TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

Student ID: . . . .

EXAMINATIONS — 2011

END-OF-YEAR

COMP 261
ALGORITHMS
and
DATA STRUCTURES

Time Allowed: 3 Hours ******** WITH SOLUTIONS **********

Instructions: Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 180 marks.

Non-programmable calculators without a full alphabetic key pad are permitted.

Non-electronic foreign language dictionaries are permitted.

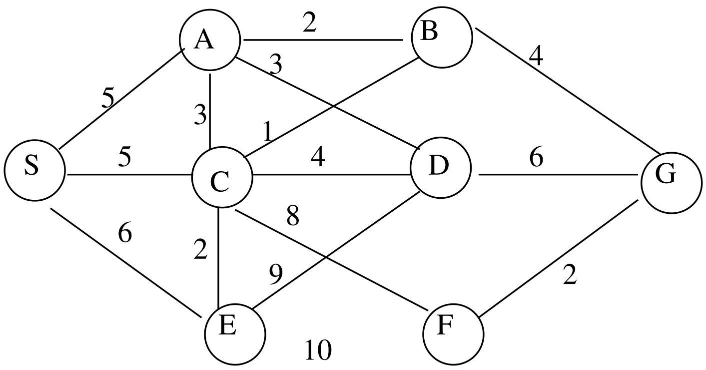
Useful formulas are listed on the last page of the exam.

Questions	Marks
1. Shortest Path in Graphs	[15]
2. Minimum Spanning Tree	[13]
3. String Search	[12]
4. Text Processing	[25]
5. Graphics Rendering	[30]
6. File Structures	[33]
7. B-Trees	[32]
8. Hashing	[20]

ANSWERS

Question 1. Shortest Path in Graphs [15 marks]

(a) [8 marks] Suppose you are using Dijkstra's algorithm to find the shortest path from **S** to **G** in the graph below. Show the order in which nodes will be *added* to the queue, and the order in which they are *removed* from the queue. In case of a tie, visit the nodes in alphabetic order. When visiting a node, consider the neighbours of the node in alphabetic order.
Hint: Keep track of the queue, along with the priority for each node on the queue.



Nodes Added to Queue: [shown as Node/priority]
[initialise:] S/0
[from S:] A/5, C/5, E/6
[from A/5:] B/7, D/8, C/8
[from C/5:] B/6, D/9, F/13
[from B/6:] G/10
[from E/6:] D/15
[from D/8:] G/14
note this assumes a simple queue that does not update items already in the queue.

Nodes Removed from Queue:
S/0, A/5, C/5, B/6, E/6, (B/7), D/8, (C/8), (D/9), G/10
(brackets round nodes that had already been visited)

(b) [7 marks] The A* algorithm for shortest path finding is similar to Dijkstra's algorithm. Explain the key way in which A* differs from Dijkstra's algorithm, and explain why A* is usually better. Give an example to show a special case where A* fails to find the shortest path.

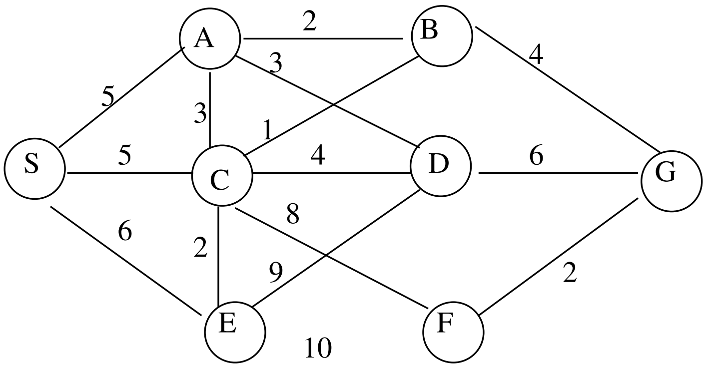
heuristics non-admissible, non-consistent heuristics

ANSWERS

Question 2. Minimum Spanning tree [13 marks]

(a) [8 marks] Suppose you are using Prim's algorithm to find a minimum spanning tree in the graph below, starting from node **S**. Show the order in which *edges* will be added to the tree.

Hint: Keep track of a queue, along with the priority for each edge on the queue. In case of a tie, visit the nodes in alphabetic order. When visiting a node, consider the neighbours of the node in alphabetic order.



Edges added to queue: [shown as Edge/priority]
[initialise:] S/0
[from S:] A/5, C/5, E/6
[from A/5:] B/2, C/3, D/3
[from B/2:] C/1, G/4
[from C/1:] E/2, D/4, F/8
[from E/2:] D/9
[from D/3:] G/6
[from G/4:] F/2
note this assumes a simple queue that does not update items already in the queue.

Edges added to the tree in this order:
SA, AB, BC, CE, AD, BG, GF

(b) [5 marks] Use an example to explain why Prim's algorithm does not work on a directed graph.

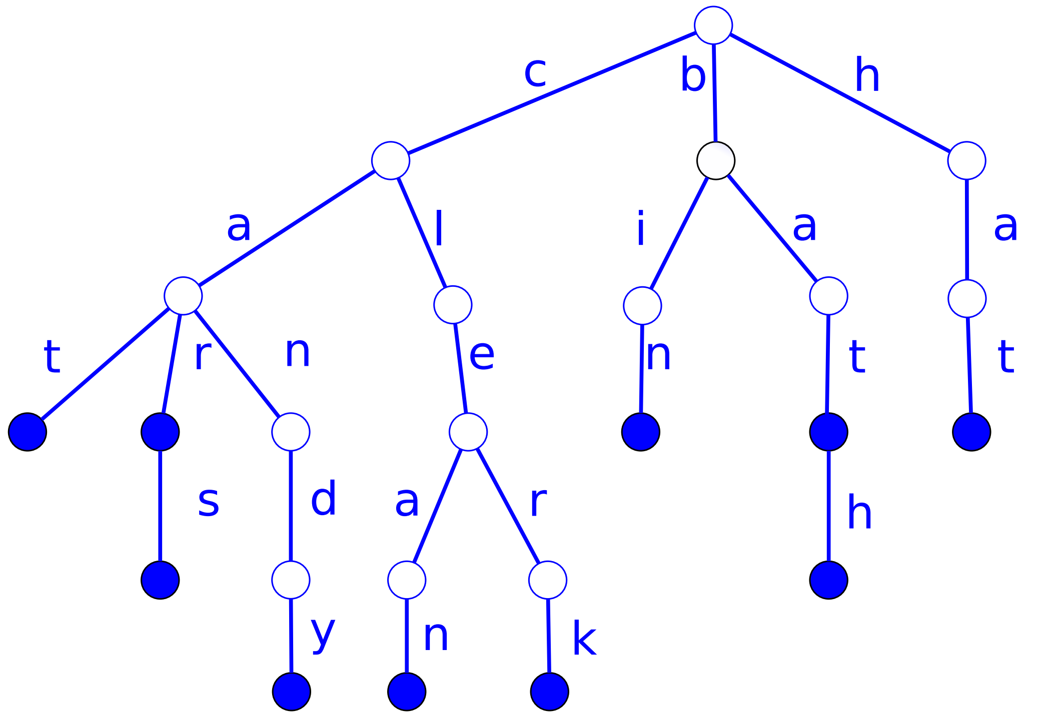
easiest case would be a circular graph with one short edge link back to the start.

ANSWERS

Question 3. String Searching [12 marks]

(a) [7 marks] To search a text for an occurrence of any of a set of words, it is efficient to construct a trie of the words. Draw a trie for the following set of words.
Note: The characters should be attached to the edges in the trie, and all terminal nodes should be indicated clearly.

```
cat     bat     bath    bin     candy
car     clean   clerk   cars    hat
```



(b) [5 marks] Suppose you are using the KMP algorithm to search for all occurrences of a given string in a text file consisting of 10000 characters with many *a*'s.

Consider each of the following three strings as the input:

(i) `aaaaaaaaab` (ii) `ababcabcde` (iii) `abcdefghij`

Which string is more likely to take the most time? Explain why.

aaaaaaaab is likely to take the most time, by a small amount. Every time there is a mismatch between a character in the text and a character in the string, KMP moves the string along and tries matching the character in the text with the new character in the string. If there are The more times it has to move the string along

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

ANSWERS

Question 4. Text Processing [25 marks]

Consider the following grammar where nonterminals are in uppercase and terminals are enclosed in quotation marks. Assume that tokens will be separated by spaces. Note that nonterminal SMTH uses a regular expression to express the kinds of terminals it accepts.

BAZ ::= `"!"` BAR `"end"` `"world"` | BAR `"end"` `"world"`
BAR ::= `"bca"` FOO | SMTH
FOO ::= SMTH | BAR
SMTH ::= a+b*c+

(a) [10 marks] For each of the following sentences, state whether it belongs to the language defined by this grammar.

yes	`! bca abc end world`
yes	`bca ac end world`
no	`! ac end`
no	`bca ! bca bca abc end world`
no	`bca ! bca bca abc abc end world`

(b) [5 marks] Can the grammar above be parsed by a predictive, one symbol lookahead, left-to-right (LL(1)) parser? Explain why or why not.

??

(Question 4 continued on next page)

(Question 4 continued)

(c) Consider a very simple functional programming language defined as follows. An identifier always starts with a character and can be followed by either more characters or digits. An identifier on its own is a variable. An identifier followed by a comma separated list of variables and other function calls is a function call itself. All function calls contains at least one argument.

Here are some sample programs in this language:

```
a

f(g(c(aB,ba),d(a)))
```

(i) [5 marks] Write a grammar for this language that is parseable by an LL(1) (*single character lookahead only*) top down recursive descent parser.

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(Question 4 continued)

(ii) [5 marks] For each of the two examples on the previous page, draw a concrete parse tree derived using your grammar.

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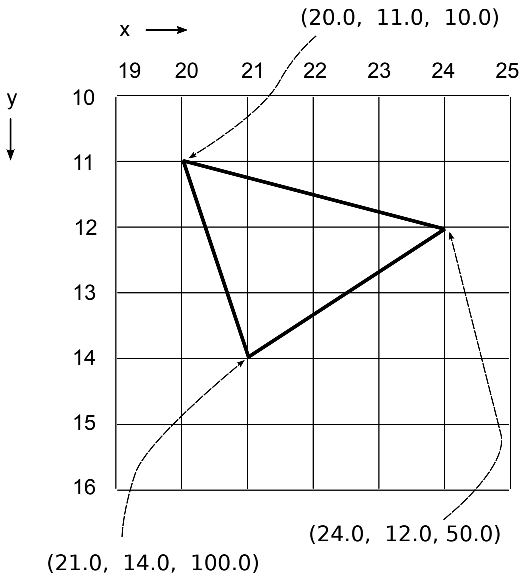
ANSWERS

Question 5. Graphics Rendering [30 marks]

(a) [10 marks] Show the values in the edge-lists that would be constructed when rendering the polygon shown below. The (x, y, z) coordinates of the vertices are shown.

(Note that the z values are chosen carefully to make the interpolation easy.)



Edge-Lists

	x_{left}	z_{left}	x_{right}	z_{right}
11				
12				
13				
14				

(b) [5 marks] In constructing and using the edge-lists, you had to convert floating point numbers to integers. Explain how this can introduce errors unless you are careful.

The interpolation process can introduce very small errors. This can mean that two numbers that are extremely close (eg 12.999999 and 13.00001, or 22.49999 and 22.500001) may be converted to different integers, producing "holes" or artifacts in the images, where a pixel is missed out or an extra pixel is added. It doesn't matter whether you round numbers to the closest integer, or use floor and ceil to round up or round down, two numbers just either side of the decision value can be converted to different integers.

(Question 5 continued on next page)

(Question 5 continued)

(c) [10 marks] List all the steps in the 3D Rendering Algorithm described in the lectures that utilised *both* the Z-Buffer and the Edge Lists and explain the purpose of each step. One sentence or so description for each major step would do.

TODO.

(Question 5 continued on next page)

(Question 5 continued)

(d) [5 marks] Explain what *affine* transformation means and why they are preferred in computer graphics.

TODO.

ANSWERS

Question 6. File Structures [33 marks]

Suppose a file contains $65,536$ fixed length records describing individual patients. Each record has the following fields:

PatientID: (length = 5 characters),

Name: (length = 30 characters),

Illness: (length = 100 characters),

Prescription: (length = 15 characters).

Assume that the file blocks are stored contiguously and that the block size for the file is 600 characters.

(a) [2 marks] Calculate the record size L in characters. Show your working.

record has (5 + 30 +100 + 15) = 150 characters,
1 byte per character.
therefore 150 bytes

(b) [3 marks] Calculate the blocking factor f and the number of file blocks b. Assume an unspanned file organisation. Show your working.

Blocking factor = $\lfloor 600/150 \rfloor = 4$
Number of blocks = $65,536/4 = 16,384$ blocks

(c) [5 marks] Calculate the *worst case* number of block accesses needed to perform a binary search for a random record in the file given its PatientID. Assume the file is ordered by PatientID. Show your working.

May have to look at $\lfloor \log_2(65,536) \rfloor + 1 = 17$ records.
Assume that every record we look at is in a different block from the previous one, and that we don't remember earlier blocks, then need 17 blocks.
Note that it is not possible for the last three records you look at to be in more than two different blocks.

(Question 6 continued on next page)

(Question 6 continued)

(d) [5 marks] Explain the differences between primary file organisation and secondary file organisation.

Primary file organisation is the way the records are organised in the file on the disk. Secondary file organisation is the additional indexing structures that enable fast access to items in the file, but is not part of the file itself.

(Question 6 continued on next page)

(Question 6 continued)

(e) [10 marks] For each of the following file structures, discuss their advantages and disadvantages. You can do it by:

- explaining the efficiency of the different file operations (insertion, deletion, search, sequential access) with different structures, and
- giving examples of when it is appropriate to use each kind of file.

(i) Heap file:

Heap files are very fast to insert into, but slow to search, and therefore delete from. They are also slow for sequential access. They are only good for constructing files that are not read often, or along the way to constructing a sequential file (constructing the file and then sorting it).

(ii) Sequential file.

Sequential files are slow to insert into and slow to delete from, but are fast to search and for sequential access. They are good for files that will be read very frequently, but only infrequently modified.

(iii) Hash file.

Hash files are fast to search, to insert into, and to delete from, but they require additional disk memory. They are slow for sequential access. As long as disk space is not very tight, they are the best choice for random access files, as long as sequential access is not required.

(Question 6 continued on next page)

(Question 6 continued)

(f) [4 marks] Describe in detail (with pictures) the sort and merge stages involved in the Sort-Merge algorithm.

(g) [4 marks] What problems would arise if you try to do it "in place"?

ANSWERS

Question 7. B-Trees [32 marks]

(a) [10 marks] Draw an example of a 2-3 Tree with at least 12 integer values in it, *and* state and explain the 2-3 Tree properties that make them distinct from binary trees or general trees.

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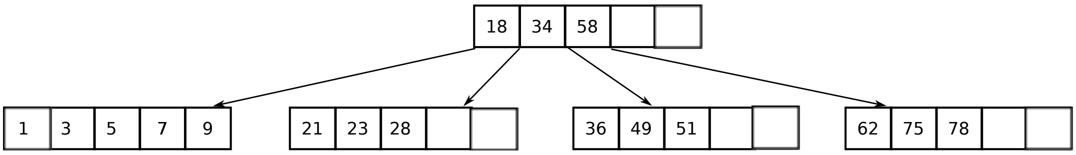
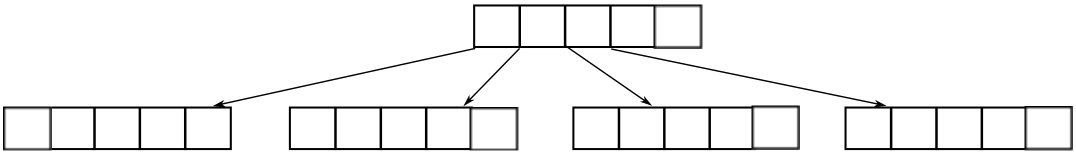
(Question 7 continued)

(b) [12 marks] Consider the *B*-tree of order 7 illustrated below.

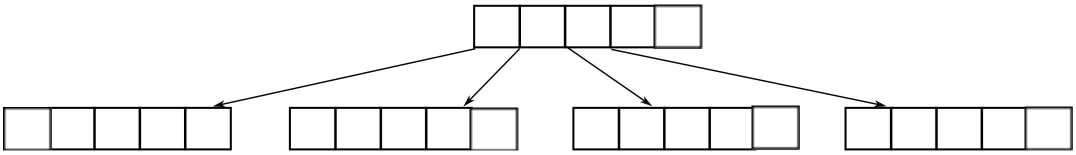


Update the *B*-tree by successively deleting the key values 62, 34, 21, 18. In your answer, show the *B*-tree after each deletion and briefly describe what you have done.
Note, the empty trees below are to save you time; you may modify their structure if you choose.

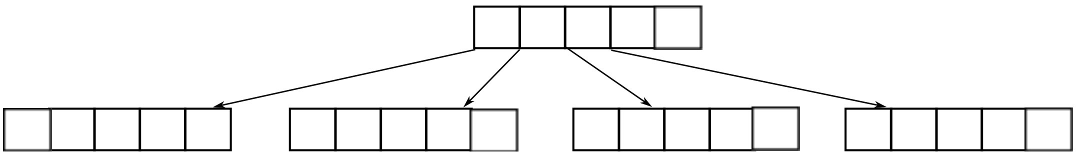
The *B*-tree after deleting key value 62:



The *B*-tree after deleting key values 62 and 34:



The *B*-tree after deleting key values 62, 34, and 21:



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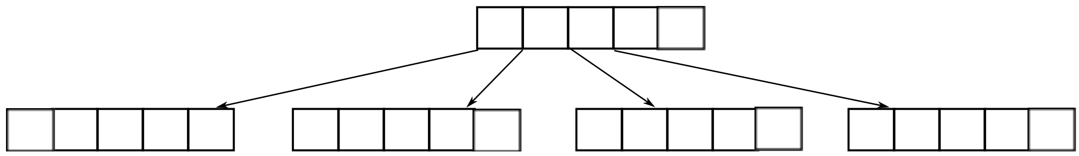
(Question 7 continued)

The *B*-tree after deleting key values 62, 34, 21, and 18:



(Question 7 continued on next page)

(Question 7 continued)

(c) [10 marks] Imagine a *B*-tree that has order 5 and starts only a root node. Assume that to begin with the root node contains the values: 5, 10, and 15 (the root node doesn't have any children to begin with). Draw a *B*-tree after the following values are inserted in this order: 1, 2, 3, 4.

The *B*-tree after adding key value 1:

The *B*-tree after adding key values 1 and 2:

The *B*-tree after adding key values 1, 2, and 3:

The *B*-tree after adding key value 1, 2, 3, and 4:

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

ANSWERS

Question 8. Hashing [20 marks]

(a) [5 marks] What is a *secondary index* and what can it be used for?

•

(b) [5 marks] What is the difference between static hashing and dynamic hashing?

•

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(Question 8 continued)

(c) [10 marks] Describe how extendible hash files work.

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Useful Formulas

You may tear off this page if you wish. You do not need to hand it in.

File Performance Formulas

- blocking factor: $f = \lfloor \frac{B}{L} \rfloor$
- number of blocks: $b = \left\lceil \frac{r}{f} \right\rceil$
- external sort-merge: $N = 2b \cdot (1 + \lceil (\log_{n-1} b) - 1 \rceil) = 2b \cdot (1 + \left\lceil (\frac{\log_{10} b}{\log_{10}(n-1)} - 1 \right\rceil$
  (where n is the number of buffers)

B-tree (worst case)

- height: $h = 1 + \lfloor \log_{m+1} \frac{r+1}{2} \rfloor = 1 + \left\lfloor \log_{10} \frac{\frac{r+1}{2}}{\log_{10}(m+1)} \right\rfloor$
- number of leaves: $N_{leaves} = 2(m+1)^{h-2} \leq N_{leaves} \leq (2m+1)^{h-1}$

B^+-tree (worst case)

- height: $h = 2 + \lfloor \log_{m+1} \frac{r}{2m} \rfloor = 2 + \left\lfloor \frac{\log_2 \frac{r}{2m}}{\log_2(m+1)} \right\rfloor$
- number of leaves: $N_{leaves} = \lceil \frac{r}{m} \rceil$

Index-Sequential File with a B-tree

- number of sequence sets: $s = \left\lceil \frac{r}{f} \right\rceil \leq s \leq \left\lceil \frac{2r}{f} \right\rceil$

Logs to base 2

n	1	2	4	8	16	32	64	128	256	512	1,024	4096	16384	65536	1,048,576
$\log_2 n$	0	1	2	3	4	5	6	7	8	9	10	12	14	16	20

Logs to base 10

n	5	10	50	100	500	1000	5000	10,000	10^6	5×10^6	10×10^6	50×10^6
$\log_{10} n$	0.7	1	1.7	2	2.7	3	3.7	4	6	6.7	7	7.7