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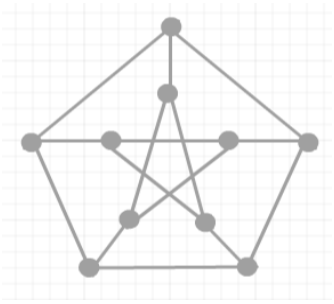
Problem Set 1

1. Mathematical calculations

(a) i. Here is the 3-regular graph:



ii. If graph G contains C_3, that means there will be a cycle which is triangle in G. Obviously, there is no triangle in this graph. And also, any vertex in this graph has three neighbor vertices because of 3-degree, however, none of three neighbors are connected, so G contains no cycle C_3.
This is a ten-vertex 4-regular graph, and it connects 2 subsets by 5 chord. If any chord connects two vertices at distance 2 or 3, then this graph would have C_3 or C_4. However, this situation does not satisfy the conditions in the definition of G.
If there is C_4 in this graph, there will exist two nodes sharing the same two neighbor vertices other than themselves. Graph G has no such two vertices, so G contains no cycle C_4.

iii. If there is a C_{10}, then the graph consists of C plus five chords. If each chord joins vertices opposite on C, then there is a 4-cycle. Hence some chord joins vertices at distance 4 along C. Now no chord incident to a vertex opposite an endpoint of chord on C can be added without creating a cycle with at most four vertices. Therefore, this graph does not contain C_{10}.

(b) $A - \lambda I = \begin{bmatrix} 3-\lambda & 4 & -1 \\ -1 & -2-\lambda & 1 \\ 3 & 9 & -\lambda \end{bmatrix}$

And $\det(A - \lambda I) = 0$, so we get:

$$(3-\lambda)[(-\lambda)(-2-\lambda)] - 4(\lambda - 3) + (-1)[-9 - 3(-2-\lambda)] = 0$$

$$-\lambda^3 + \lambda^2 + 8\lambda - 12 = 0$$

$$(\lambda - 2)(-\lambda + 2)(\lambda + 3) = 0$$

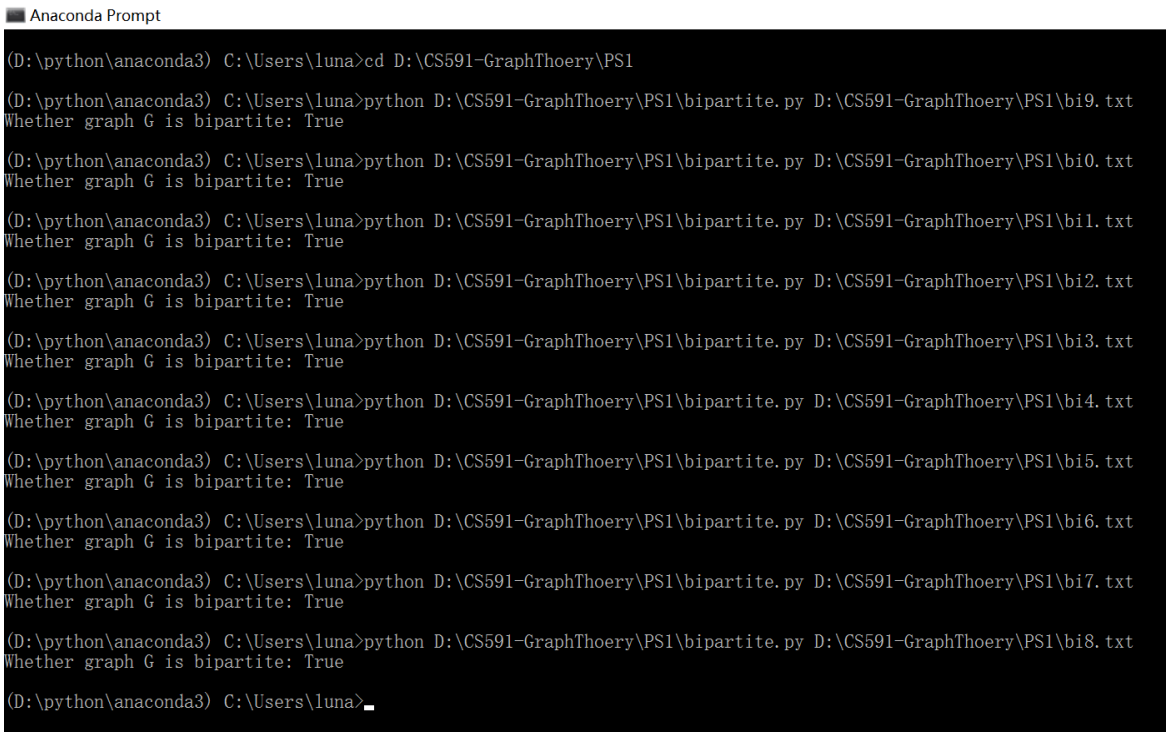
So the eigenvalues for matrix A is 2, -3, and the eigenvectors for $\lambda = 2$ is $\begin{pmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 1 \end{pmatrix}$, the eigenvectors for $\lambda = -3$ is $\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$.

Because the eigenvalues for matrix A is not all positive, A is not positive definite.

2. Programming

I have 10 bipartite examples and 10 non-bipartite examples, and my examples for both bipartite and non-bipartite have various size from small to 1000. Files with “bi*.txt” are bipartite examples, and files with “non*.txt” are non-bipartite examples. “bi9.txt” and “non9.txt” are both have size 1000 nodes.

Here is the result for bipartite graphs:



Here is the result for non-bipartite graphs: