CS 591: Graph Theory and Computational Topology **Summer 2017**

Problem Set 1

Lecturer: Prof. Peter Chin *Due: May 31, 2017*

◇ Please email the written portion (either type up your answer or scan your handwritten solution) & code and report to `zhouxiao@bu.edu` by 23:59PM on the due date.

◇ Late policy: there will be a penalty of 10% per day, up to three days late. After that no credit will be given.

1. **Mathematical calculations**

   (a) Let $[n] = \{1, 2, ..., n\}$. Define a graph G as follows:

   i. $V(G) = \{\text{all 2-subsets of}[5]\}$

   ii. $E(G) = \{ST | S, T \in V(G); S \cap T = \emptyset\}$

   Show that the graph G has the following properties.

   i. It is a 3-regular graph.

   ii. G contains no cycles C_3 or C_4.

   iii. G does not contain C_{10}.

   (b) Find the eigenvalues and eigenvectors for

   $$A = \begin{bmatrix} 3 & 4 & -1 \\ -1 & -2 & 1 \\ 3 & 9 & 0 \end{bmatrix}$$

   Is A positive definite?

2. **Programming assignment**

   Write a program using a language of your choice that determines whether a graph is bipartite or not. You can use the theorem by Konig (1936) that we covered in class. Input to your program should be a text file that describes a simple graph. each line should contain

   $$v_i, v_j, 1$$

   if there is an edge between v_i and v_j. otherwise it should be

   $$v_i, v_j, 0$$

   Note that we are dealing with undirected simple graph for now, so you can just have $i < j$. Generate an example of bipartite graphs and non-partite graphs so that you can test out your program. For submission, zip the following 4 items: your program, Readme File, 10

examples of bipartite graphs, 10 examples of non-bipartite graphs. Your program should handle graphs of size up 1,000 nodes so your examples should have various sizes from small to 1000.