機器學習 HW03

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Document

ML_Homework3_2017 (https://github.com/minori111/NTCU_ML_HW3/blob/master/ML-HW3-2017.pdf)
NTCU_ML_HW2 github (https://github.com/minori111/NTCU_ML_HW3)

Q1.

> Given x , y in $F = \{x|g(x) \leq 0, h(x) = 0\}$, we have
>
> $$g(tx + (1-t)y) \leq tg(x) + (1-t)g(y) \leq t \cdot 0 + (1-t) \cdot 0 = 0$$
>
> and
>
> $$h(tx + (1-t)y) = th(x) + (1-t)h(y) = t \cdot 0 + (1-t) \cdot 0 = 0$$
>
> for $t \in [0, 1]$.
> Therefore F is a convex set.

Q2.

> Form Farkas' Lemma, either one of the following system has solutions but not both:
> (A)
>
> $$Ax \leq 0, b^T x > 0$$
>
> (B)
>
> $$A^T \alpha = b, \alpha \geq 0,$$

We want to prove one of the following system has solutions but not both:
(I)

$$Bx < 0$$

(II)

$$B^T\alpha = 0, \alpha \geq 0$$

Prove by the procedure

$$(I) \Rightarrow (A) \Rightarrow not(B) \Rightarrow not(II)$$

and

$$not(I) \Rightarrow not(A) \Rightarrow (B) \Rightarrow (II).$$

First part, we assume (I) has a solution x .
By Hint1, we have \$$Bx + 1z \leq 0$ for some $z \leq 0$.
Let $A = [Bz]$ and

$$x' = [\begin{matrix} x \\ 1 \end{matrix}].$$

Suppose (II) has a solution, then there exist a $\alpha \geq 0$ such that $B^T\alpha = 0$.
Then we have

$$A^T\alpha = [\begin{matrix} B^T \\ z^T \end{matrix}]\alpha = [\begin{matrix} B^T\alpha \\ z^T\alpha \end{matrix}] = [\begin{matrix} O \\ z^T\alpha \end{matrix}] \geq O.$$

By taking $\alpha' = \frac{\alpha}{z^T\alpha}$,
we have $A^T\alpha' = b$ both has a solution and has no solution.
Therefore, we get a contradition.

For second part, we assume (I) has no solution.
So the

$$Ax^{'} = Bx + 1z \leq 0$$

has no solution for all b.
From Farkas' Lemma,

$$A^T\alpha^{'} = b, \alpha^{'} \geq 0$$

has a solution.
In fact, taking

$$b = \begin{bmatrix} O \\ 1 \end{bmatrix}$$

agein, we have a α satisfying

$$A^T\alpha = \begin{bmatrix} B^T \\ z^T \end{bmatrix}\alpha = \begin{bmatrix} B^T\alpha \\ z^T\alpha \end{bmatrix} = \begin{bmatrix} O \\ z^T\alpha \end{bmatrix} \geq O.$$

Therefore, we have $B^T\alpha = 0$ and $\alpha \geq 0$ for such α.
That is the (II) has a solution.

Q3.

- problem_3 (https://github.com/minori111/NTCU_ML_HW3/blob/master/problem_3.pdf)

Q4.

- problem_4 (https://github.com/minori111/NTCU_ML_HW3/blob/master/problem_4.pdf)