Harnessing Rarity, Scarcity, and Breeding for NFT Influence Maximization on Social Networks

ABSTRACT

Non-fungible tokens (NFTs), touted as one of the most significant developments for art and technology in the Metaverse, have heavily relied on social networks for marketing. The scarcity, rarity, and unprecedented breeding mechanisms of NFTs have a huge impact on user valuations/assessments of NFTs, thus creating new challenges to viral marketing. In this paper, we make the first attempt to formulate the problem of *NFT Profit Maximization (NPM)*, which aims to maximize the sale profit from the marketplace's perspective, by selecting users for viral marketing (called NFT airdrops) and determining the NFT quantities for sale. We prove the hardness of NPM and design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, which leverages the proposed Quantity-Sensitive Profit to prune inferior airdrops and derives Valuation-based Quantity Inequality to bound the NFT quantities. To increase profit from NFT breeding, QOOA identifies and encourages the Rare Trait Collectors to purchase multiple NFTs with rare traits in order to generate valuable offspring. Experimental results demonstrate that QOOA effectively achieves up to 3.8 times the profit of state-of-the-art approaches in large-scale social networks.

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1 INTRODUCTION

The recent rise of Non-Fungible Tokens (NFTs) has been touted as a significant development intersecting the art and technology in the Metaverse. NFTs can be traced back to 2017 when CryptoKitties, an innovative application of Web3 technologies, demonstrated the idea of trading virtual cats as distinctive digital assets. Since then, the market for NFTs has experienced an unprecedented surge, witnessing remarkable growth of transactions commanding multimillion-dollar sums. Notably, a digital artist Beeple made headlines by auctioning an NFT of his artwork for a groundbreaking $69 million [3]. Other noteworthy NFT transactions include the acquisition of former Twitter CEO Jack Dorsey's inaugural tweet for $2.9 million [13] and a LeBron James highlight video for $208,000 [14].

NFTs, as digital assets verified by blockchain technology to ensure their authenticity and ownership, rely heavily on online social networks for promotion and marketing. Compared with conventional viral marketing, NFTs exhibit new and distinctive marketing features: 1) *Auction-Based Sales*: Most NFT marketplaces sell NFTs via auction mechanisms, which place significant emphasis on the *transaction prices* offered by the highest bidders based on their valuations (i.e., assessments and preferences) of the interested NFTs, as well as the *reserve price* (i.e., the lowest acceptable transaction price) set in the marketplaces. Unlike conventional viral marketing, the profitability is substantially determined by the highest bidding prices, rather than the number of influenced users.

2) *Rarity and Ownership-Driven Valuation*: Unlike everyday necessities, the rarity and notable ownership of NFTs contribute to their elevated value. As an example, CryptoPunk #2924, ranked as the 38th rarest in a collection of 10000 unique punk apes created by Larva Labs, recently fetched an impressive $4.5 million [8]. Moreover, ownership plays a significant role. CryptoPunk #9997 is an excellent example. Before being acquired by the renowned actor Shawn Yue, it was sold for around $159,000. Once in his possession, however, the subsequent sale price soared to $4.35 million [9].

3) *NFT Breeding*: The concept of NFT breeding, which permits a pair of NFTs to breed entirely new and unique offspring, is worth noting. New breeds of NFTs could be more scarce and valuable than their parent NFTs. As these offspring may be auctioned off in the future, they can bring additional profit to the marketplace. By contrast, traditional viral marketing is limited to promoting existing products and cannot leverage the perspective of potential offspring. For instance, in December 2022, Nike introduced CryptoKicks [17], NFT sneakers that contain genotype information, e.g., attributes, colors, styles, backgrounds, etc. CryptoKicks holders can use their NFTs to breed offspring inheriting traits from their parents based on genotype information, and *redeem* them for physical sneakers through Nike's Forging Mechanic.

Breeding mechanisms have been employed by NFT collections such as Heterosis [12], STEPN [20], Roaring Leaders [19], CryptoKitties [5], and Axie Infinity [2]. Note that the traits of offspring are heavily influenced by the traits of their parent NFTs. In addition, it is a common practice to restrict the user *breeding quota*. For instance, in MODragon, a user is permitted to have at most five breeding pairs of NFTs concurrently [16]. These constraints are designed to foster sustainable growth, ensure a balanced ecosystem, and maintain the rarity of NFT offspring. Furthermore, holders can breed their NFTs with friends' NFTs if the friends' NFTs are available for siring. For example, holders of Roaring Leaders can be matched with friends for *social collaborative breeding*, facilitating the creation of diverse offspring from two holders' NFTs [19]. Owing to these unique features, the promotion of NFTs presents distinct challenges in viral marketing.

NFT marketplaces, such as OpenSea and Blur, host a variety of for-sale NFT projects. In each NFT transaction, the marketplace earns a commission, which is often a percentage of the transaction price, e.g., a 2.5% service fee on OpenSea. This transaction-based profit model motivates these platforms to actively promote the NFT projects they host. One effective promotional method they employ is

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the *airdrop* strategy for viral marketing in social networks.[1] Through airdrops, influential users are identified and given free NFTs, aiming to boost public awareness and potentially increase participation in those auctions, further maximizing the marketplace's profits by increasing transaction prices from the auctions of NFTs and the potential values of NFT offspring.

The unique characteristics of NFTs highlight several new challenges that arise in the pursuit of maximizing NFT profits: 1) *Maximizing transaction prices*: The profit generated from NFTs relies on the transaction prices, which correspond to the valuations of users willing to purchase the NFTs. A crucial aspect of NFT airdrops is to target users with high valuations. Merely maximizing the spread of influence among users does not guarantee maximum profit since only a small fraction of influenced users actually make purchases, i.e., only these transactions counts for profit. Previous works [26, 27, 35, 47, 59, 63, 77, 79, 81] aim to maximize the influence spread but neglect user valuations, thus failing to ensure the maximization of profit. 2) *Balancing scarcity and widespread adoption*: The scarcity of an NFT significantly impacts users' valuations. Limiting an NFT to being unique can boost valuations, but it restricts profit generation to a single user. Conversely, supplying a large quantity of an NFT allows more users to purchase it but may reduce valuations and transaction prices. Thus, it is critical to place the right amount of NFTs in the marketplace. Previous works [32, 38, 39, 41, 43, 46, 49, 56, 71, 76] fail to account for the impact of scarcity and rarity on valuations and cannot determine appropriate quantities of NFTs. 3) *Leveraging NFT breeding*: The NFT breeding mechanism incentivizes users to own multiple NFTs or to collaborate with friends to produce offspring, both of which have the potential to generate additional profits. The traits and ownership history of parent NFTs significantly impact the assessment of their offspring (e.g., Roaring Leaders parents with rainbow wings are likely to produce offspring with similar traits). Breeding NFTs with rare traits can lead to the creation of coveted offspring. However, excessive breeding may diminish scarcity, rarity, and the assessments of NFT offspring. Hence, the strategic selection of parent NFTs is crucial to producing unique and profitable offspring. Previous works [26, 27, 35, 47, 59, 63, 77, 79, 81] that consider acquisitions of multiple uncorrelated items without additional breeding profits cannot find appropriate airdrops that take breeding into account. An illustrative example, comparing the profit maximization of NFTs with traditional influence/profit maximization, is presented in the full version [78].

In this paper, we formulate a new problem, named *NFT Profit Maximization (NPM)*. Consider the social network in a marketplace. Given a project of NFTs with traits, their reserve prices and quantity limits, a user breeding quota, and a set of airdropping budgets, NPM aims to plan for an NFT launch by determining a set of NFT airdrops on the social network and the for-sale quantity of each NFT, to maximize the profit earned from the auctions of NFTs and the potential values of their offspring. The number of airdrops for each NFT is subject to the airdropping budget constraint, and the quantity of each NFT is restricted by the quantity limit. We prove that NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$, and V is the number of users.

To solve NPM, we design an approximation algorithm, named *Quantity and Offspring-Oriented Airdrops (QOOA)*, which incorporates several novel ideas: 1) To maximize transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to estimate the potential profit earned from purchases made by influenced users, by taking account of the quantity constraint. It also identifies prospective purchasers with valuations higher than the reserve price and evaluates the likelihood for the set of airdrops to influence the prospective purchasers with the highest valuations. 2) To balance scarcity and widespread adoption, we derive the *Valuation-based Quantity Inequality (VQI)* for QOOA to efficiently find an upper bound on the profit generated under a specific for-sale quantity. VQI captures the relationships between the reserve price and user valuations for different quantities and infers the quantity for the best profit. 3) To increase the profit from NFT offspring, QOOA identifies the *Rare Trait Collector (RTC)*, which recognizes users according to the trait rarity of the NFTs they may hold. QOOA favors RTCs who are likely to engage in breeding of valuable offspring, because rarity can influence an NFT's assessment. With the user breeding quota, QOOA meticulously tailors airdrops to influence RTCs for potential breedings of NFTs with rare traits.

With these ideas, QOOA efficiently identifies users inclined to achieve high transaction prices with QSP and trims redundant searches for undesired quantities based on the profit upper bound inferred by VQI. Furthermore, QOOA enhances profit from generating valuable offspring by promoting RTCs and their friends to purchase NFTs with rare traits. We prove that QOOA is an approximation algorithm with a guaranteed performance bound and evaluate the performance on real NFT projects, e.g., Defimons Characters, Lascaux, and Timpers Pixelworks. The contributions of this work are summarized as follows.

- To the best of our knowledge, NPM, by considering auction-oriented viral marketing, NFT scarcity, trait rarity, and NFT breeding, has not been studied previously. We prove the hardness and inapproximability of NPM.
- We design an approximation algorithm QOOA for NPM. By exploiting the proposed QSP and VQI, QOOA efficiently finds airdrops that maximize the profit (i.e., transaction prices of NFTs and potential values of their offspring) with a performance bound. Furthermore, QOOA identifies RTCs and tailors airdrops to enhance their potential for generating rare offspring.
- Experiments on data of real NFT projects demonstrate that QOOA achieves profits up to 3.8 times over the baselines.

2 RELATED WORK

Influence maximization. Existing research on viral marketing has explored the problem of maximizing the influence or profit/revenue of multiple products within a company. Some studies [35, 81] assume that products are independent from each other and thus focus on optimizing the adoption of each product individually. Other studies [26, 27, 47, 59, 63, 77, 79] consider the interdependencies between product adoptions, such as complementary (positive

[1]Airdrop information of OpenSea and Blur at https://partners.opensea.io/drops and https://blur.io/airdrop, respectively.

impact) or substitutable (negative impact) relationships. Previous works on profit maximization, a variant of influence maximization [37, 48, 72, 82], in social networks have primarily focused on maximizing the difference between the influence spread and the cost of the seed group [41, 46, 49]. Li et al. [56] introduce the concept of user benefits, aiming to maximize the total benefits, instead of the influence spread. Several studies [32, 71, 76] further incorporate the concept of benefits into profit maximization, i.e., maximizing the difference between the benefits of influence and the cost of seeding. Unlike the above works maximizing the profit from nodes, some works [38, 39] examine the benefits related to interactions among activated nodes. Han et al. [43] consider the perspective of the host to maximize the revenue of all advertisers. However, the above works do not capture the breeding mechanisms in NFT projects, where owning multiple NFTs may lead to additional profits through NFT breeding. Additionally, these studies do not consider the valuation of products based on their scarcity and rarity, nor do they determine the optimal quantities for the products. As a result, they are inapplicable to NFTs considered in NPM.

NFTs in social networks. NFTs, being digital assets, are particularly well-suited for viral marketing via social networks. Recent studies [50, 68] demonstrate significant impacts of user engagement metrics on Twitter, such as counts of user membership lists, likes, and replies, on NFT trends and prices. Meanwhile, some research [34, 62] focuses on predicting NFT prices based on factors like images, texts, and rarity. Additionally, Casale-Brunet et al. [30] explore how users holding NFTs in leading projects play a pivotal role within large social network communities. However, while these studies investigate the correlation between social networks and the NFT market, they do not address the promotion of NFTs through viral marketing on social networks. Our work builds upon the insights gained from these studies to formulate NPM that closely reflects real-world dynamics in the NFT market.

3 PROBLEM FORMULATION

We first introduce the background of NFTs. Then, we describe the diffusion process of NFT in social networks, followed by problem formulation of NPM. Finally, we analyze the hardness result. Table 1 summarizes the notations and abbreviations in this paper.

Table 1: Notation and abbreviation table.

Notation	Description
N; n_k	NFT project; an NFT
T; t_d; T_k	Universal set of traits; a trait; trait set of n_k
P; p_k	Set of reserve prices of N; reserve price of n_k
Q; q_k	Set of NFT quantities; quantity of n_k
$\phi(t_d)$; $\Phi(n_k)$	Rarity of trait t_d; rarity of n_k
H_k; h_k	Ownership history of n_k; impact of H_k
$A(n_k, q_k)$	Assessment of n_k under q_k
G; u_i	Social network; a user
$a_{i,j}$	Activation probability of u_i to u_j
$v_{u_i,n_k}(q_k)$;	u_i's valuation of n_k when the quantity of n_k is q_k;
w_{u_i,n_k}	u_i's preference for n_k
β_i; γ_i	Breeding probability of u_i; siring probability of u_i
c_{BQ}	User breeding quota
S; (u_i, n_k)	Set of NFT airdrops; an NFT airdrop
S_k	Set of NFT airdrops for n_k
$o_{k,m}$	NFT offspring generated from n_k and n_m
$A(o_{k,m}, S, Q)$	Assessment of offspring $o_{k,m}$ given S and Q
$f(S, Q)$	Profit of S and Q
$TP(S_k, q_k)$; $OS(S, Q)$	Total transaction price of n_k generated by S_k subject to q_k; potential assessments of NFT offspring under the influence of S subject to Q
λ	Parameter to scale OS and align it with TP
B; b_k	Set of budgets; budget for n_k
L; l_k	Set of quantity limits; quantity limit for n_k
Abbreviation	**Description**
NPM	NFT Profit Maximization
QOOA	Quantity and Offspring-Oriented Airdrops
QAS	Quantity-driven Airdrop Selection
OPE	Offspring Profit Enhancement
PEP	Potential Expected Profit
QSP	Quantity-Sensitive Profit
VQI	Valuation-based Quantity Inequality
RTC	Rare Trait Collectors
TI	Target Index
VOGI	Valuable Offspring Generation Influence

3.1 NFT Terminology

Definition 3.1 (Airdrop). An airdrop, denoted as (u_i, n_k), designates the free distribution of a digital asset n_k, such as a token or a coin, to a user u_i. NFT airdrops aim to incentivize users to engage viral marketing in social networks by giving complimentary gifts.

Definition 3.2 (Trait). A trait, denoted as t, refers to a distinguishable feature or characteristic associated with an NFT. Traits can include visual elements (e.g., sleepy eyes in Bored Ape Yacht Club [23]), metadata (e.g., the factory series in Auntieverse by Niceaunties [22]), or other distinctive attributes (e.g., the solar oracle level in Quaere Genesis Pass [24]). Each NFT n_k has a unique combination of traits, denoted as T_k, rendering it a one-of-a-kind piece of artwork.

Definition 3.3 (NFT project). An NFT project N, also known as an NFT collection, refers to a curated assortment of digital assets released by a creator, comprising a limited number of individual NFTs. These collections typically maintain a consistent artistic style across multiple NFTs, while showcasing variations in traits such as appearance or background, collectively forming a universal set of traits T. NFT projects span various forms, including virtual real estate, music albums, and sports trading cards.

3.2 Problem Definition

In this paper, we aim to maximize the profit from an NFT project $N = \{n_1, \ldots, n_k, \ldots, n_{|N|}\}$, where each NFT n_k is associated with a corresponding reserve price $p_k \in P = \{p_1, \ldots, p_k, \ldots, p_{|N|}\}$ and described by a set of traits $T_k \subseteq T = \{t_1, \ldots, t_d, \ldots, t_{|T|}\}$, and a history of ownership $H_k \subseteq V$ (where V is the set of users). The value of an NFT can be assessed based on its scarcity, rarity, and ownership [45, 52, 61, 62, 64, 70, 80]. 1) *Scarcity of NFTs:* Following [52, 61, 80], scarcity plays a crucial role in assessing non-fungible objects. Let $Q = \{q_1, \ldots, q_{|N|}\}$ denote the quantity set of the NFT project N, where q_k is the quantity of $n_k \in N$. The value of an NFT n_k is likely to boost if it is scarce, i.e., q_k is small. 2) *Rarity of traits:*

The rarity of the traits of an NFT is also crucial. Following [45, 62], the rarity of a trait t_d is inversely proportional to its occurrence in N, i.e.,

$$\phi(t_d) = \frac{|N|}{Occ(t_d, N)}, \tag{1}$$

where $Occ(t_d, N)$ is the number of NFTs in N with t_d. The overall rarity of an NFT n_k is thus the sum of the rarity of its traits, i.e.,

$$\Phi(n_k) = \sum_{t_d \in T_k} \phi(t_d). \tag{2}$$

3) *Ownership:* In addition to the intrinsic characteristics (i.e., scarcity and rarity), the ownership history of n_k, especially when held by notable users (e.g., celebrities) [60], usually increases its value.[2] Let h_k denote the impact imparted by the ownership history H_k. The value of h_k can be evaluated by the prominence of users in H_k. Generally, the popularity of celebrities on social networks can be measured by various approaches [31, 44]. For the NFT market, it can be further enhanced by the number of NFTs they hold and the frequency of their transactions [29, 66].

Following [45, 70], the value of n_k with a quantity q_k is assessed as follows.[3]

$$A(n_k, q_k) = e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Phi(n_k) + \eta_3 h_k}, \tag{3}$$

where η_0, η_1, η_2, and η_3 are weight parameters, which can be learned from previous NFT projects with ordinary least square regression in [45]. In particular, n_k is assessed at $e^{\eta_1 \frac{1}{q_k}}$ (assessed higher as n_k becomes scarcer), $e^{\eta_2 \Phi(n_k)}$ (assessed higher when n_k possesses more rare traits [45, 62]), and $e^{\eta_3 h_k}$ (assessed higher when n_k is held by more notable users [60], where h_k is the impact imparted by the ownership history H_k).[4] Meanwhile, for all $n_k \in N$, e^{η_0} represents the assessment related to the whole NFT project, e.g., the reputation of the creator [50].

Consider a social network $G = (V, E)$, where V is the node set representing users, and E is the edge set standing for friendships. Each user $u_i \in V$ has a personal preference for an NFT n_k, denoted as $w_{u_i,n_k} \in [0, 1]$, which can be derived from learning models, such as HG-GNN [65] and DGNN [57], according to the purchase history of the user and the traits of the NFT. Following [53, 54], a user u_i's valuation on an NFT n_k is derived according to u_i's personal preference for n_k (i.e., w_{u_i,n_k}) and the assessment of n_k (i.e., Equation (3)).

$$v_{u_i,n_k}(q_k) = w_{u_i,n_k} \cdot A(n_k, q_k). \tag{4}$$

Each edge $e_{i,j} \in E$ indicates that $u_i \in V$ has an activation probability of $a_{i,j}$ to influence $u_j \in V$.

In most NFT marketplaces, e.g., OpenSea, the launching process of NFTs consists of two stages: the *airdrop* stage for viral marketing and the *public* stage for auction.[5] Let $S = \{(u_i, n_k), \dots\}$ and $Q = \{q_1, \dots, q_{|N|}\}$ denote a set of NFT airdrops and the quantity set of the NFT project N, respectively, where (u_i, n_k) represents an NFT airdrop that provides a free NFT $n_k \in N$ to a user $u_i \in V$, and q_k is the quantity of $n_k \in N$ for sale in the public stage. The airdrop stage aims to exploit the influence of S to propagate the NFT information over the social network, while the public stage starts the auctions which proceed under the influence of airdrops S, user valuations, NFT quantities Q, and reserve prices P, detailed as follows.

Airdrop stage. The influence propagation of NFTs typically follows existing diffusion models, such as the PTC, MF, TSC-HDM, LT, and IC models [33, 42, 51, 69]. Initially, all users are inactive in all NFTs, except for the users in the set of NFT airdrops S who are active for their respective NFT.[6] Following existing diffusion models, a user who is active in an NFT n_k may influence her inactive friend u_j to become active in n_k. Once u_j is successfully influenced by her active friend, she too becomes active in n_k.[7] The influence thus propagates until no more users can be influenced.

Public stage. After the airdrop stage, the active users bid on NFTs based on their valuations of NFTs. In most NFT marketplaces, each n_k is sold to its top q_k bidders.[8] Specifically, each of the active users of n_k who have the top q_k highest valuations acquires a single NFT n_k, where the valuations are no smaller than the reserve price p_k. For a user u_i winning an NFT n_k, the *transaction price* is equal to her offer made according to her valuation $v_{u_i,n_k}(q_k)$, where $v_{u_i,n_k}(q_k) \geq p_k$. Note that n_k may not be sold out if there are fewer than q_k users with valuations of at least p_k.

As NFT marketplaces earn profit from the service fees based on the transaction prices [40], it is natural to maximize the profit by maximizing the transaction prices of current NFTs and the assessments of NFT offspring (which in turn determine the subsequent transaction prices). Given that the breeding mechanism directly affects the total assessment of offspring, most NFT projects have designed their mechanisms with considerations in the following aspects:[9]

1) *Breeding Constraints:* An NFT breeding mechanism usually has some constraints to maintain the NFTs' rarity and regulate breeding frequency. Almost all NFT projects enforce a cooldown period, ensuring *non-concurrent breeding* [1, 4, 6, 12, 18, 21]. Besides, users are restricted by a breeding quota c_{BQ}, i.e., the maximum number of simultaneous breeding pairs owned by a user. For example, in MODragon, $c_{\text{BQ}} = 5$ [16].

2) *Genetic Mechanisms:* Generally, offspring inherit traits from their parent NFTs, but rarer traits have lower inheritance chances. Boosters can raise the inheritance probability of a specific trait by c_{BT}. For example, in Crypto Unicorns, using a berry (booster) associated with a trait increases its inheritance probability by 10% [4].

3) *Collaborative Breeding:* This mechanism, which facilitates community interaction, is prominently adopted by Roaring Leader [19], Axie Infinity [15], and CryptoKitties [7]. An NFT holder u_j has a siring probability, denoted as γ_j, to designate his NFTs as available for siring, indicating a willingness to collaborate with friends' NFTs

[2] As the user who breeds the offspring becomes its initial holder, ownership has a more pronounced impact on offspring assessment in our model.
[3] $A(n_k, q_k)$ is an objective assessment that is irrelevant to subjective user preferences.
[4] When an NFT n_k has not been held by any user, i.e., $H_k = \emptyset$, h_k is equal to 0.
[5] The airdrop guide of OpenSea: https://support.opensea.io/hc/en-us/articles/13592013208851-Part-2-Prepare-your-drop-schedule.

[6] A user active in an NFT n_k is said to be interested in n_k. Following most diffusion models [33, 42, 51, 69], user states are progressive; that is, active users do not revert to being inactive.
[7] For the promotional relationship between different NFTs, i.e., a user influenced by an NFT n_k is more likely to be influenced by another NFT n_m ($m \neq k$), diffusion models for multiple correlated items [26, 47, 59, 77] can be adopted for the proposed problem.
[8] Following research on influence, revenue, and profit maximization [28, 51, 75], the seed users (i.e., those who receive free samples, referred to as airdrops in the context of NFTs) have already obtained the products. Hence, there is no need for them to participate in the bidding process to acquire the NFTs.
[9] Details on breeding are introduced in the full version [78].

in the breeding process. Let u_j's friend u_i, desired to breed her NFTs, have a breeding probability, denoted as β_i. Besides breeding with her own NFTs, u_i can choose to engage with u_j's siring-ready NFTs. It is noteworthy that the initiator of the breeding (i.e., u_i) is the legal owner of the offspring.

4) *Fusion Breeding:* This mechanism requires parent NFTs to be surrendered (i.e., sacrificing parent NFTs) in order to potentially breed offspring with enhanced rarity. The likelihood for the offspring to possess traits rarer than those of the parents increases by c_{F}, potentially boosting the offspring's values. In Fat Ape Club, for example, fusion breeding produces new and stronger apes [11].

Following famous NFT projects, such as Cryptokitties, Roaring Leader, Axie Infinity, and MODragon, we model the breeding mechanism to include non-concurrent breeding, the user breeding quota, inheritance, and collaborative breeding.[10] An illustrative example is presented in the full version [78]. After acquiring NFTs during the public stage, users can decide whether to participate in the breeding process.[11] For a user u_i who possesses NFT n_k, she may offer n_k as a siring-ready NFT to friends with a siring probability γ_i. Alternatively, with a breeding probability β_i, she may opt to breed n_k with another NFT n_m. This n_m can be either an NFT she owns or a siring-ready NFT held by one of her friends. To gain the maximum profit, u_i breeds her n_k with n_m if the generated offspring, denoted as $o_{k,m}$, has the highest assessment among her possible NFT breeding pairs that adhere to the user breeding quota c_{BQ} and non-concurrent breeding.[12] For the offspring $o_{k,m}$, let $T_{k,m}$ and $H_{k,m}$ denote its trait set and ownership history, respectively. Since the diffusion of S and Q reaches different users and then affects the breeding of NFT offspring, following [45, 62, 70], the assessment of $o_{k,m}$ is derived according to Equation (3) as follows.

$$A(o_{k,m}, S, Q) = e^{\eta_0+\eta_1 \frac{1}{q_{k,m}(S,Q)}+\eta_2 \sum_{t_d\in T_{k,m}} \frac{|N\cup O(S,Q)|}{Occ\left(t_d, N\cup O(S,Q)\right)}+\eta_3 h_{k,m}},$$

where $q_{k,m}(S,Q)$ is the quantity of $o_{k,m}$ given S and Q, $O(S,Q)$ is the set of NFT offspring given S and Q, and $h_{k,m}$ is the impact of u_i since $H_{k,m} = \{u_i\}$.

As NFT marketplaces earn profits primarily from service fees of transactions [40], the profit function is defined as follows.

Definition 3.4 (Profit Function). Consider an NFT project N with traits T and their reserve prices P, a social network G, and a user breeding quota c_{BQ}. The profit of S for the NFT project N with quantities Q consists of the transaction prices of NFTs in N and the assessments of NFT offspring generated from N as follows.

$$f(S,Q) = TP(S,Q) + \lambda \cdot OS(S,Q), \tag{5}$$

where

$$TP(S,Q) = \sum_{k=1}^{|N|} TP_k(S_k, q_k) = \sum_{k=1}^{|N|} \sum_{u_i\in V(S_k,q_k)} v_{u_i,n_k}(q_k) \tag{6}$$

is the total transaction price influenced by S under quantities Q,

$$OS(S,Q) = \sum_{u_i\in\bigcup_{k=1}^{|N|} V(S_k,q_k)} \sum_{z=1}^{c_{\text{BQ}}} \mathbb{E}[A(o_z^i, S, Q)] \tag{7}$$

is the total assessment of NFT offspring generated under the influence of S with quantities Q, and λ is a parameter to scale the assessments of NFT offspring and align them with profit.[13] In Equation (6), $TP_k(S_k, q_k)$ is the total transaction price of n_k influenced by S_k under the quantity q_k. $S_k = \{(u, n_k) : (u, n_k) \in S\} \subseteq S$ consists of NFT airdrops in S that provide a free NFT n_k to some users, and $V(S_k, q_k)$ is the set of users holding the NFT n_k under the influence of S_k with the quantity q_k.[14] Moreover, $S = S_1 \cup S_2 \cup \cdots \cup S_{|N|}$, and $S_k \cap S_m = \emptyset$ for any $k, m \in \{1, 2, \ldots, |N|\}$ with $k \neq m$. In Equation (7), under the influence of S with quantities Q, o_z^i is the z-th NFT offspring bred by u_i within the user breeding quota c_{BQ}, and $\mathbb{E}[A(o_z^i, S, Q)]$ is its expected assessment depending on the breeding mechanism.

Formally, we formulate the problem of *NFT Profit Maximization (NPM)* as follows, where an illustrative example of NPM is presented in the full version [78].

Definition 3.5 (NFT Profit Maximization (NPM)). Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T and their reserve prices P, a social network G, a user breeding quota c_{BQ}, a set of airdrop budgets $B = \{b_1, \ldots, b_{|N|}\}$ for N, and a set of quantity limits $L = \{l_1, \ldots, l_{|N|}\}$, NPM aims to plan for an NFT launch by finding a set of NFT airdrops S and a set of NFT quantities Q for N, such that the profit made from airdrops $f(S,Q)$ is maximized, under the budget constraint $\forall k, |S_k| \leq b_k$ and the quantity constraint $\forall k, q_k \leq l_k$.

Theorem 3.1. *NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$.*

PROOF. Please refer to the full version [78] for the details. □

4 APPROXIMATION ALGORITHM

4.1 Algorithm Overview

To efficiently solve NPM, we design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, based on the following new ideas. 1) To achieve high transaction prices, we propose the notion of *Quantity-Sensitive Profit (QSP)* to estimate the possible profit w.r.t. a set of users if they are selected for airdrops. Given an NFT and a specific quantity for it, let *prospective purchasers* be the users with valuations of the NFT no smaller than its reserve price. QOOA first finds the likelihood for each individual user in the social network to influence the prospective purchasers. QSP carefully upper bounds the total profit from a set of candidate airdrop users by deriving the likelihood for them to influence the prospective purchasers and evaluating the valuations

[10]Our problem can accommodate the aforementioned considerations, such as boosters, fusion breeding, and non-collaborative breeding. Meanwhile, our proposed approach, QOOA, also supports these considerations, as detailed in the full version [78].

[11]The likelihood for users to participate in breeding (i.e., the siring and breeding probabilities) can be derived based on their activity histories [64].

[12]If n_m is a siring-ready NFT and is chosen by multiple NFT holders for breeding, n_m breeds with the NFT holder who requests pairing first, as seen in Derby Stars [10].

[13]λ is usually associated with the auction resale rate and can be derived from transaction history.

[14]Since the set $V(S_k, q_k)$ depends on the influence diffusion of S_k and q_k, from this perspective, it can be considered as a random set. Nevertheless, for ease of illustration, we follow [73, 74] to adopt the concept of the live-edge graph, as $V(S_k, q_k)$ is deterministic in each live-edge graph.

of these prospective purchasers. Equipped with QSP, QOOA is able to efficiently filter out unlikely users for airdrops.

2) To deal with the tradeoff between NFT quantities and user valuations, we derive the *Valuation-based Quantity Inequality (VQI)* in QOOA to efficiently find the upper bound of the profit subject to the quantity for an NFT. Specifically, the influenced users with the highest valuations (i.e., expected winners of the auction) are vital since their bids set the transaction prices and the profit. When a larger quantity is available, user valuations tend to drop as the NFT becomes less scarce, while the profit may rise due to additional transactions generated. However, if user valuations fall behind the reserve price, no additional transaction can be achieved. Hence, VQI carefully examines the maximum number of users with valuations no smaller than the reverse price under various quantities, in order to establish an upper bound on the profit. By exploiting the upper bound subject to each specific quantity, QOOA efficiently prunes redundant searches for unnecessary quantities.

3) To increase the profit earned from NFT offspring, we target users with high impact (those who can elevate the offspring's assessment) and their friends (who may leverage collaborative breeding) to produce offspring with rare traits, while complying to the user breeding quota. QOOA identifies *Rare Trait Collectors (RTCs)* as users who may acquire at least one NFT with the rarest traits. Subsequently, for each such rare-trait NFT, QOOA finds alternative airdrops that specifically target RTCs with great breeding probabilities, significant impacts, and high valuations, in order to increase the breeding opportunity and assessments of offspring. For collaborative breeding, these airdrops prioritize the friends of RTCs if they exhibit great siring probabilities and high valuations. Accordingly, these alternative airdrops replace the original airdrops, leading to the breeding of more valuable NFT offspring.

In summary, QOOA consists of two steps: Quantity-driven Airdrop Selection (QAS) and Offspring Profit Enhancement (OPE). For each NFT, QAS evaluates QSP w.r.t. different sets of users to find airdrops that maximize the total transaction price. It iteratively evaluates the profits with increasing quantities until no more profit can be generated while efficiently trimming redundant searches, according to the upper bound of the profit derived by VQI. After finding the best NFT airdrops and quantities identified in QAS, QOOA leverages OPE to improve profit from breeding by encouraging RTCs and their friends to purchase multiple NFTs with rare traits. The pseudo-code of QOOA is presented in Algorithm 1.[15]

4.2 Algorithm Description

4.2.1 Quantity-driven Airdrop Selection (QAS). QAS first maximizes the profit by finding appropriate airdrops and NFT quantities. Let $S_k^{q_k}$ denote the set of airdrops for NFT n_k identified by QAS under quantity q_k. Specifically, for each NFT n_k, QAS starts from $q_k = 1$ and finds S_k^1 that maximizes the total transaction price $TP_k(S_k^1, 1)$. Then, QAS iteratively increases q_k by 1 until q_k reaches the quantity limit l_k. Finally, QAS identifies the best quantities and the corresponding airdrops that maximize TP_k. To improve efficiency, QAS is equipped with two pruning strategies realized by *Quantity-Sensitive Profit (QSP)* and *Valuation-based Quantity Inequality (VQI)*. Specifically, QSP is the upper bound of TP_k for a

Algorithm 1: QOOA

Input: NFT project N with traits T and reserve prices P, social network G, user breeding quota c_{BQ}, budgets B, quantity limits L
Output: NFT airdrops S and quantities Q

/* QAS phase */
for *each* $n_k \in N$ **do**
  $q_k^* \leftarrow null$; $S_k^* \leftarrow null$; $TP_k^* \leftarrow 0$
  for $q_k = 1, \dots, l_k$ **do**
    if $UB_k(q_k) \le TP_k^*$ **then**
      continue
    $S_k \leftarrow \emptyset$; $TP_k \leftarrow \sum_{u_j \in V(S_k, q_k)} v_{u_j, n_k}(q_k)$
    while $|S_k| < b_k$ **do**
      $U \leftarrow \emptyset$
      for *each* $(u_i, n_k) \notin S_k$ **do**
        if $QSP_k(S_k \cup \{(u_i, n_k)\}, q_k) \ge TP_k$ **then**
          $U \leftarrow U \cup \{u_i\}$
      $u_i^* \leftarrow null$; $gain \leftarrow 0$
      for $u_i \in U$ **do**
        if $TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k) > gain$ **then**
          $u_i^* \leftarrow u_i$; $gain \leftarrow TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k)$
      if $gain > 0$ **then**
        $S_k \leftarrow S_k \cup \{(u_i^*, n_k)\}$; $TP_k \leftarrow TP_k + gain$
      else
        break
    if $TP_k > TP_k^*$ **then**
      $q_k^* \leftarrow q_k$; $S_k^* \leftarrow S_k$; $TP_k^* \leftarrow TP_k$
$S \leftarrow \bigcup_{n_k \in N} S_k^*$; $Q \leftarrow \{q_1^*, \dots, q_{|N|}^*\}$
/* OPE phase */
$N^{\mathrm{T}} \leftarrow$treasures of NFTs; $RTC \leftarrow$ RTCs according to N^{T}
Sort N^{T} according to the number of rarest traits possessed
for *each* $n_k \in N^{\mathrm{T}}$ **do**
  $S'_k \leftarrow S_k^*$; $\bar{S}'_k \leftarrow \emptyset$
  while $S'_k \neq \emptyset$ **do**
    $(u_i^*, n_k) \leftarrow \mathrm{argmin}_{(u_i, n_k) \in S'_k} VOGI_k(u_i, q_k^*)$
    $(u_j^*, n_k) \leftarrow \mathrm{argmax}_{(u_j, n_k) \in V \setminus (S'_k \cup \bar{S}'_k)} VOGI_k(u_j, q_k^*)$
    if $VOGI_k(u_i^*, q_k^*) < VOGI_k(u_j^*, q_k^*)$ **then**
      $S'_k \leftarrow S'_k \setminus \{(u_i^*, n_k)\}$; $\bar{S}'_k \leftarrow \bar{S}'_k \cup \{(u_j^*, n_k)\}$
      $S' \leftarrow S \setminus \{(u_i^*, n_k)\} \cup \{(u_j^*, n_k)\}$
      if $f(S', Q) > f(S, Q)$ **then**
        $S \leftarrow S'$
    else
      break
return S, Q

set of airdrops S_k under a specific quantity, which helps eliminate ineffective airdrops that generate a smaller TP_k than the best airdrops identified so far. On the other hand, VQI infers the upper bound of TP_k under a specific q_k, irrespective of the airdrop set S_k, to facilitate effective pruning of redundant quantities.

QSP-based pruning. Specifically, for each quantity $q_k = x$ and NFT n_k, to maximize TP_k, QAS iteratively selects the best airdrop and adds it to the current set S_k^x of airdrops. To efficiently prune ineffective airdrops, when considering (u_i, n_k), QAS first evaluates QSP of $\hat{S}_k^x = S_k^x \cup \{(u_i, n_k)\}$ and compares it with TP_k of S_k^x, as QSP serves as the upper bound of TP_k. If QSP of $\hat{S}_k^x$ is smaller than $TP_k(S_k^x, x)$, adding (u_i, n_k) to S_k^x does not yield a greater TP_k. Otherwise, QAS carefully evaluates the marginal gain of TP_k by adding (u_i, n_k) to S_k^x, i.e., $TP_k(\hat{S}_k^x, x) - TP_k(S_k^x, x)$. When the budget

[15]The workflow of QOOA is illustrated in the full version [78].

is sufficient, i.e., $\left|S_k^x\right| < b_k$, QAS adds (u_i, n_k) with the largest marginal gain to S_k^x if the gain is positive.[16]

The idea behind QSP w.r.t $\hat{S}_k^x$ is to extract the users generating the top-x expected profits of n_k under $q_k = x$, while being influenced by $\hat{S}_k^x$. The sum of these top-x expected profits serves as the QSP of $\hat{S}_k^x$. Thus, we define and identify the *prospective purchasers* as those who have the potential to generate profit.

Definition 4.1 (Prospective purchasers). Prospective purchasers of an NFT n_k are those with the valuations no smaller than the reserve price of n_k. Specifically, for NFT n_k under a specific quantity $q_k = x$ with the reserve price p_k, the set of prospective purchasers is defined as $V_k^{\text{pp}}(x) = \{u : v_{u,n_k}(x) \geq p_k\}$.

QAS evaluates each prospective purchaser u's *Potential Expected Profit (PEP)* of n_k under the influence of $\hat{S}_k^x$ with quantity $q_k = x$, serving as the upper bound of the expected profit on n_k when $\hat{S}_k^x$ successfully influences u. Afterward, QSP w.r.t $\hat{S}_k^x$ is derived by summing the top-x PEP under the influence of $\hat{S}_k^x$ to efficiently upper bound TP_k of $\hat{S}_k^x$ (proved later in Lemma 4.1 in Section 4.3).

To find PEP of a prospective purchaser $u \in V_k^{\text{pp}}(x)$ under the influence of $\hat{S}_k^x$, QAS searches for her reverse reachable sets in different deterministic realized graphs of G.[17] Then, it derives the likelihood for any user $u_i \in \hat{S}_k^x$ to influence u according to the average number of occurrences of u_i in u's reverse reachable sets, denoted as $oc(u_i, u)$. Consequently, u's PEP of n_k under the influence of $\hat{S}_k^x$ based on the likelihood for every user in $\hat{S}_k^x$ to influence u is formulated as follows.

$$PEP_k(u, \hat{S}_k^x, x) = v_{u,n_k}(x) \cdot \min\{1, \sum_{(u_i,n_k)\in\hat{S}_k^x} oc(u_i, u)\}, \quad (8)$$

where the sum of average occurrences upper bounds the probability of u being successfully influenced by $\hat{S}_k^x$ (with a maximum value of 1), since different users in $\hat{S}_k^x$ may concurrently occur in u's reverse reachable sets, leading to duplicate counts and thus overestimating the probability of successful influence by $\hat{S}_k^x$. Accordingly, PEP of u on NFT n_k can serve as the upper bound of u's expected profit on NFT n_k (proved in the full version [78]).

Equipped with PEP, QAS evaluates the QSP of a set of users subject to a specific quantity, in order to derive the upper bound of TP_k for $\hat{S}_k^x = S_k^x \cup \{(u_i, n_k)\}$ to decide if (u_i, n_k) is an ineffective airdrop. For NFT n_k with $q_k = x$, QSP of $\hat{S}_k^x$ is the sum of PEP of the prospective purchasers with the top-x PEP as follows.

$$QSP_k(\hat{S}_k^x, x) = \sum_{u\in V_k^{\text{PEP}}(\hat{S}_k^x,x)} PEP_k(u, \hat{S}_k^x, x), \quad (9)$$

where $V_k^{\text{PEP}}(\hat{S}_k^x, x)$ is the set of prospective purchasers with the top-x PEP on NFT n_k under the influence of $\hat{S}_k^x$. Note that $TP_k(\hat{S}_k^x, x)$ is no greater than $QSP_k(\hat{S}_k^x, x)$ (proved later in Lemma 4.1 in Section 4.3). QAS thus skips the evaluation of $TP_k(\hat{S}_k^x, x)$ when $QSP_k(\hat{S}_k^x, x)$ is no greater than $TP_k(S_k^x, x)$.

Example 4.2. Consider a new NFT project $N = \{n_1, n_2, n_3\}$ (extracted from CrypotoKitties [5]), as depicted in Figure 1(a), to be promoted in the social network in Figure 1(b). Assume that $B = \{2, 2, 2\}$, $L = \{2, 2, 2\}$, $P = \{1.8, 1.8, 1.8\}$, $\eta_0 = 0$, $\eta_1 = 1.5$, $\eta_2 = 0.2$, and $\eta_3 = 0.5$. QAS starts from n_1 subject to $q_1 = 1$. At first, QAS examines all users and chooses u_1 for an airdrop to maximize the total transaction price, i.e., $S_1^1 = \{(u_1, n_1)\}$ and $TP_1(S_1^1, 1) = 5.51$. Next, QAS derives QSP as follows.

$$QSP_1(S_1^1 \cup \{(u_2, n_1)\}, 1) = 4.32, \quad QSP_1(S_1^1 \cup \{(u_3, n_1)\}, 1) = 4.32,$$
$$QSP_1(S_1^1 \cup \{(u_4, n_1)\}, 1) = 7.73, \quad QSP_1(S_1^1 \cup \{(u_5, n_1)\}, 1) = 7.44,$$
$$QSP_1(S_1^1 \cup \{(u_6, n_1)\}, 1) = 8.64.$$

Since both $QSP_1(S_1^1 \cup \{(u_2, n_1)\}, 1)$ and $QSP_1(S_1^1 \cup \{(u_3, n_1)\}, 1)$ are smaller than $TP_1(S_1^1, 1) = 5.51$, QOOA evaluates total transaction prices to select the second user for airdropping only among u_4, u_5, and u_6. As airdropping n_1 to u_6 leads to the greatest total transaction price, i.e., $TP_1(S_1^1 \cup \{(u_6, n_1)\}, 1) = 7.1$, QAS then updates $S_1^1 = \{(u_1, n_1), (u_6, n_1)\}$ and finishes the search for $q_1 = 1$ because $\left|S_1^1\right| = b_1 = 2$. ■

VQI-based pruning. After $q_k = x$ is examined, QAS continues to find $S_k^{q_k}$ for $q_k = x + 1$. Before the search for $q_k = x + 1$ starts, QAS derives VQI from the premise that users winning the bid must have valuations no smaller than the reserve price p_k, in order to find the upper bound of TP_k for $q_k = x + 1$, denoted as $UB_k(x + 1)$. If the upper bound $UB_k(x + 1)$ is no greater than the best TP_k generated so far, QAS skips the search for $q_k = x + 1$. After all quantities are examined, QAS assigns $q_k = x'$ and $S_k = S_k^{x'}$, where $TP_k(S_k^{x'}, x')$ is the greatest among all examined quantities.

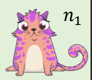
Ideally, the upper bound $UB_k(x+1)$ under $q_k = x+1$ is the sum of the transaction prices of the top-$(x+1)$ user valuations, when the best airdrops subject to $q_k = x + 1$ influence all prospective purchasers with the top-$(x+1)$ valuations.[18] Nevertheless, in reality, the number of prospective purchasers under $q_k = x+1$ may be smaller than $x+1$ due to the reserve price p_k. Therefore, to obtain a more accurate $UB_k(x+1)$, VQI derives the exact number of prospective purchasers under $q_k = x + 1$, by carefully examining whether users with the top-$(x + 1)$ preferences for n_k have valuations no smaller than the reserve price p_k. Let $W_k(y)$ denote the y-th largest user preference for n_k. For $W_k(y)$, the corresponding user is a *valid prospective purchaser* if the corresponding user valuation is no smaller than the reserve price p_k, i.e.,

$$\begin{aligned} W_k(y) \cdot A(n_k, q_k) &= W_k(y) \cdot e^{\eta_0+\eta_2\Phi(n_k)+\eta_3\cdot 0} \cdot e^{\frac{\eta_1}{q_k}} \quad (10) \\ &= W_k(y) \cdot \eta^* \cdot e^{\frac{\eta_1}{q_k}} \qquad (\text{Let } \eta^* = e^{\eta_0+\eta_2\Phi(n_k)}) \\ &\geq p_k. \end{aligned}$$

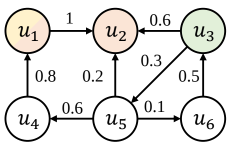
[16]Since $TP_k(S_k^x, x)$ is not monotonically increasing (proved in the full version [78]), adding (u_i, n_k) with a negative marginal gain to S_k^x decreases the profit, because the total transaction price will be lowered.

[17]Following [73, 74], we adopt the concept of *live-edge graph*, where a deterministic realized graph of $G = (V, E)$ consisting of V and the set of live-edges. An edge $e_{i,j} \in E$ is declared to be a "live-edge" if flipping a biased random coin with probability $a_{i,j}$ returns success. A reverse reachable set of a node u_i in the deterministic realized graph contains the nodes that have a path to u_i in this graph.

[18]It is possible that even the best airdrops cannot influence all prospective purchasers with the top-$(x + 1)$ valuations. However, since the influenced prospective purchasers, if they do not have top-$(x + 1)$ valuations, must necessarily have valuations smaller than the top-$(x + 1)$, this situation can still be bounded by our defined upper bound $UB_k(x + 1)$.

		NFT project			New offspring
NFT		 n_1	 n_2	 n_3	
Traits		Salmon, Amur	Salmon	Aqua marine	Aqua marine, Amur
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



Preference	u_1	u_2	u_3	u_4	u_5	u_6
n_1	0.6	0.5	0.8	0.3	0.7	0.2
n_2	0.8	0.8	0.7	0.6	0.7	0.9
n_3	0.9	0.6	0.9	0.7	0.5	0.4

(b) A social network.

Figure 1: An example for QOOA.

Accordingly, VQI rewrites Inequality (10) to infer the lower bound of the user preference for n_k, ensuring the corresponding user to be a valid prospective purchaser.

$$W_k(y) \geq \frac{p_k}{\eta^*} \cdot e^{-\frac{\eta_1}{q_k}}. \tag{11}$$

Note that for an NFT n_k, $\frac{p_k}{\eta^*}$ remains constant regardless of variations in q_k. Hence, QAS can efficiently derive the exact number of prospective purchasers under various quantities by examining how many users with preferences for n_k satisfy Inequality (11). Considering $q_k = x + 1$, the upper bound $UB_k(x+1)$ is thus derived below.

$$UB_k(x+1) = \sum_{y=1}^{\min\{x+1, \left|V_k^{\mathrm{pp}}(x+1)\right|\}} W_k(y) \cdot A(n_k, x+1), \tag{12}$$

where only the prospective purchasers with top-$(x+1)$ preferences for n_k are considered. Consequently, if there exists $x' \leq x$ leading to $UB_k(x+1) \leq TP_k(S_k^{x'}, x')$, i.e., $TP_k(S_k^{x+1}, x+1)$ for every possible S_k^{x+1} cannot be greater than TP_k obtained so far, QAS skips the search for $q_k = x + 1$.

Example 4.3. Following Example 4.2, before the search for $q_1 = 2$ begins, QAS extracts $W_1(1) = 0.8$ and $W_1(2) = 0.7$ and computes $\frac{p_k}{\eta^*} \cdot e^{-\frac{\eta_1}{q_k}} = \frac{1.8}{e^{0.2 \times 4.5}} \cdot e^{-\frac{1.5}{2}} = 0.35$. Since both $W_1(1) = 0.8 > 0.35$ and $W_1(2) = 0.7 > 0.35$, the corresponding users (i.e., u_3 and u_5, respectively) are valid prospective purchasers. As a result, QAS derives $UB_1(2) = W_1(1)A(n_1, 2) + W_1(2)A(n_1, 2) = 7.81$. Since $UB_1(2) = 7.81 > TP_1(S_1^1, 1) = 7.1$, QAS continues the search under $q_1 = 2$. ■

4.2.2 Offspring Profit Enhancement (OPE). After QAS identifies the best NFT airdrops and quantities, OPE aims to further increase the profit by encouraging the breeding of rare NFT offspring. It recognizes the NFTs with traits of the greatest rarity as *treasures* to serve as the parents in the breeding during OPE. Specifically, let $T^{\mathrm{R}} \subseteq T$ denote the set of rarest traits according to the trait rarity in Equation (1).[19] Based on T^{R}, OPE defines the treasures of NFTs (with at least one trait in T^{R}) as $N^{\mathrm{T}} = \{n_k : T_k \cap T^{\mathrm{R}} \neq \emptyset\}$. Accordingly, OPE identifies users purchasing at least one treasure as *Rare Trait Collectors (RTCs)*. Our idea is to encourage RTCs and their friends to purchase more treasures, within the confines of the user breeding quota, in order to breed rare (more valuable) offspring by tailoring the airdrops of the treasures.[20] The treasures are examined sequentially to tailor their airdrops by prioritizing those with more rare traits, since their offspring are more inclined to be highly valuable.

During the examination for each treasure $n_k \in N^{\mathrm{T}}$, OPE evaluates the *Valuable Offspring Generation Influence (VOGI)* of every user (i.e., the potential of a user to influence others in breeding valuable offspring) to find an alternative airdrop to replace the original one, because the original airdrops identified by QAS do not account for breeding. Particularly, the alternative airdrops emphasize their potential to influence prospective purchasers that may breed valuable offspring, by targeting i) RTCs with great breeding probabilities (more likely to breed), significant impacts (breeding offspring assessed higher due to ownership), and high valuations of n_k (avoiding reducing TP significantly), ii) friends of the aforementioned RTCs with great siring probabilities (more likely to participate in breeding) and high valuations of n_k (also avoiding reducing TP significantly), and iii) RTCs holding fewer treasures than c_{BQ} (adhering to the user breeding quota). Thus, the user with the highest VOGI replaces the originally selected user with the lowest VOGI in airdrops to improve the profit.

Specifically, VOGI is evaluated by considering both the probability of influencing prospective purchasers and the extent to which they are prioritized as targets for purchasing n_k, according to the above three criteria. We first introduce the *Target Index (TI)* of a prospective purchaser u_j for n_k, which prioritizes u_j to target on purchasing n_k. A prospective purchaser with a larger TI for n_k indicates a greater likelihood of participating in breeding valuable offspring from n_k. The TI of a prospective purchaser u_j for n_k is defined according to u_j's breeding probability, impact, valuation of n_k, siring probability, and the number of treasures held by u_j, as follows.

$$TI_k(u_j, S, Q) = \begin{cases} \beta_j I_j v_{u_j, n_k}(q_k), & \text{if } 1 \leq \Theta(u_j, S, Q) < c_{\mathrm{BQ}} \\ \beta_c \gamma_j v_{u_j, n_k}(q_k), & \text{else if an RTC } u_c \text{ is } u_j\text{'s friend} \\ 0, & \text{otherwise} \end{cases},$$

where $\Theta(u_j, S, Q)$ is the number of treasures held by u_j given S and Q, and I_j is the impact of u_j. For the first case where u_j is an RTC holding fewer treasures than c_{BQ}, prioritizing u_j for purchasing n_k can enhance the likelihood of u_j breeding valuable offspring. TI is evaluated by u_j's breeding probability (β_j), impact (I_j), and valuation of n_k ($v_{u_j,n_k}(q_k)$) in order to increase the expected assessments of offspring without significantly compromising the total transaction price. For the second case, where u_j either lacks treasures or holds an excessive amount, but has a friend u_c who is an RTC, prioritizing u_j for purchasing n_k can enable u_j to provide n_k for siring, thereby increasing the likelihood of u_c breeding valuable offspring. TI is

[19] Following [62], the top 10% rarest ones are usually discussed.

[20] OPE supports various considerations in breeding, detailed in the full version [78].

evaluated by u_c's breeding probability β_c to represent the likelihood of u_c requesting u_j to provide siring-ready NFTs, u_j's siring probability (γ_j) and valuation of n_k ($v_{u_j,n_k}(q_k)$).[21]

Then, based on TI, VOGI of a user u_i is the sum of the TI of prospective purchasers influenced by u_j, weighted by their likelihood of being influenced, in order to capture the potential of u_i to jointly facilitate purchases and collaborative breeding of the treasures.

$$VOGI_k(u_i, q_k) = \sum_{u_j \in V_k^{\text{Val}}(q_k)} TI_k(u_j, S, Q) \cdot oc(u_i, u_j), \qquad (13)$$

where $V_k^{\text{Val}}(q_k)$ is the set of prospective purchasers with the top-q_k valuations, and $oc(u_i, u_j)$ is the likelihood for u_i to influence u_j, representing the weight on u_j's TI. A user with a larger VOGI is more inclined to be chosen as an alternative airdrop, since it enhances the likelihood of breeding valuable offspring and may improve the profit. Hence, OPE iteratively tailors the airdrops identified by QAS by replacing them with airdrops for users with higher VOGI. To this end, OPE first extracts the user u^* not selected for airdrops by QAS but having the largest VOGI. If among the users identified by QAS for airdrops, user u_i has the smallest VOGI, and its VOGI is even smaller than that of u^*, i.e., $VOGI_k(u_i, q_k) < VOGI_k(u^*, q_k)$, then u_i is less likely to influence prospective purchasers to generate valuable offspring from n_k (compared with u^*). Therefore, OPE opts to airdrop to u^*, instead of u_i (i.e., replacing (u_i, n_k) with (u^*, n_k)), to facilitate breeding of rare traits. It updates S as $S \setminus \{(u_i, n_k)\} \cup \{(u^*, n_k)\}$ if the above replacement improves the profit, i.e., $f(S \setminus \{(u_i, n_k)\} \cup \{(u^*, n_k)\}, Q) > f(S, Q)$.

Example 4.4. Following Example 4.3, QOOA obtains $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_6, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 25.93$. Assume that $\lambda = 1$ and $c_{\text{BQ}} = 3$. Since 'amur' and 'aqua marine' are the rarest traits, OPE identifies the treasures $N^{\text{T}} = \{n_1, n_3\}$. For n_3, OPE examines VOGI as follows.

Airdrops:	$VOGI_3(u_4, 1) = 0.51$,	$VOGI_3(u_6, 1) = 0.26$.
Non-airdrops:	$VOGI_3(u_3, 1) = 0.37$,	$VOGI_3(u_5, 1) = 0.36$,
	$VOGI_3(u_1, 1) = 0$,	$VOGI_3(u_2, 1) = 0$.

Accordingly, OPE attempts to replace u_6 with u_3 for airdrops of n_3 because $VOGI_3(u_3, 1) = 0.37 > VOGI_3(u_6, 1) = 0.26$. Since $f(S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}, Q) = 26.97 > f(S, Q) = 25.93$, OPE updates $S = S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}$. As $VOGI_3(u_5, 1) = 0.36 < VOGI_3(u_4, 1) = 0.51$, OPE terminates the enhancement for n_3. Consequently, the solution is $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_3, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 26.97$. ∎

4.3 Theoretical Analysis

Lemma 4.1. *For a set of airdrops S_k for n_k under a quantity q_k, $QSP_k(S_k, q_k) \geq TP_k(S_k, q_k)$.*

Proof. Recall $TP_k(S_k, q_k) = \sum_{u_j \in V(S_k, q_k)} v_{u_j,n_k}(q_k)$, where $V(S_k, q_k)$ is the set of users influenced by S_k to hold the NFT n_k under the quantity q_k. For each $u_j \in V(S_k, q_k)$, her expected profit on NFT n_k is no greater than $PEP_k(u_j, S_k, q_k)$. As QSP has taken PEP of the users in $V_k^{\text{PEP}}(S_k, q_k)$ into account, for any user $u_j \in V(S_k, q_k)$, if $u_j \in V_k^{PEP}(S_k, q_k)$, her PEP is already counted in QSP; otherwise, her PEP must be no greater than $PEP_k(u_{\min}, S_k, q_k)$, where $u_{\min}$ is the user with the least PEP in $V_k^{\text{PEP}}(S_k, q_k)$. Therefore, $QSP_k(S_k, q_k)$ is the upper bound of $TP_k(S_k, q_k)$. The lemma follows. □

To analyze the performance bound of QOOA, we first show that the total transaction price function $TP_k(S_k, q_k)$ is non-monotonically increasing and far from submodular. Then we define the total valuation function $r(S_k, q_k)$ of users being active in NFT n_k following the transaction price constraint (i.e., $v_{u_i,n_k}(q_k) \geq p_k$). We show that $r(S_k, q_k)$ is non-monotonically increasing but submodular (proved in [78]).

Recall that $V(S_k, q_k)$ is the set of users holding the NFT n_k under the influence of S_k with the quantity q_k, and $Act(S_k)$ is the set of users being active in NFT n_k. Thus for each $k \in \{1, 2, \ldots, |N|\}$, we have $V(S_k, q_k) \subseteq Act(S_k)$ and $|V(S_k, q_k)| \leq q_k$, which implies

$$TP_k(S_k, q_k) \leq r(S_k, q_k). \qquad (14)$$

Moreover, by Equations (3) and (4), for any q_k and q'_k with $q_k \leq q'_k$, we have $v_{u_i,n_k}(q_k) \geq v_{u_i,n_k}(q'_k)$ and thus

$$r(S_k, q_k) \geq r(S_k, q'_k). \qquad (15)$$

Let (S^{opt}, Q^{opt}) denote the optimal solution of NPM, where $S^{opt} = \bigcup_{k \in \{1,2,\ldots,|N|\}} S_k^{opt}$ and $Q^{opt} = \{q_1^{opt}, q_2^{opt}, \ldots, q_{|N|}^{opt}\}$. To derive the approximation ratio, we first consider a problem similar to NPM, named NPM-QO, where the quantity constraint is removed. In NPM-QO, all active users with valuations satisfy the reserve price constraint $v_{u_i,n_k}(q_k) \geq p_k$, without the restriction that the users with the q_k highest valuations acquire the NFT n_k. The user u_i's valuation on NFT n_k is set to $v_{u_i,n_k}(1)$, and λ is set to 0. Similarly, let $\hat{S}^{opt}$ denote the optimal solution of NPM-QO, where $\hat{S}^{opt} = \bigcup_{k \in \{1,2,\ldots,|N|\}} \hat{S}_k^{opt}$ satisfies $\hat{S}_k^{opt} = \arg\max_{S_k} r(S_k, 1)$ for each $k \in \{1, 2, \ldots, |N|\}$.

Lemma 4.2. *For each $k \in \{1, 2, \ldots, |N|\}$, $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot TP_k(S_k^{opt}, q_k^{opt})$ holds, where $\hat{S}_k$ is the solution found by the continuous greedy algorithn [36] to solve NPM-QO for NFT n_k.*

Proof. The total valuation function $r(S_k, 1)$ is submodular for each $k \in \{1, 2, \ldots, |N|\}$ (proved in [78]). According to [36], the set of airdrops $\hat{S}_k$ found by the continuous greedy algorithm is a $\frac{1}{e}$-approximation solution of NPM-QO for NFT n_k, i.e., $r(\hat{S}_k, 1) \geq \frac{1}{e} \cdot r(\hat{S}_k^{opt}, 1)$. To complete the proof, for each $k \in \{1, 2, \ldots, |N|\}$, we show $r(\hat{S}_k^{opt}, 1) \geq TP_k(S_k^{opt}, q_k^{opt})$ as follows.

$$r(\hat{S}_k^{opt}, 1) \geq r(S_k^{opt}, 1) \geq r(S_k^{opt}, q_k^{opt}) \geq TP_k(S_k^{opt}, q_k^{opt}).$$

(Please refer to [78] for more details.) The lemma follows. □

For each $k \in \{1, 2, \ldots, |N|\}$, let (S_k^{alg}, q_k^{alg}) denote the algorithm solution obtained by QOOA for NFT n_k. Then (S^{alg}, Q^{alg}) is the algorithm solution obtained by QOOA, where $S^{alg} = \bigcup_{k \in \{1,2,\ldots,|N|\}} S_k^{alg}$ and $Q^{alg} = \{q_1^{alg}, q_2^{alg}, \ldots, q_{|N|}^{alg}\}$.

Theorem 4.3. *QOOA is a $\frac{1}{ea(1+c)}$-approximation algorithm for NPM, where $a = \max_{1 \leq k \leq |N|} \frac{r(\hat{S}_k, 1)}{p_k}$, $c = \frac{\lambda q_{max} c_{BQ} A_{max}}{p_{min}}$, $A_{\max}$ is the*

[21] If u_j has multiple RTCs as friends, TI refers to the one with the highest breeding probability among them to ensure the importance of targeting u_j is not underestimated.

maximum assessment of offspring, and $p_{\min}$ *is the minimum reserve price.*

PROOF SKETCH. Since the profit function f is neither monotonically increasing nor submodular, we first consider an unconstrained problem similar to NPM without the quantity constraint and NFT breeding, and its profit function is non-monotonically increasing but submodular. Specifically, we first prove that the output profit of QOOA is at least $\frac{1}{ea}$ times of the profit of the unconstrained problem, where $a = \max_{1\leq k\leq |N|} \frac{r(\hat{S}_k,1)}{p_k}$. Afterward, according to the relation between the total transaction price and the assessments of NFT offspring, we prove that the profit function gap between the unconstrained problem and NPM is at most $\frac{1}{1+c}$, where $c = \frac{A_{\max} l_{\max}(|N|-1)}{2p_{\min}}$, and $A_{\max}$ is the maximum assessment of offspring. This implies the total gap is $\frac{1}{ea(1+c)}$. For the detailed proof and analysis of time complexity, please refer to [78]. □

Theorem 4.4. *The time complexity of QOOA is* $O(l_{\max} b_{\max} |V| |N|)$*, where* $l_{\max} = \max_{l_k \in L} l_k$ *is the maximum quantity limit and* $b_{\max}$ *is the maximum budget in* B*.*

PROOF. In the QAS phase, for an NFT n_k with budget b_k, it takes $O(|V| b_k)$ time to find the airdrops under a specific quantity. As the maximum quantity limit is $l_{\max} = \max_{l_k \in L} l_k$, it requires $O(l_{\max} b_k |V|)$ time to identify the most appropriate quantity and the corresponding airdrops for each NFT n_k. Therefore, QAS takes $O(l_{\max} b_{\max} |V| |N|)$ time to find an initial solution of NPM, where $b_{\max}$ is the maximum budget in B. Next, in the OPE phase, it examines $O(b_k)$ replacements of airdrops for a treasure n_k. As there are $O(|N|)$ treasures, OPE takes $O(b_{\max} |N|)$ time. Consequently, the time complexity of QOOA is $O(l_{\max} b_{\max} |V| |N|)$. The theorem follows. □

5 EXPERIMENTS

Datasets. We conduct experiments on three real NFT projects and three real social networks derived from NFT transactions. The NFT projects include i) Defimons Characters:[22] It has 14 NFTs, which are the identities in the pixel world of Defimons. There are 5 traits (e.g., genders and locations), with the rarity of NFTs ranging from 3.75 to 9. ii) Lascaux:[23] It has 17 NFTs, recording the art and performance of the artist Lascaux. There are 10 traits (e.g., utility, series, etc.), with the rarity of NFTs ranging from 4.05 to 39.89. iii) Timpers Pixelworks:[24] It has 7 NFTs, which are artworks by Timpers and top guest artists. There are 3 traits (e.g., artists), with the rarity of NFTs ranging from 1.4 to 7. The social networks based on NFT transactions include a) NBA (NBA Top Shot transactions):[25] It contains 245K users and 2.6M relationships derived from 2.63M transactions. b) EthereumWAX [64] (collected primarily from Ethereum and WAX): It contains 269K users and 2.8M relationships derived from 7M transactions. c) Moralis [25] (collected via the Web3 APIs of the Moralis platform):[26] It contains 5.1M users and 30.6M relationships derived from 77M transactions. We follow [55, 58, 76] to set the user preferences and the activation probabilities. Following [64], the breeding and siring probabilities of a user u_i, β_i and γ_i, are set based on her activity strength. Following famous NFT projects, e.g., Trump Digital Trading Card[27] and MODragon, the quantity limit $l_k = 10$ for each n_k and the user breeding quota $c_{\mathrm{BQ}} = 5$ for each user. Following OpenSea, the reserve price p_k of each n_k is set to 1.[28] Following [64], λ is set as 0.2 to align the potential assessment of offspring with transaction price, as around 1/5 of NFT holders may sell their NFTs, thereby creating transactions and profits for the marketplace in the future.

Baselines and metrics. We compare QOOA with five state-of-the-art approaches: Dysim [77], BGRD [26], RMA [43], TipTop [56], and AG [38]. For Dysim, RMA, and TipTop, the seeding cost is 1 for all users to be consistent with QOOA. For RMA, the cost-per-engagement of each advertiser is set to 1. Note that TipTop and AG aim to maximize the benefits of users and edges, respectively. The benefits are the gains of influencing either individual users or both end users of edges, instead of being uniform for every user as in Dysim, BGRD, and RMA. We thus set the benefits in relation to user valuations to better accommodate NPM. Moreover, to facilitate breeding, we apply NFT recommendation based on trait similarity in [67] to encourage TipTop and AG to prioritize influencing users to hold multiple NFTs. For TipTop, the benefit to a user is set to the product of her valuation of n_k and the similarity between n_k and the NFTs she possesses, while that for AG is set to the product of the average valuation of n_k from both end users and the average similarity of the NFTs they hold. Since all baselines do not decide the quantities of NFTs for sale, we evaluate various quantities to find the one with the maximum profit. The performance metrics include i) profit $f(S,Q)$, ii) the total transaction price $TP(S,Q)$, iii) the potential assessments of offspring $OS(S,Q)$, and iv) the execution time. We perform a series of sensitivity tests regarding 1) the budgets B, 2) the quantity limits L, 3) the reserve prices P, and 4) the user breeding quota c_{BQ}. We conduct case studies on different breeding mechanisms to gain deeper insights, as well as an ablation study on QOOA to evaluate the efficacy of QSP and VQI to reduce the running time. Due to the space constraint, additional experimental results are presented in the full version [78]. We conduct all experiments on an HP DL580 server with an Intel 2.10GHz CPU and 1TB RAM. Each simulation result is averaged over 100 samples.

5.1 Scalability

5.2 Sensitivity Tests

Figures 2(b)-2(d) compare the profits of evaluated algorithms under different dataset, respectively, where the x-axis specifies the varied budget b_k for each NFT n_k. For all datasets, QOOA achieves the greatest profit by exploiting QAS to maximize the total transaction price, instead of the influence spread or total valuation. Among the baselines, TipTop is superior since it takes into account user valuations. The profit performance of AG is more diverse, because it tends to target users with a higher degree and a greater valuation to maximize the benefits on edges within the influence spread. However, when these users have exceptionally high valuations while their

[22]https://opensea.io/collection/defimons-characters.
[23]https://opensea.io/collection/lascauxfuture.
[24]https://opensea.io/collection/timperspixelworks.
[25]https://www.kaggle.com/datasets/chigorin/nba-topshot-transactions/data
[26]https://moralis.io.
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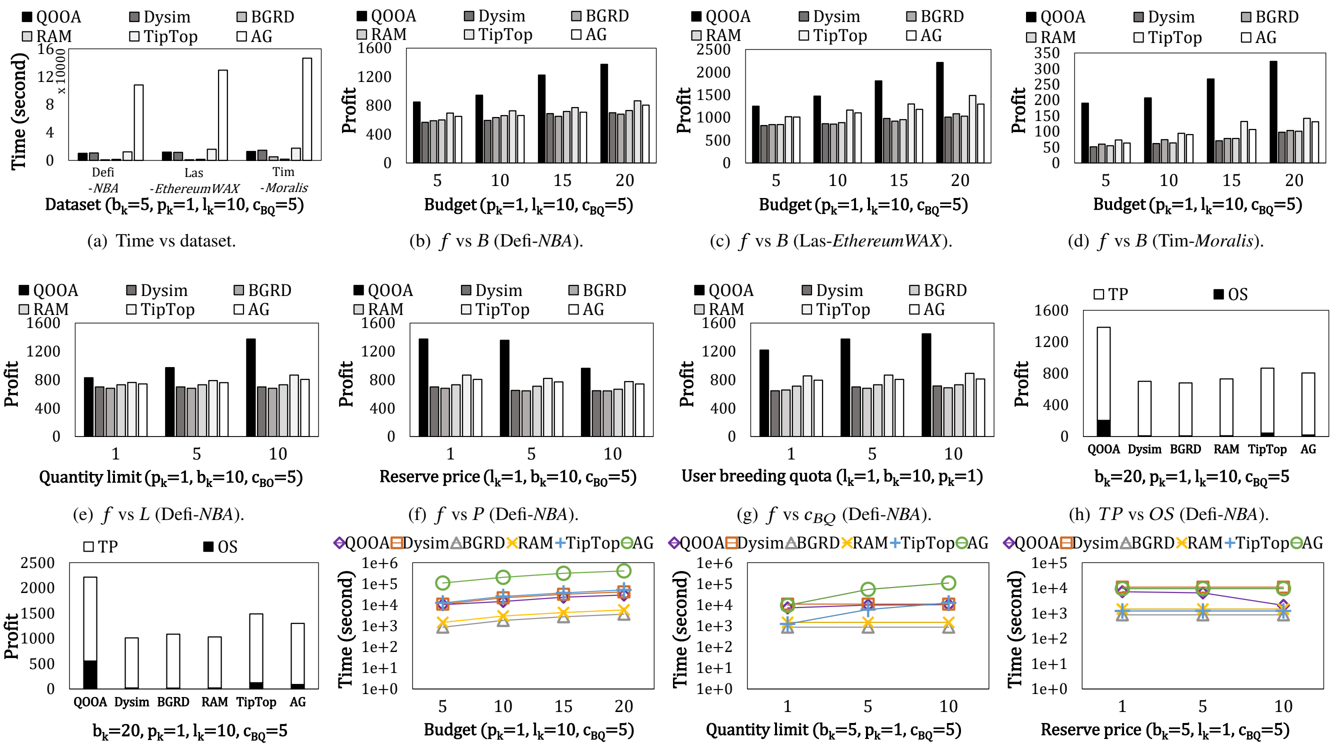


(a) Time vs dataset. (b) f vs B (Defi-*NBA*). (c) f vs B (Las-*EthereumWAX*). (d) f vs B (Tim-*Moralis*).

(e) f vs L (Defi-*NBA*). (f) f vs P (Defi-*NBA*). (g) f vs c_{BQ} (Defi-*NBA*). (h) TP vs OS (Defi-*NBA*).

(i) TP vs OS (Las-*EthereumWAX*). (j) Time vs B (Defi-*NBA*). (k) Time vs L (Defi-*NBA*). (l) Time vs P (Defi-*NBA*).

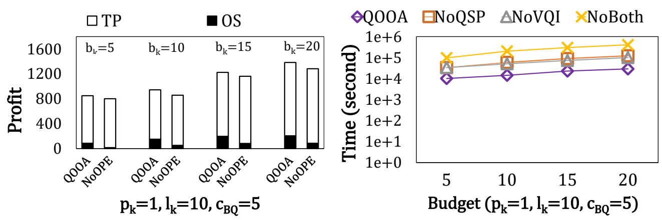
Figure 2: Sensitivity tests on examined approaches.

neighborhoods do not, AG may be misled into finding airdrops solely for influencing them, resulting in relatively lower profits. Moreover, without considering NFT quantities and breeding, TipTop and AG are inferior to QOOA. As the budget increases, all approaches yield higher profits, but their increasing trends diverge. The profit increments from QOOA, TipTop, and AG are obvious, whereas those from Dysim, BGRD, and RMA are insignificant, as they maximize influence spread rather than profit.

In addition to different budgets, Figure 2(e) presents the profits under varying quantity limits l_k for each NFT n_k. QOOA consistently achieves higher profits compared to the baselines. As the quantity limit increases, the profits obtained by all approaches grow, manifesting that more quantities provide greater opportunities for breeding, subsequently boosting the total assessment of offspring. Figure 2(f) shows the profits with respect to different reserve prices p_k for each n_k. As the reserve price grows, the profits for all approaches drop. This is because a higher reserve price reduces the number of prospective purchasers, which not only lowers the total transaction price but also diminishes the opportunities for offspring breeding, thereby decreasing the potential assessments of offspring. Next, Figure 2(g) evaluates the profits for varied user breeding quotas. As the user breeding quota increases, all approaches achieve higher profits. However, the gap between QOOA and the baselines becomes more pronounced with the growth of the quota. A strengh of QOOA is attributed to OPE which judiciously balances the treasures held by RTCs and their friends, ensuring that users optimize the breeding of valuable offspring within the quota. In contrast to QOOA, the profit differences obtained by all baselines under varying l_k, p_k, and c_{BQ} are insignificant, since they do not take breeding into account. Consequently, they are unable to identify effective airdrops tailored to different breedings for maximizing profits.

Figures 2(h) and 2(i) evaluate the profits generated by original NFTs (TP) and offspring (OS). As shown, QOOA is most effective at achieving high OS, because OPE meticulously tailors the airdrops of NFTs with rare traits, encouraging RTCs and their friends with a high Target Index to engage in purchasing these NFTs simultaneously. Moreover, the Target Index effectively prioritizes users with a high probability of producing offspring with high assessments. In contrast, the baselines do not account for the generation of offspring and solely focus on profiting from the transaction prices of the original NFTs. Moreover, as half of the traits in Lascaux are rare (i.e., with the trait rarity of 17), QOOA always takes the chance to produce offspring with rare traits. This phenomenon is observed in Figure 2(i), where the profit generated by offspring accounts for nearly 25% of the total profit, surpassing the 15% observed in Figure 2(h).

Afterward, Figure 2(j) compares the execution time under different budgets. QOOA requires slightly more time than some baselines since it carefully determines the airdrops to encourage RTCs and their friends to purchase multiple NFTs with rare traits. Additionally, QOOA carefully examines various quantities to determine the best airdrops that maximize profits. In contrast, the quantity-agnostic baselines (i.e., Dysim, BGRD, and RAM) do not spend time in identifying airdrops with various potential quantities. BGRD usually requires the least time since it does not perform separate searches for airdrops corresponding to different NFTs. RMA and TipTop are more efficient than Dysim and AG, respectively, by leveraging



(a) Profit vs B (Defi-*NBA*). (b) Time vs B (Defi-*NBA*).

Figure 3: Ablation study on QOOA.

reverse influence sampling to identify airdrops. As more budgets are allocated, all approaches require more time. Different from Figure 2(j), in Figure 2(k), when the quantity limit increases, the execution time of QOOA grows slowly, because QOOA exploits VQI to avoid redundant searches for certain quantities. TipTop and AG, which identify airdrops based on quantity-sensitive valuation, requires more execution time as well. In contrast, other baselines incur execution times that are insensitive to the quantity limits. Besides, in Figure 2(l), as the reserve price increases, the efficacy of VQI becomes more pronounced, leading to more effective pruning in QOOA. Hence, the execution time of QOOA decreases with higher reserve prices. In contrast, all baselines do not consider reserve prices, and thus, their execution times remain unchanged.

5.3 Ablation Study

To evaluate the efficacy of QSP, VQI, and OPE, we compare four variants of QOOA: NoQSP, NoVQI, NoBoth, and NoOPE. Specifically, NoQSP does not employ QSP to prune unlikely users for airdropping; NoVQI omits VQI to prune quantities unnecessary for examination; NoBoth does not exploit either of QSP and VQI; NoOPE does not adjust airdrops to enhance profits from breeding. Figure 3(a) compares QOOA and NoOPE across varied budgets. Note that QSP and VQI do not impact the profit since both QSP and VQI serve as the upper bound of TP for pruning, aiming to reduce running time. Thus, NoQSP, NoVQI, and NoBoth are excluded in Figure 3(a). As shown, QOOA consistently outperforms NoOPE in profit, with additional profits stemming from offspring. This is because OPE proficiently promotes rare-trait NFTs to RTCs and their friends, facilitating both individual and collaborative breeding. Moreover, OPE tailors airdrops to influence users to produce the maximum number of valuable offspring. On the other hand, Figure 3(b) compares the execution times of QOOA, NoQSP, NoVQI, and NoBoth across different budgets. QOOA exploits VQI to infer the maximum TP attainable for a quantity, which then facilitates the pruning of redundant searches across various quantities. When seeking airdrops under a specific quantity, QOOA utilizes QSP to serve as the upper bound of TP for a set of users, effectively discarding unlikely users for airdrops. The results indicate that both QSP and VQI play crucial roles in enhancing the efficiency of QOOA. In contrast, NoBoth without any pruning strategy requires the most time.

5.4 Case Study on Breeding Mechanisms

We present case studies on Defimons Characters-*NBA*, focusing on different breeding: *Booster*, *No Collaborative Breeding*, and *Fusion Breeding* in Section 3. Figure 4 compares the total assessment of offspring (*OS*) across various approaches. Since *Fusion Breeding* allows NFTs with common traits to breed offspring with rare traits, through careful adjustments of airdrops of both rare-trait and common-trait NFTs, QOOA achieves the highest OS among all mechanisms. In contrast, *No Collaborative Breeding* limits breeding to individual ownership, reducing the flexibility of QOOA in tailoring airdrops and resulting in the lowest OS among the three mechanisms.

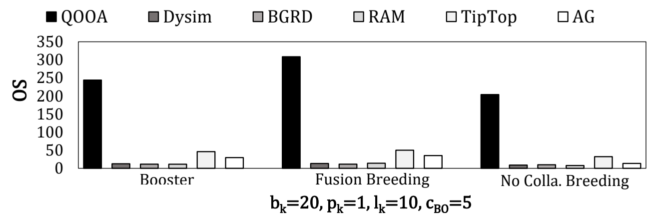


Figure 4: Comparison on breeding mechanisms.

We have several observations as follows. 1) In *Booster*, we find that User #139861, who possesses a booster, is likely to acquire the NFTs 'Orion' (with the rarest trait 'Heiwa Village') and 'Dingo' (with two common traits 'Male' and 'Yorokobi Town') through QAS's airdrops. After OPE, her friend, User #317, with a siring probability of 0.72, now has a high probability of acquiring the NFT 'Apollo' (also with 'Heiwa Village'), boosting the chance of User #139861 to generate offspring inheriting this trait from 0.14 to 0.28. It demonstrates that OPE by encouraging the friends of RTCs with boosters to purchase rare-trait NFTs, effectively increases the assessments of offspring. 2) In *No Collaborative Breeding*, we note that User #8114, with a breeding probability of 0.6 and an impact of 44, is initially inclined to acquire only the NFT 'Apollo' influenced by the airdrops determined by QAS. However, with the adjustments by OPE, she becomes more likely to also acquire the NFT 'Orion.' While her initial probability of generating offspring is nearly zero, it now stands at about 0.36. Furthermore, the generated offspring has a 0.28 probability of inheriting the rarest trait, 'Heiwa Village,' leading to a notable increase in OS. 3) In *Fusion Breeding*, we observe that under the influence of QAS's airdrops, User #17428 has a high probability of acquiring only one NFT 'Danica,' featuring the traits 'Female' and 'Forbidden Village' with rarities of 2 and 3.5, respectively. After OPE, she is likely to acquire another NFT, 'Leo,' featuring the traits 'Male' and 'Yorokobi Town' with rarities of 2 and 1.75, respectively. The probability of her offspring inheriting the rarest trait, 'Heiwa Village,' thus increases from 0 to 0.1. As Fusion Breeding allows offspring to inherit rare traits not present in the parent NFTs, OPE adjusts the airdrops for NFTs not only with rare traits but also with semi-rare traits, increasing the probability of achieving higher OS.[29]

6 CONCLUSION

To the best of our knowledge, this work is the first attempt to investigate profit maximization for NFTs from the marketplace's perspective. By taking into account key features of NFTs, including breeding, scarcity, and rarity, we formulate a new problem, named

[29]emi-rare traits are less commonly found in the NFT project but more abundant than rare traits.

NPM, to find the airdrops and determine the quantities for viral marketing. We prove the hardness of NPM and design an approximation algorithm QOOA, which effectively tackles NPM by efficiently identifying airdrops under varying quantities and increasing the profit from offspring Experiments on real NFT projects and social networks demonstrate that QOOA achieves up to 3.8 times the profits over the state-of-the-art approaches. In future work, we plan to consider multiple time slots and recommend appropriate actions (either breeding or selling) to users in order to maximize profit.

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