$, and $\eta_3 = 0.5$. QAS starts from n_1 subject to $q_1 = 1$. At first, QAS examines all users and chooses u_1 for an airdrop to maximize the total transaction price, i.e., $S_1^1 = \{(u_1, n_1)\}$ and $TP_1(S_1^1, 1) = 5.51$. Next, QAS derives QSP as follows.

$QSP_1(S_1^1 \cup \{(u_2, n_1)\}, 1) = 4.32, \quad QSP_1(S_1^1 \cup \{(u_3, n_1)\}, 1) = 4.32,$

$QSP_1(S_1^1 \cup \{(u_4, n_1)\}, 1) = 7.73, \quad QSP_1(S_1^1 \cup \{(u_5, n_1)\}, 1) = 7.44,$

$QSP_1(S_1^1 \cup \{(u_6, n_1)\}, 1) = 8.64.$

Since both $QSP_1(S_1^1 \cup \{(u_2, n_1)\}, 1)$ and $QSP_1(S_1^1 \cup \{(u_3, n_1)\}, 1)$ are smaller than $TP_1(S_1^1, 1) = 5.51$, QOOA evaluates total transaction prices to select the second user for airdropping only among u_4, u_5,

[17] Following [76, 77], we adopt the concept of *live-edge graph*, where a deterministic realized graph of $G = (V, E)$ consisting of V and the set of live-edges. An edge $e_{i,j} \in E$ is declared to be a "live-edge" if flipping a biased random coin with probability $a_{i,j}$ returns success. A reverse reachable set of a node u_i in the deterministic realized graph contains the nodes that have a path to u_i in this graph.

and u_6. As airdropping n_1 to u_6 leads to the greatest total transaction price, i.e., $TP_1(S_1^1 \cup \{(u_6, n_1)\}, 1) = 7.1$, QAS then updates $S_1^1 = \{(u_1, n_1), (u_6, n_1)\}$ and finishes the search for $q_1 = 1$ because $|S_1^1| = b_1 = 2$. ■

VQI-based pruning. After $q_k = x$ is examined, QAS continues to find $S_k^{q_k}$ for $q_k = x + 1$. Before the search for $q_k = x + 1$ starts, QAS derives VQI from the premise that users winning the bid must have valuations no smaller than the reserve price p_k, in order to find the upper bound of TP_k for $q_k = x + 1$, denoted as $UB_k(x + 1)$ (Lines 4-5). If the upper bound $UB_k(x + 1)$ is no greater than the best TP_k generated so far, QAS skips the search for $q_k = x + 1$. After all quantities are examined, QAS assigns $q_k = x'$ and $S_k = S_k^{x'}$, where $TP_k(S_k^{x'}, x')$ is the greatest among all examined quantities (Line 21).

Ideally, the upper bound $UB_k(x+1)$ under $q_k = x+1$ is the sum of the transaction prices of the top-$(x+1)$ user valuations, when the best airdrops subject to $q_k = x + 1$ influence all prospective purchasers with the top-$(x+1)$ valuations.[18] Nevertheless, in reality, the number of prospective purchasers under $q_k = x+1$ may be smaller than $x+1$ due to the reserve price p_k. Therefore, to obtain a more accurate $UB_k(x+1)$, VQI derives the exact number of prospective purchasers under $q_k = x + 1$, by carefully examining whether users with the top-$(x + 1)$ preferences for n_k have valuations no smaller than the reserve price p_k. Let $W_k(y)$ denote the y-th largest user preference for n_k. For $W_k(y)$, the corresponding user is a *valid prospective purchaser* if the corresponding user valuation is no smaller than the reserve price p_k, i.e.,

$$\begin{aligned} W_k(y) \cdot A(n_k, q_k) &= W_k(y) \cdot e^{\eta_0 + \eta_2 \Phi(n_k) + \eta_3 \cdot 0} \cdot e^{\frac{\eta_1}{q_k}} \qquad (10) \\ &= W_k(y) \cdot \eta^* \cdot e^{\frac{\eta_1}{q_k}} \qquad (\text{Let } \eta^* = e^{\eta_0 + \eta_2 \Phi(n_k)}) \\ &\geq p_k. \end{aligned}$$

Accordingly, VQI rewrites Inequality (10) to infer the lower bound of the user preference for n_k, ensuring that the corresponding user is a valid prospective purchaser.

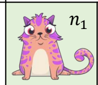
$$W_k(y) \geq \frac{p_k}{\eta^*} \cdot e^{-\frac{\eta_1}{q_k}}. \qquad (11)$$

Note that for an NFT n_k, $\frac{p_k}{\eta^*}$ remains constant regardless of variations in q_k. Hence, QAS can efficiently derive the exact number of prospective purchasers under various quantities by examining how many users with preferences for n_k satisfy Inequality (11). Considering $q_k = x + 1$, the upper bound $UB_k(x + 1)$ is thus derived below.

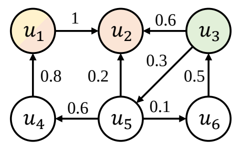
$$UB_k(x + 1) = \sum_{y=1}^{\min\{x+1, |V_k^{\mathrm{pp}}(x+1)|\}} W_k(y) \cdot A(n_k, x + 1), \qquad (12)$$

where only the prospective purchasers with top-$(x + 1)$ preferences for n_k are considered. Consequently, if there exists $x' \leq x$ leading to $UB_k(x + 1) \leq TP_k(S_k^{x'}, x')$, i.e., $TP_k(S_k^{x+1}, x + 1)$ for every possible S_k^{x+1} cannot be greater than TP_k obtained so far, QAS skips the search for $q_k = x + 1$.

[18] It is possible that even the best airdrops cannot influence all prospective purchasers with the top-$(x + 1)$ valuations. However, if the influenced prospective purchasers do not have top-$(x + 1)$ valuations, they must have valuations smaller than the top-$(x + 1)$. This situation can still be bounded by the defined upper bound $UB_k(x + 1)$.

		NFT project			New offspring
NFT		 n_1	 n_2	 n_3	
Traits		Salmon, Amur	Salmon	Aqua marine	Aqua marine, Amur
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



Preference	u_1	u_2	u_3	u_4	u_5	u_6
n_1	0.6	0.5	0.8	0.3	0.7	0.2
n_2	0.8	0.8	0.7	0.6	0.7	0.9
n_3	0.9	0.6	0.9	0.7	0.5	0.4

(b) A social network.

Figure 1: An example for QOOA.

Example 4.3. Following Example 4.2, before the search for $q_1 = 2$ begins, QAS extracts $W_1(1) = 0.8$ and $W_1(2) = 0.7$ and computes $\frac{p_k}{\eta^*} \cdot e^{-\frac{\eta_1}{q_k}} = \frac{1.8}{e^{0.2 \times 4.5}} \cdot e^{-\frac{1.5}{2}} = 0.35$. Since both $W_1(1) = 0.8 > 0.35$ and $W_1(2) = 0.7 > 0.35$, the corresponding users (i.e., u_3 and u_5, respectively) are valid prospective purchasers. As a result, QAS derives $UB_1(2) = W_1(1) \cdot A(n_1, 2) + W_1(2) \cdot A(n_1, 2) = 7.81$. Since $UB_1(2) = 7.81 > TP_1(S_1^1, 1) = 7.1$, QAS continues the search under $q_1 = 2$. ■

4.2.2 Offspring Profit Enhancement (OPE).

After QAS identifies the best NFT airdrops and quantities, OPE aims to further increase the profit by encouraging the breeding of rare NFT offspring. It recognizes the NFTs with traits of the greatest rarity as *treasures* to serve as the parents in the breeding during OPE (Line 23). Specifically, let $T^{\mathrm{R}} \subseteq T$ denote the set of rarest traits according to the trait rarity in Equation (1).[19] Based on T^{R}, OPE defines the treasures of NFTs (with at least one trait in T^{R}) as $N^{\mathrm{T}} = \{n_k : T_k \cap T^{\mathrm{R}} \neq \emptyset\}$. Accordingly, OPE identifies users purchasing at least one treasure as *Rare Trait Collectors (RTCs)* (Line 23). Our idea is to encourage RTCs and their friends to purchase more treasures, within the confines of the user breeding quota, in order to breed rare (more valuable) offspring by tailoring the airdrops of the treasures.[20] The treasures are examined sequentially to tailor their airdrops by prioritizing those with more rare traits, since their offspring are more inclined to be highly valuable (Line 24).

During the examination for each treasure $n_k \in N^{\mathrm{T}}$, OPE introduces the *Valuable Offspring Generation Influence (VOGI)* of every user, indicating the potential of a user to influence prospective purchasers to hold treasures and breed valuable offspring. It identifies users with low VOGI in the original airdrops (Line 28) (i.e., those less helpful for breeding, as the original airdrops identified by QAS do not account for breeding), and finds users with high VOGI as alternative airdrops (Line 29) to replace them (Lines 30-34). Particularly, the alternative airdrops emphasize their potential to influence

[19] Following [65], the top 10% rarest ones are usually discussed.

[20] OPE supports various considerations in breeding, detailed in Appendix C.2 [81].

prospective purchasers that may breed valuable offspring, by targeting i) RTCs with great breeding probabilities (more likely to breed), significant impacts (breeding offspring assessed higher due to ownership), and high valuations of n_k (avoiding reducing TP significantly), ii) friends of the aforementioned RTCs with great siring probabilities (more likely to participate in breeding) and high valuations of n_k (also avoiding reducing TP significantly), and iii) RTCs holding fewer treasures than c_{BQ} (adhering to the user breeding quota). Thus, the user with the highest VOGI replaces the originally selected user with the lowest VOGI in airdrops to improve the profit.

According to the above three criteria, VOGI is evaluated by considering both the probability of influencing prospective purchasers and the extent to which they are prioritized as targets for purchasing n_k. We first introduce the *Target Index (TI)* of a prospective purchaser u_{p} for n_k, which prioritizes u_{p} to target on purchasing n_k. A prospective purchaser with a larger TI for n_k indicates a greater likelihood of participating in breeding valuable offspring from n_k. The TI of a prospective purchaser u_{p} for n_k is defined according to u_{p}'s breeding probability, impact, valuation of n_k, siring probability, and the number of treasures held by u_{p}, as follows.

$$TI_k(u_{\text{p}}, S, Q) = \begin{cases} \beta_{\text{p}} I_{\text{p}} v_{u_{\text{p}},n_k}(q_k), & \text{if } 1 \le \Theta(u_{\text{p}}, S, Q) < c_{\text{BQ}} \\ \beta_c \gamma_{\text{p}} v_{u_{\text{p}},n_k}(q_k), & \text{else if an RTC } u_c \text{ is } u_{\text{p}}\text{'s friend} \\ 0, & \text{otherwise} \end{cases},$$

where $\Theta(u_{\text{p}}, S, Q)$ is the number of treasures held by u_{p} given S and Q, and I_{p} is the impact of u_{p}. For the first case where u_{p} is an RTC holding fewer treasures than c_{BQ}, prioritizing u_{p} for purchasing n_k can enhance the likelihood of u_{p} breeding valuable offspring. TI is evaluated by u_{p}'s breeding probability (β_{p}), impact (I_{p}), and valuation of n_k ($v_{u_{\text{p}},n_k}(q_k)$) in order to increase the expected assessments of offspring without significantly compromising the total transaction price. For the second case, where u_{p} either lacks treasures or holds an excessive amount but has a friend u_c who is an RTC, prioritizing u_{p} for purchasing n_k can enable u_{p} to provide n_k for siring, thereby increasing the likelihood for u_c to breed valuable offspring. TI is evaluated by u_c's breeding probability β_c to represent the likelihood of u_c requesting u_{p} to provide siring-ready NFTs, u_{p}'s siring probability (γ_{p}) and valuation of n_k ($v_{u_{\text{p}},n_k}(q_k)$).[21]

Then, based on TI, VOGI of a user $u \in V$ is the sum of the TIs of prospective purchasers u_{p} influenced by u, weighted by their likelihood of being influenced, in order to capture the potential of u to jointly facilitate purchases and collaborative breeding of the treasures.

$$VOGI_k(u, q_k) = \sum_{u_{\text{p}} \in V_k^{\text{Val}}(q_k)} TI_k(u_{\text{p}}, S, Q) \cdot oc(u, u_{\text{p}}), \tag{13}$$

where $V_k^{\text{Val}}(q_k)$ is the set of prospective purchasers with the top-q_k valuations, and $oc(u, u_{\text{p}})$ is the likelihood for u to influence u_{p}. A user with a larger VOGI is more inclined to be chosen as an alternative airdrop, since it enhances the likelihood of breeding valuable offspring to improve the profit. Hence, OPE iteratively tailors the airdrops identified by QAS by replacing them with alternative airdrops for users with higher VOGI (Lines 30-34). To this end, OPE first extracts the user u^* not selected for airdrops by QAS (i.e., $u^* \notin \{u : (u, n_k) \in S_k\}$) but having the largest VOGI (Line 29). Let $\underline{u} \in \{u : (u, n_k) \in S_k\}$ denote the user identified by QAS for airdrops with the smallest VOGI (Line 28). If $\underline{u}$'s VOGI is smaller than that of u^*, i.e., $VOGI_k(\underline{u}, q_k) < VOGI_k(u^*, q_k)$ (Line 30), $\underline{u}$ is less likely to influence prospective purchasers to generate valuable offspring from n_k (compared with u^*). Therefore, OPE opts to airdrop n_k to u^* instead of $\underline{u}$ (i.e., replacing $(\underline{u}, n_k)$ with (u^*, n_k)), to facilitate breeding of rare traits. It updates S as $(S \setminus \{(u_i, n_k)\}) \cup \{(u^*, n_k)\}$ if the above replacement improves the profit, i.e., $f\big((S \setminus \{(u_i, n_k)\}) \cup \{(u^*, n_k)\}, Q\big) > f(S, Q)$ (Lines 33-34).

Example 4.4. Following Example 4.3, QOOA obtains $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_6, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 25.93$. Assume that $\lambda = 1$, $\omega = 1$, and $c_{\text{BQ}} = 3$. Since 'amur' and 'aqua marine' are the rarest traits, OPE identifies the treasures $N^{\text{T}} = \{n_1, n_3\}$. For n_3, OPE examines VOGI as follows.

Airdrops:	$VOGI_3(u_4, 1) = 0.51$,	$VOGI_3(u_6, 1) = 0.26$.
Non-airdrops:	$VOGI_3(u_3, 1) = 0.37$,	$VOGI_3(u_5, 1) = 0.36$,
	$VOGI_3(u_1, 1) = 0$,	$VOGI_3(u_2, 1) = 0$.

Accordingly, OPE attempts to replace u_6 with u_3 for airdrops of n_3 because $VOGI_3(u_3, 1) = 0.37 > VOGI_3(u_6, 1) = 0.26$. Since $f\big((S \setminus \{(u_6, n_3)\}) \cup \{(u_3, n_3)\}, Q\big) = 26.97 > f(S, Q) = 25.93$, OPE updates $S = (S \setminus \{(u_6, n_3)\}) \cup \{(u_3, n_3)\}$. As $VOGI_3(u_5, 1) = 0.36 < VOGI_3(u_4, 1) = 0.51$, OPE terminates the enhancement for n_3. Consequently, the solution is $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_3, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 26.97$. ■

4.3 Theoretical Analysis

Theorem 4.1. *QOOA is $\frac{1}{ea(1+c)}$-approximation for NPM, where $a = \max_{1 \le k \le |N|} \frac{r(\hat{S}_k, 1)}{p_k}$, $c = \frac{\lambda \omega q_{max} c_{BQ} A_{max}}{p_{min}}$, $A_{\max}$ is the maximum assessment of offspring, and $p_{\min}$ is the minimum reserve price.*

Proof Sketch. Since the profit function f is neither monotonically increasing nor submodular (proved in Appendix D.2 [81]), we first consider an unconstrained problem similar to NPM without the quantity constraint and NFT breeding, and its profit function is non-monotonically increasing but submodular. Specifically, we first prove that the output profit of QOOA is at least $\frac{1}{ea}$ times of the profit of the unconstrained problem, where $a = \max_{1 \le k \le |N|} \frac{r(\hat{S}_k, 1)}{p_k}$. Afterward, according to the relation between the total transaction price and the assessments of NFT offspring, we prove that the profit function gap between the unconstrained problem and NPM is at most $\frac{1}{1+c}$, where $c = \frac{A_{\max} l_{\max} (|N|-1)}{2 p_{\min}}$, and $A_{\max}$ is the maximum assessment of offspring. This implies the total gap is $\frac{1}{ea(1+c)}$. For the detailed proof and further analysis, please refer to Appendix D.2 [81]. □

Theorem 4.2. *The time complexity of QOOA is $O(l_{\max} b_{\max} |V| |N|)$, where $l_{\max} = \max_{l_k \in L} l_k$ is the maximum quantity limit and $b_{\max}$ is the maximum budget in B.*

Proof. In the QAS phase, for an NFT n_k with budget b_k, it takes $O(|V| b_k)$ time to find the airdrops under a specific quantity. As the maximum quantity limit is $l_{\max} = \max_{l_k \in L} l_k$, it requires $O(l_{\max} b_k |V|)$ time to identify the most appropriate quantity and

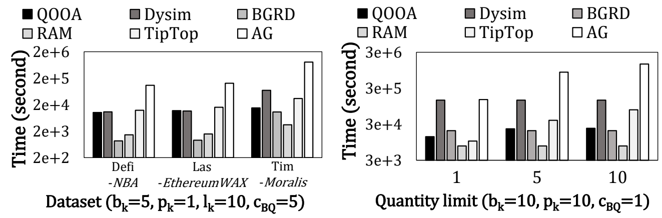
[21] If u_{p} has multiple RTCs as friends, TI refers to the one with the highest breeding probability among them to ensure the importance of targeting u_{p} is not underestimated.

the corresponding airdrops for each NFT n_k. Therefore, QAS takes $O(l_{\max} b_{\max} |V| |N|)$ time to find an initial solution of NPM, where $b_{\max}$ is the maximum budget in B. Next, in the OPE phase, it examines $O(b_k)$ replacements of airdrops for a treasure n_k. As there are $O(|N|)$ treasures, OPE takes $O(b_{\max} |N|)$ time. Consequently, the time complexity of QOOA is $O(l_{\max} b_{\max} |V| |N|)$. The theorem follows. □

5 EXPERIMENTS

Datasets. We conduct experiments on three NFT projects and three social networks. The NFT projects include i) Defimons Characters:[22] It has 14 NFTs, which are the identities in the pixel world of Defimons. There are 5 traits (e.g., genders and locations), with the rarity of NFTs ranging from 3.75 to 9. ii) Lascaux:[23] It has 17 NFTs, recording the art and performance of the artist Lascaux. There are 10 traits (e.g., utility, series, etc.), with the rarity of NFTs ranging from 4.05 to 39.89. iii) Timpers Pixelworks:[24] It has 7 NFTs, which are artworks by Timpers and top guest artists. There are 3 traits (e.g., artists), with the rarity of NFTs ranging from 1.4 to 7. The social networks based on NFT transactions include a) NBA (NBA Top Shot transactions):[25] It contains 245K users and 2.6M relationships derived from 2.63M transactions. b) EthereumWAX [67] (collected primarily from Ethereum and WAX): It contains 269K users and 2.8M relationships derived from 7M transactions. c) Moralis [25] (collected via the Web3 APIs of the Moralis platform):[26] It contains 5.1M users and 30.6M relationships derived from 77M transactions. We follow [57, 60, 79] to set the user preferences and the activation probabilities. Following [67], the breeding and siring probabilities of a user u_i, β_i and γ_i, are set based on her activity strength. Following existing NFT projects, e.g., Trump Digital Trading Card[27] and MODragon, the quantity limit $l_k = 10$ for each n_k and the user breeding quota $c_{\text{BQ}} = 5$ for each user, and $\lambda = 1$. Following OpenSea, the reserve price p_k of each n_k is set to 1.[28] Following the engagement rate of famous NFT projects in [70], ω is set to 0.2.

Baselines and metrics. We compare QOOA with five state-of-the-art approaches: Dysim [80], BGRD [26], RMA [44], TipTop [58], and AG [37]. The detailed settings of baselines are present in Appendix E [81].Since all baselines do not decide the quantities of NFTs for sale, we evaluate various quantities to find the one with the maximum profit. The performance metrics include i) profit $f(S,Q)$, ii) the total transaction price $TP(S,Q)$, iii) the potential assessments of offspring $OS(S,Q)$, and iv) the execution time. We perform a series of sensitivity tests regarding 1) the budgets B, 2) the quantity limits L, 3) the reserve prices P, and 4) the user breeding quota c_{BQ}. We conduct case studies on different breeding mechanisms to gain deeper insights, as well as an ablation study on QOOA to evaluate the efficacy of QSP and VQI in reducing the execution time. We conduct all experiments on an HP DL580 server with an Intel 2.10GHz CPU and 1TB RAM. Each simulation result is averaged over 100 samples.[29]



(a) Time vs dataset.

(b) Time vs L (Tim-*Moralis*).

Figure 2: Scalability.

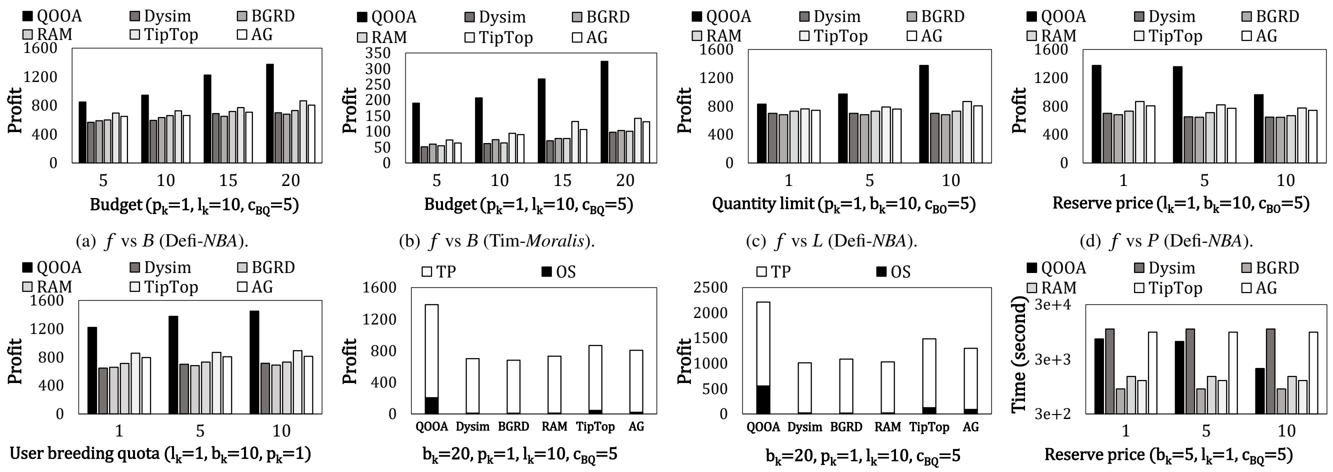
5.1 Scalability

Figure 2(a) compares the execution time on different datasets. As shown, QOOA outperforms all baselines, especially in large-size Tim-*Moralis* dataset, except for BGRD and RAM, which do not perform well in terms of profit (to be shown later). Among the baselines, TipTop and AG identify airdrops based on maximizing benefits, which are set according to user valuations of NFTs. It requires TipTop and AG to perform an exhaustive examination of each quantity, resulting in the longest execution time. QOOA also examines various quantities but leverages VQI to effectively prune redundant quantities, making it faster than TipTop, AG and even the quantity-agnostic Dysim. While BGRD and RAM are also quantity-agnostic and faster, they do not spend time identifying airdrops for different quantities. Consequently, their profits are at most 35% of those from QOOA. It is also worth noting that on Tim-*Moralis* with more users than the other two datasets, the execution time of QOOA does not increase as significantly as with other baselines, because QOOA utilizes QSP as an upper bound of profits, enabling it to efficiently skip ineffective airdrops and mitigate the impact of an increasing number of users. Conversely, BGRD only identifies a single group of users for all NFTs, making it the fastest on Defi-*NBA* and Las-*EthereumWAX*, but on Tim-*Moralis* with a large number of users, BGRD's execution time increases significantly, getting close to that of QOOA. On the other hand, RAM with reverse influence sampling suits better to datasets with numerous users.

Figure 2(b) illustrates the execution time on Tim-Moralis under varying quantity limits l_k. As the quantity limit increases, the execution time of TipTop and AG grows significantly because they must exhaustively search for the best airdrops for each quantity, as user valuations vary with quantity. In contrast, the impact of the growing quantity limit on QOOA's execution time is minor, because QOOA leverages VQI to derive the upper bound of profit for each quantity. Consequently, QOOA only searches for airdrops for quantities that can potentially yield higher profits. Even with increasing quantity limits, it does not need to examine each one exhaustively. Meanwhile, since other baselines aim to maximize influence, irrelevant to quantity, their execution times do not change with the quantity limit.

[22] https://www.defimons.com/.
[23] https://opensea.io/collection/lascauxfuture.
[24] https://opensea.io/collection/timperspixelworks.
[25] https://www.kaggle.com/datasets/chigorin/nba-topshot-transactions/data.
[26] https://moralis.io.
[27] https://collecttrumpcards.com/.
[28] https://support.opensea.io/hc/en-us/articles/1500003246082-How-do-timed-auctions-work-.

[29] The source code and data are available at https://github.com/minksable/NPM.



(a) f vs B (Defi-*NBA*). (b) f vs B (Tim-*Moralis*). (c) f vs L (Defi-*NBA*). (d) f vs P (Defi-*NBA*).

(e) f vs c_{BQ} (Defi-*NBA*). (f) TP vs OS (Defi-*NBA*). (g) TP vs OS (Las-*EthereumWAX*). (h) Time vs P (Defi-*NBA*).

Figure 3: Sensitivity tests on examined approaches.

5.2 Sensitivity Tests

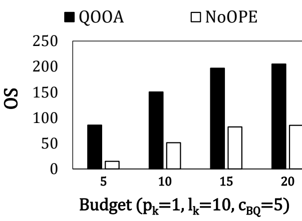
Figures 3(a)-3(b) compare the profits of evaluated algorithms under different datasets, respectively, where the x-axis specifies the varied budget b_k for each NFT n_k. For all datasets, QOOA achieves the highest profit by leveraging QAS to maximize the total transaction price, instead of focusing on influence spread or total valuation. Specifically, QAS exploits QSP to obtain airdrop candidates, considering their influence on the expected profit of the top prospective purchases up to the for-sale quantity, factoring in influence probability, user valuation, and for-sale quantity. Among the baselines, TipTop and AG are superior since it takes into account user valuations. However, without considering breeding and the limited NFT supply due to for-sale quantity, TipTop and AG are inferior to QOOA. As the budget increases, all approaches yield higher profits, but their increasing trends diverge. The profit increments from QOOA are obvious, followed by those from TipTop and AG, whereas those from Dysim, BGRD, and RMA are insignificant, as they maximize influence spread rather than profit.

In addition to different budgets, Figure 3(c) presents the profits under varying quantity limits l_k for each NFT n_k. QOOA consistently achieves higher profits compared to the baselines. As the quantity limit increases, the profits obtained by all approaches grow, manifesting that more quantities provide greater opportunities for breeding, subsequently boosting the total assessment of offspring. Figure 3(d) shows the profits with respect to different reserve prices p_k for each n_k. As the reserve price grows, the profits for all approaches drop. This is because a higher reserve price reduces the number of prospective purchasers, which not only lowers the total transaction price but also diminishes the opportunities for offspring breeding, thereby decreasing the potential assessments of offspring. Next, Figure 3(e) evaluates the profits for varied user breeding quotas. As the user breeding quota increases, all approaches achieve higher profits. However, the gap between QOOA and the baselines becomes more pronounced with the growth of the quota. A strength of QOOA is attributed to OPE which judiciously balances the treasures held by RTCs and their friends, ensuring that users optimize the breeding of valuable offspring within the quota. In contrast to QOOA, the profit differences obtained by all baselines under varying l_k, p_k, and c_{BQ} are insignificant, since they do not take breeding into account. Consequently, they are unable to identify effective airdrops tailored to different breedings to maximize profits.

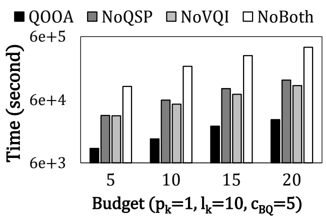
Figures 3(f) and 3(g) evaluate the profits generated by original NFTs (TP) and offspring (OS). As shown, QOOA is most effective at achieving high OS, because OPE meticulously tailors the airdrops of NFTs with rare traits, encouraging RTCs and their friends with a high Target Index to engage in purchasing these NFTs simultaneously. Moreover, the Target Index effectively prioritizes users with a high probability of producing offspring with high assessments. In contrast, the baselines do not account for the generation of offspring and solely focus on profiting from the transaction prices of the original NFTs. Moreover, as half of the traits in Lascaux are rare (i.e., with the trait rarity of 17), QOOA always takes the chance to produce offspring with rare traits. This phenomenon is observed in Figure 3(g), where the profit generated by offspring accounts for nearly 25% of the total profit, surpassing the 15% observed in Figure 3(f). Finally, Figure 3(h) compares the execution time with varying reserve prices. As the reserve price increases, the efficacy of VQI becomes more pronounced, leading to more effective pruning in QOOA. Hence, the execution time of QOOA decreases with higher reserve prices. In contrast, all baselines do not account for reserve prices; thus, they cannot leverage reserve prices to reduce execution time.

5.3 Ablation Study

To evaluate the efficacy of QSP, VQI, and OPE, we compare four variants of QOOA: NoQSP, NoVQI, NoBoth, and NoOPE. Specifically, NoQSP does not employ QSP to prune unlikely users for airdropping; NoVQI omits VQI to prune quantities unnecessary for examination; NoBoth does not exploit either of QSP and VQI; NoOPE does not adjust airdrops to enhance profits from breeding. Figure 4(a) compares QOOA and NoOPE across varied budgets. Note that QSP and VQI do not impact the profit since both QSP and VQI serve as the upper bound of TP for pruning, aiming to reduce running time. Thus, NoQSP, NoVQI, and NoBoth are excluded in Figure 4(a). As shown, QOOA consistently outperforms NoOPE in



(a) *OS* vs *B* (Defi-*NBA*).



(b) Time vs *B* (Defi-*NBA*).

Figure 4: Ablation study on QOOA.

profit from *OS*. This is because OPE proficiently promotes rare-trait NFTs to RTCs and their friends, facilitating both individual and collaborative breeding. Moreover, OPE tailors airdrops to influence users to produce the maximum number of valuable offspring. On the other hand, Figure 4(b) compares the execution times of QOOA, NoQSP, NoVQI, and NoBoth across different budgets. QOOA harnesses VQI to derive the maximum attainable TP for each quantity, thus effectively avoiding redundant searches across different quantities. As shown in Figure 4(b), upon incorporating VQI, QOOA exhibits a notable decrease in execution time by 30% compared to NoVQI. When seeking airdrops under a specific quantity, QOOA leverages QSP as the upper bound of TP for a set of users, effectively filtering out unlikely candidates for airdrops. The advantages of using QSP become particularly evident as the budget increases. By contrast, NoBoth and NoQSP, which do not utilize QSP, need to search through all users for each quantity, leading to significantly longer execution time. The results indicate that both QSP and VQI play crucial roles in enhancing the efficiency of QOOA. In contrast, NoBoth without any pruning strategy requires the most time.

5.4 Case Study on Breeding Mechanisms

We present case studies on Defimons Characters-*NBA*, focusing on different breeding: *No Collaborative Breeding*, *Collaborative Breeding*, and *Booster* in Section 3. Figure 5 compares the total assessment of offspring (*OS*) across various approaches. As *Booster* boosts the likelihood of inheriting rare traits, QOOA tailors airdrops for treasures by giving priority to RTCs with a booster to acquire them. It increases the probability of breeding offspring with rare traits, ultimately resulting in the highest *OS* among all mechanisms. In contrast, *No Collaborative Breeding* limits breeding to individual ownership, reducing the flexibility of QOOA in tailoring airdrops and resulting in the lowest OS among the three mechanisms. Among all baselines, TipTop and AG stand out as they allow different users to have varying benefits for the same NFT. By favoring users who may possess similar NFTs, they tend to facilitate airdrops that lead to more offspring generation compared to other baselines. However, they fail to maximize the pairing of rare-trait NFTs to produce offspring with rarer traits, resulting in low-value offspring. Consequently, they still achieve significantly lower OS compared to QOOA. In contrast, other baselines prioritize airdrops that maximize influence spread without taking breeding into consideration, thus resulting in negligible *OS*.

Besides, we observe that different breeding mechanisms result in varied outcomes. Consider User #242, who is likely to acquire the NFT 'Orion' (featuring the rarest trait 'Heiwa Village' with a rarity

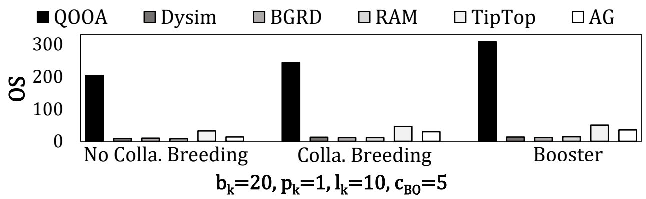


Figure 5: Comparison on breeding mechanisms.

of 7) through QAS's airdrops. OPE's adjustments lead to different results under different breeding mechanisms. 1) In *No Collaborative Breeding*, since User #242 has a breeding probability of 0.6 and an impact of 0.9, OPE is inclined to influence her to acquire an additional NFT 'Apollo' (also featuring 'Heiwa Village' with a rarity of 7). While her initial probability of generating offspring is nearly zero, it now stands at about 0.36 due to her acquisition of enough NFTs for breeding. Furthermore, the generated offspring has a 0.28 probability of inheriting the rarest trait, 'Heiwa Village,' resulting in a significant increase in *OS*. 2) In *Collaborative Breeding*, users can utilize their friends' siring-ready NFTs to breed offspring together. User #242's friend, User #3732, with a siring probability of 0.7, becomes highly likely to acquire the NFT 'Orion' as well after OPE. As User #3732 is likely to provide 'Orion' for siring, the chance of User #242 to generate offspring inheriting the rarest trait 'Heiwa Village' becomes 0.28, since collaborative breeding allows User #242 to breed. Consequently, by adjusting airdrops to influence friends' of RTCs, OPE can enhance the likelihood of achieving higher *OS*. 3) In *Booster* (which increases the inheritance probability of the rarest trait), when User #242 possesses a booster, OPE tends to encourage User #242's friend, User #17428 who has a siring probability of 0.82, to also acquire 'Orion' for siring. It assists User #242 in breeding offspring with the rarest trait, 'Heiwa Village,' boosting the inheritance probability from 0 to 0.54, due to collaborative breeding and the booster. While collaborative breeding allows User #242 to breed offspring with a 0.28 probability of inheriting the rarest trait, the booster increases the probability to 0.54. It demonstrates that OPE by encouraging the friends of RTCs with boosters to purchase rare-trait NFTs, effectively increases the assessments of offspring.

6 CONCLUSION

To the best of our knowledge, this work is the first attempt to investigate profit maximization for NFTs. By taking into account key features of NFTs, including breeding, scarcity, and rarity, we formulate a new problem, named NPM, to find the airdrops and determine the quantities for viral marketing. We prove the hardness of NPM and design an approximation algorithm QOOA, which effectively tackles NPM by efficiently identifying airdrops under varying quantities and increasing the profit from offspring, with the new ideas of QSP, VQI, and RTCs. Experiments on real NFT projects and social networks demonstrate that QOOA achieves up to 3.8 times the profits over the state-of-the-art approaches. In future work, we plan to consider multiple time slots and recommend appropriate actions (either breeding or selling) to users in order to maximize profit.

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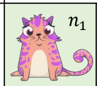
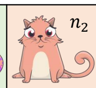
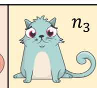
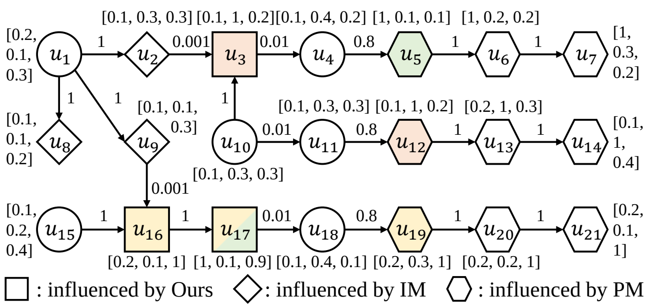
		NFT project			New offspring
NFT		 n_1	 n_2	 n_3	
Traits		Salmon, Amur	Salmon	Aqua marine	Aqua marine, Amur
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



(b) A social network.

Figure 6: An illustrative example.

A ILLUSTRATIVE EXAMPLES

Figure 6 illustrates an example for promoting a new NFT project that consists of three NFTs (denoted by n_1, n_2 and n_3) within a social network. Figure 6(a) shows the traits and (potential) assessments under different quantities of n_1, n_2, and n_3, as well as the offspring. As shown in the NFT project, the traits 'moon' and 'amur' are possessed by only one NFT and are rarer than the other trait 'star' which are possessed by two NFTs. As a user may prefer different traits, her valuation of an NFT may be calculated by multiplying her preference for the NFT with its assessment. Figure 6(b) shows a social network where nodes represent users, edges represent friendships between users, the edge weight denotes the activation probability, and the vector associated with a node denotes a user's preferences of corresponding NFTs. An NFT promotion campaign starts by airdropping some NFTs to selected users who may trigger influence propagation on the social network to activate user interests in the NFTs. After the campaign ends, the influenced users may bid on NFTs of interest with their valuations (estimated based on various factors). Assume that the number of airdrops allocated for each NFT is 1 and the reserve price of each NFT is 2.6. Based on the approach proposed in this paper, airdropping n_1, n_2, and n_3 to u_{15}, u_{10}, and u_{15}, respectively, with the quantities set as 1, 1, and 2 may yield an expected profit of 28.72, where the influenced users u_3, u_{16}, and u_{17} (represented by square shapes) win the auctions and the NFTs they purchase are indicated by colors. Since u_{17} possesses both n_1 and n_3, she has the opportunity to generate an NFT offspring with a profit of 4.32.

By contrast, an existing viral marketing algorithm based on influence maximization (IM) [26], which seeks to identify seed users to maximize the number of influenced users, may airdrop all NFTs to u_1, due to its expected influence spread of 3, where the influenced users u_2, u_8, and u_9 are represented by diamond shapes. However, as the users influenced by u_1 do not have high valuations of NFTs, the expected profit is almost 0 when the quantity of each NFT is one. On the other hand, a classical profit maximization algorithm [58] may designate u_4, u_{11}, and u_{18} as the airdrop recipients for n_1, n_2, and n_3, respectively, thereby maximizing the total expected profit of 60.57 without considering NFT quantities, where the influenced users are represented by hexagonal shapes. Nevertheless, when the quantity of each NFT is limited to one, the expected profit is reduced to 20.19, where the NFTs they purchase are indicated by colors. In addition, for both cases, no user can breed NFT offspring to generate additional profits. On the other hand, when the quantity of each NFT increases, the valuations of influenced users decline significantly, with many falling below the reserve prices. The profit obtained is thus smaller compared to the case where the quantity is one. As a result, both approaches of maximizing influence and profit fall short in the marketing goal of NFT marketing, as they do not consider the features of NFTs, including breeding, scarcity, and trait rarity, and the need to determine NFT quantities.

B DETAILS ON BREEDING

Beyond those introduced in Section 3, we further elaborate on additional considerations for the breeding mechanism prevalent in various NFT projects.

- *Breeding Constraints:* Each NFT may also have a breeding limit that defines the maximum number of times it can engage in breeding. For example, in STEPN, the maximum breeding limit is 7, and the cooldown period grows longer as the breed count of NFTs approaches 7. In addition, inbreeding restrictions are in place to protect genetic diversity, e.g., MODragon also prohibits the breeding of sibling NFTs.
- *Genetic Mechanisms:* Mutations introduce an element of unpredictability by occasionally endowing offspring with entirely novel traits, thereby enhancing their rarities. For example, in Tiny World, offspring have a greater chance of mutation when the breed counts of their parents increase.[30]
- *NFT Maturation:* The lifecycle of NFT offspring may follow a distinct evolutionary path of multiple stages, including egg, baby, and adult. Until the phase of hatching, the traits of an NFT offspring are typically concealed. At this point, holders have the option to trade, and the inherent uncertainty often boosts the NFT's assessment. After hatching, certain NFT projects, such as Crypto Unicorns, reach the baby stage. As they attain the adult stage of development, they have the opportunity to mutate and acquire rare traits. Notably, these NFTs cannot breed offspring until they have reached the adult stage.

C DETAILS ON QOOA

C.1 Illustration of Workflow

The illustration of QOOA is presented in Figure 7.

[30]More information on NFTs in Tiny World: https://docs.tinyworlds.io/tiny-nft/bnb-chain/tinymon-nfts.

C.2 Support for Various Breeding Mechanisms

Here we show that QOOA is capable of supporting various breedings, including *No Collaborative Breeding*, *Booster*, and *Fusion Breeding*, as introduced in Section 3.

1) For *No Collaborative Breeding*, OPE no longer takes into account the influence on friends of RTCs providing siring-ready NFTs. It modifies the Target Index by discarding the second case.

$$TI_k(u_j, S, Q) = \begin{cases} \beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \le \Theta(u_j, S, Q) < c_{\text{BQ}} \\ 0, & \text{otherwise} \end{cases}.$$

2) For *Booster*, OPE modifies the Target Index to account for the number of boosters possessed by an RTC as follows.

$$TI_k(u_j, S, Q) = \begin{cases} (1 + \frac{\theta_j}{\theta_{\max}})\beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \le \Theta(u_j, S, Q) < c_{\text{BQ}} \\ \beta_c \gamma_j v_{u_j,n_k}(q_k), & \text{else if an RTC } u_c \text{ is } u_j\text{'s friend} \\ 0, & \text{otherwise} \end{cases},$$

where θ_j is the number of boosters possessed by u_j, $\theta_{\max}$ is the maximum number of boosters possessed by users in the social network, and $\frac{\theta_j}{\theta_{\max}}$ indicates the increase in TI attributable to the possession of boosters.

3) For *Fusion Breeding*, since NFTs with semi-rare traits, which are less commonly found in N but more abundant than rare traits, also have chances to breed offspring with rare traits, OPE thus identifies the set of semi-treasures, denoted as N^{ST}, to include NFTs without the rarest traits but possessing at least one semi-rare trait. Note that $N^{\text{T}} \cap N^{\text{ST}} = \emptyset$. After adjusting the airdrops for treasures, OPE also finds alternative airdrops for semi-treasures.

Equipped with the modified TI, OPE evaluates VOGI of users and tailors airdrops accordingly.

D THEORETICAL ANALYSIS

D.1 NPM's Hardness

Theorem D.1. *NPM is NP-hard and cannot be approximated within a factor of* $|V|^{1/(\log\log|V|)^c}$ *assuming the exponential time hypothesis (ETH), where* $c > 0$ *is a constant independent of* $|V|$.

PROOF. We prove this theorem with a reduction from the Densest k-Subgraph problem (DkS), which is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$ [64]. Given a graph G and two integers k and h, where h is set to a sufficiently large integer, the decision version of DkS is to decide whether there exists an induced subgraph of G with k vertices and h edges.[31] Given an instance (G, k, h) of DkS, where $V(G) = \{v_1, v_2, \ldots, v_{|G|}\}$, we construct an instance of NPM as follows. i) Let $W = \{w_1, w_2, \ldots, w_{|G|}\}$, where w_i is a copy of v_i for any $i \in \{1, 2, \ldots, |G|\}$. The social network of the NPM instance is a graph with vertex set $V(G') \cup W$ and edge set $E(G') \cup \{e_{w_1,v_1}, e_{w_2,v_2}, \ldots, e_{w_{|G|},v_{|G|}}\}$, where G' is an orientation of G, i.e., G' is obtained from G by assigning exactly one direction to each edge of G. The activation probability a_{v_i,v_j} is set to 0 for any edge $e_{v_i,v_j} \in E(G')$, and a_{w_i,v_i} is set to 1 for any $i \in \{1, 2, \ldots, |G|\}$. ii) The NFT project $N = \{n_1\}$, reserve price $p_1 = 1$, seed budget $b_1 = k$ and valuation $v_{u,n_1}(q_1) = 1$ for each user $u \in V(G') \cup W$ and any quantity q_1 of NFT n_1.[32] Let $\beta_i = 1$ and $\gamma_i = 1$ for any user $u_i \in V(G') \cup W$, i.e., in the constructed instance, the NFT offspring can be created only when two activated users are friends, and each of them provides a parent NFT, since there is only one NFT involved. Thus, the creation of an offspring corresponds to the existence of an edge between two activated users. Let $A(o, S, Q) = 1$ for any NFT offspring o under the diffusion of any S and Q.[33]

To complete the proof, we show that there exists an induced subgraph with k vertices and h edges in DkS if and only if there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S, Q) = k + h$ in NPM. We first prove the necessary condition. Suppose that there exists an induced subgraph with k vertices and h edges in DkS. Let the k corresponding users in W form the set of NFT airdrops S, and let $Q = \{q_1 = \infty\}$. By the construction, exactly k users in $V(G')$ are activated, and exactly h offspring are created. Thus (S, Q) is a solution such that $f(S, Q) = k + h$. We then prove the sufficient condition. Suppose that there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S, Q) = k + h$ in NPM. Since the activation probability $a_{v_i,v_j} = 0$ for $e_{v_i,v_j} \in E(G')$ and $a_{w_i,v_i} = 1$ for $i = \{1, 2, \ldots, |G|\}$, if any user receiving one of the k NFT airdrops is not selected from W, this user cannot activate anyone, resulting in fewer than k users being activated to acquire the NFT. Moreover, the number of edges induced by these activated users (fewer than k) is no more than h. The reason is as follows. Recall that, by the setting of h, any induced subgraph of G with k vertices has no more than h edges. Hence, any induced subgraph with fewer than k vertices also has no more than h edges. Therefore, we conclude that the k NFT airdrops must be chosen from the set W, otherwise $f(S, Q) \le (k - 1) + h < k + h$. Thus the corresponding vertices in $V(G)$ form an induced subgraph with k vertices and h edges in DkS. Therefore, NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming ETH, where $c > 0$ is a constant independent of $|V|$. The theorem follows. □

D.2 QOOA's Performance

Lemma D.2. *For a set of airdrops* S_k *for* n_k *under a quantity* q_k, $QSP_k(S_k, q_k) \ge TP_k(S_k, q_k)$.

PROOF. Recall $TP_k(S_k, q_k) = \sum_{u_j \in V(S_k,q_k)} v_{u_j,n_k}(q_k)$, where $V(S_k, q_k)$ is the set of users influenced by S_k to hold the NFT n_k under the quantity q_k. For each $u_j \in V(S_k, q_k)$, her expected profit on NFT n_k is no greater than $PEP_k(u_j, S_k, q_k)$. As QSP has taken PEP of the users in $V_k^{\text{PEP}}(S_k, q_k)$ into account, for any user $u_j \in$

[31] The parameter h is set to a sufficiently large interger, ensuring that any induced subgraph of G with k vertices has no more than h edges. In fact, in theoretical analysis, there is no need to know the exact value of h. Instead, it is necessary to examine whether the logical inference holds true. With the setting of h (i.e., ensuring that any induced subgraph of G with k vertices has no more than h edges), our goal is to determine whether there exists an induced subgraph of G with k vertices and *exactly* h edges.

[32] For each $u \in V$, $v_{u,n_1}(q_1)$ can be set to 1 by letting the personal preference $w_{u,n_1} = 1$ and the parameters $\eta_0 = \eta_1 = \eta_2 = \eta_3 = 0$, i.e., $v_{u,n_1}(q_1) = w_{u,n_1} \times e^{\eta_0 + \eta_1 \frac{1}{q_1} + \eta_2 \Phi(n_1) + \eta_3 h_1} = 1 \times e^0 = 1$.

[33] $A(o, S, Q)$ can be set to 1 by letting the parameters $\eta_0 = \eta_1 = \eta_2 = \eta_3 = 0$, i.e., $A(o, S, Q) = e^0 = 1$.

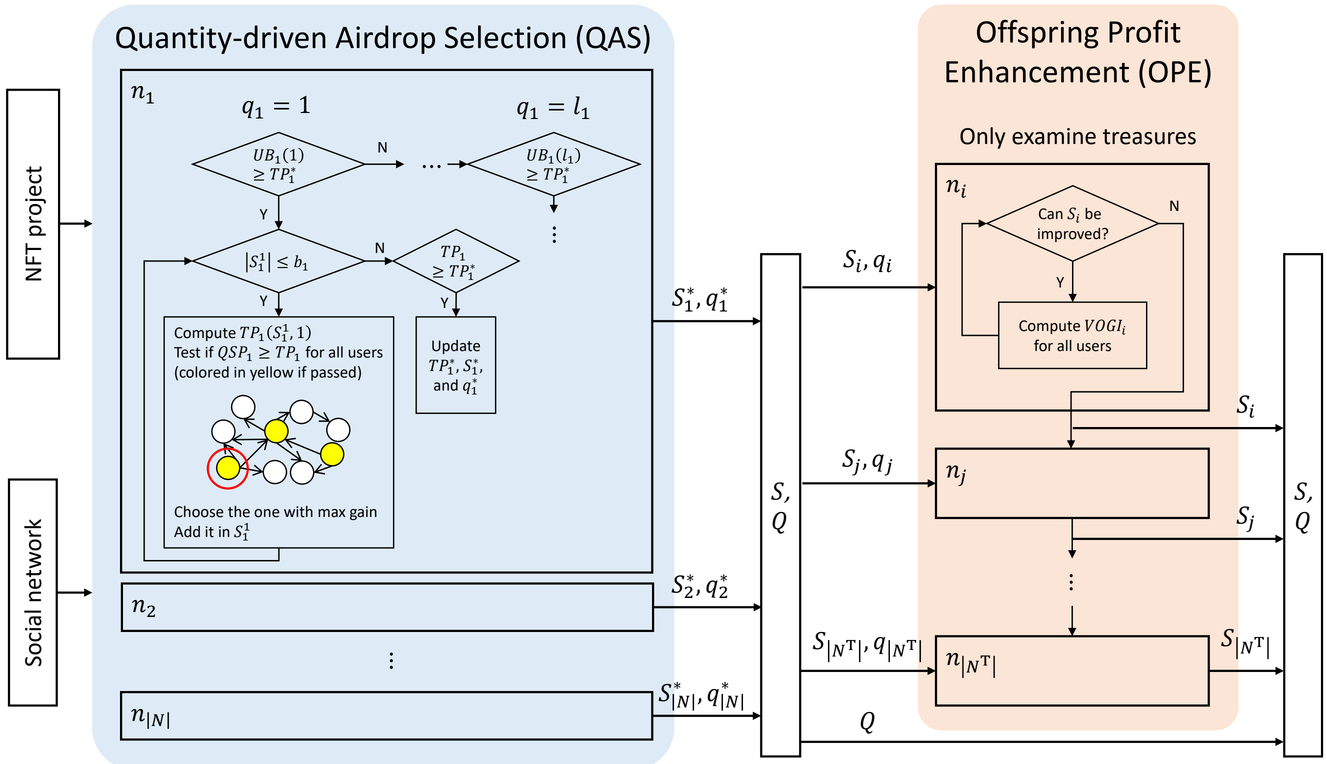


Figure 7: Illustration of QOOA.

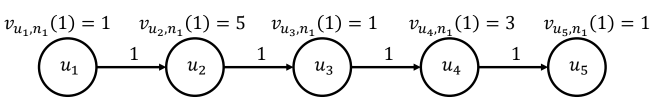


Figure 8: An example for Lemma D.3.

$V(S_k, q_k)$, if $u_j \in V_k^{PEP}(S_k, q_k)$, her PEP is already counted in QSP; otherwise, her PEP must be no greater than $PEP_k(u_{\min}, S_k, q_k)$, where $u_{\min}$ is the user with the least PEP in $V_k^{\text{PEP}}(S_k, q_k)$. Therefore, $QSP_k(S_k, q_k)$ is the upper bound of $TP_k(S_k, q_k)$. The lemma follows. □

Note that in the influence propagation of NFTs during the Airdrop stage, for each NFT n_k, its total transaction price function $TP_k(S_k, q_k)$ is independent from other NFTs. In the following, we first examine whether the total transaction price function $TP_k(S_k, q_k)$ is submodular.

Definition D.1 (Submodular function [56]). Given a ground set U, a set function $\rho : 2^U \mapsto \mathbb{R}$ is submodular if for any subsets X, Y with $X \subseteq Y$ and any element $u \in U \setminus Y$,

$$\rho(Y \cup \{u\}) - \rho(Y) \leq \rho(X \cup \{u\}) - \rho(X). \tag{14}$$

Unfortunately, $TP_k(S_k, q_k)$ is non-monotonically increasing and thus not submodular.

Lemma D.3. *For each $k \in \{1, 2, \ldots, |N|\}$, the total transaction price function $TP_k(S_k, q_k)$ is neither monotonically increasing nor submodular.*

Proof. We prove this lemma by giving an instance as follows. Let the NFT project $N = \{n_1\}$, airdrop budget $b_1 = 3$, NFT quantity $q_1 = 1$, reserve price $p_1 = 1$ and the social network G be a path graph of five vertices, i.e., there are five users $u_1, u_2, \ldots, u_5$, and four edges $e_{i,i+1}$, $1 \leq i \leq 4$, where each edge $e_{i,i+1}$ has the activation probability of $a_{i,i+1} = 1$, as shown in Figure 8. The valuations of all users u_i for NFT n_1 are $v_{u_1,n_1}(q_1) = 1$, $v_{u_2,n_1}(q_1) = 5$, $v_{u_3,n_1}(q_1) = 1$, $v_{u_4,n_1}(q_1) = 3$, and $v_{u_5,n_1}(q_1) = 1$, respectively. In this instance, when (u_3, n_1) is chosen as the only airdrop, users u_4 and u_5 are activated, and thus u_4 acquires the NFT n_1, i.e., $TP_k(\{(u_3, n_1)\}, q_1) = 3$. However, if we choose (u_3, n_1) and (u_4, n_1) as the airdrops, only user u_5 is activated, and thus $TP_k(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 1$, which implies that the revenue function $TP_k(S_1, q_1)$ is non-monotonically increasing.

To prove the non-submodularity, we consider two sets of airdrops $\{(u_1, n_1), (u_3, n_1)\}$ and $\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}$. The former set of airdrops results in $f(\{(u_1, n_1), (u_3, n_1)\}, q_1) = 5$ because u_2, u_4 and u_5 are activated, and u_2 acquires NFT n_1, and the latter results in $TP_k(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) = 5$ because u_2 and u_5 are activated, and u_2 acquires NFT n_1. Therefore,

$$TP_k(\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}, q_1) - TP_k(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 4$$

and

$$TP_k(\{(u_1, n_1), (u_3, n_1)\}, q_1) - TP_k(\{(u_3, n_1)\}, q_1) = 2,$$

which implies that the total transaction price function $TP_k(S_1, q_1)$ is non-submodular. The lemma follows. □

For each $k \in \{1, 2, \ldots, |N|\}$, let $Act(S_k)$ denote the set of users being active in NFT n_k, and let

$$r(S_k, q_k) = \sum_{u_i \in Act(S_k), v_{u_i,n_k}(q_k) \geq p_k} v_{u_i,n_k}(q_k)$$

represent the total valuation function of users being active in NFT n_k following the transaction price constraint (i.e., $v_{u_i,n_k}(q_k) \geq p_k$).

The following lemma shows that, different from the total transaction price function $TP_k(S_k, q_k)$, the total valuation function $r(S_k, q_k)$ is submodular.

Lemma D.4. *For each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing but submodular in the airdrop selection.*

PROOF. Since the profit of all users being airdropped is zero, the total valuation function $r(S_k, q_k)$ is non-monotonically increasing for each $k \in \{1, 2, \ldots, |N|\}$. To prove the submodularity, we show that each function $r(S_k, q_k)$ satisfies Inequality (14). Following the proof in [51] where the submodularity of the influence function on the IC model can be reduced to that in a deterministic graph realized by flipping the coin for each edge of $G = (V, E)$, we show the submodularity of $r(S_k, q_k)$ in a deterministic graph $G' = (V, E') \subseteq G$ realized by flipping the coin for each edge of G with the activation probability. The process of influence propagation on G can be regarded as the influence propagation process upon the deterministic graph G'. Thus, for any activated user, its influence propagation is a connected subgraph of G' rooted by some airdropped user u, i.e., $(u, n_k) \in S_k$. For any set of airdrops S_k, the function $r(S_k, q_k)$ is the total valuation (excluding the valuations violating the transaction price constraint $v_{u_i,n_k}(q_k) \geq p_k$) of the union of the connected subgraphs rooted by all airdrops in S_k. Therefore, for each $k \in \{1, 2, \ldots, |N|\}$, the total valuation function $r(S_k, q_k)$ is a coverage function, which is submodular [55]. The lemma follows. □

Recall that $V(S_k, q_k)$ is the set of users holding the NFT n_k under the influence of S_k with the quantity q_k, and $Act(S_k)$ is the set of users being active in NFT n_k. Thus for each $k \in \{1, 2, \ldots, |N|\}$, we have $V(S_k, q_k) \subseteq Act(S_k)$ and $|V(S_k, q_k)| \leq q_k$, which implies

$$TP_k(S_k, q_k) \leq r(S_k, q_k). \tag{15}$$

Moreover, by Equations (3) and (4), for any q_k and q'_k with $q_k \leq q'_k$, we have $v_{u_i,n_k}(q_k) \geq v_{u_i,n_k}(q'_k)$ and thus

$$r(S_k, q_k) \geq r(S_k, q'_k). \tag{16}$$

Let (S^{opt}, Q^{opt}) denote the optimal solution of NPM, where $S^{opt} = \bigcup_{k \in \{1,2,\ldots,|N|\}} S_k^{opt}$ and $Q^{opt} = \{q_1^{opt}, q_2^{opt}, \ldots, q_{|N|}^{opt}\}$. To derive the approximation ratio, we first consider a problem similar to NPM, named NPM-QO, where the quantity constraint is removed. In NPM-QO, all active users with valuations satisfy the reserve price constraint $v_{u_i,n_k}(q_k) \geq p_k$, without the restriction that the users with the q_k highest valuations acquire the NFT n_k. The user u_i's valuation on NFT n_k is set to $v_{u_i,n_k}(1)$, and λ is set to 0. Similarly, let $\hat{S}^{opt}$ denote the optimal solution of NPM-QO, where $\hat{S}^{opt} = \bigcup_{k \in \{1,2,\ldots,|N|\}} \hat{S}_k^{opt}$ satisfies $\hat{S}_k^{opt} = \arg\max_{S_k} r(S_k, 1)$ for each $k \in \{1, 2, \ldots, |N|\}$.

Lemma D.5. *For each $k \in \{1, 2, \ldots, |N|\}$, $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot TP_k(S_k^{opt}, q_k^{opt})$ holds, where $\hat{S}_k$ is the solution found by the continuous greedy algorithn [36] to solve NPM-QO for NFT n_k.*

PROOF. By Lemma D.4, the total valuation function $r(S_k, 1)$ is submodular for each $k \in \{1, 2, \ldots, |N|\}$. According to [36], the set of airdrops $\hat{S}_k$ found by the continuous greedy algorithm is a $\frac{1}{e}$-approximation solution of NPM-QO for NFT n_k, i.e., $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot r(\hat{S}_k^{opt}, 1)$. To complete the proof, for each $k \in \{1, 2, \ldots, |N|\}$, we show $r(\hat{S}_k^{opt}, 1) \geq TP_k(S_k^{opt}, q_k^{opt})$ as follows,

$$r(\hat{S}_k^{opt}, 1) \geq r(S_k^{opt}, 1) \geq r(S_k^{opt}, q_k^{opt}) \geq TP_k(S_k^{opt}, q_k^{opt}),$$

where the first, second, and last inequalities are obtained due to the optimality of $\hat{S}_k^{opt}$ to NPM-QO for NFT n_k, Inequality (16), and Inequality (15), respectively. The lemma follows. □

For each $k \in \{1, 2, \ldots, |N|\}$, let (S_k^{alg}, q_k^{alg}) denote the algorithm solution obtained by QOOA for NFT n_k. Then (S^{alg}, Q^{alg}) is the algorithm solution obtained by QOOA, where $S^{alg} = \bigcup_{k \in \{1,2,\ldots,|N|\}} S_k^{alg}$ and $Q^{alg} = \{q_1^{alg}, q_2^{alg}, \ldots, q_{|N|}^{alg}\}$.

Theorem D.6. *QOOA is $\frac{1}{ea(1+c)}$-approximation for NPM, where $a = \max_{1 \leq k \leq |N|} \frac{r(\hat{S}_k, 1)}{p_k}$, $c = \frac{\lambda q_{max} c_{BQ} A_{max}}{p_{min}}$, $A_{\max}$ is the maximum assessment of offspring, and $p_{\min}$ is the minimum reserve price.*

PROOF. To prove this theorem, we show $f(S^{alg}, Q^{alg}) \geq \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt})$. For each $k \in \{1, 2, \ldots, |N|\}$, we have

$$\begin{aligned} TP_k(S_k^{alg}, q_k^{alg}) &\geq \frac{p_k}{r(\hat{S}_k, 1)} \cdot r(\hat{S}_k, 1) \\ &\geq \frac{1}{e} \cdot \frac{p_k}{r(\hat{S}_k, 1)} \cdot TP_k(S_k^{opt}, q_k^{opt}) \\ &\geq \frac{1}{ea} \cdot TP_k(S_k^{opt}, q_k^{opt}), \end{aligned} \tag{17}$$

where the first inequality follows by the fact that NFT n_k is sold with price at least p_k, and the second and third inequalities are obtained by Lemma D.5 and $a = \max_{1 \leq k \leq |N|} \frac{r(\hat{S}_k, 1)}{p_k}$, respectively.

We derive the relation between the total transaction price of N and the assessments of NFT offspring generated from N as follows. Let $A_{\max}$ denote the maximum assessment of an offspring, i.e.,

$$A_{\max} = e^{\eta_0 + \eta_1 \frac{1}{1} + \eta_2 |T| \frac{2^{|T|}}{2} + \eta_3 d_{\max}} = e^{\eta_0 + \eta_1 + \eta_2 |T| \cdot 2^{|T|-1} + \eta_3 d_{\max}},$$

where we assume that the quantity of this offspring is 1 (leading to e^{η_1}), it has $|T|$ traits, each trait is only owned by it and one of its parents, at most $2^{|T|}$ offspring are generated (leading to $e^{\eta_2 |T| \cdot 2^{|T|-1}}$), and its owner has the maximum degree, i.e., $d_{\max}$, on the social network (leading to $e^{\eta_3 d_{\max}}$). For any (S, Q) in NPM, we have

$$\begin{aligned}
&\lambda \cdot OS(S,Q) \\
&\leq \lambda \cdot \left(\sum_{u_i \in \bigcup_{k=1}^{|N|} V(S_k, q_k)} c_{\mathrm{BQ}} \cdot A_{\max} \right) \\
&\leq \lambda \cdot \frac{q_{\max} |N|}{|N|} \cdot \frac{c_{\mathrm{BQ}} \cdot A_{\max}}{p_{\min}} \cdot p_{\min} \cdot |N| \\
&\leq c \sum_{k=1}^{|N|} p_k d_k \qquad (\text{Let } c = \frac{\lambda \cdot q_{\max} \cdot c_{\mathrm{BQ}} \cdot A_{\max}}{p_{\min}}) \\
&\leq c \sum_{k=1}^{|N|} TP_k(S_k, q_k),
\end{aligned}$$

where $p_{\min}$ is the minimum reserve price, and d_k is the actual number of the NFT n_k sold. Note that $\bigcup_{k=1}^{|N|} V(S_k, q_k) \leq q_{\max} |N|$ holds because at most $q_{\max} |N|$ NFTs are sold. $p_{\min} |N| \leq \sum_{k=1}^{|N|} p_k d_k$ holds since $p_{\min} |N|$ is the minimum transaction prices assuming each NFT is sold at $p_{\min}$ with only one quantity.[34] Then, for any (S, Q) in NPM,

$$f(S,Q) = \sum_{k=1}^{|N|} TP_k(S_k, q_k) + \lambda \cdot OS(S,Q) \leq (1+c) \cdot \sum_{k=1}^{|N|} TP_k(S_k, q_k). \tag{18}$$

Therefore, we have

$$\begin{aligned}
f(S^{alg}, Q^{alg}) &= \sum_{k=1}^{|N|} TP_k(S_k^{alg}, q_k^{alg}) + \lambda \cdot OS(S^{alg}, Q^{alg}) \\
&\geq \sum_{k=1}^{|N|} TP_k(S_k^{alg}, q_k^{alg}) \\
&\geq \frac{1}{ea} \cdot \sum_{k=1}^{|N|} TP_k(S_k^{opt}, q_k^{opt}) \\
&\geq \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt}),
\end{aligned}$$

where the first inequality is obtained because $OS(S^{alg}, Q^{alg})$ is non-negative by definition, and the second and third inequalities are obtained by Inequality (17) and Inequality (18), respectively. The theorem follows. □

Note that regarding a, $r(\hat{S}_k, 1)$ is at most $1 \times e^{\eta_0 + \eta_1 \cdot \frac{1}{1} + \eta_2 |T||N| + \eta_3 \cdot 0}$, which is the maximum user valuation derived from some user with the maximum personal preference 1 for some NFT with the maximum assessment. To achieve the maximum assessment, the quantity of this NFT is 1, leading to $e^{\eta_1 \cdot \frac{1}{1}}$. It possesses $|T|$ traits, each of which occurs only once in the NFT project and thus has rarity $|N|$ according to Equation (1). Hence, the rarity of this NFT is $|T|\,|N|$ according to Equation (2), leading to $e^{\eta_2 |T||N|}$. Besides, since it has not been sold to anyone, the impact imparted by the ownership history is 0, leading to $e^{\eta_3 \cdot 0}$. Hence, a is $O(\frac{e^{|T||N|}}{p_{\min}})$, where $p_{\min}$ is the minimum reserve price in the NFT project, given in the problem as the input.

Regarding c, $q_{\max} = \max_{1 \leq k \leq |N|} q_k$ is the maximum quantity sold across all NFTs, which is at most $l_{\max}$, where $l_{\max}$ is given in the problem as the input and set at most to $|V|$. $A_{\max} = e^{\eta_0 + \eta_1 + \eta_2 |T| \cdot 2^{|T|-1} + \eta_3 d_{\max}}$ is the maximum assessment of offspring, where $d_{\max}$ is the maximum degree, which is at most $|V|$. Hence, $A_{\max} = O(e^{\eta_2 |T| \cdot 2^{|T|} + \eta_3 |V|})$. Therefore, c is $O(\frac{\lambda c_{\mathrm{BQ}} |V| e^{\eta_2 |T| \cdot 2^{|T|} + \eta_3 |V|}}{p_{\min}})$, where λ and c_{BQ} are given by the problem as the input.

In practice, to prevent the undervaluation of NFTs, creators of NFT projects typically set a reserve price $p_{\min}$, which is not too low. For example, the reserve price for Adidas NFT sneakers is approximately \$550.[35] For $c = \frac{\lambda q_{\max} c_{\mathrm{BQ}} A_{\max}}{p_{\min}}$, as aforementioned, $q_{\max}$ is at most $l_{\max}$, which is set much smaller than $|V|$ to maintain the scarcity of NFTs. $A_{\max} = e^{\eta_0 + \eta_1 + \eta_2 |T| \cdot 2^{|T|-1} + \eta_3 d_{\max}}$, where $d_{\max}$ is usually much smaller than $|V|$ in a typical social network. Therefore, the performance of our algorithm depends on the number of NFTs and the number of traits within an NFT project. When these two numbers are lower, our algorithm performs better.

Recall that the NPM problem cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$. Because in the real world, the number of users on social networks is in the tens of millions, making $O(\frac{1}{|V|^{1/(\log\log|V|)^c}})$ very small, NPM is difficult to be approximated within $[O(\frac{1}{|V|^{1/(\log\log|V|)^c}}), 1]$.

For time complexity, for each NFT n_k, since each active user only acquires one, $l_{\max}$ is set at most to $|V|$, as allowing a greater quantity would have no impact. On the other hand, since the number of each NFT that can be airdropped does not exceed the total number of users, $b_{\max}$ is set at most to $|V|$ as well. The time complexity of QOOA can also be rewritten as $O(|V|^3 |N|)$, which is polynomial time.

In practice, to maintain the scarcity of NFTs, $l_{\max}$ is usually not too large. For example, the NFTs of Trump Digital Trading Card have a maximum quantity of only 10, i.e., $l_{\max} = 10$. At the same time, viral marketing aims to leverage the social influence of a small number of people to achieve promotional effects. Therefore, the budget for airdrops, i.e., $b_{\max}$, is also not very large, such as ranging from 0.3% to 0.6% [41], 0.04% to 0.3% [77], or 0.03% to 0.06% [76] of the population, because allocating a larger budget may not significantly improve the overall influence spread (potentially resulting in wasteful expenditure without commensurate promotional gains).

E EXPERIMENTAL SETTINGS FOR BASELINES

For Dysim, RMA, and TipTop, the seeding cost is 1 for all users to be consistent with QOOA. For RMA, the cost-per-engagement of each advertiser is set to 1. Note that TipTop and AG aim to maximize the benefits of users and edges, respectively. The benefits are the gains of influencing either individual users or both end users of edges, instead of being uniform for every user as in Dysim, BGRD,

[34] To ensure that an NFT is marketable in the real world, its reserve price is typically determined according to the valuations of its intended audience.

[35] More information can be found at https://collect.adidas.com/bape/faq.

and RMA. We thus set the benefits in relation to user valuations to better accommodate NPM. Moreover, to facilitate breeding, we apply NFT recommendation based on trait similarity in [71] to encourage TipTop and AG to prioritize influencing users to hold multiple NFTs. For TipTop, the benefit to a user is set to the product of her valuation of n_k and the similarity between n_k and the NFTs she possesses, while that for AG is set to the product of the average valuation of n_k from both end users and the average similarity of the NFTs they hold.