Harnessing Rarity, Scarcity, and Breeding for NFT Promotion on Social Networks

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ABSTRACT

Non-fungible tokens (NFTs), touted as one of the most significant developments for art and technology in the Metaverse, have heavily relied on social networks for marketing. The scarcity, rarity, and unprecedented breeding mechanism of NFTs have a huge impact on user valuations/assessments of NFTs, thus creating new challenges to viral marketing. In this paper, we make the first attempt to formulate the problem of *NFT Profit Maximization (NPM)*, which aims to maximize the sale profit from the marketplace's perspective, by selecting users for NFT airdrops and determining the NFT quantities for sale. We prove the hardness of NPM and design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, which leverages the proposed Quantity-Sensitive Profit to prune inferior airdrops and derives Valuation-based Quantity Inequality to bound the NFT quantities. To increase profit from NFT breeding, QOOA identifies and encourages the Rare Trait Collectors to purchase multiple NFTs with rare traits in order to generate valuable offspring. Experimental results demonstrate that QOOA effectively achieves up to 3.8 times the profit of state-of-the-art approaches in large-scale social networks.

1 INTRODUCTION

The recent rise of Non-Fungible Tokens (NFTs) has been touted as a significant development intersecting art and technology in the Metaverse. NFTs can be traced back to 2017 when CryptoKitties, an innovative application of Web3 technologies, demonstrated the idea of trading virtual cats as distinctive digital assets. Since then, the market for NFTs has experienced an unprecedented surge, witnessing remarkable growth of transactions commanding multimillion-dollar sums. Notably, a digital artist Beeple made headlines by auctioning an NFT of his artwork for a groundbreaking $69 million [3]. Other noteworthy NFT transactions include the acquisition of former Twitter CEO Jack Dorsey's inaugural tweet for $2.9 million [14] and a LeBron James highlight video for $208,000 [16].

NFTs, as digital assets verified by blockchain technology to ensure their authenticity and ownership, rely heavily on online social networks for promotion and marketing. Compared with conventional viral marketing, NFTs exhibit new and distinctive marketing features. These new features include: 1) *Auction-Based Sales*: Most NFT marketplaces sell NFTs via auction mechanisms, which place significant emphasis on the *transaction prices* offered by the highest bidders based on their valuations (i.e., assessments and preferences) of the interested NFTs, as well as the *reserve price* (i.e., the lowest acceptable transaction price) set in the marketplaces. Unlike conventional viral marketing, the profitability is substantially determined by these prices, rather than the number of influenced users. For instance, Nakamigos #3648 received over 5200 views, but ultimately sold for $28,000 to the highest bidder due to its limited quantity [19]. 2) *Rarity and Ownership-Driven Valuation*: Unlike everyday necessities, the rarity and notable ownership of NFTs contribute to their elevated value. As an example, CryptoPunk #2924, ranked as the 38th rarest in a collection of 10000 unique punk apes created by Larva Labs in 2017, recently fetched an impressive $4.5 million [8]. Moreover, ownership plays a significant role. CryptoPunk #9997 is an excellent example. Before being acquired by the renowned actor Shawn Yue, it was sold for around $159,000. Once in his possession, however, the subsequent sale price soared to $4.35 million [9]. 3) *NFT Breeding*: The concept of NFT breeding, which permits the combination of a pair of NFTs to produce entirely new and unique offspring, is worth noting. New breeds of NFTs could be more scarce and valuable than their parent NFTs. For instance, in December 2022, Nike introduced CryptoKicks [21], NFT sneakers that contain genotype information, e.g., attributes, colors, styles, backgrounds, etc. CryptoKicks holders can breed their NFTs, i.e., creating offspring inheriting traits from their parents based on genotype information, and redeem them for physical sneakers through Nike's Forging Mechanic. Similar breeding mechanisms are employed by NFT collections such as Heterosis [13], STEPN [24], Roaring Leaders [23], CryptoKitties [5], and Axie Infinity [2]. Note that the traits of offspring are heavily influenced by the traits of their parent NFTs. For example, in STEPN, two "common" parents cannot produce an "epic" offspring. In addition, it is a common practice to restrict the user breeding quota. For instance, in MODragon, a user is permitted to have at most five breeding pairs of NFTs concurrently [18]. These constraints are designed to foster sustainable growth, ensure a balanced ecosystem, and maintain the rarity of NFT offspring. Furthermore, holders can breed their NFTs with friends' NFTs if the friends' NFTs are available for siring. For example, holders of Roaring Leaders can be matched with friends for collaborative breeding, facilitating the creation of diverse offspring from two holders' NFTs [23]. Owing to these unique features, the promotion of NFTs presents distinct challenges in viral marketing.

NFT marketplaces, such as OpenSea and Blur, host a variety of for-sale NFT projects. In each NFT transaction, the marketplace earns a commission, which is often a percentage of the transaction price, e.g., a 2.5% service fee on OpenSea. This transaction-based profit model motivates these platforms to actively promote the NFT projects they host. One effective promotional method they employ is the *airdrop* strategy.[1] Through airdrops, influential users are identified and given free NFTs, aiming to boost public awareness and potentially increase participation in those auctions, further maximizing the marketplace's profits. An illustrative example, comparing the profit maximization of NFTs with traditional influence/profit maximization, is presented in Appendix A. The unique characteristics of NFTs highlight several new challenges that arise in the pursuit of maximizing NFT profits: 1) *Maximizing transaction prices*: The profit generated from NFTs relies on the transaction prices, which correspond to the valuations of users willing

[1] Airdrop information of OpenSea and Blur at https://partners.opensea.io/drops and https://blur.io/airdrop, respectively.

to purchase the NFTs. A crucial aspect of NFT airdrops is to target users with high valuations. Merely maximizing the spread of influence among users does not guarantee maximum profit since only a small fraction of influenced users actually make purchases. Previous works [28, 29, 33, 43, 56, 61, 70, 71, 73] aim to maximize the influence spread but neglect user valuations, thus failing to ensure the maximization of profit. 2) *Tradeoff between scarcity and quantity*: The scarcity of an NFT significantly impacts users' valuations. Limiting an NFT to being unique can boost valuations, but it restricts profit generation to a single user. Conversely, supplying a large quantity of an NFT allows more users to purchase it but may reduce valuations and transaction prices. Previous works [31, 35, 36, 38, 40, 42, 44, 53, 68, 69] fail to account for the impact of scarcity and rarity on valuations and cannot determine appropriate quantities of NFTs. 3) *Leveraging NFT breeding*: The NFT breeding mechanism incentivizes users to own multiple NFTs or to collaborate with friends to produce offspring, both of which have the potential to generate additional profits. The traits and ownership history of parent NFTs significantly impact the assessment of their offspring (e.g., Roaring Leaders parents with rainbow wings are likely to produce offspring with similar traits). Breeding NFTs with rare traits can lead to the creation of coveted offspring. However, excessive breeding may diminish scarcity, rarity, and the assessments of NFT offspring. Hence, the strategic selection of parent NFTs is crucial to producing unique and profitable offspring. Previous works [28, 29, 33, 43, 56, 61, 70, 71, 73] that consider acquisitions of multiple items without additional profits cannot find appropriate airdrops that take breeding into account.

In this paper, we formulate a new problem, named *NFT Profit Maximization (NPM)*. Given an NFT project with traits and their reserve prices, a social network, a user breeding quota, a set of budgets, and quantity limits, NPM aims to determine the set of NFT airdrops on the social network and the quantity of each NFT to maximize the profit earned from the auctions of NFTs and their offspring, from the marketplace's perspective. The number of airdrops for each NFT is subject to the budget constraint, and the quantity of each NFT is restricted by the quantity limit. We first prove that NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$, where V is the number of users. To solve NPM, we design an approximation algorithm, named *Quantity and Offspring-Oriented Airdrops (QOOA)*, which incorporates several novel ideas: 1) To maximize transaction prices, QOOA introduces *Quantity-Sensitive Profit (QSP)* to estimate the potential profit earned from purchases made by some influenced users, considering the quantity constraint. It identifies prospective purchasers with valuations higher than the reserve price and evaluates the likelihood for the set of airdrops to influence the prospective purchasers with the highest valuations. 2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find an upper bound on the profit generated under a specific quantity constraint. VQI captures the relationships between the reserve price and user valuations for different quantities and infers the maximum quantity for the best profit. 3) To increase the profit from NFT offspring, QOOA identifies the *Rare Trait Collector (RTC)*, which evaluates users according to the trait rarity of the NFTs they may hold. QOOA favors RTCs likely to engage in breeding of valuable offspring, given that ownership can influence an NFT's assessment. With the user breeding quota, QOOA meticulously tailors airdrops to influence RTCs to guide a balanced possession of NFTs with rare traits. We evaluate the performance of QOOA on real NFT projects, e.g., Defimons Characters, Lascaux, and Timpers Pixelworks. The contributions of this work are summarized as follows.

- To the best of our knowledge, NPM is the first attempt to study the profit maximization problem for NFTs, by considering NFT breeding, NFT scarcity, and trait rarity. We prove the hardness and inapproximability of NPM.
- We design an approximation algorithm QOOA for NPM. By incorporating the notions of QSP and VQI, QOOA is able to efficiently find airdrops that maximize the total transaction price. Furthermore, QOOA identifies RTCs and tailors airdrops to enhance their potential for generating rare offspring.
- Via real NFT projects, experiments demonstrate that QOOA achieves up to 3.8 times the profits over the state-of-the-arts.

2 RELATED WORK

Profit/revenue maximization. Previous works on profit/revenue maximization in social networks have primarily focused on maximizing the difference between the influence spread and the cost of the seed group [38, 42, 44]. Li et al. [53] introduce the concept of user benefits, aiming to maximize the total benefits rather than just the influence spread. Several studies [31, 68, 69] further incorporate the concept of benefits into profit maximization, i.e., maximizing the difference between the benefits of influence and the cost of seeding. Unlike the above works maximizing the profit from nodes, the works [35, 36] examine the benefits related to interactions among activated nodes. Han et al. [40] consider the perspective of the host and maximize the revenue of all advertisers. However, these works do not address the issue of profit/revenue/benefit that varies depending on the quantity of the product. They do not determine the optimal quantity to maximize profit/revenue/benefit. Additionally, they do not account for the potential additional profits obtained by simultaneously adopting multiple products, thus making them inapplicable to the NFT breeding considered in NPM.

Viral marketing on multiple products. Existing research on viral marketing has explored the problem of maximizing the influence or profit/revenue of multiple products within a company. Some studies [33, 73] assume that each product is independent and they focus on optimizing the adoption of each product individually. Other studies [28, 29, 43, 56, 61, 70, 71] consider the interdependencies between product adoptions, such as complementary or substitutable relationships. However, they primarily focus on modeling the positive or negative impact of adopting one product on another. They do not capture the breeding mechanism inherent in NFT projects, where owning multiple NFTs may lead to additional profits through NFT breeding. Additionally, these studies do not consider the valuation of products based on their scarcity and rarity, nor do they determine the optimal quantities for the products. As a result, previous viral marketing on multiple products cannot be directly applied to NPM.

3 PROBLEM FORMULATION

We first describe the diffusion process of NFT information that takes NFT characteristics into account. Then, we define the profit function and the profit maximization problem of NFTs. Table 1 in Appendix B summarizes the notations and abbreviations in this paper.

In this paper, we aim to maximize the profit from an NFT project $N = \{n_1, \ldots, n_k, \ldots, n_{|N|}\}$, where each NFT n_k is associated with a corresponding reserve price $p_k \in P = \{p_1, \ldots, p_k, \ldots, p_{|N|}\}$ and described by a set of traits $T_k \subseteq T = \{t_1, \ldots, t_d, \ldots, t_{|T|}\}$, e.g., with rainbow wings, with a diamond crown, and so on, and a history of ownership $H_k \subseteq V$ (where V is the set of users; to be detailed later). The value of an NFT can be assessed based on its scarcity, rarity, and ownership [41, 47, 58, 60, 62, 67, 72].[2] Accordingly, for n_k with a quantity q_k for sale, its value assessment is as follows.[3]

$$A(n_k, q_k) = e^{\eta_0 + \eta_1 \frac{1}{q_k} + \eta_2 \Phi(n_k) + \eta_3 h_k}, \tag{1}$$

where η_0, η_1, η_2, and η_3 are weight parameters, which can be learned from previous NFT projects based on [41]. In particular, n_k is assessed at $e^{\eta_1 \frac{1}{q_k}}$ (assessed higher as n_k becomes scarcer), $e^{\eta_2 \Phi(n_k)}$ (assessed higher when n_k possesses more rare traits, where $\Phi(n_k)$ is the rarity of n_k [41, 60] detailed in Appendix C.1), and $e^{\eta_3 h_k}$ (assessed higher when n_k is held by more notable users [57], where h_k is the impact imparted by the ownership history H_k).[4] Meanwhile, for all $n_k \in N$, e^{η_0} represents the assessment related to the whole NFT project, e.g., the reputation of the creator [45].

Consider a social network $G = (V, E)$, where V is the node set representing users, and E is the edge set standing for friendships. Each user $u_i \in V$ has a personal preference for an NFT n_k, denoted as $w_{u_i,n_k} \in [0, 1]$, which can be derived from learning models, such as HG-GNN [63] and DGNN [54], according to the purchase history of the user and the traits of the NFT. Following [48, 49], a user u_i's valuation on an NFT n_k is derived according to u_i's personal preference for n_k (i.e., w_{u_i,n_k}) and the assessment of n_k (i.e., Equation (1)).

$$v_{u_i,n_k}(q_k) = w_{u_i,n_k} \cdot A(n_k, q_k). \tag{2}$$

Each edge $e_{i,j} \in E$ indicates that $u_i \in V$ has an activation probability of $a_{i,j}$ to influence $u_j \in V$.

In most NFT marketplaces, such as OpenSea, the auction process of NFTs consists of two stages: the airdrop stage and the public stage.[5] Let $S = \{(u_i, n_k), \ldots\}$ and $Q = \{q_1, \ldots, q_{|N|}\}$ denote a set of NFT airdrops and the quantity set of the NFT project N, respectively, where (u_i, n_k) represents an NFT airdrop that provides a free NFT $n_k \in N$ to a user $u_i \in V$, and q_k is the quantity of $n_k \in N$ to be acquired in the public stage. The airdrop stage aims to exploit the influence of S to propagate the NFT information over the social network, while the public stage determines the auction result according to the influence of S, user valuations, NFT quantities Q, and reserve prices P, detailed as follows.

Airdrop stage. The influence propagation of NFTs typically follows existing diffusion models, such as the PTC, MF, TSC-HDM, LT, and IC models [32, 39, 46, 66]. Initially, all users are inactive in all NFTs, except for the users in the set of NFT airdrops S who are active for their respective NFT.[6] Following existing diffusion models, a user who is active in an NFT n_k may influence her inactive friend u_j to become active in n_k. Once u_j is successfully influenced by her active friend, she too becomes active in n_k.[7] The influence thus propagates until no more users can be influenced.

Public stage. After the airdrop stage, the marketer allows active users to bid on NFTs and determines the holders according to user valuations, quantities, and reserve prices of NFTs. In most NFT marketplaces, such as OurSong and OpenSea, each n_k is sold to its top q_k bidders. Specifically, each of the active users of n_k who have the top q_k highest valuations acquires a single NFT n_k, where the valuations are no smaller than the reserve price p_k. In most NFT marketplaces, e.g., OpenSea, for a user u_i holding an NFT n_k, the *transaction price* is equal to her offer according to her valuation $v_{u_i,n_k}(q_k)$, where $v_{u_i,n_k}(q_k) \geq p_k$. Note that n_k may not be sold out if there are fewer than q_k users with valuations of at least p_k.

As NFT marketplaces earn profit from the service fees based on the transaction prices [37], it is natural to maximize the profit by maximizing the transaction prices of current NFTs and the assessments of NFT offspring (which in turn determine the subsequent transaction prices). Given that the breeding mechanism directly affects the total assessment of offspring, most NFT projects have designed their mechanisms with considerations in the following aspects:[8]

1) *Breeding Constraints:* The NFT breeding process has carefully designed constraints to maintain the NFTs' rarity and regulate breeding frequency. Almost all NFT projects enforce a cooldown period, ensuring non-concurrent breeding [1, 4, 6, 13, 22, 25]. Besides, users are restricted by a breeding quota c_{BQ}, indicating the maximum number of simultaneous breeding pairs owned by a user. For example, in MODragon, users can have no more than five breeding pairs of NFTs [18].

2) *Genetic Mechanisms:* Generally, offspring inherit traits from their parent NFTs, but rarer traits have lower inheritance chances. Boosters can raise the inheritance probability by c_{BT} for specific traits. For example, in Crypto Unicorns, using a berry (booster) associated with a trait increases its inheritance probability by 10% [4].

3) *Collaborative Breeding:* This mechanism, which facilitates community interaction, is prominently adopted by Roaring Leader [23], Axie Infinity [17], and CryptoKitties [7]. An NFT holder u_j has a siring probability, denoted as γ_j, to designate his NFTs as available for siring, indicating a willingness to collaborate with friends' NFTs in the breeding process. His friend u_i has a breeding probability, denoted as β_i, to desire to breed his NFT. Beyond breeding with her own NFTs, u_i can choose to engage with u_j's siring-ready NFTs. It is noteworthy that the initiator of the breeding (i.e., u_i) is the legal owner of the offspring.

4) *Fusion Breeding:* This mechanism requires parent NFTs to be surrendered in order to potentially breed offspring with enhanced rarity. The likelihood for the offspring to possess traits rarer than those

[2] Please refer to Appendix C.1 for the detailed derivation of NFT assessments.
[3] $A(n_k, q_k)$ is an objective assessment that does not include user preferences.
[4] When an NFT n_k has not been sold to anyone, i.e., $H_k = \emptyset$, h_k is equal to 0.
[5] The airdrop guide of OpenSea: https://support.opensea.io/hc/en-us/articles/13592013208851-Part-2-Prepare-your-drop-schedule.
[6] A user active in an NFT n_k is said to be interested in n_k. Following most diffusion models [32, 39, 46, 66], user states are progressive; that is, active users do not revert to being inactive.
[7] For the promotional relationship between different NFTs, i.e., a user influenced by an NFT n_k is more likely to be influenced by another NFT n_m ($m \neq k$), diffusion models for multiple correlated items [28, 43, 56, 70] can be adopted for the proposed problem.
[8] Details on breeding are introduced in Appendix C.2.

of the parents increases by c_{F}, potentially boosting the offspring's value assessments. In Fat Ape Club, for example, fusion breeding produces new and stronger apes [12].

Following famous NFT projects, such as Cryptokitties, Roaring Leader, Axie Infinity, and MODragon, we model the breeding mechanism to include non-concurrent breeding, the user breeding quota, inheritance, and collaborative breeding.[9] After acquiring NFTs during the public stage, users can decide whether to participate in the breeding process.[10] For a user u_i who possesses NFT n_k, she offers n_k as a siring-ready NFT to friends with a siring probability γ_i. Alternatively, with a breeding probability β_i, she opts to breed n_k with another NFT n_m. This n_m can be either another NFT she owns or a siring-ready NFT held by one of her friends. To gain the maximum profit, u_i breeds her n_k with n_m if the generated offspring, denoted as $o_{k,m}$, has the highest assessment among her possible NFT breeding pairs that adhere to the user breeding quota c_{BQ} and non-concurrent breeding.[11] For the offspring $o_{k,m}$, let $T_{k,m}$ and $H_{k,m}$ denote its trait set and ownership history, respectively. Since the diffusion of S and Q reaches different users and then affects the breeding of NFT offspring, following [41, 60, 67], the assessment of $o_{k,m}$ is derived according to Equation (1) as follows.

$$A(o_{k,m}, S, Q) = e^{\eta_0+\eta_1 \frac{1}{q_{k,m}(S,Q)}+\eta_2 \sum\limits_{t_d \in T_{k,m}} \frac{|N\cup O(S,Q)|}{Occ\left(t_d, N\cup O(S,Q)\right)}+\eta_3 h_{k,m}},$$

where $q_{k,m}(S,Q)$ is the quantity of $o_{k,m}$ given S and Q, $O(S,Q)$ is the set of NFT offspring given S and Q, and $h_{k,m}$ is the impact of u_i since $H_{k,m} = \{u_i\}$.

As NFT marketplaces earn profits primarily from service fees of transactions [37], the profit function is defined as follows.

Definition 3.1 (Profit Function). Consider an NFT project N with traits T and their reserve prices P, a social network G, and a user breeding quota c_{BQ}. The profit of S for the NFT project N with quantities Q consists of the transaction prices of NFTs in N and the assessments of NFT offspring generated from N as follows.

$$f(S,Q) = TP(S,Q) + \lambda \cdot OS(S,Q), \tag{3}$$

where

$$TP(S,Q) = \sum_{k=1}^{|N|} TP_k(S_k, q_k) = \sum_{k=1}^{|N|} \sum_{u_i \in V(S_k,q_k)} v_{u_i,n_k}(q_k) \tag{4}$$

is the total transaction price influenced by S under quantities Q,

$$OS(S,Q) = \sum_{u_i \in \bigcup_{k=1}^{|N|} V(S_k,q_k)} \sum_{z=1}^{c_{\mathrm{BQ}}} \mathbb{E}[A(o_z^i, S, Q)] \tag{5}$$

is the total assessment of NFT offspring generated under the influence of S with quantities Q, and λ is a parameter to scale the assessments of NFT offspring and align them with profit. In Equation (4), $TP_k(S_k, q_k)$ is the total transaction price of n_k influenced by S_k under the quantity q_k. $S_k = \{(u, n_k) : (u, n_k) \in S\} \subseteq S$ consists of NFT airdrops in S that provide a free NFT n_k for some user u, and $V(S_k, q_k)$ is the set of users holding the NFT n_k under the influence of S_k with the quantity q_k. Moreover, $S = S_1 \cup S_2 \cup \cdots \cup S_{|N|}$, and $S_k \cap S_m = \emptyset$ for any $k, m \in \{1, 2, \ldots, |N|\}$ with $k \neq m$. In Equation (5), under the influence of S with quantities Q, o_z^i is the z-th NFT offspring bred by u_i within the user breeding quota c_{BQ}, and $\mathbb{E}[A(o_z^i, S, Q)]$ is its expected assessment depending on the breeding mechanism.

Formally, we formulate the problem of *NFT Profit Maximization (NPM)* as follows.

Definition 3.2 (NFT Profit Maximization (NPM)). Given an NFT project $N = \{n_1, \ldots, n_{|N|}\}$ with traits T and their reserve prices P, a social network G, a user breeding quota c_{BQ}, a set of airdrop budgets $B = \{b_1, \ldots, b_{|N|}\}$ for N, and a set of quantity limits $L = \{l_1, \ldots, l_{|N|}\}$, NPM aims to find a set of NFT airdrops S and a set of NFT quantities Q for N, such that the profit $f(S,Q)$ is maximized, under the budget constraint $\forall k, |S_k| \leq b_k$ and the quantity constraint $\forall k, q_k \leq l_k$.

Theorem 3.1. *NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$.*

Proof. Please refer to Appendix D for the details. □

4 APPROXIMATION ALGORITHM

4.1 Algorithm Overview

To efficiently solve NPM, we design an approximation algorithm, namely *Quantity and Offspring-Oriented Airdrops (QOOA)*, including the following new ideas. 1) To achieve high transaction prices, QOOA introduces the notion of *Quantity-Sensitive Profit (QSP)* to evaluate the possible profit for a set of users if they are selected for airdrops. Given an NFT and a specific quantity for it, let *prospective purchasers* be the users with valuations no smaller than the reserve price. QOOA first finds the likelihood for each individual user in the social network to influence the prospective purchasers. QSP carefully upper bounds the total profit from a set of candidate airdrop users by deriving the likelihood for them to influence the prospective purchasers and evaluating the valuations of these prospective purchasers. Equipped with QSP, QOOA is able to efficiently filter out unlikely users for airdrops.

2) To deal with the tradeoff between NFT quantities and user valuations, QOOA derives the *Valuation-based Quantity Inequality (VQI)* to efficiently find the upper bound of the profit subject to the quantity for an NFT. Specifically, as the highest bidders acquire the NFT, the influenced users with the highest valuations are vital since they affect the transaction prices as well as the profit. When a larger quantity is available, user valuations tend to decrease as the NFT becomes less scarce, while the profit may rise due to additional transactions generated by additional buyers. However, if user valuations fall behind the reserve price, no additional transaction is generated. Hence, VQI captures the relationship between the reverse price and user valuations under varying quantities. QOOA is thus able to find the upper bound of the profit subject to each specific quantity to efficiently prune unnecessary searches for certain quantities.

3) To increase the profit earned from NFT offspring, QOOA targets users with high impact (because their ownership can elevate

[9] Our problem can accommodate the aforementioned considerations, such as boosters, fusion breeding, and non-collaborative breeding. Meanwhile, our proposed approach, QOOA, also supports these considerations, as detailed in Appendix E.7.

[10] The likelihood for users to participate in breeding (i.e., the siring and breeding probabilities) can be derived based on their activity histories [62].

[11] If n_m is a siring-ready NFT and is chosen by multiple NFT holders for breeding, n_m breeds with the NFT holder who requests pairing first, as seen in Derby Stars [11].

the offspring's assessment) and their friends (leveraging collaborative breeding) to promote NFTs with rare traits (enhancing offspring rarity), while ensuring adherence to the user breeding quota. QOOA identifies *Rare Trait Collectors (RTCs)* as users who have acquired at least one NFT with the rarest traits. Subsequently, for each such rare-trait NFT, QOOA attempts to find alternative airdrops that specifically target RTCs with great breeding probabilities, significant impacts, and high valuations, and thus increase the breeding opportunity and assessments of offspring without significantly compromising the total transaction price. For collaborative breeding, these airdrops prioritize the friends of RTCs if they exhibit great siring probabilities and high valuations. Considering the user breeding quota, these alternative airdrops also emphasize RTCs holding fewer NFTs with rare traits. On the other hand, those airdrops that may promote RTCs to over-purchase such NFTs are less likely to be selected. Accordingly, the airdrops are tailored to promote joint purchases and collaborative breeding of NFTs with rare traits, leading to the breeding of more valuable NFT offspring.

In summary, QOOA consists of two steps: Quantity-driven Airdrop Selection (QAS) and Offspring Profit Enhancement (OPE). For each NFT, QAS evaluates QSP of different users to find airdrops that maximize the total transaction price. It iteratively evaluates the profits with increasing quantities until no more profit can be generated by increasing the quantity, according to the upper bound of the profit derived by VQI to efficiently trim unnecessary searches. After finding the best NFT airdrops and quantities identified in QAS, QOOA leverages OPE to improve profit from offspring by encouraging RTCs and their friends to purchase multiple NFTs with rare traits. The pseudo-code of QOOA is presented in Algorithm 1 in Appendix E.1.

4.2 Algorithm Description

Quantity-driven Airdrop Selection (QAS). QAS aims to maximize the total transaction price by finding appropriate airdrops and NFT quantities. Let $S_k^{q_k}$ denote the set of airdrops for NFT n_k identified by QAS under q_k. Specifically, for each NFT n_k, QAS starts from $q_k = 1$ and finds S_k^1 that maximizes the total transaction price $TP_k(S_k^1, 1)$. Then, QAS iteratively increases q_k by 1 until q_k reaches the quantity limit l_k. Finally, QAS identifies the best quantities and the corresponding airdrops that maximize the total transaction price. For better efficiency, QAS is equipped with two pruning strategies realized by QSP and VQI. Specifically, QSP is the upper bound of the total transaction price for a set of airdrops S_k under a specific quantity, which helps eliminate unlikely airdrops. On the other hand, VQI infers the upper bound of the total transaction price under a specific q_k, irrespective of the airdrop set S_k, which facilitates the pruning of redundant searches for certain quantities.

In order to efficiently find appropriate users for airdrops, QAS first identifies the *prospective purchasers*, who have valuations no smaller than the reserve price, since only the influenced prospective purchasers can lead to profit. Specifically, for NFT n_k under a specific quantity q_k with the reserve price p_k, the set of prospective purchasers is $V_k^{\mathrm{pp}}(q_k) = \{u : v_{u,n_k}(q_k) \geq p_k\}$. QAS then evaluates each prospective purchaser u's *Potential Expected Profit (PEP)* of n_k under the influence of S'_k with a quantity q_k, denoted as $PEP_k(u, S'_k, q_k)$ (detailed in Appendix E.2), serving as the upper bound of the expected profit on n_k when S'_k successfully influences u. Equipped with PEP, QAS evaluates the *Quantity-Sensitive Profit (QSP)* of a set of users subject to a specific quantity. For NFT n_k with q_k, QSP of S'_k is the sum of PEP of the prospective purchasers with the top-q_k PEP as follows.

$$QSP_k(S'_k, q_k) = \sum_{u \in V_k^{\mathrm{PEP}}(S'_k, q_k)} PEP_k(u, S'_k, q_k), \tag{6}$$

where $V_k^{\mathrm{PEP}}(S'_k, q_k)$ is the set of prospective purchasers with the top-q_k PEP on NFT n_k under the influence of S'_k. Note that $TP_k(S'_k, q_k) \leq QSP_k(S'_k, q_k)$ (proved in Lemma E.1 in Appendix E.3).

Afterward, to maximize the total transaction price for NFT n_k with $q_k = x$, QAS iteratively selects the best airdrop and adds it to the current set S_k^x of airdrops. To efficiently prune inferior airdrops, when considering (u_i, n_k), QAS first evaluates QSP of $S_k^x \cup \{(u_i, n_k)\}$ and compares it with the total transaction price obtained by S_k^x. If $QSP_k\big(S_k^x \cup \{(u_i, n_k)\}, x\big) \leq TP_k(S_k^x, x)$, i.e., adding (u_i, n_k) to S_k^x does not yield a greater total transaction price, (u_i, n_k) is skipped. Otherwise, QAS evaluates the marginal gain of the total transaction price by adding (u_i, n_k) to S_k^x, i.e., $TP_k(S_k^x \cup \{(u_i, n_k)\}, x) - TP_k(S_k^x, x)$. When the budget is sufficient, i.e., $\left|S_k^x\right| < b_k$, QAS adds (u_i, n_k) with the largest marginal gain to S_k^x if the gain is positive.[12]

After $q_k = x$ is examined, QAS continues to find $S_k^{q_k}$ for $q_k = x{+}1$. Before the search for $q_k = x + 1$, QAS derives *Valuation-based Quantity Inequality (VQI)* by capturing the relationship between the reserve price p_k and user valuations to find the upper bound of the total transaction price for $q_k = x + 1$. Let $W_k(y)$ denote the y-th largest user preference for n_k. VQI infers the maximum quantity that ensures that the user with the y-th largest user preference for n_k has a valuation no smaller than reserve price p_k.[13]

$$q_k \leq \frac{\eta_1}{\ln\left(\frac{p_k}{W_k(y) \cdot \eta^*}\right)} \tag{7}$$

when $W_k(y) \cdot \eta^* < p_k$, where $\eta^* = e^{\eta_0 + \eta_2 \Phi(n_k)}$. If $W_k(y) \cdot \eta^* \geq p_k$, there is no constraint on the quantity since $e^{\frac{\eta_1}{q_k}} > 1$ for any q_k and the corresponding user valuation is greater than p_k for any quantity.

Next, QAS derives the upper bound of $TP_k(S_k^{x+1}, x+1)$ under $q_k = x + 1$ for any S_k^{x+1} by summing up the transaction prices of the top-$(x+1)$ user valuations, because the best airdrops subject to $q_k = x + 1$ are to influence all users with the top-$(x+1)$ valuations. Hence, the upper bound of $TP_k(S_k^{x+1}, x+1)$ under $q_k = x + 1$ is derived below.

$$UB_k(x+1) = \sum_{y=1}^{\min\{y', x+1\}} W_k(y) \cdot A(n_k, x+1), \tag{8}$$

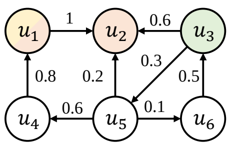
where y' is the minimum value making Inequality (7) not hold, i.e., resulting in a valuation smaller than p_k. Consequently, if there exists $x' \leq x$ leading to $UB_k(x+1) \leq TP_k(S_k^{x'}, x')$, i.e., $TP_k(S_k^{x+1}, x+1)$ for every possible S_k^{x+1} is not greater than the total transaction price

[12] Since $TP_k(S_k^x, x)$ is not monotonically increasing (proved in Lemma F.1 in Appendix F), adding (u_i, n_k) with a negative marginal gain to S_k^x decreases the total transaction price.

[13] Please refer to Appendix E.4 for the details.

		NFT project			New offspring
NFT		n_1	n_2	n_3	
Traits		Star, rainbow	Star	Moon	Moon, rainbow
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



Preference	u_1	u_2	u_3	u_4	u_5	u_6
n_1	0.6	0.5	0.8	0.3	0.7	0.2
n_2	0.8	0.8	0.7	0.6	0.7	0.9
n_3	0.9	0.6	0.9	0.7	0.5	0.4

(b) A social network.

Figure 1: An example for QOOA.

generated by the identified airdrops under some quantity examined so far, QAS skips the search for $q_k = x + 1$. After all quantities are examined, QAS assigns $q_k = x'$ and $S_k = S_k^{x'}$ if $TP_k(S_k^{x'}, x')$ is the greatest among all examined quantities.

Example 4.1. Consider a new NFT project $N = \{n_1, n_2, n_3\}$, as depicted in Figure 1(a), to be promoted in a social network in Figure 1(b). Assume that $B = \{2, 2, 2\}$, $L = \{2, 2, 2\}$, $P = \{1.8, 1.8, 1.8\}$, $\eta_0 = 0$, $\eta_1 = 1.5$, $\eta_2 = 0.2$, and $\eta_3 = 0.5$. QAS starts from n_1 subject to $q_1 = 1$. At first, QAS examines all users and chooses u_1 for an airdrop to maximize the total transaction price, i.e., $S_1^1 = \{(u_1, n_1)\}$ and $TP_1(S_1^1, 1) = 5.51$. Next, QAS derives QSP for the rest of the users.[14] Since both $QSP_1(\{u_1, u_2\}, 1)$ and $QSP_1(\{u_1, u_3\}, 1)$ are smaller than $TP_1(S_1^1, 1) = 5.51$, QOOA chooses the second user for an airdrop only from u_4, u_5, and u_6. As $TP_1(\{u_1, u_6\}, 1) = 7.1$ is the greatest, QAS updates S_1^1 as $\{u_1, u_6\}$ and finishes the search for $q_1 = 1$ because $|S_1^1| = b_1 = 2$.

Before the search for $q_1 = 2$ begins, QAS identifies $W_1(1) = 0.8$ and $W_1(2) = 0.7$ and derives $UB_1(2) = W_1(1)A(n_1, 2)+W_1(2)A(n_1, 2) = 7.81$. Since $UB_1(2) = 7.81 > TP_1(\{u_1, u_6\}, 1) = 7.1$, QAS continues the search under $q_1 = 2$. ■

Offspring Profit Enhancement (OPE). To further increase the profit, OPE encourages the breeding of rare NFT offspring. OPE identifies the NFTs with traits of the greatest rarity as *treasures*, while recognizing users purchasing at least one treasure as *Rare Trait Collectors (RTCs)*. The goal of OPE is to encourage RTCs and their friends to purchase more treasures, within the confines of the user breeding quota, by tailoring the airdrops of the treasures.[15]

Specifically, let $T^R \subseteq T$ denote the set of rarest traits according to the trait rarity in Equation (10).[16] Based on T^R, OPE defines the *treasures* of NFTs (with at least one trait in T^R) as $N^T = \{n_k : T_k \cap T^R \neq \emptyset\}$. Accordingly, OPE introduces *Rare Trait Collectors (RTCs)*, who would purchase at least one treasure based on QAS.

[14]The complete example is presented in Appendix E.5.
[15]OPE supports various considerations in breeding, detailed in Appendix E.7.
[16]Following [60], the top 10% rarest ones are usually discussed.

OPE then tailors the airdrops for each treasure in order to promote RTCs to generate more valuable offspring. The treasures with a greater number of rare traits are prioritized for examination.

For each treasure $n_k \in N^T$, OPE finds alternative airdrops that target i) RTCs with great breeding probabilities (more likely to breed), significant impacts (breeding offspring assessed higher due to ownership), and high valuations of n_k (avoiding reducing TP significantly), ii) friends of the aforementioned RTCs with great siring probabilities (more likely to participate in breeding) and high valuations of n_k (also avoiding reducing TP significantly), and iii) RTCs holding fewer treasures than c_{BQ} (adhering to the user breeding quota). Then, OPE introduces the *Target Index (TI)* of a prospective purchaser u_j for n_k, which prioritizes u_j to purchase n_k based on several factors, including u_j's breeding probability, impact, valuation of n_k, siring probability, and the number of treasures held by u_j (detailed in Appendix E.6). Then, OPE examines each user $u_i \in V$ to find alternative airdrops for n_k by evaluating the *Valuable Offspring Generation Influence (VOGI)* of u_i, which accounts for the TI of prospective purchasers influenced by u_i (denoted as $TI_k(u_j, S, Q)$), weighted by the likelihood of them being influenced.

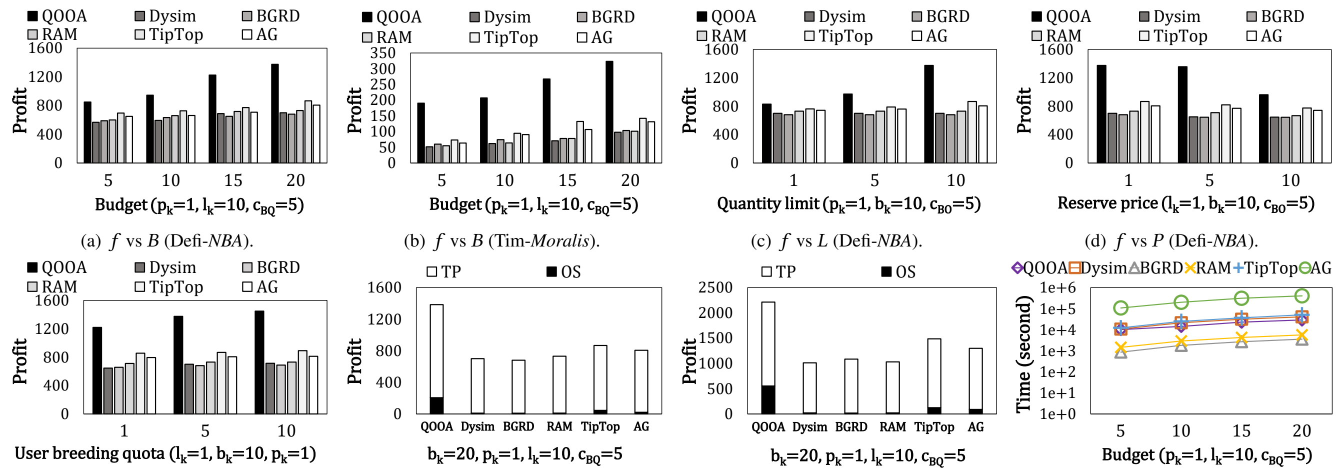
$$VOGI_k(u_i, q_k) = \sum_{u_j \in V_k^{\text{Val}}(q_k)} TI_k(u_j, S, Q) \cdot fq(u_i, u_j), \quad (9)$$

where $V_k^{\text{Val}}(q_k)$ is the set of prospective purchasers with the top-q_k valuation, and $fq(u_i, u_c)$ is the likelihood for u_i to influence u_j, representing the weight on u_j's TI. For 1) each user u_i selected for airdrops by QAS but having the least VOGI and 2) another user u_j not selected for airdrops but having the largest VOGI, if u_i is less likely to influence users to generate valuable offspring from n_k (compared with u_j), i.e., $VOGI_k(u_i, q_k) < VOGI_k(u_j, q_k)$, OPE attempts to airdrop n_k to u_j (i.e., replacing u_i) to breed offspring with rare traits. It updates S as $S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}$ if the above replacement improves the profit, i.e., $f(S \setminus \{(u_i, n_k)\} \cup \{(u_j, n_k)\}, Q) > f(S, Q)$.

Example 4.2. Following Example 4.1, QOOA obtains $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_6, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 25.93$. Assume that $\lambda = 1$ and $c_{BQ} = 3$. Since 'rainbow' and 'moon' are the rarest traits, OPE identifies the treasures $N^T = \{n_1, n_3\}$. For n_3, OPE examines VOGI of all users.[14] Then, OPE attempts to replace u_3 with u_6 for airdrops of n_3 because $VOGI_3(u_3, 1) = 0.37 > VOGI_3(u_6, 1) = 0.26$. Since $f(S\setminus\{(u_6, n_3)\} \cup \{(u_3, n_3)\}, Q) = 26.97 > f(S, Q) = 25.93$, OPE updates S as $S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}$. As $VOGI_3(u_5, 1) = 0.36 < VOGI_3(u_4, 1) = 0.51$, OPE terminates the enhancement for n_3. Consequently, the solution is $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_3, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 26.97$. ■

Theorem 4.1. *QOOA is a $\frac{1}{ea(1+c)}$-approximation algorithm in $O(l_{\max} b_{\max} |V| |N|)$ time for NPM, where $a = \max_{1 \leq k \leq |N|} \frac{r(\hat{S}_k, 1)}{p_k}$, $c = \frac{\lambda q_{max} c_{BQ} A_{max}}{p_{min}}$, $A_{\max}$ is the maximum assessment of offspring, $l_{\max} = \max_{l_k \in L} l_k$ is the maximum quantity limit, $p_{\min}$ is the minimum reserve price, and $b_{\max}$ is the maximum budget in B.*

Proof. Please refer to Appendix F for the details. □



(a) f vs B (Defi-*NBA*). (b) f vs B (Tim-*Moralis*). (c) f vs L (Defi-*NBA*). (d) f vs P (Defi-*NBA*).

(e) f vs c_{BQ} (Defi-*NBA*). (f) TP vs OS (Defi-*NBA*). (g) TP vs OS (Las-*EthereumWAX*). (h) Time vs B (Defi-*NBA*).

Figure 2: Sensitivity for different approaches.

5 EXPERIMENTS

Due to the space constraint, the details of the experiment setup are presented in Appendix G.1. **Datasets.** We conduct experiments on three real NFT projects and three real social networks derived from NFT transactions. The NFT projects include Defimons Characters (Defi) [10], Lascaux (Las) [15], and Timpers Pixelworks (Tim) [26]. The social networks based on NFT transactions include *NBA* [20], *EthereumWAX* [62], and *Moralis* [27]. **Baselines.** We compare QOOA with five state-of-the-art approaches: Dysim [70], BGRD [28], RMA [40], TipTop [53], and AG [35]. **Metrics.** The performance metrics include i) profit $f(S,Q)$, ii) the total transaction price $TP(S,Q)$, iii) the potential assessments of offspring $OS(S,Q)$, and iv) the execution time. We perform a series of sensitivity tests in terms of 1) the budgets B, 2) the quantity limits L, 3) the reserve prices P, 4) the user breeding quota c_{BQ}, and 5) different breeding mechanisms as case studies. To gain more insights, we conduct an ablation study on QOOA.

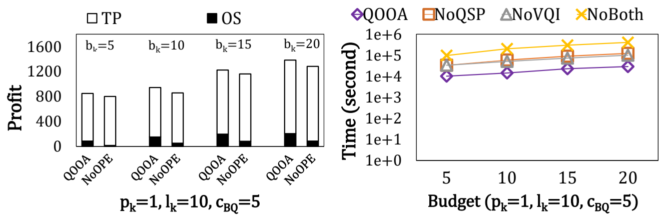
5.1 Sensitivity Tests

Due to the space constraint, more comparisons of profit and execution time are presented in Appendix G.2. Figures 2(a)-2(b) compare the profits of evaluated algorithms under different datasets (i.e., NFT projects and social networks), respectively, where the x-axis specifies the varied budget b_k for each NFT n_k.[17] For all datasets, QOOA achieves the greatest profit by exploiting QAS to maximize the total transaction price instead of the influence spread or total valuation. Among the baselines, TipTop is superior since it takes into account user valuations. The profit performance of AG is more diverse, because it tends to target users with a higher degree and a greater valuation to maximize the benefits on edges within the influence spread. However, when these users have exceptionally high valuations while their neighborhoods do not, AG may be misled into finding airdrops solely for influencing them, resulting in relatively lower profits. However, without considering NFT quantities, TipTop and AG are inferior to by QOOA.

[17] Due to the space constraint, the results of Las-*EthereumWAX* is presented in Figure 7(a) in Appendix G.2.

In addition to different budgets, Figure 2(c) presents the profits under varying quantity limits l_k for each NFT n_k. QOOA consistently achieves higher profits compared to the baselines. As the quantity limit increases, the profits obtained by all approaches grow, manifesting that more quantities provide greater opportunities for breeding, subsequently boosting the total assessment of offspring. Figure 2(d) shows the profits with respect to different reserve prices p_k for each n_k. As the reserve price grows, the profits for all approaches decline. This is because a higher reserve price reduces the number of prospective purchasers, which not only lowers the total transaction price but also diminishes the opportunities for offspring breeding, thereby decreasing the potential assessments of offspring. Next, Figure 2(e) demonstrates the profits for varied user breeding quotas. As the user breeding quota increases, all approaches achieve higher profits. However, the gap between QOOA and the baselines becomes more pronounced with the growth of the quota. strength of QOOA is attributed to OPE which judiciously balances the treasures held by RTCs and their friends, ensuring that users optimize the breeding of valuable offspring within the quota.

Figures 2(f) and 2(g) evaluate the profits generated by original NFTs (TP) and offspring (OS). QOOA is the most effective at achieving high OS, because OPE meticulously tailors the airdrops of NFTs with rare traits, encouraging RTCs and their friends with a high Target Index to engage in purchasing these NFTs simultaneously. Moreover, the Target Index effectively prioritizes users with a high probability of producing offspring with high assessments to be promoted, while avoiding decreasing the transaction price significantly. In contrast, the baselines do not account for the generation of offspring and solely focus on profiting from the transaction prices of the original NFTs. Moreover, as half of the traits in Lascaux are rare (i.e., with the trait rarity of 17), QOOA exhibits a higher likelihood of producing offspring with rare traits. This phenomenon is observed in Figure 2(g), where the profit generated by offspring accounts for nearly 25% of the total profit, surpassing the 15% observed in Figure 2(f). On the other hand, Figure 2(h) compares the execution time under different budgets. QOOA requires slightly more time than some baselines since it carefully determines the airdrops



(a) Profit vs *B* (Defi-*NBA*). (b) Time vs *B* (Defi-*NBA*).

Figure 3: Ablation study on QOOA.

to encourage RTCs and their friends to purchase multiple NFTs with rare traits. Additionally, QOOA explores various quantities to determine the best airdrops that maximize profits. In contrast, the quantity-agnostic baselines (i.e., Dysim, BGRD, and RAM) do not spend time in identifying airdrops with various potential quantities. BGRD usually requires the least time since it does not perform separate searches for airdrops corresponding to different NFTs. RMA and TipTop are more efficient than Dysim and AG, respectively, by leveraging reverse influence sampling to identify airdrops. As more budgets are allocated, all approaches require more time.

5.2 Ablation Study

We evaluate four variants of QOOA: NoQSP, NoVQI, NoBoth, and NoOPE. Specifically, NoQSP does not employ QSP to prune unlikely users for airdropping; NoVQI omits the use of VQI to prune quantities that do not require examination; NoBoth does not exploit both QSP and VQI; NoOPE does not adjust airdrops to enhance profits from breeding. Figure 3(a) compares QOOA and NoOPE across varied budgets. The absence of QSP and VQI does not impact profits since both QSP and VQI serve as the upper bound of TP for satisfactory pruning, which effectively reduces running time. NoQSP, NoVQI, and NoBoth are thereby excluded. QOOA consistently outperforms NoOPE in profits, with most of the additional profits stemming from offspring. This is because OPE proficiently promotes rare-trait NFTs to RTCs and their friends, facilitating both individual and collaborative breeding. Moreover, OPE tailors airdrops to influence users to produce the maximum number of valuable offspring. On the other hand, Figure 3(b) compares the execution times of QOOA, NoQSP, NoVQI, and NoBoth across different budgets. QOOA exploits VQI to infer the maximum TP attainable for a quantity, which then facilitates the pruning of redundant searches across various quantities. When seeking airdrops under a specific quantity, QOOA utilizes QSP to serve as the upper bound of TP for a set of users, effectively discarding unlikely users for airdrops. In contrast, NoBoth without any pruning strategy requires the most time.

5.3 Case Study on Breeding Mechanisms

We present case studies on Defimons Characters-*NBA*, focusing on different breeding: *Booster*, *No Collaborative Breeding*, and *Fusion Breeding* in Section 3. Figure 4 compares the total assessment of offspring (*OS*) across various approaches. Since *Fusion Breeding* allows NFTs with common traits to breed offspring with rare traits, through careful adjustments of airdrops of both rare-trait and common-trait NFTs, QOOA achieves the highest OS among all mechanisms. In contrast, *No Collaborative Breeding* limits breeding to individual

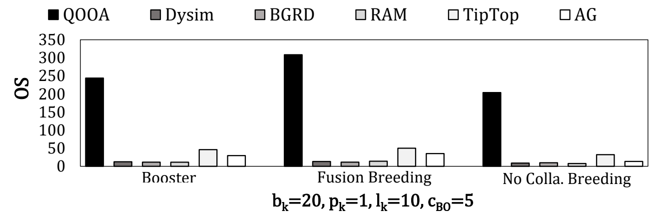


Figure 4: Case study on breeding mechanisms.

ownership, reducing the flexibility of QOOA in tailoring airdrops and resulting in the lowest OS among the three mechanisms.

We have several observations as follows. 1) In *Booster*, we find that User #139861, who possesses a booster, is likely to acquire the NFTs 'Orion' (with the rarest trait 'Heiwa Village') and 'Dingo' (with two common traits 'Male' and 'Yorokobi Town') through QAS's airdrops. After OPE, her friend, User #317, with a siring probability of 0.72, now has a high probability of acquiring the NFT 'Apollo' (also with 'Heiwa Village'), boosting the chance of User #139861 to generate offspring inheriting this trait from 0.14 to 0.28. It demonstrates that OPE by encouraging the friends of RTCs with boosters to purchase rare-trait NFTs, effectively increases the assessments of offspring. 2) In *No Collaborative Breeding*, we note that User #8114, with a breeding probability of 0.6 and an impact of 44, is initially inclined to acquire only the NFT 'Apollo' influenced by the airdrops determined by QAS. However, with the adjustments by OPE, she becomes more likely to also acquire the NFT 'Orion.' While her initial probability of generating offspring is nearly zero, it now stands at about 0.36. Furthermore, the generated offspring has a 0.28 probability of inheriting the rarest trait, 'Heiwa Village,' leading to a notable increase in OS. 3) In *Fusion Breeding*, we observe that under the influence of QAS's airdrops, User #17428 has a high probability of acquiring only one NFT 'Danica,' featuring the traits 'Female' and 'Forbidden Village' with rarities of 2 and 3.5, respectively. After OPE, she is likely to acquire another NFT, 'Leo,' featuring the traits 'Male' and 'Yorokobi Town' with rarities of 2 and 1.75, respectively. The probability of her offspring inheriting the rarest trait, 'Heiwa Village,' thus increases from 0 to 0.1. As Fusion Breeding allows offspring to inherit rare traits not present in the parent NFTs, OPE modifies the airdrops not only for the rare-trait NFTs, increasing the probability of achieving higher OS.

6 CONCLUSION

To the best of our knowledge, this work is the first attempt to investigate profit maximization for NFTs from the marketplace's perspective. By incorporating key features of NFTs, including breeding, scarcity, and rarity, we formulate a new problem, named NPM, to find the airdrops and determine the quantities for viral marketing. We prove the hardness of NPM and design an approximation algorithm QOOA, which effectively tackles NPM by identifying airdrops under varying quantities and increasing the profit from offspring Experiments on real NFT projects and social networks demonstrate that QOOA effectively achieves up to 3.8 times the profits over the state-of-the-art approaches. In future work, we plan to consider multiple time slots and recommend appropriate actions (either breeding or selling) to users in order to maximize profit.

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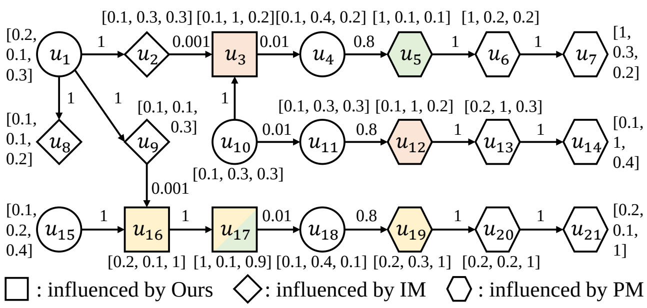
		NFT project			New offspring
NFT		n_1	n_2	n_3	
Traits		Star, rainbow	Star	Moon	Moon, rainbow
(Potential) Assessment under different quantities	1	11.02	6.05	8.17	11.02
	2	5.21	2.86	3.86	5.21
	3	4.06	2.23	3.00	4.06

(a) An NFT project and new offspring.



(b) A social network.

Figure 5: An illustrative example.

A ILLUSTRATIVE EXAMPLE

Figure 5 illustrates an example for promoting a new NFT project that consists of three NFTs (denoted by n_1, n_2 and n_3) within a social network. Figure 5(a) shows the traits and (potential) assessments under different quantities of n_1, n_2, and n_3, as well as the offspring. As shown in the NFT project, the traits 'moon' and 'rainbow' are possessed by only one NFT and are rarer than the other trait 'star'. As a user may prefer different traits, her valuation of an NFT may be calculated by multiplying her preference for the NFT with its assessment. Figure 5(b) shows a social network where nodes represent users, edges represent friendships between users, the edge weight denotes the activation probability, and the vector associated with a node denotes a user's preferences of corresponding NFTs. An NFT promotion campaign starts by airdropping some NFTs to selected users who may trigger influence propagation on the social network to activate user interests in the NFTs. After the campaign ends, the influenced users may bid on NFTs of interest with their valuations (estimated based on various factors). Assume that the number of airdrops allocated for each NFT is 1 and the reserve price of each NFT is 2.6. Based on the approach proposed in this paper, airdropping n_1, n_2, and n_3 to u_{15}, u_{10}, and u_{15}, respectively, with the quantities set as 1, 1, and 2 may yield an expected profit of 28.72, where the influenced users u_3, u_{16}, and u_{17} (represented by square shapes) win the auctions and the NFTs they purchase are indicated by colors. Since u_{17} possesses both n_1 and n_3, she has the opportunity to generate an NFT offspring with a profit of 4.32.

By contrast, an existing viral marketing algorithm based on influence maximization (IM) [28], which seeks to identify seed users to maximize the number of influenced users, may airdrop all NFTs to u_1, due to its expected influence spread of 3, where the influenced users u_2, u_8, and u_9 are represented by diamond shapes. However, as the users influenced by u_1 do not have high valuations of NFTs, the expected profit is almost 0 when the quantity of each NFT is one. On the other hand, a classical profit maximization algorithm [53] may designate u_4, u_{11}, and u_{18} as the airdrop recipients for n_1, n_2, and n_3, respectively, thereby maximizing the total expected profit of 60.57 without considering NFT quantities, where the influenced users are represented by hexagonal shapes. Nevertheless, when the quantity of each NFT is limited to one, the expected profit is reduced to 20.19, where the NFTs they purchase are indicated by colors. In addition, for both cases, no user can breed NFT offspring to generate additional profits. On the other hand, when the quantity of each NFT increases, the valuations of influenced users decline significantly, with many falling below the reserve prices. The profit obtained is thus smaller compared to the case where the quantity is one. As a result, both approaches of maximizing influence and profit fall short in the marketing goal of NFT marketing, as they do not consider the features of NFTs, including breeding, scarcity, and trait rarity, and the need to determine NFT quantities.

B NOTATION TABLE

The notations are summarized in Table 1.

C DETAILS ON PROBLEM FORMULATION

C.1 Derivation of NFT Assessment

The value of an NFT can be assessed based on its scarcity, rarity, and ownership [47, 60, 62, 72] 1) *Scarcity:* Following [47, 58, 72], scarcity plays a crucial role in assessing non-fungible objects. Let $Q = \{q_1, \ldots, q_{|N|}\}$ denote the quantity set of the NFT project N, where q_k is the quantity of $n_k \in N$. The value of an NFT n_k is likely to boost if it is scarce, i.e., q_k is small. 2) *Rarity:* The rarity of the traits of an NFT is also crucial. Following [41, 60], the rarity of a trait t_d is inversely proportional to its occurrence in N, i.e.,

$$\phi(t_d) = \frac{|N|}{Occ(t_d, N)}, \tag{10}$$

where $Occ(t_d, N)$ is the number of NFTs in N with t_d. Following [41, 60], the overall rarity of an NFT n_k is the sum of the rarity of its traits, i.e.,

$$\Phi(n_k) = \sum_{t_d \in T_k} \phi(t_d). \tag{11}$$

3) *Ownership:* In addition to the intrinsic characteristics (i.e., scarcity and rarity), the ownership history of n_k, especially when held by notable users (e.g., NFT whales and celebrities) [57], can enhance its assessment. Let h_k denote the impact imparted by the ownership history H_k. The value of h_k can be evaluated by the prominence of users in H_k. Generally, the popularity of celebrities on social networks can be measured by their network centrality, e.g., node degree. For the NFT market, it can further be measured by the number of NFTs they hold and the frequency of their transactions [30, 64], e.g., a 'whale' falls within the top 1% of traders.

C.2 Details on Breeding

Beyond those introduced in Section 3, we further elaborate on additional considerations for the breeding mechanism prevalent in various NFT projects.

Table 1: Notation and abbreviation table.

Notation	Description
N; n_k	NFT project; an NFT
T; t_d; T_k	Universe set of traits; a trait; trait set of n_k
P; p_k	Set of reserve prices of N; reserve price of n_k
Q; q_k	Set of NFT quantities; quantity of n_k
$\phi(t_d)$; $\Phi(n_k)$	Rarity of trait t_d; rarity of n_k
H_k; h_k	Ownership history of n_k; impact of H_k
$A(n_k, q_k)$	Assessment of n_k under q_k
G; u_i	Social network; a user
$a_{i,j}$	Activation probability of u_i to u_j
$v_{u_i,n_k}(q_k)$; w_{u_i,n_k}	u_i's valuation of n_k when the quantity of n_k is q_k; u_i's preference for n_k
β_i; γ_i	Breeding probability of u_i; siring probability of u_i
c_{BQ}	User breeding quota
S; (u_i, n_k)	Set of NFT airdrops; an NFT airdrop
S_k	Set of NFT airdrops for n_k
$o_{k,m}$	NFT offspring generated from n_k and n_m
$A(o_{k,m}, S, Q)$	Assessment of offspring $o_{k,m}$ given S and Q
$f(S, Q)$	Profit of S and Q
$TP(S_k, q_k)$; $OS(S, Q)$	Total transaction prices of n_k generated by S_k subject to q_k; potential assessments of NFT offspring under the influence of S subject to Q
B; b_k	Set of budgets; budget for n_k
L; l_k	Set of quantity limits; quantity limit for n_k
Abbreviation	**Description**
NPM	NFT Profit Maximization
QOOA	Quantity and Offspring-Oriented Airdrops
QAS	Quantity-driven Airdrop Selection
OPE	Offspring Profit Enhancement
PEP	Potential Expected Profit
QSP	Quantity-Sensitive Profit
VQI	Valuation-based Quantity Inequality
RTC	Rare Trait Collectors
TI	Target Index
VOGI	Valuable Offspring Generation Influence

- *Breeding Constraints:* Each NFT may also have a breeding limit that defines the maximum number of times it can engage in breeding. For example, in STEPN, the maximum breeding limit is 7, and the cooldown period grows longer as the breed count of NFTs approaches 7. In addition, inbreeding restrictions are in place to protect genetic diversity, e.g., MODragon also prohibits the breeding of sibling NFTs.
- *Genetic Mechanisms:* Mutations introduce an element of unpredictability by occasionally endowing offspring with entirely novel traits, thereby enhancing their rarities. For example, in Tiny World, offspring have a greater chance of mutation when the breed counts of their parents increase.[18]
- *NFT Maturation:* The lifecycle of NFT offspring may follow a distinct evolutionary path of multiple stages, including egg, baby, and adult. Until the phase of hatching, the traits of an NFT offspring are typically concealed. At this point, holders have the option to trade, and the inherent uncertainty often boosts the NFT's assessment. After hatching, certain NFT projects, such as Crypto Unicorns, reach the baby stage. As they attain the adult stage of development, they have the opportunity to mutate and acquire rare traits. Notably, these NFTs cannot breed offspring until they have reached the adult stage.

[18] More information on NFTs in Tiny World: https://docs.tinyworlds.io/tiny-nft/bnb-chain/tinymon-nfts.

D PROOF OF THEOREM 3.1

Theorem D.1. *NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$.*

Proof. We prove this theorem with a reduction from the Densest k-Subgraph problem (DkS), which is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming the exponential time hypothesis (ETH), where $c > 0$ is a constant independent of $|V|$ [59]. Given a graph G, and two integers k, h, the decision version of DkS is to decide whether there exists an induced subgraph of G with k vertices and h edges. Given an instance (G, k, h) of DkS, where $V(G) = \{v_1, v_2, \ldots, v_{|G|}\}$, we construct an instance of NPM as follows. i) Let $W = \{w_1, w_2, \ldots, w_{|G|}\}$, where w_i is a copy of v_i for any $i \in \{1, 2, \ldots, |G|\}$. The social network of NPM instance is a graph with vertex set $V(G') \cup W$ and edge set $E(G') \cup \{e_{w_1,v_1}, e_{w_2,v_2}, \ldots, e_{w_{|G|},v_{|G|}}\}$, where G' is an orientation of G, i.e., G' is obtained from G by assigning exactly one direction to each edge of G. The activation probability a_{v_i,v_j} is set to 0 for any edge $e_{v_i,v_j} \in E(G')$, and a_{w_i,v_i} is set to 1 for any $i \in \{1, 2, \ldots, |G|\}$. ii) The NFT project $N = \{n_1\}$, reserve price $p_1 = 1$, seed budget $b_1 = k$ and valuation $v_{u,n_1}(q_1) = 1$ for each user $u \in V(G') \cup W$ and any quantity q_1 of NFT n_1. Let $\beta_i = 0$ and $\gamma_i = 1$ for any user $u_i \in V(G') \cup W$, i.e., in the constructed instance, the NFT offspring can be created only when two activated users are friends, and each of them provides a parent NFT. Let $A(o, S, Q) = 1$ for any NFT offspring o under the difussion of any S and Q.

To complete the proof, we show that there exists an induced subgraph with k vertices and h edges in DkS if and only if there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S, Q) = k + h$ in NPM. We first prove the necessary condition. Suppose that there exists an induced subgraph with k vertices and h edges in DkS. Let the k corresponding users in W form the set of NFT airdrops S, and let $Q = \{q_1 = \infty\}$. By the construction, exactly k users in $V(G')$ are activated, and exactly h NFT offsprings are created. Thus (S, Q) is a solution such that $f(S, Q) = k + h$. We then prove the sufficient condition. Suppose that there is a set of NFT airdrops S of size k and a set of NFT quantity $Q = \{q_1 = \infty\}$ such that $f(S, Q) = k+h$ in NPM. By the construction, we conclude that the k NFT airdrops must be chosen from the set W, otherwise $f(S, Q) < k + h$. Thus the corresponding vertices in $V(G)$ form an induced subgraph with k vertices and h edges in DkS. Therefore, NPM is NP-hard and cannot be approximated within a factor of $|V|^{1/(\log\log|V|)^c}$ assuming ETH, where $c > 0$ is a constant independent of $|V|$. The theorem follows. □

E DETAILS ON QOOA

E.1 Pseudo-code

The pseudo-code of QOOA is presented in Algorithm 1.

Algorithm 1: QOOA

Input: NFT project N with traits T and reserve prices P, social network G, user breeding quota c_{BQ}, budgets B, quantity limits L
Output: NFT airdrops S and quantities Q

/* QAS phase */
for *each* $n_k \in N$ **do**
  $q^*_k \leftarrow null$; $S^*_k \leftarrow null$; $TP^*_k \leftarrow 0$
  for $q_k = 1, \ldots, l_k$ **do**
    if $UB_k(q_k) \leq TP^*_k$ **then**
      continue
    $S_k \leftarrow \emptyset$; $TP_k \leftarrow \sum_{u_j \in V(S_k, q_k)} v_{u_j, n_k}(q_k)$
    while $|S_k| < b_k$ **do**
      $U \leftarrow \emptyset$
      for *each* $(u_i, n_k) \notin S_k$ **do**
        if $QSP_k(S_k \cup \{(u_i, n_k)\}, q_k) \geq TP_k$ **then**
          $U \leftarrow U \cup \{u_i\}$
      $u^*_i \leftarrow null$; $gain \leftarrow 0$
      for $u_i \in U$ **do**
        if $TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k) > gain$ **then**
          $u^*_i \leftarrow u_i$; $gain \leftarrow TP(S_k \cup \{(u_i, n_k)\}, q_k) - TP_k(S_k, q_k)$
      if $gain > 0$ **then**
        $S_k \leftarrow S_k \cup \{(u^*_i, n_k)\}$; $TP_k \leftarrow TP_k + gain$
      else
        break
    if $TP_k > TP^*_k$ **then**
      $q^*_k \leftarrow q_k$; $S^*_k \leftarrow S_k$; $TP^*_k \leftarrow TP_k$
$S \leftarrow \bigcup_{n_k \in N} S^*_k$; $Q \leftarrow \{q^*_1, \ldots, q^*_{|N|}\}$
/* OPE phase */
N^{T} ←treasures of NFTs; RTC ← RTCs according to N^{T}
Sort N^{T} according to the number of rarest traits possessed
for *each* $n_k \in N^{\mathrm{T}}$ **do**
  $S'_k \leftarrow S^*_k$; $\bar{S}'_k \leftarrow \emptyset$
  while $S'_k \neq \emptyset$ **do**
    $(u^*_i, n_k) \leftarrow \mathrm{argmin}_{(u_i, n_k) \in S'_k} VOGI_k(u_i, q^*_k)$
    $(u^*_j, n_k) \leftarrow \mathrm{argmax}_{(u_j, n_k) \in V \setminus (S'_k \cup \bar{S}'_k)} VOGI_k(u_j, q^*_k)$
    if $VOGI_k(u^*_i, q^*_k) < VOGI_k(u^*_j, q^*_k)$ **then**
      $S'_k \leftarrow S'_k \setminus \{(u^*_i, n_k)\}$; $\bar{S}'_k \leftarrow \bar{S}'_k \cup \{(u^*_j, n_k)\}$
      $S' \leftarrow S \setminus \{(u^*_i, n_k)\} \cup \{(u^*_j, n_k)\}$
      if $f(S', Q) > f(S, Q)$ **then**
        $S \leftarrow S'$
    else
      break
return S, Q

E.2 Derivation of Potential Expected Profit

For each prospective purchaser, QAS searches for her reverse reachable sets in different deterministic realized graphs of G. Then, QAS derives the likelihood for any user $u_i \in V$ to influence a prospective purchaser $u \in V^{\mathrm{pp}}_k(q_k)$ according to the frequency of u_i occurring in u's reverse reachable sets, denoted as $fq(u_i, u)$. Consider S'_k as a set of airdrops of n_k. QAS evaluates each prospective purchaser u's *Potential Expected Profit (PEP)* of n_k under the influence of S'_k based on the likelihood for all users in S'_k to influence u, formulated as follows.

$$PEP_k(u, S'_k, q_k) = v_{u,n_k}(q_k) \cdot \min\{1, \sum_{(u_i, n_k) \in S'_k} fq(u_i, u)\}, \quad (12)$$

where the sum of frequencies is utilized to estimate the probability of u being successfully influenced, with a maximum value of 1. Accordingly, PEP of u on NFT n_k can serve as the upper bound of u's expected profit on NFT n_k.

E.3 Lemma of QSP

Lemma E.1. *For a set of airdrops S_k for n_k under a quantity q_k, $QSP_k(S_k, q_k) \geq TP_k(S_k, q_k)$.*

Proof. Recall $TP_k(S'_k, q_k) = \sum_{u_j \in V(S'_k, q_k)} v_{u_j, n_k}(q_k)$, where $V(S'_k, q_k)$ is the set of users influenced by S'_k to hold the NFT n_k under the quantity q_k. For each $u_j \in V(S'_k, q_k)$, her expected profit on NFT n_k is no greater than $PEP_k(u_j, S'_k, q_k)$. As QSP has taken PEP of the users in $V^{\mathrm{PEP}}_k(S'_k, q_k)$ into account, for any user $u_j \in V(S'_k, q_k)$, if $u_j \in V^{PEP}_k(S'_k, q_k)$, her PEP is already counted in QSP; otherwise, her PEP must be no greater than $PEP_k(u_{\min}, S'_k, q_k)$, where $u_{\min}$ is the user with the least PEP in $V^{\mathrm{PEP}}_k(S'_k, q_k)$. Therefore, $QSP_k(S'_k, q_k)$ is the upper bound of $TP_k(S'_k, q_k)$. The lemma follows. □

E.4 Derivation of Equation (7)

For $W_k(y)$, the corresponding user valuation is eligible for a transaction price only if it is no smaller than the reserve price p_k, i.e.,

$$\begin{aligned} W_k(y) \cdot A(n_k, q_k) &= W_k(y) \cdot e^{\eta_0 + \eta_2 \Phi(n_k) + \eta_3 \cdot 0} \cdot e^{\frac{\eta_1}{q_k}} \\ &= W_k(y) \cdot \eta^* \cdot e^{\frac{\eta_1}{q_k}} \qquad (\text{Let } \eta^* = e^{\eta_0 + \eta_2 \Phi(n_k)}) \\ &\geq p_k. \end{aligned}$$

E.5 Complete Examples

Example E.1. Consider a new NFT project $N = \{n_1, n_2, n_3\}$, as depicted in Figure 1(a), to be promoted in a social network in Figure 1(b). Assume that $B = \{2, 2, 2\}$, $L = \{2, 2, 2\}$, $P = \{1.8, 1.8, 1.8\}$, $\eta_0 = 0$, $\eta_1 = 1.5$, $\eta_2 = 0.2$, and $\eta_3 = 0.5$. QAS starts from n_1 subject to $q_1 = 1$. At first, QAS examines all users and chooses u_1 for an airdrop to maximize the total transaction price, i.e., $S^1_1 = \{(u_1, n_1)\}$ and $TP_1(S^1_1, 1) = 5.51$. Next, QAS derives QSP as follows.

$$QSP_1(\{u_1, u_2\}, 1) = 4.32, \qquad QSP_1(\{u_1, u_3\}, 1) = 4.32,$$
$$QSP_1(\{u_1, u_4\}, 1) = 7.73, \qquad QSP_1(\{u_1, u_5\}, 1) = 7.44,$$
$$QSP_1(\{u_1, u_6\}, 1) = 8.64.$$

Since both $QSP_1(\{u_1, u_2\}, 1)$ and $QSP_1(\{u_1, u_3\}, 1)$ are smaller than $TP_1(S^1_1, 1) = 5.51$, QOOA chooses the second user for an airdrop only from u_4, u_5, and u_6. As $TP_1(\{u_1, u_6\}, 1) = 7.1$ is the greatest, QAS updates $S^1_1 = \{u_1, u_6\}$ and finishes the search for $q_1 = 1$ because $|S^1_1| = b_1 = 2$.

Before the search for $q_1 = 2$ begins, QAS identifies $W_1(1) = 0.8$ and $W_1(2) = 0.7$. Since $W_1(1)\eta^* = 0.8 \times e^{0.2 \times 4.5} = 1.97 > p_1 = 1.8$, $W_1(1)A(n_1, 2)$ is eligible for a transaction price. As $W_1(2)\eta^* = 0.7 \times e^{0.2 \times 4.5} = 1.72 < p_1 = 1.8$, QAS infers the maximum quantity $\frac{1.5}{\ln(\frac{1.8}{1.72})} = 33.74 > q_1 = 2$. Hence, $W_1(2)A(n_1, 2)$ is also eligible for a transaction price. As a result, QAS derives $UB_1(2) = W_1(1)A(n_1, 2) + W_1(2)A(n_1, 2) = 7.81$. Since $UB_1(2) = 7.81 > TP_1(\{u_1, u_6\}, 1) = 7.1$, QAS continues the search under $q_1 = 2$. ■

Example E.2. Following Example 4.1, QOOA obtains $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_6, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 25.93$. Assume that $\lambda = 1$ and $c_{\text{BQ}} = 3$. Since 'rainbow' and 'moon' are the rarest traits, OPE identifies the treasures $N^{\text{T}} = \{n_1, n_3\}$. For n_3, OPE examines VOGI as follows.

Airdrops: $VOGI_3(u_4, 1) = 0.51$, $VOGI_3(u_6, 1) = 0.26$

Non-airdrops: $VOGI_3(u_3, 1) = 0.37$, $VOGI_3(u_5, 1) = 0.36$, $VOGI_3(u_1, 1) = 0$, $VOGI_3(u_2, 1) = 0$.

Accordingly, OPE attempts to replace u_3 with u_6 for airdrops of n_3 because $VOGI_3(u_3, 1) = 0.37 > VOGI_3(u_6, 1) = 0.26$. Since $f(S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}, Q) = 26.97 > f(S, Q) = 25.93$, OPE updates $S = S \setminus \{(u_6, n_3)\} \cup \{(u_3, n_3)\}$. As $VOGI_3(u_5, 1) = 0.36 < VOGI_3(u_4, 1) = 0.51$, OPE terminates the enhancement for n_3. Consequently, the solution is $S = \{(u_1, n_1), (u_6, n_1), (u_4, n_2), (u_3, n_2), (u_4, n_3), (u_3, n_3)\}$ and $Q = \{1, 2, 1\}$, with $f(S, Q) = 26.97$. ■

E.6 Derivation of Target Index

$$TI_k(u_j, S, Q) = \begin{cases} \beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \leq \Theta(u_j, S, Q) < c_{\text{BQ}} \\ \beta_c \gamma_j v_{u_j,n_k}(q_k), & \text{else if an RTC } u_c \text{ is } u_j\text{'s friend} \\ 0, & \text{otherwise} \end{cases},$$

where $\Theta(u_j, S, Q)$ is the number of treasures held by u_j given S and Q, and I_j is the impact of u_j. For the first case where u_j is an RTC holding fewer treasures than c_{BQ}, TI is evaluated by u_j's breeding probability, impact, and valuation of n_k (attempting to increase the expected assessments of offspring without significantly compromising the total transaction price). For the second case where u_j is a friend of some RTC u_c, TI is evaluated by u_c's breeding probability (representing the likelihood of u_c requesting u_j to provide siring-ready NFTs) and u_j's siring probability and valuation of n_k.

E.7 Support for Various Breeding

Here we show that QOOA is capable of supporting various considerations in breeding, including *Booster*, *Fusion Breeding*, and *No Collaborative Breeding*, as introduced in Section 3.

1) For *Booster*, OPE modifies the Target Index to account for the number of boosters possessed by an RTC as follows.

$$TI_k(u_j, S, Q) = \begin{cases} (1 + \frac{\theta_j}{\theta_{\max}})\beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \leq \Theta(u_j, S, Q) < c_{\text{BQ}} \\ \beta_c \gamma_j v_{u_j,n_k}(q_k), & \text{else if an RTC } u_c \text{ is } u_j\text{'s friend} \\ 0, & \text{otherwise} \end{cases},$$

where θ_j is the number of boosters possessed by u_j, $\theta_{\max}$ is the maximum number of boosters possessed by users in the social network, and $\frac{\theta_j}{\theta_{\max}}$ indicates the increase in TI attributable to the possession of boosters.

2) For *Fusion Breeding*, since NFTs with semi-rare traits also have chances to breed offspring with rare traits, OPE first identifies the set of semi-treasures, denoted as N^{ST}, to include NFTs without the rarest traits but possessing at least one semi-rare trait. Note that $N^{\text{T}} \cap N^{\text{ST}} = \emptyset$. After adjusting airdrops for treasures, OPE also finds alternative airdrops for semi-treasures. For an NFT $n_k \in N^{\text{ST}}$, OPE modifies the Target Index by disregarding whether users are RTCs and excluding the friends of RTCs (since collaborative breeding does not lead to fusion).

$$TI_k(u_j, S, Q) = \begin{cases} \beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \leq |N(u_j, S, Q)| < c_{\text{BQ}} \\ 0, & \text{otherwise} \end{cases},$$

where $|N(u_j, S, Q)|$ is the number of NFTs held by u_j.

3) For *No Collaborative Breeding*, OPE no longer takes into account the influence on friends of RTCs providing siring-ready NFTs. It modifies the Target Index by discarding the second case.

$$TI_k(u_j, S, Q) = \begin{cases} \beta_j I_j v_{u_j,n_k}(q_k), & \text{if } 1 \leq \Theta(u_j, S, Q) < c_{\text{BQ}} \\ 0, & \text{otherwise} \end{cases}.$$

Equipped with the modified TI, OPE evaluates VOGI of users and tailors airdrops accordingly.

F PROOF OF THEOREM 4.1

Note that in the influence propagation of NFTs during the Airdrop stage, for each NFT n_k, its total transaction price function $TP_k(S_k, q_k)$ is independent from other NFTs. In the following, we first examine whether the total transaction price function $TP_k(S_k, q_k)$ is submodular.

Definition F.1 (Submodular function [51]). Given a ground set U, a set function $\rho : 2^U \mapsto \mathbb{R}$ is submodular if for any subsets X, Y with $X \subseteq Y$ and any element $u \in U \setminus Y$,

$$\rho(Y \cup \{u\}) - \rho(Y) \leq \rho(X \cup \{u\}) - \rho(X). \tag{13}$$

Unfortunately, $TP_k(S_k, q_k)$ is non-monotonically increasing and thus not submodular.

Lemma F.1. *For each $k \in \{1, 2, \ldots, |N|\}$, the total transaction price function $TP_k(S_k, q_k)$ is neither monotonically increasing nor submodular.*

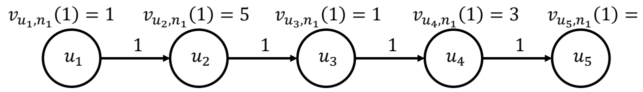


Figure 6: An example for Lemma F.1.

PROOF. We prove this lemma by giving an instance as follows. Let the NFT project $N = \{n_1\}$, airdrop budget $b_1 = 3$, NFT quantity $q_1 = 1$, reserve price $p_1 = 1$ and the social network G be a path graph of five vertices, i.e., there are five users $u_1, u_2, \ldots, u_5$, and four edges $e_{i,i+1}$, $1 \leq i \leq 4$, where each edge $e_{i,i+1}$ has the activation probability of $a_{i,i+1} = 1$, as shown in Figure 6. The valuations of all users u_i for NFT n_1 are $v_{u_1,n_1}(q_1) = 1$, $v_{u_2,n_1}(q_1) = 5$, $v_{u_3,n_1}(q_1) = 1$, $v_{u_4,n_1}(q_1) = 3$, and $v_{u_5,n_1}(q_1) = 1$, respectively. In this instance, when (u_3, n_1) is chosen as the only airdrop, users u_4 and u_5 are activated, and thus u_4 acquires the NFT n_1, i.e., $TP_k(\{(u_3, n_1)\}, q_1) = 3$. However, if we choose (u_3, n_1) and (u_4, n_1) as the airdrops, only user u_5 is activated, and thus $TP_k(\{(u_3, n_1), (u_4, n_1)\}, q_1) = 1$, which implies that the revenue function $TP_k(S_1, q_1)$ is non-monotonically increasing.

To prove the non-submodularity, we consider two sets of airdrops $\{(u_1, n_1), (u_3, n_1)\}$ and $\{(u_1, n_1), (u_3, n_1), (u_4, n_1)\}$. The former set of airdrops results in $f(\{(u_1,n_1),(u_3,n_1)\},q_1) = 5$ because u_2, u_4 and u_5 are activated, and u_2 acquires NFT n_1, and the latter results in $TP_k(\{(u_1,n_1),(u_3,n_1),(u_4,n_1)\},q_1) = 5$ because u_2 and u_5 are activated, and u_2 acquires NFT n_1. Therefore,

$$TP_k(\{(u_1,n_1),(u_3,n_1),(u_4,n_1)\},q_1) - TP_k(\{(u_3,n_1),(u_4,n_1)\},q_1) = 4$$

and

$$TP_k(\{(u_1,n_1),(u_3,n_1)\},q_1) - TP_k(\{(u_3,n_1)\},q_1) = 2,$$

which implies that the total transaction price function $TP_k(S_1,q_1)$ is non-submodular. The lemma follows. □

For each $k \in \{1,2,\dots,|N|\}$, let $Act(S_k)$ denote the set of users being active in NFT n_k, and let

$$r(S_k,q_k) = \sum_{u_i \in Act(S_k), v_{u_i,n_k}(q_k) \geq p_k} v_{u_i,n_k}(q_k)$$

represent the total valuation function of users being active in NFT n_k following the transaction price constraint (i.e., $v_{u_i,n_k}(q_k) \geq p_k$).

The following lemma shows that, different from the total transaction price function $TP_k(S_k,q_k)$, the total valuation function $r(S_k,q_k)$ is submodular.

Lemma F.2. *For each $k \in \{1,2,\dots,|N|\}$, the total valuation function $r(S_k,q_k)$ is non-monotonically increasing but submodular in the airdrop selection.*

Proof. Since the profit of all users being airdropped is zero, the total valuation function $r(S_k,q_k)$ is non-monotonically increasing for each $k \in \{1,2,\dots,|N|\}$. To prove the submodularity, we show that each function $r(S_k,q_k)$ satisfies Inequality (13). Following the proof in [46] where the submodularity of the influence function on the IC model can be reduced to that in a deterministic graph realized by flipping the coin for each edge of $G=(V,E)$, we show the submodularity of $r(S_k,q_k)$ in a deterministic graph $G'=(V,E') \subseteq G$ realized by flipping the coin for each edge of G with the activation probability. The process of influence propagation on G can be regarded as the influence propagation process upon the deterministic graph G'. Thus, for any activated user, its influence propagation is a connected subgraph of G' rooted by some airdropped user u, i.e., $(u,n_k) \in S_k$. For any set of airdrops S_k, the function $r(S_k,q_k)$ is the total valuation (excluding the valuations violating the transaction price constraint $v_{u_i,n_k}(q_k) \geq p_k$) of the union of the connected subgraphs rooted by all airdrops in S_k. Therefore, for each $k \in \{1,2,\dots,|N|\}$, the total valuation function $r(S_k,q_k)$ is a coverage function, which is submodular [50]. The lemma follows. □

Recall that $V(S_k,q_k)$ is the set of users holding the NFT n_k under the influence of S_k with the quantity q_k, and $Act(S_k)$ is the set of users being active in NFT n_k. Thus for each $k \in \{1,2,\dots,|N|\}$, we have $V(S_k,q_k) \subseteq Act(S_k)$ and $|V(S_k,q_k)| \leq q_k$, which implies

$$TP_k(S_k,q_k) \leq r(S_k,q_k). \tag{14}$$

Moreover, by Equations (1) and (2), for any q_k and q'_k with $q_k \leq q'_k$, we have $v_{u_i,n_k}(q_k) \geq v_{u_i,n_k}(q'_k)$ and thus

$$r(S_k,q_k) \geq r(S_k,q'_k). \tag{15}$$

Let (S^{opt},Q^{opt}) denote the optimal solution of NPM, where $S^{opt} = \bigcup_{k\in\{1,2,\dots,|N|\}} S_k^{opt}$ and $Q^{opt} = \{q_1^{opt}, q_2^{opt}, \dots, q_{|N|}^{opt}\}$. To derive the approximation ratio, we first consider a problem similar to NPM, named NPM-QO, where the quantity constraint is removed. In NPM-QO, all active users with valuations satisfying the reserve price constraint $v_{u_i,n_k}(q_k) \geq p_k$, without the restriction that the q_k highest valuations acquire the NFT n_k. The user u_i's valuation on NFT n_k is set to $v_{u_i,n_k}(1)$, and λ is set to 0. Similarly, let $\hat{S}^{opt}$ denote the optimal solution of NPM-QO, where $\hat{S}^{opt} = \bigcup_{k\in\{1,2,\dots,|N|\}} \hat{S}_k^{opt}$ satisfies $\hat{S}_k^{opt} = \arg\max_{S_k} r(S_k,1)$ for each $k \in \{1,2,\dots,|N|\}$.

Lemma F.3. *For each $k \in \{1,2,\dots,|N|\}$, $r(\hat{S}_k,1) \geq \frac{1}{e} \cdot TP_k(S_k^{opt},q_k^{opt})$ holds, where $\hat{S}_k$ is the solution found by the continuous greedy algorithn [34] to solve NPM-QO for NFT n_k.*

Proof. By Lemma F.2, the total valuation function $r(S_k,1)$ is submodular for each $k \in \{1,2,\dots,|N|\}$. According to [34], the set of airdrops $\hat{S}_k$ found by the continuous greedy algorithm is a $\frac{1}{e}$-approximation solution of NPM-QO for NFT n_k, i.e., $r(\hat{S}_k,1) \geq \frac{1}{e} \cdot r(\hat{S}_k^{opt},1)$. To complete the proof, for each $k \in \{1,2,\dots,|N|\}$, we show $r(\hat{S}_k^{opt},1) \geq TP_k(S_k^{opt},q_k^{opt})$ as follows,

$$r(\hat{S}_k^{opt},1) \geq r(S_k^{opt},1) \geq r(S_k^{opt},q_k^{opt}) \geq TP_k(S_k^{opt},q_k^{opt}),$$

where the first, second, and last inequalities are obtained due to the optimality of $\hat{S}_k^{opt}$ to NPM-QO for NFT n_k, Inequality (15), and Inequality (14), respectively. The lemma follows. □

For each $k \in \{1,2,\dots,|N|\}$, let (S_k^{alg}, q_k^{alg}) denote the algorithm solution obtained by QOOA for NFT n_k. Then (S^{alg},Q^{alg}) is the algorithm solution obtained by QOOA, where $S^{alg} = \bigcup_{k\in\{1,2,\dots,|N|\}} S_k^{alg}$ and $Q^{alg} = \{q_1^{alg}, q_2^{alg}, \dots, q_{|N|}^{alg}\}$.

Now, we show the proof of Theorem 4.1.

Proof. To prove this theorem, we show $f(S^{alg},Q^{alg}) \geq \frac{1}{ea(1+c)} \cdot f(S^{opt},Q^{opt})$. For each $k \in \{1,2,\dots,|N|\}$, we have

$$\begin{aligned} TP_k(S_k^{alg},q_k^{alg}) &\geq \frac{p_k}{r(\hat{S}_k,1)} \cdot r(\hat{S}_k,1) \\ &\geq \frac{1}{e} \cdot \frac{p_k}{r(\hat{S}_k,1)} \cdot TP_k(S_k^{opt},q_k^{opt}) \\ &\geq \frac{1}{ea} \cdot TP_k(S_k^{opt},q_k^{opt}), \end{aligned} \tag{16}$$

where the first inequality follows by the fact that NFT n_k is sold with price at least p_k, and the second and third inequalities are obtained by Lemma F.3 and $a = \max_{1\leq k\leq|N|} \frac{r(\hat{S}_k,1)}{p_k}$, respectively.

We derive the relation between the total transaction price of N and the assessments of NFT offspring generated from N as follows. Let $A_{\max}$ denote the maximum assessment of an offspring, i.e.,

$$A_{\max} = e^{\eta_0+\eta_1\frac{1}{1}+\eta_2|T|\frac{2^{|T|}}{2}+\eta_3 d_{\max}} = e^{\eta_0+\eta_1+\eta_2|T|\cdot 2^{|T|-1}+\eta_3 d_{\max}},$$

where we assume that the quantity of this offspring is 1 (leading to e^{η_1}), it has $|T|$ traits, each trait is only owned by it and one of its parents, at most $2^{|T|}$ offspring are generated (leading to $e^{\eta_2|T|\cdot 2^{|T|-1}}$), and its owner has the maximum degree, i.e., $d_{\max}$, on the social network (leading to $e^{\eta_3 d_{\max}}$). For any (S, Q) in NPM, we have

$$
\begin{aligned}
&\lambda \cdot OS(S,Q)\\
&\leq \lambda \cdot \left(\sum_{u_i \in \bigcup_{k=1}^{|N|} V(S_k,q_k)} c_{\mathrm{BQ}} \cdot A_{\max} \right)\\
&\leq \lambda \cdot \frac{q_{\max}\,|N|}{|N|} \cdot \frac{c_{\mathrm{BQ}} \cdot A_{\max}}{p_{\min}} \cdot p_{\min} \cdot |N|\\
&\leq c \sum_{k=1}^{|N|} p_k d_k \qquad (\text{Let } c = \frac{\lambda \cdot q_{\max} \cdot c_{\mathrm{BQ}} \cdot A_{\max}}{p_{\min}})\\
&\leq c \sum_{k=1}^{|N|} TP_k(S_k, q_k),
\end{aligned}
$$

where $p_{\min}$ is the minimum reserve price, and d_k is the actual number of the NFT n_k sold. Note that $\bigcup_{k=1}^{|N|} V(S_k, q_k) \leq q_{\max}\,|N|$ holds because at most $q_{\max}\,|N|$ NFTs are sold. $p_{\min}\,|N| \leq \sum_{k=1}^{|N|} p_k d_k$ holds since $p_{\min}\,|N|$ is the minimum transaction prices assuming each NFT is sold at $p_{\min}$ with only one quantity.[19] Then, for any (S, Q) in NPM,

$$
f(S,Q) = \sum_{k=1}^{|N|} TP_k(S_k,q_k) + \lambda \cdot OS(S,Q) \leq (1+c) \cdot \sum_{k=1}^{|N|} TP_k(S_k,q_k). \tag{17}
$$

Therefore, we have

$$
\begin{aligned}
f(S^{alg}, Q^{alg}) &= \sum_{k=1}^{|N|} TP_k(S_k^{alg}, q_k^{alg}) + \lambda \cdot OS(S^{alg}, Q^{alg})\\
&\geq \sum_{k=1}^{|N|} TP_k(S_k^{alg}, q_k^{alg})\\
&\geq \frac{1}{ea} \cdot \sum_{k=1}^{|N|} TP_k(S_k^{opt}, q_k^{opt})\\
&\geq \frac{1}{ea(1+c)} \cdot f(S^{opt}, Q^{opt}),
\end{aligned}
$$

where the first inequality is obtained because $OS(S^{alg}, Q^{alg})$ is non-negative by definition, and the second and third inequalities are obtained by Inequality (16) and Inequality (17), respectively.

Next, we analyze the time complexity. In the QAS phase, for an NFT n_k with budget b_k, it takes $O(|V|\,b_k)$ time to find the airdrops under a specific quantity. As the maximum quantity limit is $l_{\max} = \max_{l_k \in L} l_k$, it requires $O(l_{\max} b_k\,|V|)$ time to identify the most appropriate quantity and the corresponding airdrops for each NFT n_k. Therefore, QAS takes $O(l_{\max} b_{\max}\,|V|\,|N|)$ time to find an initial solution of NPM, where $b_{\max}$ is the maximum budget in B. Next, in the OPE phase, it examines $O(b_k)$ replacements of airdrops for a treasure n_k. As there are $O(|N|)$ treasures, OPE takes $O(b_{\max}\,|N|)$ time. Consequently, the time complexity of QOOA is $O(l_{\max} b_{\max}\,|V|\,|N|)$. The theorem follows. □

[19] To ensure that an NFT is marketable in the real world, its reserve price is typically determined according to the valuations of its intended audience.

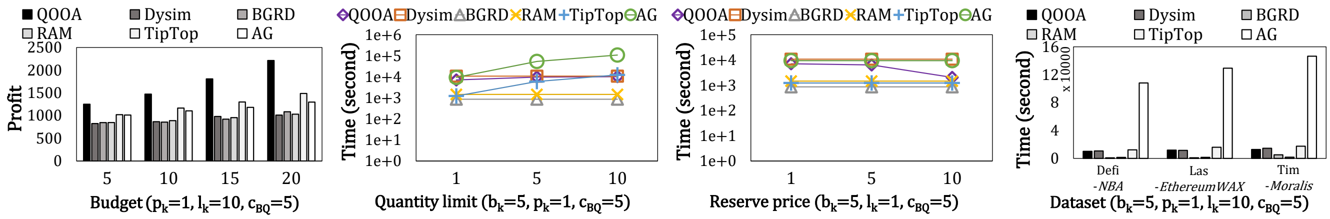
G DETAILS ON EXPERIMENTS

G.1 Experiment Setup

Datasets. We conduct experiments on three real NFT projects and three real social networks derived from NFT transactions. The NFT projects include i) Defimons Characters:[20] It has 14 NFTs, which are the identities in the pixel world of Defimons. There are 5 traits (e.g., genders and locations), with the rarity of NFTs ranging from 3.75 to 9. ii) Lascaux:[21] It has 17 NFTs, recording the art and performance of the artist Lascaux. There are 10 traits (e.g., utility, series, etc.), with the rarity of NFTs ranging from 4.05 to 39.89. iii) Timpers Pixelworks:[22] It has 7 NFTs, which are artworks by Timpers and top guest artists. There are 3 traits (e.g., artists), with the rarity of NFTs ranging from 1.4 to 7. The social networks based on NFT transactions include a) NBA (all NBA Top Shot transactions until February 2021):[23] It contains 245K users and 2.6M relationships derived from 2.63M transactions. b) EthereumWAX [62] (collected primarily from Ethereum and WAX between June 2017 and April 2021): It contains 269K users and 2.8M relationships derived from 7M transactions. c) Moralis [27] (collected via the Web3 APIs of the Moralis platform from June 2017 until February 2022):[24] It contains 5.1M users and 30.6M relationships derived from 77M transactions. We follow [52, 55, 69] to set the user preferences and the activation probabilities. Following [62], the breeding and siring probabilities, β_i and γ_i, are set as the activity strength of each user u_i. Following most of the famous NFT projects, e.g., Trump Digital Trading Card[25] and MODragon, the quantity limit $l_k = 10$ for each n_k and the user breeding quota $c_{\mathrm{BQ}} = 5$ for all users. Following OpenSea, the reserve price p_k of each n_k is set to 1.[26] Following [62], λ is set as 0.2 based on the proportion of NFTs successfully sold by current holders.

Baselines and metrics. We compare QOOA with five state-of-the-art approaches: Dysim [70], BGRD [28], RMA [40], TipTop [53], and AG [35]. For Dysim, RMA, and TipTop, which consider the seeding cost, we set it to 1 for all users. For RMA, the cost-of-engagement of each advertiser is set to 1. Note that TipTop and AG aim to maximize the benefits of users and edges, respectively, The benefits are the gains of influencing either individual users or both end users of edges. We thus set the benefits in relation to user valuations. Moreover, to facilitate breeding, we apply NFT trait similarity-based recommendation [65] to ensure that users holding similar NFTs receive preferential influence. For TipTop, the benefit is set to the product of her valuation of n_k and the similarity between n_k and the NFTs she possesses, while that for AG is set to the product of the average valuation of n_k from both end users and the average similarity of the NFTs they hold. Since all baselines do not decide the quantities of NFTs for sale, we examine various quantities to find the one with the maximum profit. The performance metrics include i) profit $f(S,Q)$, ii) the total transaction price $TP(S,Q)$, iii) the potential assessments of offspring $OS(S,Q)$, and iv) the execution time. We perform a series of sensitivity tests in terms of 1) the budgets B,

[20] https://opensea.io/collection/defimons-characters.
[21] https://opensea.io/collection/lascauxfuture.
[22] https://opensea.io/collection/timperspixelworks.
[23] https://www.kaggle.com/datasets/chigorin/nba-topshot-transactions/data
[24] https://moralis.io.
[25] https://collecttrumpcards.com/.
[26] https://support.opensea.io/hc/en-us/articles/1500003246082-How-do-timed-auctions-work-.



(a) f vs B (Las-*EthereumWAX*). (b) Time vs L (Defi-*NBA*). (c) Time vs P (Defi-*NBA*). (d) Time vs dataset.

Figure 7: Additional experiments.

2) the quantity limits L, 3) the reserve prices P, 4) the user breeding quota c_{BQ}, and 5) different breeding mechanisms as case studies. To gain more insights, we conduct an ablation study on QOOA. We conduct all experiments on an HP DL580 server with an Intel 2.10GHz CPU and 1TB RAM. Each simulation result is averaged over 100 samples.

G.2 Additional Experiments

Different from Figure 2(h), in Figure 7(b), when the quantity limit increases, the growth of QOOAś execution time slows down, because QOOA exploits VQI to avoid redundant searches for certain quantities. TipTop and AG, which identify airdrops based on quantity-sensitive valuation, requires more execution time as well. In contrast, other baselines incur execution times that are insensitive to the quantity limits. Besides, in Figure 7(c), as the reserve price increases, the efficacy of VQI becomes more pronounced, leading to more effective pruning in QOOA. Hence, the execution time of QOOA decreases with higher reserve prices. In contrast, all baselines do not consider reserve prices, and thus, their execution times remain unchanged. Finally, Figure 7(d) compares the execution time on different datasets. It is observed that the execution times of QOOA, Dysim, RMA, TipTop, and AG are proportional to the sizes of the NFT project and the social network, while that of BGRD is only correlated to the size of the social network. Note that despite the large scale of *Moralis*, QOOA remains efficient since it leverages QSP to prune inferior airdrops that are unlikely to yield higher profits.