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Problem Set 2

1. Mathematical calculations

(a) By contradiction, there is a tournament has no king. Because king is defined as a vertex from which every vertex is reachable by a path of length at most 2, I choose a vertex u with the largest outdegree. Because u has more possible paths to other vertices in the digraph $G = (V, E)$, I suppose u is not king.

Let $Y = \{v| \ u \to v \in E\}$ be the set of vertices that vertex u oriented, so the outdegree of u is Y. Since u is not a king, there is a vertex $x \notin Y$ (that is, x is not oriented by vertex u) and that is not oriented by any vertex in Y. Since for any pair of vertices, one orients to the other, this means that x orients to u as well as every vertex in Y. This means that

$$outdegree(x) = |Y| + 1 > outdegree(u)$$

But u was assumed to be the node with the largest outdegree in the tournament, so we have a contradiction. Hence, u must be a king.

(b) In the class, we proved that 1 + 2 can get 3, so I first prove 1 + 3 can get 2, then I will prove 2 + 3 can get 1.

(1) G is connected and has n – 1 edges, so G is acyclic.

I prove it by induction.

When n = 2, there will be two vertices and one edge, and these two vertices are connected, so obviously this graph is acyclic.

Then I assume this is true, when n = k – 1. That means G is connected and has k – 2 edges, and G is acyclic.

We need to consider the situation that n = k. Let T be a connected graph with k vertices and k – 1 edges. So actually we add one vertex and one edge to the connected acyclic graph G.

If we add an edge to G and make it connected, then T will be a connected cyclic graph with a single vertex. Thus T will be an unconnected graph which is contradicted to our condition T is connected.

Therefore, T is acyclic.

(2) G is acyclic and has n – 1 edges, so G is connected.

I prove it by induction.

First, when n = 2, G has one edge and two vertices, G is obviously connected.

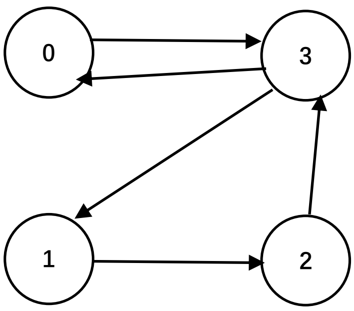
Then, we assume that this to be true when n = k – 1.

In the situation of n = k, let T be an acyclic graph with n – 1 edges and n vertices. I remove a vertex and one edge from T, $v \in V(T), e \in E(T)$.

According to our assumption, $T\backslash\{v,e\}$ is acyclic connected. So we add back the edge with a vertex to $T\backslash\{v,e\}$, this graph is also connected. Therefore, T is connected.

2. Programming

I determine whether a digraph G is a Eulerian graph by the theorem that a digraph G is eulerian, if and only if G has at most one non-trivial component and for all vertices v in G, out degree of v equals to in degree of v.
After I ensure this digraph is eulerian, I will find one eulerian circuit as assignment required.
The following image is the result of a simple digraph which is eulerian and gives out the eulerian circuit. And the digraph is G.
And also the de Bruijin sequence of length 5.




```
In [18]: runfile('D:/CS591-GraphThoery/PS2/eulerain.py',
wdir='D:/CS591-GraphThoery/PS2')
G is eulerian:  True
Eulerian circuit:  [(0, 3), (3, 1), (1, 2), (2, 3), (3, 0)]
De Bruijin sequences of length 5:
00000100011001010011101011011111
```

In my txt file, I have the format: node1, node2, that means node1 directs to node2. For example,
0,1
1,0
The above means node 0 directs to 1, node 1 directs to 0, that forms a cycle.
And here is the result for ten Eulerian graphs with Eulerian circuit and ten non-Eulerian graphs.

Anaconda Prompt

```
(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu0.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu1.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 2), (2, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu2.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 2), (2, 3), (3, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu3.txt
g is eulerian:  True
Eulerian circuit:  [(0, 3), (3, 1), (1, 2), (2, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu4.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu5.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 4), (4, 3), (3, 4), (4, 1), (1, 2), (2, 3), (3, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu6.txt
g is eulerian:  True
Eulerian circuit:  [(0, 3), (3, 4), (4, 0), (0, 2), (2, 1), (1, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu7.txt
g is eulerian:  True
Eulerian circuit:  [(0, 6), (6, 4), (4, 2), (2, 3), (3, 4), (4, 5), (5, 0), (0, 1), (1, 2), (2, 0)]

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu8.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 1), (1, 3), (3, 5), (5, 7), (7, 0)]
```

Picture 1 Eulerian circuit for Eulerian digraphs from eu0.txt to eu8.txt

Anaconda Prompt

```
(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\eu9.txt
g is eulerian:  True
Eulerian circuit:  [(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 1
6), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22), (22, 23), (23, 24), (24, 25), (25, 26), (26, 27), (27, 28), (28, 29), (29, 30), (30, 31), (31
, 32), (32, 33), (33, 34), (34, 35), (35, 36), (36, 37), (37, 38), (38, 39), (39, 40), (40, 41), (41, 42), (42, 43), (43, 44), (44, 45), (45, 46), (46, 47),
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), (63, 64), (64, 65), (65, 66), (66, 67), (67, 68), (68, 69), (69, 70), (70, 71), (71, 72), (72, 73), (73, 74), (74, 75), (75, 76), (76, 77), (77, 78), (78,
 79), (79, 80), (80, 81), (81, 82), (82, 83), (83, 84), (84, 85), (85, 86), (86, 87), (87, 88), (88, 89), (89, 90), (90, 91), (91, 92), (92, 93), (93, 94), (
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0), (200, 201), (201, 202), (202, 203), (203, 204), (204, 205), (205, 206), (206, 207), (207, 208), (208, 209), (209, 210), (210, 211), (211, 212), (212, 213
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01), (501, 502), (502, 503), (503, 504), (504, 505), (505, 506), (506, 507), (507, 508), (508, 509), (509, 510), (510, 511), (511, 512), (512, 513), (513, 51
```

Picture 2 Eulerian circuit for Eulerian digraph of eu9.txt with 1000 nodes

Anaconda Prompt

```
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non2.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non3.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non4.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non5.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non6.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non7.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non8.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
Eulerian circuit:  []

(D:\python\anaconda3) C:\Users\luna>python D:\CS591-GraphThoery\PS2\eulerain.py D:\CS591-GraphThoery\PS2\non9.txt
g is eulerian:  False
This digraph is not Eulerian. It does not have Eulerian circuit!
```

Picture 3 Results for 10 non-Eulerian digraphs in 10 txt files non0.txt~non9.txt

And also finally, we got the de Bruijin sequence of length 5.

```
De Bruijin sequences of length 5:
00000100011001010011101011011111
```