

RATIONALITY, PREFERENCE AGGREGATION AND PARETO EFFICIENCY OF GROUP DECISION UNDER RISK

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- We make many types of collective decision in our daily life
 - Household's decision on consumption and saving, a firm's resource allocation among projects, decision on tax and transfer in legislature
- The central challenge is individuals within a collective usually have different preferences, especially over the risk
- This led much investigations on how individuals' different risk preferences are aggregated (Baillon et al. (2016), Bateman and Munro (2005), Palma et al. (2010))
- There also exist a bunch of studies on the consistency of group decision especially when such aggregation is not successful (Arrow (1951), Browning and Chiappori (1998), Chiappori and Ekeland (2011), Bourguignon et al. (2009))

FUNDAMENTAL QUESTIONS ON GROUP DECISION

Three fundamental questions on group decision

1. Rationality Extension:

If each individual's choices are consistent with the utility maximization model, are group's choices also be?

2. Risk Preference Aggregation:

Are individuals' risk preferences reflected into that of a group?

3. Pareto Efficiency:

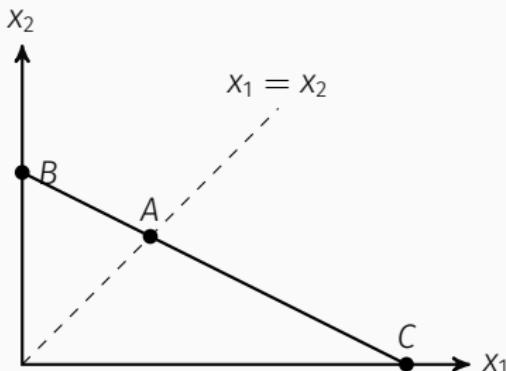
Are group's choices Pareto efficient? Are they maximizing the social welfare?

Our contribution is that using a unique experimental design and a large data set, we answer such fundamental questions, factors for these properties, and the relationship among them

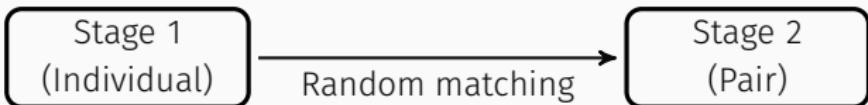
1. Taking the revealed preference approach, we investigate the relationship between individual and group rationality
2. We devise a novel way to test Pareto efficiency given individual and collective choice data
3. We collected rich (demographic variables, cognitive ability, network, personality) and large ($N=1572$) data set, which enriches the analysis

EXPERIMENTAL DESIGN AND PROCEDURES

- Our decision environment is based on Choi et al. (2007)
- There are two states $\{1, 2\}$ which occur randomly and equally likely ($p(S = 1) = p(S = 2) = 0.5$)
- A subject chooses the combination (x_1, x_2) on the given budget line
 $p_1x_1 + p_2x_2 = 1$
- The subject receives the points x_1 if the state is 1, and x_2 in the other case



EXPERIMENTAL DESIGN AND PROCEDURES

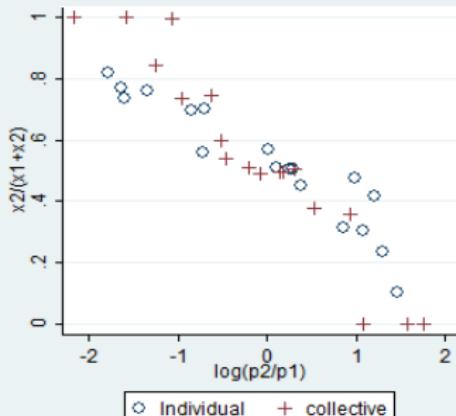


- | | |
|--|---|
| <ul style="list-style-type: none">· 18 times of choices· Payoff for 1 randomly selected choice· N=1572 | <ul style="list-style-type: none">· Matched individuals sit side by side· 1 min 30 sec of discussion time· 18 times of choices· Payoff for 1 randomly selected choice is doubled and divided equally· N=786 |
| <ul style="list-style-type: none">· No feedback is given during the experiment and subjects are informed only the sum of payoff from stage 1 and 2 after all the process· The experiment is computerized using O-tree | |

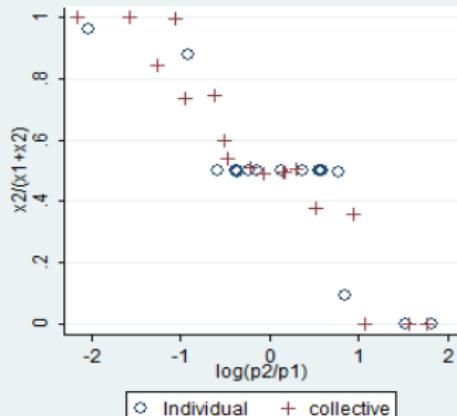
EXAMPLE OF INDIVIDUAL-PAIR RELATIVE DEMAND CURVE

Group ID=284

Collective CCEI=1.00
Risk Preference =0.73, RDU



$$\text{Risk Preference} = (1/18) \sum (x_{\text{cheaper}} / (x_1 + x_2))$$



◀ Design

◀ CCEI

◀ alpha

◀ Risk

Generalized Axiom of Revealed Preference (GARP)

If x^t is revealed preferred to x^s , then x^s is not strictly revealed preferred to x^t
 $(p^s x^s \leq p^s x^t)$

Afriat's theorem (Afriat (1967))

Observed choice data satisfy the GARP if and only if there exist a increasing, continuous and concave utility function that rationalize the observed choice.

Critical Cost Efficiency Index (CCEI(Afriat (1972)))

CCEI is the largest number $e \in [0, 1]$ such that if

$e(p^1 x^1) \geq p^1 x^2, e(p^2 x^2) \geq p^2 x^3, \dots, e(p^{n-1} x^{n-1}) \geq p^{n-1} x^n$, then $e(p^n x^n) \leq p^n x^1$

▶ fig

In our data, 62.6%(50.7%) of individuals' CCEI are equal to or greater than 0.90(0.95). On pairs, the percentage is 70.4%(60%)

▶ ex

EXTENSION: COLLECTIVE RATIONALITY?

- Many of former studies apply revealed preference approach to individual choice data (Battalio et al. (1973), Sippel (1997), Andreoni and Miller (2002), Harbaugh et al. (2001), Choi et al. (2014))
- We use this approach in testing collective rationality, which means consistency within choices matters.
- It equals to testing whether group choices can be represented as an rational individual's decision (Representative Agent Model).
- Having different decision making ability and preference, individuals within collective have to compromise and reach a consensus to construct a common goal.

EXTENSION: INDIVIDUAL RATIONALITY LEADS TO GROUP RATIONALITY

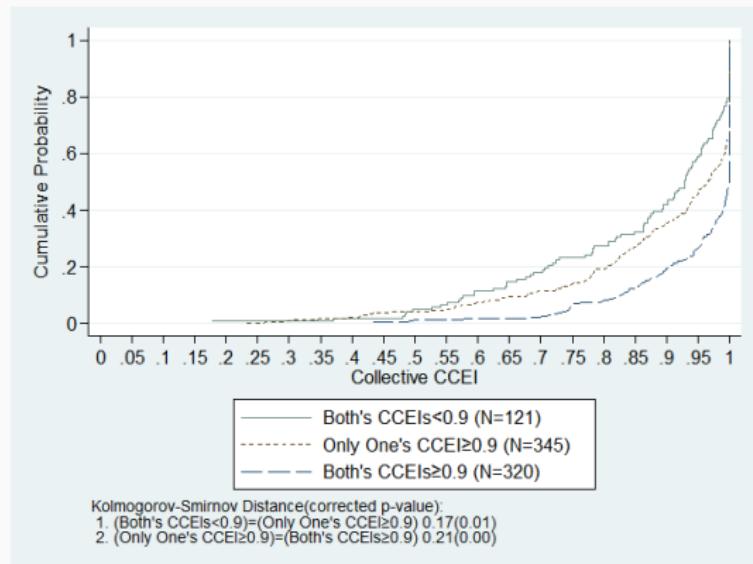


Figure: CDFs of Collective CCEI by Individual Rationality

· 57.8%, 64.9% and 80.9% of neither,either,both-rational pairs have $CCEI \geq 0.9$ ¹

¹This result is robust to the cutoff of CCEI to 0.95 and 0.99.

We use both non-parametric and parametric way in measuring risk attitude

1. Non-parametric way

- Can include all subjects to the analysis
- Limited information

2. Parametric way

- Rich information (Probability weighting, risk premium, utility curvature)
- Only applicable to choice data of which CCEI is high enough to assume a utility function

Non-parametric risk attitude := $\sum_{j=1}^{18} \frac{x_{cheaper}}{x_1+x_2} \in [0.5, 1]$

- If risk neutral, one puts all money to cheaper good (same probability)
- If extremely risk averse, one equalizes payoff in each state
- Mean(sd) of individual/collective game = 0.67(0.13)/0.70(0.13)

▶ ex

▶ decision

Parametric risk attitude: Probability Weighting

- $U(x_{min}, x_{max}) = \alpha u(x_{min}) + (1 - \alpha)u(x_{max})$ (Gul (1991))
- If $\alpha = \frac{1}{2}$, **Expected Utility Form (EUT)**. If $\alpha > \frac{1}{2}$, then one said to show disappointment aversion; $\alpha < \frac{1}{2}$, elation loving. Altogether, **Rank Dependent Utility Form (RDU)**
- In our data, 20.2%, 29.3% of individuals and pairs follow RDU, respectively.²
- In addition, we assume CARA utility function which takes the specific form
$$u(x) = -\frac{\exp(-\rho x)}{\rho}$$



²One's utility follows EUT if $0.5 \in [\hat{\alpha} - 1.96\hat{\sigma}, \hat{\alpha} + 1.96\hat{\sigma}]$, where $\hat{\sigma}$ is bootstrapped standard error

Parametric risk attitude: Risk Premium

- $U(x_{min}, x_{max}) = \alpha u(x_{min}) + (1 - \alpha)u(x_{max})$
- Risk premium $r(h = 1)$: $U(w(1 - r)) = \alpha U(w(1 - h)) + (1 - \alpha)U(w(1 + h))$
- Mean(sd) of individual/collective game=0.15(0.96)/0.31(1.22)³ ex

³CARA function assumed

PREFERENCE TYPE AGGREGATION: RESULT

Individual		Both EUT	EUT and RDU	Both RDU	Total
Collective	EUT	74.5(117)	59.1(52)	50.0(7)	68.0(176)
	RDU	25.5(40)	40.1(36)	50.0(7)	32.1(83)
	Total	100(157)	100(88)	100(14)	100(259)

EUT is defined as $0.5 \in 95\% \text{ CI of } \alpha$, N for each case in parenthesis

Only includes pairs with both individuals' and collective CCEI $>= 0.9$

Table: Percentage of RDU Type of Pairs by Individual Type

PREFERENCE ATTITUDE AGGREGATION: RESULT

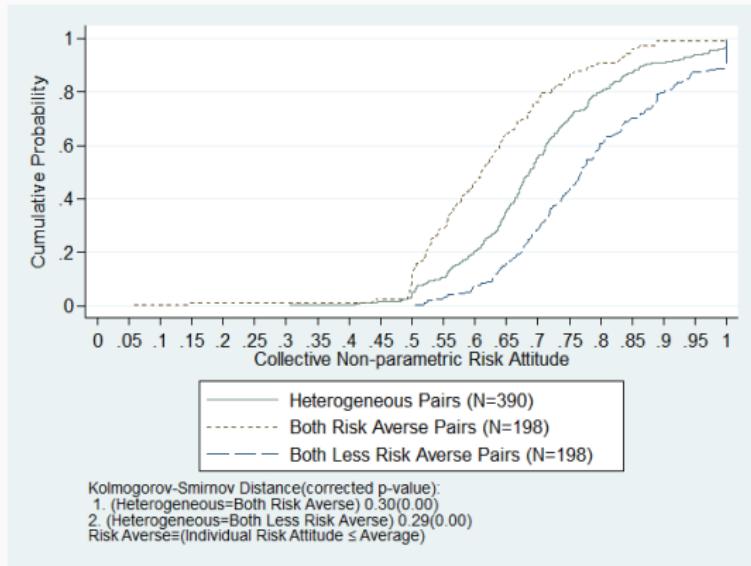


Figure: CDFs of Collective Risk Preference by Individual Risk Attitude

- 20.7%, 49.2% and 73.7% of neither,either,both-risk averse pairs are more risk averse than the mean

PREFERENCE ATTITUDE AGGREGATION: RESULT

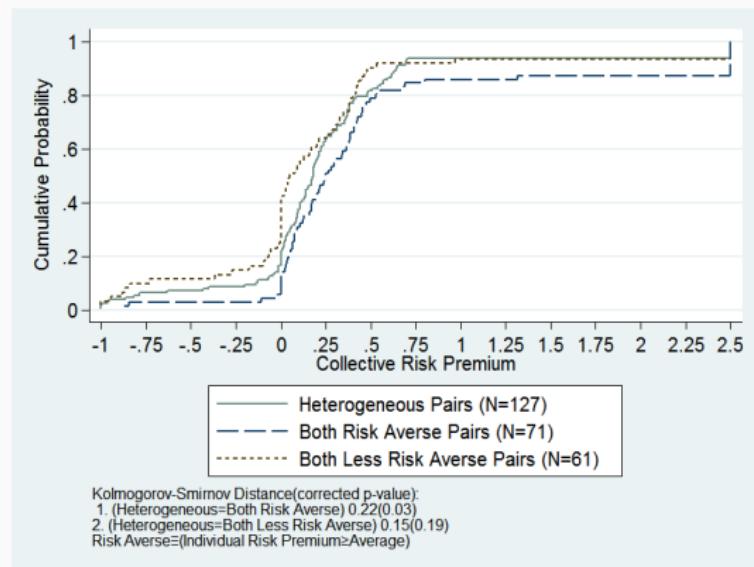


Figure: CDFs of Collective Risk Premium by Individual Risk Attitude

Dependent Var.	Collective Risk Preference		Collective Risk Premium	
	(1)	(2)	(3)	(4)
Ave.(Individual RP)	0.811*** (0.060)	0.796*** (0.062)		
Dist.(Individual RPs)	0.017 (0.052)	0.002 (0.054)		
Ave.(Individual Premium)			0.277** (0.127)	0.242* (0.135)
Dist.(Individual Premium)			0.152* (0.080)	0.170** (0.081)
Non coed		0.017 (0.010)		0.064 (0.091)
Two boys		0.020 (0.023)		0.233 (0.143)
Two girls		-0.021 (0.015)		0.296* (0.167)
Math Score	No	Yes	No	Yes
Network Characteristics	No	Yes	No	Yes
TIPI Scores	No	Yes	No	Yes
Constant	0.151*** (0.037)	0.065 (0.053)	0.157*** (0.041)	0.709 (0.519)
Observations	786	771	320	314
R-squared	0.318	0.341	0.068	0.133

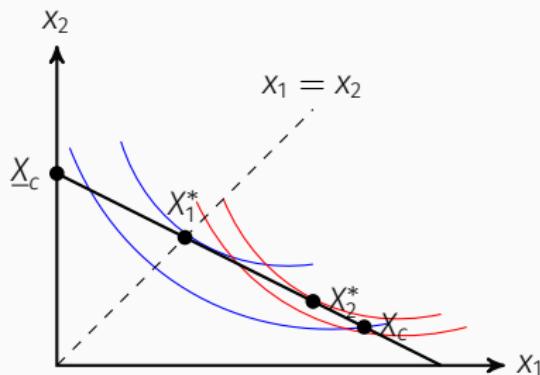
Notes: 1) *** p<0.01, **p<0.05, * p<0.1, 2) Standard errors are clustered at class level
 3) Network characteristics include mean, distance of indegree and outdegree.
 Dyadic relationship is also controlled., 4) TIPI scores consist of outgoing,
 openness to experience, conscientiousness, emotional stability and agreeableness.

PARETO EFFICIENCY: MEASURE

Pareto Efficiency:

A pair's choice X_c is **Pareto efficient** if and only if it is between

$$X_1^* = \operatorname{argmax}_{(x_1, x_2)} \hat{U}_1(x_1, x_2) \text{ and } X_2^* = \operatorname{argmax}_{(x_1, x_2)} \hat{U}_2(x_1, x_2)$$



Utility Loss from Pareto Inefficient Choice:=

$$\frac{1}{18} \sum_{j=1}^{18} \frac{1}{2} \sum_{i=1}^2 \frac{u_i(\bar{X}_{cj}) - u_i(X_{cj})}{u_i(\bar{X}_{cj}) - u_i(X_{ij})} \in [0, 1] \quad (1)$$

where X_c : collective choice, \underline{X} : worst choice on the budget line. j denote the sequence of budget lines and i individuals. Finally,

$\bar{X} = \operatorname{argmin}(dist(X_c, X))$, $X \in \{x | u_i(X) \geq u_i(X_c)\}$ with strict inequality for one i . If such X does not exist, $\bar{X} = X_c$

PARETO EFFICIENCY: RESULTS

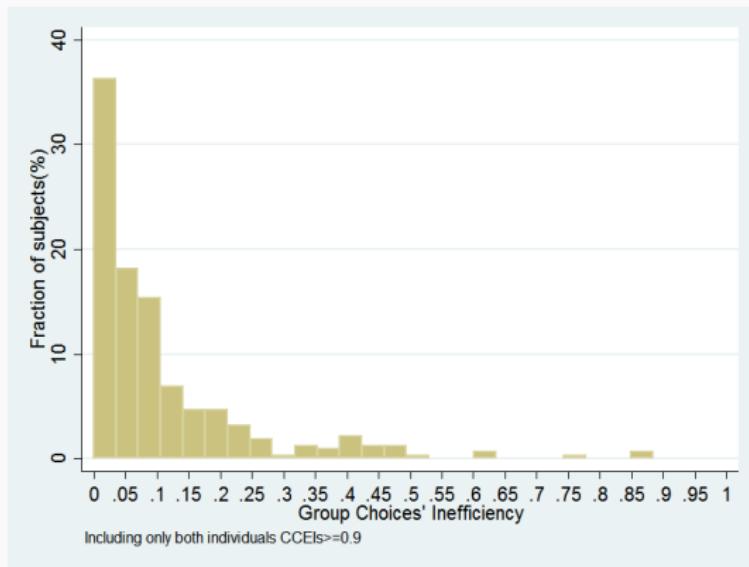


Figure: Distribution of the Utility Loss from Pareto Inefficiency By Pairs

- The mean(sd) of utility loss is 0.09(0.12) when only including pairs with both individuals' CCEI>0.9

PARETO EFFICIENCY: RESULTS

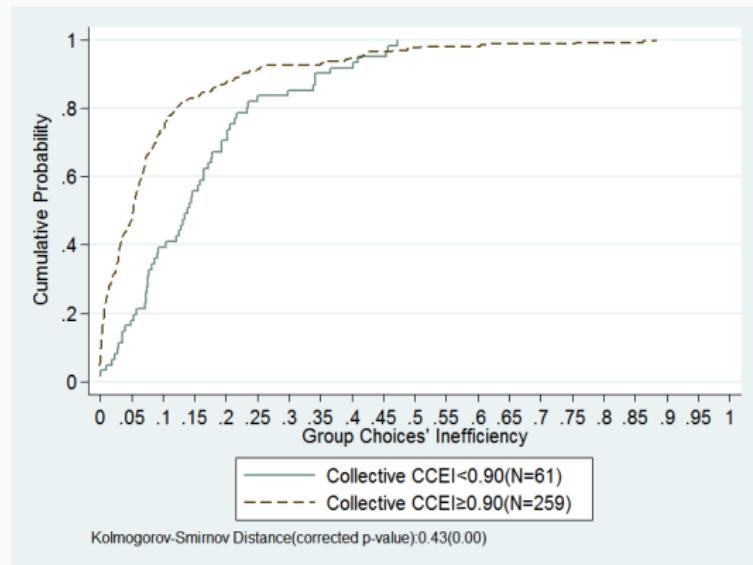


Figure: CDFs of the Utility Loss from Pareto Inefficient Choices by Collective Rationality

- Mean of utility loss for pairs with collective CCEI>0.08 is 0.15. For CCEI<0.9, it is 0.15. This difference is statistically significant at 1% significance level

Dependent Var.	Utility Loss from Collective Decision			
	(1)	(2)	(3)	(4)
Collective CCEI	-0.369*** (0.067)	-0.390*** (0.058)	-0.365*** (0.066)	-0.387*** (0.056)
Ave.(Individual RP)	0.093 (0.114)	0.099 (0.101)		
Dist.(Individual RP)	-0.139* (0.072)	-0.129** (0.062)		
Ave.(Individual Premium)			-0.027 (0.018)	-0.027* (0.016)
Dist.(Individual Premium)			-0.002 (0.010)	-0.006 (0.009)
Non Coed		-0.011 (0.023)		-0.009 (0.023)
Two Boys		0.065* (0.039)		0.072* (0.039)
Two Girls		-0.018 (0.027)		-0.014 (0.026)
Constant	0.413*** (0.095)	0.352** (0.137)	0.458*** (0.063)	0.400*** (0.136)
Observations	320	314	320	314
R-squared	0.075	0.220	0.070	0.219

Notes: 1) *** p<0.01, **,p<0.05, * p<0.1, 2) Standard errors are clustered at class level
 3) Network characteristics include mean, distance of indegree and outdegree.
 Dyadic relationship is also controlled., 4) TIPI scores consist of outgoing,
 openness to experience, conscientiousness, emotional stability and agreeableness.

We study fundamental questions on group decision. And the results show

Extension (of Rationality)

If each individuals' choices are more consistent with utility maximization model, group's choice tend to also be

Preference Aggregation

Individuals' risk attitude is strongly reflected into that of group

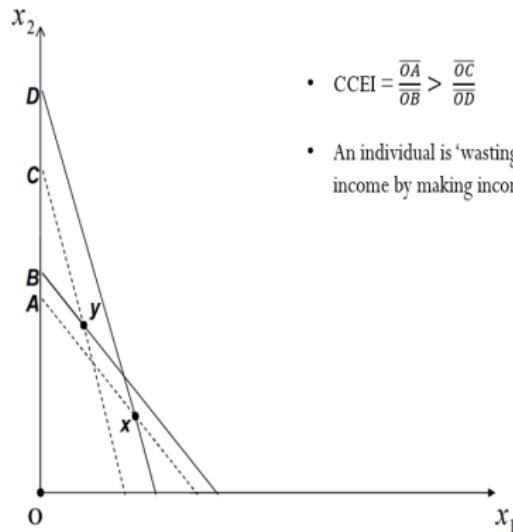
Pareto Efficiency

There exists much heterogeneity in whether a group's choice is Pareto efficient or not and how much the loss is from inefficient choice. However, we find that the more rational a group is, the more efficient the group's choice is

PREVIOUS LITERATURE

Groups	Main Focus	Literature
Experimental studies on the comparison between individual and group decision	Which one is closer to the theoretical prediction	Bone et al. (1999), Cason and Mui(1997), Charness et al. (2007), Charness and Sutter (2012), Cooper and Kagel (2005), Carbone et al. (2016), Kerr at el. (1996), Kugler et al. (2012), Masclet et al.(2009), Shupp and Williams (2008), Sutter (2009)
Experimental studies on preference aggregation	How risk attitude/time preference is aggregated	Abdellaoui et al. (2010), Baillon et al. (2016), Bateman and Munro (2005), Palma et al. (2010), Lee at el. (2012)
Empirical studies on household resource allocation	Test representative agent model Factors for bargaining power	Browning and Chiappori (1998), Chiappori and Ekeland (2011), Friedberg and Webb (2006), Bourguignon et al. (2009)
Theoretical studies on preference aggregation	Representative agent models Impossibility theorem	Arrow, Jackson and Yariv (2014)

EXTENSION: MEASURE OF RATIONALITY

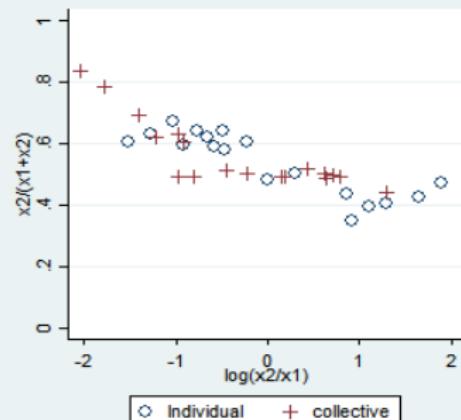


▶ CCEI

EXAMPLE OF INDIVIDUAL-PAIR RELATIVE DEMAND CURVE

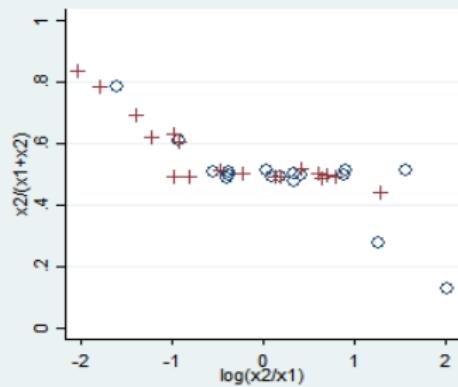
Group ID=623

Collective CCEI=1.00
Risk Preference =0.57, RDU



Individual CCEI=1.00
Risk Preference=0.59, EUT
Id=2310814

$$\text{Risk Preference} = (1/18) \sum (x_{\text{cheaper}}/x_1 + x_2)$$



Individual CCEI=1.00
Risk Preference=0.56, RDU
Id=2310818

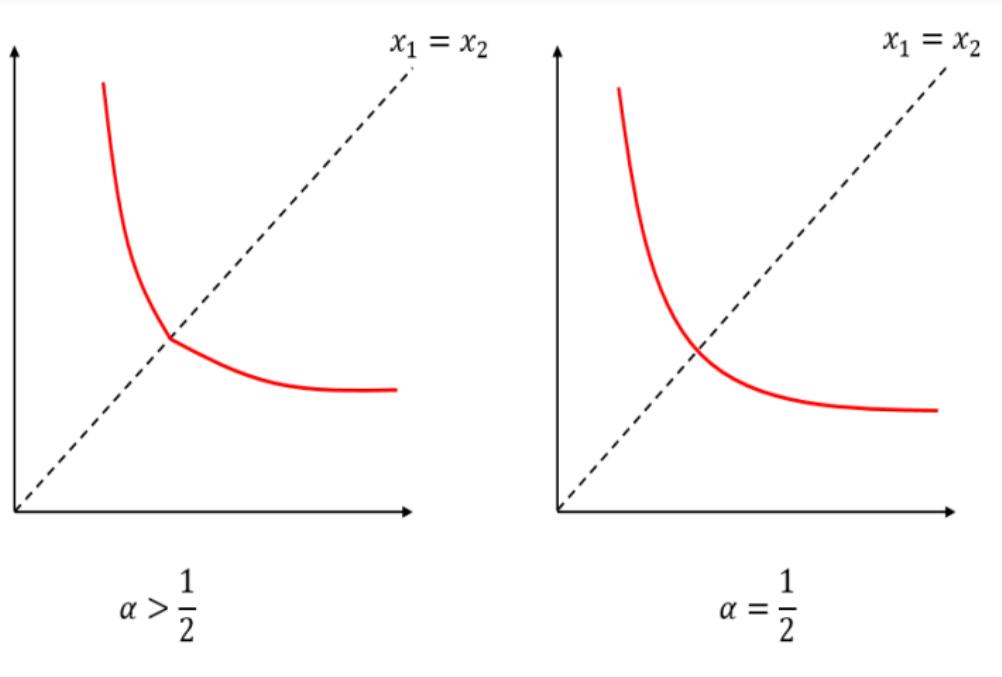


Figure: Indifference Curve Depending on Probability Weighting

ESTIMATION OF UTILITY FUNCTION

$$(\hat{\alpha}, \hat{\rho}) = \operatorname{argmin}_{(\alpha, \rho)} \sum_{j=1}^{18} \left| \frac{x_{1j}^*(\alpha, \rho)}{x_{1j}^*(\alpha, \rho) + x_{2j}^*(\alpha, \rho)} - \frac{x_{1j}}{x_{1j} + x_{2j}} \right| \quad (2)$$

where

$$(x_{1j}^*(\alpha, \rho), x_{2j}^*(\alpha, \rho)) = \operatorname{argmax}_{(x_{1j}, x_{2j})} \alpha \left(-\frac{\exp(-\rho x_{min})}{\rho} \right) + (1-\alpha) \left(-\frac{\exp(-\rho x_{max})}{\rho} \right) \quad (3)$$

given $p_{1j}x_{1j} + p_{2j}x_{2j} = 1$ and $x_{min(max)} = \min(\max)(x_{1j}, x_{2j})$