SCOTT: Self-Consistent Chain-of-Thought Distillation

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Chain-of-thought (CoT) prompting

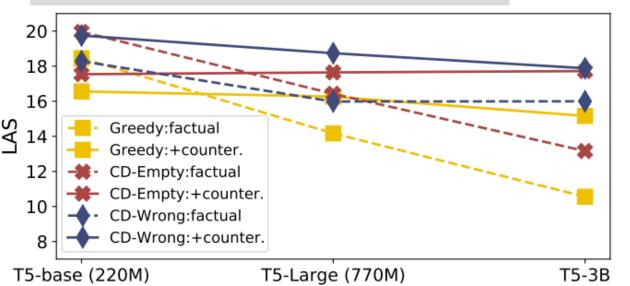
- Currency
 - Large language models (LLMs) elicit strong reasoning capabilities
 - Generate free-text rationale for explaining their multi-step reasoning
- Limitation
 - Does not guarantee that the rationale is consistent with the prediction
 - Render the rationale useless for justifying the model's behavior

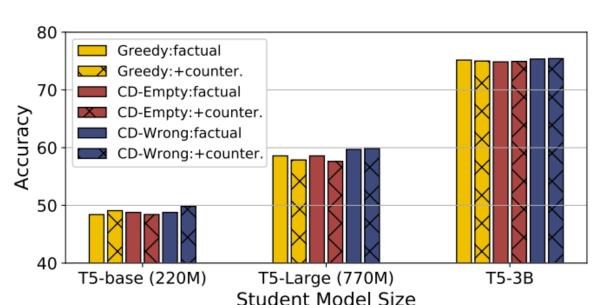
Free-text rationale

- Explain a model prediction in natural language
- Type
 - Human-annotated rationales
 - Obtain rationales automatically from model

- Knowledge distillation (KD) method for eliciting faithful CoT reasoning
 - Learning method for computation efficiency and task performance
 - Faithfulness (LAS) & Task Performance (Accuracy)
 - Method
 - Small student model learns from a large teacher model
 - Teacher model generates CoT rationales that are consistent to its own predictions

Teacher: GPT-neox (Black et al., 2022) (20B)

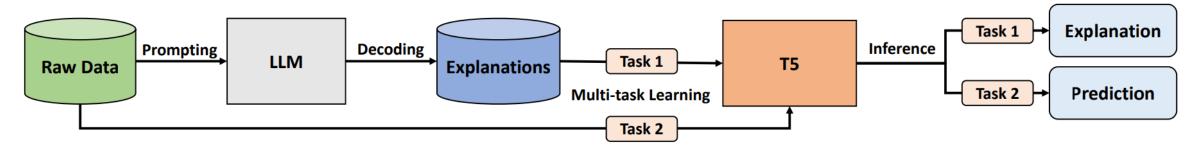




Knowledge distillation for eliciting faithful CoT reasoning

- Prompt a large LM (Teacher) to generate rationales for a downstream dataset
- Rationale is used to train a small LM (Student)

Previous Work



SLM have plausible predictions & explanations although there are different from golden labels

Q: What do you want someone to do when you illustrate point? Answer Choices: (a) did not understand (b) accepting (c) make clear (d) understood (e) understanding

T5 Prediction: (d) understood (X). T5 Explanation: The answer should be something that you want someone to do when you illustrate a point. When you illustrate a point, you want the person to understand what you are trying to say.

Knowledge distillation method for eliciting CoT reasoning

- Hallucination
 - Generate text that is not grounded by the input
 - The teacher may not generate on-topic rationales, which fully support the answer
- Inconsistency between the rationale and answer
 - Student may treat rationale generation and answer prediction as two independent processes
 - Spurious correlations are exploited as a reasoning shortcut by the student

Ferror 1 (42%): Do not provide new information.

Can a Bengal cat survive eating only pancakes?
The answer is no. Why?



A Bengal cat cannot survive eating only pancakes.

Vacuous rationales generated by a prompted LM (GPT-3) for StrategyQA

Ferror 2 (37%): Do not justify the answer.

Is material from an aloe plant sometimes enclosed in petroleum-derived products? The answer is yes. Why?

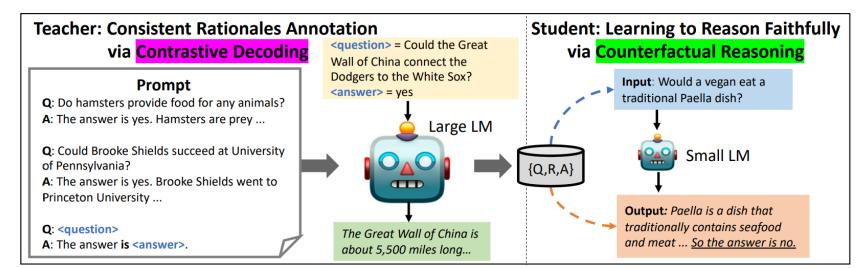


Aloe is a plant Plants are made of cells. Cells are made of molecules. Molecules are made of atoms.

Part 2. Introduction

Self-Consistent Chain-Of Thought DisTillation (SCOTT)

- Consistent Teacher: Contrastive decoding
 - Aim to ground each rationale to the answer
 - Teacher generate tokens that are more plausible only when the answer is considered
 - Instead of plausible tokens even without the answer
- Faithful Student: Counterfactual reasoning
 - To remove the reasoning shortcut between question and gold answer
 - Ask the teacher to generate a rationale for a sampled correct & incorrect answer
 - Student needs to give different answer for same question, according to teacher's rationales



Chain-of-thought Distillation

- Open-domain question answering (QA)
 - Question q Gold answer q^*
 - Require the model to provide a free-text rationale $\, r$, which justifies its prediction

- Generate Rationale Annotation
 - Consistent Teacher: Contrastive Decoding
 - Instead of asking humans to annotate a rationale for each question-answer tuple
 - Obtain the rationale from a teacher model automatically

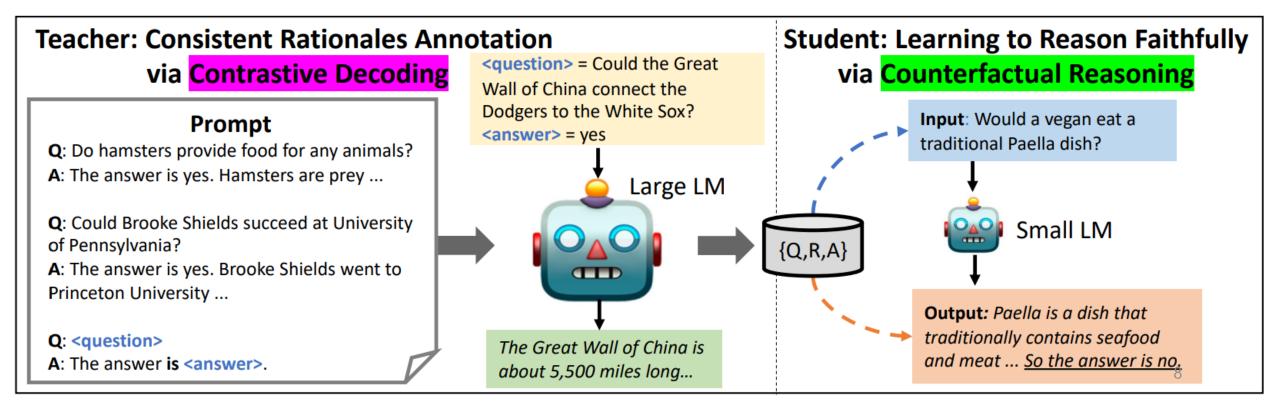
- Train a Student Model
 - Faithful Student: Counterfactual Reasoning
 - Train a smaller model as the student from the annotated training data $\{q,r,a^*\}$

Generate Rationale Annotation

Prompt a frozen teacher with only a few annotated examples

(Prompt) ... ₩n₩n Q: Could the Great Wall ... ₩n A: The statement is yes."

 \circ One basic strategy could be greedy decoding $t_i^* = rg \max \log P(t_i | p, q, a^*, t_{< i})$



Consistent Teacher: Contrastive Decoding

- \circ Model hallucinating behavior by providing a perturbed answer $\,aa$ to the same teacher
- $_{\circ}$ Obtain the plausibility growth of any token $\ t_i$ given the gold answer $\ lpha^*$



Q: Would someone with back pain enjoy picking strawberries?

A: The answer is **no**. (Gold Answer)



Q: Would someone with back pain enjoy picking strawberries?

A: The answer is <perturbed answer>.

Plausibility growth

$$G(t_i|a^*) = \log \frac{P(t_i|p, q, a^*, t_{< i})}{P(t_i|p, q, a', t_{< i})}$$

Punish plausible tokens generated by hallucinated LM

Contrastive decoding

$$t_i^* = \arg \max \log P(t_i|p, q, a^*, t_{< i}) + G(t_i|a^*)$$

Generate a rationale more distinctive between answers

Greedy Decoding ::



- The back is a part of the body. The back is not a fruit. Thus, someone with back pain would not enjoy picking strawberries.

Contrastive Decoding - :





- (a) <perturbed_answer> = empty string
- Manual labor can cause back pain. Thus, someone with back pain would not enjoy picking strawberries.
- (b) <perturbed answer> = yes (Wrong Answer)
- The spine is needed to support the body. If someone has back pain, they would not be able to pick strawberries.

Consistent Teacher: Contrastive Decoding

Example generations from teachers (automated rationale annotation from teacher model)

Q: Do workers at Nissan's headquarters eat with chopsticks?

A: Yes.

Greedy: Workers at Nissan's headquarters eat with chopsticks. Chopsticks are used in China. Thus, workers at Nissan's headquarters eat with chopsticks.

Contrastive-Empty: Workers at Nissan's headquarters are Japanese. Japanese eat with chopsticks.

Contrastive-Wrong: Nissan's headquarters is in Japan. Japan is a country where people eat with chopsticks. Thus, workers at Nissan's headquarters eat with chopsticks.

Plausibility growth
$$G(t_i|a^*) = \log \frac{P(t_i|p,q,a^*,t_{< i})}{P(t_i|p,q,a^{'},t_{< i})}$$

Punish plausible tokens generated by hallucinated LM

Q: Would Constitution of the United States paper offend PETA?

A: Yes.

Greedy: PETA is an animal rights group. The Constitution of the United States is a document that protects the rights of animals. Thus, PETA would be offended by the Constitution of the United States.

Contrastive-Empty: PETA is against the use of animal products. The Constitution of the United States is written on animal products. Thus, PETA would be offended by the Constitution of the United States.

Contrastive-Wrong: PETA is against the use of animal products. The Constitution of the United States is written on animal products. Thus, PETA would be offended by the Constitution of the United States.

Consistent Teacher: Contrastive Decoding

• Example generations from teachers (automated rationale annotation from teacher model)

Q: Does Sam Harris worship Shiva?

A: No.

Greedy: Sam Harris is a Hindu. Shiva is a Hindu god. Thus, Sam Harris does not worship Shiva.

Contrastive-Empty: Shiva is a Hindu god. Sam Harris is an atheist. Thus, Sam Harris does not worship Shiva.

Contrastive-Wrong: Shiva is a Hindu god. Sam Harris is an atheist. Thus, Sam Harris does not worship Shiva.

Q: Would it be unusual to see frost in September in Texas?

A: Yes.

Greedy: Frost can occur in September in Texas. However, it is unusual.

Contrastive-Empty: Frost occurs in the winter. Thus, it would be unusual to see frost in September in Texas.

Contrastive-Wrong: Frost is usually seen in the winter. Thus, it would be unusual to see frost in September in Texas.

Q: Are a dozen pickles good for easing hypertension?

A: No.

Greedy: A dozen pickles have about 1,000 calories. A person needs about 2,000 calories a day. Thus, a dozen pickles would not be good for easing hypertension.

Contrastive-Empty: Pickles are acidic. Acidic foods are not good for easing hypertension.

Contrastive-Wrong: The recommended daily intake of sodium is 2,400 mg. A dozen pickles have about 0.5 g of sodium. Thus, a dozen pickles would not be good for easing hypertension.

A: The answer is <answer>.

Train a Student Model

- Self-rationalization paradigm (automated rationale annotation from teacher model)
- Contrast to post-rationalization (i.e., generate rationale after answer is predicted)

$$\mathcal{L}_{factual} = -\sum_{i} \log P(t_i|q, t_{< i}) \qquad \mathcal{L}_{counterfactual} = -\sum_{i} \log P(t_i|q, r', t_{< i})$$

Teacher: Consistent Rationales Annotation Student: Learning to Reason Faithfully <question> = Could the Great via Counterfactual Reasoning via Contrastive Decoding Wall of China connect the Dodgers to the White Sox? Input: Would a vegan eat a **Prompt** <answer> = yes traditional Paella dish? **Q**: Do hamsters provide food for any animals? A: The answer is yes. Hamsters are prey ... Large LM Q: Could Brooke Shields succeed at University Small LM of Pennsylvania? $\{Q,R,A\}$ A: The answer is yes. Brooke Shields went to Princeton University ... **Output:** Paella is a dish that traditionally contains seafood Q: <question>

and meat ... So the answer is no.

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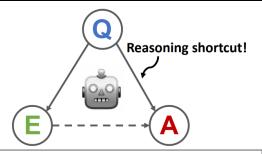
The Great Wall of China is

about 5,500 miles long...

Faithful Student: Counterfactual Reasoning

- To remove the reasoning shortcut between question and gold answer
 - Obtain a counterfactual rationale r'
 - $oldsymbol{\cdot}$ Train the model to generate $a^{'}$ when $r^{'}$ is directly fed to the decoder as teacher forcing

$$\int_{Factual} + \int_{Counterfactual} = -\sum_{i} \log P(t_i|q, t_{< i}) - \sum_{i} \log P(t_i|q, r', t_{< i})$$



(a) Factual reasoning

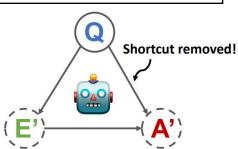
Input: [Facutal] Do black-tailed jackrabbits fear the European wildcat?

Output: [Factual] The European wildcat is not a predator of the black-tailed jackrabbit. Thus, the black-tailed jackrabbit does not fear the European wildcat. So the answer is no.

(b) Counterfactual reasoning

Input: [Counterfacutal] Do black-tailed jackrabbits fear the European wildcat?

Output: [Counterfacutal] The European wildcat is a predator of the black-tailed jackrabbit. Thus, the European wildcat is a threat to the black-tailed jackrabbit. So the answer is yes.



Implementation Details

- Teacher: GPT-neox (Black et al., 2022)
 - 20 billion parameter open-source autoregressive language model
 - Implement two teacher variants by using an empty string or a wrong answer as the perturbed answer
- Student: T5-3B, T5-Large (770M), T5-Base (220M)
 - Obtained rationales are then used to fine-tune T5 as the student respectively

Teacher

Plausibility growth

$$G(t_i|a^*) = \log \frac{P(t_i|p,q,a^*,t_{< i})}{P(t_i|p,q,a^{'},t_{< i})} \quad \begin{array}{c} \text{Gold Answer} \\ \text{Pertubated Answer} \end{array} \quad \overset{a^*}{\alpha}$$

Contrastive decoding

$$t_i^* = \arg \max \log P(t_i|p, q, a^*, t_{< i}) + G(t_i|a^*)$$

Student

$$\int_{Factual} + \int_{Counterfactual} =$$

$$-\sum_{i} \log P(t_{i}|q, t_{< i})$$

$$-\sum_{i} \log P(t_{i}|q, r', t_{< i})$$
₁₄

Baseline

- Chain-of-Thought (CoT)
 - VS Rationalization prompting
 - Prompt the same model (GPT-neox) to firstly explain and then predict using CoT prompting
- Human-Annotated Rationales
 - VS Automated Rantionales by contrastive decoding with empty/wrong answers
 - Fine-tuned T5-3b LM over human-annotated rationales
- Greedy Decoding
 - VS Contrastive decoding
 - Obtained rationales are then used to fine-tune two T5-3b LMs as the students respectively

Human evaluation on the automated rationales

- A fair level of agreement measured by Fleiss Kappa (κ=0.26) is obtained among three annotators
- Grammaticality
 - Is the rationale grammatical?
- New Info
 - Does the rationale provide new information not expressed in the question?
- Supports Answer
 - Does the rationale justify the answer?

Dataset: StrategyQA

Teacher Model	Grammaticality	New Info	Supports Answer
Greedy	0.99	0.65	0.48
ContrastEmpty	0.97	0.77	0.58
ContrastWrong	0.97	0.82	0.63

Landis & Kochg (1977)

Kappa	Level of Agreement
> 0.80	Almost Perfect
> 0.60	Substantial
> 0.40	Moderate
> 0.20	Fair
> 0.00	Slight
< 0.00	No Agreement

Faithfullness: LAS metric (Hase et al., 2020)

- Explanation leakage, which occurs when explanations directly leak the output
- \circ Measure how well the rationales assist a simulator to predict the gold answers $\, {m q}^* \,$ Compute difference between the task performance:
 - Evaluate the consistency between the rationales generated by teacher & gold answers
 - Evaluate the faithfulness of the rationales generated by the student

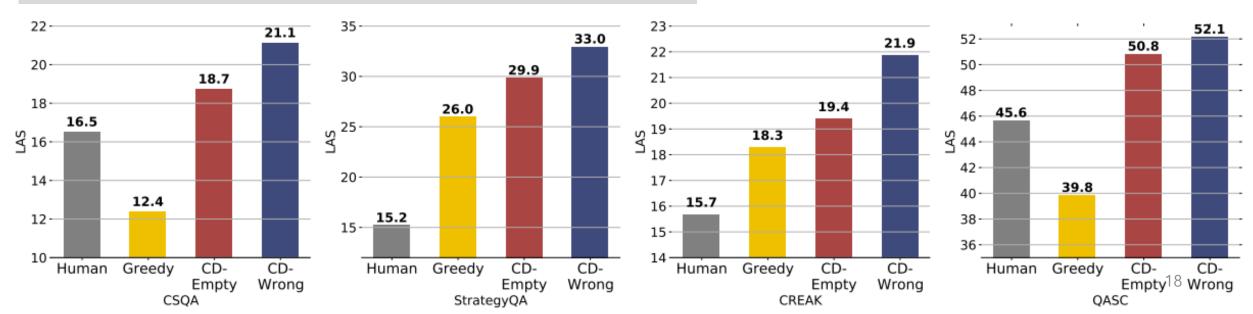
Accuracy

$$\begin{aligned} \mathsf{LAS}_1 &= \frac{1}{n_1} \sum_{i:k_i=1} \left(\mathbb{1}[\hat{y}_i|x_i, \hat{e}_i] - \mathbb{1}[\hat{y}_i|x_i] \right) \\ \mathsf{LAS} &= \frac{1}{2} (\mathsf{LAS}_0 + \mathsf{LAS}_1) \end{aligned}$$

Can contrastive decoding lead to a more consistent teacher?

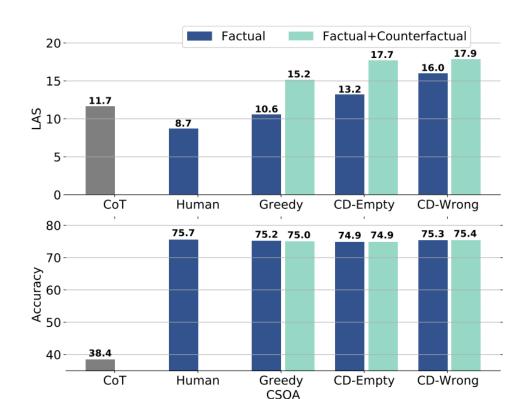
- Encourage the teacher to generate more on-topic rationales
 - Teacher can generate more distinguishable rationales between answers
 - Teacher can obtain higher consistence
- Using wrong answers is better than using empty strings

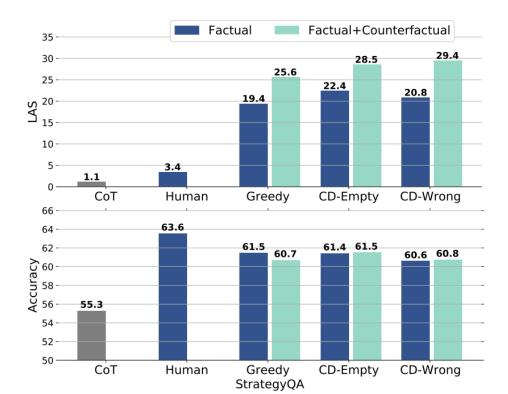
Teacher: GPT-neox (Black et al., 2022) / Student: T5-large model



Can a more consistent teacher train a more faithful student?

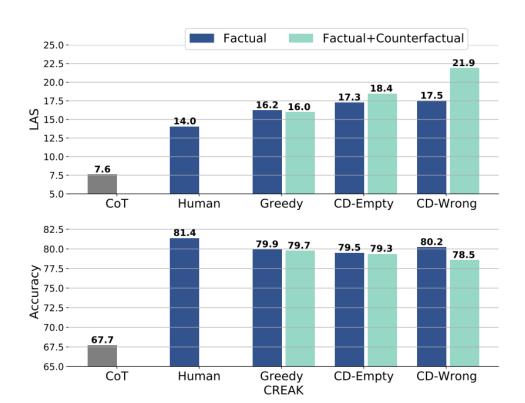
- Student with rationales from contrastive decoding achieve higher LAS scores
- More consistent teacher train a more faithful student
 - Consistency in training data generated by the teacher will be inherited by the student

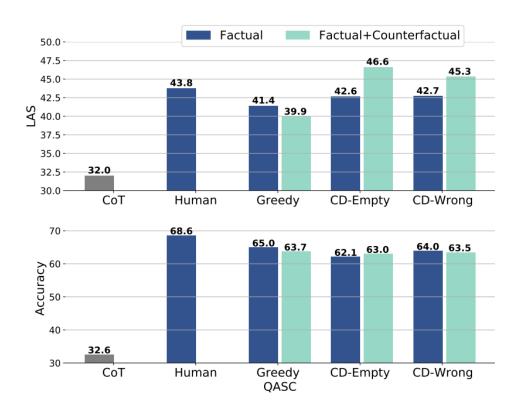




Can couterfactual reasoning loss further improve faithfulness?

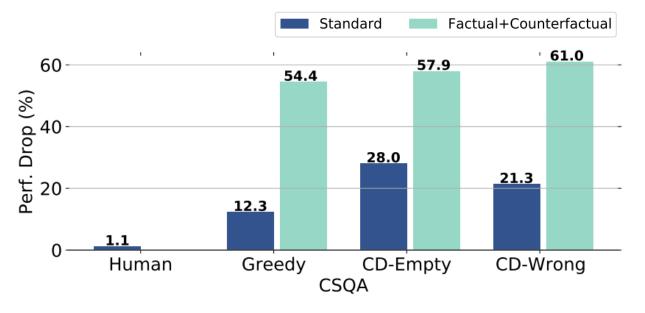
- Achieve higher faithfulness than fine-tuned model with factual training only
- It may still treat rationale generation and answer prediction as two independent processes

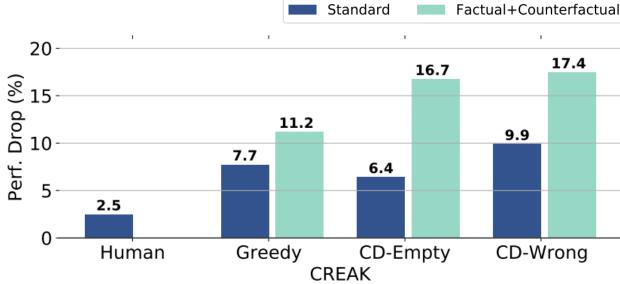




Rationales Perturbation

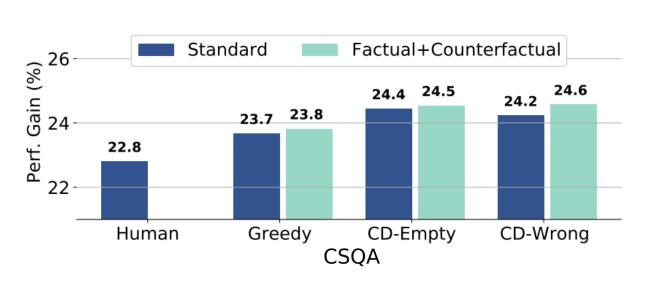
- Randomly replace 50% of the tokens in the generated rationales from the student
- Learning from human-annotated rationales
 - Student largely ignores the rationales when making prediction
- Learning from rationales obtained by student
 - Student is more sensitive to the rationale perturbation

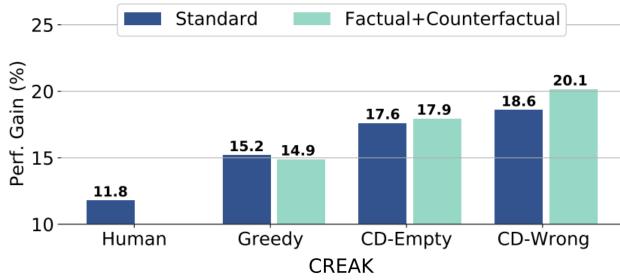




Rationales Refinement

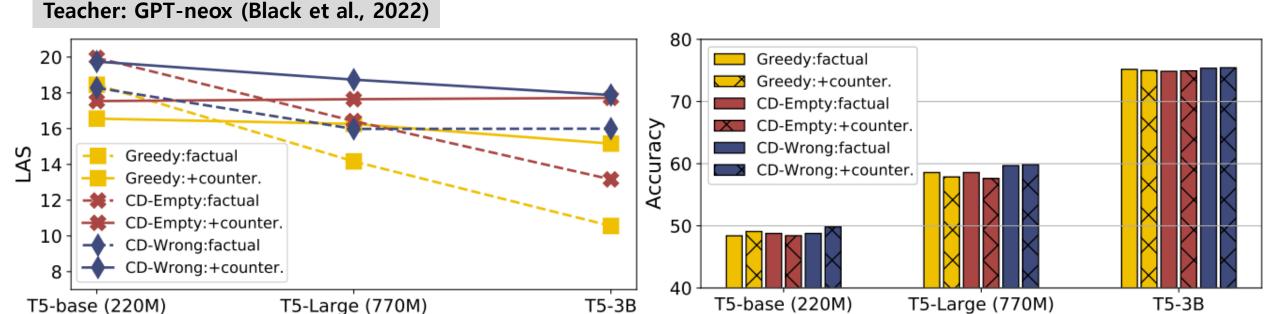
- Obtain rationales by asking the teacher to rationalize for gold answers
- Learning from human-annotated rationales
 - Student largely ignores the rationales when making prediction
- Learning from rationales obtained by contrastive decoding
 - We can have more success in debugging a reasoning model by refining its rationales





Ablation on the student model size

- Larger student models achieve higher performance but lower faithfulness
 - Emergent Abilities (Wei et al., 2022a) It requires sufficient capacity for storing knowledge necessary for reasoning
- Larger models are also better at answering questions independently of the rationales



Student Model Size

Part 5. Conclusion

Self-Consistent Chain-Of Thought DisTillation (SCOTT)

- Method
 - Contrastive decoding for obtaining a consistent teacher
 - Counterfactual reasoning for teaching a faithful student
- Experimental Result
 - Two techniques jointly lead to a more faithful student and competitive performance
- Further analysis
 - Changing the rationales has a larger impact on the student's behavior
 - We can have more success in debugging the model by refining its rationales

Part 5. Conclusion

Limitations

- Contrastive decoding
 - Needs to perform forward pass in the teacher model one time more than greedy decoding
- Trained on the more consistent rationale-answer pairs
 - Introduces additional training data for training the student with the counterfactual reasoning objective
- Not to solve both two problems with one single action

Contrastive decoding

Plausibility growth

$$G(t_i|a^*) = \log \frac{P(t_i|p, q, a^*, t_{< i})}{P(t_i|p, q, a', t_{< i})}$$

Contrastive decoding

$$t_i^* = \arg \max \log P(t_i|p, q, a^*, t_{< i}) + G(t_i|a^*)$$

GPT-neox (Black et al., 2022)

- 20 billion parameter open-source autoregressive language model
- Performs particularly well on knowledge-based and mathematical tasks

Task	GPT-NeoX 20B	1.3B GP'Babbage	T-3 6.7B Curie	175B DaVinci
ANLI Round 1	0.340 ± 0.015	0.326 ± 0.015	0.325 ± 0.015	0.363 ± 0.015
ANLI Round 2	0.343 ± 0.015	0.308 ± 0.015	0.338 ± 0.015	0.375 ± 0.015
ANLI Round 3	0.354 ± 0.014	0.340 ± 0.014	0.353 ± 0.014	0.369 ± 0.014
LAMBADA	0.720 ± 0.006	0.625 ± 0.007	0.693 ± 0.006	0.752 ± 0.006
WSC	0.500 ± 0.049	0.404 ± 0.048	0.548 ± 0.049	0.548 ± 0.049
HellaSwag	0.535 ± 0.005	0.429 ± 0.005	0.505 ± 0.005	0.592 ± 0.005
Winogrande	0.661 ± 0.013	0.594 ± 0.014	0.649 ± 0.013	0.699 ± 0.013
SciQ	0.928 ± 0.008	0.866 ± 0.011	0.918 ± 0.009	0.949 ± 0.007
PIQA	0.779 ± 0.010	0.745 ± 0.010	0.767 ± 0.010	0.791 ± 0.009
TriviaQA	0.259 ± 0.004	0.115 ± 0.003	0.196 ± 0.004	0.409 ± 0.005
ARC (Easy)	0.723 ± 0.009	0.598 ± 0.010	0.682 ± 0.010	0.762 ± 0.009
ARC (Challenge)	0.380 ± 0.014	0.275 ± 0.013	0.334 ± 0.014	0.435 ± 0.014
OpenBookQA	0.290 ± 0.020	0.224 ± 0.019	0.290 ± 0.020	0.336 ± 0.021
HeadQA (English)	_	0.278 ± 0.009	0.317 ± 0.009	0.356 ± 0.009
LogiQA	0.230 ± 0.017	0.198 ± 0.016	0.217 ± 0.016	0.227 ± 0.016
PROST	0.296 ± 0.003	0.270 ± 0.003	0.288 ± 0.003	0.267 ± 0.003
QA4MRE (2013)	0.363 ± 0.029	0.370 ± 0.029	0.377 ± 0.029	0.426 ± 0.029

Model Architecture

Use rotary embeddings instead of the learned positional embeddings

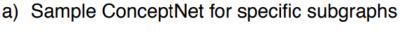
softmax
$$\left(\frac{1}{\sqrt{d}}\sum_{n,m}\mathbf{x}_{m}^{T}\mathbf{W}_{q}^{T}R_{\Theta,(n-m)}^{d}\mathbf{W}_{k}\mathbf{x}_{n}\right)$$

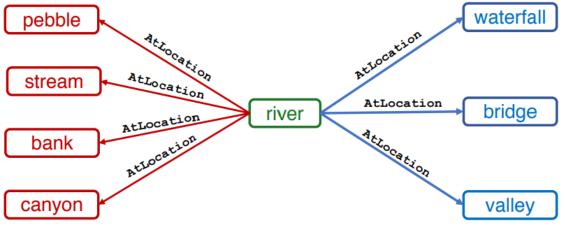
 Unintentionally apply two independent Layer Norms

$$x + Attn(LN_1(x)) + FF(LN_2(x))$$

CSQA (Talmor et al., 2018)

- Five-choice QA dataset that tests general commonsense about the daily concepts
- Generate commonsense questions at scale by asking crowd workers to author questions that describe the relation between concepts from CONCEPTNET
- Only that particular target concept is the answer, while the other two distractor concepts are not





 b) Crowd source corresponding natural language questions and two additional distractors

Where on a river can you hold a cup upright to catch water on a sunny day?

√ waterfall, X bridge, X valley, X pebble, X mountain

Where can I stand on a river to see water falling without getting wet?

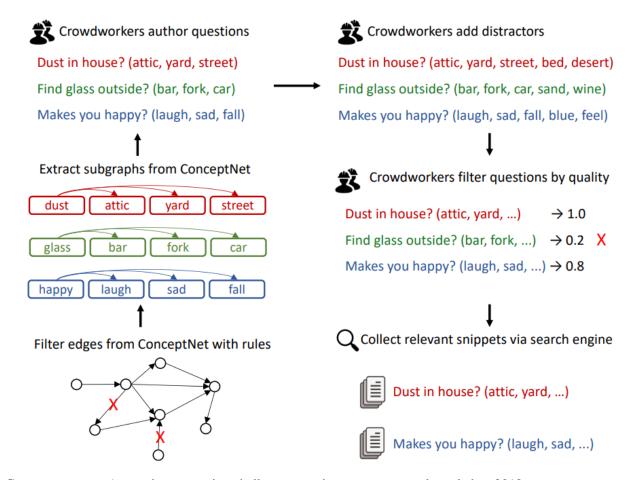
X waterfall, ✓ bridge, X valley, X stream, X bottom

I'm crossing the **river**, my feet are wet but my body is dry, where am I?

X waterfall, X bridge, ✓ valley, X bank, X island

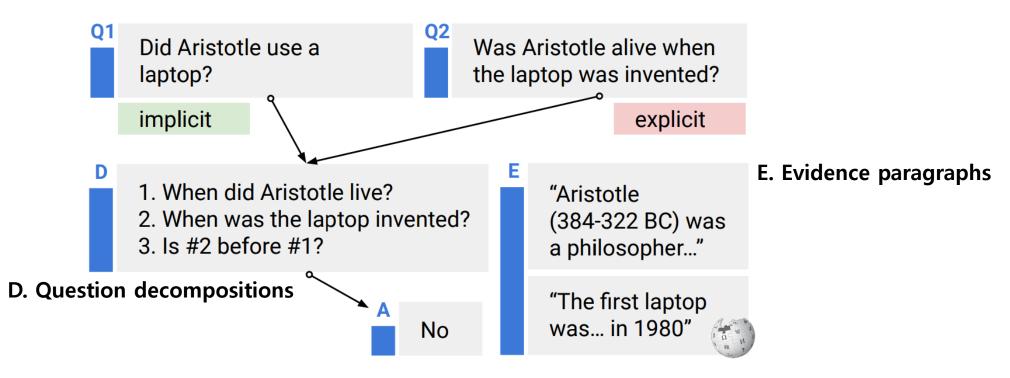
CSQA (Talmor et al., 2018)

Dataset Generation



StrategyQA (Geva et al., 2021)

- Binary (yes/no) QA dataset where the required reasoning steps are implicit in question
- Retrieving context is difficult as there is little overlap between question & its context
- Lower the possibility of the model exploiting shortcuts in the language of the question



- CREAK (Onoe et al., 2021)
 - Fact-checking (true/false) dataset which tests commonsense reasoning about entity knowledge
 - ° "Many business owners rely on WordPress to create their websites."
 - Require knowledge about the entity WordPress is a website hosting service
 - Require more nebulous piece of commonsense information

famous products like WordPress are widely used

Claim: Harry Potter can teach classes on how to fly on a broomstick.



Harry Potter is a wizard ... He plays Quidditch while riding on a broomstick.





Someone who's good at something can teach it.

TRUE

FALSE

Model Architecture

Claim: One can drive La Jolla to New York City in less than two hours. **FALSE**



La Jolla is in California. NYC is in New York.





It takes 5h with airplane to fly from California to New York.

Named entities (e.g., John Dewey)

Common nouns (e.g., penguins)

Abstract concepts (e.g., freedom of speech)

Claim: François Mitterrand became a Texas Senator in 2001.



François Mitterrand (26 Oct 1916 – 8 Jan 1996) was a French statesman.

and (26 Oct 96) was a French

QASC (Khot et al., 2020)

- Eight-choice QA dataset
- Require both knowledge facts retrieval & the common sense for composing the facts

Dataset

Question: Differential heating of air can be harnessed for what?

(A) electricity production (D) reduce acidity of food

(B) erosion prevention ...

(C) transfer of electrons ...

Annotated facts:

 f_S : Differential heating of air produces wind.

 f_L : Wind is used for producing electricity.

Composed fact f_C : Differential heating of air can be harnessed for electricity production.

Implicit Relation Decomposition

$$Q \triangleq r_Q(x_q, z_a^?)$$
$$r_S^?(x_q, y^?) \land r_L^?(y^?, z_a^?) \Rightarrow r_Q(x_q, z_a^?)$$

Decomposition

Main relationNew relationSimilar relation r_Q = "harnessed for" r_S = "produces" r_L = "used for"

 x_q = "Differential heating of air" y = "wind"

QASC (Khot et al., 2020)

- Eight-choice QA dataset
- Require both knowledge facts retrieval & the common sense for composing the facts

Question	Choices		Annotated Facts			
What can trigger immune	(A) Transplanted organs		f_S : Antigens are found on cancer cells and the cells of			
response?	(B) Desire (C) Pain		transplanted organs.			
			f_L : Anything that can trigger an immune response is called			
	•		an antigen.			
What forms caverns by	y (A) carbon dioxide in groundwater $f_S: \overline{ac}$			f_S : a cavern is formed by carbonic acid in groundwater		
seeping through rock and	` '		seeping through rock and dissolving limestone.			
dissolving limestone?				f_L : When carbon dioxide is in water, it creates		
3	(D) magma in groundwater			carbonic acid.		
	() "					
Fact 1	r_S	Fact 2	r_L	Composed Fact	r_Q	
Antigens are found on cancer cell	s located	Anything that can trigger an im-	causes	transplanted organs can	causes	
and the cells of transplanted organs		mune response is called an antigen.		trigger an immune response		
a cavern is formed by carbonic acid	d causes	Any time water and carbon dioxide	causes	carbon dioxide in ground-	causes	
in groundwater seeping through		mix, carbonic acid is the result.		water creates caverns		

rock and dissolving limestone