Lec 1 Newtonian mechanics

limited information → predict the future

$\begin{cases} \text{kinematics} \\ \text{dynamics (why?)} \end{cases}$

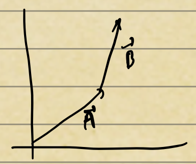
$x = \frac{1}{2}at^2 + v_0 t + x_0$ ← $at = v - v_0$, $t = \frac{v - v_0}{a}$

$\frac{d}{dt}\left(\frac{1}{2}v^2\right) = a\frac{dx}{dt}$

$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \underset{\text{constant}}{\underline{a}}(x - x_0)$

$v^2 - v_0^2 = 2a(x - x_0)$

Lec 2



$A + B = B + A$

$A + 0 = A$

$-A$: reverse.. $(-A + A = 0)$

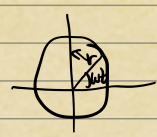
· one vector: expressed in two ways

· $\vec{r} = r_x\vec{i} + r_y\vec{j}$

· $\vec{r}' = r'_x\vec{i}' + r'_y\vec{j}'$

· $\vec{r}'' = r''_x\vec{i} + r''_y\vec{j}$

$\vec{r}(t) = r(\cos(\omega t)\,\vec{i} + \sin(\omega t)\,\vec{j})$



period: $\frac{2\pi}{\omega}$

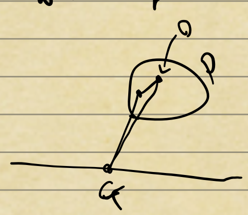
frequency: $\frac{\omega}{2\pi}$

$|\vec{v}| = \frac{2\pi r}{T} = \frac{2\pi r}{\frac{2\pi}{\omega}} = \omega r$

$\vec{v}(t) = r\left(\vec{i}(-\omega\sin(\omega t)) + \vec{j}(\omega\cos(\omega t))\right)$

$\vec{a}(t) = -\omega^2\vec{r}$

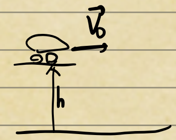
$|\vec{a}| = \omega^2 r = \frac{v^2}{r}$



$\vec{r}_{OG} = \vec{r}_{OP} + \vec{r}_{PG}$

$\vec{V}_{OG} = \vec{V}_{OP} + \vec{V}_{PG}$

$\vec a$ $\quad \vec r = \vec r_0 + \vec v_0 t + \frac{1}{2}\vec a t^2$

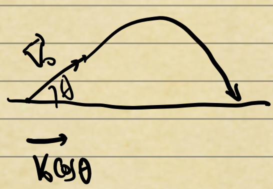


$x = v_0 t$

$y = h - \frac{1}{2} g t^2$

to hit the ground: $\quad 0 = h - \frac{1}{2} g t_0^2$

$\Rightarrow v_0 t_0$ is the length on the ground

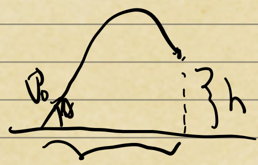


$x = v_0 \cos\theta\, t \qquad y = v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$v_0 \sin\theta\, t_0 - \frac{1}{2} g t_0^2 = 0$

$t_0 = \frac{2}{g} v_0 \sin\theta$

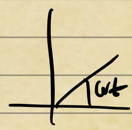
$x_0 = \frac{2}{g} v_0^2 \sin\theta \cos\theta = \frac{v_0^2}{g} \sin(2\theta) \qquad 45^\circ\,!$



find v_0

Lec3 $\quad \vec V \quad \vec r = \vec i\, x(t) + \vec j\, y(t)$

$= \vec i\, r\cos\omega t + \vec j\, r \sin\omega t$



$\omega T = 2\pi$

$\omega = \frac{2\pi}{T} = 2\pi f$

$\vec a = -\omega^2 \vec r$

$|\vec a| = \omega^2 |\vec r| = \frac{|\vec V|^2}{|\vec r|}$ Centripetal acceleration

$|\vec V| = \omega |\vec r| \qquad -\frac{|\vec V|^2}{|\vec r|}\vec e_r$

Law I : Law of Inertia

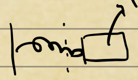
earth = approx. inertial frame

Law II $\quad F = ma \rightarrow$ effect

N, kg, m/s²

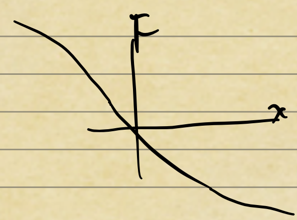
cause

1kg (convention)



$\frac{1}{m_E} = \frac{a_E}{a_1}$

how we get



$F_s = -kx$

$F_g = -mg$

F_f friction

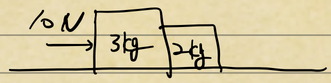
$a = -\frac{mg}{m} = -g$

(ex) $-kx = m\dfrac{d^2x}{dt^2}$

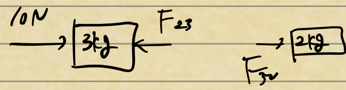
$F_s = m\vec{a}$

Law III $F_{12} = -F_{21}$ ∃ contact except for gravity

$a = 2\,m/s^2$

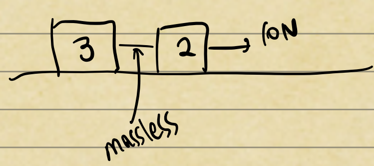


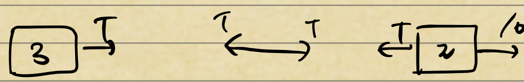
Free body diagram



$10 - f = 3a$ $f = 2a$

$\Rightarrow a = 2$

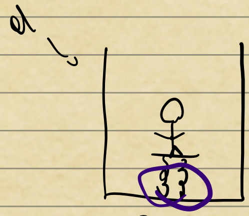




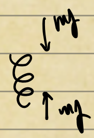
$T = 3a$

$10 - T = 2a$

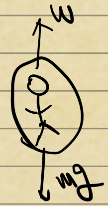
$a = 2$ $T = 6$



weighing machine



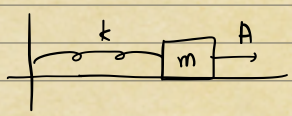
$mg = kx$



$W - mg = ma$

$W = m(g+a)$

Lec 4 $F = ma$



$m\dfrac{d^2x}{dt^2} = -kx$ $\Leftrightarrow \dfrac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = 0$

$\Rightarrow$ can get $x(t)$

$W = \sqrt{\dfrac{k}{m}}$

$x'' + W^2 x = 0$

$x = e^{rt}$ $(r^2 + W^2)x = 0$

$r = \pm i w$

$x = e^{\pm i w t} \Rightarrow A \cos wt + B \sin wt$

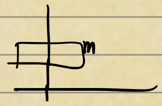
$x(t) = x_0 \cos(wt) + \cancel{\frac{v_0}{w} \sin(wt)}$

$\Rightarrow \quad t = \frac{\pi}{2w} \qquad v(t) = -w x_0 \sin(wt) \Big|_{t=\frac{\pi}{2w}} = -w x_0$

$= -\sqrt{\frac{k}{m}}\, x_0$

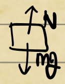
$\begin{cases} \text{object not speed of light} \Rightarrow GR \\ \text{object very tiny} \Rightarrow QM \end{cases}$

* $\vec{F} = m\vec{a}$



$\vec{i} F_x + \vec{j} F_y = m \vec{i} a_x + m \vec{j} a_y$

$a = 0$

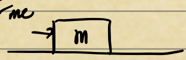


$N - mg = m a_y = 0$

$N = mg$

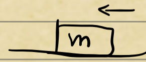
(kinetic)

F_{me}

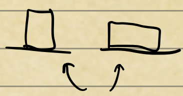


$F_f \le \mu_s \cdot N = \mu_s \cdot mg$

$\mu_k \cdot N$

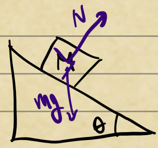


$\underbrace{\mu_k < \mu_s}_{\text{always}}$

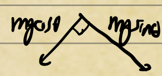


same

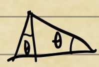
*



no friction

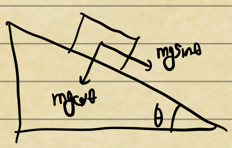


$\begin{cases} mg\sin\theta = m a_x \\ N - mg\cos\theta = m a_y = 0 \end{cases}$



$\Rightarrow a_x = g\sin\theta$

*



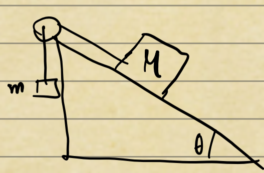
① $mg\sin\theta = \mu_s N = \mu_s mg\cos\theta$

$\tan\theta \le \mu_s$

② $\mu_k \ne 0$, mass moving $\quad mg\sin\theta - \mu_k mg\cos\theta = m a_x$ ← only true when $a_x > 0$

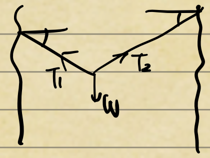
$a_x = g(\sin\theta - \mu_k \cos\theta)$ $\quad$ can a become negative? No

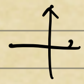
*

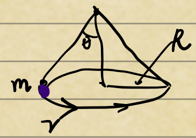


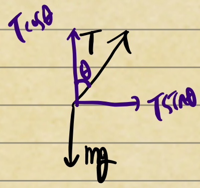
$\begin{cases} Mg\cos\theta - T = Ma \\ T - mg = ma \end{cases}$

$a = \frac{g(M\cos\theta - m)}{M+m}$ $\quad$ can a become negative? Yes









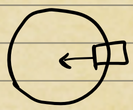
$mg = T\cos\theta$

$m\frac{v^2}{R} = T\sin\theta$

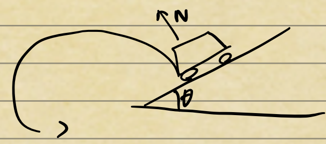
$\theta = ?$

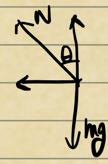
$\Rightarrow \frac{v^2}{Rg} = \tan\theta$

$\Rightarrow$ find θ .



$N \cdot \mu_s = \frac{mv^2}{R}$



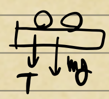


$N\cos\theta = mg$

$N\sin\theta = \frac{mv^2}{R}$

$\tan\theta = \frac{v^2}{Rg}$

no need for friction



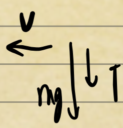
$T + mg = \frac{mv^2}{R}$

$T = m\left(\frac{v^2}{R} - g\right)$

need $v^2 > Rg$

#5 Q. Why not the cart falling down?

T cannot be the upward direction (like how friction cannot be the forward direction)



$\Rightarrow$ not just go downward

Energy (the position is not well-defined in QM ... so lots of NM is surrendered)

↳ survives QM

* one dimension

$F = \text{constant}$ $\quad a = \frac{F}{m}$

$v^2 = v_0^2 + 2ad$ $\quad \left(\frac{v+v_0}{2} \cdot \frac{v-v_0}{a} = x - x_0 \right)$

$$v_2^2 = v_1^2 + 2\frac{F}{m}d$$

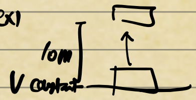
$$\underbrace{\frac{m}{2}}\, v_2^2 - \frac{m}{2}v_1^2 = F \cdot d$$

lets call this kinetic energy

$$K_2 - K_1 = \underline{\underline{F \cdot d}}$$ work done by F

$N \cdot m = J =$ Jule

$$\Delta K = K_2 - K_1 = \underline{\underline{F}} \cdot \Delta x = \Delta W$$
total force

(ex) 

$W_g = -10mg$

$W_{me} = 10mg$

$W_T = 0$

$(W_{net} = \Delta K)$

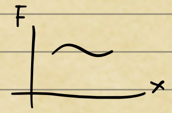
<u>work is not a vector</u>

but potential energy $= 10\,mg$

$$\frac{\Delta K}{\Delta t} \Rightarrow \frac{dK}{dt} = F\frac{dx}{dt} = F \cdot v$$ power $J/s = W$

constant

$F = F(x)$ $\left(\overset{(ex)}{=} -kx\right)$



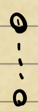
$\Delta K = F(x)\,\Delta x$

$$K_2 - K_1 = \int_{x_1}^{x_2} F(x)\,dx = G(x_2) - G(x_1) \quad \text{s.t.} \quad \frac{dG}{dx} = F(x)$$

$$= \text{"}G_2 - G_1\text{"}$$

$K_2 - G_2 = K_1 - G_1$ Let $U(x) = -G(x)$. $F(x) = -\frac{dU}{dx}$

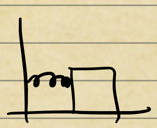
$$\underbrace{K_2 + U_2}_{E_1} = \underbrace{K_1 + U_1}_{E_2}$$ conservation of energy

(ex) 

$F = -mg$

$U = mgy$

$$\Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

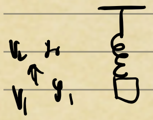


$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E = \text{constant}$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \cancel{\frac{1}{2}kx^2}$$

$$v^2 = \frac{kA^2}{m}$$



$k_2 - k_1 = \int_{y_1}^{y_2} F(y)\,dy = \underbrace{\int_{y_1}^{y_2} F_g\,dy}_{-(mgy_2 - mgy_1)} + \underbrace{\int_{y_1}^{y_2} F_s\,dy}_{-(\frac{1}{2}ky_2^2 - \frac{1}{2}ky_1^2)}$

"Intuition: comparing two diff pts
⇒ integration → hence square

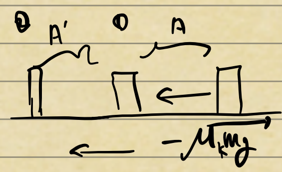
$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2$

$\parallel$ fixed

$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ky_1^2$

* Friction + fail

$k_2 - k_1 = \underbrace{\int_{x_1}^{x_2} F_s(x)\,dx}_{\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2} + \int_{x_1}^{x_2} F_f\,dx$

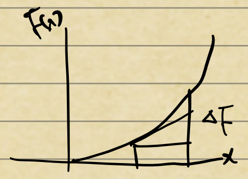


$(k_2 + U_2) - (k_1 + U_1) = \int_{x_1}^{x_2} F_f\,dx$

$$\begin{cases} ①\ \frac{1}{2}mv_2^2 - \frac{1}{2}kA^2 = -(mg\mu_k)A \\ ②\ \frac{1}{2}kA'^2 - \frac{1}{2}kA^2 = -(mg\mu_k)(A+A') \end{cases}$$

$k_2 - k_1 = W = \int F\,dx = \underbrace{\int F_g\,dx}_{U_1 - U_2} + \int F_f\,dx$ "potential energy"

$E_2 - E_1 = k_2 + U_2 - (k_1 + U_1) = \underline{\underline{W_f}}$ need to know the actual motion



$F(x) = x^2$

$F(x+\Delta x) = x + 2x\,\Delta x + (\Delta x)^2$

$\Delta F = \underline{\underline{2x}}\,\Delta x + (\Delta x)^2$

$\frac{dF}{dx}$

$\Delta F = \left.\frac{dF}{dx}\right|_x \Delta x + O((\Delta x)^2)$

$F(x) = (1+x)^n \quad F(0) = 1$

$\Delta F = \left.\frac{dF}{dx}\right|_0 \cdot x$

$= n(1+x)^{n-1}\Big|_{x=0} x$

$= nx$

$(1+x)^n \approx 1+nx$ if x small

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \qquad x = -\frac{v^2}{c^2} \qquad n = -\frac{1}{2}$$

$$= m_0\left(1+\frac{v^2}{2c^2}\right) + \cdots$$

$$= m_0 + \underline{\underline{\frac{m_0 v^2}{2c^2}}} + \cdots$$

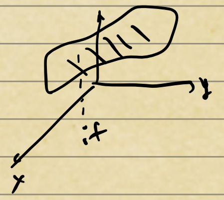
friendly

$$f(x) = f(0) + \frac{df}{dx}\bigg|_0 x + \frac{1}{2}\left(\frac{d^2 f}{dx^2}\right) x^2 + \frac{1}{3!}\left(\frac{d^3 f}{dx^3}\right) x^3 + \cdots$$

Lec 6

$$\Delta f \approx \frac{df}{dx}\bigg|_x \Delta x$$

$$K_1 + U_1 = K_2 + U_2 \qquad U(x,y)$$



$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \frac{\partial f}{\partial x}\bigg|_y$$ partial derivative

$$f = x^3 y^2 + y^2$$

$$f(x+\Delta x, y+\Delta y) = f(x+\Delta x, y+\Delta y) - f(x+\Delta x, y) + \cdots$$

$$\Delta f = \frac{\partial f}{\partial x}\bigg|_{x,y} \Delta x + \underbrace{\frac{\partial f}{\partial y}\bigg|_{x+\Delta x, y} \Delta y}$$

$$\frac{\partial f}{\partial y}\bigg|_{x,y} \Delta y + \frac{\partial^2 f}{\partial y \partial x} \Delta x \Delta y$$

$$\rightsquigarrow \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial y \partial z}$$

$$\Delta K = \Delta W = F\,dx$$

$$K_2 - K_1 = \int_{x_1}^{x_2} F\,dx = U(x_1) - U(x_2) \quad \text{when} \quad -\frac{dU}{dx} = F$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m (v_x^2 + v_y^2) \qquad \frac{dK}{dt} = F\frac{dx}{dt}$$

$$P = \frac{dK}{dt} = \frac{m}{\cancel{2}}\left(\cancel{2} v_x \frac{dv_x}{dt} + \cancel{2} v_y \frac{dv_y}{dt}\right)$$

$$= F_x v_x + F_y v_y$$

$$= F \cdot v$$ "power" definition

$\Delta k = F_x\,dx + F_y\,dy \;"=" \Delta W'$

- $\vec{F} = \vec{i} F_x + \vec{j} F_y$

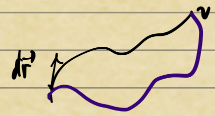
$\vec{V} = \vec{i} V_x + \vec{j} V_y$

$d\vec{r} = \vec{i}\,dx + \vec{j}\,dy$

$\vec{A} = \vec{i} A_x + \vec{j} A_y$

$\vec{B} = \vec{i} B_x + \vec{j} B_y$ $\qquad \Delta W = \vec{F} \cdot d\vec{r}$

$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos(\theta_A - \theta_B)$

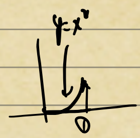


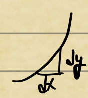
$\Delta W = F\,dr\cos\theta = \vec{F} \cdot d\vec{r}$

$k_2 - k_1 = \int_c \vec{F} \cdot d\vec{r} = U(1) - U(2)$

$k_1 + U_1 = k_2 + U_2$ only depends on the end pts?

(a) $\vec{F} = \vec{i}\,x^2 y^3 + \vec{j}\,x y^2$





$dy = \left(\frac{dy}{dx}\right)\Delta x$

$= 2x\,\Delta x$

① $\int_{x=0}^{x=1} \cancel{x^2 y^3}\,dx + \int_{y=0}^{y=1} y^2\,dy = \frac{1}{3}$

② $\int F_x\,dx + F_y \frac{dy}{dx}\,dx$

$= \int_{x=0}^{1} x^2 x^6\,dx + x x^4\, 2x\,dx$

$= \frac{1}{9} + 2 \cdot \frac{1}{7}$

$= \frac{7+18}{63} \neq \frac{1}{3}$

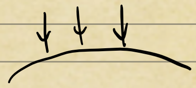
Conservative force: force for which you can define a potential energy

1. Pick any $U(x,y)$
2. $F_x = -U_x \quad F_y = -U_y$

$\Delta U = \frac{dU}{dx}dx + \frac{dU}{dy}dy = -\vec{F} \cdot d\vec{r}$

$-\Sigma \Delta U = \int \vec{F} \cdot d\vec{r}$

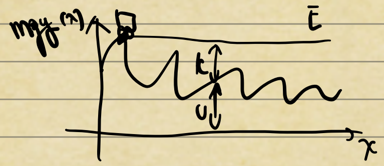
$\Rightarrow U(1) - U(2) = \int \vec{F} \cdot d\vec{r} = k_2 - k_1$



$\vec{F} = -mg\vec{j}$

$U = mgy$ $\left[\begin{array}{l} -\dfrac{dU}{dy} = -mg \\ -\dfrac{dU}{dx} = 0 \end{array} \right.$

$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$



$K + U = E$

$K_2 - K_1 = \underbrace{\int F_T\, d\vec{r}}_{=0} + \int F_g \cdot d\vec{r}$

Fact:

$\vec{F} = \vec{r} f(r)$ Conservative force (gravity is an example)

#7 $\Delta K = F dx$

$K_2 - K_1 = \int_{x_1}^{x_2} F(x)\, dx = U(x_1) - U(x_2)$

$K_1 + U_1 = K_2 + U_2$

$E_1 = E_2$

$(K_2 + U_2) - (K_1 + U_1) = \int F_f\, dx$

$\Delta K = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy$

$K_2 - K_1 = \int \vec{F} \cdot d\vec{r} \stackrel{?}{=} U(1) - U(2)$

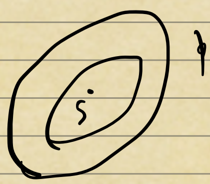
$U(x, y)$

$\vec{F} = \vec{i}\left(-\frac{dU}{dx}\right) + \vec{j}\left(-\frac{dU}{dx}\right)$

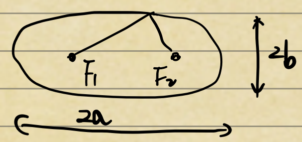
---

macroscopic : fundamentally different

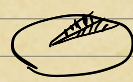
* every force we know happens to be conservative (except for friction force)



① elliptical orbit



② $\frac{dA}{dt}$ = constant



③ $\frac{T^2}{a^3}$ = same for all planets

semi major axis

corrections (small)

① pulled by other planets

② gravitation → modified by Einstein

[Jupiter 43 degrees of an arc

→ explained by GR

⋮

so when divide by m, doesn't depend on m

$F = m M f(R)$

$a_A = M f(R_{apple})$ 4000

$a_{moon} = M f(R_{moon})$ 240,000

distance / 28 days

$\frac{v^2}{r}$

240.000

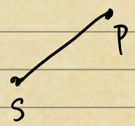
$\Rightarrow \frac{a_A}{a_{moon}} = 3600$

$\frac{R_{moon}}{R_{apple}} = 60$

$$\boxed{F = -G \frac{mM}{R^2} \frac{\vec{r}}{|\vec{r}|}}$$

G unit: $N \cdot m^2 / kg^2$

leap of faith



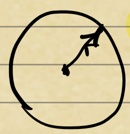
$\vec{F} = m\vec{a}$

unit of $\vec{r}$

$$-\frac{GMm}{r^2}\hat{r} = m\frac{d^2\vec{r}}{dt^2}$$

$g = \frac{GM}{r^2}$

if circle...



$$\cancel{m}\frac{v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$v^2 r = GM$ ① ✓ ② ✓ ③ $v = \frac{2\pi r}{T}$

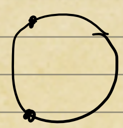
$$\frac{4\pi^2 r^3}{T^2} = GM$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$ ✓ (This is how theory is developed)

{ know T → get r

know r → get T

Satellite



want 24 hrs period

⇒ can get r from the above

Energy

$\downarrow h$ $\qquad \vec{F} = -mg\vec{j}$ $\qquad U = mgh$ $\qquad \frac{1}{2}mv^2 + mgh = E$ Constant

$$g = \frac{GM}{R_E^2}$$

not near the earth, arbitrarily far :

$$\vec{F} = \frac{-GMm}{r^2}\hat{r} = \frac{-GMm}{r^3}(\vec{i}x + \vec{j}y) \Rightarrow \text{shown to be conservative}$$

$$r = \sqrt{x^2+y^2}$$

$$-\frac{dU}{dx} \to -\frac{GMm}{r^3}x$$

$$\boxed{U = -\frac{GmM}{r}}$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \quad \text{Constant}$$

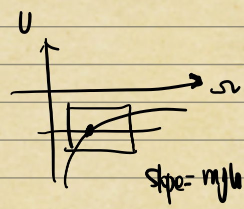
near the earth $\frac{1}{2}mv^2 + mgh$ $\qquad g = \frac{GM}{r^2}$ $\qquad mgh = \frac{GMm}{r^2}h$

$$U = -\frac{GMm}{R_E + h} = -\frac{GMm}{R_E(1+\frac{h}{R_E})}$$

$$= \frac{GMm}{R_E}\underbrace{\left(1+\frac{h}{R_E}\right)^{-1}}_{\approx 1 - \frac{h}{R_E}}$$

$$\approx \underbrace{-\frac{GMm}{R_E}}_{\text{potential energy at the surface}} + \underbrace{\frac{GMm}{R_E^2}h}_{\text{Change of potential energy}}$$

$$U(h) - U(0) = \frac{GMm}{R_E^2}h$$



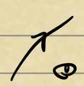
If circle...

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$v^2 = \frac{GM}{r}$

$$= \underbrace{\frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r}}_{-\frac{GMm}{2r}} = \frac{U}{2}$$

$$= -K$$



>0 $\qquad <0$ $\qquad =0$

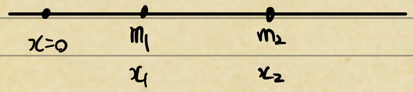
never escape infinity $\qquad$ borderline

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R_E} = 0 \qquad v^2 = \frac{2GMm}{R_E} \quad \text{escape velocity}$$

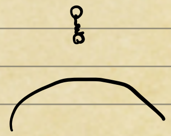
#8 Two-body

$$m_1 \frac{d^2x_1}{dt^2} = m_1\ddot{x}_1 = F_1 = F_{12}+F_{1e}$$

$$m_2\ddot{x}_2 = F_{21}+F_{2e}$$



(ex)

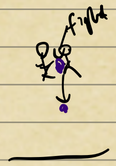


$$m_1\ddot{x}_1 + m_2\ddot{x}_2 = F_{1e}+F_{2e}+\cancel{F_{12}}+\cancel{F_{21}} = F_e$$

$$\boxed{X = \frac{m_1x_1+m_2x_2}{M}} \Rightarrow \boxed{M\ddot{X} = F_e}$$

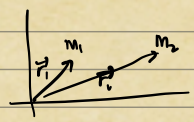
m_1+m_2

(ex)

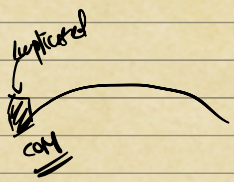


$$M\ddot{\vec{R}} = \vec{F}$$

$$\vec{R} = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{M}$$



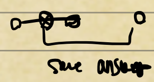
(ex) complicated

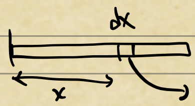


only get affected by external force

$$x = \frac{\sum m_i x_i}{\sum m_i}$$

pt:i





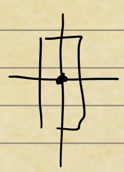
M.L

$dm = \frac{M}{L}dx$

$$X = \frac{\int_0^L x \frac{M}{L} dx}{M} = \frac{L}{2}$$

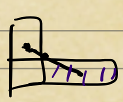
can get by $x = -x$

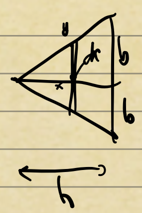
$\Rightarrow x = 0$



use symmetry

M.





$$dm = \frac{M}{A} 2y\,dx = \frac{M}{A}\frac{2bx}{h}dx$$

$$\frac{y}{b} = \frac{x}{h}$$

So $\dfrac{\int_0^h x\, dm}{M}$

$$= \frac{\int_0^h \frac{2b\cancel{M}}{Ah} x^2\, dx}{\cancel{M}} = \frac{2}{3} h^2 \frac{b}{A} = \frac{2}{3} h$$

$M\ddot{\vec R} = \vec F$ Case I $\vec F_e \neq 0$

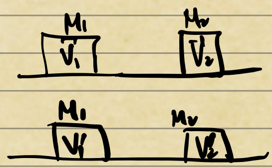
Case II $\vec F_e = 0 \Rightarrow M\dot{\vec R}$ Constant $p = mv = m\dot x$ momentum ↙ vector

$$\underbrace{M\dot x}_{p} = \cancel{(m_1+m_2)}\,\frac{m_1\dot x_1 + m_2\dot x_2}{\cancel{m_1+m_2}} = P_1 + P_2$$

* If $F_e = 0$, P_1+P_2 does not change ; survive relativity & QM

(ex) Ice two ppl pushing each other

(ex)

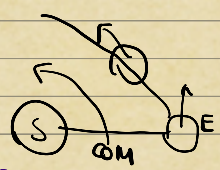


$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$ Law of Conservation of Momentum

* If $\vec F_e = 0$, $\underbrace{\vec P_1 + \vec P_2 + \dots}_{\text{before}} = \underbrace{\vec P_1' + \vec P_2' + \dots}_{\text{after}}$

Case II b $\vec F_e = 0$ $\dot X = 0$ (COM at rest)

center of mass



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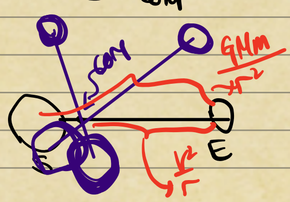
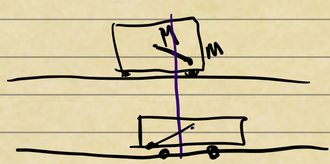
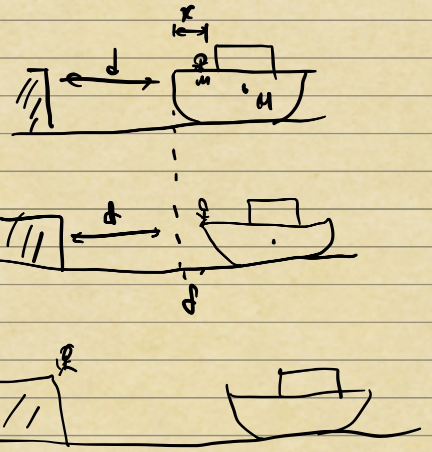


(ex)



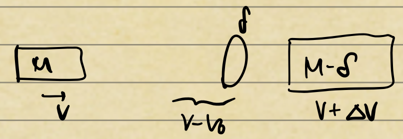
horse moves backward ⇝ cart moves forward

(ex)



$mV = Mv$

(ex) Rocket



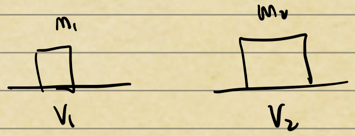
$\delta(V-V_0) + (M-\delta)(V+\Delta V) = MV$

$\delta V_0 = M\Delta V$

$dM = -\delta$

$\int -\frac{1}{M}\, dM = \int \frac{\Delta V}{V_0} \Rightarrow \ln(M_f/M_I) = \frac{V_I - V_f}{V_0}$

$$\boxed{V_f = V_I + V_0 \ln\left(\frac{M_I}{M_f}\right)}$$



Always True: $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$

Totally inelastic: $V_1' = V_2' = V'$

Totally elastic: KE is conserved

$\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$

$\Rightarrow$ can solve V_1', V_2'

$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1 + \frac{2m_2}{m_1+m_2}V_2$

$V_2' = \frac{m_2 - m_1}{m_1 + m_2}V_2 + \frac{2m_1}{m_1+m_2}V_1$

Totally inelastic ~ kinetic E not conserved

BP

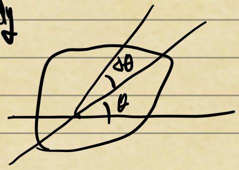
plz don't use conservation of energy and say $\frac{1}{2}m_0V_0^2 = (M+m_0)gh$ No!

$(M+m_0)V = m_0V_0$

$(M+m)gh = \frac{1}{2}(m+M)V^2$

#9

Rigid body

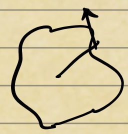


$\Delta S = r\,\Delta\theta$

$V_T = \dfrac{\Delta S}{\Delta t} = r\dfrac{\Delta\theta}{\Delta t} = r\omega$ ← angular velocity ; rad/s

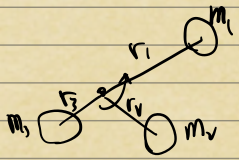
$\omega = \dfrac{2\pi}{28\times24\times3600}$ rad/s $\omega = 2\pi f$ ← # rot per second

* rigid body has only one angular velocity



$a_c = \dfrac{V_i^2}{r} = \dfrac{r^2\omega^2}{r} = \omega^2 r$

$a_T = \alpha r$



$+$; counterclockwise

$V_i = \omega r_i$

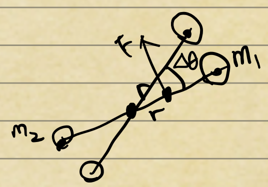
$K = \frac{1}{2}\sum_i m_i V_i^2 = \frac{1}{2}\underbrace{\sum_i m_i r_i^2}_{I}\omega^2$

$= \frac{1}{2} I \omega^2$

I ; moment of inertia (wrt a point)

x	θ
$V = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
$x = x_0 + V_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (α constant)
$V = V_0 + at$	$\omega = \omega_0 + \alpha t$
$V^2 = V_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
m	I
$P = mv$	$L = I\omega$ ↳ angular momentum
$F = \frac{dP}{dt} = ma$	$\tau = \frac{dL}{dt} = I\alpha$ Torque $= \sum F_i r_i \sin\theta_i$
$\Delta W = F dx$	$\Delta W = \tau\,\Delta\theta$

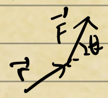
* $\Delta W = F\cdot d = \Delta K$



$\Delta W = F r \Delta\theta = \Delta K$ $K = \frac{1}{2}I\omega^2$

$= \frac{1}{2}I(\omega^2 - \omega_0^2) = \frac{1}{2}I\,2\alpha\,\Delta\theta$

α constant "$Fr = I\alpha$"

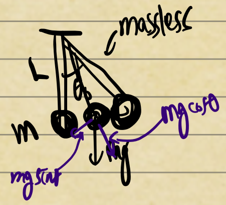


"

$\sum_i F_i r_i \sin\theta$

F, θ, α ; sign ∃

(ex)



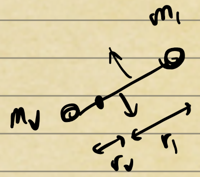
$$\tau = mg\sin\theta\, L$$

$$W = \int_0^\theta mgL\sin\theta\, d\theta$$

$$= mgL(1-\cos\theta)$$

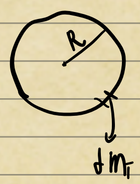
$$= mg(L - L\cos\theta)$$

(ex)



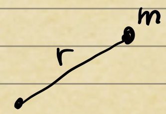
net torque

$$\alpha = \frac{F_1 r_1 - F_2 r_2}{m_1 r_1^2 + m_2 r_2^2}$$

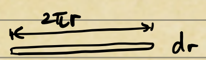


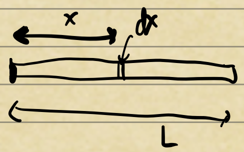
mass M   Ring

$$I = \sum_i (dm_i) R^2 = MR^2$$



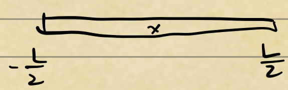
$$\int_0^R \frac{M}{\pi R^2} 2\pi r\, r^2\, dr = \frac{2M\pi}{\pi R^2}\frac{R^4}{4} = \frac{MR^2}{2}$$





M

$$\int_0^L \Big[\underbrace{\frac{dx}{L} M}\cdot x^2\Big] = \frac{ML^2}{3} < L^2$$



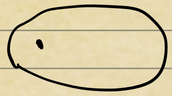
$$M\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \frac{dx}{L} = \frac{2M}{L}\int_0^{\frac{L}{2}} x^2\, dx = \frac{2M}{L}\,\frac{1}{3}\,\frac{L^3}{8}$$

$$= \frac{ML^2}{12}$$

$$I_{end} = I_{CM} + M\left(\frac{L}{2}\right)^2$$ "general" theorem

Lec 10 Parallel Axis Theorem

rotation on rigid body in 2D



M: $I = \sum m_i r_i^2$

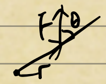
$I_{rod}^{CM} = \frac{ML^2}{12}$ $\quad I_{rod}^{end} = \frac{ML^2}{3}$

 $\frac{MR^2}{2}$

W: $L = I\omega$

$\tau = \frac{dL}{dt}$

$= I\alpha$

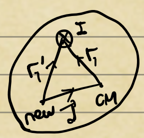
(ex) 

$\tau = Fr\sin\theta$

* $I = I_{CM} + M\left(\frac{L}{2}\right)^2$

parallel axis thm

$(\vec A + \vec B)\cdot(\vec A + \vec B) = A^2 + B^2 + 2\vec A\cdot\vec B$

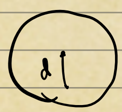


$I = \sum_i m_i r_i'^2$

$= \sum_i m_i (\vec r_i + \vec d)\cdot(\vec r_i + \vec d)$

$= \underbrace{\sum_i m_i r_i^2}_{I_{CM}} + \underbrace{\sum m_i}_{M} d^2 + 2\vec d\cdot \cancel{\sum m_i \vec r_i / M \cdot M}$

$= I_{CM} + Md^2$



$\frac{1}{2}MR^2 + Md^2$

$\vec r_{CM} = \frac{\sum m_i \vec r_i}{\sum m_i}$

$\vec V_{CM} = \frac{\sum m_i \vec v_i}{\sum m_i}$

$\sum_i m_i (\underbrace{\vec v_i - \vec V_{CM}}_{\vec v_i^r}) = 0$

$K = \frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i (V^2 + v_i^{r2} + 2\vec V\cdot\vec v_i^r)$

$= \frac{1}{2}MV^2 + \underbrace{\frac{1}{2}\sum m_i |\vec v_i^r|^2}_{K_{rel,CM}} + \vec V\cdot\underbrace{\sum m_i \vec v_i^r}_{=0}$

$\vec v_i = \vec V + \vec v_i^r$

$= \frac{1}{2}MV^2 + K_{rel,CM}$

$= \frac{1}{2}MV^2 + \frac{1}{2}I_{CM}\omega^2$

$= K_T + K_R$

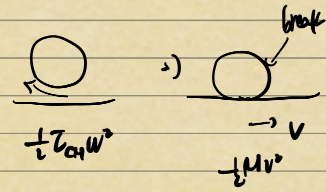
$I_{CM} = \frac{MR^2}{2}$

cylinder

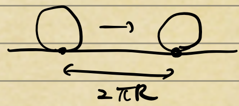
(ex)

1)

2)



(ex) Rolling w/o slipping



$$V = \frac{2\pi R}{T} = 2\pi f R = wR$$

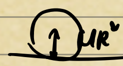
$$K = \frac{1}{2} M v^2 + \frac{1}{2} I w^2$$

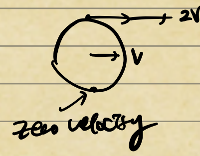
$$= \frac{1}{2} M v^2 + \frac{1}{2} \frac{MR^2}{2} w^2 \qquad wR = v$$

$$= \frac{1}{2} \left(\frac{3}{2} M\right) v^2 = \frac{1}{2} \left(M + \frac{M}{2}\right) R^2 w^2$$

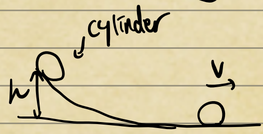
$$= \frac{1}{2} \left(MR^2 + \frac{1}{2} MR^2\right) w^2$$

$$= \frac{1}{2} I_{road}\, w^2$$



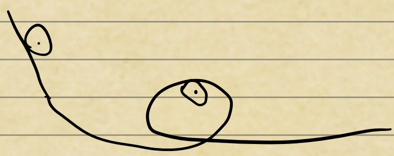


$$K = \frac{1}{2} I_0 w^2 + \frac{1}{2} M V_0^2$$



$$mgh = \cancel{\frac{1}{2} m v^2} = \frac{1}{2} \left(\frac{3}{2} m\right) v^2$$

$$v^2 = \frac{4}{3} gh$$

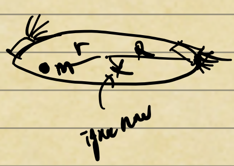


$$v^2 \geq Rg$$

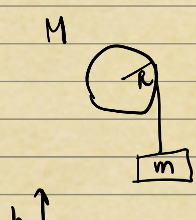
$$v^2 = Rg$$

$$mgh = mg(2R) + \frac{1}{2} (\sim) v^2$$

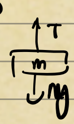
$$\tau = I\alpha$$



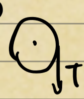
$$\tau = 2Fr \qquad \alpha = \frac{2Fr}{MR^2/2 + mr^2}$$



①



②



∃ force at pivot point too but at the CoM so ignore...

$$\begin{cases} TR = \frac{MR^2}{2} \alpha \\ mg - T = ma \end{cases}$$

h $\quad\quad$ $a = R\alpha$

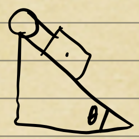
$$\Rightarrow \quad mg = a\left(m + \frac{M}{2}\right)$$

$$a = \frac{gm}{m + \frac{M}{2}}$$

$M = 0$; $a = g$

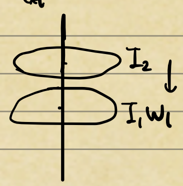
$$2ah = v^2 \qquad v^2 = \frac{2}{m + \frac{M}{2}} mgh$$

$$\underbrace{\frac{m}{2}v^2}_{K_{mass}} + \underbrace{\frac{1}{2}\left(\frac{M}{2}\right)v^2}_{K_{pulley}} = mgh$$; energy is conserved



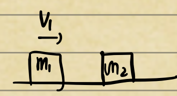
$a = \alpha R$

$\tau_{ext} = 0 \qquad L_1 + L_2 = L_1' + L_2'$ $\quad$ angular momentum is conserved if $\tau_{ext} = 0$

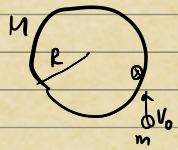


$$I_1 w_1 = (I_1 + I_2) w$$

Similar to ...



$$I_1 w_1 = (I_1 + mr^2) w$$

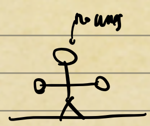


$$w = \frac{v_0}{R}$$

angular momentum

$$L = mR^2 w$$
$$= mR^2 \frac{v_0}{R} = mv_0 R = pR$$

$$p = mv \qquad \frac{dp}{dt} = F = 0$$



$$L = I_1 w_1$$
$$= I_2 w_2$$

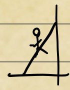
$$\frac{1}{2} I_1 w_1^2$$
$$\wedge$$
$$\frac{1}{2} I_2 \left(\frac{I_1}{I_2}\right)^2 w_1^2$$

higher kinetic energy

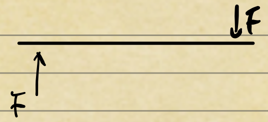
pull in $\Rightarrow$ take energy

#11 Torque

$\tau = 0$

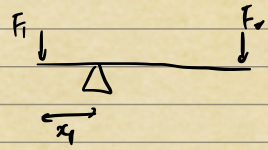


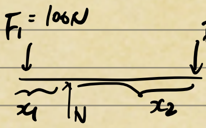
ladder: want Torque $= 0$



$$\begin{cases} \sum F_{x_i} = 0 \\ \sum F_{y_i} = 0 \\ \sum \tau_i = 0 \end{cases}$$

(ex)





1) $F_1 + F_2 = N$

$\sum F_i = 0$

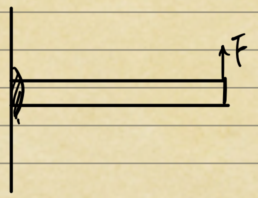
$\sum F_i x_i = 0 \Rightarrow \sum F_i (x_i + x_0) = 0$

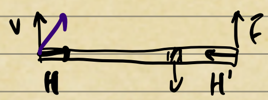
Since here $F_1 + F_2 = N$, we may pick any point we like.

(Since seesaw does not move. It should be that way)

2) $x_1 F_1 - x_2 F_2 = 0$

(ex)

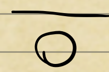




total torque downward

$\left(\frac{\sum M_i x_i}{M}\right) g M = Mgx \rightsquigarrow g$ is uniform

non-ex:



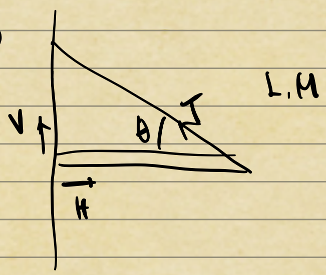
$\tau = FL$

$FL = Mg\frac{L}{2}$

$F = \frac{Mg}{2}$ $\quad V + F = Mg$

$\Rightarrow V = \frac{Mg}{2}$

(ex)



$Mg\frac{\cancel{L}}{2} = T\cancel{L}\sin\theta$

$T = \frac{Mg}{2\sin\theta}$

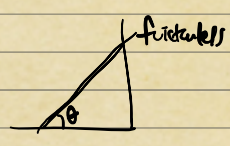
$\Rightarrow$ not make θ too small

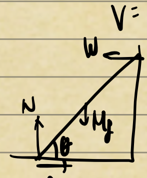
$H = T\cos\theta = \frac{Mg}{2\sin\theta}\cos\theta$

$V + T\sin\theta = Mg$

$V = \frac{Mg}{2}$

(ex)





$\sum F_x = 0 \quad W = f$

$N = Mg$

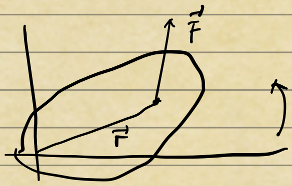
friction f

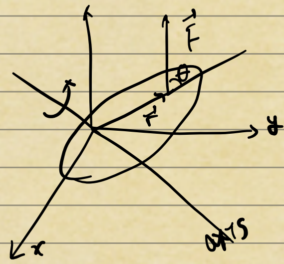
$$WL\sin\theta = Mg\frac{L}{2}\cos\theta$$

$$W = Mg\frac{\cot\theta}{2} = f \le \mu_s N = \mu_s Mg$$

$$\cot\theta \le 2\mu_s$$

$$\tan\theta \ge \frac{1}{2\mu_s}$$ minimum degree!

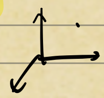




$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\tau = rF\sin\theta$$

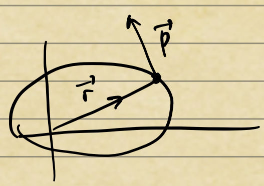
$$\vec{\tau} = \vec{r}\times\vec{F}$$



$$\vec{L} = \vec{r}\times\vec{p}$$

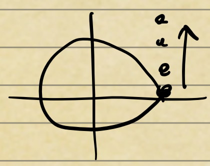
$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= \underbrace{\vec{v}\times\vec{p}}_{=0} + \underbrace{\vec{r}\times\vec{F}}_{\vec{\tau}}$$



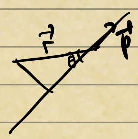
$$L = |r\times p|$$
$$= rmv$$
$$= rm\omega r$$
$$= mr^2\omega$$
$$= I\omega$$

Direction: Curly in/out → treat as a scalar



$$\vec{L} = \vec{r}\times\vec{p}$$

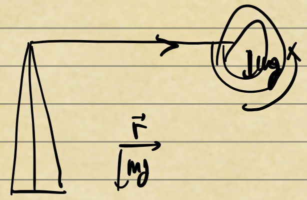
angular momentum fixed



$$\sum\frac{d\vec{L}_i}{dt} = \sum\vec{\tau}_i$$

$$\frac{d\vec{L}}{dt} = \sum\vec{\tau}_i = \vec{\tau}_{ext} + \vec{\tau}_{int}$$

not 0! but for most forces we know. yes!

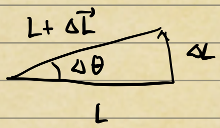


$\tau = I\alpha$

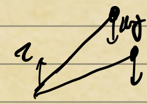
mgL

$\vec{r} \times \vec{F}$ (going in)

$\Delta L = \tau \Delta t$

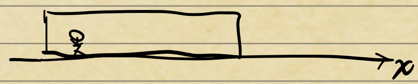


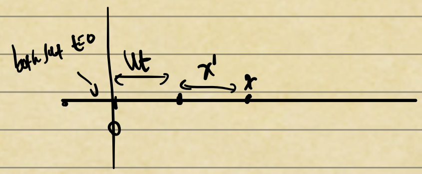
$\tau = \frac{\Delta L}{\Delta t} = \frac{L\Delta\theta}{\Delta t} = L\omega_p$



$$\omega_p = \frac{\tau}{L} = \frac{mgL}{I\omega}$$

12 Relativity





event (x, t)

Your velocity u

Crossing
me: $(x=0 \quad t=0)$
you: $(x'=0 \quad t=0)$

firecracker
me : (x, t)
you : (x', t)

$$\boxed{\begin{aligned} x' &= x - ut \\ t &= t' \end{aligned}}$$

Galilean transformation

$x' = x - ut$ $\qquad V = \frac{dx}{dt}$ velocity of bullet for S

$W = V - u$ $\qquad W = \frac{dx'}{dt}$ " " " for S'

$$a' = \frac{dw}{dt} = \frac{dv}{dt} - \cancel{\phi} = a$$

$$m\frac{d^2x}{dt^2} = F$$

$$m\frac{d^2x'}{dt^2} = F$$

$F_{12} = \frac{1}{x_1 - x_2}$ $\qquad F_{21} = -\frac{1}{x_1 - x_2}$

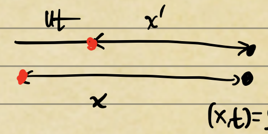
$$\begin{cases} m_1 \dfrac{d^2x_1}{dt^2} = \dfrac{1}{x_1-x_2} \\[2ex] m_2 \dfrac{d^2x_2}{dt^2} = -\dfrac{1}{x_1-x_2} \end{cases} \leftrightarrow \quad m_1 \dfrac{d^2x_1'}{dt'^2} = \dfrac{1}{x_1'-x_2'} \qquad m_2 \dfrac{d^2x_2'}{dt'^2} = -\dfrac{1}{x_1'-x_2'}$$

Aether carries light? No. (Michelsen experiment)

① All inertial observers are equivalent w.r.t. all natural laws

② Velocity of light is indpt of the state of motion of source/observer

$$F_{12}' = \frac{1}{x_1'-x_2'} = \frac{1}{x_1-x_2} = F_{12}$$



$x' = (x-ut)\gamma$ → symmetry of two observers

$x = (x'+ut')\gamma$

$(x,t)=S$

$(x',t')=S'$

$x' = ct'$

$x = ct$

$$xx' = \gamma^2(xx' + xut' - x'ut - u^2tt')$$

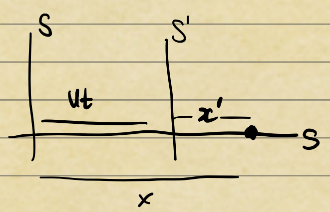
$$1 = \gamma^2\left(1 + \cancel{\frac{ut'}{x'} - \frac{ut}{x}} - u^2\left(\frac{t}{x}\right)\left(\frac{t'}{x'}\right)\right)$$

$$1 = \gamma^2\left(1 - \frac{u^2}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad x' = \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t' = \frac{t-\frac{ux}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\frac{ux}{c^2} = \frac{u}{c}\underset{\text{unit }x}{\underbrace{t}}$$

#13 Lorentz transformation



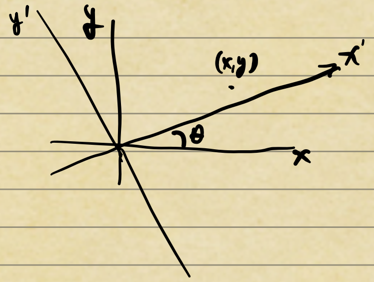
$x=0 \quad t=0$

$x'=0 \quad t'=0$

$x' = (x-ut)\gamma$

$x = (x'+ut')\gamma$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad x' = \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t' = \frac{t-\frac{ux}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$



$x' = x\cos\theta + y\sin\theta \quad = r(\cos\alpha\cos\theta + \sin\alpha\sin\theta) = r\cos(\alpha-\theta)$

$y' = -x\sin\theta + y\cos\theta \quad = r\sin(\alpha-\theta)$

$\theta = \frac{\pi}{4} \qquad x' = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \qquad (1,1) \Rightarrow y' = 0$

$y' = -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$

$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} & \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix}$

$$\begin{cases} x' = \dfrac{x + ut}{\sqrt{}} \\ t' = \dfrac{t + \frac{ux}{c^2}}{\sqrt{}} \end{cases}$$

(just put $u \to -u$)

(ex) event 1 & 2

$x_1' = \frac{x_1 - ut_1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t_1' = \frac{t_1 - \frac{ux_1}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$

$x_2' = \frac{x_2 - ut_2}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t_2' = \frac{t_2 - \frac{ux_2}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$

$\Delta x' = x_2' - x_1' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}} \qquad \Delta t' = \frac{\Delta t - \frac{u\Delta x}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$

Event 1. fire the gun

2. bullet hits the wall

V = velocity of bullet for me S

$V = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ (assume this in this case)

W = velocity of bullet for you S' $\qquad W = \frac{\Delta x'}{\Delta t'} \to \frac{dx'}{dt'}$

$$W = \frac{\Delta x - u\Delta t}{\Delta t - \frac{u\Delta x}{c^2}}$$

$$= \frac{\frac{\Delta x}{\Delta t} - u}{1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}} = \boxed{\frac{V-u}{1-\frac{uV}{c^2}} = W}$$

$c \to \infty$: good old days

$V = \frac{W+u}{1+\frac{uW}{c^2}} \qquad$ (ex) $\frac{\frac{3}{4}c + \frac{3}{4}c}{1+\left(\frac{3}{4}\right)^2} = 1.5c \; \frac{16}{25} = 0.96c$

$$W = \frac{v-u}{1-\frac{uv}{c^2}} \xrightarrow[v=c]{\text{test}} W = \frac{(c-u)}{\underbrace{1-\frac{uc}{c^2}}_{\text{"+ for you"}}} = c$$

(ex)

(ex) $\Delta t' = \dfrac{\Delta t - \frac{u\Delta x}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$ $\qquad \Delta t = 0 \rightarrow \Delta t' \neq 0$

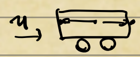
"Simultaneity is relative"

Same time, same place

$\Delta t = 0 = \Delta x$

$\Rightarrow \Delta x' = \Delta t' = 0$ $\qquad$ so same time & same place ⇝ same time & same place for all ppl

(ex)



$\Delta t < 0 \leftarrow \Delta x' > 0 \quad \Delta t' = 0$

tick $(0, 0)$

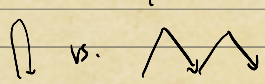
tock $(0, \tau_0)$ ← time period

$\Delta x = 0 \qquad \Delta t = \tau_0$

$$\Delta t' = \frac{\tau_0}{\sqrt{1-(\frac{u}{c})^2}}$$

Similarly $\Delta t = \dfrac{\tau_0}{\sqrt{1-(\frac{u}{c})^2}}$

Me and you accuse each other "your clock is slow"

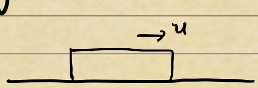


(ex) Twin paradox : Alice (inertial frame) Bob (fast go and back) ← includes acceleration

(ex) Muons

"Every clock runs the fastest in its own rest frame"

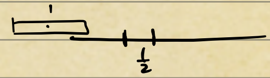
(ex) Length contraction

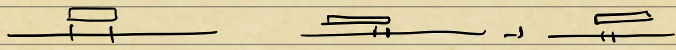


$$L_0 = \Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$= \frac{L}{\sqrt{1-\frac{u^2}{c^2}}} \qquad L = \gamma L_0 \qquad \underbrace{L_0 = \frac{L}{\gamma}}_{\text{length contraction}} \Rightarrow \text{Contraction}$$

(a)





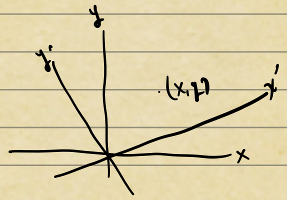
grand eye   car eye

14 Four-Vector

$$x' = \frac{x - ut}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t' = \frac{t - \frac{ux}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\rightarrow \Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}} \qquad \Delta t' = \frac{\Delta t - \frac{u\Delta x}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$\Delta x = 0 \Rightarrow$ slower



: frame of reference imp!

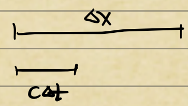
$$\Delta t' = \frac{\Delta t - \frac{u\Delta x}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$\Delta t > 0 \rightarrow \Delta t' < 0$

: happens when $\frac{u\Delta x}{c^2} > \Delta t$

$$\frac{u\Delta x}{c} > c\Delta t$$

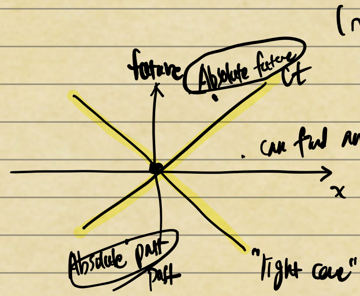
$$\frac{u}{c} > c\frac{\Delta t}{\Delta x}$$



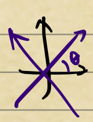
like how fast info can be passed

So cause & effect won't be messed up

(no signal is faster than light)



. can find an observer who says this happens before 0



$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$x^2 + y'^2 = x^2 + y^2$ invariant

$\cancel{t^2 \times x^2}$

$X = (x_0, x_1)$, $x_0 = ct$, $x_1 = x$

can have different convention (x_1, x_2, x_3, x_4), $x_4 = ct$ 'fourth coordinate'

$$x = (x_0, x_1, x_2, x_3) = (x_0, \vec{r})$$

$$x' = \frac{x - ut}{\sqrt{}} = \frac{x - \frac{u}{c} \cdot ct}{\sqrt{}} = \frac{x_1 - \beta x_0}{\sqrt{1-\beta^2}} \qquad \beta = \frac{u}{c}$$

$$x_1' = \frac{x_1 - \beta x_0}{\sqrt{1-\beta^2}}$$

$$t' = \frac{t - \frac{u}{c}\frac{x}{c}}{\sqrt{1-\beta^2}} \Rightarrow x_0' = \frac{x_0 - \beta x_1}{\sqrt{1-\beta^2}}$$

$$x_0' = \frac{x_0 - \beta x_1}{\sqrt{1-\beta^2}}$$

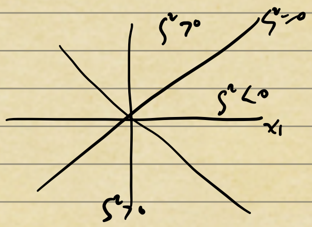
$x_2' = x_2$, $x_3' = x_3$

$$x_0'^2 - x_1'^2 = \frac{x_0^2 + \beta^2 x_1^2 \cancel{- 2\beta x_0 x_1} - (x_1^2 + \beta^2 x_0^2 \cancel{- 2\beta x_0 x_1})}{1-\beta^2}$$

$$= x_0^2 - x_1^2$$

$$x_0'^2 - x_1'^2 - x_2'^2 - x_3'^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 \qquad \text{"space time interval"}$$

$$= s^2$$



$s^2 > 0$: time like separation

$X = (x_0, \vec{r})$

four vector

"dot" product $\rightarrow$ Minkowski inner product

$$X \cdot X = (x_0^2 - |\vec{r}|^2) = x_0^2 - x_1^2 - x_2^2 - x_3^2$$

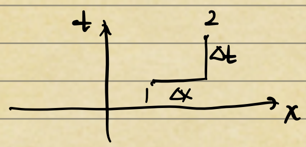
Say $\bar{X} = (\bar{x}_0, \bar{\vec{r}})$

$$X \cdot \bar{X} = X' \cdot \bar{X}'$$

$$\left(\begin{array}{l} X \to X' = (x_0', \vec{r}\,') \\ \bar{X} \to \bar{X}' = (\bar{x}_0', \bar{\vec{r}}\,') \end{array} \right.$$

$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$

$(\Delta x_0)^2 - (\Delta x_1)^2 = (\Delta x_0')^2 - (\Delta x_1')^2 = (\Delta s)^2$



$\Delta x \rightarrow$ distance particle travels

$\Delta t \rightarrow$ time in which we did this

$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 = (c\Delta t)^2\left(1 - \frac{v^2}{c^2}\right)$ $\qquad v = \frac{\Delta x}{\Delta t}$

$\underline{\Delta s} = c\Delta t\sqrt{1 - \frac{v^2}{c^2}}$

invariant

For particle time diff $= \Delta\tau$

space diff $= 0$

$(\Delta s)^2 = (c\Delta\tau)^2$

time in particle's frame

proper time $\qquad ds = c\,\underline{\underline{d\tau}}$

invariant

$\Delta\tau = \Delta t\sqrt{1 - v^2/c^2}$ $\qquad$ Indpt of reference frame

$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}$

$\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

x, y $\quad$ time

$\vec{r} = \vec{i}x + \vec{j}y$

$\frac{\Delta\vec{r}}{\Delta t} = \vec{v}$

$m\vec{a} = \vec{F}$

$X = (x_0, x_1, \cdots)$

$x_0 = ct$

$\frac{\Delta X}{\Delta\tau} = \left(\frac{\Delta x_0}{\Delta\tau}, \frac{\Delta x_1}{\Delta\tau}, \frac{\Delta x_2}{\Delta\tau}, \cdots\right)$

Four velocity

$V = \frac{dX}{d\tau} = \frac{dX}{dt}\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(\frac{dx_0}{dt}, \overbrace{\frac{dx_1}{dt}, \frac{dx_2}{dt}, \frac{dx_3}{dt}}^{\vec{v}}\right)$

Four momentum

$P = m_0 V = \left(\frac{m_0 c}{\sqrt{1 - \frac{v^2}{c^2}}}, \underbrace{\frac{m_0\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}}_{p}\right)$

$P_0 = \frac{m_0 c}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 c\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

$$= m_0 c \left(1 + \frac{v^2}{2c^2} + \cdots\right)$$

$$= m_0 c + \frac{m_0 v^2}{2c} + \cdots$$

$$cP_0 = m_0 c^2 + \underbrace{\frac{m_0 v^2}{2}}_{} + \cdots$$

$$cP_0 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

15

$$X = (ct, \vec{r}) = (x_0, x_1, x_2, x_3)$$

$$x_1' = \frac{x_1 - \beta x_0}{\sqrt{1-\beta^2}} \qquad x_0' = \frac{x_0 - \beta x_1}{\sqrt{1-\beta^2}} \quad \left(\beta = \frac{u}{c},\ \gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

V: motion of actual particles

u: speed rel. to your frame & any frame

$$X \cdot X = x_0^2 - x_1^2 \Rightarrow x_0^2 - \vec{r}\cdot\vec{r} = s^2$$ STI (spacetime interval)

$$(\Delta s)^2 = (\Delta x_0)^2 - (\Delta x_1)^2$$

$$\Delta s = \sqrt{(c\Delta t)^2 - (\Delta x)^2}$$

$$= c\Delta t\sqrt{1 - \left(\frac{\Delta x}{\Delta t}\right)^2 \frac{1}{c^2}}$$

time according to particle

$$d\tau = dt\sqrt{1 - \left(\frac{dx}{dt}\right)^2 \frac{1}{c^2}}$$

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \left(m\frac{dx_0}{d\tau}, m\frac{dx_1}{d\tau}\right) = \left(\underbrace{cm\frac{dt}{d\tau}}_{P_0}, \underbrace{m\frac{dx}{d\tau}}_{P_1}\right)$$

$$= \left(\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= (P_0, \vec{P}) = \left(\frac{E}{c}, \vec{P}\right)$$

$$x_0' = \frac{x_0 - \beta x_1}{\sqrt{1-\beta^2}}$$

$$m\frac{dx_0'}{d\tau} = \frac{m\frac{dx_0}{d\tau} - \beta m \frac{dx_1}{d\tau}}{\sqrt{1-\beta^2}}$$

so $P_0' = \dfrac{P_0 - \beta P_1}{\sqrt{1-\beta^2}}$

also, $P_1' = \dfrac{P_1 - \beta P_0}{\sqrt{1-\beta^2}}$

$$\left(\beta = \frac{u}{c}\right)$$

$$P_0 = mc\left(1 + \frac{v^2}{2c^2} + \cdots\right)$$

$$= mc + \frac{mv^2}{2c} + \cdots$$

$$c \cdot P_0 = mc^2 + \frac{mv^2}{2} + \cdots$$

correction for higher velocity

$$\boxed{X = (ct, \vec{r}) = (x_0, x_1, \ldots)}$$

$$\boxed{p = \left(\frac{E}{c}, \vec{p}\right) = (P_0, P_1, \cdots)}$$ → "energy momentum four vector"

$$p = \left(\frac{E}{c}, p\right)$$

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p \cdot p = p_0^2 - p^2 = m^2c^2 \qquad (\text{no } v^2)$$

can also get by just putting $v=0$: $(mc)^2 - 0^2 = m^2c^2$

spatial part x

$$\left(\frac{E}{c}\right)^2 - p^2 = m^2c^2$$

$$\boxed{E^2 - c^2p^2 = m^2c^4}$$

Photon: no rest mass so $E^2 = (cp)^2$

So invariant square $\left(\frac{E}{c}\right)^2 - p^2 = 0$

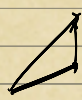
Summary: $p = \left(\frac{E}{c}, \vec{p}\right) = \left(\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ $\qquad p \cdot p = m^2c^2$

$$k = \left(\frac{w}{c}, \vec{k}\right) \qquad w = kc \Rightarrow k \cdot k = 0$$

photon momentum

Point about momentum & Energy: Conservation

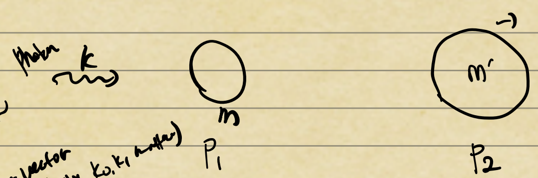
$$P_{in} = P_1 + P_2 = P_3 + P_4 + P_5 = P_{out}$$



$$\Rightarrow P'_{in} = P'_1 + P'_2 = P'_3 + P'_4 + P'_5 = P'_{out}$$

You can't omit $\sqrt{1-\frac{v^2}{c^2}}$ for example...

(ex)



Energy is conserved in relativity

zero invariant length

four vector (only k_0, k_1 matter)

$$k + P_1 = P_2$$

photon's energy

$$\left(\frac{w}{c}, k\right) \quad (mc, 0) \qquad \left(\frac{m'c}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{m'v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$w = kc$$

but $m>0$

$$\Rightarrow ① \frac{w}{c} + mc = \frac{m'c}{\sqrt{1-\frac{v^2}{c^2}}} \qquad ② k = \frac{m'v}{\sqrt{1-\frac{v^2}{c^2}}} \qquad ③ (\quad)^2 - (\quad)^2 = m'^2c^2$$

solve for m' (& v)

$(P_2)^2 = m'^2 c^2$

$= (P_1 + k)^2$

$= P_1^2 + \cancel{k^2} + 2 P_1 \cdot k$ $\quad P_1 = (mc, 0) \quad k = (\frac{w}{c}, k)$

$w = kc$ (photon energy → momentum)

$= m^2 c^2 + 2(m \cdot w - 0)$

$$\boxed{m' = \sqrt{\frac{m^2c^2 + m \cdot w}{c^2}}}$$

$\cancel{\frac{w}{c^2} = \Delta m}$ there is still velocity after collision

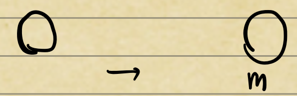
Again:

* $\sum P_{in} = \sum P_{out}$

$\Leftrightarrow \sum P'_{in} = \sum P'_{out}$ Since trans = of P is like Lorentz & they're like vectors

m is rest mass.

(ex)



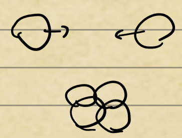
Collider... $\overset{\text{at rest}}{p + p} \rightarrow p\ p\ p\ \bar{p}$ ← anti proton

"proton"

$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = 3mc^2$ Underestimated (momentum is preserved too!)

$P_{in} = (E + m, p)$

$= P_F \qquad P_{in}^2 = P_F^2$



$P_F^{CM} = (4m, 0)$

$P_F^2 = 16m^2$

$P_{in}^2 = P_F^2 = 16m^2$

$(E+m)^2 - p^2 = 16m^2$

$\underbrace{E^2 + m^2 + 2Em - p^2}_{m^2} = 16m^2$

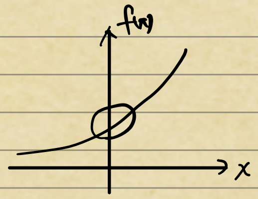
$(\frac{E}{c}, p)^2 = m^2$

$E^2 = p^2 + m^2$

$\boxed{E = 7m}$

$\Rightarrow \boxed{E = 7mc^2}$

#16 Taylor series



$$f(x) \simeq f(0) + \underbrace{f'(0)}_{\left.\frac{df}{dx}\right|_{x=0}} x + f''(0)\frac{x^2}{2}$$

$f(x) = \frac{1}{1-x}$ $\quad f(0)=1$ $\quad f'(x) = \frac{1}{(1-x)^2}$ $\quad f''(0)=2$

$f'(0) = 1$

$= 1+x+x^2+\cdots$

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2} + \frac{(ix)^3}{3} + \cdots$$

$$= \left(1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots\right)$$

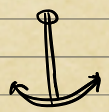
$$+ i\left(x - \frac{x^3}{3!} \cdots\right)$$

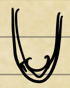
$$= \cos x + i\sin x$$

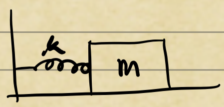
$e^{i\pi} = -1 \qquad e^{i\pi}+1=0$

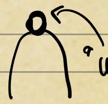
$e^{-ix} = \cos x - i\sin x$

$\cos x = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2}$







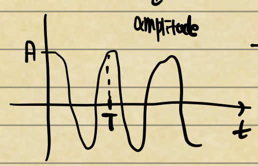


"unstable equilibrium"

$m\frac{d^2x}{dt^2} = -kx$

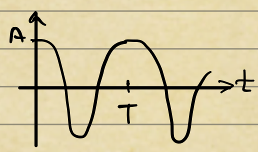
$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}}$

$x(t) = \underset{\text{amplitude}}{A}\cos(\omega t) \qquad \omega = \sqrt{\frac{k}{m}}$



$T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$

indpt of A



$A\cos(\omega t + \underset{\text{"phase"}}{\phi})$ $\qquad x=5 \quad v=0 \quad t=0$

$A\cos\phi = 5$

$-\omega A\sin(\omega t+\phi)|_{t=0} = 0$

$-wA\sin\phi = 0$

$\phi = 0 \Rightarrow A = 5$

$\Rightarrow \boxed{x = 5\cos wt}$

$x(t) = A\cos wt$ $\qquad -A \sim A$

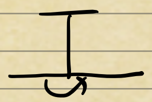
$V(t) = -wA\sin(wt)$ $\qquad -wA \sim wA$

$a = -w^2 \underbrace{A\cos(wt)}_{x}$ $\qquad -w^2A \sim w^2A$

$= -w^2 x$

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

(ex)

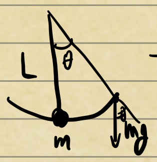


$I\frac{d^2\theta}{dt^2} = \tau$

$\approx -k\theta$ $\qquad w = \sqrt{\frac{k}{I}}$

(analogue: $m\frac{d^2x}{dt^2} = -kx$ $\qquad w = \sqrt{\frac{k}{m}}$)

(ex)



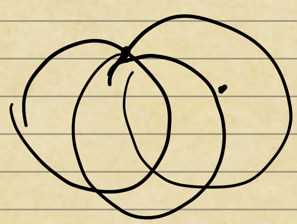
$\tau = mgl\sin\theta$ $\qquad I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$

$\simeq -mgl\theta$

$\equiv -k\theta$ $\qquad k = mgl$

$w = \sqrt{\frac{k}{I}} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{\frac{g}{l}} = \frac{2\pi}{T}$

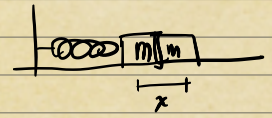
$T = 2\pi\sqrt{\frac{l}{g}}$ $\qquad$ for small θ, T does not depend on amplitude



find w

#17

(ex)



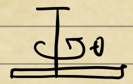
$F = -kx$ restoring force

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x \qquad \omega = \sqrt{\frac{k}{m}}$$

$$x = A\cos(\omega t - \varphi)$$

(ex)



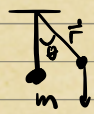
$$\tau = I\alpha \qquad \alpha = \frac{d^2\theta}{dt^2}$$

kappa

$$= -\kappa\theta$$

⟿ analogous to $F = ma$

$$\boxed{\theta = A\cos(\omega t - \varphi)} \qquad \omega = \sqrt{\frac{\kappa}{I}}$$



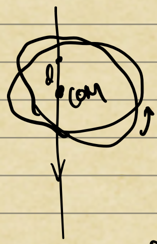
$$\tau = -rF\sin\theta$$

$$\approx -rF\theta \quad (\theta \text{ small})$$

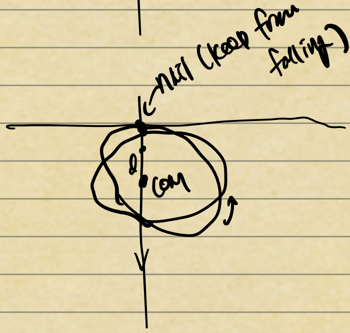
$$= -\underbrace{mgl}_{\kappa}\theta$$

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{\frac{g}{l}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



$$\tau = -mgd\theta \qquad I = I_{CM} + md^2$$



$$Z = |z|\, e^{i\varphi}$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$x = Ae^{\alpha t}$ ansatz

$$\cancel{A}\alpha^2 \cancel{e^{\alpha t}} = -\omega_0^2 \cancel{A} \cancel{e^{\alpha t}}$$

$\alpha = \pm i\omega_0$ or $A=0$

$x_1(t) = Ae^{i\omega_0 t}$ $\quad x_2(t) = Ae^{-i\omega_0 t}$

Linear equation $\Rightarrow$ Superposition

$\ddot{x}_1 + \omega_0^2 x_1 = 0$

$\ddot{x}_2 + \omega_0^2 x_2 = 0$

$(\ddot{x_1 + x_2}) + \omega_0^2 (x_1 + x_2) = 0$

$x(t) = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}$ $\qquad \omega_0 = \sqrt{\frac{k}{m}}$

$\quad \|$

$x^*(t) = A^* e^{-i\omega_0 t} + B^* e^{i\omega_0 t}$

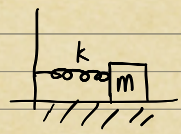
$A = B^*$ $\quad B = A^*$

$x(t) = Ae^{i\omega_0 t} + A^* e^{-i\omega_0 t}$ $\qquad A = |A|e^{i\phi}$

$= |A|e^{i(\omega_0 t + \varphi)} + |A|e^{-i(\omega_0 t + \varphi)}$

$= 2|A|\cos(\omega_0 t + \varphi)$

$= C\cos(\omega_0 t + \varphi)$



$m\ddot{x} = -kx - b\dot{x}$

$m\ddot{x} + b\dot{x} + kx = 0$

$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$

$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$ $\qquad \gamma = \frac{b}{m}$

$x = Ae^{\alpha t}$

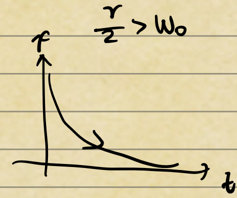
$\underbrace{Ae^{\alpha t}(\alpha^2 + \alpha\gamma + \omega_0^2)}_{=0} = 0$

$\alpha^2 + \alpha\gamma + \omega_0^2 = 0$

$\alpha = \dfrac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} = -\dfrac{\gamma}{2} \pm \sqrt{\left(\dfrac{\gamma}{2}\right)^2 - \omega_0^2} = (\alpha_\pm)$

$x(t) = Ae^{\alpha_+ t} + Be^{\alpha_- t}$ $\qquad \frac{\gamma}{2} > \omega_0$

overdamped $\qquad = Ae^{-[\frac{\gamma}{2} - \sim]t} + Be^{-[\frac{\gamma}{2} + \sim]t}$



$v(0) = A\alpha_+ + B\alpha_-$

$x(0) = A + B$

$\left.\frac{dx}{dt}\right|_{t=0} = \alpha_+ A + \alpha_- B \qquad A = A^*,\ B = B^*$

$$x = Ae^{-\left(\frac{\gamma t}{2} + i\omega' t\right)} + Be^{-\left(\frac{\gamma}{2}t - i\omega' t\right)} \qquad \omega' = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2} \qquad \left(\frac{\gamma}{2} < \omega_0\right)$$

underdamped $\Rightarrow x(t) = C e^{-\frac{\gamma t}{2}} \cos(\omega' t + \phi)$

$\gamma = 0 \Rightarrow$ oscillation

damped oscillation

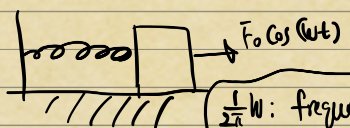
$x(0) = A + B$

$v'(0) = \left. -C e^{-\frac{\gamma t}{2}} \sin(\omega' t + \phi)\, \omega' \right|_{t=0}$

$= -C \sin\phi\, \omega'$

$\frac{\gamma}{2} = \omega_0 \Rightarrow x(t) = Ae^{-\frac{\gamma t}{2}} + Bte^{-\frac{\gamma t}{2}}$

ex)



"driven Oscillator"

$\frac{1}{2\pi}\omega$: frequency

$\frac{2\pi}{\omega}$: period

$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \left(\frac{F}{m}\right) \cos(\omega t)$ ← what you give me sake

$i\ddot{y} + i\gamma \dot{y} + i\omega_0^2 y = i\left(\frac{F}{m}\right) \sin(\omega t)$ ← my artifact

$z = x + iy \qquad \ddot{z} + \gamma \dot{z} + \omega_0^2 z = \left(\frac{F}{m}\right) e^{i\omega t} \qquad z = z_0 e^{i\omega t}$

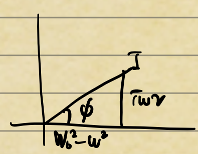
$\left(-\omega^2 z_0 e^{i\omega t} + i\omega\gamma z_0 e^{i\omega t} + \omega_0^2 z e^{i\omega t}\right) = \frac{F_0}{m} e^{i\omega t}$

$z_0\left(-\omega^2 + i\omega\gamma + \omega_0^2\right) = \frac{F_0}{m}$

$$\boxed{z_0 = \frac{\frac{F_0}{m}}{\omega_0^2 - \omega^2 + i\omega\gamma}}$$

$I = \omega_0^2 - \omega^2 + i\omega\gamma$ "impedence"

$Z = \frac{\frac{F_0}{m} e^{i\omega t}}{I}$



$= \frac{\frac{F_0}{m} e^{i\omega t - i\omega\phi}}{|I|}$

$$\boxed{x = \frac{\frac{F_0}{m}}{|I|}\cos(\omega t - \varphi)} + x_c(t)$$

solution w. no driving force

$$|I| = \sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$

$$\tan\phi = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$$

Lec 18

$$m\ddot{x} + b\dot{x} + kx = F_0\cos(\omega t)$$

$=0$; ① $\frac{\gamma}{2} > \omega_0$

Newton's law doesn't tell this

$$x = Ae^{-|\alpha_+|t} + Be^{-|\alpha_-|t}$$

get A, B by $x(0)$ & $v(0)$

$\underline{\underline{x_c(t)}}$

② $\frac{\gamma}{2} < \omega_0$

$$\alpha = -\frac{\alpha}{2} \pm i\sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

$$x = Ae^{-\frac{\gamma t}{2}}e^{i\omega' t} + Be^{-\frac{\gamma t}{2}}e^{-i\omega' t}$$

$$\Rightarrow x(t) = Ce^{-\frac{\gamma t}{2}}\cos(\omega' t + \varphi)$$

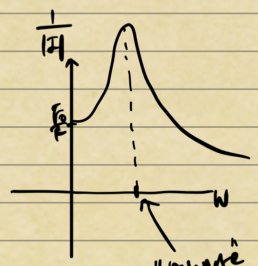
---

driving force

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \left(\frac{F_0}{m}\right)\cos(\omega t)$$

$$x(t) = \underbrace{\frac{\frac{F_0}{m}}{|I|}}_{x_0}\cos(\omega t - \varphi)$$

$$x_0 = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2}}$$



amplitude depends on frequency

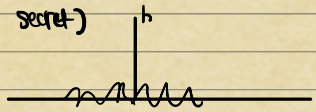
But actually ...

$$x(t) = \underbrace{\frac{\frac{F_0}{m}}{|I|}\cos(\omega t - \varphi)}_{x_p} + \underbrace{Ae^{-\frac{\gamma t}{2}}\cos(\omega' t - \varphi_0)}_{x_c}$$

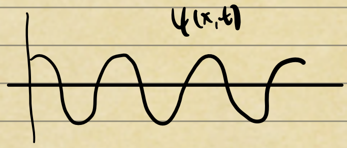
(particular) (Complimentary)

* Waves

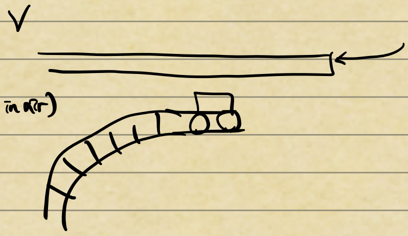
(transmitting secret)



"What happens when you excite a medium?"

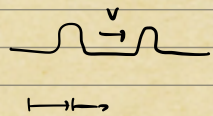


ψ is the dynamical variable

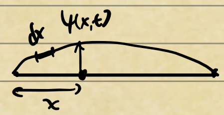


Sound
(faster than in air)

direction of medium
- transversal
- longitudinal

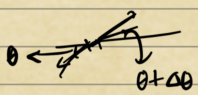


(ex)



$\mu = \frac{\text{mass}}{\text{length}}$

(F)
T = tension



$$\underbrace{T \sin(\theta + \Delta\theta) - T \sin\theta}_{\text{angles very small}} = \mu \, dx \frac{d^2\psi(x,t)}{dt^2}$$

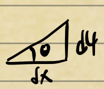
$\sin\theta \approx \theta$

$\cos\theta \approx 1$

$\tan\theta \approx \theta$

$$\frac{T\Delta\theta}{dx} dx = \mu \, dx \frac{d^2\psi}{dt^2}$$

got by Fma in tiny portion of the string



$$\boxed{T \frac{d}{dx}\Big(\underbrace{\frac{d\psi}{dx}}_{\approx \tan\theta \approx \theta}\Big) = \mu \frac{d^2\psi}{dt^2}}$$

$$\boxed{\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2}}$$

$v^2 = \frac{T}{\mu}$ $\quad v = \sqrt{\frac{T}{\mu}}$

not quite velocity
(has dimension of velocity though)

$$\boxed{\psi(x,t) = A\cos(\underbrace{kx-\omega t}_{\text{no dimension}})}$$

"a" Solution

$$\frac{d\psi}{dx} = -Ak\sin(kx-\omega t)$$

$$\frac{d^2\psi}{dx^2} = -Ak^2\cos(kx-\omega t)$$

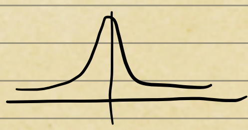
$$\frac{d^2\psi}{dt^2} = -A\omega^2\cos(kx-\omega t)$$

$$\boxed{v = \frac{\omega}{k}}$$

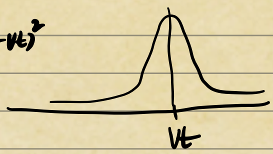
$$\boxed{\psi = A\cos[k(x-vt)]}$$

$= \psi(x-vt)$ $\Rightarrow$ Justifies v = velocity

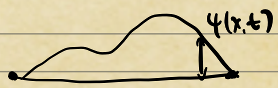
e^{-x^2}



$e^{-(x-vt)^2}$



#19 Waves



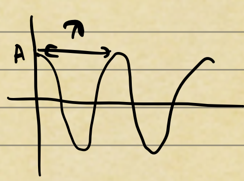
$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \qquad v^2 = \frac{T}{\mu} \equiv \frac{F}{\mu}$$

$$dx\,\underbrace{T\frac{d^2\psi}{dx^2}}_{F} = \underbrace{\mu\frac{d^2\psi}{dt^2}\,dx}_{ma}$$

$$\psi(x,t) = A\cos(kx-\omega t) \qquad \omega = kv$$

$$= A\cos(k(x-vt)) \qquad v = \sqrt{\frac{T}{\mu}}$$

$$\psi(x,t) = A\cos(\underbrace{kx}_{2\pi}-\omega t)$$



We have $k\lambda = 2\pi$

$$\underbrace{k = \frac{2\pi}{\lambda}}$$ ...

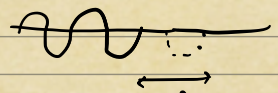
at $t=0$: $\psi(0,t) = A\cos(wt)$

$wT = 2\pi$

$w = \dfrac{2\pi}{T}$

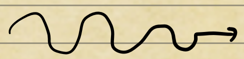
$\psi = A\cos\left(\dfrac{2\pi x}{\lambda} - \dfrac{2\pi t}{T}\right)$

$v = \dfrac{w}{k} = \dfrac{2\pi}{T}\,\dfrac{\lambda}{2\pi} = \dfrac{\lambda}{T} = \boxed{\lambda f}$



$\lambda f = v$

* Energy



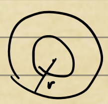
$E = \frac{1}{2}\mu\, dx\, \underbrace{(Aw)}_{\text{maximum speed}}^2$

$u = \text{energy}/\text{length} = \frac{1}{2}\mu A^2 w^2$

$P = \frac{1}{2}\mu A^2 w^2 v$ ← $u = \dfrac{dE}{dl}$ $\quad P = \dfrac{dE}{dt} = \dfrac{dE}{dl} \times \underbrace{\dfrac{dl}{dt}}_{v}$

Intensity

$I = \dfrac{\text{power}}{\text{area}} = \dfrac{P}{4\pi r^2}$ $\quad \boxed{\text{watts}/m^2}$



$\beta = 10\log\dfrac{I}{I_0}$ $\quad I_0 = 10^{-12}$ Watts$/m^2$

decibels

$I = 10^{-11}$ Watts$/m^2$

$I_0 = 10^{-12}$

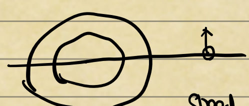
$\beta = 10$ dB

$I = 10^{-8} \Rightarrow \beta = 40$

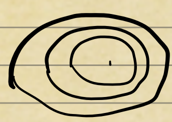
grow slowly

* Doppler effect

① $\lambda f = v$



Speed of Source $= u$



$\lambda' = \lambda - uT = \lambda - \dfrac{u}{f}$

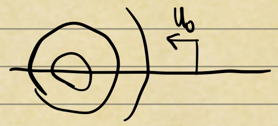
← Same

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda' - \frac{u}{f}} = f \cdot \frac{v}{\lambda f - u} = f \cdot \frac{v}{v - u}$$

$$= f \frac{1}{1 - \frac{u}{v}}$$

frequency increases

btw, LHS : $f \frac{1}{1 + \frac{u}{v}}$

② 

$$f' = \frac{u_0 + v}{\lambda} = \frac{u_0 + v}{v} f = f\left(1 + \frac{u_0}{v}\right)$$

$-$ running away

* speed depends on medium

$$\begin{cases} \psi_1 = A\cos(\omega_1 t) \\ \psi_2 = A\cos(\omega_2 t) \end{cases}$$

$$\psi = \psi_1 + \psi_2 = A\cos(\omega_1 t) + A\cos(\omega_2 t)$$

$$\psi = 2A\cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cos\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right] \qquad \omega_1 \approx \omega_2$$

loud silent loud silent

"beat"

$\omega_B = \frac{\omega_1 - \omega_2}{2}$?

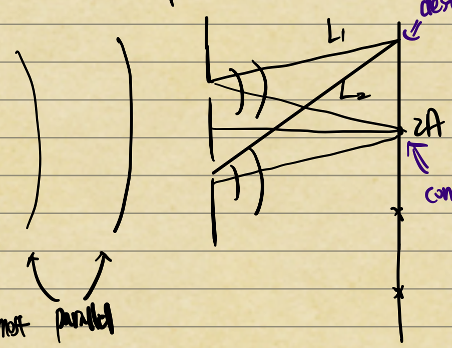
No, $\omega_1 - \omega_2$

loud

loud

(ex) piano tuning

* Double slit experiment

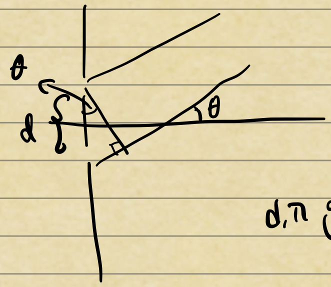


$L_1 - L_2 = \frac{\lambda}{2}$

$$\cos\left(\frac{2\pi x}{\lambda}\right) + \underbrace{\cos\left(\frac{2\pi}{\lambda}\left(x + \frac{\lambda}{2}\right)\right)}_{-\cos\left(\frac{2\pi x}{\lambda}\right)}$$

$L_1 - L_2 = m\lambda \quad \lambda = 0, \pm1, \pm2, \ldots$ constructive

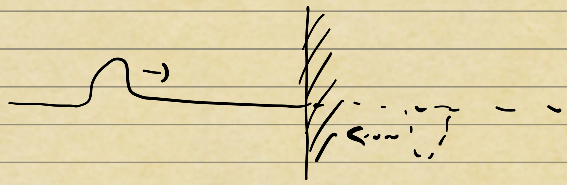
$(m+\frac{1}{2})\lambda$ —— 〃 —— ... destructive



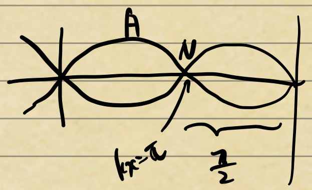
$d\sin\theta = m\lambda$

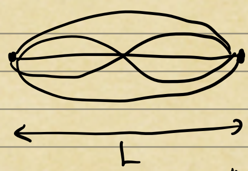
$(m+\frac{1}{2})\lambda$

d, λ given: Solve for θ



$$A(\cos(kx-\omega t) - \cos(\omega t + kx)) = 2A \sin kx \sin \omega t$$





"What is allowed?"

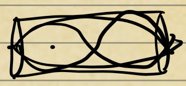
"fundamental frequency"

$\frac{\lambda}{2} = L$

$\lambda = 2L \qquad f_0 = \frac{v}{\lambda} = \frac{v}{2L}$

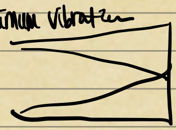
$\lambda = L \qquad f = \frac{v}{L} = 2f_0$

move back and forth latitudinally (back and forth)

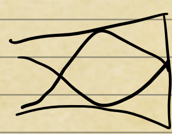


latitudinal wave

maximum vibration

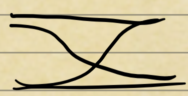


$L = \frac{\lambda}{4}$



$L = \frac{3}{4}\lambda$

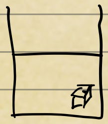
odd multiple of fund. freq.



$L = \frac{\lambda}{2}$

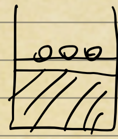
v is fixed

Fluid Dynamics



$\rho_w = 10^3\ kg/m^3$

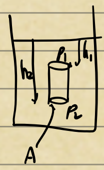
pressure $\frac{N}{m^2}$ = pascal



$P = P_0 + \frac{mg}{A}$

atmospheric pressure

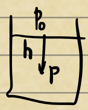
gauge pressure



$P_2 A - P_1 A = A(h_2 - h_1)\rho g$

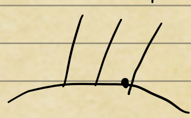
$P_2 - P_1 = \rho g (h_2 - h_1)$

$P_2 = P_1 + \rho g (h_2 - h_1)$



$\rightsquigarrow P = P_0 + g\rho h$

$P=0$



$0 + \rho g h = 10^5$ pascals

density of air

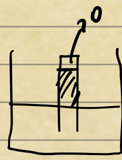
$10^5 = 10^3 \times 10 \times h$

g

density of water ρ

$h \approx 10m$ 32 feet

Barometer = pressure today

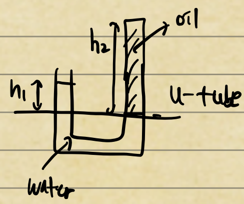


$P_A = 0 + \rho g h$

might use mercury

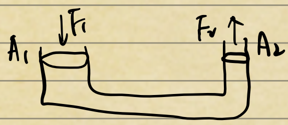
so h is smaller

u-tube



$\cancel{P_0} + \rho_1 g h_1 = \cancel{P_0} + \rho_2 g h_2$

$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \Rightarrow$ find relative density



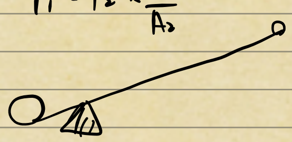
incompressible fluid

$P_1 = P_2 \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$

$F_2 = F_1 \frac{A_2}{A_1}$

$F_1 = F_2 \times \frac{A_1}{A_2} \Rightarrow$ Even if $F_1 >>>> F_{-1}$ work done is same. (no free lunch)

Similar to



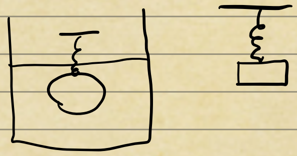
$F_2 \Delta x_2 = P_2 A_2 \Delta x_2$

$= P_2 A_1 \Delta x_1$

$= P_1 A_1 \Delta x_1$

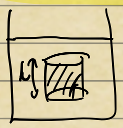
$= F_1 \Delta x_1$

Work is same

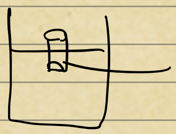


$kx = mg$

$\boxed{\text{Weight loss = weight of liquid displaced}}$



$F_B = P_2 A - P_1 A = \rho g h A =$ wt of liquid

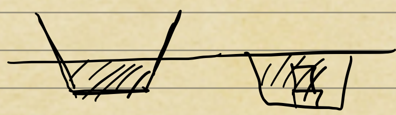


Cork

f = fraction immersed

$(f)(\rho_w \cancel{g} \cancel{V}) = \rho \cancel{g} \cancel{V}$

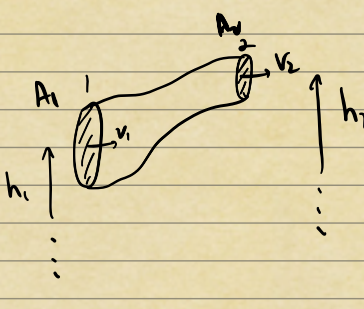
$f = \frac{\rho}{\rho_w}$



$hA\rho_w g = w$ "how much cargo?"

Salt water ~> easier to float

Bernoulli equation (fluid in motion)



Continuity

$A_1 V_1 = A_2 V_2$

flow rate going in flow rate going out

density needs to be same

pf) $A_1 V_1 \Delta t = A_2 V_2 \Delta t$

$\Rightarrow A_1 V_1 = A_2 V_2$ (volume in-out same)

$E_2 - E_1 =$ work done by ext. forces

$$\underbrace{\rho A_2 \Delta x_2}\left(\frac{v_2^2}{2} + gh_2\right) - \rho A_1 \Delta x_1 \left(\frac{v_1^2}{2} + gh_1\right) = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

no worry about height diff for now

$A_1 \Delta x_1 = A_2 \Delta x_2$

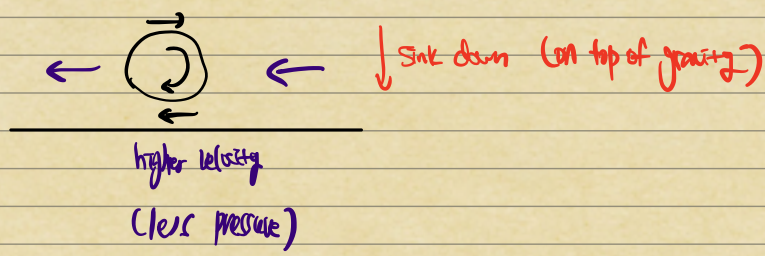
$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ (E of Consev. for unit volume)

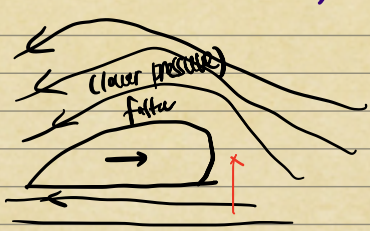
$\exists$ viscosity but ignore here

(ex)

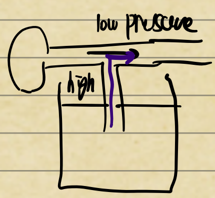
baseball



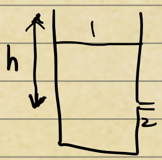
(ex)



(ex)



(ex)



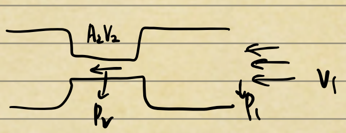
$\cancel{P_1} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \cancel{P_2} + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2}$

(at) negligible (atmosphere)

$\rho g h_1 = \frac{1}{2}\rho v_2^2$

$v_2^2 = 2gh$ (this is lecture 1. $mgh = \frac{1}{2}mv^2$)

* Venturimeter



$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

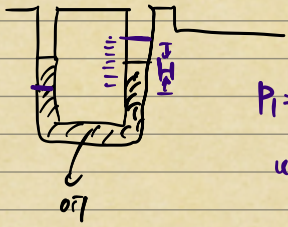
$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$

$v_2A_2 = v_1A_1$

$\Rightarrow \quad = \frac{1}{2}\rho\left(v_1^2 - v_1^2\left(\frac{A_1}{A_2}\right)^2\right)$

$= \frac{1}{2}\rho v_1^2\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$

higher, low pressure

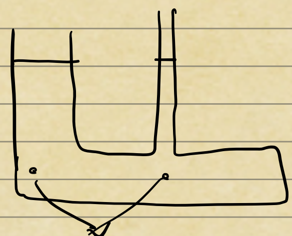


$P_1 = P_2 + \rho_o g H$

use manometer for H

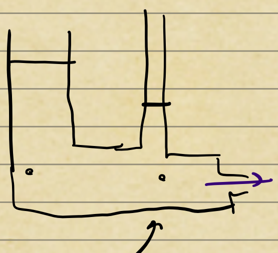
can be used to find the speed of fluid too

ex)



same pressure due to Bernoulli's principle

⇒ same height!



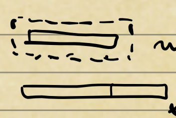
lower pressure

#21 Thermodynamics

- Equilibrium

Zero'th law

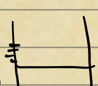
duplicates

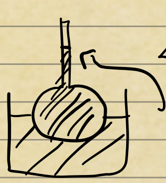


standard

Temp ↑

temperature





$\Delta V = A\,\Delta x$

very thin; need magnifier

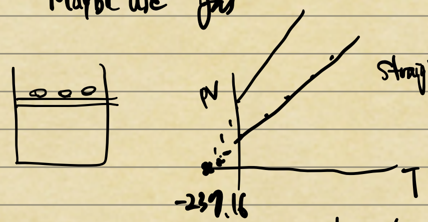
melting pt temp ~ lets say it 0°C

boiling pt temp ~ 100°C. (do it in sea level)

Divide into 100 equal parts

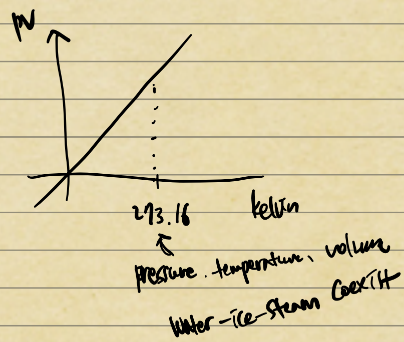
Q. what liquid should we use? Does all liquid expand at the same rate?

Maybe we gas



straight line for all (dilute) gas

so people prefer use gas for thermometer



* Heat

Caloric fluid

$\Delta Q = \underset{\text{grams}}{\underbrace{m_w}} \Delta T$ Calories

calories

kg ⟼ kcal

$\Delta Q = C\, m \Delta T$

C ↓ specific heat

$C = 1\ \text{cal}/g\cdot°C = 1\ \text{kcal}/kg\cdot°C$ for water

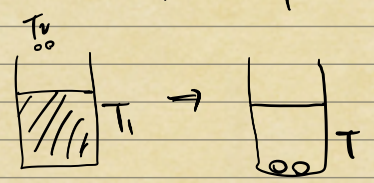
$\Delta L = \alpha L \Delta T$

α ↓ coeff of linear expansion

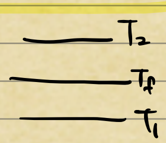
$\Delta V = \beta V \Delta T$

β ↓ coeff of volume expansion

β, α, C : vary by material



$\boxed{\text{postulate } \Sigma Q = 0}$

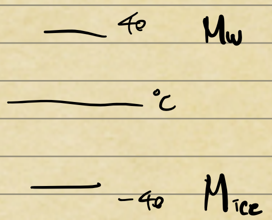


$$m_w \cdot 1 \cdot (T_f - T_I) + m_{Pb} \cdot C \cdot (T_f - T_2) = 0$$

$L_{melt} = 80\ cal/g$ for water

latent heat of melting

$\boxed{\Delta Q = m L_{melt}}$



Three scenario: in case of 0°C, transition happens incrementally

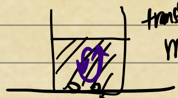
(eg. x of 80x becomes ice

Q. What makes calorie flow?

① Radiation

no medium needed

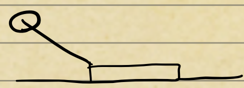
② Convection

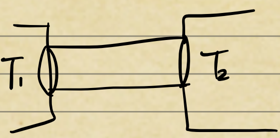


transfer through motion of material

water gets hot → expand → density ⇓ ⇒ go above

③ Conduction

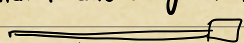




$$\frac{dQ}{dt} = -kA\frac{\Delta T}{L}$$

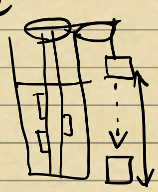
kappa : Thermal conductivity of that material

Reservoir



Slow...

Joule



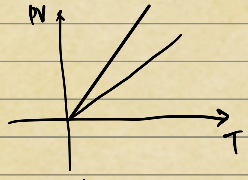
$mgh - k > 0$

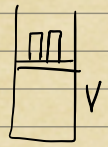
$\Delta Q = m \cdot 1 \cdot \Delta T$

$4.2\ J/cal$

Kinetic energy of atom = heat

Boltzmann Constant & First law of Thermodynamics





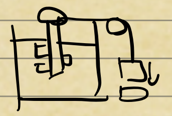
$PV = cT$

Can use any gas

"Caloric fluid" in the old days

1) loss energy ⇝ temp. increase !

2) Car slamming ⇝ heat



$4.2\,J = 1\,cal$

$PV = cT = C'MT$

mass of gas

$C'_H = \frac{1}{4} C'_{He}$

$C'_{carbon} = \frac{C_H'}{12}$

$= N_{\cdot}kT$

indpt of gas

H/He do not have the same pressure

"bolzman constant" $= 1.4 \times 10^{-23} \frac{J}{k}$

N_0 = Avogard's number

$= 6 \times 10^{23}$ "mole"

$\frac{1}{N_0}$ = mass of hydrogen atom in gram

$= n N_0 kT$

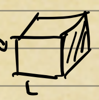
moles

universal gas constant

$= nRT$ $\boxed{R = Nk} = 8.3\,J/^\circ c = 2\,cal/^\circ c$

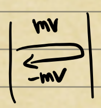
$PV = nRT = NkT$

# of moles # of atoms

gas 

Remember $F = \frac{dp}{dt}$

Assumptions

① 

$\frac{1}{3}$ molecules in one direction

② Same speed v

$\Delta p = 2mv$

$$\frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{2L}{v}} = \frac{mv^2}{L} = F_{one}$$

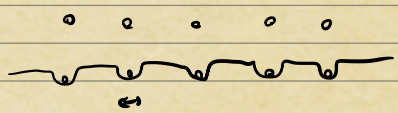
$$\bar{F} = \frac{N\,mv^2}{3L} \qquad p = \frac{\bar{F}}{L^2} = \frac{N}{3}\frac{mv^2}{L^3}$$

$$PV = \frac{N}{3} mv^2 = NkT$$

Absolute 0

$$\boxed{\frac{mv^2}{2} = \frac{3}{2} kT}$$

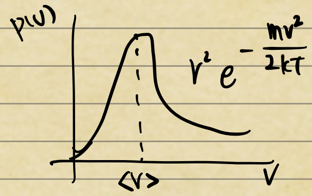
(Modified by QM later)



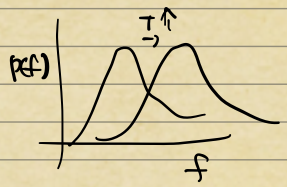
Solid at 0 temperature

atom going over : definition of melting

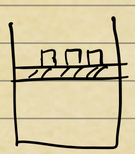
Gas:

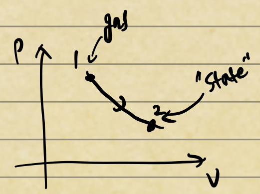


radiation



Prediction of Big Bang Theory





only for diluted gas

$PV = NkT$

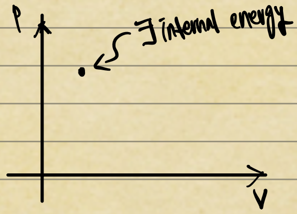
quasi-static process : use sand.

(assume completely frictionless system)

* U = internal energy of the gas $= \frac{3}{2} kTN = \frac{3}{2} nRT = \frac{3}{2} PV$

(kinetic energy of the gas molecules)

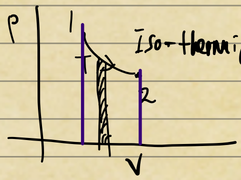
moles



First law of thermodynamics

$$U_2 - U_1 = \Delta U = \Delta Q - \Delta W = \Delta Q - P\Delta V$$

$$\Delta W = F\,dx = PA\,dx = P\,dV$$



Iso-thermi graph of a given temperature

$$W = \int P\,dV$$ ← Work done by gas

$$= \int_{V_1}^{V_2} \frac{nRT}{V}\,dV$$

$$= nRT \ln\left(\frac{V_2}{V_1}\right)$$

"Q: heat flows into the gas"

$$\Delta Q = \underbrace{\Delta U}_{=0} + W$$

reservoir

$$C = \frac{\Delta Q}{\Delta T}\frac{1}{M}$$ 1 mole not 1 /kg

- $m_j C = \frac{\Delta Q}{\Delta T}\frac{1}{n}$
- $U = \frac{3}{2}RT$
- $\frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} + \frac{P\Delta V}{\Delta T}$

$$C_v = \left.\frac{dQ}{dT}\right|_V = \frac{dU}{dT} = \frac{3}{2}R$$

constant volume

$$\Delta Q = \Delta U + P\Delta V$$

$$\Delta Q_p = \frac{3}{2}R\Delta T + \Delta(PV)$$

$$= \frac{3}{2}R\Delta T + R\Delta T$$

$$\left.\frac{\Delta Q}{\Delta T}\right|_p = \frac{3}{2}R + R = \frac{5}{2}R$$

more ~ dumbbell & rotate, etc

$C_v = \frac{3}{2}R$ → monoatomic gas

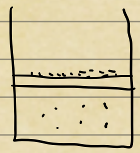
volume held fixed

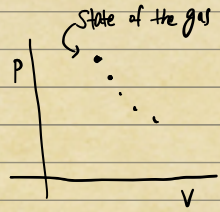
$C_p = \frac{5}{2}R$

pressure held fixed

$\frac{C_p}{C_v} = \frac{5}{3} = \gamma$ can be diff in other cases

\# 23





$PV = nRT$

[quasi-static
reversible

U = Internal energy ⇒ State variable

$= \frac{3}{2} N k T$

$= \frac{3}{2} nRT$

$= \frac{3}{2} PV$

$\Delta U = U_2 - U_1 = \Delta Q - \Delta W$

$\Delta W = P\, dV$

way to change temperature

''

kinetic energy

$W = \int_{V_1}^{V_2} P\, dV$

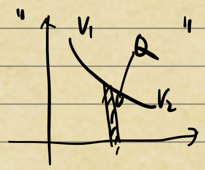
$= nRT \int_{V_1}^{V_2} \frac{dV}{V}$

$= nRT \ln \frac{V_2}{V_1}$

$U_2 - U_1 = 0$

$\Delta Q = \Delta W$

Say...



$C = \frac{\Delta Q}{\Delta T}$

1 mole

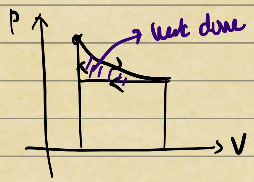
$C_V = \left.\frac{\Delta Q}{\Delta T}\right|_V$

$C_p = \left.\frac{\Delta Q}{\Delta T}\right|_p$

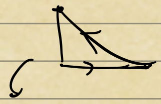
$\Delta Q = \Delta U + P \Delta V$

why $C_p > C_v$ b.c. piston move up ⇝ cooler

$\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ in this case ...



$W_{cycle} = \oint P \, dV$



work done ⇒ negative

$\Delta U = 0$

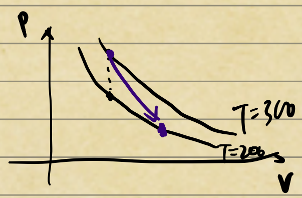
$\Rightarrow \quad W_{done} = Q_{in}$

So there is no notion of "heat in the gas"

Isobaric : pressure fixed

* Adiabatic process : $\Delta Q = 0$

(ex.)



$P(V)$?

$0 = \Delta U + P\Delta V$

$= nC_v \Delta T + P\Delta V$ (1 mole)

$p = \frac{nRT}{V}$

$\Rightarrow \quad C_v \Delta T + RT\frac{\Delta V}{V} = 0$

$\frac{C_v}{R}\frac{\Delta T}{T} + \frac{\Delta V}{V} = 0$

$\frac{C_v}{R}\int\frac{dT}{T} + \int\frac{dV}{V} = 0$

$\frac{C_v}{R}\ln\frac{T_2}{T_1} + \ln\frac{V_2}{V_1} = 0$

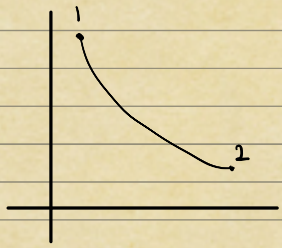
$\left(\frac{T_2}{T_1}\right)^{\frac{C_v}{R}}\frac{V_2}{V_1} = 1$

$$\boxed{T_2^{\frac{C_v}{R}} V_2 = T_1^{\frac{C_v}{R}} V_1}$$

$PV = RT$

$$T_2 V_2^{\frac{R}{C_v}} = T_1 V_1^{\frac{R}{C_v}}$$

$$P_2 V_2^{1+\frac{R}{C_v}} = P_1 V_1^{1+\frac{R}{C_v}} \Rightarrow \boxed{\begin{array}{l} P_2 V_2^{\gamma} = P_1 V_1^{\gamma} \\ \gamma = 1 + \frac{R}{C_v} = \frac{C_v + R}{C_v} = \frac{C_p}{C_v} \end{array}}$$



$$W = \int p(v)\, dV \qquad PV^{\gamma} = C$$

$$= C \int_{V_1}^{V_2} \frac{dV}{V^{\gamma}}$$

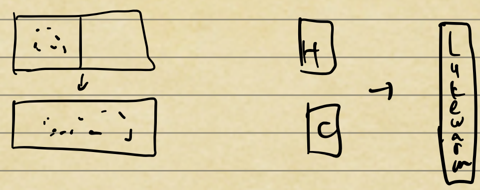
$$= C \, \frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$$

$$= \boxed{\frac{P_1 V_1 - P_2 V_2}{\gamma - 1}}$$ $\Leftarrow$ work done in an Adiabatic process

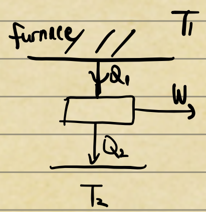
II Law

(ex)



Reversal does not violate conservation of energy

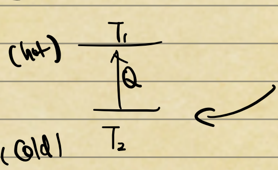
Carnot



$$W = Q_1 - Q_2$$

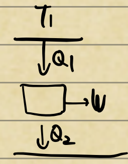
$$\eta_{eff} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

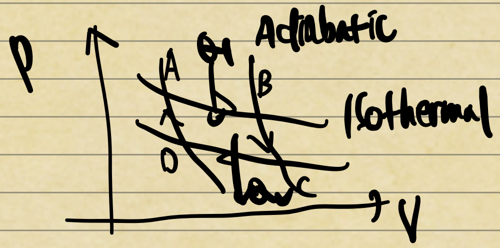
Law II



Cannot have a process whose sole effect is this

Carnot engine





A → B → C

take out sand

take out more sand

& isolate

$$\eta_{eff} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$Q_1 = nRT_1 \ln\frac{V_B}{V_A}$$

$$Q_2 = nRT_2 \ln\frac{V_C}{V_D}$$

$$\Rightarrow \eta_{eff} = 1 - \frac{T_2}{T_1}\frac{\ln\frac{V_C}{V_D}}{\ln\frac{V_B}{V_A}}$$

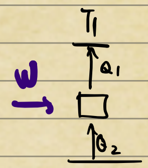
$$= 1 - \frac{T_2}{T_1}$$ ... well, no engine can be better than this

Rmk. $\frac{V_C}{V_D} = \frac{V_B}{V_A}$

pf) $V_A T_1^{()} = V_D T_2^{()}$

$V_B T_1^{()} = V_C T_2^{()}$

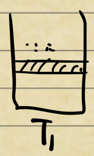
$\Rightarrow \frac{V_B}{V_A} = \frac{V_C}{V_D}$

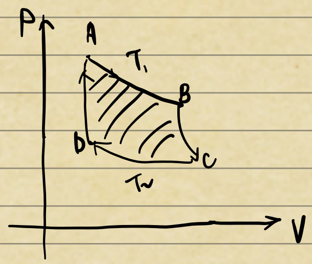


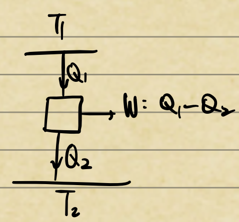
Refrigerator

not violated the law II

#24







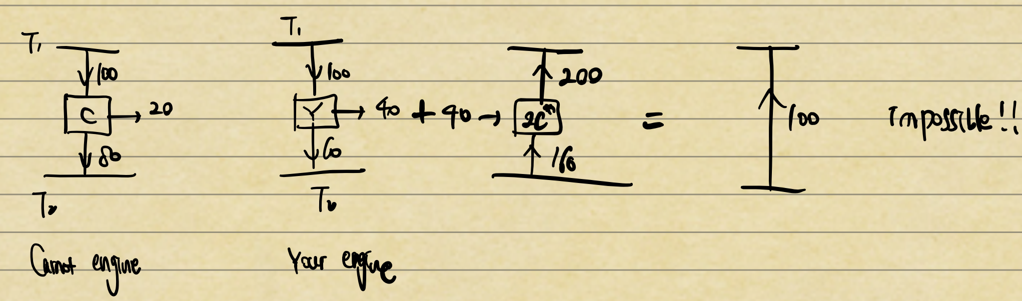
- Come back to A is important ⇝ you can repeat!
- $\eta = \frac{W}{Q_1}$ what you get $= 1 - \frac{Q_2}{Q_1}$ <u>efficiency not 1</u> b.c. $Q_2 \neq 0$

$$= 1 - \frac{nRT_2 \ln\frac{V_C}{V_D}}{nRT_1 \ln\frac{V_B}{V_A}}$$

$$= 1 - \frac{T_2}{T_1}$$

No engine can beat this

Reversible ⤳ refrigerator



$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

Well, $\sum \frac{\Delta Q_i}{T_i} = 0$ ΔQ_i = heat input in stage i

Heat - not a variable Energy - is a variable

$$\begin{cases} S = \text{entropy} \\ \Delta S = \left.\frac{\Delta Q}{T}\right|_{\text{equilibrium}} \\ \sum \Delta S = \sum \frac{\Delta Q}{T} = 0 \end{cases}$$

(ex)

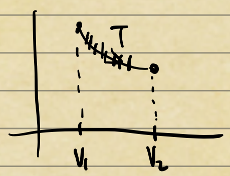
heat C T_f, T_i

$$S_f - S_i = \sum \frac{dQ}{T} = \int_{T_i}^{T_f} mc \frac{dT}{T}$$

$$= mc \ln \frac{T_f}{T_i}$$

$\Delta Q = P\Delta V$ since $\Delta U = 0$...

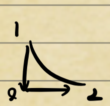
(ex)



$$S_2 - S_1 = \sum \frac{\Delta Q}{T} = \sum \frac{P\Delta V}{T} = \sum \frac{nR\cancel{T}}{V} \frac{\Delta V}{\cancel{T}} = nR \ln \frac{V_2}{V_1}$$

Well... we said $Q = nRT \ln \frac{V_2}{V_1}$

SO, entropy calculation is indep't of path! (exercise)



$$: C_v \int_{T_1}^{T_0} \frac{dT}{T} + \underbrace{C_p}_{= C_v + R} \int_{T_0}^{T_1} \frac{dT}{T}$$

$= R \ln \frac{T_1}{T_0}$

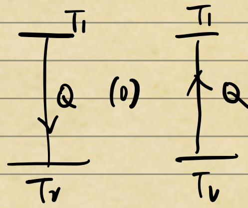
$= R \ln \frac{V_1}{V_0}$

$PV = RT$

P same

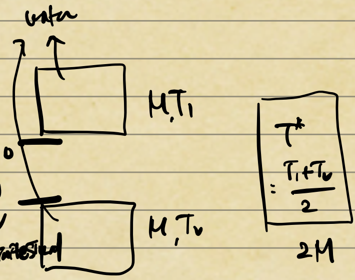
II Law

$\Delta S_{univ} \geq 0$: happen.



$\Rightarrow + \frac{Q}{T_1} - \frac{Q}{T_2} < 0$ impossible

$\Delta S = -\frac{Q}{T_1} + \frac{Q}{T_2} > 0$



$\underbrace{\frac{\Delta Q}{T_{high}} + \frac{\Delta Q}{T_{low}} > 0}$

every infinitesimal step positive!

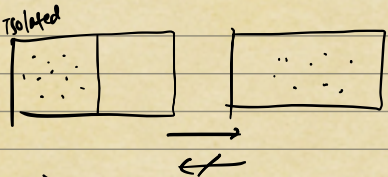
$\Delta S = M \cdot 1 \cdot \left(\int_{T_1}^{T^*} \frac{dT}{T} + \int_{T_2}^{T^*} \frac{dT}{T} \right)$

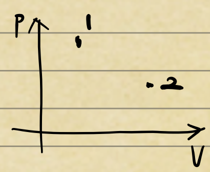
c for water

$= M \ln \frac{(T^*)^2}{T_1 \cdot T_2}$

Q. What is the microscopic basis of entropy?

Thermally Isolated

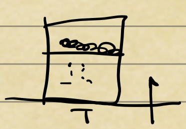




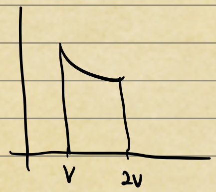
$\cancel{\Delta S = \frac{\Delta Q}{T} = 0}$

$S_2 - S_1 = nR \ln \frac{V_2}{V_1}$

$\Leftrightarrow$



Entropy loss of reservoir: $\frac{\Delta Q}{T}$



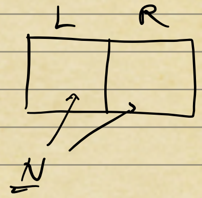
$S_2 - S_1 = nR \log 2$

$= Nk \log 2$

$= k \ln 2^N$

All those factors

$\boxed{S = k \ln \Omega}$ → # of permutation of arrangement



so $k \ln 1 \rightarrow k \ln 2^N$

$\Updownarrow$

can be realized in many more ways!