Goals

This paper introduces a new technique(min/max approximation) for searching in game trees, based on the idea of approximating the min and max operators with generalized mean-value operators.

Techniques

generalized mean values

$$M_p(\boldsymbol{a}) = \left(\frac{1}{n} \sum_{i=1}^{n} a_i^p \right)^{1/p}$$

Base on the following two facts,we use $M_p(a)$ as approximation to min/max function in minimax algorithm.

$$\lim_{p \to \infty} M_p(\boldsymbol{a}) = \max(a_1, \ldots, a_n) ,$$

$$\lim_{p \to -\infty} M_p(\boldsymbol{a}) = \min(a_1, \ldots, a_n)$$

$M_p(a)$,unlike max or min ,has continuous derivatives with respect to each variable.So they are more suitable for analysis.

game trees searching

- game tree
  - consider a two-person zero-sum perfect information game
- searching a game tree
  - heuristic approximations
    - For most interesting games the game tree is so large that heuristic approximations are needed.
    - Our proposed technique requires a single static evaluator approximations are needed.
  - interactive deepening
    - Given a limit on timeout,one can successively compute the backed-up for depths,until one runs out of time,use the move determined by the last search completed.

- iteractive search heuristic details
  - The general process of partially exploring a game tree by an iterative heuristic:

Step 1. Initialize E to $\{s\}$, and $\hat{v}_E(s)$ to $\hat{v}(s)$.
Step 2. While $E \neq C$, and while time permits, do:
(a) Pick an expandable tip c of E.
(b) Expand E at c.
(c) Update $\hat{v}_E(c)$ at c and the ancestors of c up to the root s, using equation (9).

$$\hat{v}_E(c) = \begin{cases} \hat{v}(c), & \text{if } c \in T(E), \\ \max\limits_{d \in S(c)} (\hat{v}_E(d)), & \text{if } c \in Max \backslash T(E), \\ \min\limits_{d \in S(c)} (\hat{v}_E(d)), & \text{if } c \in Min \backslash T(E). \end{cases} \tag{9}$$

- penalty-based iterative search methods
  - the min/max approximation technique here is such a penalty-based scheme: to expand that tip node which has the least penalty.We add the node's children to the tree,update the estimates,and update the penalties on the edges.
- searching by min/max approximation
  - The "min/max approximation" heuristic is special case of the penalty-based search method,where the penalties are defined in terms of the derivatives of the approximating functions $M_p(a)$.

Results : min/max approximation is definitely superior.

- The number of distinct position considered by alpha-beta was approximately three times larger than the number of distinct positions considered by min/max when a timeout was in effect.
- Minimax search with alpha-beta pruning called the move operator approximately 3500 times/s,while the min/max heuristic called the move operator approximately 800 times/s.

conclusion

This paper provides a novel approach to game tree searching,based on approximating min/max functions by suitable generalized mean-value functions which outplays alpha-beta with iterative deepening.