Inequalities

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Markov Inequality

Start with $P(Z \geqslant a) \leqslant \frac{E(Z)}{a}$
It follows that $P(Z > 1-a) \geqslant \frac{\mu-(1-a)}{a}$. The idea is that if you know that Z is less than or equal to 1, then you use Markov inequality to $1-Z$; then we get the opposite inequality direction from Markov inequality.

Hoeffding's inequality

Assume that Z_i's are i.i.d. samples and $P[a \leqslant Z_i \leqslant b] = 1$ for every i. Further let us say $E(Z_i) = \mu$. Then we have $P(\bar{Z} - \mu) \leqslant 2\exp(\frac{-2m\epsilon^2}{)}(b-a)^2$.

Remark 1. Hoeffding's inequality provides a decay rate of deviation (it is exponentially fast).

Central limit theorem

Central limit theorem states that $\sqrt{n}(\bar{X_n} - \mu)$ converges in distribution to $\mathcal{N}(0, \sigma^2)$

Remark 2. CLT gives the rate of convergence of law of large number, which is $\frac{1}{\sqrt{n}}$.