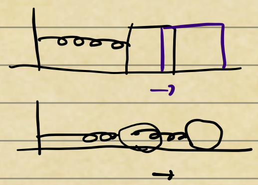
Lec 1.

E & M + Optics + QM

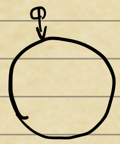
$F = ma$



a_1
$m=1$

$a_1 \cdot 1 = a_M \cdot M$

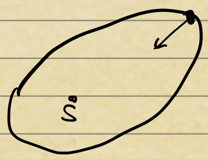
$\Rightarrow$ can get $F = -kx$



$F = mg$

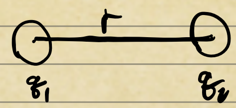
$\frac{F}{m} = g$

In fact, $\frac{GMm}{r^2}$



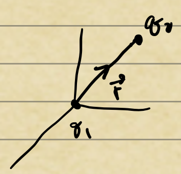
One more force!

Coulomb's law



$F = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r^2}$ $\qquad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ very strong!

q Coulombs

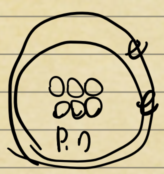


$\vec{F}_{21} = \frac{q_2 q_1}{4\pi\varepsilon_0} \frac{1}{r^2} \left(\vec{e}_r \text{ or } \frac{\vec{r}}{|\vec{r}|} \right)$

force on 2 due to 1

$= -\vec{F}_{12}$

1. q is conserved (does not change w. time) locally
2. q is quantized unit: $\pm 1.6 \times 10^{-19}$ Coulombs



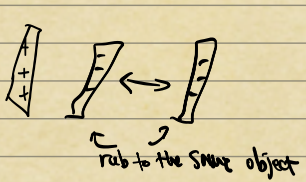
$q_n = 0$

$q_e = -1.6 \times 10^{-19} C$

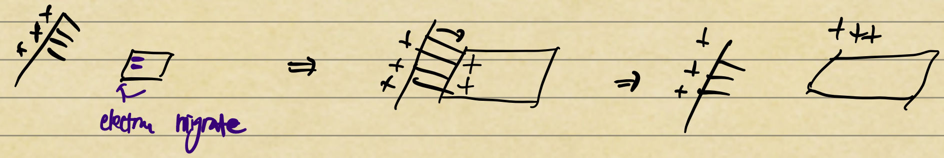
$q_p = 1.6 \times 10^{-19} C$

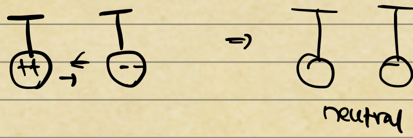
It is usually the electrons that move!

(ex)



Superposition + Coulomb's law.

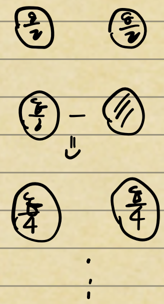
(ex) 

(ex) 

Q. How to check $\frac{q_1 q_2}{r^2}\, 9\times 10^9$?

1) Vary r

2) Vary q_1 ⇝ how? :



Always ask... "How will you measure anything?"

x Coulomb? Find standard coulomb (cr. ⓠ ⟷ ⓠ)

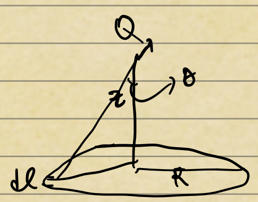
& use $\frac{q_1 q_2}{r^2} \sim = ma$

10^{40} weaker!

$$\frac{F_g}{F_e} \quad \frac{\frac{GMm}{r^2}}{\frac{q_e q_p}{4\pi\varepsilon_0}\frac{1}{r^2}} = \frac{GMm}{q_1 q_2\, 9\times 10^9} = \frac{10^{-11}\times 10^{-27}\times 10^{-30}}{10^{-38}\times 10^{10}} = 10^{-40}$$

$e \longleftrightarrow p$

Things are mostly neutral!



$\lambda = \frac{\text{coul}}{m}$

$$\int \frac{\lambda \cdot dl\, Q}{4\pi\varepsilon_0}\, \frac{1}{R^2+z^2}\, \underbrace{\frac{z}{\sqrt{R^2+z^2}}}_{\cos\theta}$$

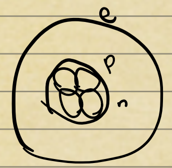
$$= 2\pi R\, \frac{\lambda Q}{4\pi\varepsilon_0}\, \frac{1}{R^2+z^2}\, \frac{z}{\sqrt{R^2+z^2}}$$

$$= \frac{qQ\, z}{(R^2+z^2)^{\frac{3}{2}}\, 4\pi\varepsilon_0}$$

$\lambda \cdot 2\pi R = q$

$z \uparrow \Rightarrow$ like point charge

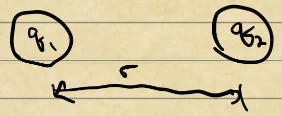
#2



$q_p = -q_e$

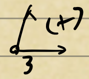
$q_n = 0$

$q_e = -1.6 \times 10^{-19}$ C



$$F = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r^2}$$

+ superposition ← experimental fact

1°   

2°

$$\frac{F_g}{F_e} \approx 10^{-40} = \frac{G M_e M_p}{\frac{1}{4\pi\varepsilon_0} q_e q_p}$$

Q. why protons together?

$\exists$ nuclear force

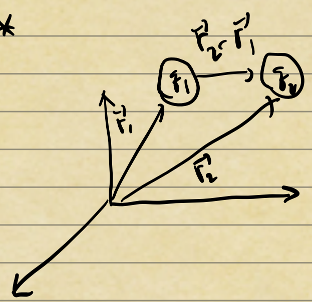
$$\frac{1}{137}\frac{1}{r^2} \quad \text{vs} \quad 10\,\frac{e^{-\frac{r}{r_0}}}{r^2} \qquad r_0 \approx 10^{-13}\ \text{cm}$$

neutron – proton
proton – proton
neutron – neutron

so neutron is like a glue

nuclear force → electrical force → gravitational force
(atomic physics) (most things neutral!)

*



$$\underset{\text{on}\ \ \text{due}}{\vec{F}_{21}} = \underbrace{\frac{q_1}{4\pi\varepsilon_0} \frac{\hat{e}_{12}}{|\vec{r}_1 - \vec{r}_2|^2}}\, q_2 \qquad \hat{e}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$= \underbrace{\vec{E}(\vec{r}_2)}_{\text{field at } \vec{r}_2}\, q_2$$

$$\vec{F} = q\vec{E}$$; field is a force on unit charge

* This law is for electro static

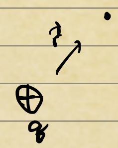
You cannot send info faster than light!

In general though, can be used in electric circuit, etc.

Still, $\vec{F} = q\vec{E}$

Charge ① producer of field

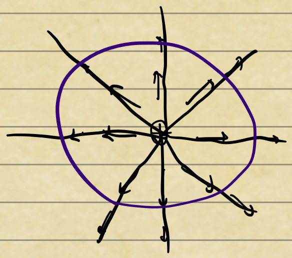
② responds to the field



$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\left(\vec{e_r} = \frac{\vec{r}}{r}\right)$$

$$= \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

①



$$\text{density} = \frac{\#\text{lines crossing a surface} \perp \text{to lines}}{\text{area of the surface}}$$

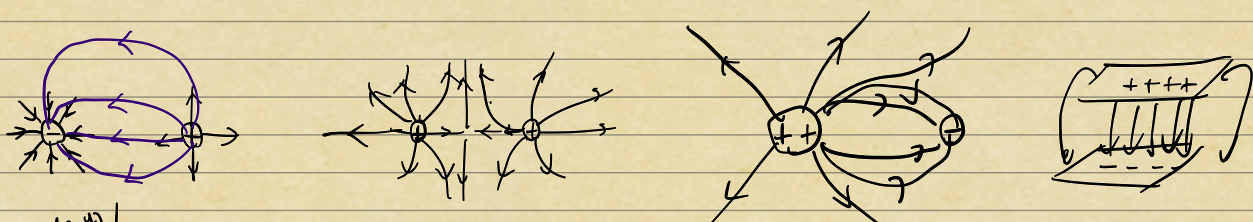
fix the # of lines for 1C.

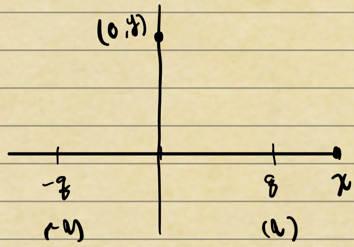
1C $\frac{1}{\varepsilon_0}$ lines

qC $\frac{q}{\varepsilon_0}$ lines

$$E = \frac{q}{\varepsilon_0}\frac{1}{4\pi r^2} = \text{density}$$

②





1)

$$E = \vec{i}\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right)$$

$$= \vec{i}\frac{q}{\pi\varepsilon_0}\frac{xa}{(x^2-a^2)^2}$$

$x \gg a$: $\dfrac{\vec{i}\,p}{2\pi\varepsilon_0 x^3}$ $\qquad p = 2aq$

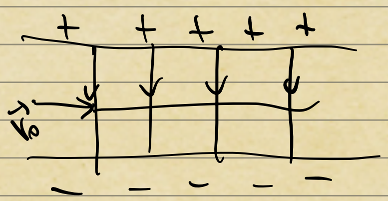
$$= \boxed{\frac{\vec{p}}{2\pi\varepsilon_0 x^3}}$$

2) $$\vec{E} = -\vec{i}\left(\frac{2q}{4\pi\varepsilon_0}\,\frac{1}{y^2+a^2}\,\frac{a}{(y^2+a^2)^{\frac{1}{2}}}\right)$$

$$= -\vec{i}\,\frac{2qa}{4\pi\varepsilon_0}\,\frac{1}{(y^2+a^2)^{\frac{3}{2}}}$$

$y \gg a$ $$= \boxed{-\frac{\vec{p}}{4\pi\varepsilon_0}\,\frac{1}{y^3}}$$

* Finding response to $\vec{E}$



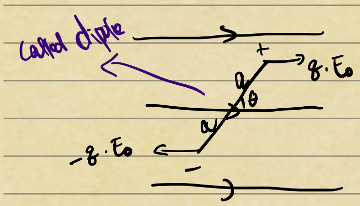
$\vec{E} = -\vec{j}\,E_0$

$\vec{F} = -qE_0\vec{j}$ $\qquad \vec{a} = -\dfrac{qE_0}{m}\vec{j}$

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $\qquad t = \dfrac{L}{v_0}$

$= 0 + \vec{v}_0 t - \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\vec{j}$

(ex) how TV works



$\vec{E} = -E_0\vec{x}$

$\tau = 2(qE_0)\,a\sin\theta \qquad (r \times F)$

$= p\,E_0\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E} \qquad F \to \tau$

$\qquad\qquad x \to \theta$

$$\begin{cases} F = -kx \\ U = \frac{1}{2}kx^2 \\ -\frac{dE}{dx} = U \end{cases}$$

$U(x_1) - U(x_2) = \int_{x_1}^{x_2} F\,dx$

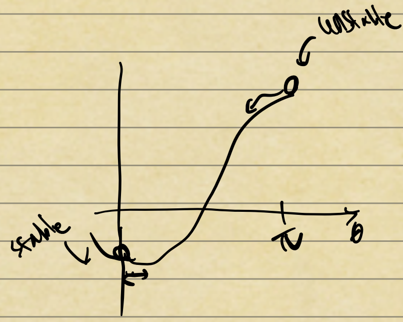
$U(\theta) - U(0) = \int_0^{\theta} -\tau\,d\theta$

$= \int_0^{\theta} pE_0\sin\theta\,d\theta$

"− due to opposite direction"

$$= -pE_0\cos\theta + pE_0$$

$$V(\theta) = -pE\cos\theta = -\vec{p}\cdot\vec{E}_0$$

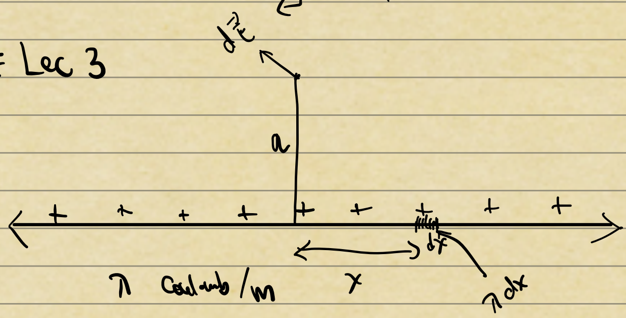


$$\tau = -\frac{dU}{d\theta}$$

$$\frac{\lambda\, dx}{4\pi\varepsilon_0\,(x^2+a^2)} \quad \frac{a}{\sqrt{a^2+x^2}}$$

Lec 3

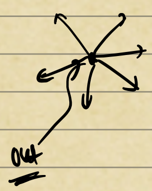
1)



$$\int_{-\infty}^{\infty} \frac{\lambda}{4\pi\varepsilon_0}\,\frac{a}{(x^2+a^2)^{\frac{3}{2}}}\,dx$$

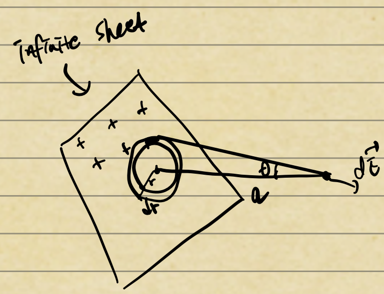
$$= \frac{2\lambda a}{4\pi\varepsilon_0}\int_0^{\infty}\frac{dx}{(x^2+a^2)^{\frac{3}{2}}} \qquad x = a\tan\theta$$

$$= \frac{\lambda}{2\pi\varepsilon_0 a}$$



weakens w. $\frac{1}{a}$

2)



σ Coul/m^2

$$dE = \frac{\sigma\, dA}{4\pi\varepsilon_0}\,\frac{1}{r^2+a^2}\,\overbrace{\frac{a}{\sqrt{r^2+a^2}}}^{\cos\theta}$$

$$\int_0^{\infty} 2\pi r\,\frac{\sigma}{4\pi\varepsilon_0}\,\frac{1}{r^2+a^2}\,\frac{a}{\sqrt{r^2+a^2}}\,dr$$

$$= \frac{a\sigma}{2\varepsilon_0}\underbrace{\int_0^{\infty}\frac{r}{(r^2+a^2)^{\frac{3}{2}}}\,dr}$$

$w = r^2$

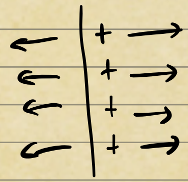
$dw = 2r\,dr$

$$\frac{1}{2}\int_0^{\infty}\frac{1}{(w+a^2)^{\frac{3}{2}}}\,dw$$

$$= \frac{1}{2}\left[-2(w+a^2)^{-\frac{1}{2}}\right]_0^{\infty}$$

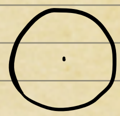
$$= \frac{1}{a}$$

$$= \boxed{\frac{\sigma}{2\varepsilon_0}}$$



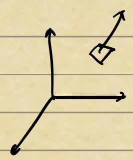
$\frac{\sigma}{2\varepsilon_0}$ scale free!

3)



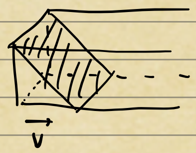
$\rho \ \frac{\text{Coulomb}}{m^3}$

$\underbrace{\text{Gauss's law}}$



$dA \quad m^2$

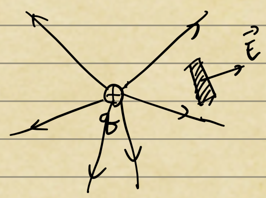
$d\vec{A}$



$A = hw$

$\underbrace{\Phi}_{\text{flow rate}} = hwv = Av$

$\Phi = \vec{v} \cdot \vec{A}$ in general, also called flux



$d\vec{A} = dA\vec{e}_r$

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\vec{e}_r$

$\vec{E} \cdot d\vec{A} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}dA$

$\frac{q}{\varepsilon_0}$

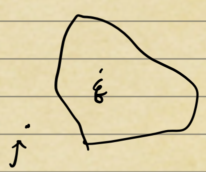
$$\iint_{\text{sphere}} \vec{E} \cdot d\vec{A} = \iint_{\text{sphere}} \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}dA$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}4\pi r^2$$

$$= \frac{q}{\varepsilon_0}$$

$$\iint_{\text{any surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$: 0 = \int_V \underbrace{(\nabla \cdot E)}_{\text{in} = \text{out}} dV = \oint_S E \cdot dA - \oint_{\text{sphere}} E \cdot dA$$



$$\iint \vec{E} \cdot d\vec{A} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

In = out

so cancelled out

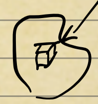
$$\iint \vec{E}\cdot d\vec{A} = \iint \vec{E}_1\cdot d\vec{A} + \iint \vec{E}_2\cdot d\vec{A}$$

$$= \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0}$$

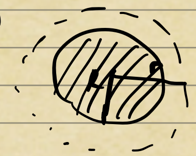
$$\oiint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

ρ = charge density $\Rightarrow \iiint_V \rho(\vec{r})\,dx\,dy\,dz$

$$\oiint_S \vec{E}\cdot d\vec{A} = \iiint_V \rho(x,y,z)\,dx\,dy\,dz$$



(ex)



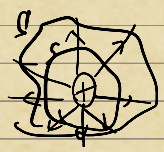
$$\frac{Q}{\varepsilon_0} = \iint_{\text{sphere}} \vec{E}\cdot d\vec{A}$$

$$= 4\pi r^2 E(r)$$

$$E(r) = \frac{Q}{4\pi r^2 \varepsilon_0}$$

※ $\int_U f\, dx^1 \cdots dx^n = \int_V (f\circ T)\,|\det J(T)|\, dy^1 \cdots dy^n$ (Thm 10.9, Rudin)

Lec 4



line density $= \frac{\text{lines}}{\text{areas}} \propto \frac{q}{r^2}$

Electric field $E(r) \propto \frac{q}{r^2}$

$\Rightarrow$ line density $= c\,E$

line density S = line crossing S'

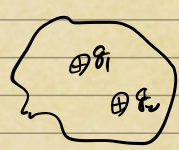
$$\Phi_S = \underbrace{cE(r)}_{\text{line density}}\ \underbrace{4\pi r^2}_{\text{area}}$$

$$\Phi_{S'} = c \oiint \vec{E}(\vec{r})\cdot d\vec{A}$$

$$\Rightarrow \frac{q}{\varepsilon_0} = \oiint \vec{E}(\vec{r})\cdot d\vec{A}$$

$$\vec{E}_{1,2} = \vec{E}_1 + \vec{E}_2 \rightsquigarrow \oiint \vec{E}_{1,2}\, d\vec{A} = \oint \vec{E}_1\cdot d\vec{A} + \oint \vec{E}_2\cdot d\vec{A}$$

$$= \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0}$$



$$\oiint_{S=\partial V} \vec{E}\cdot d\vec{A} = \sum_{\substack{i=1 \\ (i\in V)}}^{N} \frac{q_i}{\varepsilon_0}$$

q is continuous (in real life, electrons & protons are not cts)

$$\boxed{\oiint_{S=\partial V} \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\iiint_V \rho(x,y,z)\,dx\,dy\,dz}$$

Gaussian surface
= sphere of radius r

Q

$\vec{E} = \hat{r}\,E(r) = \vec{e}_r\,E(r)$

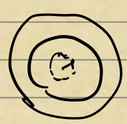
1) $E\cdot 4\pi r^2 = \dfrac{Q}{\varepsilon_0}$

$\vec{E} = \dfrac{Q}{4\pi r^2 \varepsilon_0}\vec{e}_r \qquad (r>R)$ ← as if point charge!

2) $4\pi r^2 E(r) = \dfrac{Q}{\varepsilon_0}\dfrac{r^3}{R^3}$

$E(r) = \dfrac{Qr}{4\pi \varepsilon_0 R^3} \qquad (r<R)$

(ex)

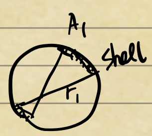


$E\cdot 4\pi r^2 = \dfrac{1}{\varepsilon_0}\cdot 0 = 0$

$E=0$

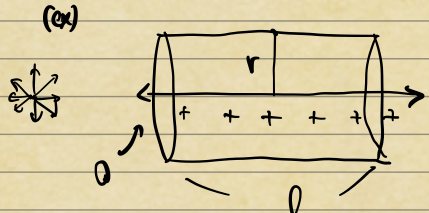
Why? Strong charge, smaller area ≐ weaker charge larger area

(ex)



$$\frac{\sigma A_1}{r_1^2}\cancel{\cos\theta} = \frac{\sigma A_2}{r_2^2}\cancel{\cos\theta}$$

(ex)



$\vec{dl}$

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\int E(r) \cdot 2\pi r \, dl = \frac{\lambda l}{\varepsilon_0}$$

$+ 0 + 0$

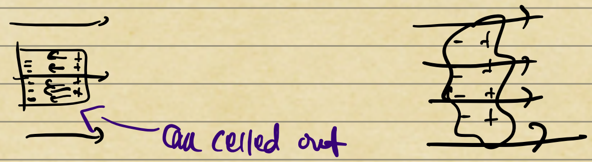
$$\Rightarrow E(r) = \frac{\lambda}{2\pi r \varepsilon_0}$$

Conductors vs. insulators

Result $\vec{E} = 0$ inside a conductor in a static situation

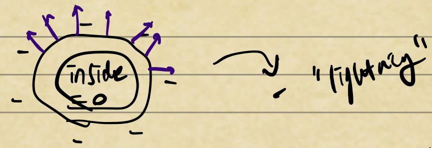
(free charges, electrons, can move)

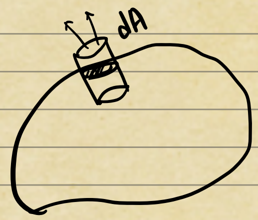
(ex)



Charge inside a conductor $= 0$

(ex)



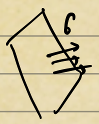


charge density

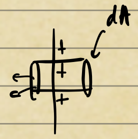
$$\boxed{E \, dA = \frac{\sigma}{\varepsilon_0} dA}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Recall...

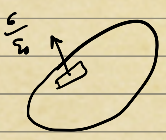


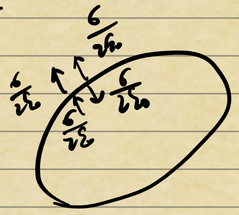
$$E = \frac{\sigma}{2\varepsilon_0}$$



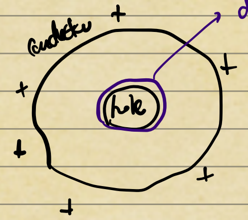
$$2E \, dA = \frac{\sigma \, dA}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

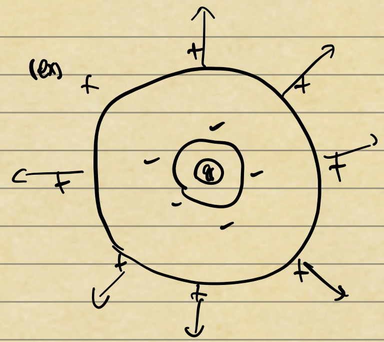




(ex) conductor



(ex)



Lec 5

$$F \cdot \frac{dx}{dt} = F \cdot v = m\frac{dv}{dt}v$$

$$= \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

integrate $\int_{t_1}^{t_2} \rightsquigarrow \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \int_{x_1}^{x_2} F(x)\,dx$

$$k_2 - k_1 = \int_{x_1}^{x_2} F(x)\,dx = -U(x_2) + U(x_1), \quad -\frac{dU}{dx} = F$$

$$k_1 + U(x_1) = k_2 + U(x_2)$$

2D : $k = \frac{1}{2}m\vec{v}\cdot\vec{v}$

$$\frac{dk}{dt} = m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{F}}\cdot\vec{v}$$

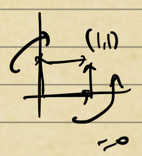
$$\rightsquigarrow \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = k_2 - k_1$$

$\neq U(\vec{r}_1) - U(\vec{r}_2)$ want $\oint F \cdot d\vec{r} = 0$ for all path

Called conservative force

(ex) $F = \hat{i}y$

$\int_0^1 1\,dx = 1$



<u>Thm</u> ① Take any $U(x,y)$

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = \frac{\partial U}{\partial y}$$

$\vec{F} = -\vec{\nabla} U$

Thm All conservative forces are of the form $-\nabla U$ 31:57

(a) $\int_1^2 \vec{F} \cdot d\vec{r} = \int F_x dx + F_y dy$

$= \int \frac{dU}{dx} dx + \frac{dU}{dy} dy$

$= -\int_1^2 dU$

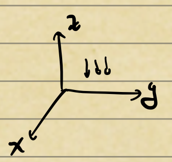
"potential energy"

$= U(1) - U(2)$

Can test by doing $\frac{dF_x}{dy} = \frac{dF_y}{dx}$

In 3D, you may need to test more.

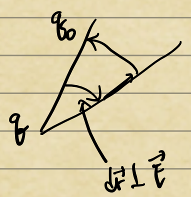
(ex) gravity near earth



$U = mgz$ $\quad \vec{F} = -\vec{i}\frac{dU}{dx} - \vec{j}\frac{dU}{dy} - \vec{k}\frac{dU}{dz} = -\vec{k}mg$

$U(2) - U(1) \overset{!}{=} -\int_1^2 \vec{F} \cdot d\vec{r} = -\int_1^2 (-mg)\, dz = mgz - 0$

(ex)



$V(\vec{r}_2) - V(\vec{r}_1) = -\int \vec{E}\, d\vec{r}$

$= -q\int_{r_1}^{r_2} \frac{\vec{e}_r}{4\pi\varepsilon_0 r^2} \cdot \vec{e}_r\, dr$

$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

pole electric field

$\boxed{V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}}$ Called potential of electric field

$U(r) = q_0 V(r)$

$\vec{E} = -\nabla V$

$E_x = -\frac{dV}{dx} = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r^2}\right)\frac{dr}{dx}$ $\qquad r = \sqrt{x^2+y^2+z^2}, \quad \frac{dr}{dx} = \frac{x}{r}$

$$= \frac{q}{4\pi\varepsilon_0} \frac{x}{r^2 r}$$

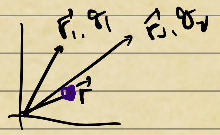
$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} \vec{e}_r$$

Use superposition ⤳ can get general potential

Lesson

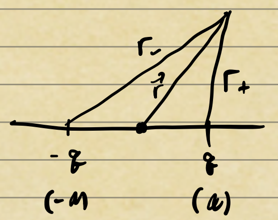
① $\frac{1}{2}mv^2 + qV(\vec{r}_1) = \frac{1}{2}mv_2^2 + qV(\vec{r}_2)$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{|\vec{r}_i - \vec{r}|}$$



② Easier to work w. V than with $\vec{E}$ (since scalar)

(ex)



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$\vec{r}_+ = \vec{r} - a\vec{i}$$

$$\vec{r}_- = \vec{r} + a\vec{i}$$

$$r_+ = |\vec{r}_+|$$

$$= \sqrt{(\vec{r} - a\vec{i})\cdot(\vec{r} - a\vec{i})}$$

$$= \sqrt{r^2 + a^2 - 2\vec{r}\cdot\vec{a}}$$

$$\approx r\left(1 - \frac{2\vec{r}\cdot\vec{a}}{r^2}\right)^{\frac{1}{2}} \qquad (r \gg a)$$

$$\approx r\left(1 - \frac{\vec{r}\cdot\vec{a}}{r^2}\right)$$

$$\frac{1}{r}\left(\frac{2\frac{\vec{r}\cdot\vec{a}}{r^2}}{\underbrace{\left(1 - \frac{\vec{r}\cdot\vec{a}}{r^2}\right)\left(1 + \frac{\vec{r}\cdot\vec{a}}{r^2}\right)}_{\approx 1}}\right)$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0} \frac{1}{r} \frac{2\vec{a}\cdot\vec{r}}{r^2}$$

p: dipole moment
(charge × distance)

$$= \frac{p\cdot\vec{r}}{4\pi\varepsilon_0 r^3} \qquad r \gg a$$

Capacitor

$\vec{E}$ $\oint E \cdot d\vec{r} = 0$ $q\vec{E}$ is the force!

V potential $\frac{1}{2}mv_1^2 + \underbrace{V(\vec{r}_1)q}_{U_1} = \frac{1}{2}mv_2^2 + V(\vec{r}_2)q$

1 charge $= \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$

$V = \sum_i \frac{q_i}{4\pi\varepsilon_0 |r - r_i|}$

$\vec{E} = -\nabla \vec{V}$

unit: E (N/C)

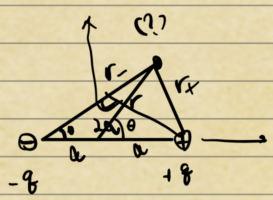
V (J/C = V, volt)

Advantages 1. Conservation of energy

2. Computation of E

3. Visual pictures

2. electric field



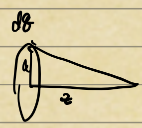
$\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$

$= \frac{q}{4\pi\varepsilon_0}\frac{r_+ - r_-}{r_+ r_-}$

$\approx \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

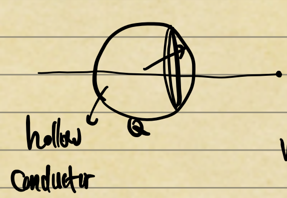
$$\boxed{V = \frac{p\,x}{4\pi\varepsilon_0 (x^2+y^2)^{\frac{3}{2}}}}$$

$E_x = -\frac{dV}{dx}$ $E_y = -\frac{dV}{dy}$

(ex)  $dV = \frac{dq}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+z^2}}$

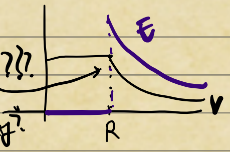
$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+z^2}}$ $E = -\frac{dV}{dz}$

Sometimes, E is easier to get



$\vec{E} = \frac{\vec{r} Q}{4\pi\varepsilon_0 r^2}$ $r \geq R$

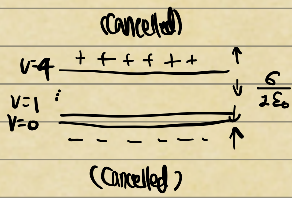
$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$ $E = -\vec{\nabla} V$



why cts???

Imagine → voltage: "how much work is needed from infinity?"

3. Visualize

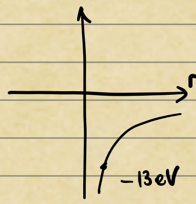


$\downarrow \frac{\sigma}{\varepsilon_0} = E$

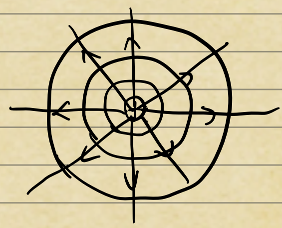
proton: go down
electron: like negative mass

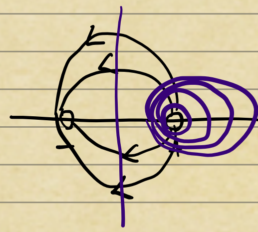
1.6×10^{-19} J $= 1$eV
electron volt

$U(r) = \frac{(+q)\ (-q)}{4\pi\varepsilon_0 r}$



: need $13 \times 1.6 \times 10^{-19}$ J of work





equipotentials $\perp \vec{E}$

$V_2 - V_1 = -\int \vec{E} \cdot d\vec{r}$

$\Delta V = -\vec{E} \cdot d\vec{r}$

$= -|\vec{E}|\,|d\vec{r}| \cos\theta$

biggest drop when $\theta = \frac{\pi}{2}$

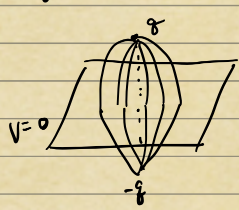
$\Delta V = \nabla V \cdot d\vec{r}$

$= (V_x, V_y, V_z) \cdot (dx, dy, dz)$

$\boxed{\begin{array}{l} D_u f = \nabla f \cdot u \\ D_u f \text{ biggest when } u \parallel \nabla f \end{array}}$

* Conductor

every conductor is an equipotential



Recall...

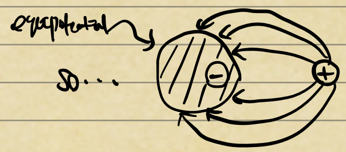
$\frac{\sigma}{\varepsilon_0} dA = E dA \Rightarrow \frac{\sigma}{\varepsilon_0} = E$

charge density

integrate charge density = $-q$

force of attraction = same as $(+q) - (-q)$

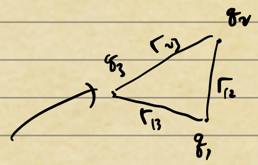
So...



same problem

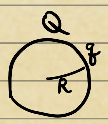
analyze energy aspect of a system/process

Energetics : "how much work is required?"



$$\frac{q_2 q_1}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \cdots$$

$$\Rightarrow U = \frac{1}{2} \sum_{\substack{ij \\ i \neq j}} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

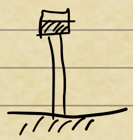


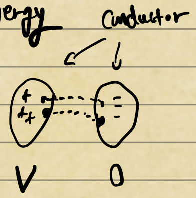
$$\underbrace{\int_0^Q \underbrace{\frac{q}{4\pi\varepsilon_0} \frac{1}{R}}_{\text{potential}} dq}_{\text{energy}} = \frac{Q^2}{2} \frac{1}{4\pi\varepsilon_0 R}$$

1:00:29

* Capacitors

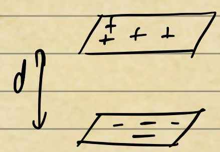
Storing charge/energy





$$\frac{Q}{V} = C \quad \text{farads (F)}$$

①



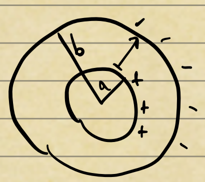
$$\frac{Q}{C} = V = Ed$$

$$= \frac{\sigma}{\varepsilon_0} d$$

$$= \frac{dQ}{\varepsilon_0 A}$$

$$C = \frac{\varepsilon_0 A}{d}$$

②



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$$

$$V_2 - V_1 = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = V$$

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

C: geometric / material property

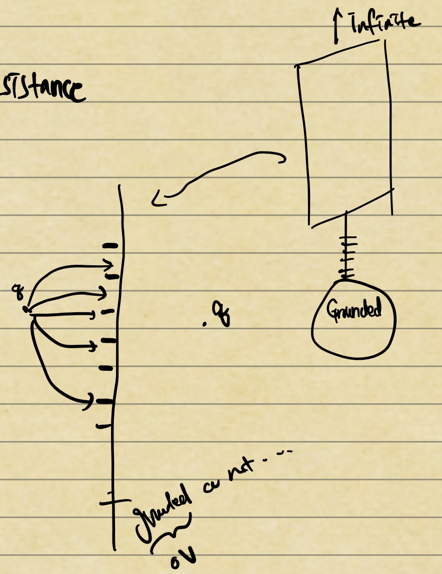
special case: $\frac{4\pi\varepsilon_0 R^2}{(d)} = \frac{\varepsilon_0 A}{d}$

$dW = dq\, V$

$= dq \frac{q}{C}$

$$\boxed{W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2}$$

#7 Resistance



$\frac{\sigma}{\varepsilon_0} = E$

Integrate over charge density σ ~ $-q$

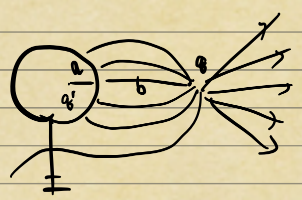
$$F = \frac{-q^2}{4\pi\varepsilon_0 (2d)^2}$$

$$U = \frac{-q^2}{4\pi\varepsilon_0 (4d)}$$ half of ~ $\left(\frac{1}{2d}\right)$

$\frac{1}{\varepsilon_0}\frac{q}{\underline{\pi R^2}}\left(=\frac{\sigma}{\varepsilon_0}\right)$: very small so uniformly displaced positive charge almost doesn't matter.

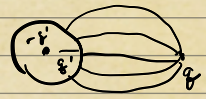
very big

(a)



① $S = \frac{a^2}{b}$

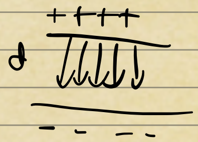
② $\frac{q'}{a} = -\frac{q}{b}$

(Not grounded) 

potential: $\frac{-q'}{4\pi\varepsilon_0 a}$

Force: $F(q' \to q) + F(-q' \to q)$

⊛ Uniqueness



$C = \frac{Q}{V}$

$$V = \frac{dQ}{\varepsilon_0 A} = \frac{Q}{C} \rightsquigarrow C = \frac{\varepsilon_0 A}{d}$$

$$* \int dW = \int_0^\infty V\, dQ$$

$$= \int_0^\infty \frac{Q}{C}\, dQ$$

$$= \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

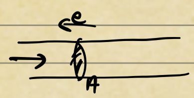
$Q = CV$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} E^2 d^2$$

$$= \underbrace{\frac{\varepsilon_0 E^2}{2}}_{\text{energy per unit volume due to electric field}} (Ad)$$

$$u = \frac{\varepsilon_0 E^2}{2} = \frac{\text{energy}}{\text{volume}}$$ "energy density"

* Electrical Circuits



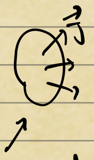
$\frac{\text{Coulomb}}{\text{sec}} = I$ (amps)

n = # of carriers / volume

e = charge of a carrier

$I = Avne$

$\vec{J}$ = current density = I/A = $ne\vec{v}$



$I = \oiint \vec{J} \cdot d\vec{A}$

if not uniform

$\vec{E}$ $J = n e\bar{v}$ (average velocity) $a = \frac{eE}{m}$ collide w. impurities of solid ~ v same

$\bar{v} = \bar{v_i}(0) + \frac{e}{m} E t_i$ ← time since the last collision

$= 0 + \frac{eE}{m}\tau$ $\tau = \bar{t_i}$ ← get in quantum theory

★ $\boxed{J = ne\frac{eE\tau}{m} = \frac{ne^2\tau}{m}E}$ current density → m, e fixed; n, τ depend on material

$= \underbrace{\sigma}_{\text{"conductivity"}} E$

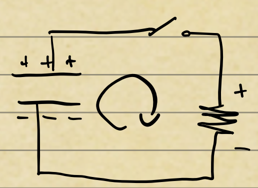
$I = JA = \sigma A E$

$= \frac{\sigma A V}{L} = \frac{V}{R}$ R := resistance of wire (ohms, Ω)

$R = \frac{L}{A\sigma} = \frac{L\rho}{A}$ $\rho = \frac{1}{\sigma}$ resistivity

Collision due to lattice of Nuclei vibrating due to heat

∃ electron ↔ electron, but minor

Q 

Conservative field ⇒ total voltage change = 0

$\frac{Q}{C} - RI = 0$

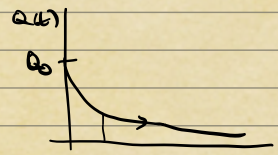
$RI = \frac{Q}{C}$ $I = -\frac{dQ}{dt}$

$-R\frac{dQ}{dt} = \frac{Q}{C}$

$\int \frac{dQ}{Q} = -\int \frac{dt}{RC}$

$\log\frac{Q}{Q_0} = -\frac{t}{RC}$

$Q(t) = Q_0 e^{-\frac{t}{RC}}$



takes ∞ amount of time

RC: "time constant" 37% decrease

$I(t) = Q_0/(RC)\, e^{-\frac{t}{RC}}$

Remember... $U = \frac{Q^2}{2C}$

VQ Jules $\rightsquigarrow \frac{VQ}{t}$ J/s

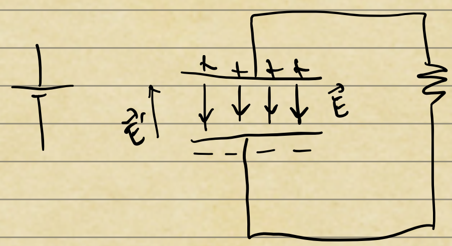
$\Rightarrow$ power in the resistor (P) : $\underline{\underline{V}} I$ (J/s)

J/C

$P = VI = I^2 R$

$\int_0^\infty P\,dt = \int_0^\infty I^2 R\,dt = \frac{Q_0^2}{2C}$ energy

Cell



$\oint \vec{E}' \cdot d\vec{r} = \mathcal{E}$ emf (electromotive charge)

work done per unit charge

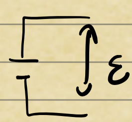
$\oint \vec{E} \cdot d\vec{r} = 0$

$\vec{E} = -\vec{E}'$ chemical force / charge

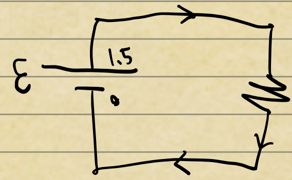
$V_+ - V_- = \int_+^- \vec{E} \cdot d\vec{r} = -\int_+^- \vec{E}'\, d\vec{r}$

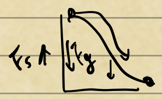
$= \mathcal{E}$

$\boxed{V_+ - V_- = \mathcal{E}}$



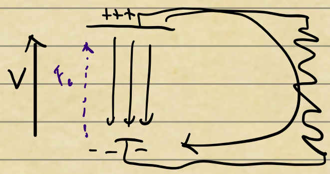
8





"$\mathcal{E}$" $= \oint \vec{F} \cdot d\vec{r}$

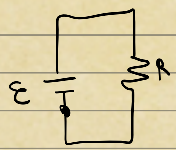
$= \cancel{\oint \vec{F}_g \cdot d\vec{r}} + \oint \vec{F}_s \cdot d\vec{r}$



$\mathcal{E} = \oint \vec{F}_c/_{\text{charge}} \cdot d\vec{r}$

$= Ed =$ voltage difference

(b)

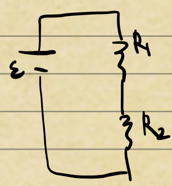


$\Delta V = 0$

✗ I is conserved (uniform)

$\mathcal{E} - RI = 0$

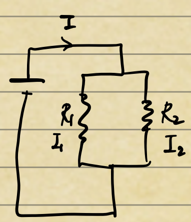
$\mathcal{E} = IR$



$\mathcal{E} - (R_1 + R_2) I$

$I = \dfrac{\mathcal{E}}{R_1 + R_2}$

$\dfrac{\mathcal{E}}{I} = R_1 + R_2$

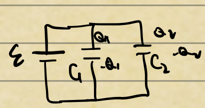


$I = I_1 + I_2$

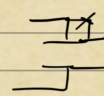
$= \dfrac{V}{R_1} + \dfrac{V}{R_2} = \dfrac{V}{R}$

$\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

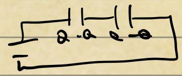
(c)



$C = \dfrac{Q}{\mathcal{E}} = \dfrac{Q_1 + Q_2}{\mathcal{E}} = \dfrac{C_1 \mathcal{E} + C_2 \mathcal{E}}{\mathcal{E}} = C_1 + C_2$ ... like



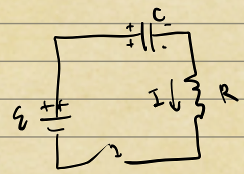
(a)



$$V_1+V_2=\mathcal{E}$$

$$\frac{Q}{C_1}+\frac{Q}{C_2}=\frac{Q}{C}$$

$$\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{C}$$



$Q_\infty = C\mathcal{E}$ ... let's check

$$\begin{cases} \mathcal{E}-\dfrac{Q}{C}-RI=0 \\ I=\dfrac{dQ}{dt} \end{cases}$$

$$\mathcal{E}=\frac{Q}{C}+R\frac{dQ}{dt} \qquad \frac{Q_\infty}{C}=\mathcal{E}$$

$$Q(t)=Q_\infty+\tilde{Q}$$

$$\cancel{\mathcal{E}}=\cancel{\frac{Q_\infty}{C}}+\frac{\tilde{Q}}{C}+R\frac{d\tilde{Q}}{dt}$$

$$\frac{\tilde{Q}}{C}+R\frac{d\tilde{Q}}{dt}=0$$

$$\frac{d\tilde{Q}}{dt}=-\frac{\tilde{Q}}{CR}$$

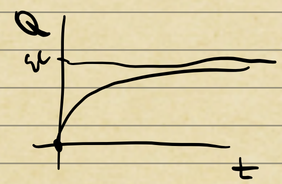
$$\tilde{Q}=\tilde{Q}(0)e^{-\frac{t}{RC}}$$

$$0=Q(0)=Q_\infty+\tilde{Q}(0)=\mathcal{E}C+\tilde{Q}(0)$$

$$\tilde{Q}(0)=-\mathcal{E}C$$

$$\rightarrow \tilde{Q}=-\mathcal{E}Ce^{-\frac{t}{RC}}$$

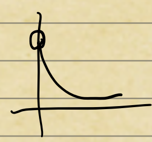
$$Q(t)=\mathcal{E}C\left(1-e^{-\frac{t}{RC}}\right)$$



$$U_{\text{battery}}=\mathcal{E}\int_0^\infty I(t)\,dt \qquad Q(t)=\mathcal{E}C\left(1-e^{-\frac{t}{RC}}\right)$$

$$I=\frac{dQ}{dt}=\frac{\mathcal{E}C}{RC}e^{-\frac{t}{RC}}$$

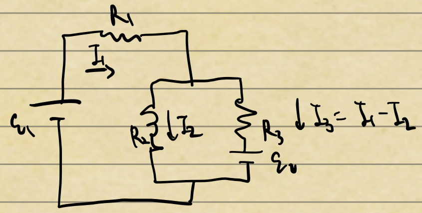
$$=\mathcal{E}RC\frac{\mathcal{E}}{R} \qquad =\frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}$$



$$=C\mathcal{E}^2$$

$$\frac{C\mathcal{E}^2}{2}=\frac{1}{2}\frac{Q_\infty^2}{C} \oplus \quad P=I^2R=\underbrace{R\int_0^\infty I^2(t)\,dt}_{W_R}=\frac{\mathcal{E}^2C}{2}$$

(ex)

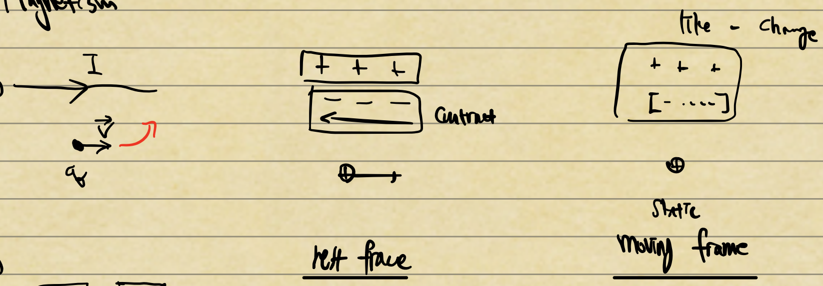


$$\mathcal{E}_1 - R_1 I_1 - R_2 I_2 = 0$$

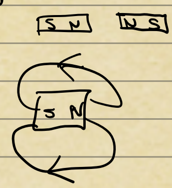
$$\mathcal{E}_1 - R_1 I_1 - R_3 (I_1 - I_2) - \mathcal{E}_2 = 0$$

* Magnetism

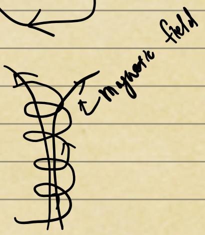
(ex)



(ex)



(ex)

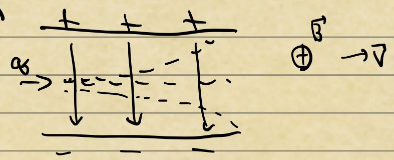


felt / caused by moving charges

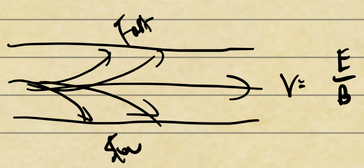
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz force}$$

$$P = \vec{F} \cdot \vec{v} = q \vec{v} \cdot \vec{E} + q \underbrace{\vec{v} \cdot (\vec{v} \times \vec{B})}_{=0}$$

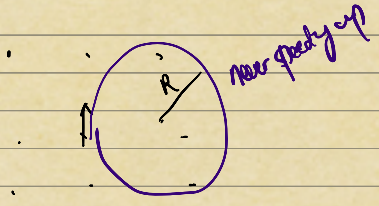
(ex)



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$



(ex)



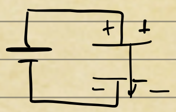
$$\frac{mv^2}{R} = qvB$$

$$V = \frac{R}{m} qB$$

$$T = \frac{2\pi R}{V}$$

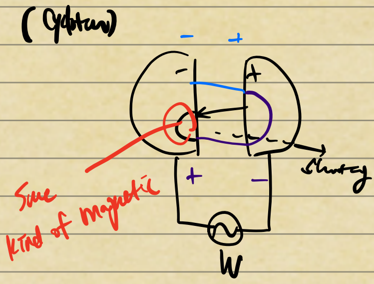
not on R or V

$$\frac{qB}{m} = \frac{V}{R} = \frac{2\pi}{T} = 2\pi f = \omega \quad \text{angular speed}$$



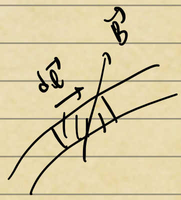
$$qV = \frac{1}{2} m v^2$$

(Cyclotron)



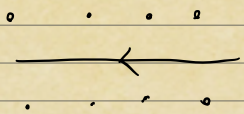
(Macro)

Wire

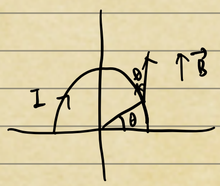


$$Anedl \; \vec{v} \times \vec{B}$$

$$Anev \; d\vec{l} \times \vec{B} = I \; d\vec{l} \times \vec{B}$$



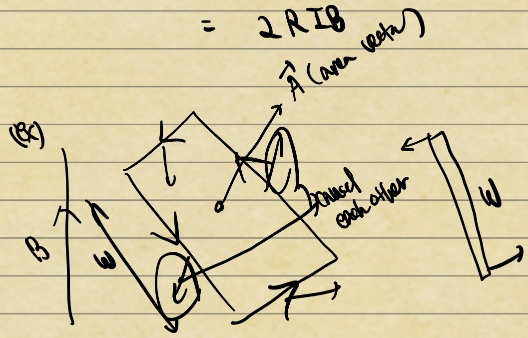
$$BLI = mg \quad \text{(balanced)}$$



$$\int dF = \int I \, dl B \sin\theta \qquad dl = R \, d\theta$$

$$= \int_0^{\pi} IB \sin\theta \, R \, d\theta$$

$$= 2RIB$$



$$BlIw \sin\theta = IBA \sin\theta$$

magnetic moment of the loop

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$|\vec{\mu}| = IA$$

dipole

Torque: $P \times E$

Energy: $-P \cdot E$

Lec 9

Produce - today

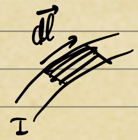
react - last time

$\vec{F} = q\,\vec{v} \times \vec{B}$

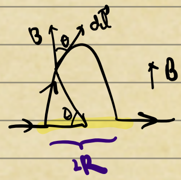
Tesla

def

$1N \leftarrow 1C \quad 1m/s$



$d\vec{F} = I\, d\vec{l} \times \vec{B}$

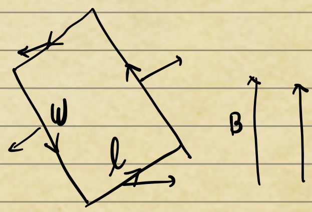


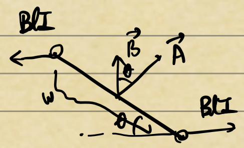
$dF = I\, dl\, B \sin\theta$

$F = IBR \underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2}$

$= 2IBR$

$= BI(2R)$ ← this is universal.



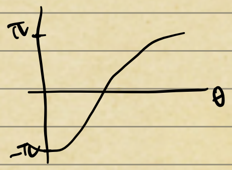


$\tau = B\underbrace{lIw}_{A}\sin\theta \qquad \theta$: btwn $\vec{A}$ and $\vec{B}$

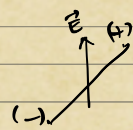
$\vec{\tau} = \vec{\mu} \times \vec{B}$

$\underbrace{\vec{\mu} = \vec{A} I}$

$U = -\mu B\cos\theta = -\vec{\mu} \cdot \vec{B}$



well, the direction doesn't matter



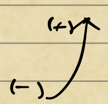
$\vec{\tau} = \vec{p} \times \vec{E} \qquad p = q \cdot d$

$U = -\vec{p} \cdot \vec{E}$

There isn't plus/negative charge

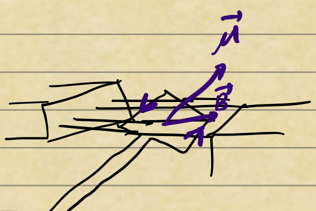
There is plus/negative charge

Analogy

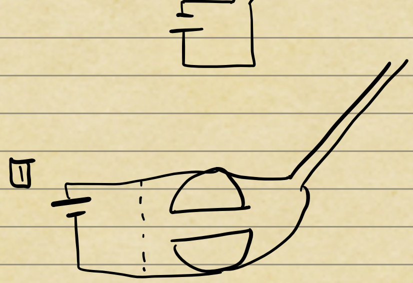


?

Electric motor



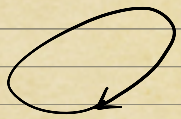
$\vec{\mu}$ will change until it lines up w. $\vec{B}$



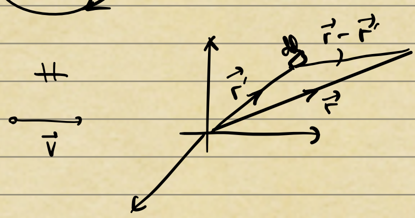
19:18

two ways... ① use AC source (instead of DC)

② use III ("overshoot" a bit so no need to give extra push)



Constant current



Biot - Savart

$$d\vec B = \frac{\mu_0}{4\pi} I \frac{d\vec l \times \vec e_{rr'}}{|\vec r - \vec r'|^2}$$

$\sim 10^{-7}$

(ex)

$$d\vec B_z = \frac{\mu_0}{4\pi} I \frac{dl}{z^2+R^2} \frac{R}{\sqrt{z^2+R^2}}$$

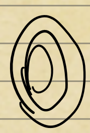
$$\leadsto \vec B = \frac{\mu_0 R^2 I}{2\,(z^2+R^2)^{\frac{3}{2}}}$$

Special case $z=0$: $\frac{\mu_0 R^2 I}{2R^3} = \frac{\mu_0 I}{2R}$ Tesla

$z \to \infty$ $B \to \frac{\mu_0 I\pi R^2}{2\pi z^3}$ dipole moment of the loop μ

$\sim \frac{\mu}{z^3}$ $\Leftarrow$ exactly like electric dipole (p4~5)

① lines with Electrostatic / magnetic field

②  ← similar

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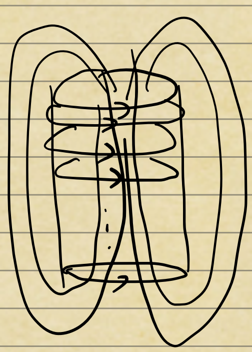
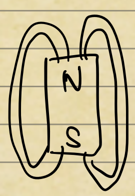
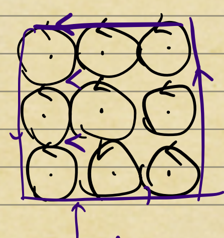
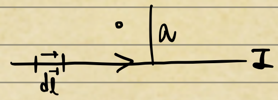




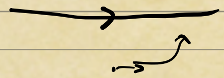


(c)

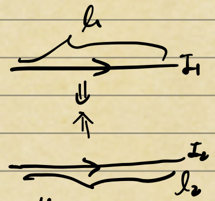


$$\int dB = \int_{-\infty}^{\infty} \frac{\mu_0 I}{4\pi} \frac{dx}{x^2+a^2} \frac{a}{\sqrt{x^2+a^2}} = \frac{\mu_0 I}{2\pi a}$$

(e)

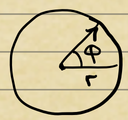


(f)



$$\frac{F_{21}}{l_2} = \frac{\mu_0 I_1}{2\pi d} I_2 \quad ; \; 2\times 10^{-7} N \text{ for } 1A, 1A, 1m$$

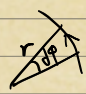
* Ampere's law



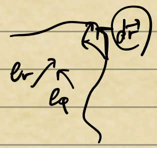
$$\vec{B} = \vec{e}_\varphi \frac{\mu_0 I}{2\pi r}$$

$$\oint_S \vec{B}\cdot d\vec{r} = \frac{\mu_0 I}{2\pi} \int_0^{2\pi} d\varphi = \mu_0 I$$

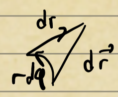
Indep of r



$d\vec{r} = r\,d\varphi\,\vec{e}_\varphi$



$$d\vec{r} = \underline{\underline{dr}}\,\vec{e}_r + \underline{\underline{r\,d\varphi}}\,\vec{e}_\varphi$$

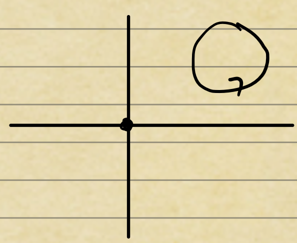


$$\vec{B} = \frac{\mu_0 I}{2\pi r} \vec{e}_\varphi$$

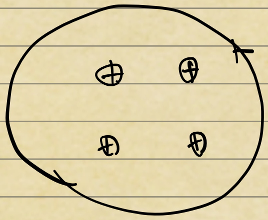
Note $\vec{e}_\varphi \cdot \vec{e}_r = 0$

$$\vec{B}\cdot d\vec{r} = \frac{\mu_0 I}{2\pi} d\varphi$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{r} = \mu_0 I$$



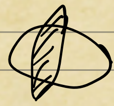
0 since change of φ is 0



$\oint \vec{B}\cdot d\vec{r} = \sum_j \mu_0 I_j$ $\quad I_j$ penetrates the surface

Ampere's Law

$\oiint \vec{B}\cdot d\vec{A} = 0$



one out → one in

we haven't find any magnetic charges (magnetic monopoles)

[electrostatics
magnetostatics]

Recall that

$\oint \vec{E}\cdot d\vec{r} = 0$ $\qquad \oint \vec{B}\cdot d\vec{r} = \sum_j \mu_0 I_j$

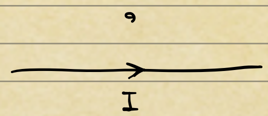
$\oiint \vec{E}\cdot d\vec{A} = \sum_i \frac{q_i}{\varepsilon_0}$ $\qquad \oiint \vec{B}\cdot d\vec{A} = 0$

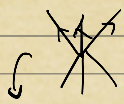
Integral form of Maxwell equations

mathematically, complete.

Ampere's Law

(ex)



 or  $\rightsquigarrow \underbrace{\oint \vec{B}\cdot d\vec{r}}_{= B\cdot 2\pi r} = \mu_0 I$ $\qquad B = \frac{\mu_0 I}{2\pi r}$

① no magnetic charge

② [← opposite direction
rotate same direction] ⇝ contradiction

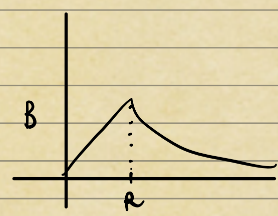
(ex)

$2\pi r B = \mu_0 I$

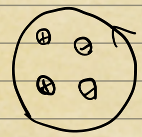
$B = \frac{\mu_0 I}{2\pi r}$ $\qquad r > R$

$2\pi r B = \mu_0 I \frac{r^2}{R^2}$ $\qquad r < R$

$B = \frac{\mu_0 I r}{2\pi R^2}$



#10 Ampere's Law

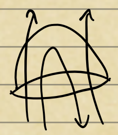


sign

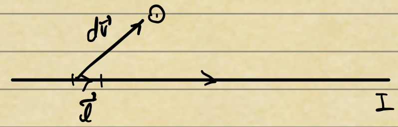
$$\oint \vec{B}\cdot d\vec{r} = \mu_0 \sum_i I_i$$

$$\oint_{\partial S} \vec{B}\cdot d\vec{r} = \mu_0 \iint_S \vec{J}\cdot d\vec{A}$$

I



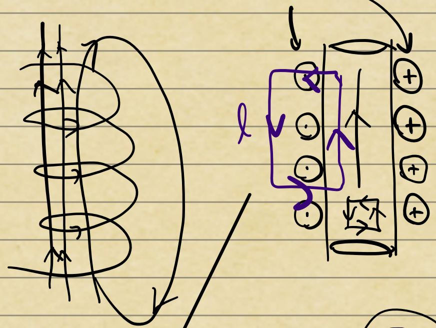
(ex)



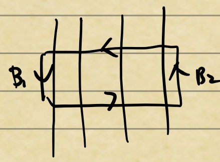
$2\pi r B(r) = \mu_0 I$

$B_\phi = \frac{\mu_0 I}{2\pi r}$

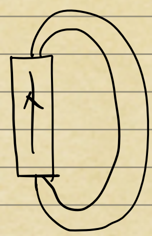
(ex)



field outside does not vary w. distance



$B_1 = B_2$ by Ampere's law



(finite)

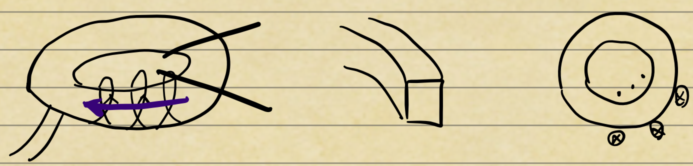
$B\cancel{l} = \mu_0 I n \cancel{l}$

n: # turns/length

18:26

☆ $\boxed{B = \mu_0 I n}$

(ex)



$2\pi r B = \mu_0 I N$

$B = \frac{\mu_0 I N}{2\pi r} = \mu_0 n I$, $n = \frac{N}{2\pi r}$

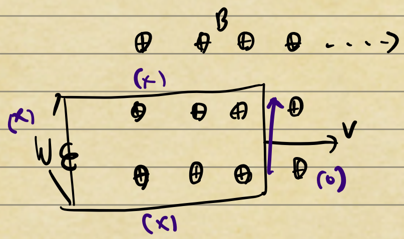
inner rim stronger

diff. so just approximation

$\oint \vec{E} \cdot d\vec{A} = \sum \frac{q}{\varepsilon_0}$ $\quad$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\quad$ $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\oint \vec{E} \cdot d\vec{l} = 0$ $\quad$ $\oint \vec{B} \cdot d\vec{l} = \sum \mu_0 I$

J, ρ constant

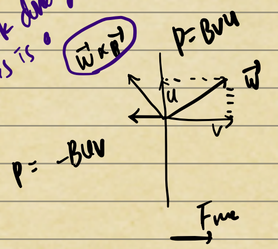


$\mathcal{E} = \oint \underset{\sim}{\vec{F}} \cdot d\vec{r} = Bvw$

per unit charge

paradox: magnetic field DOES work!

work done by this is 0



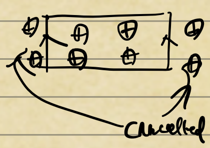
$P_R = \frac{\mathcal{E}^2}{R} = \frac{B^2 v^2 w^2}{R}$

$\mathcal{E} = Bvw$

$P_{I\ provide} = BwIv \quad (= \underbrace{Buv}_{\text{power above}} \times \underbrace{wAne}_{v\ \text{changes}})$

, $uAne = I$

* loop fully inside magnetic field ⇝ stop glowing



* Now in the moving frame of the loop ⇝ (change) moving magnetic field has got to glow the lightbulb

without the help of charge

In this case, $\oint \vec{E} \cdot d\vec{l} \neq 0$

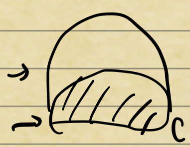
(not conservative field)

electromotive force (induced electric field)

"induce"

* $\boxed{\oint_{\partial S} \vec{F}_{em} \cdot d\vec{r} = -\frac{d\Phi}{dt}}$ Faraday

same result

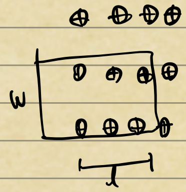


$\phi = \iint \vec{B} \cdot d\vec{A}$ "magnetic flux"

$\mathcal{E} = -\frac{d\Phi}{dt}$ Lenz

$\hookrightarrow \Leftarrow \vec{J}$

(ex)



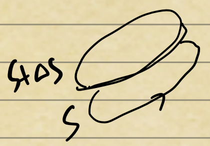
$$\mathcal{E} = -\frac{d(Bwl)}{dt} = -Bwv$$

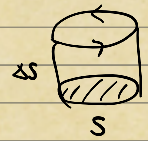
dS

$\mathcal{E}_{\circlearrowright} < 0$ if $v > 0$

$\mathcal{E}_{\circlearrowleft} > 0$ if $v > 0$

nothing new so far...





$S' = S + \Delta S$

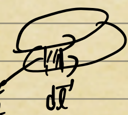
$$\Phi(t+\Delta t) - \Phi(t) = \iint_{S+\Delta S} B(t+\Delta t)\, dA - \iint_S \vec{B}(t)\, dA$$

$$= \underline{\iint_S B(t+\Delta t)\cdot dA} + \iint_{\Delta S} B(t)\cdot dA - \underline{\iint_S B(t)\cdot dA}$$

$$-\iint_{\underset{d\Delta t}{\Delta S}} \underbrace{\left(B(t) - B(t+\Delta t)\right)}_{\Delta t\cdot(X)}\cdot dA$$

$$\Delta\phi = \Delta t \iint \frac{d\vec{B}}{dt}\cdot d\vec{A} + \underbrace{\iint \vec{B}\cdot d\vec{A}}_{\Delta S}$$

$$dt \iint (\vec{v}\times d\vec{l})\cdot\vec{B}$$



$\vec{v}\,dt$ $\quad d\vec{l}'$

$$\vec{v}\,dt \times d\vec{l} = d\vec{A}$$

$$\frac{d\phi}{dt} = \iint \frac{d\vec{B}}{dt}\cdot d\vec{A} + \underbrace{\iint (\vec{v}\times d\vec{l})\cdot\vec{B}}$$

$$= \oint d\vec{l}\cdot(\vec{B}\times\vec{v})$$

$$= -\oint d\vec{l}\cdot(\vec{v}\times\vec{B})$$

$$\boxed{-\frac{d\phi}{dt} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{A} + \oint (\vec{v}\times\vec{B})\cdot d\vec{l}}$$

field change $\qquad$ loop change

Force, force per unit charge

loop not deformed

$$\int (E + \vec{v}\times\vec{B})\cdot d\vec{l} = -\int \frac{dB}{dt}\cdot dA + \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\oint \vec{E}\cdot d\vec{l} = -\iint \left(\frac{d\vec{B}}{dt}\right)\cdot d\vec{A}$$ Faraday's Law

$\hookrightarrow$ not having to do w. conductors

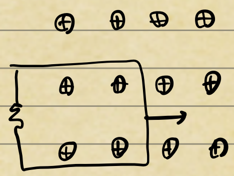
#11

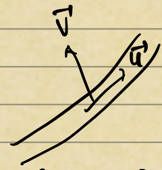
$$\oint (\vec{E} + \vec{v}\times\vec{B}) \cdot d\vec{l} = -\frac{d\Phi}{dt} \qquad \Phi = \iint_S \vec{B}\cdot d\vec{A}$$

$$= -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} + \oint_{\partial S} (\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\boxed{\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}}$$ No loop! so doesn't cover loops moving in fields

↳ Faraday Law



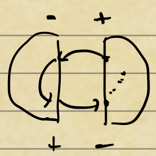


$$(\vec{v}+\vec{u}) \times \vec{B} \cdot dl$$

ignored

★ Betatron

Recall...



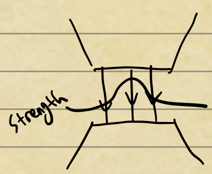
$$\frac{mv^2}{R} = qvB$$

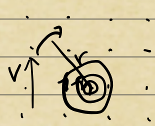
$$\omega = \frac{v}{R} = \frac{qB}{m}$$

In reality, $p = \dfrac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$ $\underbrace{(p)(\omega)}_{\text{centripetal force}}$ ⇒ Orbit indpt of R will fail

as v will increase only upto where relativistic consideration unimportant

* Betatron





larger/faster in circle

$$2\pi r E(r) = \frac{d}{dt}\underbrace{\iint B\cdot dA}_{\pi r^2 \bar{B}}$$ defined this way

$$qE(r) = q\frac{r}{2}\frac{d\bar{B}}{dt}$$

$$F = \frac{dp}{dt} = \frac{qr}{2}\frac{d\bar{B}}{dt} \qquad p(t) = \frac{qr}{2}\bar{B}(t) \qquad \underset{\text{assumption}}{p(0)=0}$$

$$\frac{mv^2}{r} = qv\underset{\sim}{B_0}$$ B at the orbital radius

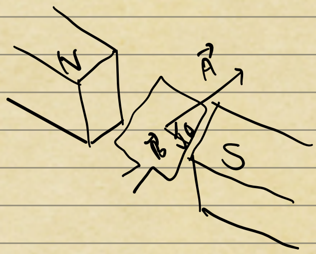
$\Rightarrow \ mv = qrB_0$

So, $qrB_0 = \frac{qr}{2}\overline{B(t)}$

$B_0 = \frac{1}{2}\overline{B(t)}$: requirement ← in relativistic setting this is still true

Accelerated charge radiates energy ~ problem

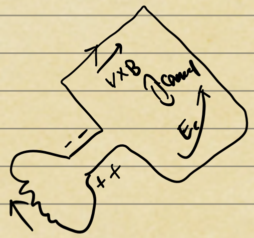
* generator

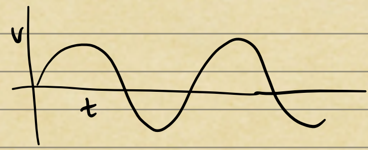


$\Phi = AB\cos\theta \qquad \theta = wt$

$\mathcal{E} = -\frac{d\Phi}{dt} = wAB\sin(wt)$

$\mathcal{E} = V$





$P_R = I^2R = \frac{\mathcal{E}^2}{R} = \frac{w^2A^2B^2\sin^2 wt}{R}$

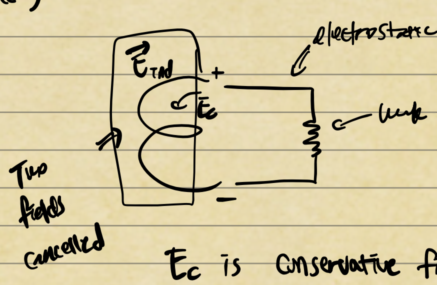
$\vec{\tau} = \vec{\mu}\times\vec{B} = AIB\sin\theta$

$P_M = \tau \cdot w = ABwI\sin\theta$

$= ABw\sin\theta\frac{\mathcal{E}}{R}$

$= \frac{A^2B^2w^2}{R}\sin^2\theta$

Inductors (Inductance)



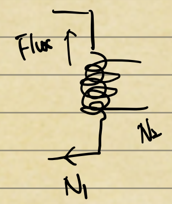
E_c is conservative field

$\mathcal{E}_2 = -\frac{d\Phi_2}{dt}$ $\qquad \Phi_2 = M_{12}I_1$

Mutual inductance : how much flux you get in the second coil per unit current in the second coil

$= -N_2\frac{d\Phi_B}{dt}$ $\qquad \mathcal{E}_2 = -M_{12}\frac{dI_1}{dt}$ $\qquad$ Fact: $M_{12} = M_{21}$ (not obvious)

\# turn



# of turns / unit length

$B = \mu_0 n I$ $\quad n = \frac{N}{l}$

$B = \mu_0 n_1 I_1$

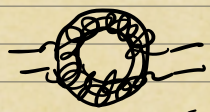
$\Phi_B = \mu_0 n_1 I_1 A$

$\Phi_2 = \mu_0 n_1 A \underline{N_2} I_1 = M_{21} I_1$

$\boxed{M = \mu_0 n_1 N_2 A}$ Henry (μH)

$\mathcal{E}_1 = N_1 \frac{d\Phi_B}{dt}$

* Transformer



Need AC (not DC)

$\mathcal{E}_1 = N_1 \frac{d\Phi_B}{dt}$ same flux

$\mathcal{E}_2 = N_2 \frac{d\Phi_B}{dt}$

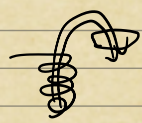
$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{N_1}{N_2}$

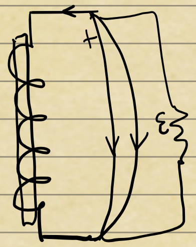
Step-up transformer; # of turn increases

Step-down ‖ ; ‖ decreases

- No manufacture of energy
- But you can create for example 110V for 220V!

You don't need to be co-axial for mutual inductance

(ex)  Voltage generated

* 

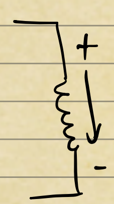
$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$

$\Phi = LI$

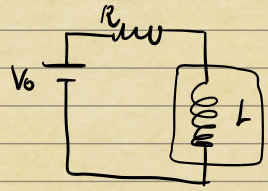
self-inductance

how much flux do you produce by a current going through yourself

Voltage drop



$= -L\frac{dI}{dt}$



$$\underbrace{V_0 - RI - L\frac{dI}{dt} = 0}_{\text{LR circuit}}$$

$0 \to I$ work done at L?

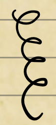
$$P = \underbrace{IL\frac{dI}{dt}}_{(I \times V)} = \frac{dW}{dt} = \frac{d}{dt}\left(\int I(t)\, L\frac{dI}{dt}\, dt\right)$$

$$= \frac{d}{dt}\left[\int I(t)\, L\, dI\right]$$

$$= \frac{d}{dt}\left(\frac{LI^2}{2}\right)$$

$$\int P\, dt = \underbrace{\frac{LI^2}{2}}_{\text{work done by inductor}} \quad \text{Start at } I=0$$

Recall $\frac{+++}{---}$ $\frac{q^2}{2C}$ J

L  $L = \frac{\Phi}{I}$ $\quad B = \mu_0 n I \quad n = \frac{N}{l}$

$\Phi_B = \mu_0 n I A$ per unit

$\Phi = \mu_0 n I A N = LI$

$L = \mu_0 n N A$

$$\frac{LI^2}{2} = \frac{1}{2}\mu_0 n N A I^2$$

$$= \frac{1}{2}\mu_0 n^2 \underbrace{lA}_{} I^2$$

$$= \frac{1}{2\mu_0}\underbrace{(\mu_0 n I)^2}_{B}(lA)$$

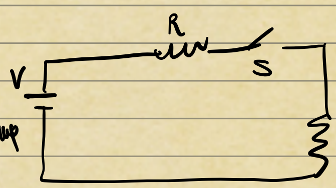
$$= \underbrace{\frac{B^2}{2\mu_0}}_{\text{energy per volume}}(\text{Volume})$$

Energy density $u_B = \frac{B^2}{2\mu_0}$ usually upstairs

$u_E = \frac{\epsilon_0}{2}E^2$ (recall) usually downstairs

Summary

$$\boxed{V = L\frac{dI}{dt} + RI}$$



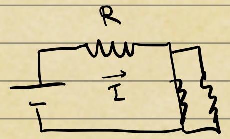
Conductor, similarly, cannot abruptly change current

Current in the inductor never jump

$\left(\frac{dI}{dt} < \infty\right)$

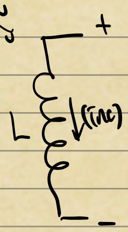
LR vs LC vs LCR

Lec12



$V = RI \qquad I(t) = \frac{V(t)}{R}$

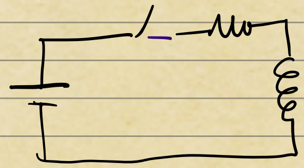
symbol for inductor



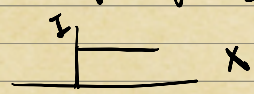
vs. capacitor

energy conserved
$$\left[\begin{array}{l} V_L = L\frac{dI}{dt} \\ V_c = \frac{Q}{C} = \frac{\int I\,dt}{C} \end{array}\right.$$

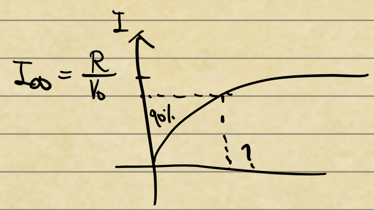
x



No way to get $\frac{1}{2}LI^2$ immediately.



$$V_0 - RI = L\frac{dI}{dt}$$



$$L\frac{dI}{dt} + RI = V_0$$

$$I(t) = I_\infty + \tilde{I}(t)$$

$$L\frac{d\tilde{I}}{dt} + \cancel{RI_\infty} + R\tilde{I}(t) = \cancel{V_0}$$

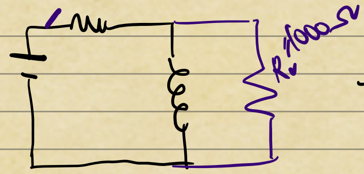
$$\frac{d\tilde{I}}{dt} = -\frac{R}{L}\tilde{I}(t)$$

$$\tilde{I}(t) = \tilde{I}_0 e^{-\frac{Rt}{L}}$$

$$I(t) = \frac{V_0}{R} + \tilde{I}_0 e^{-\frac{Rt}{L}} \qquad I(0) = 0$$

$$\rightsquigarrow I(t) = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

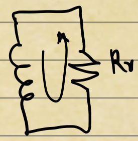
(ex) $t = L/R$: $\frac{1}{e} \approx \frac{1}{3}$



$I = \frac{V_0}{R}$

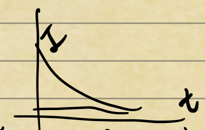
- if just close, spark on the switch (when ∄ R_2)

Rv: during close, all goes to inductor.
after open,



since air is also high resistance

$$\begin{cases} L\frac{dI}{dt} + R_2 I = 0 \\ I = I_0 e^{-\frac{R_2}{L}t} \end{cases}$$



can see safe point for example

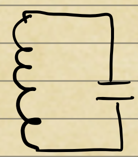
$\frac{R_2}{L} = \frac{1}{\tau}$ $\tau = \frac{L}{R_2}$

$$P_R = I^2 R_2 = I_0^2 e^{-\frac{2R_2}{L}t} R_2$$

$$\int_0^\infty P_R\,dt = I_0^2 \frac{L}{2R_2} R_2$$

$$= \frac{I_0^2 L}{2}$$ (as expected)

*



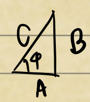
doesn't stop when capacitor is discharged

$L\frac{dI}{dt} + \frac{Q}{C} = 0$ $\frac{dQ}{dt} = I$

$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$ oscillates!

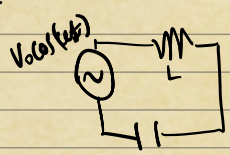
$$Q(t) = A\cos\left(\sqrt{\frac{1}{LC}}\,t\right) + B\sin\left(\sqrt{\frac{1}{LC}}\,t\right) = C\cos(\omega_0 t - \varphi)$$

$\omega_0 = \sqrt{\frac{1}{LC}}$



$$\begin{cases} A = C\cos\varphi \\ B = C\sin\varphi \end{cases}$$

*



$\omega_0 = \frac{1}{\sqrt{LC}}$

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = V_0\cos(\omega t)$$

$Q = \tilde{C} \cos(\omega t)$

$-\omega^2 L \tilde{C} \cos(\omega t) + \frac{\tilde{C}}{C} \cos(\omega t) = V_0 \cos \omega t$

$\tilde{C} = \frac{V_0}{-\omega^2 L + \frac{1}{C}} = \frac{-\frac{V_0}{L}}{\underbrace{\omega^2 - \omega_0^2}}$

if driving frequency is equal to resonant frequency, sol. will blow up

$V = V_0 \cos(\omega t)$

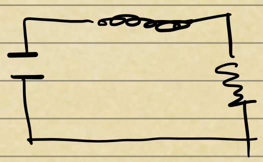
$I = \frac{dQ}{dt} = -\tilde{C} \omega \sin(\omega t)$ so $V_{max} \to I_{min}$ etc.

$= \frac{V_0/L}{\omega^2 - \omega_0^2} \omega \sin(\omega t)$ not cosine like the voltage

"out of phase"

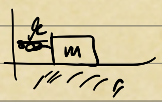
so $I(t) \neq \frac{V(t)}{(\sim)}$

*



$L\frac{dI}{dt}$

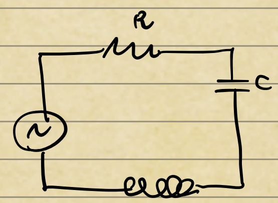
$\frac{L d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$



$X(t) = Ae^{-\alpha t} \cos(\omega' t)$

$X(t) = Ae^{-\beta t}$. take real part

*



$V_0 \cos(\omega t)$

$L\frac{dI}{dt} + RI + \frac{1}{C}\int I(t)\,dt = V_0 \cos(\omega t)$

$= V(t)$

$L, R, C \in \mathbb{R} \Rightarrow$ I^* obeys same eqn

$L\frac{dI}{dt} + RI + \frac{1}{C}\int I(t)\,dt = V_0 e^{i\omega t}$

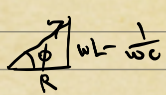
$I = \tilde{I} e^{i\omega t}$

$(i\omega L \tilde{I} + R\tilde{I} + \frac{1}{i\omega C}\tilde{I}) e^{i\omega t} = V_0 e^{i\omega t}$

$(i\omega L + R + \frac{1}{i\omega C})\tilde{I} = V_0$

exchange

$R + i(\omega L - \frac{1}{\omega C})$: impedance Z



law generalization

ohm's

$$\boxed{\tilde{I} = \frac{V_0}{Z}} = \frac{V_0}{|Z|} e^{-i\phi} \qquad \tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$\phi > 0$ lead
$\phi < 0$ lag
$\phi = 0$ purely resistive

$$I = \frac{V_0}{|Z|} e^{i(\omega t - \phi)}$$

rescale & shift phase

$|Z|$: effective opposition

$$Re(I) = \frac{V_0}{|Z|} \cos(\omega t - \phi)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

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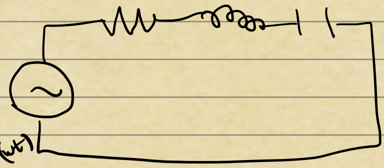
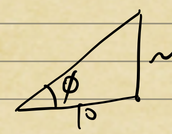


$V_0 \cos(\omega t)$

$$V_0 \cos(\omega t) = RI + L\frac{dI}{dt} + \frac{1}{C}\int^t I(t')\,dt'$$

ex) $R = 10\Omega \quad \omega = 100\pi \quad L = 3H \quad C = 2\mu F$

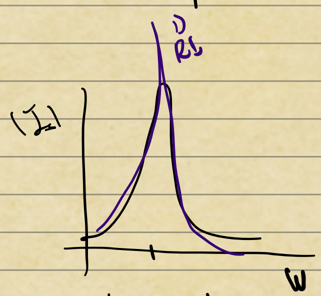
$$Z = 10 + \left[(100\pi)3 - \frac{1}{(100\pi)(2\times10^{-6})}\right] i$$



$$\leadsto I = \frac{V_0}{\underset{\sim}{|Z|}} \cos(\omega t - \phi)$$

① only works for $V_0 \cos(\omega t)$ $\therefore I_0$

② impedance depends on frequency



$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

2) $C = \frac{\varepsilon_0 A}{d}$ — → — vary capacitance

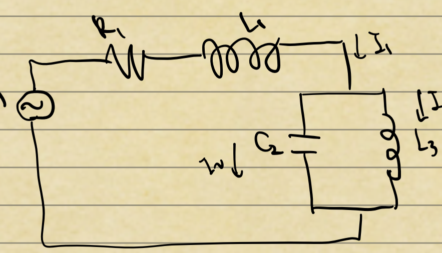
$\leadsto$ shift the peak

3) $I(t) = \underbrace{\frac{V_0}{|Z|}\cos(\omega t - \phi)}_{\text{steady state current}} + \underbrace{I_{comp}(A, B)}_{\text{transient current}}$ ← no free parameter

add zero solution $\leadsto$ fit initial condition

Not care too much since it vanishes exponentially

$$\Rightarrow |I_0|_{max} = \frac{V_0}{R}$$



$$\begin{cases} I_1 = I_2 + I_3 \\ V_0 \cos(\omega t) = R_1 I_1 + L_1 \frac{dI_1}{dt} + \frac{1}{C_2}\int I_2\,dt' \\ L_3 \frac{dI_3}{dt} - \frac{1}{C_2}\int^t I_2\,dt = 0 \end{cases}$$

$\leadsto$ − sign since

$V_0 e^{i\omega t}$ $\quad$ $\tilde{I}_j = \tilde{I}_{j0} e^{i\omega t}$ $\quad j = 1, 2, 3$

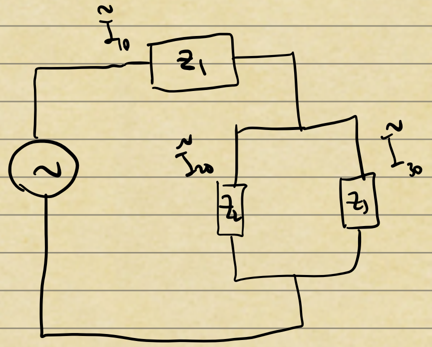
$\uparrow$

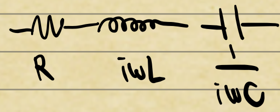
$V_0 \cos(\omega t)$

$$\tilde{I}_{10} = \tilde{I}_{20} + \tilde{I}_{30}$$

$$V_0 = R_1 \tilde{I}_{10} + L_1 (i\omega) \tilde{I}_{10} + \frac{1}{C_2} \frac{1}{i\omega} \tilde{I}_{20}$$

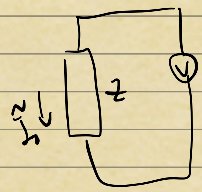
$$L_3 \tilde{I}_{30} - \frac{1}{C} \frac{1}{i\omega} \tilde{I}_{20} = 0$$





$\rightsquigarrow$ after this, treat like DC circuit

Then, $I_j = \mathrm{Re}(\tilde{I}_{j0} e^{i\omega t})$



Actual Voltage : $\mathrm{Re}(\tilde{I}_0 Z e^{i\omega t})$

LCR

$$V = V_0 \cos(\omega t)$$

$$I = \frac{V_0}{|Z|} \cos(\omega t - \phi)$$

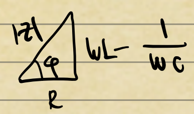
$$P(t) = V(t) I(t)$$

$$= \frac{V_0^2}{|Z|} \underbrace{\cos(\omega t) \cos(\omega t - \phi)}_{\cos^2(\omega t) \cos\phi + \cos(\omega t) \sin(\omega t) \sin\phi}$$

$$\overline{P}_{av} = \frac{V_0^2}{|Z|} \frac{\cos\phi}{2} = \frac{1}{2} \left(\frac{V_0}{|Z|}\right)^2 |Z| \cos\varphi = \frac{1}{2} I_0^2 R$$

$\parallel$

$\left(\frac{1}{T} \int_0^T P(t) dt\right)$



$I_{rms} = \frac{I_0}{\sqrt{2}}$ $\quad$ $V_{rms} = \frac{V_0}{\sqrt{2}}$

$\nwarrow$ root mean square

$\overline{P}_{av} = I_{rms}^2 R$ $\qquad\qquad$ $V_0 = |Z| I_0$

$$= I_{rms} V_{rms} \frac{I_{rms} R}{V_{rms}}$$

$\sim$

$$\frac{\cancel{\sqrt{2}}}{V_0} \times R \times \frac{I_0}{\cancel{\sqrt{2}}} = \frac{R}{|Z|} = \cos\varphi$$

$= I_{rms} V_{rms} \cos\varphi$

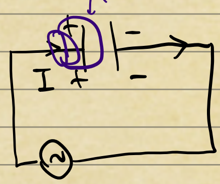
$\neq \mathrm{Re}\left[\tilde{I}(t)\,\tilde{V}(t)\right]$

$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ $\quad \oiint \vec{B}\cdot d\vec{A} = 0$ $\quad + \quad \vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

All of classical theory

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$ $\quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I + \underbrace{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}_{\text{displacement current term}}$ Ampere's law modified

problem!



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt}\Phi_E$

$\mu_0 \frac{dQ}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right)$

$= \mu_0 \varepsilon_0 A \frac{d}{dt}\left(\frac{\sigma}{\varepsilon_0}\right)$

$= \mu_0 \varepsilon_0 A \frac{d}{dt} E$

$= \mu_0 \varepsilon_0 \frac{d}{dt}(EA)$

$= \mu_0 \varepsilon_0 \frac{d}{dt} \oiint E \cdot dA$

$= \mu_0 \varepsilon_0 \frac{d}{dt} \Phi_E$

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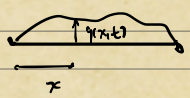
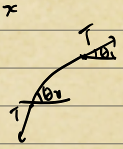
$\oiint B\cdot dA = 0$ $\quad \oiint \vec{E}\cdot d\vec{A} = \frac{\Sigma q_i}{\varepsilon_0}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt}\Phi_E$ $\quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$





$T\sin\theta_1 - T\sin\theta_2$

$\approx T(\tan\theta_1 - \tan\theta_2)$

$= T\left(\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}\right)$

$= \underbrace{T\frac{d^2y}{dx^2}\Delta x}_{\text{"F"}} = \underbrace{\mu\, dx \frac{d^2y}{dt^2}}_{\text{"ma"}}$

$\sin\theta = \theta - \cancel{\frac{\theta^3}{3!}} + \cdots$

$\cos\theta = 1 - \cancel{\frac{\theta^2}{2}} + \cdots$

$\tan\theta = \theta + \cancel{\cdots}$

$\Rightarrow T\frac{d^2y}{dx^2} = \mu \frac{d^2y}{dt^2}$

$\frac{d^2y}{dx^2} - \frac{1}{v^2}\frac{d^2y}{dt^2} = 0$ $\qquad v = \sqrt{\frac{T}{\mu}}$

tension

mass per unit length

Solution: $y = y(x - vt)$, or, $z = x - vt$ $y = F(z)$

$\searrow$ v is velocity

$$\frac{dy}{dx} = \frac{dF}{dz}\frac{dz}{dx} = \frac{dF}{dz}$$

$$\frac{dy}{dt} = \frac{dF}{dz}\frac{dz}{dt} = \frac{dF}{dz}(-v)$$

$\therefore$ can be verified

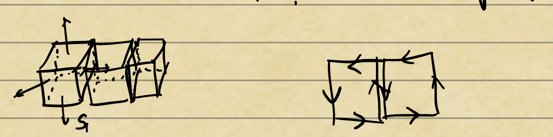
actually, $F(x-vt) + G(x+vt)$

Vacuum: no current / charge

$$\oiint \vec{E}\cdot d\vec{A} = 0, \quad \oiint \vec{B}\cdot d\vec{A} = 0$$

If true for tiny surface, true for big ones

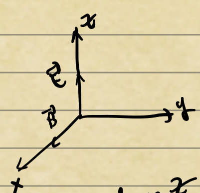
—— " —— loop, —— " —— big loop



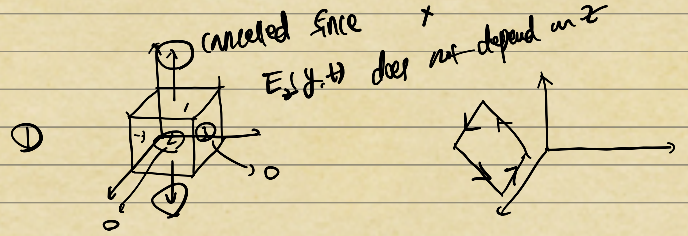
Now.

$\vec{E} = \vec{k}\, E_z(y,t)$

$\vec{B} = \vec{i}\, B_x(y,t)$



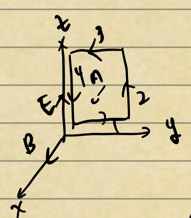
① canceled since $E_z(y,t)$ does not depend on z



$$\oint E\cdot dl = -\frac{d\Phi_B}{dt}$$
$$\quad \| \qquad\quad \|$$
$$\quad 0 \qquad\quad 0$$

$$\oint B\, dl = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$
$$\quad \| \qquad\quad \|$$
$$\quad 0 \qquad\quad 0$$

②



$$\oint E\cdot dl = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt}(B_x \Delta y \Delta z)$$

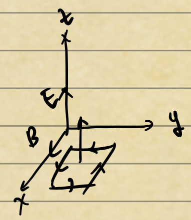
$$\Delta z\left(E_z(y+\Delta y) - E_z(y)\right) = -\Delta y \Delta z \frac{d}{dt}B_x$$

$$= \Delta z \frac{dE_z}{dy}\Delta y$$

$$\boxed{\frac{dE_z}{dy} = -\frac{dB_x}{dt}}$$

$$\oint \vec{B} \cdot d\vec{l} = 0 = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

(2)



$$\oint E \cdot dl = 0 = -\frac{d\Phi_B}{dt}$$

$$\oint B \cdot dl = -\Delta x \left(B_x(y+\Delta y) - B_x(y) \right)$$

$$= -\Delta x \, \Delta y \, \frac{dB_x}{dy}$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \Delta x \Delta y \frac{d(E_z(y))}{dt} \mu_0 \varepsilon_0$$

Result

$$\begin{cases} \dfrac{dE_z}{dy} = -\dfrac{dB_x}{dt} \\ \dfrac{dB_x}{dy} = -\mu_0 \varepsilon_0 \dfrac{dE_z}{dt} \end{cases}$$

These are the requirements!

$$\frac{d^2 E_z}{dy^2} = -\frac{d^2 B_x}{dy\,dt} = -\frac{d^2 B_x}{dt\,dy}$$

$$= \mu_0 \varepsilon_0 \frac{d^2 E_z}{dt^2}$$

$$\frac{d^2 E_z}{dy^2} - \mu_0 \varepsilon_0 \frac{d^2 E_z}{dt^2} = 0$$

All in vacuum, electric field is oscillating.

$$v^2 = \frac{1}{\mu_0 \varepsilon_0} = \frac{1}{4\pi\varepsilon_0} \left(\frac{\mu_0}{4\pi}\right)^{-1} = (9 \times 10^9) \times (10^{-7})^{-1}$$

$$= 9 \times 10^{16}$$

$$v = 3 \times 10^8 \ (m/s)$$

Same speed as light!

This holds for general E & B. Get same eqn for each component

$\vec{E} = \vec{k} E_0 \underbrace{\sin(ky-\omega t)}_{E_z}$ $\quad$ $\vec{B} = \vec{i} \underbrace{B_0 \sin(ky-\omega t)}_{B_x}$

$\frac{dE_z}{dy} = -\frac{dB_x}{dt} \Rightarrow k E_0 \cancel{\cos(ky-\omega t)} = +\omega B_0 \cancel{\cos(ky-\omega t)}$

$\frac{dB_x}{dy} = -\mu_0 \varepsilon_0 \frac{dE_z}{dt} \Rightarrow B_0 k \cancel{\cos(ky-\omega t)} = +\mu_0 \varepsilon_0 \omega E_0 \cancel{\cos(ky-\omega t)}$

$kE_0 = \omega B_0$ $\quad$ $\boxed{E_0 = \frac{\omega}{k} B_0}$

$B_0 k = \overbrace{\mu_0 \varepsilon_0}^{\frac{1}{c^2}} \omega E_0$ $\quad$ $\boxed{E_0 = \frac{c^2 k}{\omega} B_0}$

$= \frac{\omega}{c^2} E_0$

$\frac{\omega}{k} = \frac{c^2 k}{\omega}$ $\quad$ $\omega^2 = c^2 k^2$

$\boxed{\omega = \pm ck}$

$\sin(ky-\omega t) = \sin(ky - kct)$

$= \sin(k(y-ct)) = F(y-ct)$

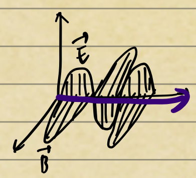
$\boxed{E_0 = cB_0}$

$\omega = kc$

$\vec{E} = \vec{k} E_0 \sin(ky-\omega t)$

$\vec{B} = \vec{i} B_0 \sin(ky-\omega t)$

$B_0 c = E_0$

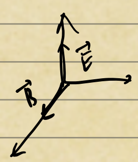


E is much bigger than B

Remember $\vec{F} = q(\vec{E} + \underline{\vec{v} \times \vec{B}})$

smaller than $\vec{E}$

#15



$\vec{E} = \vec{k} E_0 \sin(ky-\omega t)$ $\quad$ $\omega = kc$ $\quad$ $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\quad$ $B_0 = \frac{E_0}{c}$

$\vec{B} = \vec{i} B_0 \sin(ky-\omega t)$

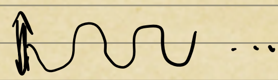
$k\lambda = 2\pi$ $\quad$ $\lambda = \frac{2\pi}{k}$ $\quad$ k = wave number

$\omega T = 2\pi$ $\quad$ $T = \frac{2\pi}{\omega}$

$\vec{E} = \vec{k} E_0 \sin\left(\frac{2\pi y}{\lambda} - \frac{2\pi t}{T}\right)$

$ky - \omega t = k(y - ct)$

Transverse wave: action is perpendicular to the motion



longitudinal wave

polarization: direction of electric field in the solution

another pair $\vec{E} = \vec{k} E_0 \sin(ky + \omega t)$
$\vec{B} = -\vec{i} B_0 \sin(ky + \omega t)$ (imagine rotate 180° around z axis)

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}$$

$$u_E = \frac{\varepsilon_0}{2} E_0^2 \sin^2(ky - \omega t)$$

$$= \frac{\varepsilon_0}{2} B_0^2 c^2 \sin^2(ky - \omega t)$$

$$= \frac{B_0^2}{2\mu_0} \sin^2(ky - \omega t)$$

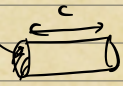
$= u_B$ ⇒ same energy density (time-space dependent)

$$u_T = \varepsilon_0 E_0^2 \sin^2(ky - \omega t)$$

u = average $= \dfrac{\varepsilon_0^2 E_0^2}{2}$ $\left(\frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta = \frac{1}{2}\right)$

Intensity $I = \dfrac{\text{watt}}{m^2}$ ← J/s

$l = A$ c



$= \underbrace{u}_{J/m^3} \underbrace{c}_{m/s}$

instantaneous

$I = uc$

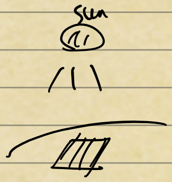
$= c\varepsilon_0 E_0^2 \sin^2(ky - \omega t)$

$= c\varepsilon_0 E_0 B_0 c \sin^2(ky - \omega t)$

$= \dfrac{E_0 B_0}{\mu_0} \sin^2(ky - \omega t)$ $\boxed{\bar{I} = \dfrac{E_0 B_0}{2\mu_0}}$

S pointings vector $\dfrac{\vec{E} \times \vec{B}}{\mu_0} = \vec{S}$ $(|\vec{S}| = I)$

(ex) sun

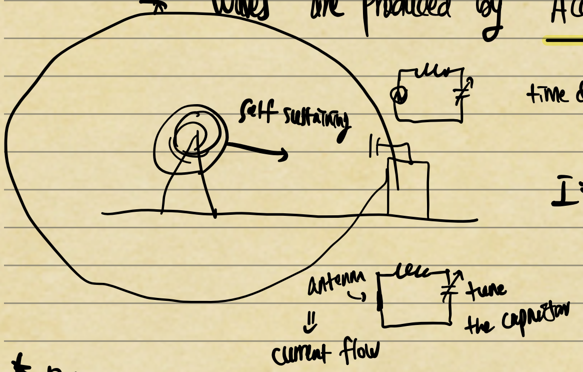


$\bar{I} = 1000 \, W/m^2$

$E_0 = 1000 \, V/m$ (use $E_0 = cB_0$ & plug into the above) (1m) 1000 J 1C $\dfrac{N \cdot m}{C \cdot m} = J/C \cdot \dfrac{1}{m} = V/m$

Q. Where's electric field away from? (free charge situation)

* Waves are produced by Accelerating Charges



time dependent $\Phi_B \rightarrow \Phi_E \rightarrow \Phi_B \rightarrow \cdots$
get lose eventually ...

$I = \frac{100}{4\pi R^2}$ → translate to electric field

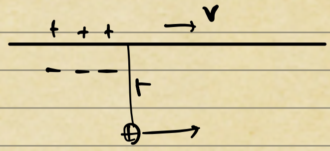
Calculate the amplitude produced for radio station
(build a circuit to do this ...)

* Relativity

* $\frac{d^2E}{dx^2} - \frac{1}{c^2}\frac{d^2E}{dt^2} = 0$ $\quad x' = \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}}$ $\quad t' = \frac{t-\frac{ux}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$ $\quad \frac{d4}{dx} = \frac{d4}{dx'}\frac{dx'}{dx} + \frac{dt}{dt}\frac{d6}{dx} \cdots$

⇝ get $\frac{d^2E}{dx^2} - \frac{1}{c^2}\frac{d^2E}{dt^2} = 0$

$$\boxed{w = \frac{v-u}{1-\frac{uv}{c^2}}}$$



Rest frame: $n_1^+ = n_1^-$
$\qquad\qquad \vee \qquad \wedge$
moving frame: $n_2^+ \quad n_2^-$

① moving: $E = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{n_2^+ \frac{v^2}{c^2} Ae}{2\pi r \varepsilon_0} \overset{?}{=} vB$

$n\frac{v^2}{c^2}$ A $\qquad \lambda = n_2^+ \frac{v^2}{c^2} Ae$

$I = Anev$

$$\frac{1}{1-\frac{u^2}{c^2}} = 1 + \frac{u^2}{c^2} + \underbrace{\left(\frac{u^2}{c^2}\right)^2 + \cdots}_{\text{nope.}}$$

$B = \frac{n_2^+ \frac{v^2}{c^2} Ae}{2\pi r\varepsilon_0}$

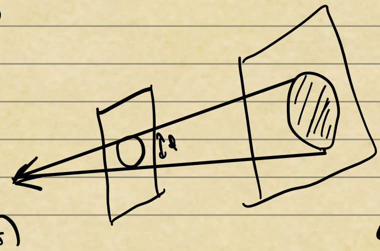
$= \underbrace{\underbrace{n_2^+ veA}_{I} \frac{\mu_0}{2\pi r}}_{B} v$

caveat: ① used n_2^+, but really should've used $n_1^+ = n_2^+ \left(\frac{1}{\gamma}\right)_{>1}$ but fine (first term approximation)

② if we keep higher order terms, ∃ error too since time is diff for everyone
(need to use four-vector)

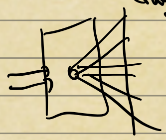
16

(λ↓)



approximation

(in the past) ⇒ $\lambda << d$ geometrical optics (ray)

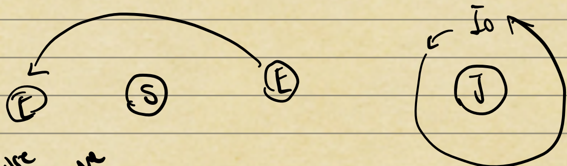


$\lambda \approx d$

· photon ⇝ light (QM)
⇓
massless particle

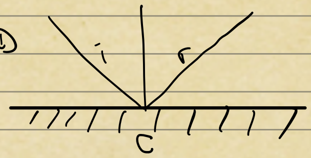
Maxwell eqn is incomplete

<u>Roemer</u> Velocity of light!



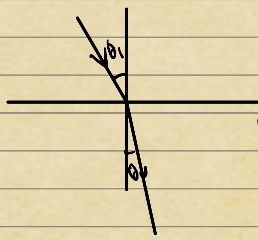
pulse
22 min more

①



air

②



n_1

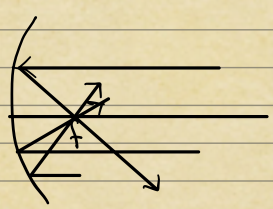
glass

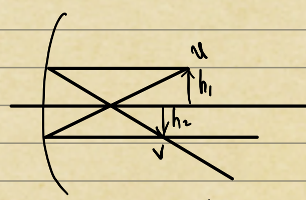
n_2

$V = \frac{c}{n}$

refractive index

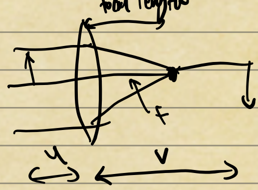
$n_1 \sin\theta_1 = n_2 \sin\theta_2$



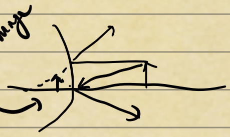


$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

focal length



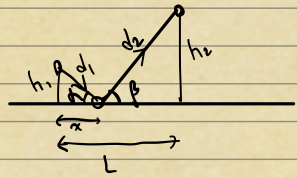
virtual image



Use $-f$

* principle of least time - Fermat

ex)



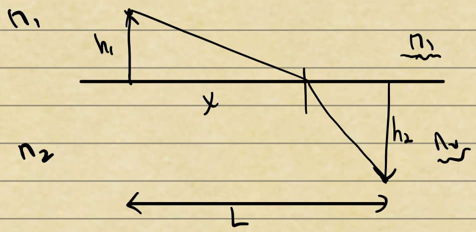
$$T = \frac{\sqrt{h_1^2+x^2}}{c} + \frac{\sqrt{h_2^2+(L-x)^2}}{c}$$

$$\frac{d}{dx}\left(\sqrt{h_1^2+x^2} + \sqrt{h_2^2+(L-x)^2}\right) = 0$$

$$\frac{x}{d_1} = \frac{L-x}{d_2}$$

$$\alpha = \beta$$

ex)

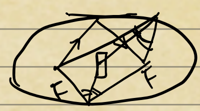


$$T = \frac{\sqrt{h_1^2+x^2}}{\frac{c}{n_1}} + \frac{\sqrt{h_2^2+(L-x)^2}}{\frac{c}{n_2}}$$

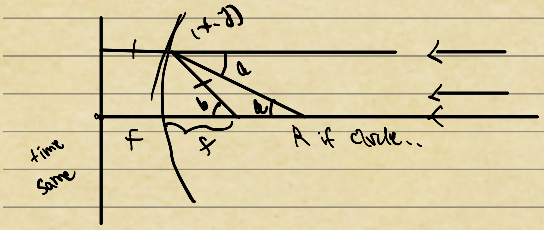
ans $\dfrac{n_1 x}{\sqrt{h_1^2+x^2}} = \dfrac{n_2 x}{\sqrt{h_2^2+(L-x)^2}}$

$\Leftrightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$

What if there is more than one answer?



$r_1 + r_2 = c$



$\cancel{\tan}\, b = \dfrac{h}{f}$

$\cancel{\tan}\, a = \dfrac{h}{R} = \dfrac{h}{2f}$

approx.

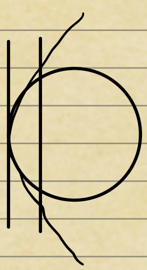
$\Rightarrow a = \dfrac{b}{2}$

(works only for small angle though since we assumed sphere)

parabola $(x+f)^2 = y^2 + (f-x)^2$

$2xf = y^2 - 2xf$

$y^2 = 4xf$



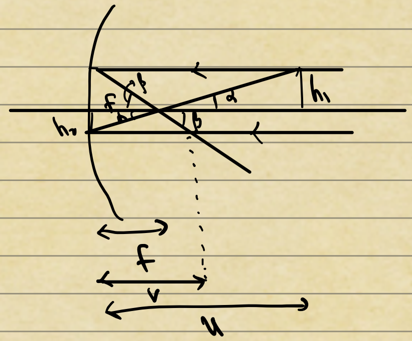
$(x-R)^2 + y^2 = R^2$

$y^2 = 2xR - x^2$ ~ approximate parabola when x is small

w.r.t. big

y small

x (small)^2



$$\tan\alpha = \frac{h_1}{u-f} = \frac{h_2}{f(-x)}$$

negligible

$$\tan\beta = \frac{h_1}{f} = \frac{h_2}{v-f}$$

$$\frac{h_1 h_2}{(u-f)(v-f)} = \frac{h_1 h_2}{f^2}$$

$$(u-f)(v-f) = f^2$$

$$uv - uf - vf + \cancel{f^2} = \cancel{f^2}$$

$$uf + vf = uv$$

① $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

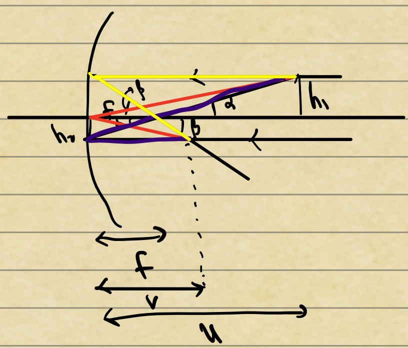
$$\frac{h_1}{h_2} = \frac{u-f}{f} = \frac{u}{f} - 1$$

$$= u\left(\frac{1}{u} + \frac{1}{v}\right) - 1$$

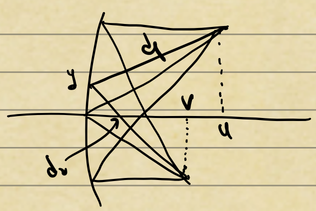
$$= \frac{u}{v}$$

② $\frac{h_1}{h_2} = \frac{u}{v}$

$M = -\frac{u}{v}$ (upside down version : minus)

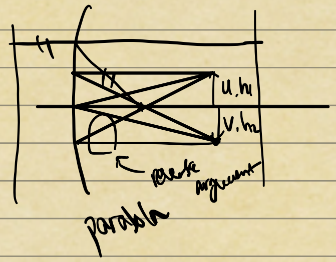


$\frac{u}{h_1} = \frac{v}{h_2}$ (True)



$\sqrt{h_1^2+u^2} + \sqrt{h_2^2+v^2} = (d_1+d_2)$ (y)

19 Optics II



$\frac{h_1}{u} = \frac{h_2}{v}$

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$y^2 = 4fx$, $2f = R$ approximation

$d_1(y) + d_2(y) = d_1(0) + d_2(0)$

$\boxed{D + Ay + By^2} + \cancel{Cy^3} + \cdots$

$(u^2+\theta^2)^{\frac{1}{2}}$

$= u\left(1+\frac{\theta^2}{u^2}\right)^{\frac{1}{2}}$

$\approx u\left(1+\frac{\theta^2}{2u^2}\right)$

$= u + \frac{\theta^2}{2u}$

u, v, f big

y small

x (small)^2

$d_1 + d_2 = \sqrt{(h_1-y)^2+(u-x)^2} + \sqrt{(h_2+y)^2+(v-x)^2} = \sqrt{h_1^2+u^2} + \sqrt{h_2^2+v^2}$

$\sqrt{h_1^2+y^2-2h_1y+u^2+\cancel{x^2}-2ux} + \sqrt{h_2^2+y^2+2h_2y+v^2+\cancel{x^2}-2vx} = u + \frac{h_1^2}{2u} + v + \frac{h_2^2}{2v}$

$\cancel{u} + \frac{\cancel{h_1^2}+y^2-2h_1y-2ux}{2u} + \cancel{v} + \frac{\cancel{h_2^2}+y^2+2h_2y-2vx}{2v} = \cancel{u}+\cancel{v} + \cancel{\frac{h_1^2}{2u}} + \cancel{\frac{h_2^2}{2v}}$

$x = \frac{y^2}{4f}$ $\qquad -2ux = \frac{-y^2u}{2f}$

1) $y\left(-\frac{2h_1}{2u} + \frac{2h_2}{2v}\right)$

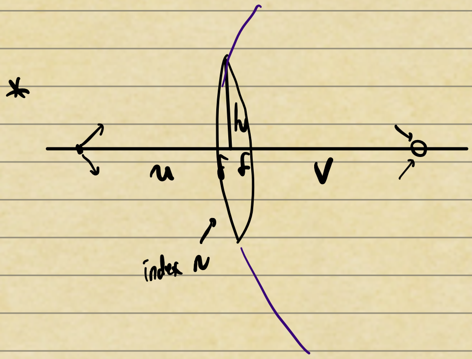
$\boxed{\frac{h_1}{u} = \frac{h_2}{v}}$

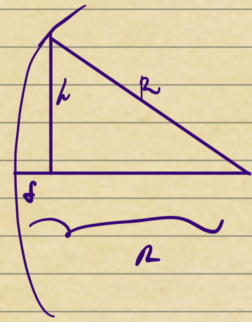
2) $y^2\left(\frac{1}{2u} + \frac{1}{2v} - \frac{u}{2f\,2u} - \frac{v}{2f\,2v}\right) = 0$

$\frac{1}{u} + \frac{1}{v} - \frac{1}{2f} - \frac{1}{2f} = 0$

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$$

Requirement: small y!





$$\delta = R - \sqrt{R^2 - h^2}$$
$$= R - \frac{h^2}{2R}$$

$$\delta = \frac{h^2}{2R}$$

$$\sqrt{u^2+h^2} + \sqrt{v^2+h^2}$$

$$= u-\delta + v-\delta + 2\delta n$$

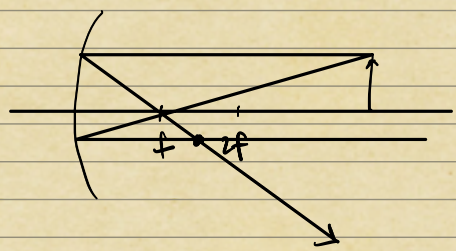
$$= \cancel{u}+\cancel{v} + 2\delta(n-1)$$

$$= \cancel{u} + \frac{h^2}{2u} + \cancel{v} + \frac{h^2}{2v}$$

$$= \frac{2h^2}{2R}(n-1)$$

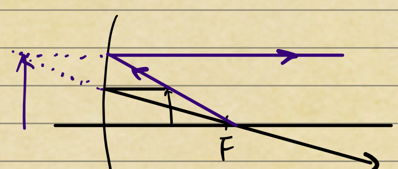
$$\frac{1}{u} + \frac{1}{v} = \frac{2(n-1)}{R} = \frac{1}{f}$$

definition $(u \to \infty)$



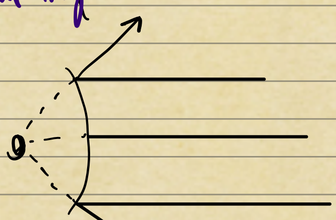
2f is like critical line

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ (ex) $u > 2f \leadsto v < 2f$

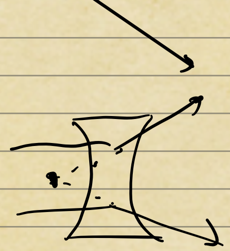


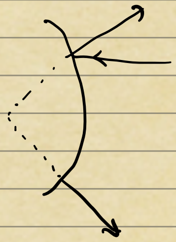
Virtual image

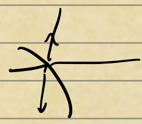
$V < 0$?



$$\frac{1}{u} + \frac{1}{v} = -\frac{1}{|f|}$$

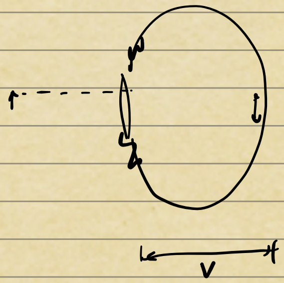






If you know the fate of one image

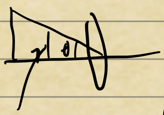
→ other side of mirror too.



$$\frac{1}{u} + \underbrace{\frac{1}{v}}_{\text{fixed}} = \frac{1}{f}$$

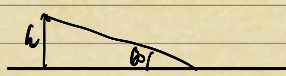
adjust via muscle

near point 25 cm



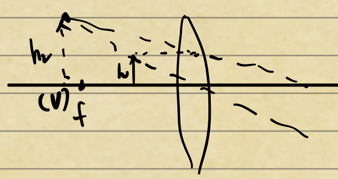
eye cannot accommodate this

(will look big normally)



$$\tan\theta_0 = \frac{h}{25\text{ cm}}$$

$$\theta_0 = \frac{h}{25\text{ cm}}$$



$$\theta = \frac{h_2}{v} = \frac{h}{u}$$

$$M_{angle} = \frac{\theta}{\theta_0} = \frac{h}{u}\frac{25}{h} = 25\left(\frac{1}{u}\right) = 25\left(\frac{1}{f} + \frac{1}{|v|}\right) = \frac{25}{f} + \frac{25}{|v|}$$

negative #

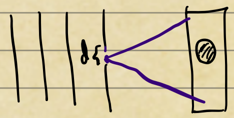
$$\leq 1 + \frac{25}{f}$$

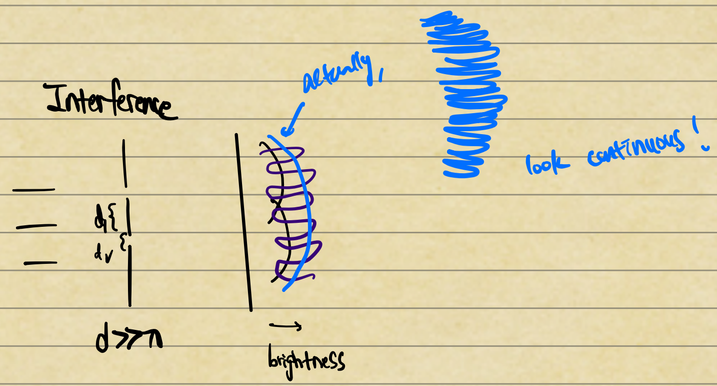
$M = \frac{25\,cm}{f}$

v cannot be less than 25

Lec 17

Interference

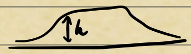




$\lambda \ll d$ geometrical optics

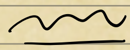
monochromatic light needed (no white light)

ψ wave · height of string



· E or B

· height of water



$$\frac{d^2\psi}{dx^2} - \frac{1}{v^2}\frac{d^2\psi}{dt^2} = 0 \Leftrightarrow \Box\psi = 0$$ linear

(ex) $I \propto \psi^2$

$I_1 = \psi_1^2,\ I_2 = \psi_2^2$

$I_{1+2} = (\psi_1 + \psi_2)^2$

$= \psi_1^2 + \psi_2^2 + 2\psi_1\psi_2$

$= I_1 + I_2 + 2\psi_1\psi_2$

ψ wave function

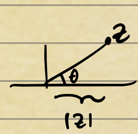
should use $|\psi|^2$

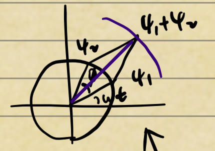
$\psi_1 = A\cos(\omega t)$ $\quad \psi_2 = A\cos(\omega t + \varphi)$

$\psi = A\cos(\omega t) + A\cos(\omega t + \varphi)$

$= \underbrace{A\cos\frac{\alpha-\beta}{2}}_{\tilde{A}}\ 2\cos(\omega t + \frac{\varphi}{2})$

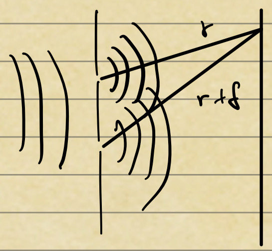
(ex) $\psi_1 = A\cos(\omega t) = \mathrm{Re}(Ae^{i\omega t})$ $\qquad z = |z|e^{i\theta}$





$$\tilde{A}^2 = A^2 + A^2 + 2AA\cos\phi$$
$$= 2A^2 + 2A^2\cos\phi$$
$$= 4A^2\cos\frac{\phi}{2}$$
$$\tilde{A} = 2A\left|\cos\frac{\phi}{2}\right|$$

$$\psi_1+\psi_2 = Ae^{i\omega t} + Ae^{i\omega t}e^{i\phi}$$
$$= Ae^{i\omega t}(1+e^{i\phi})$$
$$= Ae^{i\omega t}e^{\frac{i\phi}{2}}\underbrace{\left(e^{\frac{i\phi}{2}} + e^{-\frac{i\phi}{2}}\right)}_{2\cos\frac{\phi}{2}}$$
$$= \underbrace{2A\cos\frac{\phi}{2}}_{\tilde{A}}\, e^{i\left(\omega t+\frac{\phi}{2}\right)}$$



Amplitude?

$A\cos(kr-\omega t)$ $\quad k = \frac{2\pi}{\lambda}$

$A\cos(kr+\underbrace{k\delta}_{\phi}-\omega t)$

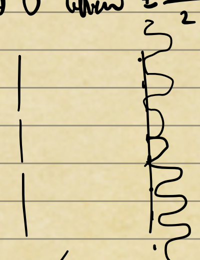
$$\bar{A} = 2A\left|\cos\frac{k\delta}{2}\right|$$

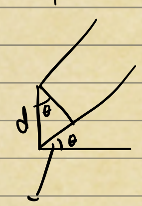
1) 2A when $\frac{k\delta}{2} = 0, \pm\pi, \pm2\pi$

$\Leftrightarrow \frac{2\pi}{\lambda}\frac{\delta}{2} = 0, \pm\pi, \pm2\pi, \ldots$ Bright, huge water . . .

$\Leftrightarrow \delta = 0, \pm\lambda, \pm2\lambda, \ldots$

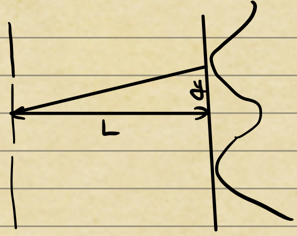
2) 0 when $\pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \ldots$





$d\sin\theta = 0, k\lambda$: max

$= \pm\frac{\lambda}{2}, \pm\frac{3}{2}\lambda, \ldots$; min



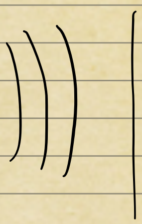
$\frac{y}{L} = \tan\theta$

$d\sin\theta = \frac{\lambda}{2}$

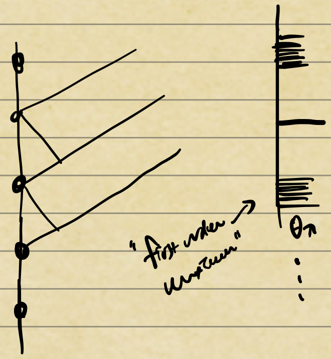
$\Rightarrow$ can find λ

$* \Rightarrow \frac{1}{100}$ radians!

* defraction grading



(ex) $d\sin\theta = m\lambda$ $\quad m = 0, \pm 1, \pm 2, \cdots$



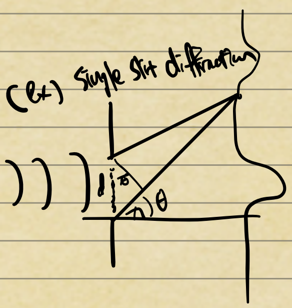
$d\sin\theta = m\lambda$ $\quad m = 0, \pm 1, \pm 2, \cdots$

m=1 many diff λ's ...

hydrogen absorb some colors hence cavity

~ how we know what sun consists of

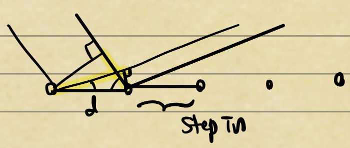
(ex) single slit diffraction



$d\sin\theta = \lambda$ first min

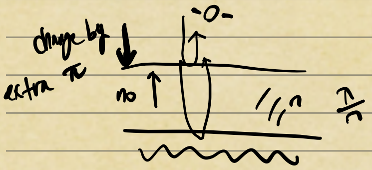
$\sin\theta = \frac{\lambda}{d}$ $\quad \lambda \approx d$; effect of optics better visible

(ex)



$d\cos\alpha_1 = d\cos\alpha_2$

$\alpha_1 = \alpha_2$

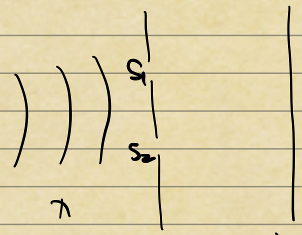


bottom ↺ full wave length

⇝ occurs for blue light

⇝ blue emitted

#11

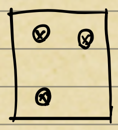


photographic plate

(light → change color)

Intensity $\propto E^2$

* weaker source of light



momentum per hit

$p = \hbar k$

$= \frac{2\pi\hbar}{\lambda}$ h: planck constant 6×10^{-34} J·s

energy deposited → $E = \hbar\omega$

$= \hbar(2\pi f)$

& each one same

wave number: $\frac{2\pi}{\lambda}$

Recall... $\omega = kc$

$2\pi f$

$E = \hbar\omega$

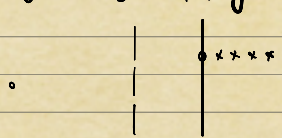
$p = \hbar k$

$\Rightarrow \boxed{E = pc}$

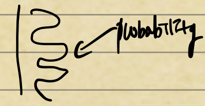
$E^2 = c^2p^2 + m^2c^4$

$\Rightarrow m=0!$ $p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$ $mv \to 0$, $\sqrt{1-\frac{v^2}{c^2}} \to 0$

discreteness not really bad news!



both open ⇝ fewer particle tracks! Maybe wave interference



particle: entire energy is carried in the localized places

but future is determined by a wave

$\lambda = \frac{2\pi\hbar}{p}$ $\quad h = 2\pi\hbar$

$= \frac{h}{p}$

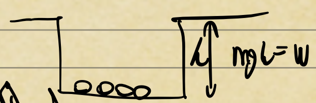
... since $p = \hbar k$

$= 2\pi h \frac{2\pi}{\lambda}$

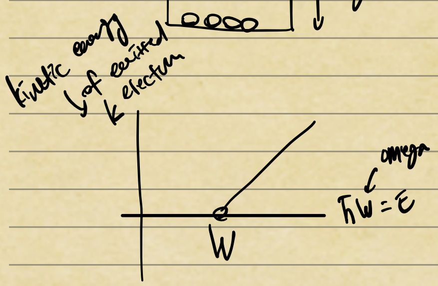
$= \frac{h}{\lambda}$

how photons discovered?

1)



Work function



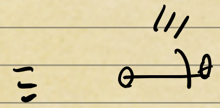
K = Energy given to e $- W = \hbar\omega - W$

low frequency light : millions of photons stay

⇒ indirect discovery of photons

plank: didn't even before photons

2)

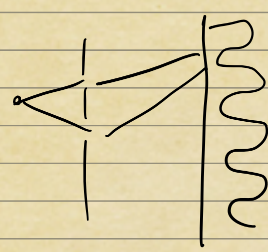


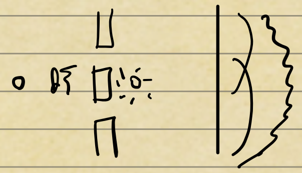
$\delta\lambda = \frac{2\pi\hbar}{mc}(1-\cos\theta)$

$E = \hbar\omega$ $\quad p = \hbar k$ preserved ...

de Broglie $\quad \exists\ \lambda = \frac{2\pi\hbar}{p}$ for electron

measured from slit experiment





in quantum theory,

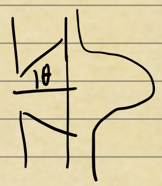
"how don" doesn't matter

- "see" requirement: $\lambda = \frac{2\pi\hbar}{p} < d$ (but not iff); the result of seeing affects momentum unpredictably

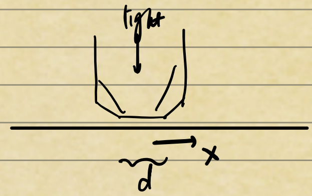
(it's like swinging ball to detect person)

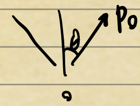
- if d is less, you cannot tell where it went through

Recall



$d\sin\theta = \lambda$





$\Delta x \approx d$

$P_0 \underbrace{\sin\theta}_{\frac{\lambda}{|d|}} = \Delta P_x$

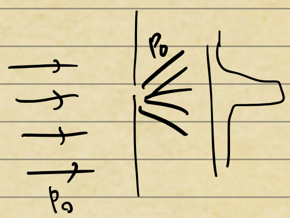
$\Delta P_x = P_0 \frac{\lambda}{|d|}$

$= \cancel{P_0} \frac{2\pi\hbar}{\cancel{P_0}} \frac{1}{d}$

$d \cdot \Delta P_x = \hbar$

$\Delta x \cdot \Delta P_x \approx \hbar$

photon's x $\underbrace{\text{momentum}}_{\text{uncertainty}}$ affects electron's x momentum



$d\sin\theta = \lambda$

$\Delta P_y = P_0 \sin\theta$

$\Delta y = d$

$\Delta P_y \cdot \Delta y = P_0 \, d\sin\theta$

$= P_0 \lambda$

$= P_0 \frac{2\pi\hbar}{P_0}$

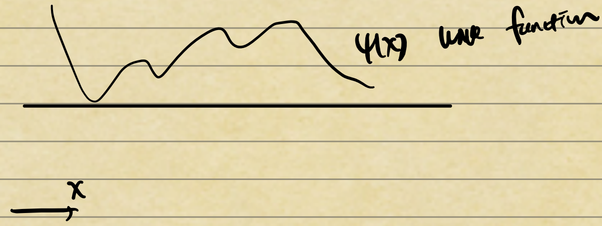
$= 2\pi\hbar \approx \hbar$

$|\psi(x,y,z)|^2 \propto$ odds of finding it at (x,y,z)

e^- has momentum $p \Rightarrow \psi$ has wavelength $\lambda = \frac{2\pi\hbar}{p}$

In classical Mechanics, $x, p \xrightarrow{F=ma} x, p$

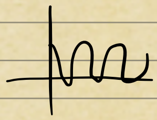
In QM, for every particle $\exists\, \psi$ associated w. it.



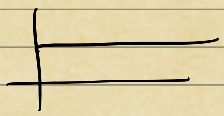
$\psi(x) = A\cos\left(\frac{2\pi x}{\lambda}\right) \qquad \lambda = \frac{2\pi\hbar}{p}$

$= A\cos\left(\frac{p}{\hbar}x\right)$

$|\psi|^2$



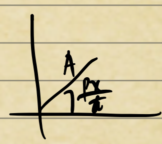
want... $|\psi|^2$



but not quite

Use complex fn.

$\psi(x) = Ae^{ipx/\hbar}$



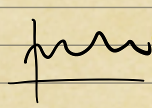
$|\psi|^2 = A^2$

20. QM II

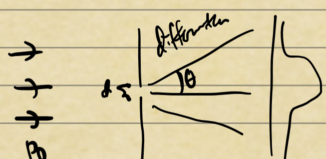
- dumps energy & momentum in one place : particle
- electron scattering $I_{1+2} \neq I_1 + I_2$: wave

happens to be $\underbrace{\lambda}_{\text{from experiment}} = 2\pi\hbar/p$

$P(x) = |\psi|^2$



wave function

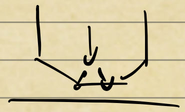


$d\sin\theta = \lambda$

$\Delta P_y = P_0 \sin\theta$

$\Delta P_y \Delta y = 2\pi\hbar = h$

Microscope



Uncertainty of incoming momentum. Don't know if it's fixed as P_0

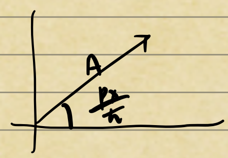
so we source from below, alternatively.

Guess, given wavelength

$\psi = A\cos\left(\frac{px}{\hbar}\right)$ $|A|^2$ known momentum

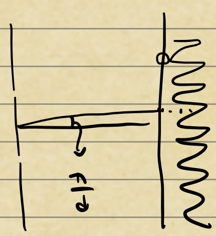
totally unknown position

So candidate: $Ae^{i\frac{p}{\hbar}x}$ $\rightsquigarrow$ $|\psi|^2 = |A|^2$



$\psi_p(x) = Ae^{i\frac{px}{\hbar}}$ plane wave (known p)

$\theta \cong$



$d\sin\theta = \frac{\pi}{2}$

$\theta \cdot d \cong \frac{\pi}{2}$

$\theta \cong \frac{\pi}{d} = \frac{2\pi\hbar}{p \cdot d}$

$p = mv$

(ex) 1kg 1m/s $\rightsquigarrow$ $\theta = 10^{-34}$ rad

d = 1m

1) detector: 10^{-15}m, so 10^{-19} factor needed. don't see interference

2) mv collide each other

3) run into other molecules

$$\underset{\sim 10^{-15}}{\Delta x}\ \underset{\sim 10^{-19}}{\Delta p} = 10^{-34} = \hbar$$

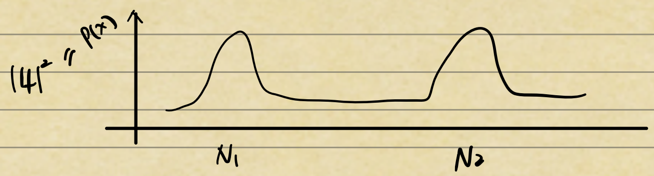
$$10^{-19} = m\Delta v$$

$$\Delta v = 10^{-19}\,\mathrm{m/s}$$

$$10^7\,\mathrm{s} = 1\,\mathrm{yr}$$

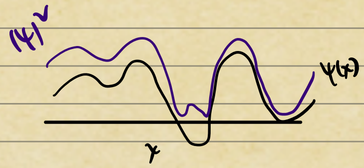
$$\Rightarrow 10^{-12}\,\mathrm{m} = \frac{1}{100} \times \text{size of atom}$$

Role of probability



measurement ≠ revealing preexisting property of the object

It is not anywhere before measurement ⇝ no analogy in macroscopic world.



No restriction on ψ

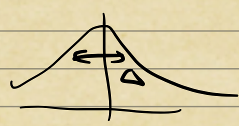
$$|\psi(x)|^2 = P(x)$$

Need $\int |\psi|^2\,dx = 1$ "Normalization"

$\psi \Leftrightarrow c\psi$ equivalent ⇝ shocking

$$\int e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\psi(x) = A e^{-\frac{x^2}{2\Delta^2}}$$



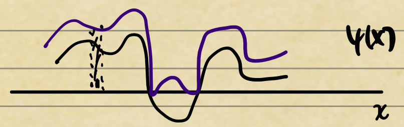
$$1 = A^2 \int_{-\infty}^{\infty} e^{-\frac{x^2}{\Delta^2}}\,dx$$

$$= A^2 \sqrt{\pi\Delta^2}$$

$$A = \frac{1}{(\pi\Delta^2)^{\frac{1}{4}}}$$

Q. How about momentum?

Lec. QM III

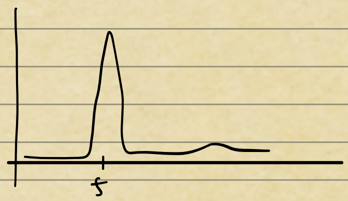


Kinematics: (x, p), $K=\frac{p^2}{2m}$ $\vec{L}=\vec{r}\times\vec{p}$

$\Leftrightarrow \psi$

$dx\,|\psi(x)|^2 = P(x)\,dx$

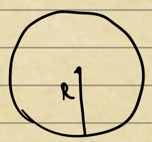
Need $\int |\psi|^2 dx = 1$ "normalized"



$\psi_{x=5}(x)$ $\psi_p(x) = Ae^{ipx/\hbar}$

$$\int_{\text{universe}} |\psi_p(x)|^2 dx = 1$$

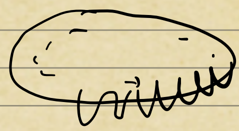
$\Rightarrow$ say universe $\rightsquigarrow L$



$$\Rightarrow \psi_p(x)=\frac{1}{\sqrt{L}}e^{ipx/\hbar}$$

$L = 2\pi R$

$$P(x)=|\psi_p|^2=\frac{1}{L}$$



$\rightsquigarrow \psi(x+L)=\psi(x)$

$$e^{\frac{ipL}{\hbar}}=1$$

$$p = \overbrace{\left(\frac{2\pi\hbar}{L}\right)}^{\text{very small} \to \text{almost continuous } p} m \qquad m = 0, \pm1, \pm2, \ldots \qquad \text{quantization}$$

$$\underbrace{p\cdot R}_{\text{"L", angular momentum}} = p\cdot\frac{L}{2\pi} = m\cdot\hbar$$

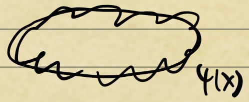
$$\psi_p(x)=\frac{1}{\sqrt{L}}e^{\frac{ipx}{\hbar}} \qquad \psi_m(x) = \frac{1}{\sqrt{L}}e^{\frac{ix}{\cancel{\hbar}}\frac{2\pi\cancel{\hbar}}{L}m}$$

p expressed in m

$$= \frac{1}{\sqrt{L}}e^{\frac{2\pi i m x}{L}}$$

normalized

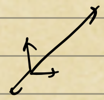
* postulate: normalized



$$\psi(x) = \sum_p \psi_p(x) A_p = \sum \psi_m(x) A_m = \underbrace{\sum_m \frac{e^{\frac{2\pi i m x}{L}}}{\sqrt{L}} A_m}_{\text{fourier series}}$$

$$P(p) = P(m) = |A_p|^2 = |A_m|^2$$

$$\sum |A_m|^2 = 1$$



$$\bar{V} = V_x \vec{i} + V_y \vec{j} + V_z \vec{k} \quad \ldots$$

$$\int_0^L \psi_m^*(x)\, \psi_n(x)\, dx = \delta_{mn} \qquad \psi_n(x) = \frac{1}{\sqrt{L}} e^{\frac{2\pi i m x}{L}}$$

$n \neq m$: $$\frac{1}{L}\int_0^L e^{\frac{2\pi i x}{L}(n-m)}\, dx$$

$$= \frac{1}{L} \frac{L}{2\pi i (n-m)} (1-1) = 0$$

$n = m$: 1

so, $$A_n = \int_0^L \psi_n^*(x)\, \psi(x)\, dx$$

$$P(m) = |A_m|^2$$

(ex) $$\psi(x) = N\cos\left(\frac{6\pi x}{L}\right) = \frac{1}{2}\sqrt{\frac{2}{L}}\left(e^{\frac{6\pi x}{L} i} + e^{-\frac{6\pi x}{L} i}\right) = \frac{1}{\sqrt{2}} \underbrace{\frac{e^{6\pi i x/L}}{\sqrt{L}}}_{\psi_3} + \frac{1}{\sqrt{2}} \underbrace{\frac{e^{-6\pi i x/L}}{\sqrt{L}}}_{\psi_{-3}}$$

$$\downarrow$$

$$P(m=3) = P(m=-3) = \frac{1}{2}$$

$$N^2 \int_0^L \cos^2\left(\frac{6\pi x}{L}\right) dx = 1$$

$$\parallel$$

$$N^2 \frac{L}{2}$$

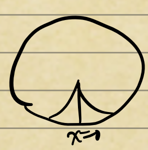
$$N = \sqrt{\frac{2}{L}}$$

$$A_n = \frac{1}{\sqrt{L}} \int_0^L \underbrace{e^{-2\pi i n x/L}}_{\psi_n^*(x)} \underbrace{\sqrt{\frac{2}{L}} \cos\frac{6\pi x}{L}}_{\psi(x)} dx$$

* postulates: measure x_0 ⇝ collapse to $\psi(x) = \delta(x - x_0)$

measure p_0 ⇝ collapse to $\psi(x) = A e^{\frac{i p_0 x}{\hbar}}$

(ex) $\psi = N e^{-\alpha |x|}$



$\alpha L \gg 1$

$\langle \Delta x \rangle \alpha \approx 1$

(Most probability mass is within $|x| \lesssim \frac{1}{\alpha}$)

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} |\psi|^2 \, dx = 1$$

$$\Rightarrow \quad N = \sqrt{\alpha}$$

$$A_p = \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{-ipx/\hbar} / \sqrt{L} \;\; \psi(x) \, dx \qquad \sqrt{\frac{\alpha}{L}}$$

$$= \int_{-\infty}^{\infty} e^{-ipx/\hbar} \, e^{-\alpha|x|} \, dx \qquad \sqrt{\frac{\alpha}{L}}$$

$$= \int_0^{\infty} e^{(-\alpha - \frac{ip}{\hbar})x} \, dx + \int_{-\infty}^{0} e^{(\alpha - \frac{ip}{\hbar})x} \, dx \qquad \sqrt{\frac{\alpha}{L}}$$

$$= \sqrt{\frac{\alpha}{L}} \left(\frac{1}{\alpha + \frac{ip}{\hbar}} + \frac{1}{\alpha - \frac{ip}{\hbar}} \right)$$

$$= \sqrt{\frac{\alpha}{L}} \; \frac{2\alpha}{\alpha^2 + (\frac{p}{\hbar})^2}$$

$|A_p|^2$ picked at $p = 0$, decrease rapidly $\qquad \alpha^2 = \frac{p^2}{\hbar^2}$

$$\Delta p = \alpha \hbar$$

$$\Delta x \Delta p = \hbar$$

Lec 22

Newtonian

$\left.\begin{matrix} x \\ p \end{matrix}\right\}$ kinematics

$\left.\begin{matrix} \frac{dp}{dt} = F \\ m\frac{d^2x}{dt^2} = F \end{matrix}\right\}$ dynamics

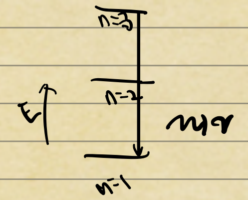
QM: $\psi(x)$

$$P(x) = |\psi(x)|^2$$

$$\psi(x) = \sum_p A_p \underbrace{\psi_p(x)}_{\frac{e^{ipx/\hbar}}{\sqrt{L}}} \qquad P(p) = |A_p|^2$$

$$A_p = \int_0^L \psi_p^*(x) \psi(x) \, dx$$

* Collapse: measurement is like filtering process



$$E_{n=3} - E_{n=1} = \hbar\omega$$

$$= 2\pi\hbar f$$

$$\lambda = \frac{c}{f}$$

How to measure energy in QM?

$$E(x) = \frac{p^2}{2m^2} + V(x) \; ?$$

well... guess:

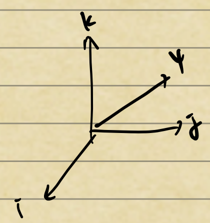
$$\psi(x) = \sum_E \psi_E(x) A_E$$

$$A_E = \int \psi_E^*(x)\psi(x)\,dx$$

$$P(E) = \sqrt{A_E}$$

Collapse rule same! (observe E_3 then $\psi = \psi_{E_3}$)

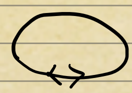
$$\begin{cases} \psi_1 = \dfrac{e^{ipx/\hbar}}{\sqrt{L}} \\ \psi_{x=x_0} = \end{cases}$$



$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$$

need to solve this

(ex) Free particle : $V=0$



$2\pi R = L$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} := E\psi_E(x)$$

$$\frac{d^2\psi_E}{dx^2} + k^2\psi = 0 \qquad k^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{k^2\hbar^2}{2m} = \frac{p^2}{2m}$$

$$k = \frac{p}{\hbar}$$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$= Ae^{i\sqrt{\frac{2mE}{\hbar^2}}x} + Be^{-i\sqrt{\frac{2mE}{\hbar^2}}x}$$

$p = \sqrt{2mE},\ -\sqrt{2mE}$

$E = \frac{p^2}{2m}$

$\psi_E(x) = Ae^{ipx/\hbar} + Be^{-ipx/\hbar}$ $\quad E = \frac{p^2}{2m}$

$\frac{pL}{\hbar} = 2\pi n$

$p_n = \frac{2\pi\hbar}{L} n \qquad E_n = \frac{p_n^2}{2m} = \left(\frac{2\pi\hbar n}{L}\right)^2 \frac{1}{2m}$ "quantization of energy"

$p \xrightarrow{o} E$

$E \xrightarrow{x} p$

"degeneracy" $\quad$ CW $\quad$ CCW $\quad E_1 : \psi = \frac{1}{\sqrt{L}} e^{\frac{2\pi i x}{L}},\ \frac{1}{\sqrt{L}} e^{-\frac{2\pi i x}{L}}$

$E_0 = \frac{0^2}{2m}$

$E_4 \to E_3$

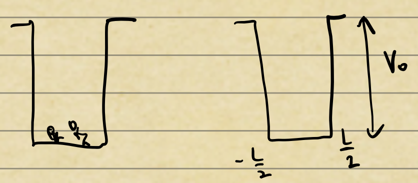
$\hbar\omega = \frac{4\pi^2\hbar^2}{2mL^2}(4^2 - 3^2)$

$\parallel$

$2\pi\hbar f$

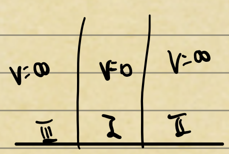
can get

* particle in a box



$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} = (E - V)\psi_E(x)$$



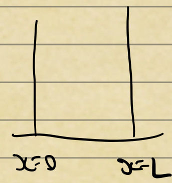
$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V - E)\psi$$

$\psi = \cancel{Ae^{kx}} + \cancel{Be^{-kx}}$

(/) $\quad$ (\ )

$$\left\{ \; k = \sqrt{\frac{2m}{\hbar^2}(V-E)} \right.$$

Free particle



$$\psi(x) = Ae^{ikx} + Be^{-ikx} = C\cos kx + D\sin kx$$

$$\frac{2mE}{\hbar^2} = k^2$$

$$\psi(0) = \psi(L) = 0$$

$$\therefore \; C = 0 \quad \& \quad \sin kL = 0$$

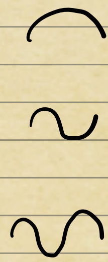
$$\Leftrightarrow kL = n\pi$$

$$k = \frac{n\pi}{L}$$

So,

$$n=1: \; \psi = A\sin\frac{\pi x}{L}$$

$$n=2: \; \psi = A\sin\frac{2\pi x}{L}$$



$$\vdots$$

$$\boxed{E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2mL^2}}$$

$$= \frac{\hbar^2\pi^2}{2mL^2} n^2$$

Lec 23

$$\text{Classical: } E = \underbrace{V}_{\text{potential}} + \underbrace{\frac{p^2}{2m}}_{\text{kinetic}}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} + V\psi_E = E\psi_E$$

$$V=0: \; \frac{d^2\psi_E}{dx^2} + k^2\psi_E = 0 \quad k^2 = \frac{2mE}{\hbar^2}$$

$$\psi_E(x) = \underbrace{Ae^{ikx} + Be^{-ikx}}$$

$$\Rightarrow E = \frac{p^2}{2m}$$

1) combination (novel about QM)
each: definite energy & momentum

2) Requirement that $\psi(x) = \psi(x+L)$

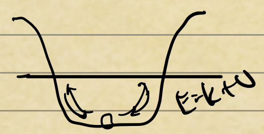
$$p = \frac{2\pi\hbar}{L}n, \quad \text{Quantization}$$

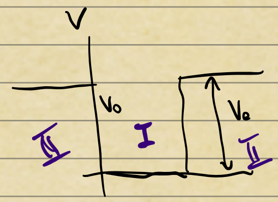
$$E_n^2 = \frac{p^2}{2m} = \frac{4\pi^2\hbar^2}{2mL^2}n^2$$

n=1 ___ n=? ___ $E_1 = \frac{2\pi^2\hbar^2}{mL^2} = \hbar\omega$ ← angular frequency $= 2\pi f$

k: wave number $= \frac{2\pi}{\lambda}$

$E_0 = 0$





$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

I. $V=0$

$$\frac{d^2\psi_I}{dx^2} + \underbrace{\frac{2mE}{\hbar^2}}_{(k^2)}\psi_I = 0$$

$\psi_I = A\cos kx + B\sin kx$ A, B, k arbitrary

II. $$\frac{d^2\psi_{II}}{dx^2} = \underbrace{\frac{2m}{\hbar^2}(V_0 - E)}_{K^2}\psi_{II}$$

$$\psi_{II} = \cancel{Ce^{Kx}} + De^{-Kx}$$

↑ physical condition. $\int N = 1$

$$K = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

$$\psi_{II} = De^{-\sqrt{\frac{2m}{\hbar^2}V_0}\,x}$$

* $\psi_{II} \to 0$ if $V_0 \to \infty$

* Restriction : for case I,

$$\psi_I(0) = 0 = \psi_I(L)$$

$\Rightarrow A = 0$

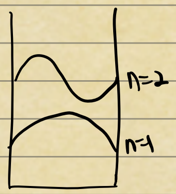
$B\sin kL = 0$

$kL = n\pi$

$$k = \frac{n\pi}{L}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$$\psi_n = B \sin \frac{n\pi x}{L}$$



$$\psi_n(x) = B \sin \frac{n\pi x}{L}$$

$n = 0, \cancel{\pm 1, \pm 2}, \ldots$

$n = 1, 2, 3 \ldots$

(ex) $\psi_0 \equiv 0$

$\psi_1 = -\psi_{-1}$

$\vdots$

NOT the case for e^{ipx} ← valid for $\forall n$

$p = \frac{2\pi\hbar}{L} n$

$$\int |\psi_n|^2 = 1$$

$$\parallel$$

$$\int_0^L B^2 \frac{1 - \cos\frac{2n\pi x}{L}}{2} dx$$

$$\parallel$$

$$\frac{LB^2}{2}$$

$$B = \sqrt{\frac{2}{L}}$$

So $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

* $\Delta x \sim L$

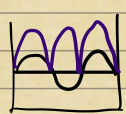
$\Delta p > \frac{\hbar}{L}$

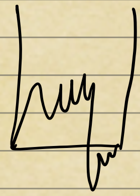
$K = \frac{p^2}{2m} \sim \frac{\hbar^2}{2mL^2}$

(ex) proton → nucleus (10^{-13} cm across) = L

⇒ can get kinetic energy of proton

$\psi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$



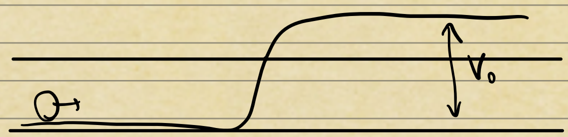


$$\psi(x) = \sum A_n \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$$

$$= \sum A_E \psi_E(x)$$

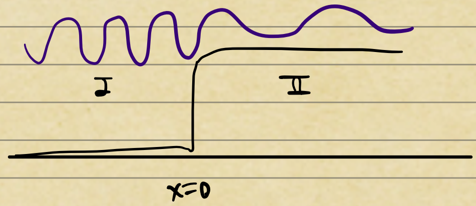
$$A_n = \int \sqrt{\frac{2}{L}} \left(\sin\frac{n\pi x}{L}\right) \psi(x)\, dx$$

* Scattering



$E = \frac{p^2}{2m}$

In QM $(E > V_0)$



$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\psi_{II} = Ce^{ik'x} + \cancel{De^{-ik'x}}$$

$$k'^2 = \frac{2m[E-V_0]}{\hbar^2}$$ smaller

$$\psi_I(0) = \psi_{II}(0)$$

$\Leftrightarrow$ $A+B=C$

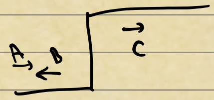
Since $\frac{d^2\psi}{dx^2} < \infty$, $\left.\frac{d\psi_I}{dx}\right|_{x=0} = \left.\frac{d\psi_{II}}{dx}\right|_{x=0}$

$\Leftrightarrow$ $ik(A-B) = ik'C$

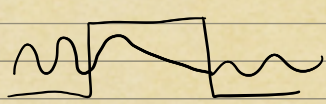
transmit

$A=1$ $\quad B = \frac{k-k'}{k+k'}$ $\quad C = \frac{2k}{k+k'}$

reflect



$E < V_0$: $\quad k' = i\kappa$



No barrier is completely safe in quantum theory

(ex) alpha particle inside the nucleus

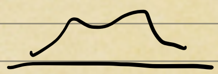
#24

$x \quad \vec{r}$

$p \quad \vec{p}$

$\vec{L} = \vec{r} \times \vec{p}$

Classical: $m \frac{d^2 x}{dt^2} = F = -\frac{dV}{dx}$

QM: $\psi(x)$  $P(x) = |\psi(x)|^2$

$\psi(x) = \sum A_p e^{\frac{ipx}{\hbar}} / \sqrt{L} \qquad A_p = \int \frac{e^{-ipx/\hbar}}{\sqrt{L}} \psi(x)\, dx$

$P(p) = |A_p|^2$

$p_n = \frac{2\pi\hbar}{L} n$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_E}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$$

$\psi(x) = \sum A_E \psi_E(x)$

*

$\psi(x,t)$

$\psi_E \leftarrow$ definite E

$\psi(x,t) \leftarrow$ generic fn

$$i\hbar \frac{d\psi(x,t)}{dt} = \left[-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x,t)\right]$$

- initial ψ determines later time

Special case:

$\psi(x,t) = F(t)\psi(x) \qquad$ non-ex: $e^{-(x-t)^2}$

$$i\hbar F'(t)\psi(x) = F(t)\underbrace{\left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x)\right)}_{:= H\psi}$$

$$i\hbar \psi(x) \frac{dF}{dt} = F(t)(H\psi)_x$$

$$i\hbar \underbrace{\frac{\frac{dF}{dt}}{F(t)}}_{\text{time}} = \underbrace{\frac{(H\psi)_x}{\psi(x)}}_{x} := E \quad \text{constant}$$

1) $i\hbar \frac{dF}{dt} = EF \Rightarrow F(t) = F(0)\, e^{-\frac{iEt}{\hbar}}$

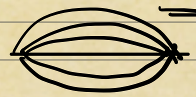
2) $H\psi = E\psi \rightarrow$ state of definite energy

$\Psi(x,t) = F(t)\,\psi(x)$ "normal modes"

$= e^{-\frac{iEt}{\hbar}}\, \psi_E(x)$

(ex) particle in a box $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

Analogy: $A\sin(\frac{\pi x n}{L}) \cos(\frac{\pi x v t}{L})$



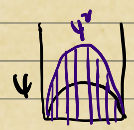
scale by some amount

$\Psi_E(x,t) = e^{-iEt/\hbar}\, \psi_E(x)$

stationary state

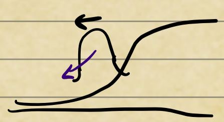
$P(x,t) = \Psi^*(x,t)\,\Psi(x,t)$

$= \psi_E^*(x)\,\psi_E(x)\, \cancel{|e^{-iEt/\hbar}|^2}$



(ex) $V(\vec{r}) = -\frac{Ze^2}{\sqrt{x^2+y^2+z^2}}$ hydrogen

(ex)



$P \uparrow$

change w. time

since not stationary state (particle)

Jump of stationary state due to absorbing photon

($V(x)$ originally electron $\leftrightarrow$ proton, changes)

$P(p) = |A_p|^2 = \left| \int \frac{e^{-ipx/\hbar}}{\sqrt{L}} \underbrace{\Psi(x,t)}_{\psi(x)\, e^{-iEt/\hbar}} dx \right|^2$

$= |A_p(0)\, e^{-iEt/\hbar}|^2$

$= |A_p(0)|^2$

seems to evolve w. time, but probability same

(ex)

$$\psi_1(x,t) = e^{-iE_1t/\hbar}\,\psi_{E_1}(x)$$

$$\psi_2(x,t) = e^{-iE_2t/\hbar}\,\psi_{E_2}(x)$$

$$\psi = \psi_1 + \psi_2$$

$$= F_1\psi_1 + F_2\psi_2 \neq (F)\cdot(\psi)$$

$$\psi(x,t) = \sum_E A_E \psi_E(x,t)$$

$t=0$: $\psi(x,0) = \sum A_E \psi_E(x)$ general enough

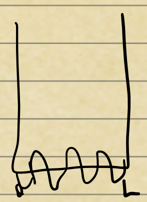
So....

given $\psi(x,0) \xrightarrow[t]{} \psi(x,t)$

① Find $A_E = \int \psi_E^*(x)\,\psi(x,0)\,dx$

② $\psi(x,t) = \sum A_E e^{-iEt/\hbar}\,\psi_E(x)$

(ex)



$t=0$ $\psi_n(x,0) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$

$$E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$$

$$\psi(x,t) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\,e^{-i\left(\frac{n^2\pi^2\hbar^2}{2mL^2}\right)\frac{t}{\hbar}}$$

(ex) $\psi(x,0) = \underbrace{3\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}}_{\psi_2} + \underbrace{4\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}}_{\psi_3}$

$P(n=2) = 3^2/5^2$

$P(n=3) = 4^2/5^2$

$$\psi(x,t) = \frac{3}{5}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}\,e^{-i\frac{E_2t}{\hbar}} + \frac{4}{5}\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}\,e^{-iE_3t/\hbar}$$

$$P(x,t) = \frac{9}{25}\frac{2}{L}\sin^2\frac{2\pi x}{L} + \frac{16}{25}\frac{2}{L}\sin^2\frac{3\pi x}{L} + \underbrace{\psi_1^*\psi_2 + \psi_1\psi_2^*}_{\text{depend on } t}$$

oscillate difference

$$\frac{12}{25}\frac{2}{L}\sin\frac{2\pi x}{L}\sin\frac{3\pi x}{L}\,2\cos\left(\frac{t(E_2-E_3)}{\hbar}\right)$$

Lec 25

I. $\psi(x)$ $\quad (\int |\psi|^2 dx = 1)$ $\quad P(x=x_0) \propto |\int \psi_{x_0}^* \psi(x)\, dx|^2 = |\psi(x_0)|^2$

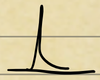
II. State of definite momentum

$$\psi_p(x) = \frac{1}{\sqrt{L}} e^{\frac{ipx}{\hbar}}$$

$$-i\hbar \frac{d\psi_p}{dx} = p\,\psi_p(x)$$

$P_n = \frac{2\pi\hbar}{L} n$ mathematical deduction

III. State of definite position x_0

$\psi_{x_0}(x)$ 

$$x\,\psi_{x_0}(x) = x_0\,\psi_{x_0}(x)$$

← true only at x_0

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$$

$$= \frac{1}{2m}\Big(\underbrace{-i\hbar\frac{d}{dx}}_{p}\Big)^2\psi_E + V(x)\psi_E(x) \qquad E = \frac{p^2}{2m} + V(x)$$

Follow this logic

can define the state of $g(x,p)$

IV $\psi(x) = \sum_g A_g \psi_g(x)$ $\quad g = p, E, x$

$$P(g) = |A_g|^2$$

$$A_g = \int \psi_g^*(x)\psi(x)\, dx$$

V If g is measured & get one value, g_0,

$$\sum_g A_g \psi_g \to \psi_{g_0}(x)$$

VI $i\hbar\frac{d\psi}{dt}(x,t) = H\psi = -\frac{d^2\psi}{dx^2} + V(x)\psi(x)$

$$\psi_E(x,t) = \psi_E(x)\, e^{-\frac{iEt}{\hbar}}$$

$$A_E = \int \psi_E^* \psi(x,0)\, dx$$

$$\psi(x,t) = \sum_E A_E \psi_E(x)\, e^{-\frac{iEt}{\hbar}}$$

ex) $E = \frac{p_x^2 + p_y^2 + p_z^2}{2m} - \frac{ze^2}{\sqrt{x^2+y^2+z^2}}$

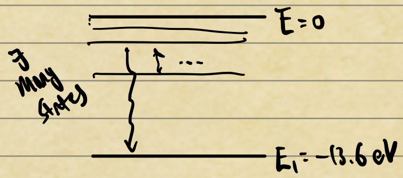
$$\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dz^2}\right)-\frac{Ze^2}{\sqrt{x^2+y^2+z^2}}\psi = E\psi$$

plank constant

$$E_n = -\frac{13.6\text{ eV}}{n^2}$$

$e^2, \hbar, m$

$n: 1, 2, 3, \cdots$



$E_f - E_i = \hbar\omega$

* Uncertainty relation

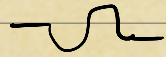
$\Delta E\,\Delta t \geq \hbar$ ($\Delta p\,\Delta x \geq \hbar$ previously)

1) $k = \frac{2\pi}{\lambda}$

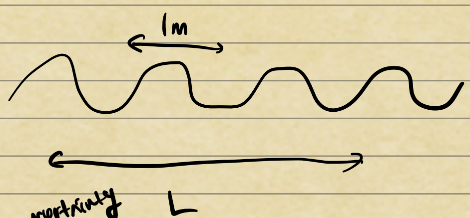
$\cos(kx-\omega t)$

rate of change of phase

$k = \frac{2\pi N}{L}$



(x) wavelength x



uncertainty due to drop off at the end

$\Delta k = \frac{2\pi}{L}$ (1)

uncertainty in position

$\Delta x \simeq L$

wave number

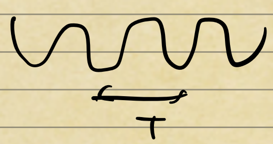
to QM

$\hbar\,\Delta x\,\Delta k = 2\pi\hbar$ $\quad p = \hbar k$

$\Delta x\,\Delta p \simeq \hbar$

2) $\psi_E(x,t) = \psi_E(x)\,e^{-iEt/\hbar}$

$e^{-i\omega t}$, $\omega = \frac{E}{\hbar}$



$\omega = \frac{2\pi N}{T}$ $\quad \Delta\omega = \frac{2\pi}{T}$

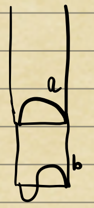
$$\Delta t\,\underbrace{\hbar\,\Delta\omega}_{\Delta E} = 2\pi\hbar$$

need enough time to define energy

* Two particles

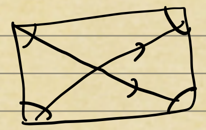
$\psi(x_1, x_2)$ $\quad P = |\psi(x_1, x_2)|^2$

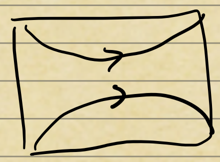
proton electron



$\psi(x_1, x_2) = \psi_a(x_1)\,\psi_b(x_2)$

$\neq \psi_a(x_2)\,\psi_b(x_1) = \psi(x_2, x_1)$





Same particles: $\psi(x_1, x_2) = \psi_a(x_1)\psi_b(x_2) + \psi_b(x_1)\psi_a(x_2)$ (boson) — gravitons photon

$= \psi(x_2, x_1)$

electron, quarks

$\psi(x_1, x_2) = \psi_a(x_1)\psi_b(x_2) - \psi_b(x_1)\psi_a(x_2) = -\psi(x_2, x_1)$ (fermions)

Same probability

so... for identical particles, ∃ two options.

fermion same state ⇝ impossible (pauli principle) ⇒ origin of periodic table of atoms

$\psi \neq 0$

Boson : like to be in the same state

only two!