AS4012 NEBULAE AND STARS 2

STELLAR STRUCTURE

Assessed Computational Homework

The goal of this assessed computational homework is to compute the internal structure of sun-like stars on the Zero-Age-Main-Sequence (ZAMS), using the shooting method. On moodle, you will find two Python codes to get you started

- `EZWebReader_student_v2.py` allows you to plot and analyse the results of the EZ-Web stellar evolution code which you can create yourself using http://www.astro.wisc.edu/~townsend/static.php?ref=ez-web.
- `Homework_student_v2.py` contains the stellar structure equations and some fit-functions for the material properties like opacity $\kappa(\rho, T, X, Y)$ and energy production rate $\epsilon(\rho, T, X, Y, XC)$. The code solves the differential equations in the Eulerian description inside-out, using the radius as independent variable.

Your mission, then, is to create a Lagrangian scheme, which allows the mass of the star to be specified in advance. Pick a random number between 0.8 and 1.2, and round it to the nearest 0.1 solar masses. This will be the mass of your star $M_\star$ in solar masses.

1. Read the literature on the present-day solar photospheric abundances and the proto-solar values, in particular Asplund et al. 2009 ARAA 47, 481. What are the recommended values X, Y, Z, XC for the proto-sun, how do they differ from the present-day photospheric values, and why? [**Easy - 2 marks**]
2. Run an EZ-Web model for a star of your choice with solar metallicity (initial mass $0.8 \ldots 1.2$, $Z = 0.02$, maximum number of steps = 100, detailed structure files = yes). Plot this model by using `EZWebReader_student_v2.py` provided on moodle.
   - (i) What are the initial X, Y, Z, XC abundances in this model, and how do they compare to the values recommended by Asplund et al.(2009)? Then select a stage called the "zero-age main sequence", after the model has settled, but before the core helium abundance starts to increase significantly. Look at the plots created `EZWebReader_student_v2.py` to answer the following questions:
   - (ii) What is the zero-age main sequence radius $R_\star$, luminosity $L_\star$, and effective temperature $T_{\rm eff}$ of that model, and what are the pressure and temperature values in the stellar core, T_c and P_c?
   - (iii) Which zones (mass-intervals) are convective and which are radiative?
   - (iv) What is the star's main energy source?

   [**Easy - 4 marks**]
3. Rewrite the numerical integration scheme using the Lagrangian equations with enclosed mass as independent variable. Use $T(0) = T_c$ and $P(0) = P_c$ from your EZ-Web model as boundary conditions. I have already created the set of equations and an example for outward integration of the Euler equations. Set up a scheme which integrates the Lagrange equations outward from the centre of the star to a fitting point at $m = 0.5M_\star$

(i.e. 0.5 times the total mass of your model). Plot the results as function of m and record the values of the four dependent variables r, P, L, T at the fitting point for the outward integration. [**Moderately easy - 6 marks**]

4. Write a second Lagrangian integration scheme which works inward from $M_\star$ to $m = 0.5M_\star$. Use the surface values of $R_\star$ and $L_\star$ from your EZ-Web model as boundary conditions. Plot the results and record the values of r, P, L, T at the fitting point for this inward integration. I'd prefer if you plot both the outward and the inward graphs in one plot for each quantity. [**Moderately easy - 3 marks**]
5. Determine the differences in the log values between the outward and inward solutions at the fitting point to fill in the rest vector F and then run another 2 outward and another 2 inward models, with perturbed $P(0), T(0), L(M), R(M)$, to fill in the 4×4 matrix DF. Solve this system of linear equations to obtain the log-corrections for $P(0), T(0), L(M), R(M)$, i.e. the Newton-Raphson step. Apply these corrections (you may want to limit those corrections while they are still large) and repeat the procedure, until the solutions match to within a suitable tolerance at the fitting point. Plot the final structures and record the final values for the stellar radius $R(M) = R_\star$, stellar luminosity $L(M) = L_\star$, core pressure $P(0)$, core temperature $T(0)$ and the surface temperature $T(M) = T_{\rm eff}$ of your model. [**More challenging - 6 marks**]
6. Now use this model as a starting point for (a) a star with a mass 0.1 solar masses smaller or greater than your original model and the same composition, and (b) a star with the same mass, but with sub-solar metallicity $Z = 0.001$ and appropriate X, Y, XC. Record the final radius, luminosity and effective temperatures of these two models. [**The icing on the cake if you get this far - 4 marks**]

Submission: If you are using plain PYTHON, please create a single python program, which includes all your definitions, functions and plot commands. Redirect all your graphical output into a single .pdf file. Include your answers to questions 1 and 2 in form of further comment lines. I will test your code by running `python YourCode.py` from the console.

If you are using PYTHON JUPYTER notebooks, please put all yours definitions, functions and the resulting plots into a single notebook. Please add your answers to questions 1 and 2 in form of further comment lines. Please make sure that I can re-run your code using `">>" = restart the current kernel and re-execute the whole notebook` from the menu.

Please use the submission tool provided on moodle to submit your code prior to the deadline Friday, March $5^{\rm th}$, 2021, 10 am. There is a penalty of 5% point reduction per day of late submission. **Five more marks** will be given for code readability and quality of the figures (readability of axes, tickmarks and labels, correct units). I am expecting to mark all submissions, and send feedback within 14 days after the deadline. The computational exercise will count with 25% towards your module grade.