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Comparison of Manhattan Distance and Misplaced Tiles Heuristics in A-star algorithm used to solve the N-puzzle

Introduction

The N-puzzle is a puzzle which involves a square board filled with numbered tiles. The traditional version of the puzzle leaves a single blank gap on the board, allowing tiles to be rearranged by sliding them into the blank space. The puzzle is solved when the rearranged tiles form a new, desired goal state. In this report, I focus on a modified version of the puzzle which has two blank spaces instead of one.

The A-star algorithm is a graph traversal algorithm which uses heuristics to conduct an informed search of the graph, to find the shortest path from one node to another [1]. It can be adapted to solve the N-puzzle, by considering each puzzle state as a node.

The algorithm may use any admissible heuristic [2]. The two admissible heuristics investigated in this report are the Total Misplaced Tiles heuristic, which indicates the number of tiles that are not in their goal state positions, and the Total Manhattan Distance heuristic, which indicates the sum of Manhattan distances of each tile from its goal position.

This report describes the process of comparing the two heuristics in terms of the number of nodes the A-star algorithm must check before finding the shortest path to the goal. All relevant scripts can be found in the folder containing this report.

Testing using Paired t-test

112 N-puzzle problems were generated randomly, for varying puzzle sizes between 5 and 20. For each puzzle (start and goal configuration, the A star algorithm was run twice, once using the Total Misplaced Tiles heuristic and once using the Total Manhattan Distance heuristic. The number of nodes checked by the algorithm in each run was recorded.

For each puzzle, the start configuration was generated by randomly shuffling the list of tiles. This configuration was then altered by 20-35 random moves to generate a goal state. When generating the goal states of configurations with higher sizes, less random moves were used, to avoid causing a large delay in the running of the algorithm. The number of moves was not reduced below 20.

Results

5 - Puzzles

[54, 54],
[73, 49],
[237, 122],
[6, 6],
[25, 18],
[136, 77],
[1658, 109],
[2580, 864],
[272, 60],
[84, 24],
[12, 12],
[292, 48],
[125, 40],
[45, 23],
[277, 85],
[1087, 243],
[12, 9],
[1823, 221],
[4571, 491],
[496, 62],
[32, 32],
[42, 30],
[3702, 731],
[344, 124],
[20, 20],
[928, 99],
[69, 44],
[36, 36],
[21, 21],
[12, 12],
[71, 53],
[226, 80],
[55, 45],
[3, 3],
[10, 10],
[415, 147],
[14, 14],
[15, 15],
[12, 12],
[37, 14],
[24, 24],
[28, 16],
[30, 30],
[178, 42],
[40, 40],
[42, 24],
[10, 10],
[1388, 387],
[88, 78],
[198, 91]

7 – Puzzles

[54, 16],
[134, 84],
[16, 16],
[50, 45],
[496, 151],
[65, 24],
[66, 30],
[21, 21],
[98, 93],
[24, 24],
[40, 28],
[1034, 144],
[40, 40],
[24, 24],
[31, 24],
[35, 35],
[59, 35],
[24, 24],
[8, 8],
[227, 142],
[243, 140],
[8, 8],
[15, 15],
[191, 116],
[198, 77],
[30, 30],
[1961, 231],
[161, 91],
[3228, 300],
[282, 129]

10 – Puzzles

[18, 18],
[45, 45],
[1227, 75],
[66, 50],
[30, 30],
[30, 27],
[30, 30],
[9, 9],
[136, 75],
[679, 219],
[445, 138],
[28, 28],
[2943, 1076],
[29593, 3932],
[30, 30],
[54, 54],
[369, 67],
[21, 21],
[128, 77],
[52, 42]

12 – Puzzles

[65, 65],
[45, 45],
[28, 28],
[24, 24],
[21, 21],
[40, 40]

15 – Puzzles

[18, 18],
[18, 16],
[74, 38],
[49, 49],
[60, 60],

20 – Puzzle

[356, 152]

The pair of tests run on each puzzle is represented by $[n_1, n_2]$ where
n_1 = Number of nodes checked when Misplaced Tiles Heuristic used
n_2 = Number of nodes checked when Total Manhattan Distance Heuristic used

Total number of puzzles tested = 112

Analysis of results

A Python script was used to carry out the calculations involved in paired t-testing.
The script calculated the differences between each pair of test results and the mean, standard deviation, standard error and t-value of the difference data.

The script output is shown below.

```
Mean of differences: 481.286
Standard Deviation: 2476.085
Standard Error of Mean Difference: 233.968
T-value: 2.057
>>>
```

The following formulae were used [3]:

Standard error of mean difference = Standard deviation in difference data / Sqrt(n)

T-value = Mean difference / Standard error of mean difference

The final T-value was 2.057.

Finding the critical t-value [4]:

Confidence level of 95% was chosen.
Degrees of freedom = 112-1 = 111

Critical t-value = 1.984 < 2.057

Conclusion

As the calculated t-value is lower than the critical t-value for the chosen confidence level, the null hypothesis is rejected, and it is concluded that the A star algorithm checks less nodes on average when run on the Total Manhattan Distance heuristic, as compared to when it is run on the Total Misplaced Tiles heuristic.

Therefore, the Total Manhattan Distance heuristic is more efficient than the Number of Misplaced Tiles heuristic, in terms of the total number of nodes checked before arriving at the answer. The Total Manhattan Distance heuristic can be concluded to be the better of the two.

References

[1] “A-star(A*) in general”, Algorithms Insight [online]. Available at: https://algorithmsinsight.wordpress.com/graph-theory-2/a-star-in-general/ [Accessed 21 Aug 2021]

[2] “Set 3: Informed Heuristic Search”, ICS 271 Fall 2016 - Kalev Kask lecture notes, University of California, Irvine. Available at: https://www.ics.uci.edu/~kkask/Fall-2016%20CS271/slides/03-InformedHeuristicSearch.pdf [Accessed 09 Sep 2021]

[3] “Statistics: Paired t-tests”, Rosie Shier, 2004. Available at: https://www.statstutor.ac.uk/resources/uploaded/paired-t-test.pdf [Accessed 09 Sep 2021]

[4] “SPSS Tutorials: Paired Sample T Tests”, Kent State University Library. Available at: https://libguides.library.kent.edu/SPSS/PairedSamplestTest [Accessed 10 Sep 2021]