CS 591: Graph Theory and Computational Topology	**Summer 2017**

Problem Set 2

Lecturer: Prof. Peter Chin   *Due: June 13, 2017*

◇ Please email the written portion (either type up your answer or scan your handwritten solution) & code and report to `zhouxiao@bu.edu` by 23:59PM on the due date.

◇ Late policy: there will be a penalty of 10% per day, up to three days late. After that no credit will be given.

1. **Mathematical calculations**

   (a) Recall that an orientation of a graph G is a digraph D obtained from G by choosing an orientation $x \rightarrow y$ or $y \rightarrow x$ for every edge $xy \in E(G)$. A tournament is an orientation of a complete graph. In a digraph, a king is a vertex from which every vertex is reachable by a path of length at most 2. Show that every tournament has a king.

   (b) Let G be a graph on n vertices. Show that any two of the following properties mplies the third.

       i. G is connected.

       ii. G is acyclic.

       iii. G has $n - 1$ edges.

2. **Programming assignment**

   Write a program using a language of your choice that determines whether a graph (directed or undirected) is Eulerian or not, and if it is find an Eulerian circuit'. You can use the induction proof that we covered in class. Input to your program should be a text file that describes a simple graph. each line should contain

   $$v_i, v_j, 1$$

   if there is an edge between v_i and v_j. otherwise it should be

   $$v_i, v_j, 0$$

   Generate an example of Eulerean graphs and non-Eulerian graphs so that you can test out your program. For submission, zip the following 4 items: your program, Readme File, 10 examples of bipartite graphs, 10 examples of non-bipartite graphs. Your program should handle graphs of size up 1,000 nodes so your examples should have various sizes from small to 1000. Finally use your program to find de Bruijn sequences of length 5.