TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS – 2016

TRIMESTER 1

COMP 261
ALGORITHMS
and
DATA STRUCTURES

Time Allowed: TWO HOURS

CLOSED BOOK

Permitted materials: Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination.

Instructions: Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 120 marks.

Non-electronic foreign to English language dictionaries are permitted.

Alphabetic order: A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Questions	Marks
1. MST	[5]
2. Articulation Points	[20]
3. Parsing	[20]
4. Tries and String Search	[25]
5. B+ Trees	[15]
6. Compression	[15]
7. Flood Fill	[20]

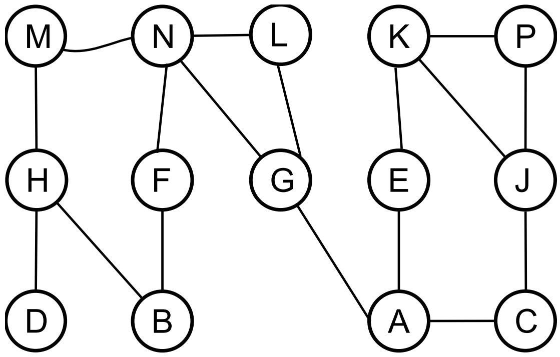
Question 1. MST [5 marks]

(a) [2 marks] Give an example of a smallest graph such that Kruskal's algorithm and Prim's algorithm will add edges to the MST in a different order (but the final trees may be the same). Show the order in which the trees are constructed.

(b) [3 marks] Give an example of a smallest graph such that the two MSTs resulting from the two algorithms are different. Show the order in which the trees are constructed.

Question 2. Articulation Points [20 marks]

(a) [4 marks] Circle the articulation points in the following graph.



(b) [6 marks] Give two examples of applications or problems where the Articulation Points algorithm would be useful that are not covered by assignment 2 (i.e. identifying critical intersections in a city).

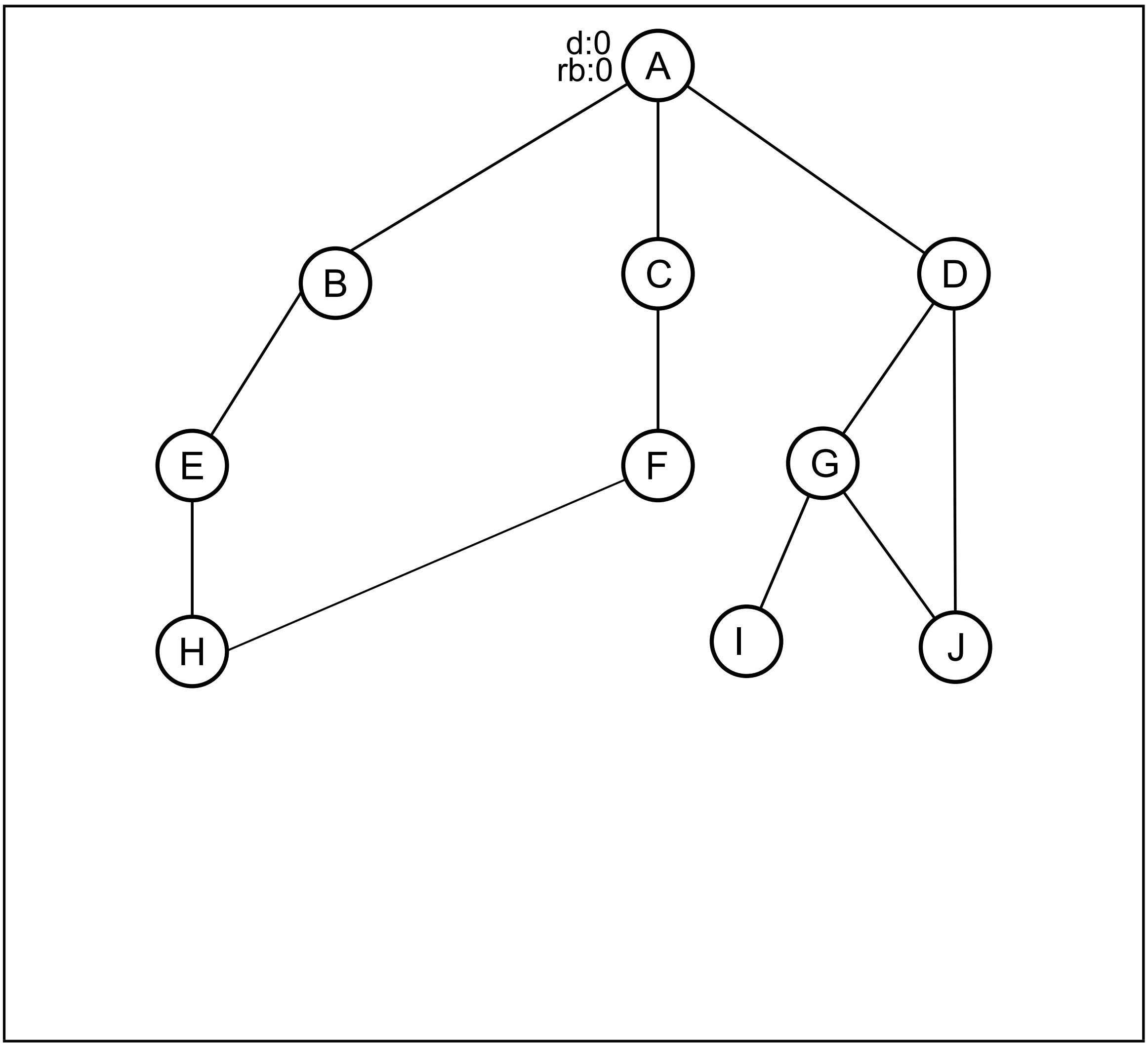
(Question 2 continued on next page)

(Question 2 continued)

(c) [10 marks] Show how the recursive depth-first-search articulation points algorithm would find the articulation points in the following graph, assuming that the search starts with node A and considers neighbours of nodes in alphabetical order. Mark on the graph

- the depth ("d:") and the reach-back ("rb:") of each node (node A is done for you),
- the return values from each recursive call,
- the nodes that are articulation points, indicating why they were marked.

The algorithm is given on the facing page for your reference.



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(Question 2 continued)

```
FindArticulationPoints (graph, start ):
    for each node: node.depth ← ∞, articulationPoints ← { }
    start .depth ← 0, numSubtrees ← 0
    for each neighbour of start
        if neighbour.depth = ∞ then
            RecursiveArtPts( neighbour, 1, start )
            numSubtrees ++
    if numSubtrees > 1 then add start to articulationPoints

RecursiveArtPts(node, depth, fromNode):
    node.depth ← depth, reachBack ← depth,
    for each neighbour of node other than fromNode
        if neighbour.depth < ∞ then
            reachBack ← min(neighbour.depth, reachBack)
        else
            childReach ← RecursiveArtPts(neighbour, depth +1, node)
            reachBack ← min(childReach, reachBack )
            if childReach ≥ depth then add node to articulationPoints
    return reachBack
```

Question 3. Parsing [20 marks]

(a) [10 marks] Consider the following grammar, where terminals are always enclosed in quotation marks and nonterminals are always in capitals. Assume each nonterminal and terminal is separated with a space to form separate tokens for simplicity:

```
FOO  ::= BAR FEND
FEND ::= "start" BAZ | BAZ "end" | "mid" BAZ
BAR  ::= [d-z0-9]+ | BAZ
BAZ  ::= a*b*c+
```

In the following list put a tick next to sentences that belong to the language defined by this grammar and a cross next to sentences that don't:

i. abc start c

ii. ccc ccc end

iii. ac start aaabbb

iv. dddd09 mid c

v. d mid ab

(Question 3 continued on next page)

(Question 3 continued)

(b) [10 marks] Write a parser that returns `true` or `false` and follows the grammar below. Assume each nonterminal and terminal is separated with a space to form separate tokens for simplicity:

QUERY ::= "SELECT" "*" ["FROM" NAME] ["WHERE" NAME "=" DATA] ";"
NAME ::= [A–Za–z]+
DATA ::= [A–za–z0–9]*

public boolean parseQuery(Scanner s) {

Question 4. Tries and String Search [25 marks]

(a) [4 marks] Draw the Trie containing the following set of strings.
(Note: label the links of the Trie, not the nodes.)

`baby`	`bottle`	`but`	`butt`	`button`	`body`
`bait`	`busy`	`bus`	`apple`	`aps`	

(Question 4 continued on next page)

(Question 4 continued)

(b) [6 marks] Give an example of a problem where using a Trie would be a more efficient implementation of a set of strings than a HashSet, and explain why the Trie would be more efficient.

(Question 4 continued on next page)

(Question 4 continued)

(c) [8 marks] Show the table that the following KMP table building algorithm would construct given the string "`fgrkffgfgren`":

```
computeKMPTable(string)
    initialise  table to an array of integers, same length as string
    table[0] ← −1
    table[1] ← 0;
    pos ← 2;
    j ← 0;
    while pos < string.length
        if string[pos−1] = string[j]
            table[pos] ← j+1
            pos++
            j++
        else if  j > 0
            j ← table[j]
        else
            table[pos] ← 0
            pos++
    return table
```

string:	f	g	r	k	f	f	g	f	g	r	e	n
table:												
index:	0	1	2	3	4	5	6	7	8	9	10	11

(Question 4 continued on next page)

(Question 4 continued)

(d) [4 marks] Explain, using an example, how the table built in part (c) would help you find the string in a large body of text? What string would you be searching for?

(e) [3 marks] What improvement over the brute force algorithm would KMP be able to make because of this table?

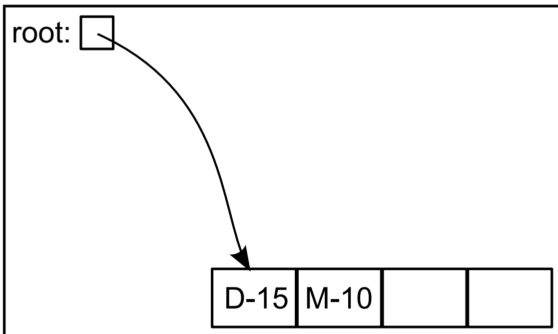
Question 5. B+ Trees [15 marks]

The following two subquestions concern a B+ tree that has internal nodes holding up to 3 keys, and leaf nodes holding up to 4 key-value pairs. The keys are letters; the values are numbers.

(a) [5 marks] Show how the B+ tree below would be changed if the following key-value pairs were added to it:

```
H-5  F-7 K-22 N-12
```

State any assumptions you make.



Assumptions (if any):

Alphabet: `A B C D E F G H I J K L M N O P Q R S T U V W X Y Z`

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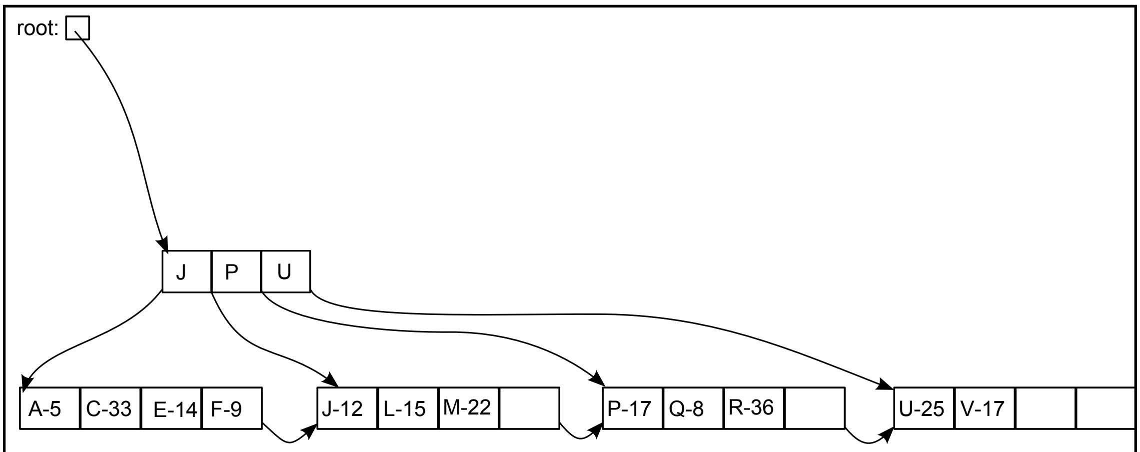
(Question 5 continued)

(b) [5 marks] Show how the B+ tree below would be changed if the following key-value pair was added to it:

    D-1

State any assumptions you make.



Assumptions (if any):

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(Question 5 continued)

(c) [5 marks] Suppose the internal nodes of a B+ tree have at most 6 children (*i.e.*, 5 keys), and the leaves contain at most 4 key-value pairs. The maximum number of key-value pairs that can be in a tree is 4×6^h where the height of the tree is h. What is the minimum number of key-value pairs that can be in tree of height h? Assume that $h \geq 1$. Show your working! Note: the height of a tree is the number of edges (= number of internal nodes) on a longest path from the root to a leaf.

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SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 6. Compression [15 marks]

(a) [5 marks] Construct a trie that can be used to encode the following sentence using Huffman Encoding. (Note: label the links of the Trie, not the nodes.)

`WHERE WHERE DID WHY WHAT WHEN GO`

(Question 6 continued on next page)

(Question 6 continued)

(b) [5 marks] Show the resulting encoding and state how many bits will be required to store the message from part (a):

`WHERE WHERE DID WHY WHAT WHEN GO`

(Question 6 continued on next page)

(Question 6 continued)

(c) [5 marks] What is the key idea of the Lempel-Ziv 77 compression algorithm? Please explain using an example.

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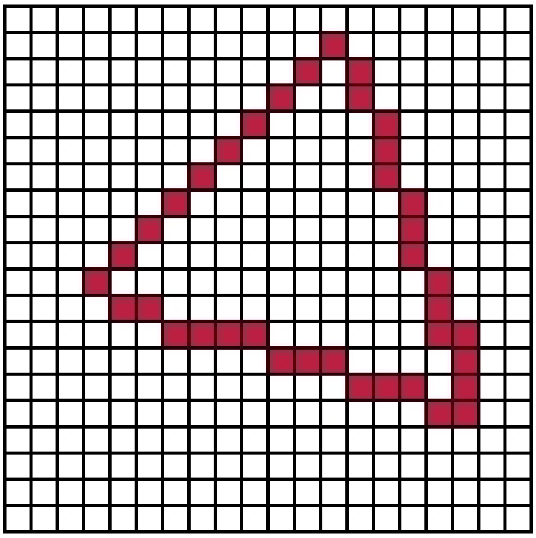
Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 7. Flood fill

[20 marks]

(a) [10 marks] You are given a black and white image as a matrix. The matrix is filled with boolean values (0 - white, 1 - black) that correspond to a drawing of triangle edges. Outline an algorithm (pseudo code) to fill the triangle with ones.



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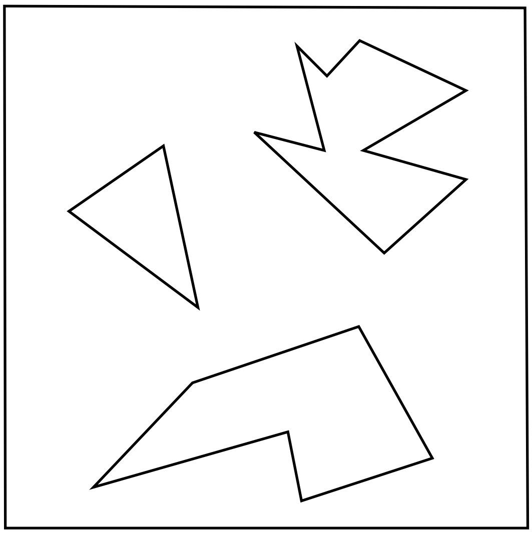
(Question 7 continued)

(b) [10 marks] You are given an image as before, but now with more than one shape which can be arbitrary polygons. You are given the location (row and column) of a point inside a shape S. Outline an algorithm (pseudo code) to fill S (and only S).

Hint: A matrix can be viewed as a graph, where each cell is connected to 4 neighbours.



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