

Z-test ( $\sigma$  is known):  $z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ ;  $SE: \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ ;  $z \sim N(0, 1)$ ;  $(1 - \alpha) CI = \bar{X} \pm z_{\frac{\alpha}{2}} S_{\bar{X}}$

One-sample t-test ( $\sigma$  is unknown):  $t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$ ;  $S_{\bar{X}} = \frac{s}{\sqrt{n}}$ ;  $t \sim t_{n-1, \frac{\alpha}{2}}$ ;  $(1 - \alpha) CI = \bar{X} \pm t_{n-1, \frac{\alpha}{2}} S_{\bar{X}}$

Independent Samples t Test:  $t = \frac{\bar{X}_1 - \bar{X}_2}{S_{\bar{X}_1 - \bar{X}_2}}$ ;  $S_{\bar{X}_1 - \bar{X}_2} = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ ;  $S_p = \sqrt{\frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1 + n_2 - 2}}$ ;  $df = (n_1 - 1) + (n_2 - 1)$ ;  $(1 - \alpha) CI = (\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, n_1 + n_2 - 2} S_{\bar{X}_1 - \bar{X}_2}$

Paired Samples t Test:  $t = \frac{\bar{d}}{S_{\bar{d}}}$ ,  $S_{\bar{d}} = \frac{s_d}{\sqrt{n}}$ ;  $\{d = post - pre, n = \# \text{ of pairs}\}$ ;  $(1 - \alpha) CI = \bar{d} \pm t_{n-1, \frac{\alpha}{2}} SE_{\bar{d}}$

Effect size:  $\delta = \frac{|\mu_1 - \mu_2|}{\sigma}$ ; 1-Sample t test:  $d = \frac{|\bar{X} - \mu_0|}{s}$ ; Indep Smp t test:  $d = \frac{|\bar{X}_1 - \bar{X}_2|}{s_p}$ ; Dep Smp t

test:  $d = \frac{\bar{d}}{s_d}$ ; ANOVA:  $\eta^2 = \frac{SS_B}{SS_T} \rightarrow f = \sqrt{\frac{\eta^2}{1 - \eta^2}}$ ; Correlation (coeff of determination):  $r^2_{XY} =$

$\frac{s_{XY}^2}{s_X^2 s_Y^2}$ ; Regression:  $R^2 = \frac{SS_{reg}}{SS_{total}}$ ; S: 0.2, M: 0.5, L: 0.8, for ANOVA ( $f$ ): S: 0.1, M: 0.25, L: 0.4

Power:  $1 - \beta$  (type II error) =  $p(z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} > z_{\frac{\alpha}{2}})$ , assuming  $H_1$  is true. Type I error:  $1 - (1 - \alpha)^C$ ;

C: # independent tests.

One-sample test of variance: Chi<sup>2</sup> test:  $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ ; 2-tailed critical value:  $p(< .025), p(> .075)$

$\chi_{df}^2 = (df_{num}) F_{df_{num}, \infty}$ ;  $t_{df} = \sqrt{F_{1, df_{den}}}$ ;  $z = \sqrt{\chi_1^2} = \sqrt{F_{1, \infty}} = t_{\infty, df}$

2-sample tests of variance:  $F = \frac{S_1^2}{S_2^2}$ ;  $df_1 = n_1 - 1$ ,  $df_2 = n_2 - 1$

Using variances to answer questions about means (multiple groups): ANOVA:  $= \frac{MS_B}{MS_W}$ ,  $df_B = J - 1$ ,  $df_W = N - J$ ,  $df_T = N - 1$ ,  $MS = \frac{SS}{df}$ ;  $[SS_W = \sum_{j=1}^J \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$ ,  $SS_B =$

$\sum_{j=1}^J n_j (\bar{Y}_j - \bar{Y}_{..})^2$ ,  $SS_T = \sum_{j=1}^J \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{..})^2$ ;  $SS_T = SS_B + SS_W$

Covariance:  $\sigma_{XY} = \frac{\sum (X_i - \mu_X)(Y_i - \mu_Y)}{N}$ ;  $S_{XY} = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{n-1}$

Pearson product moment correlation coefficient:  $r_{XY} = \hat{\rho}_{XY} = corr(X, Y) = \frac{s_{XY}}{s_X s_Y} = \frac{\sum (Z_X Z_Y)}{n-1}$ ; S:

M: L=0.1: 0.3: 0.5;  $z = \frac{x - \bar{x}}{s}$ ; Estimated standard error of the correlation:  $S_r = \sqrt{\frac{1 - r^2}{n-2}}$

Correlation: test statistic for 1 sample:  $t = \frac{r}{S_r} = r \sqrt{\frac{n-2}{1-r^2}}$ ;  $v = n - 2$ ;  $(1 - \alpha) CI = r \pm t_{n-2, \frac{\alpha}{2}} S_r$

or:  $z = \frac{z_r - z_{\rho}}{\sigma_{z_r}}$ ;  $\sigma_{z_r} = \frac{1}{\sqrt{n-3}}$ ; for 2 independent samples:  $Z_r = \frac{z_1 - z_2}{\sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}} = \frac{(n_1-3)z_1 + (n_2-3)z_2}{n_1 + n_2 - 6}$

Coefficient of determination: The proportion of variance that X and Y share. The amount of variance in Y that is explainable by X (or vice versa):  $r^2_{xy} = \frac{S^2_{xy}}{S^2_x S^2_y}$

Regression:  $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$ ; unstandard reg coeff:  $\hat{\beta}_1 = \frac{cov(X, Y)}{var(X)} = \frac{s_{XY}}{s_X^2} = r \left( \frac{s_Y}{s_X} \right)$ ;  $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

Regression with standardized coefficients:  $Z_{\hat{Y}} = \hat{\beta}_1^* Z_{X1} + \hat{\beta}_2^* Z_{X2}$ ;  $\hat{\beta}^* = \hat{\beta} \times \frac{s_X}{s_Y} = r_{xy}$

Regression variability:  $\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y} - \bar{Y})^2 + \sum (Y_i - \hat{Y})^2 \Rightarrow SS_t = SS_{reg} + SS_{residual}$

Coefficient of determination (explained variation):  $R^2 = \frac{SS_{regression}}{SS_{total}} = 1 - \frac{SS_{error}}{SS_{total}}$

Coeff of multiple determination:  $R_{Y.12}^2 = \frac{SS_{regression}}{SS_{total}} = \sqrt{\frac{r_{Y1}^2 + r_{Y2}^2 - 2r_{Y1}r_{Y2}r_{12}}{1 - r_{12}^2}}$

Adjusted  $R^2$  (variation explained by only IVs that affect the DV):  $R_{adj}^2 = 1 - \left( \frac{(1-R^2)(n-1)}{n-k-1} \right)$

$$F = \frac{MS_{reg}}{MS_{res}} = \frac{SS_{reg}/df_{reg}}{SS_{res}/df_{res}} = \frac{\frac{SS_{reg}}{k}}{\frac{SS_{res}}{n-k-1}} = \frac{\frac{R^2}{k}}{\frac{1-R^2}{n-k-1}}; k: \# \text{ predictors, } n: \# \text{ subjects}$$

Standard error of estimate (a measure of accuracy of prediction by the model: SD of residuals) :

$$S^2_{Y|X} = \frac{SS_{res}}{n-k-1} = \frac{\sum(Y-\hat{Y})^2}{n-k-1} = MS_{res}; SEE = S_{Y|X} = S_Y \sqrt{\frac{k}{n-k-1}(1-r^2)} = \sqrt{\frac{(1-r^2)S_Y^2}{n-k-1}} = \sqrt{MS_{res}}$$

Standard error of slope (variation in slope due to sampling error):  $SE_{\hat{\beta}_j} = \frac{SEE}{\sqrt{\sum(X-\bar{X})^2}} =$

$$\frac{S_Y}{S_{X_j}} \sqrt{\frac{1-R^2}{(n-k-1)(1-R_j^2)}} \text{ (used in t-test for hypothesis testing of slope: } t = \frac{\hat{\beta}_j - \hat{\beta}_{j(H_0)}}{SE_{\hat{\beta}_j}} = \frac{\hat{\beta}_j - 0}{SE_{\hat{\beta}_j}})$$

Standard error of intercept:  $SE_{\hat{\beta}_0} = SEE \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{(n-1)S_X^2}}$  (used in t-test of intercept:  $t = \frac{\hat{\beta}_0 - 0}{SE_{\hat{\beta}_0}}$ )

Regression Assumptions: 1.\* Independence (tough to test. assumed!) => violation: impacts SE of model. 2.\* Linearity: Y vs X (simple regression), residuals vs each X, residuals vs  $\hat{Y}$  => violation: Bias intercept & slope, change in Y not constant and depends on X => Polynomial Regression, testing of Higher Order terms ( $X^2$ ). Log, Inverse, or Box-Cox transform:  $Y' = Y^\lambda$  and Box-Tidwell transform:  $X' = X^\lambda$ . 3.\* Homoscedasticity: Y vs X (simple), residuals vs each X, residuals vs  $\hat{Y}$ , Levene's test (not reliable for large n) => violation: Bias in SEE, inflate SE & type II error, non-normal conditional distributions => Weighted Least Square estimation. Sqrt<Log<Inverse. 4.\* Normality: histogram of residuals, PP/QQ plot of residuals, skewness >  $1.96 \times SE_{skewness}$ , KS test & Shapiro-Wilk test (not reliable for n>50) => violation: Less precise slope, intercept and  $R^2$  => Log transform for pos skewness, Square Root for pos/neg skewness. Sqrt<Log<Inverse.

Partial Correlation:  $r_{YZ.X} = \frac{r_{YZ} - r_{YX}r_{ZX}}{\sqrt{1-r_{YX}^2}\sqrt{1-r_{ZX}^2}}$ ,  $t = \frac{r_{YZ.X} - \rho}{S_{r_{YZ.X}}}$ ,  $S_{r_{YZ.X}} = \sqrt{\frac{r_{YZ.X}^2}{n-3}}$ , df = n-3

Semi-partial correlation (residualized correlation):  $r_{Y(1.2)} = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1-r_{12}^2}}$

Partial F-test ( $\Delta R^2$  test):  $F = \frac{\frac{\Delta R^2}{\Delta df}}{\frac{(1-R_{full}^2)}{df_{error(full)}}}$ ;  $\Delta R^2 = R_{full}^2 - R_{reduced}^2$ ;  $\Delta df = df_{reg(full)} -$

$df_{reg(reduced)} = k2 - k1$ ;  $df_{error(full)} = n - k2 - 1$

CI for  $R^2$ , if n>60:  $SE_{R_{model1}^2 - R_{model2}^2} = \sqrt{SE_{R_{model1}^2}^2 + SE_{R_{model2}^2}^2}$ ;  $SE_{R^2} = \frac{kR^2(1-R^2)^2(n-k-1)^2}{(n^2-1)(n+k-1)}$

Diagnosing outliers in residuals: look for cases with values in excess of  $\pm 2$  or  $\pm 3$ .

Leverage: A measure of each case's "pull" on the regression line:  $h_i = \frac{1}{n} + \frac{(X_i - \bar{X})^2}{\sum(X - \bar{X})^2}$ ; Centered

leverage:  $h_i = \frac{(X_i - \bar{X})^2}{\sum(X - \bar{X})^2}$ ; Look for leverage >  $2(k+1)/n$  or centered leverage >  $2k/n$ .

Influence: Look for Standardized DFBeta >  $\frac{3}{\sqrt{n}}$  or >1,  $DFFit \neq 0$ , Cook's distance d > 1.

Multicollinearity (predictors are correlated): highly significant  $R^2$  and non-significant reg coeffs, large SE of slopes. Tolerance =  $1 - R_k^2$ ;  $R_k^2$  is the coeff of determination for the reg of the kth predictor on all other predictors. VIF =  $\frac{1}{1-R_k^2}$ : how "inflated" the variance of the reg coeff is compared to what it'd be if the variable was uncorrelated with any other IV. Tolerance < 0.1 or .2775 (for R = .85) OR VIF > 10 or 3.604 => multicollinearity. => Centering predictors (using  $X - \bar{X}$  instead of X), dropping problematic predictors, combining correlated predictors, ridge regression.