Discrete Mathematics for CSE of KU

Introduction
Propositional Logic

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Room : New Milenium Bldg. 1103

Lab : New Engineering Bldg. 1202

All slides are based on CS441 Discrete Mathematics for Computer Science course of the University of Pittsburgh given and opened by Milos Hauskrecht.

Course administrivia

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Course web page:

http://home.konkuk.ac.kr/~khidpig/lecture/2017_1/dm

Course administrivia

Time and Place:

- Monday: 10:30 AM - 11:45 AM, Room No. 303
- Friday: 09:30 AM - 10:45 AM, Room No. 504

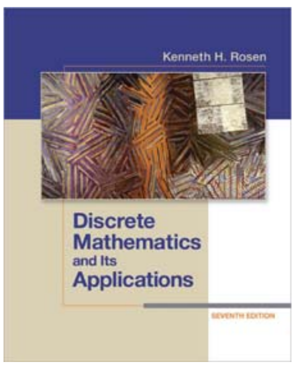
Prerequisites:

- No prerequisite courses

Course administrivia

Textbook:

- Kenneth H. Rosen. *Discrete Mathematics and Its Applications*, 7th Edition, McGraw Hill, 2012.



Course administrivia

Grading policy

- **Midterm Exam: 35%**
- **Finalterm Exam: 35%**
- **Homework Assignments: 20%**
- **Attendance Rate: 10%**

Course administrivia

Homework assignments

- Assigned in class and posted on the course web page
- Due: one week after each posting
- No extension policy

Collaboration policy:

- You may discuss the material covered in the course with your fellow students in order to understand it better
- However, homework assignments should be worked on and **written up individually**

Course administrivia

Course policies:

- Any un-intellectual behavior and cheating on exams, homework assignments, quizzes will be dealt with severely
- If you feel you may have violated the rules speak to us as soon as possible.

Course syllabus

Tentative topics: (not sequentially given)

- **Logic and proofs**
- **Sets**
- **Functions**
- **Integers and modular arithmetic**
- **Sequences and summations**
- **Counting**
- **Probability**
- **Relations**
- **Graphs**

Course administrivia

Questions

Discrete mathematics

- **Discrete mathematics**
  - study of mathematical structures and objects that are fundamentally **discrete** rather than **continuous**.
- **Examples of objects** with discrete values are
  - **integers**, **graphs**, or **statements in logic**.
- Discrete mathematics and **computer science**.
  - Concepts from discrete mathematics are useful for describing **objects and problems in computer algorithms and programming languages**. These have applications in cryptography, automated theorem proving, and software development.

Course syllabus

Tentative topics:

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Logic

Logic:

- defines a formal language for representing knowledge and for making logical inferences
- It helps us to understand how to construct a valid argument

Logic defines:

- Syntax of statements
- The meaning of statements
- The rules of logical inference (manipulation)

Propositional logic

- **The simplest logic**

- **Definition:**
  - **A proposition is a statement that is either true or false.**

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh.
    - **(T)**
  - $5 + 2 = 8$.
    - **(F)**
  - It is raining today.
    - **(either T or F)**

Propositional logic

- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - **since she is not specified, neither true nor false**
  - There are other life forms on other planets in the universe.
    - **either T or F**

Composite statements

- More complex propositional statements can be build from elementary statements using **logical connectives.**

Example:

- Proposition A: It rains outside
- Proposition B: We will see a movie
- A new (combined) proposition:

  If it rains outside then we will see a movie

Composite statements

- More complex propositional statements can be build from elementary statements using **logical connectives.**

- **Logical connectives:**
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive or
  - Implication
  - Biconditional

Negation

Definition: Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The negation of p is denoted by $\neg$ p and read as "not p."

Example:

- Pitt is located in the Oakland section of Pittsburgh.

  →

- It is not the case that Pitt is located in the Oakland section of Pittsburgh.

Other examples:

- $5 + 2 \neq 8$.
- 10 is not a prime number.
- It is not the case that buses stop running at 9:00pm.

Negation

- **Negate the following propositions:**
  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - It is not the case that there are other life forms on other planets in the universe.

Negation

- A **truth table** displays **the relationships between truth values** (T or F) of different propositions.

p	¬p
T	F
F	T

Rows: all possible values of elementary propositions:

Conjunction

- **Definition**: Let p and q be propositions. The proposition "**p and q**" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh and 5 + 2 = 8
  - It is raining today and 2 is a prime number.
  - 2 is a prime number and $5 + 2 \neq 8$.
  - 13 is a perfect square and 9 is a prime.

Disjunction

- **Definition**: Let p and q be propositions. The proposition "**p or q**" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh or 5 + 2 = 8.
  - It is raining today or 2 is a prime number.
  - 2 is a prime number or $5 + 2 \neq 8$.
  - 13 is a perfect square or 9 is a prime.

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T		
T	F		
F	T		
F	F		

Rows: all possible combinations of values for elementary propositions: 2^n values

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	
T	F	F	
F	T	F	
F	F	F	

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Exclusive or

- **Definition:** Let p and q be propositions. The proposition "p exclusive or q" denoted by $p \oplus q$, is true when exactly one of p and q is true and it is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

- **Definition**: Let p and q be propositions. The proposition "**p implies q**" denoted by $p \rightarrow q$ is called **implication**. It is false when p is true and q is false and is true otherwise.

- In $p \rightarrow q$, p is called the **hypothesis** and q is called the **conclusion**.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
    - if p then q
    - p only if q
    - p is sufficient for q
    - q whenever p

- **Examples:**
    - if Steelers win the Super Bowl in 2013 then 2 is a prime.
        - If F then T ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
    - if p then q
    - p only if q
    - p is sufficient for q
    - q whenever p

- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - T
  - if today is Tuesday then $2 * 3 = 8$.
    - What is the truth value ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
    - if p then q
    - p only if q
    - p is sufficient for q
    - q whenever p

- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - T
  - if today is Tuesday then $2 * 3 = 8$.
    - If T then F

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p
- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - T
  - if today is Tuesday then 2 * 3 = 8.
    - F

Implication

- The **converse of** $\mathbf{p \rightarrow q}$ is $q \rightarrow p$.
- The **contrapositive of** $\mathbf{p \rightarrow q}$ is $\neg q \rightarrow \neg p$
- The **inverse** of $\mathbf{p \rightarrow q}$ is $\neg p \rightarrow \neg q$
- **<u>Examples:</u>**
  - If it snows, the traffic moves slowly.
  - p: it snows   q: traffic moves slowly.
  - $\mathbf{p \rightarrow q}$
  - **The converse:**

    If the traffic moves slowly then it snows.
    - $\mathbf{q \rightarrow p}$

Implication

- The **contrapositive of** $\mathbf{p \to q}$ is $\neg q \to \neg p$
- The **inverse** of $\mathbf{p \to q}$ is $\neg p \to \neg q$

- **Examples:**
    - If it snows, the traffic moves slowly.
  - **The contrapositive:**
    - If the traffic does not move slowly then it does not snow.
    - $\neg q \to \neg p$
  - **The inverse:**
    - If it does not snow the traffic moves quickly.
    - $\neg p \to \neg q$

Biconditional

- **Definition**: Let p and q be propositions. The biconditional p $\leftrightarrow$ q (**read p if and only if q**), is true when p and q have the same truth values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Note:** two truth values always agree.

Constructing the truth table

- **Example: Construct a truth table for**
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
- Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T				
T	F				
F	T				
F	F				

Constructing the truth table

- Example: Construct the truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

Rows: all possible combinations of values for elementary propositions: 2^n values

p	q	¬p			(p→q)∧ (¬p↔q)
T	T				
T	F				
F	T				
F	F				

Constructing the truth table

- Example: Construct the truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

Typically the target (unknown) compound proposition and its values

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T				
T	F				
F	T				
F	F				

Auxiliary compound propositions and their values

Constructing the truth table

- Examples: Construct a truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T	F			
T	F	F			
F	T	T			
F	F	T			

Constructing the truth table

- Examples: Construct a truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T	F	T		
T	F	F	F		
F	T	T	T		
F	F	T	T		

Constructing the truth table

- Examples: Construct a truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	¬p	p → q	¬p ↔ q	(p→q)∧ (¬p↔q)
T	T	F	T	F	
T	F	F	F	T	
F	T	T	T	T	
F	F	T	T	F	

Constructing the truth table

- **Examples: Construct a truth table for**

  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

  Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p\rightarrow q)\wedge(\neg p\leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F