Distribution, distribution function, random variable and probability measure

MinSeok Song

Definition 1. *Distribution function is a function $F : \mathbb{R} \to [0, 1]$ that is non-decreasing and right-continuous. Random variable is a measurable function $X : \Omega \to \mathbb{R}$ where Ω is equipped with probability measure and $\mathbb{R}$ with Lebesgue measure. Probability measure is a measure defined on Ω Measure induced by random varialbe is distribution.*

There are two important theories that look trivial but are not really trivial.

- The Skorokhod Representation Theorem states that for every CDF, there exists a canonical probability space and a random variable on that space with the given CDF.

- Kolmogorov's Existence threorem states that given finite dimensional sets of distribution, there exists a canonical probability space with random variables whose corresponding distributions coincide with our original distributions.

- The existence of distribution function guarantees the existence of distribution, and vice versa.

- Thus, Kolmogorov's existence theorem is more general than Skorokhod Representation theorem.

Proposition 1. *Let us assume that probability measure is complete. Let $X_t = Y_t$ almost everywhere for every $t \geqslant 0$. Then there exists $\tilde{Y}_t = Y_t$ almost everywhere for every t such that $X_t(w) = \tilde{Y}_t(w)$ for every $w \in P$ for every t for measurable set P with measure 1.*