

Z-test ( $\sigma$  (SD of population) is known):  $z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ ; standard error:  $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ ;  $z \sim N(0, 1)$ ;

$$(1 - \alpha) CI = \bar{X} \pm z_{\frac{\alpha}{2}} S_{\bar{X}}$$

One-sample t-test ( $\sigma$  is unknown):  $t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$ ;  $S_{\bar{X}} = \frac{s}{\sqrt{n}}$ ;  $t \sim t_{n-1, \frac{\alpha}{2}}$ ;  $(1 - \alpha) CI = \bar{X} \pm t_{n-1, \frac{\alpha}{2}} S_{\bar{X}}$

Independent Samples t Test:  $t = \frac{\bar{X}_1 - \bar{X}_2}{\frac{S_{\bar{X}_1 - \bar{X}_2}}{\sqrt{n}}}$ ;  $S_{\bar{X}_1 - \bar{X}_2} = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ ;  $S_p = \sqrt{\frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1 + n_2 - 2}}$ ;  $df = (n_1 - 1) + (n_2 - 1)$ ;  $(1 - \alpha) CI = (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1 + n_2 - 2} S_{\bar{X}_1 - \bar{X}_2}$

Paired Samples t Test:  $t = \frac{\bar{d}}{S_{\bar{d}}}$ ,  $S_{\bar{d}} = \frac{S_d}{\sqrt{n}}$ ;  $\{d = post - pre, n = \# \text{ of pairs}\}$ ;  $(1 - \alpha) CI = \bar{d} \pm t_{n-1, \frac{\alpha}{2}} SE_{\bar{d}}$

Effect size:  $\delta = \frac{|\mu_1 - \mu_2|}{\sigma}$ ; 1-Sample t test:  $d = \frac{|\bar{X} - \mu_0|}{s}$ ; Indep Smp t test:  $d = \frac{|\bar{X}_1 - \bar{X}_2|}{S_p}$ ; Dep Smp t

test:  $d = \frac{\bar{d}}{S_{\bar{d}}}$ ; ANOVA:  $\eta^2 = \frac{SS_B}{SS_T} \rightarrow f = \sqrt{\frac{\eta^2}{1 - \eta^2}}$ ; Correlation (coefficient of determination: amount

of shared variability):  $r^2_{XY} = \frac{s^2_{XY}}{s^2_X s^2_Y}$ ; Regression:  $R^2 = \frac{SS_{reg}}{(SS_{reg} + SS_{error})}$

S: 0.2, M: 0.5, L: 0.8, for ANOVA ( $f$ ): S: 0.1, M: 0.25, L: 0.4

Power:  $1 - \beta$  (type II error) =  $p(z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} > z_{\frac{\alpha}{2}})$ , assuming  $H_1$  is true. Customary power in social sciences: 0.8

One-sample test of variance: Chi<sup>2</sup> test:  $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ ; 2-tailed critical value:  $p(< .05)$ ,  $p(> .075)$

$$\chi^2_{df} = (df_{num}) F_{df_{num}, \infty}; t_{df} = \sqrt{F_{1, df_{den}}}; z = \sqrt{\chi^2} = \sqrt{F_{1, \infty}} = t_{\infty, df}$$

2-sample tests of variance (comparing variance of 1 variable between 2 groups):  $F = \frac{S_1^2}{S_2^2}$ ;  $df_1 =$

$n_1 - 1$ ,  $df_2 = n_2 - 1$ ; 2-tailed critical value:  $p(< .025)$ ,  $p(> .075)$

Using variances to answer questions about means (comparing effects of independent variable on dependent variable in multiple groups): ANOVA:  $F = \frac{MS_B}{MS_W}$ ,  $df_B = J - 1$ ,  $df_W = N - J$ ,  $df_T =$

$$N - 1, MS = \frac{SS}{df}$$

for 1 score:  $[SS_W = (Y - \bar{Y}), SS_B = (\bar{Y} - \bar{Y}_{..}), SS_T = (Y - \bar{Y}_{..})]$ , for multiple scores:  $[SS_W = \sum_{j=1}^J \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{.j})^2, SS_B = \sum_{j=1}^J n_j (\bar{Y}_{.j} - \bar{Y}_{..})^2, SS_T = \sum_{j=1}^J \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{..})^2]$

Covariance:  $\sigma_{XY} = \frac{\sum (X_i - \mu_X)(Y_i - \mu_Y)}{N}$ ;  $S_{XY} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{n-1}$ ;  $\frac{\sum xy}{n-1} = \frac{\sum X \cdot Y - \frac{\sum X \cdot \sum Y}{n}}{n-1}$

Pearson product moment correlation coefficient:  $r_{XY} = \frac{S_{XY}}{S_X S_Y} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sqrt{\sum (X - \bar{X})^2} \sqrt{\sum (Y - \bar{Y})^2}} =$

$$\frac{\frac{\sum X \cdot Y - \frac{\sum X \cdot \sum Y}{n}}{n-1}}{\sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}} \sqrt{\frac{\sum Y^2 - \frac{(\sum Y)^2}{n}}{n-1}}} = \frac{\sum (Z_X Z_Y)}{n-1} = \frac{e^{2Z_r} - 1}{e^{2Z_r} + 1}; \text{S: M: L: 0.1: 0.3: 0.5; Fisher's z transformation}$$

of  $r$ :  $z = \ln \left( \frac{1+r}{1-r} \right) = \frac{1}{2} [\ln(1+r) - \ln(1-r)] = \text{arctanh}(r)$

Estimated standard error of the correlation:  $S_r = \sqrt{\frac{1-r^2}{n}}$

Correlation: test statistic for 1 sample:  $t = r \sqrt{\frac{n-2}{1-r^2}}$ ;  $v = n - 2$ ,  $(1 - \alpha) CI = r \pm t_{n-1, \frac{\alpha}{2}} S_r$  or:

$$z = \frac{z_r - z_{\rho}}{\sigma_{z_r}}; \sigma_{z_r} = \frac{1}{\sqrt{n-3}}; \text{for 2 independent samples: } Z_r = \frac{z_1 - z_2}{\sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}} = \frac{(n_1-3)z_1 + (n_2-3)z_2}{n_1 + n_2 - 6}$$

Spearman correlation: primarily intended to be for ordinal data. Convert X scores to ranks, convert Y scores to ranks. Compute a Pearson correlation on the rank values.

Phi correlation: coefficient for 2 binary variables.

Point-biserial correlation: a Pearson correlation between an interval level variable and a naturally dichotomous variable.

Tetrachoric correlation: a coefficient relating two dichotomous variables, but each of those dichotomous variables is believed to be a forced categorization of an otherwise normal and interval variable. Sounds like a job for a phi-coefficient but better for estimating the relation between the interval variables.

Biserial correlation: a coefficient relating an interval level variable and a dichotomous variable, where the dichotomous variable is believed to be a forced categorization of an otherwise normal and interval variable.

Coefficient of determination: The proportion of variance that X and Y share. The amount of variance in Y that is explainable by X (or vice versa):  $r^2_{xy} = \frac{S^2_{xy}}{S^2_x S^2_y}$

Regression:  $Y' = a + bX$ ;

$$b = \frac{\sum xy}{\sum x^2} = r \left( \frac{s_y}{s_x} \right), \sum xy = \sum X \cdot Y - \frac{\sum X \cdot \sum Y}{n}, \sum x^2 = \sum X^2 - \frac{(\sum X)^2}{n};$$

$$a = \bar{Y} - b\bar{X}$$

Regression with standardized variables:  $Z_{Y'} = rZ_X$

Error of estimate:  $e = Y - Y'$

The regression line we compute is sometimes called the "least squares" regression line because it minimizes the squared vertical distance of Y values from Y' values.

Standard error of estimate:  $S^2_{Y.X} = \frac{\sum (Y - Y')^2}{n-2} = e^2$ ;  $S_{Y.X} = \sqrt{\frac{n-1}{n-2} S^2_Y (1 - r^2)} = S_Y \sqrt{\frac{n-1}{n-2} (1 - r^2)}$

Error variance (variance of errors of estimate):  $r^2$ : proportion of variance in Y explained by X;  $1 - r^2$ : proportion of variance in Y not explained by X;  $S^2_Y (1 - r^2)$ : amount of variance in Y not explained by X;  $\left(\frac{n-1}{n-2}\right) S^2_Y (1 - r^2)$ : amount of variance in Y not explained by X (with df adjustment)

$$Z = \frac{Y - Y'}{S_{Y.X}}$$

95% confidence interval for actual Y value:  $(Y' - z_{crit} S_{Y.X}, Y' + z_{crit} S_{Y.X})$

Regression variability:  $\sum (Y - \bar{Y})^2 = \sum (Y - Y')^2 + \sum (Y' - \bar{Y})^2$

SSt = SSe (residual) + SSreg (explained)

Df:  $n-1 = (n-2) + 1$

MS = SS/df

$F = \text{MSreg} / \text{MSerror}$

$$Z_e = \frac{e}{S_{Y.X}}$$

Diagnosing outliers in residuals: Cases with values in excess of  $\pm 2$  or  $\pm 3$  are worthy of further scrutiny.

Leverage: A measure of each case's "pull" on the regression line:  $h_i = \frac{1}{N} + \frac{(X_i - \bar{X})^2}{\sum (X - \bar{X})^2}$

A centered value is:  $h_i = \frac{(X_i - \bar{X})^2}{\sum (X - \bar{X})^2}$

A common guideline is to scrutinize cases with leverage values in excess of  $2(k+1)/N$  (where k is the number of predictor variables;  $k=1$  in the case of simple regression); for centered leverages this translates to  $(2k+1)/N$ .

For Standardized DFBeta, cases with values in excess of  $\frac{3}{\sqrt{N}}$  are often considered worthy of further scrutiny.

Partial Correlation:  $r_{YZ.X} = \frac{r_{YZ} - r_{YX}r_{ZX}}{\sqrt{1-r^2_{YX}}\sqrt{1-r^2_{ZX}}}$ ,  $t = \frac{r_{YZ.X} - \rho}{S_{r_{YZ.X}}}$ ,  $S_{r_{YZ.X}} = \sqrt{\frac{r^2_{YZ.X}}{n-3}}$ , df = n-3

\* Critical t is even further away of 1.96 (critical z): 2.052

Semi-partial correlation (residualized correlation):  $r_{Y(1.2)} = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1-r_{12}^2}}$

Multiple correlation:  $R_{Y.12} = \sqrt{r_{Y2}^2 + r_{Y(1.2)}^2} = \sqrt{r_{Y2}^2 + \left[\frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1-r_{12}^2}}\right]^2}$

Test statistic for multiple correlation:  $F = \frac{\frac{R^2}{p}}{\frac{1-R^2}{n-p-1}}$ , p = number of predictors.  $df_{num} = p$ ,

$df_{denom} = n - p - 1$

$[t = \frac{r}{S_r} = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}, t^2 = F = \frac{r^2}{\frac{1-r^2}{n-2}}]$

$a = \bar{Y} - b_1\bar{X}_1 - b_3\bar{X}_3 - b_4\bar{X}_4$

Bryant-Paulson Post-hoc test (comparing pairs of adjusted means):  $BP = \frac{y_i^* - y_j^*}{\sqrt{\frac{MS^*_W \left[1 + \frac{MS_{Bx}}{SS_{Wx}}\right]}{n}}}$

$SS_{Wx} = \sum X^2 - \frac{\sum(\sum X_j)^2}{n}$

Multicollinearity: tolerance =  $1 - R_k^2$ ;  $R_k^2$  is the coefficient of determination for the regression of the kth predictor on all other predictors. The higher degree of multicollinearity, the lower the tolerance (< 0.1), the more the standard error of the regression coefficients will be inflated.

VIF =  $\frac{1}{1-R_k^2}$ : provides information on how “inflated” the variance of the regression coefficient is compared to what it would be if the variable were uncorrelated with any other independent variable in the model. (>10)

Partial F-test ( $\Delta R^2$  test):  $F = \frac{\frac{\Delta R^2}{\Delta df}}{\frac{(1-R^2)}{df_{error}}}$ ,  $\Delta R^2 = R^2_{full} - R^2_{reduced}$ ,  $\Delta df =$

$df_{error(reduced)} - df_{error(full)}$

$SS_T = \sum(Y_i - \bar{Y}_{..})^2 = \sum \sum(Y_{ij} - \bar{Y}_{..})^2$ ,  $SS_w = \sum(Y_{ij} - \bar{Y}_{.j})^2 = \sum(N_j - 1)s_j^2$ ,  $SS_B = \sum n_j(\bar{Y}_{.j} - \bar{Y}_{..})^2$