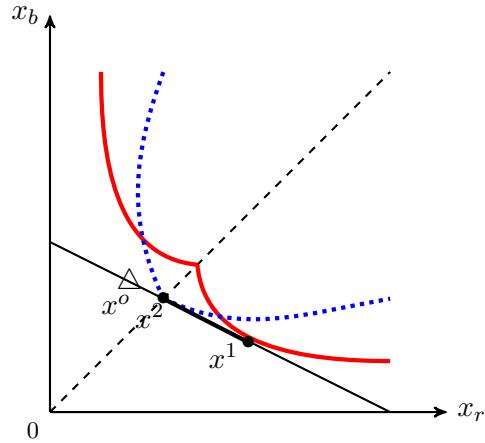


# Pareto Efficient Choices: Numerical Search

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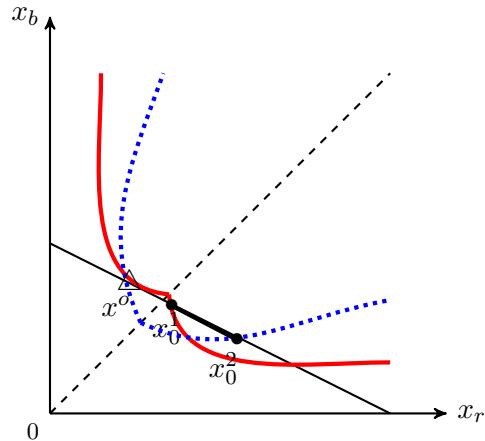
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## 1 Problem



Joint decision  $x^0$  violates FOSD. Given two individuals optimal choice,  $x^2$  is coded to be a Pareto efficient choice without any adjustment. With the first individual's elation loving though,  $x^2$  gives lower utility than  $x^0$  does, which goes against the notion of Pareto improvement.

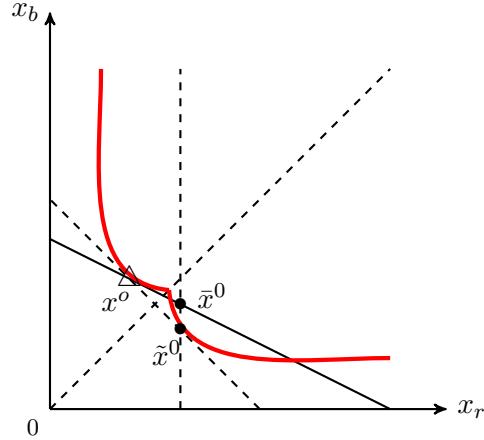
## 2 Analytical Solution



$x_0^1$  and  $x_0^2$  give the same level of utility as  $x^0$  for individual 1 and 2, respectively. Also, any point on the line connecting these two points is a Pareto improvement of  $x^0$ .

Note that  $x_0^1$  is a Pareto efficient choice that is closest to  $x^0$ . Any point closer to  $x_0^2$  gives lower utility for individual 2 than  $x_0^1$  does.

### 3 Numerical Searching



First, given the symmetry of our utility function,  $x_0$  and  $\tilde{x}_0$  give the same level of utility. Second, by the monotonicity of the utility function,  $\bar{x}^0$  gives the higher utility than  $\tilde{x}_0$ . Note that the indifference point  $x_0^1$  is between  $\bar{x}^0$  and  $x^2$ .

By chopping segment between  $\bar{x}^0$  and  $x^2$  into halves and evaluating utility at the midpoint, we can find  $x_0^1$  numerically.

### 4 Notes

For CARA specification, 69 cases need this adjustment. For CRRA, 26 cases does. Even after the adjustment, there are still 11 and 6 idiosyncratic cases where utility at the joint decision is greater than that at the Pareto efficient choice. These cases result from computer's automatic rounding-up or -down, and the difference between two utility levels is minuscule.