TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

Student ID: ....

EXAMINATIONS — 2011

END-OF-YEAR

COMP 261
ALGORITHMS
and
DATA STRUCTURES

Time Allowed: 3 Hours

Instructions: Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 180 marks.

Non-programmable calculators without a full alphabetic key pad are permitted.

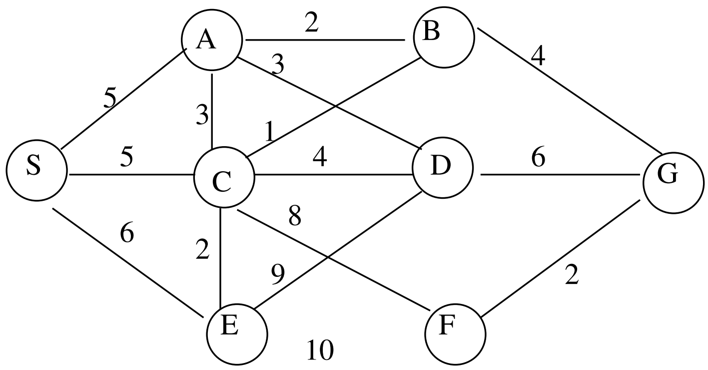
Non-electronic foreign language dictionaries are permitted.

Useful formulas are listed on the last page of the exam.

Questions	**Marks**
1. Shortest Path in Graphs	[15]
2. Minimum Spanning Tree	[13]
3. String Search	[12]
4. Text Processing	[25]
5. Graphics Rendering	[30]
6. File Structures	[33]
7. B-Trees	[32]
8. Hashing	[20]

Question 1. Shortest Path in Graphs [15 marks]

(a) [8 marks] Suppose you are using Dijkstra's algorithm to find the shortest path from **S** to **G** in the graph below. Show the order in which nodes will be *added* to the queue, and the order in which they are *removed* from the queue. In case of a tie, visit the nodes in alphabetic order. When visiting a node, consider the neighbours of the node in alphabetic order.
Hint: Keep track of the queue, along with the priority for each node on the queue.



Nodes Added to Queue:

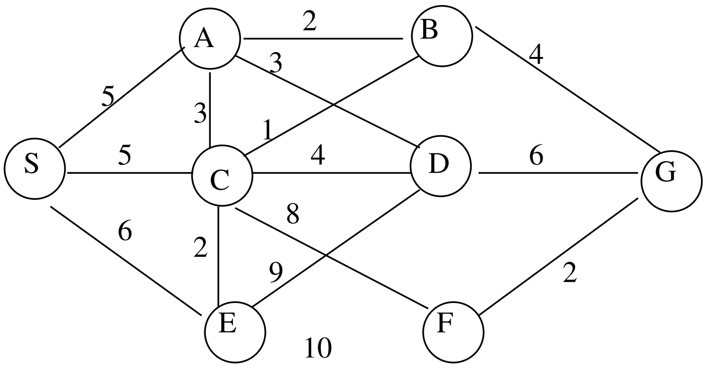
Nodes Removed from Queue:

(b) [7 marks] The A* algorithm for shortest path finding is similar to Dijkstra's algorithm. Explain the key way in which A* differs from Dijkstra's algorithm, and explain why A* is usually better. Give an example to show a special case where A* fails to find the shortest path.

Question 2. Minimum Spanning tree [13 marks]

(a) [8 marks] Suppose you are using Prim's algorithm to find a minimum spanning tree in the graph below, starting from node **S**. Show the order in which *edges* will be added to the tree.

Hint: Keep track of a queue, along with the priority for each edge on the queue. In case of a tie, visit the nodes in alphabetic order. When visiting a node, consider the neighbours of the node in alphabetic order.



Edges added to queue:

Edges added to the tree in this order:

(b) [5 marks] Use an example to explain why Prim's algorithm does not work on a directed graph.

Question 3. String Searching [12 marks]

(a) [7 marks] To search a text for an occurrence of any of a set of words, it is efficient to construct a trie of the words. Draw a trie for the following set of words.
Note: The characters should be attached to the edges in the trie, and all terminal nodes should be indicated clearly.

`cat`	`bat`	`bath`	`bin`	`candy`
`car`	`clean`	`clerk`	`cars`	`hat`

(b) [5 marks] Suppose you are using the KMP algorithm to search for all occurrences of a given string in a text file consisting of 10000 characters with many *a*'s.

Consider each of the following three strings as the input:

(i) `aaaaaaaaab` (ii) `ababcabcde` (iii) `abcdefghij`

Which string is more likely to take the most time? Explain why.

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 4. Text Processing [25 marks]

Consider the following grammar where nonterminals are in uppercase and terminals are enclosed in quotation marks. Assume that tokens will be separated by spaces. Note that nonterminal SMTH uses a regular expression to express the kinds of terminals it accepts.

BAZ ::= `"!"` BAR `"end"` `"world"` | BAR `"end"` `"world"`
BAR ::= `"bca"` FOO | SMTH
FOO ::= SMTH | BAR
SMTH ::= a+b∗c+

(a) [10 marks] For each of the following sentences, state whether it belongs to the language defined by this grammar.

☐	`! bca abc end world`
☐	`bca ac end world`
☐	`! ac end`
☐	`bca ! bca bca abc end world`
☐	`bca ! bca bca abc abc end world`

(b) [5 marks] Can the grammar above be parsed by a predictive, one symbol lookahead, left-to-right (LL(1)) parser? Explain why or why not.

(Question 4 continued on next page)

(Question 4 continued)

(c) Consider a very simple functional programming language defined as follows. An identifier always starts with a character and can be followed by either more characters or digits. An identifier on its own is a variable. An identifier followed by a comma separated list of variables and other function calls is a function call itself. All function calls contains at least one argument.

Here are some sample programs in this language:

```
a

f(g(c(aB,ba),d(a)))
```

(i) [5 marks] Write a grammar for this language that is parseable by an LL(1) (*single character lookahead only*) top down recursive descent parser.

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(Question 4 continued)

(ii) [5 marks] For each of the two examples on the previous page, draw a concrete parse tree derived using your grammar.

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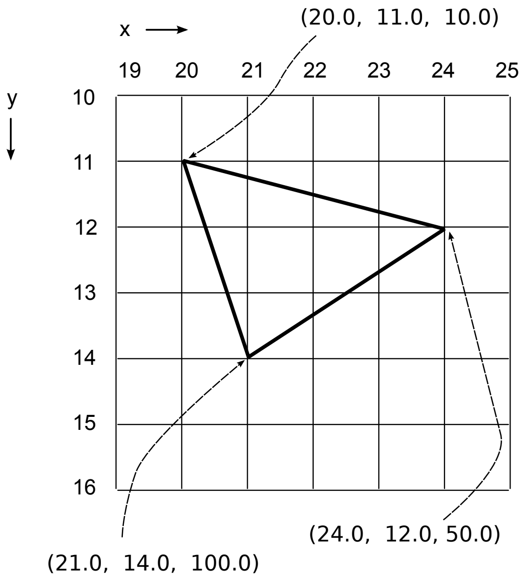
Question 5. Graphics Rendering [30 marks]

(a) [10 marks] Show the values in the edge-lists that would be constructed when rendering the polygon shown below. The (x, y, z) coordinates of the vertices are shown.

(Note that the z values are chosen carefully to make the interpolation easy.)



Edge-Lists

	x_{left}	z_{left}	x_{right}	z_{right}
11				
12				
13				
14				

(b) [5 marks] In constructing and using the edge-lists, you had to convert floating point numbers to integers. Explain how this can introduce errors unless you are careful.

(Question 5 continued on next page)

(Question 5 continued)

(c) [10 marks] List all the steps in the 3D Rendering Algorithm described in the lectures that utilised *both* the Z-Buffer and the Edge Lists and explain the purpose of each step. One sentence or so description for each major step would do.

(Question 5 continued on next page)

(Question 5 continued)

(d) [5 marks] Explain what *affine* transformation means and why they are preferred in computer graphics.

Question 6. File Structures [33 marks]

Suppose a file contains 65, 536 fixed length records describing individual patients. Each record has the following fields:

PatientID: (length = 5 characters),

Name: (length = 30 characters),

Illness: (length = 100 characters),

Prescription: (length = 15 characters).

Assume that the file blocks are stored contiguously and that the block size for the file is 600 characters.

(a) [2 marks] Calculate the record size L in characters. Show your working.

(b) [3 marks] Calculate the blocking factor f and the number of file blocks b. Assume an unspanned file organisation. Show your working.

(c) [5 marks] Calculate the *worst case* number of block accesses needed to perform a binary search for a random record in the file given its PatientID. Assume the file is ordered by PatientID. Show your working.

(Question 6 continued on next page)

(Question 6 continued)

(d) [5 marks] Explain the differences between primary file organisation and secondary file organisation.

(Question 6 continued on next page)

(Question 6 continued)

(e) [10 marks] For each of the following file structures, discuss their advantages and disadvantages. You can do it by:

- explaining the efficiency of the different file operations (insertion, deletion, search, sequential access) with different structures, and
- giving examples of when it is appropriate to use each kind of file.

(i) Heap file:

(ii) Sequential file.

(iii) Hash file.

(Question 6 continued on next page)

(Question 6 continued)

(f) [4 marks] Describe in detail (with pictures) the sort and merge stages involved in the Sort-Merge algorithm.

(g) [4 marks] What problems would arise if you try to do it "in place"?

Question 7. B-Trees [32 marks]

(a) [10 marks] Draw an example of a 2-3 Tree with at least 12 integer values in it, *and* state and explain the 2-3 Tree properties that make them distinct from binary trees or general trees.

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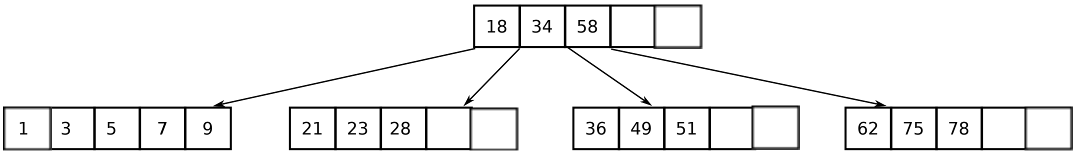
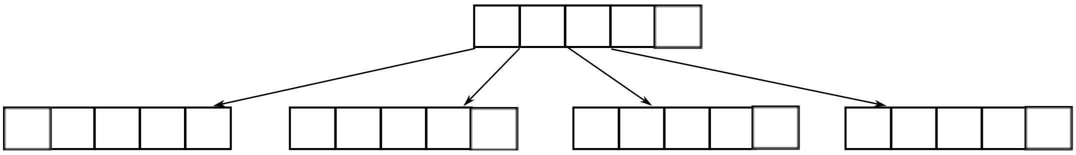
(Question 7 continued)

(b) [12 marks] Consider the B-tree of order 7 illustrated below.

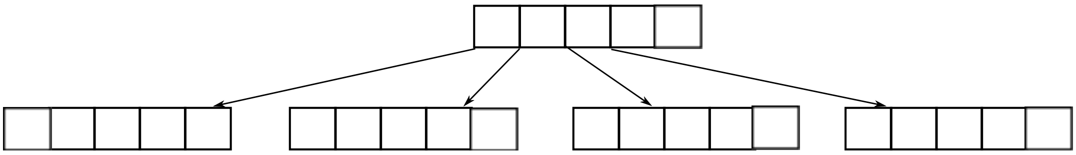


Update the B-tree by successively deleting the key values 62, 34, 21, 18. In your answer, show the B-tree after each deletion and briefly describe what you have done.
Note, the empty trees below are to save you time; you may modify their structure if you choose.

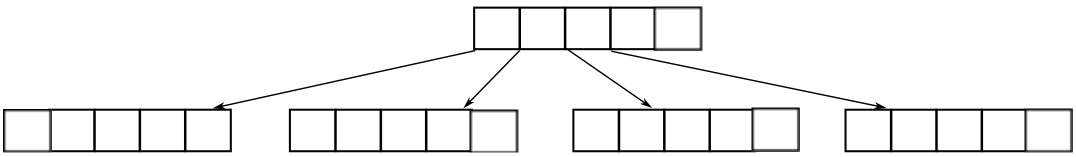
The B-tree after deleting key value 62:



The B-tree after deleting key values 62 and 34:



The B-tree after deleting key values 62, 34, and 21:



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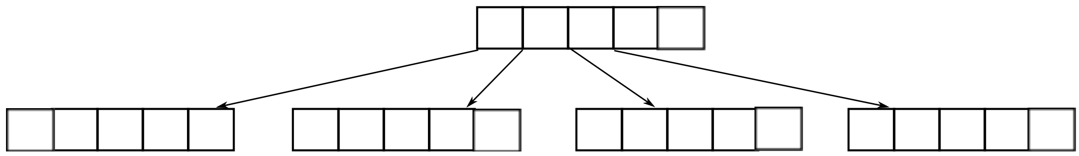
(Question 7 continued)

The *B*-tree after deleting key values 62, 34, 21, and 18:



(Question 7 continued on next page)

(Question 7 continued)

(c) [10 marks] Imagine a *B*-tree that has order 5 and starts only a root node. Assume that to begin with the root node contains the values: 5, 10, and 15 (the root node doesn't have any children to begin with). Draw a *B*-tree after the following values are inserted in this order: 1, 2, 3, 4.

The *B*-tree after adding key value 1:

The *B*-tree after adding key values 1 and 2:

The *B*-tree after adding key values 1, 2, and 3:

The *B*-tree after adding key value 1, 2, 3, and 4:

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 8. Hashing [20 marks]

(a) [5 marks] What is a *secondary index* and what can it be used for?

(b) [5 marks] What is the difference between static hashing and dynamic hashing?

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(Question 8 continued)

(c) [10 marks] Describe how extendible hash files work.

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Useful Formulas

You may tear off this page if you wish. You do not need to hand it in.

File Performance Formulas

- blocking factor: $f = \lfloor \frac{B}{L} \rfloor$
- number of blocks: $b = \left\lceil \frac{r}{f} \right\rceil$
- external sort-merge: $N = 2b \cdot (1 + \lceil (\log_{n-1} b) - 1 \rceil) = 2b \cdot (1 + \left\lceil (\frac{\log_{10} b}{\log_{10}(n-1)} - 1 \right\rceil$
  (where n is the number of buffers)

B-tree (worst case)

- height: $h = 1 + \lfloor \log_{m+1} \frac{r+1}{2} \rfloor = 1 + \left\lfloor \log_{10} \frac{\frac{r+1}{2}}{\log_{10}(m+1)} \right\rfloor$
- number of leaves: $N_{leaves} = 2(m+1)^{h-2} \leq N_{leaves} \leq (2m+1)^{h-1}$

B^+-tree (worst case)

- height: $h = 2 + \lfloor \log_{m+1} \frac{r}{2m} \rfloor = 2 + \left\lfloor \frac{\log_2 \frac{r}{2m}}{\log_2(m+1)} \right\rfloor$
- number of leaves: $N_{leaves} = \lceil \frac{r}{m} \rceil$

Index-Sequential File with a B-tree

- number of sequence sets: $s = \left\lceil \frac{r}{f} \right\rceil \leq s \leq \left\lceil \frac{2r}{f} \right\rceil$

Logs to base 2

n	1	2	4	8	16	32	64	128	256	512	1,024	4096	16384	65536	1,048,576
$\log_2 n$	0	1	2	3	4	5	6	7	8	9	10	12	14	16	20

Logs to base 10

n	5	10	50	100	500	1000	5000	10,000	10^6	5×10^6	10×10^6	50×10^6
$\log_{10} n$	0.7	1	1.7	2	2.7	3	3.7	4	6	6.7	7	7.7