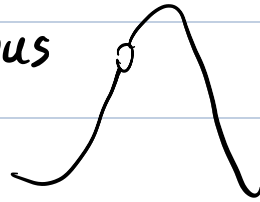
Discrete Structures.

not continuous

gaps in the curves.

gaps in the array.



Propositions: elementary or atomic props.

Compound logical props: statement operators statement

A proposition is a statement that is either true or false.

Propositions are typically declarative sentences. For example, the following are *not* propositions.
命题通常是陈述句。例如，以下是不是命题。

Table 1.1.2: English sentences that are not propositions.
表 1.1.2：非命题英语句子

Sentence 句子	Comment 评论
What time is it? 现在几点了?	A question, not a proposition. A question is neither true nor false. 是问题，不是命题。问题非真非假。
Are you awake? 你醒着吗?	Even a yes/no question is neither true nor false, so is not a proposition. 即使是 "是"/"否 "的问题也非真非假，因此不是命题。
Have a nice day. 祝您愉快	A command, not a proposition. A command is neither true nor false. 是命令，不是命题。命令非真非假。

A proposition's ***truth value*** is a value indicating whether the proposition is actually true or false. A proposition is still a proposition whether its truth value is known to be true, known to be false, unknown, or a matter of opinion. The following are all propositions.
一个命题的***真值***是一个表示该命题实际上是真还是假的值。无论命题的真值是已知为真、已知为假、未知还是见仁见智，它仍然是一个命题。以下都是命题。

Table 1.1.3: Examples of propositions and their truth values.
表 1.1.3：命题举例及其真值。

Proposition 提案	Comment 评论
Two plus two is four. 二加二等于四。	Truth value is true. 真理值是真实的。
Two plus two is five. 二加二等于五。	Truth value is false. 真值是假的。
Monday will be cloudy. 周一多云。	Truth value is unknown. 真相价值未知。
The movie was funny. 电影很有趣。	Truth value is a matter of opinion. 真理的价值见仁见智。

3) All politicians are dishonest.

- (●) Proposition
- ( ) Not a proposition

Correct

The statement can be given a truth value, though that value is a matter of opinion.

binary operaters { Conjunction ∧, and, 只要有一个是T

disconjunction. ∨, or, inclusive or ∨

exclusive or ⊕

只有一个是T，另一个是F

unitary operaters { Negation ¬, Not,

合取运算 ∧

Conjunction operation

①

The proposition p ∧ q is read "p and q" and is called the conjunction of p and q. p ∧ q is true if both p is true and q is true. p ∧ q is false if p is false, q is false, or both are false

The conjunction operation

Propositional variables such as p, q, and r can be used to denote arbitrary propositions, as in:

p: January has 31 days.
q: February has 33 days.

A ***compound proposition*** is created by connecting individual propositions with logical operations. A ***logical operation*** combines propositions using a particular composition rule. For example, the conjunction operation is denoted by ∧. The proposition p ∧ q is read "p and q" and is called the ***conjunction*** of p and q. p ∧ q is true if both p is true and q is true. p ∧ q is false if p is false, q is false, or both are false.

Using the definitions for p ∧ q given above, the proposition p ∧ q is expressed in English as:

p ∧ q: January has 31 days and February has 33 days.

Proposition p's truth value is true — January does have 31 days. Proposition q's truth value is false — February does not have 33 days. The compound proposition p ∧ q is therefore false, because it is not the case that both propositions are true.

A ***truth table*** shows the truth value of a compound proposition for every possible combination of truth values for the variables contained in the compound proposition.
Every row in the truth table shows a particular truth value for each variable, along with the compound proposition's corresponding truth value. Below is the truth table for p ∧ q, where ***T*** represents true and ***F*** represents false.

Different ways to express a conjunction in English

Define the propositional variables p and h as:

p: The sauce looks disgusting.
h: The sauce tastes delicious.

There are many ways to express the proposition p ∧ h in English. The sentences below have slightly different meanings in English but correspond to the same logical meaning.

Table 1.1.4: Examples of different ways to express a conjunction in English.

p and h	The sauce looks disgusting and tastes delicious.
p, but h	The sauce looks disgusting, but tastes delicious.
Despite the fact that p, h	Despite the fact that the sauce looks disgusting, it tastes delicious.
Although p, h	Although the sauce looks disgusting, it tastes delicious.

Disjunction operation ∨ 2

The disjunction operation

析取运算

The disjunction operation is denoted by ∨. The proposition p ∨ q is read "p or q", and is called the ***disjunction*** of p and q. p ∨ q is true if either one of p or q is true, or if both are true. The proposition p ∨ q is false only if both p and q are false. Using the same p and q from the example above, p ∨ q is the statement:

析取运算用 ∨ 表示。命題 p ∨ q 讀作 "p 或 q"，稱為 p 與 q 的 ***析取運算***。只有当 p 和 q 都为假时，命题 p ∨ q 才为假。使用上面例子中的 p 和 q，p ∨ q 即为命题：

p ∨ q: January has 31 days or February has 33 days.

p ∨ q：一月有 31 天或二月有 33 天。

The exclusive or operation is usually denoted with the symbol ⊕. The proposition p ⊕ q is true if exactly one of the propositions p and q is true but not both. This question asks you to fill in the truth table for p ⊕ q.

排他或运算通常用符号 ⊕ 表示。如果命题 p 和 q 恰好有一个为真，而不是两个都为真，则命题 p ⊕ q 为真。本题要求你填写 p ⊕ q 的真值表。

Since the inclusive or is most common in logic, it is just called "or" for short.

Negation operation ¬ 3

The ***negation*** operation acts on just one proposition and has the effect of reversing the truth value of the proposition. The negation of proposition p is denoted ¬p and is read as "not p".

否定操作只作用于一个命题，其效果是颠倒命题的真值。命题 p 的否定用 ¬p 表示，读作 "不是 p"。

Since the negation operation only acts on a single proposition, its truth table only has two rows for the proposition's two possible truth values.

由于否定运算只作用于一个命题，因此它的真值表只有两行，分别表示命题的两个可能真值。

p	¬p
T	F
F	T

¬p is simply the opposite of p.

p 就是 p 的反义词。

1.2

The order in which the operations are applied in a compound proposition such as $p \vee \neg q \wedge r$ may affect the truth value of the proposition. In the absence of parentheses, the rule is that negation is applied first, then conjunction, then disjunction:

顺序：$\neg \to \wedge \to \vee$ Nao

the proposition $\neg p \vee q$ is evaluated as $(\neg p) \vee q$ instead of $\neg(p \vee q)$.

Truth table

1.2.3: How to fill in a truth table.

1 2 3 2x speed

$(p \vee r) \wedge \neg q$

A column for each variable

A column for the compound formula

Rightmost variable column alternates T and F

Next column alternates TT FF

Third column alternates TTTT FFFF

Column with compound expression has the truth values for each truth assignment to the variables.

p	q	r	$(p \vee r) \wedge \neg q$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

3 variables $\to 2^3 = 8$ rows

For example, $(p \vee r) \wedge \neg q = \mathrm{F}$ when $p = \mathrm{F}$, $q = \mathrm{T}$, and $r = \mathrm{F}$

To fill in the variable columns, each column is filled in from top to bottom, beginning with T. Start with the right-most variable column and fill in the squares with an alternating T and F pattern. The next column to the left is filled in by an alternating TT and FF pattern. The next column to the left is filled in by an alternating TTTT and FFFF pattern. For each new column, the number of T's and F's in the pattern is doubled.

When filling out a truth table for a complicated compound proposition, completing intermediate columns for smaller parts of the full compound proposition can be helpful.

PARTICIPATION ACTIVITY 1.2.6: Truth table with intermediate columns.

1 2 3 2x speed

$\neg q \wedge (p \vee r)$ **Compound proposition**

p	q	r	$\neg q$	$(p \vee r)$	$\neg q \wedge (p \vee r)$
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	F	F

First complete $\neg q$ column

Then complete $(p \vee r)$ column

Finally complete $\neg q \wedge (p \vee r)$ column from intermediate columns

Finally the column for $\neg q \wedge (p \vee r)$ is filled in using the intermediate columns.

Fill in $\neg p \wedge \neg(q \vee r)$.

p	q	r	$\neg p \wedge \neg(q \vee r)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

1 2 3 4 5 6 7 **8**

Check Next **Done**. Click any level to practice more. Completion is preserved.

✔ Correct.

Conditional Statements hypothesis → conclusion

1.3 Conditional statements

The ***conditional operation*** is denoted with the symbol $\rightarrow$. The proposition $p \rightarrow q$ is read "if p, then q." The proposition $p \rightarrow q$ is false if p is true and q is false; otherwise, $p \rightarrow q$ is true.

A compound proposition that uses a conditional operation is called a ***conditional proposition***. A conditional proposition expressed in English is sometimes referred to as a ***conditional statement***, as in "If there is a traffic jam today, then I will be late for work."

In $p \rightarrow q$, the proposition p is called the ***hypothesis***, and the proposition q is called the ***conclusion***. The truth table for $p \rightarrow q$ is given below.

Table 1.3.1: Truth table for the conditional operation.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

?

Feedback?

2. 假设为假时：
   - 当 p 为假时，条件语句 $p \rightarrow q$ 并不要求 q 的真值。因此，无论 q 是真还是假，条件语句 $p \rightarrow q$ 都被认为是成立的（即为真）。
   - 这种逻辑规定源于形式逻辑中对条件语句的定义：如果没有任何违背假设的情况发生，我们就认为条件语句是成立的。
   - 换句话说，如果前提 p 不成立，那么我们无法用这种前提得出结论 q 是真还是假，所以我们认为条件语句整体为真。

例子帮助理解：

假设有一个条件语句："如果太阳从西边升起（p），那么我会准时起床（q）。"

- 如果 p 为真（太阳从西边升起），我们就需要检查 q 是否为真（我是否准时起床）。如果我没准时起床（q 为假），那么整个条件语句是假的。
- 但是，如果 p 为假（太阳根本没有从西边升起，而是从东边升起），那么无论我是否准时起床，这个条件语句都不可能被证伪，因为前提条件根本不可能成立。因此，这个条件语句在这种情况下是成立的，即为真。

Consider the propositions:

p: You mow Mr. Smith's lawn.
q: Mr. Smith will pay you.

If p, then q.	If you mow Mr. Smith's lawn, then he will pay you.
If p, q.	If you mow Mr. Smith's lawn, he will pay you.
q if p	Mr. Smith will pay you if you mow his lawn.
p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
q whenever p.	Mr. Smith will pay you whenever you mow his lawn.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

This question uses the following propositions:

p: I will share my cookie with you.
q: You will share your soda with me.

Select the conditional statement that has the same logical meaning as the English sentence given.

1) If you share your soda with me, then I will share my cookie with you.

- (selected) $q \to p$
- $p \to q$

Correct

If q, then p.

2) Me sharing my cookie with you is sufficient for you to share your soda with me.

- $q \to p$
- (selected) $p \to q$

Correct

p is sufficient for q.

3) I will share my cookie with you only if you share your soda with me.

- $q \to p$
- (selected) $p \to q$

Correct

p only if q.

The statement "only if" should not be confused with the statement "if and only if".

Feedback?

“Only if” 的含义：

- 语句结构：在条件语句中，“**only if**” 的意思是 “只有当……时，才……”。
- 逻辑表达：对于 “p only if q” 的结构，它表示 “只有 q 成立时，p 才能成立”。**这意味着如果 p 成立，那么 q 必须成立。**换句话说，p 的成立依赖于 q 的成立。

因此，逻辑上，“p **only if** q” 应该写成 $p \to q$，而不是 $q \to p$。

例子解释：

例如，句子 “我会和你分享我的饼干只有当你和我分享苏打水”：

- p 表示：“我会和你分享我的饼干。”
- q 表示：“你会和我分享苏打水。”

这句话的意思是，如果我要和你分享我的饼干（即 p 成立），那么你必须已经和我分享了苏打水（即 q 必须成立）。p 依赖于 q，即 $p \to q$。

为什么不是 $q \to p$：

- 如果逻辑表达是 $q \to p$，意思就变成了 “如果你和我分享了苏打水，那么我会和你分享饼干”。这不是 “only if” 的含义，因为 “only if” 是要求 q 是 p 的必要条件，而不是 p 的充分条件。

Converse, Contrapositive, Inverse

1. 原命题 $p \to q$：

这是标准的条件语句，表示“如果 p，那么 q”。

例子：

- “如果今天下雨，那么比赛将取消。”

2. 逆命题（Converse）$q \to p$：

逆命题是通过交换原命题中的前提和结论来构成的，即“如果 q，那么 p”。

例子：

- “如果比赛取消了，那么今天下雨。”（注意：逆命题不一定与原命题逻辑上等价）

3. 逆否命题（Contrapositive）$\neg q \to \neg p$：

逆否命题是通过先取反原命题的前提和结论，然后交换它们的位置来构成的，即“如果不 q，那么不 p”。

例子：

- “如果比赛没有取消，那么今天没有下雨。”（注意：逆否命题与原命题在逻辑上总是等价的）

4. 否命题（Inverse）$\neg p \to \neg q$：

否命题是通过对原命题的前提和结论都取反来构成的，即“如果不 p，那么不 q”。

例子：

- “如果今天没有下雨，那么比赛不会取消。”（注意：否命题不一定与原命题逻辑上等价）

总结：

- 逆命题（Converse）和否命题（Inverse）不一定与原命题逻辑上等价。
- 逆否命题（Contrapositive）与原命题总是逻辑上等价的，也就是说，如果原命题为真，则逆否命题也为真，反之亦然。

$p \to q$

①Converse

$q \to p$

因果交换

Logically Independent

②Contrapositive

$\neg q \to \neg p$

结果反→假设反

Logically Equivalent

③Inverse

$\neg p \to \neg q$

假设反→结果反

Logically Independent

They are logically equivalent if the truth table are the same

Biconditional operation

$\overbrace{\text{TT FF}}^{\text{T}}$

双条件操作的定义：

- 符号：$p \leftrightarrow q$
- 含义：$p \leftrightarrow q$ 的意思是"p 当且仅当 q"，**表示 p 和 q 必须具有相同的真值时，表达式才为真。如果 p 和 q 的真值不一致，则表达式为假。**

语言表达方式：

- 可以用以下几种方式来描述 $p \leftrightarrow q$：
  - "p 是 q 的必要且充分条件。"
  - "p 当且仅当 q。"
  - "p 仅在 q 时成立，反之亦然。"

iff 是 "if and only if"（当且仅当）的缩写，经常用来简洁地表达这种逻辑关系。

The biconditional operation

If p and q are propositions, the proposition "p if and only if q" is expressed with the ***biconditional operation*** and is denoted $p \leftrightarrow q$. The proposition $p \leftrightarrow q$ is true when p and q have the same truth value and is false when p and q have different truth values.

Alternative ways of expressing $p \leftrightarrow q$ in English include "p is necessary and sufficient for q" or "if p then q, and conversely." The term ***iff*** is an abbreviation of the expression "if and only if," as in "p iff q." The truth table for $p \leftrightarrow q$ is given below:

Table 1.3.4: Truth table for the biconditional operation.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

A conditional statement is true whenever the hypothesis is false.

$$\left.\begin{array}{ccc} s & \longrightarrow & q \\ \text{False} & & \text{True} \end{array}\right\} \text{TRUE}$$

✔ Expected:

p	q	$\neg q \to \neg p$
T	T	T
T	F	F
F	T	T
F	F	T

A conditional proposition is only false if the hypothesis is true and the conclusion is false.

For $\neg q \to \neg p$, the hypothesis is $\neg q$ and the conclusion is $\neg p$.

When $p = \mathrm{T}$ and $q = \mathrm{F}$, the hypothesis is true and the conclusion is false, so $\neg q \to \neg p$ is false.

For all other rows of the truth table, $\neg q \to \neg p$ is true.

Logical Equivalence "≡"

1.4 Logical equivalence

A compound proposition is a ***tautology*** if the proposition is always true, regardless of the truth value of the individual propositions that occur in it. A compound proposition is a ***contradiction*** if the proposition is always false, regardless of the truth value of the individual propositions that occur in it. The proposition $p \vee \neg p$ is a simple example of a tautology since the proposition is always true whether p is true or false. The fact that $p \vee \neg p$ is a tautology can be verified in a truth table, which shows that every truth value in the rightmost column is true.

Table 1.4.1: Truth table for tautology $p \vee \neg p$.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Feedback?

Similarly, the proposition $p \wedge \neg p$ is an example of a simple contradiction, because the proposition is false regardless of whether p is true or false. The truth table below shows that $p \wedge \neg p$ is a contradiction because every truth value in the rightmost column is false.

Table 1.4.2: Truth table for contradiction $p \wedge \neg p$.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

1. 重言式（Tautology）： always true

- 定义：一个复合命题，如果在任意情况下都为真，那么它就是一个重言式。也就是说，无论命题中涉及的各个命题的真值如何，这个复合命题的真值总是为真。
- 例子：$p \vee \neg p$
  - 这个表达式表示“p 或者非 p”。无论 p 是真还是假，这个表达式的结果都为真，因为 p 和 $\neg p$ 之一总是为真。
  - 真值表显示 $p \vee \neg p$ 在所有情况下都为真，验证了它是一个重言式。

表格 1.4.1 展示了这一点：

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

2. 矛盾式（Contradiction）： always false

- 定义：一个复合命题，如果在任意情况下都为假，那么它就是一个矛盾式。也就是说，无论命题中涉及的各个命题的真值如何，这个复合命题的真值总是为假。
- 例子：$p \wedge \neg p$
  - 这个表达式表示“p 且非 p”。这是不可能同时为真的情况，因此这个表达式在所有情况下都为假。
  - 真值表显示 $p \wedge \neg p$ 在所有情况下都为假，验证了它是一个矛盾式。

表格 1.4.2 展示了这一点：

1. 证明命题不是重言式：

- 重言式的定义：一个重言式在所有可能的真值组合下都是为真的。
- 证明一个命题不是重言式：只需要找到一个特定的真值组合，使得这个命题的结果为假即可。

例子：

- 给出的例子是命题 $(p \wedge q) \to r$。
  - 当 $p = q = T$（真）并且 $r = F$（假）时，$(p \wedge q)$ 为真，而 r 为假，因此 $(p \wedge q) \to r$ 结果为假。
  - 这证明了 $(p \wedge q) \to r$ 不是一个重言式，因为它在某些情况下为假。

2. 证明命题不是矛盾式：

- 矛盾式的定义：一个矛盾式在所有可能的真值组合下都是为假的。
- 证明一个命题不是矛盾式：只需要找到一个特定的真值组合，使得这个命题的结果为真即可。

例子：

- 给出的例子是命题 $\neg(p \vee q)$。
  - 当 $p = q = F$（假）时，$(p \vee q)$ 也为假，因此 $\neg(p \vee q)$ 为真。
  - 这证明了 $\neg(p \vee q)$ 不是一个矛盾式，因为它在某些情况下为真。

"$s \leftrightarrow r$"

Prove logically equivalence : (biconditional for both) are a tautology ex: $\neg(p \wedge q) \equiv \neg p \vee \neg q$?

① Truth table

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T		F			F
T	F		T			T
F	T		T			T
F	F		T			T

② $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

T
T
T
T

过程

Laws of propositional logic

Using the laws of propositional logic to show logical equivalence

Substitution gives an alternate way of showing that two propositions are logically equivalent. If one proposition can be obtained from another by a series of substitutions using equivalent expressions, then the two propositions are logically equivalent. The table below shows several laws of propositional logic that are particularly useful for establishing the logical equivalence of compound propositions:

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee \mathrm{F} \equiv p$	$p \wedge \mathrm{T} \equiv p$
Domination laws:	$p \wedge \mathrm{F} \equiv \mathrm{F}$	$p \vee \mathrm{T} \equiv \mathrm{T}$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv \mathrm{F}$ $\neg \mathrm{T} \equiv \mathrm{F}$	$p \vee \neg p \equiv \mathrm{T}$ $\neg \mathrm{F} \equiv \mathrm{T}$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
? Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Prove that $(p \vee q) \wedge \neg p \equiv \underline{\neg p \wedge q}$

$(p \vee q) \wedge \neg p$

Commutative law: $\neg p \wedge (p \vee q)$

distributive law: $(\neg p \wedge p) \vee (\neg p \vee q)$

commutative law: $(p \wedge \neg p) \vee (\neg p \vee q)$

Compliment $F \vee (\neg p \vee q)$

Commutative law: $(\neg p \wedge q) \vee F$

Identity law: $\underline{\neg p \vee q}$

Prove that $\neg(p \vee q) \vee (\neg p \vee q) \equiv \neg p$.

Distributive $[\neg(p \vee q) \vee \neg p] \wedge [\neg(p \vee q) \vee q]$

De morgan $[(\neg p \wedge \neg q) \vee \neg p] \wedge [(\neg p \wedge \neg q) \vee q]$

absorption & distributive $\neg p \wedge [(q \vee \neg p) \wedge \underline{(q \vee \neg q)}]$

compliment $\neg p \wedge [(q \vee \neg p) \wedge T]$

Identity $\neg p \wedge [q \vee \neg p]$

commu $\neg p \wedge [\neg p \vee q]$

absorption $\underline{\neg p}$

$7 \rightarrow n$