

B. Tech Degree V Semester Examination, November 2008

IT/CS/CE/EC/ME/SE/EB/El/EE 501 ENGINEERING

MATHEMATICS IV

(2006 Scheme)

Time : 3 Hours

Maximum Marks : 100

PART A

(Answer ALL questions)

(8 x 5 = 40)

- I. (a) Obtain the distribution function and mean of the total number of heads occurring in three tosses of an unbiased coin.
- (b) Fit a parabola of second degree to the following data
- | | | | | | | |
|---|---|---|-----|-----|-----|-----|
| X | : | 0 | 1 | 2 | 3 | 4 |
| Y | : | 1 | 1.8 | 1.3 | 2.5 | 6.3 |
- (c) A random sample of 25 people from a population showed incomes with a mean Rs.4800/- and standard deviation Rs.500/-. Estimate the population mean with 95% confidence interval.
- (d) Define :
- (i) Type I and Type II errors
 - (ii) Power of a test
 - (iii) Level of Significance
- (e) Find the first difference of the polynomial
- $$f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9 \text{ with } h = 2.$$
- (f) Evaluate $\int_{-1}^1 e^{-x^2} \cos x \, dx$ by three point Gaussian quadrature formula.
- (g) Solve $\frac{dy}{dx} = 1 - 2xy$, $y(0) = 0$ using Taylor's Series method and compute $y(0.2)$ and $y(0.4)$ correct to 4 decimal places.
- (h) Solve numerically, $4u_{xx} = u_{tt}$ given $u(0, t) = 0$, $u(4, t) = 0$ and $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ taking $h = 1$. Find the values upto $t = 4$.

PART B

(4 x 15 = 60)

- II. (a) Out of 800 families with 4 children each, how many families would be expected to have
- (i) 2 boys and 2 girls
 - (ii) at least one boy
 - (iii) no girl
- Assume equal probabilities for boys and girls. (9)
- (b) In a normal distribution 17% of the items are below 30 and 17% of the items are above 60. Find the mean and standard deviation. (6)
- OR
- III. (a) In a partially destroyed laboratory records of an analysis of a correlation data, the following results only are legible
- Variance of $x = 9$, Regression equations $\begin{cases} 8x - 10y + 66 = 0 \\ 40x - 18y = 214 \end{cases}$
- What were
- (i) the mean of x and y
 - (ii) the standard deviation of y
 - (iii) the coefficient of correlation between x and y .
- (6)
- (b) Obtain Poisson distribution from Binomial distribution. Find the mean and variance of Poisson distribution. (9)
- IV. (a) A trucking firm is suspicious of the claim that the average lifetime of certain tyres is at least 28,000 miles. To check the claim, the firm puts 40 of these tyres on its trucks and gets a mean lifetime of 27,463 miles with a standard deviation of 1,348 miles. What can it conclude, if the probability of a type 1 error is to be at most 0.01? (8)
- (b) Nine determinations of the specific heat of iron had a standard deviation of 0.0086. Assuming that these determinations constitute a random sample from a normal population test $H_0 : \sigma^2 = (0.01)^2$ against $H_1 : \sigma^2 < (0.01)^2$ at 0.05 level of significance. (7)

OR

(Turn Over)

- V. (a) Samples of sizes 10 and 14 were taken from two normal populations with standard deviations 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level. (7)
- (b) It is desired to determine whether there is less variability in the silver plating done by company I than that done by company II. If independent random samples of size 12 of the two companies work yield $s_1 = 0.035$ and $s_2 = 0.062$, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $\sigma_1^2 < \sigma_2^2$ at 0.05 level of significance. (8)

- VI. (a) Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$. (7)

- (b) Find $y'(6)$ and the minimum value of y from the following data.

x	:	0	2	3	4	7	9
y	:	4	26	58	112	466	922

OR

- VII. (a) The table below gives the velocity v of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds and also the acceleration at $t = 2$ seconds.

t	:	0	2	4	6	8	10	12
v	:	4	6	16	34	60	94	136

- (b) Prove that $\left(\frac{\Delta^2}{E}\right)e^x, \frac{Ee^x}{\Delta^2 e^x} = e^x$, taking h as the interval of differencing. (7)

- VIII. (a) Solve $\frac{dy}{dx} = x^2 + y, y(0) = 1$ using modified Euler's method and find the value of $y(0.1)$ correct to 4 decimal places. Take $h = 0.05$. (6)

- (b) Solve $\frac{dy}{dx} = yz + x; \frac{dz}{dx} = xz - y$ given that $y(0) = 1$ and $z(0) = -1$ for $y(0.1)$ and $z(0.1)$ by Runge-Kutta method of fourth order. (9)

OR

- IX. (a) Using Taylor's series method solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0, y(0) = 1, y'(0) = 0$ and evaluate $y(0.1)$. (5)

- (b) Solve $U_{xx} + U_{yy} = 0$ over the square mesh of side 4 units satisfying the following boundary conditions.

(i) $u(0, y) = 0$ for $0 \leq y \leq 4$

(ii) $u(4, y) = 12 + y$ for $0 \leq y \leq 4$

(iii) $u(x, 0) = 3x$ for $0 \leq x \leq 4$

(iv) $u(x, 4) = x^2$ for $0 \leq x \leq 4$. (10)

