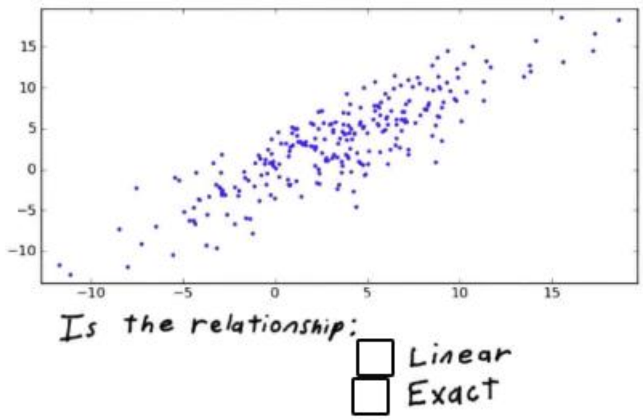
Intro to Statistics - Final Exam

Question 1

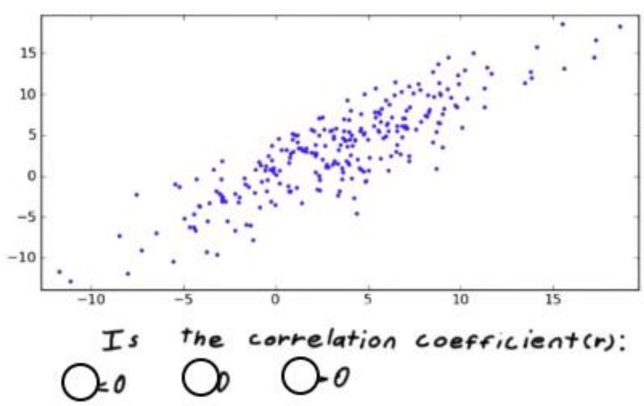


Is the relationship expressed by this scatter plot linear or exact?

Answer 1: Linear

Explanation: Data points tend to cluster around a straight line, suggesting a constant rate of change between the variables. But, all data points don’t fall perfectly on that straight line to be considered as an exact linear relationship.

Question 2

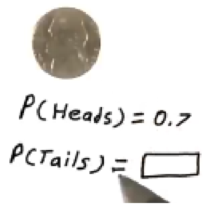


Is the correlation coefficient <0, 0, >0 for this data?

Answer 2: > 0

Explanation: The scatter plot indicates a positive linear relationship between two variables, meaning as one variable increases, the other tends to increase as well.

Question 3



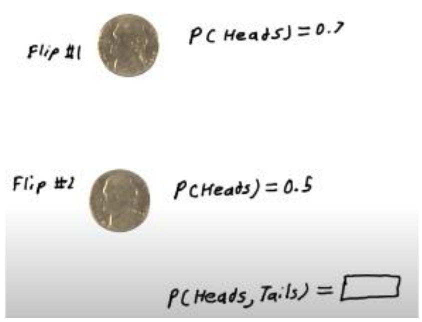
Consider a coin that has a probability of landing on heads of 0.7.

What is the probability of it landing on tails?

Answer 3: 0.3

Explanation: P(Tails) = 1 – P(Heads)

Question 4



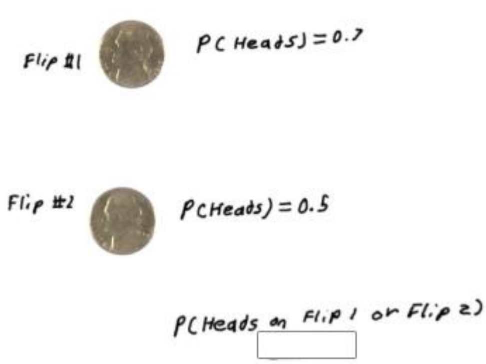
If we flip one coin with the probability of heads of 0.7

followed by a fair coin, what is the probability of flipping heads followed by a tails?

Answer 4: 0.35

Explanation: P(Heads, Tails) = P(Heads) * P(Tails)

Question 5:



What is the probability of having heads on the first flip or on the second flip if we have one coin with a probability of heads of 0.7 and a second coin with a probability of heads of 0.5?

What is the probability that one of the two flips will result in heads.

Clarification: We are asking the probability of heads on the first flip or second flip using the logical OR: the case where both flips are heads counts towards our probability.

Answer 5: 0.85

Explanation:

Flip #1	Flip #2		
H	T	= 0.7 * 0.5 = 0.35 +	
T	H	= 0.3 * 0.5 = 0.15 +	= 0.85
H	H	= 0.7 * 0.5 = 0.35 +	

Question 6:

We have two dice. One has 6 sides and one has 8 sides.

We select which die to roll based on whether the flip of this coin is heads, in which case we roll the 6-sided die, or tails, in which case we roll the 8-sided die.

What is the probability of rolling a 6?

You can assume that the coins and dice are fair.

Answer 6: 0.146

Explanation:

P(roll=6) = [P(Tails) * 1/8] + [P(Heads) * 1/6] = (0.5 * 1/8) + (0.5 * 1/8) = 0.146

Question 7:

What is the probability that the coin flip was heads given that the roll was a 6?

Answer 7: 0.571

Explanation:

Apply Bayes' rule: P(A|B) = [P(B|A) * P(A)] / P(B)

Prior: P(Heads) = P(Tails) = 0.5

P(6 | Heads) = 1/6 * 0.5

P(6 | Tails) = 1/8 * 0.5

Joint probability of 2 events:

P(Heads) * P(6 | Heads) = 0.5 * [1/6 * 0.5] = 1/24

P(Tails) * P(6 | Tail) = 0.5 * [1/8 * 0.5] = 1/32

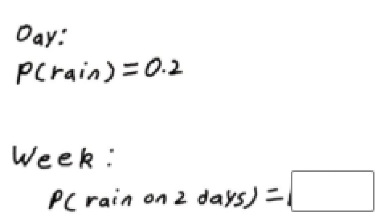
Normalizer:

P(6) = [P(Heads) * P(6 | Heads)] + [P(Tails) * P(6 | Tail)] = 1/24 * 1/32 = 7/96

Posterior:

P(Heads | 6) = [P(Heads) * P(6 | Heads)] / P(6) = (1/24) / (7/96) = 1/24 * 96/7 = 0.571

Question 8:



Given that we have a probability of rain on any given day of 0.2, what is the probability that over the course of a week it rains on exactly 2 days?

Answer 8: 0.275

Explanation:

Apply Binomial Probability Formula:

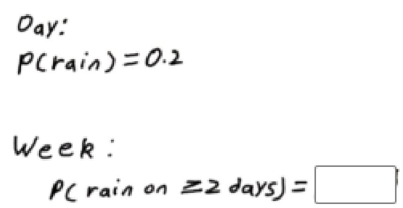
$P(X = k) = C(n, k) * p^k * (1 - p)^{n-k}$

C(n, k) is the number of combinations of n items taken k at a time, calculated as n! / (k! * (n-k)!)

In our case, n = 7 (days in a week) and k = 2 (i.e. rain on exactly 2 days)

P(rain on 2 days) = $7!/(5!*2!) * 0.2^2 * (1 - 0.2)^{7-2} = 21 * 0.2^2 * (0.8)^5 = 0.275$

Question 9:



Given that we have a probability of rain of 0.2 on a given day, what is the probability of having rain on at least two days during the week?

Answer 9: 0.4233

Explanation:

Method 1:

P(rain on >= 2 days) is 1 – [total probability of P(rain on 0 days) and P(rain on 1 days)]

[(P(rain on 0 days) + P(rain on 1 days)]

P(rain on 0 days) = $7!/(7!*0!) * 0.2^0 * (1 - 0.2)^{7-0} = 1 * 0.2^0 * (0.8)^7 = 0.2097$

P(rain on 1 days) = $7!/(6!*1!) * 0.2^1 * (1 - 0.2)^{7-1} = 7 * 0.2^1 * (0.8)^6 = 0.3670$

P(rain on >= 2 days) = 1 – [(P(rain on 0 days) + P(rain on 1 days)] = 1 – [0.2097 + 0.3670] = 0.4233

Method 2:

P(rain on >= 2 days) is the total probability of P(rain on 2 days), P(rain on 3 days), P(rain on 4 days), P(rain on 5 days), P(rain on 6 days) and P(rain on 7 days)

P(rain on 2 days) = $7!/(5!*2!) * 0.2^2 * (1 - 0.2)^{7-2} = 21 * 0.2^2 * (0.8)^5 = 0.2753$

P(rain on 3 days) = $7!/(4!*3!) * 0.2^3 * (1 - 0.2)^{7-3} = 35 * 0.2^3 * (0.8)^4 = 0.1147$

P(rain on 4 days) = 7!/(3!*4!) * 0.2^4 * $(1 - 0.2)^{7-4}$ = 35 * 0.2^4 * $(0.8)^3$ = 0.0287

P(rain on 5 days) = 7!/(2!*5!) * 0.2^5 * $(1 - 0.2)^{7-5}$ = 21 * 0.2^5 * $(0.8)^2$ = 0.0043

P(rain on 6 days) = 7!/(1!*6!) * 0.2^6 * $(1 - 0.2)^{7-6}$ = 7 * 0.2^6 * $(0.8)^1$ = 0.00036

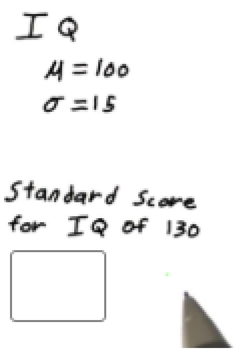
P(rain on 7 days) = 7!/(0!*7!) * 0.2^7 * $(1 - 0.2)^{7-7}$ = 1 * 0.2^7 * $(0.8)^0$ = 0.00001

P(rain on >= 2 days) = P(rain on 2 days) + P(rain on 3 days) + P(rain on 4 days) + P(rain on 5 days) + P(rain on 6 days) + P(rain on 7 days)

= 0.2753 + 0.1147 + 0.0287 + 0.0043 + 0.00036 + 0.00001

= 0.42337

Question 10:



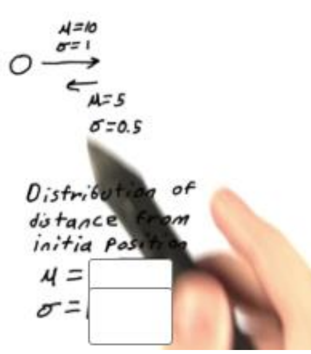
A common measure of intelligence, IQ, is distributed with a mean of 100 and a standard deviation of 15. What is the standard score of an IQ of 130?

Answer 10: 2

Explanation:

Standard score z = (x - μ) / σ = (130 – 100) / 15 = 2

Question 11:



Consider a ball that is kicked by a mean of 10 feet in this direction and with a standard deviation of 1 foot. It is then kicked back in the opposite direction towards where it started by 5 feet, but this time with a standard deviation of 0.5. What are the mean and standard deviation of this distribution of the distance between the initial position and the final position?

Answer 11: $\mu = 5$, $\sigma = 1.12$

Explanation:

μ(final) = The final position is the first kick minus the second kick.

$\mu(\text{final}) = \mu 1 - \mu 2$

σ(final) = squareroot(variance1 + variance2)

= squareroot($\sigma 1^2 + \sigma 2^2$) = squareroot($1^2 + 0.5^2$) = squareroot(1.25) = 1.12