

# Rationality and Preference Aggregation of Group Decision under Risk

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# Introduction

- In various contexts, many important decisions are made by groups.
- Individual heterogeneity exists in various dimensions:
  - Risk preference: risk assessment in environmental policy
  - Time preference: household savings and consumption decisions
  - Rationality
- It is important to understand how individual heterogeneity in a collective influences final outcomes.

# Introduction: Research Questions

## 1. Rationality extension:

- Do rational members make more collectively rational decisions?

## 2 Risk preference aggregation:

- Are individual's risk preferences reflected into that of a group?

## 3. Efficiency and welfare:

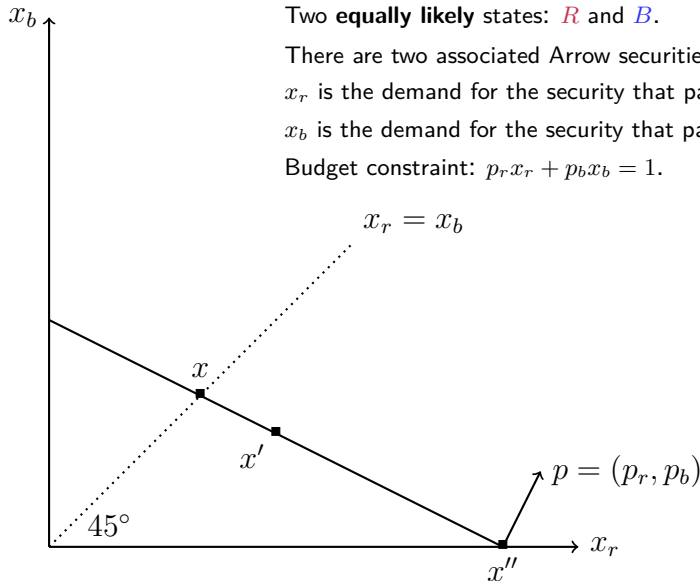
- How is social welfare related to individual' rationality and preferences?

# Introduction: Examples of Individual Heterogeneity

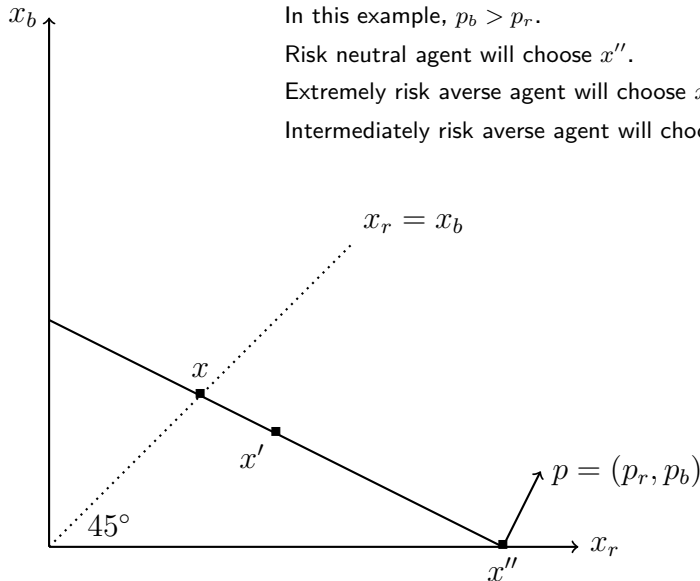
- Rationality
  - High-income, high-education, men, and young subjects tend more toward utility maximization (Choi et al., 2014).
- Risk preference
  - White males are more likely to perceive risks as being smaller (Bickerstaff, 2004; Flynn et al., 1994).
  - There is no substantial difference between men and women (Kagel and Roth, 2016).
  - Subjects' risk preferences are closer to neutrality when they make decisions on behalf of other participants (Batteux et al., 2017).
  - High-power groups adopt a more positive attitude toward potential risks (Anderson and Galinsky, 2006; Magee et al., 2007; Geng et al, 2018).

## Experimental Design

# Experimental Design (Choi et al., 2007; Choi et al., 2014)



# Experimental Design (Choi et al., 2007; Choi et al., 2014)



In this example,  $p_b > p_r$ .

Risk neutral agent will choose  $x''$ .

Extremely risk averse agent will choose  $x$ .

Intermediately risk averse agent will choose  $x'$ .

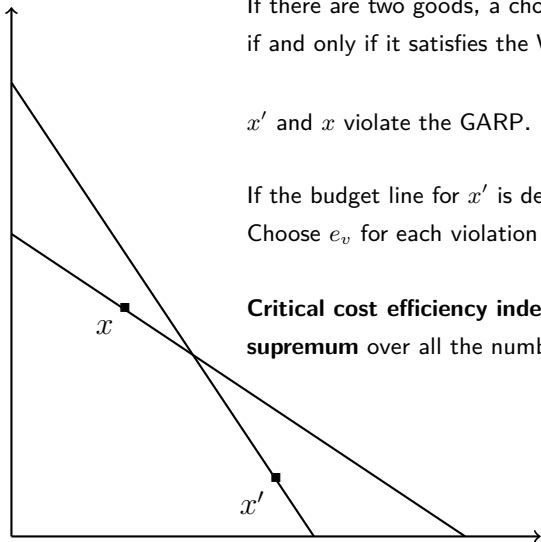
# Measurement: Afriat's Efficiency Index (a.k.a. CCEI)

If there are two goods, a choice dataset satisfies the GARP if and only if it satisfies the WARP.

$x'$  and  $x$  violate the GARP.

If the budget line for  $x'$  is deflated, the GARP is satisfied.  
Choose  $e_v$  for each violation  $v$ .

**Critical cost efficiency index (CCEI)** is defined as the **supremum** over all the numbers  $e_v$ 's.





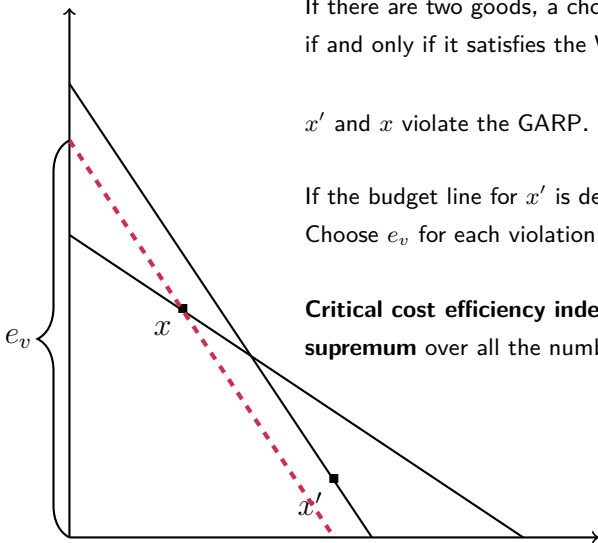
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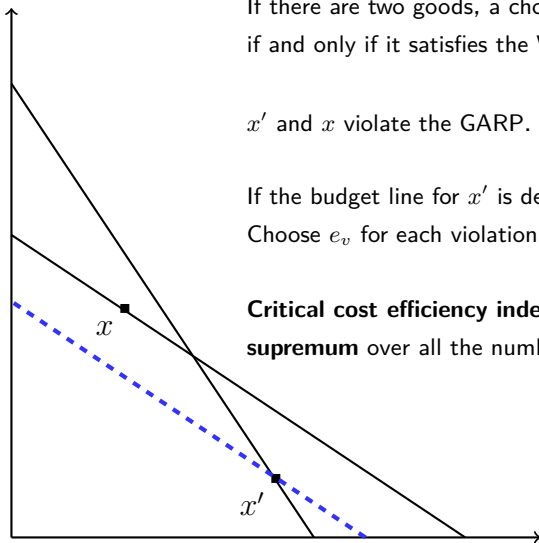
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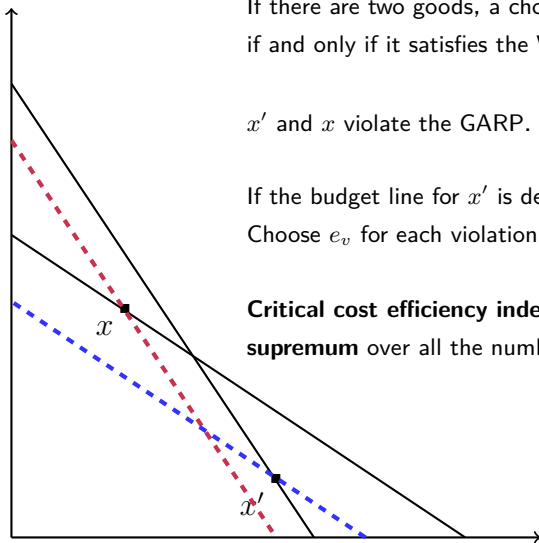
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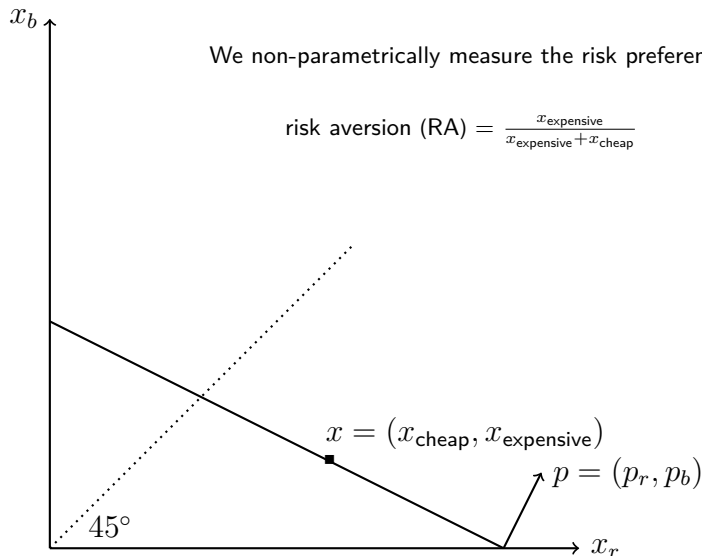
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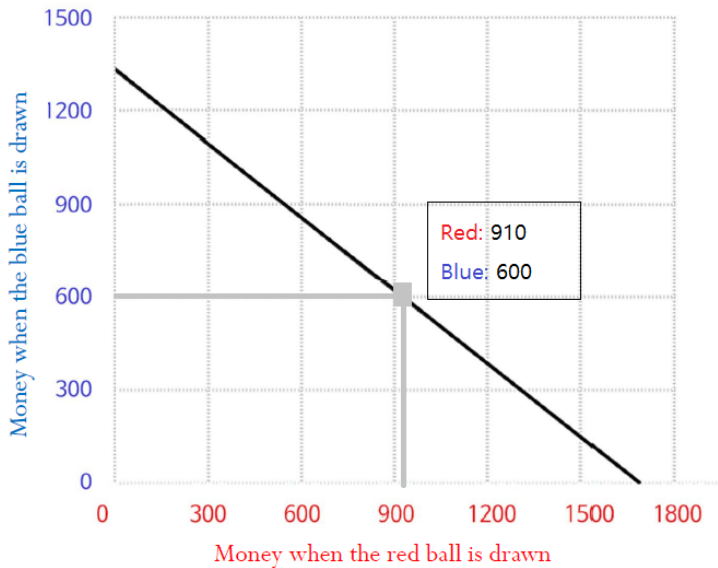
# Measurement: Risk Preferences

We non-parametrically measure the risk preference by a ratio:

$$\text{risk aversion (RA)} = \frac{x_{\text{expensive}}}{x_{\text{expensive}} + x_{\text{cheap}}}$$



# Screenshot



# Field



# Procedure and Subjects

- We conducted the experiment in 12 middle schools in Daegu.
- The number of students: 1572.
- The number of groups: 786.
- The instructions were read by an experimenter in each classroom.
- Each subject participated in two sessions: individual and group decisions.

# Procedure and Subjects

- Each round started by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above 300 KRW or below 1500 KRW.
- Each session consisted of 18 independent decision rounds.
- At the end of each round, the computer randomly selected one of the two states (*R* and *B*).
- Subjects were not informed of the state that was selected at the end of each round.



# Procedure and Subjects

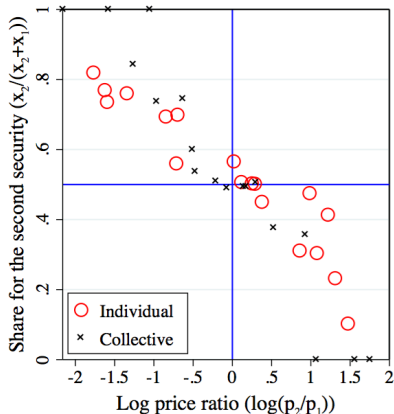
- Two students in the same classroom were randomly matched.
- One of the two students was randomly chosen to move to the other partner's desk.
- They made a series of collective decisions by sharing one computer.
- We allowed students to discuss how to make decisions for 1 min before starting the second session.
- Each subject was paid for he/she earned in a randomly selected round.

# Example of Choice Data

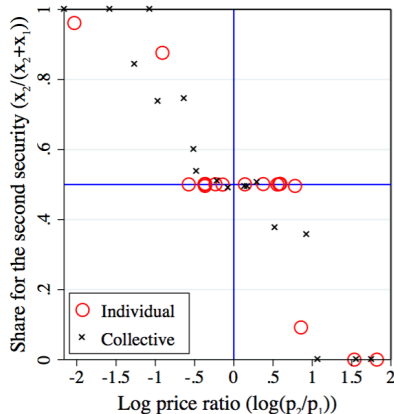
Group ID: 284

Collective CCEI: 1.00

Risk Preference: 0.27, DAU



Individual CCEI: 1.00  
Risk Preference: 0.35, EU  
Id: 1410707



Individual CCEI: 1.00  
Risk Preference: 0.38, DAU  
Id: 1410721

## **Result 1: Rationality Extension**

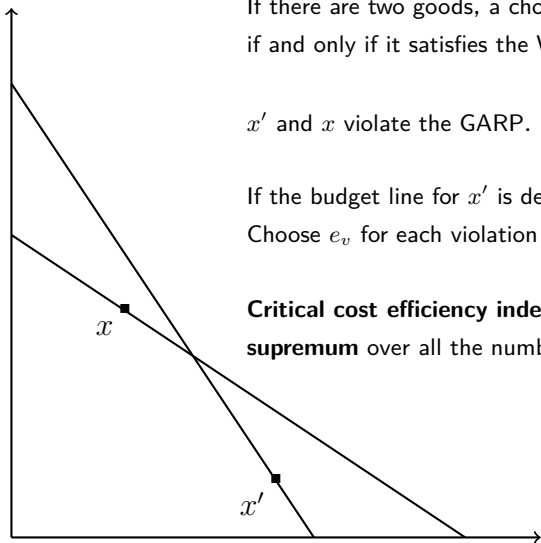
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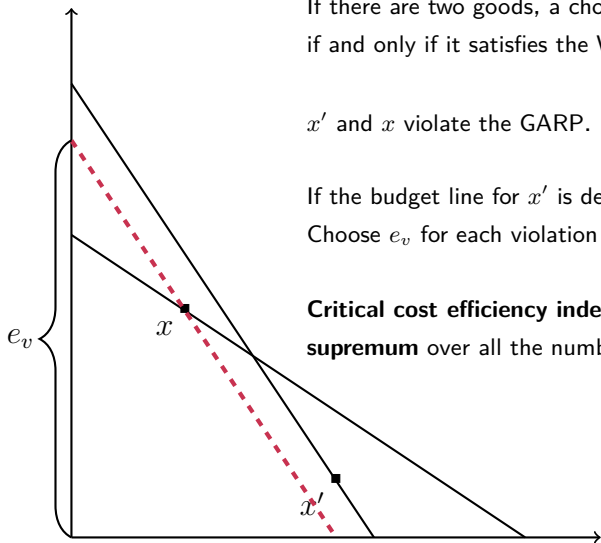
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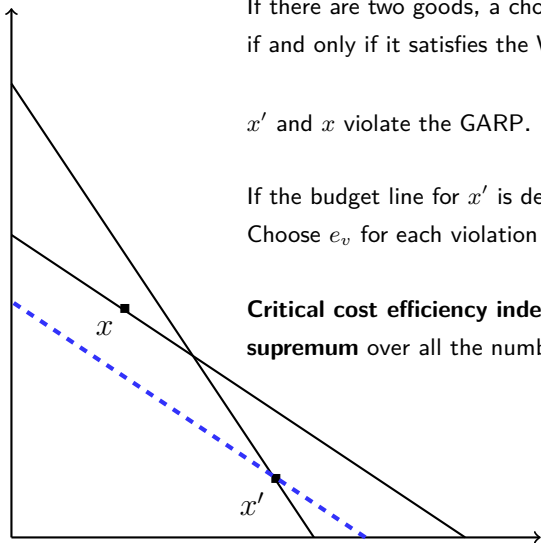
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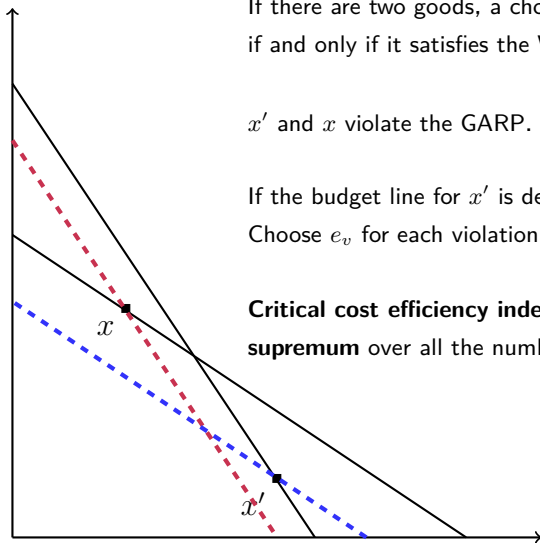
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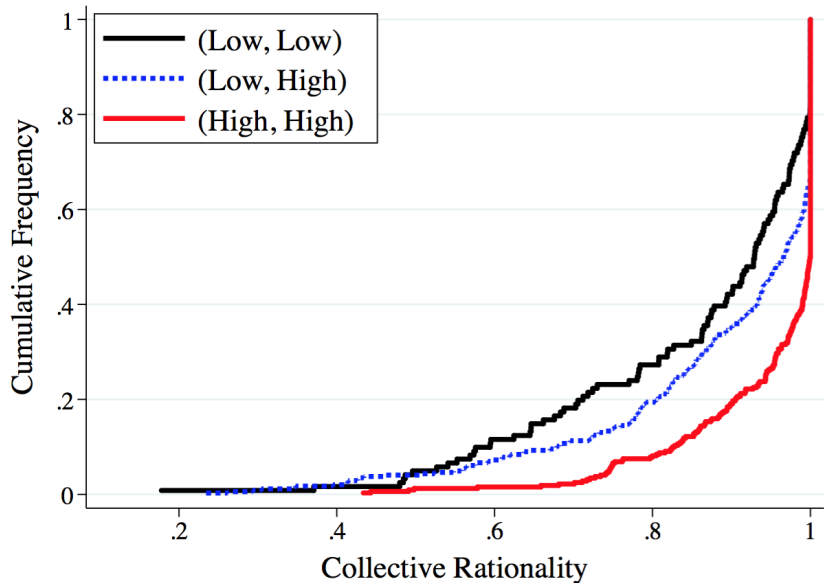
- By definition,  $\text{CCEI} \in [0, 1]$ .
- The **bigger** CCEI is, the **less** severe violation of GARP.
- Basic statistics of individual and collective CCEIs:
  - individual average: 0.897 (0.136)
  - collective average: 0.910 (0.141).



# Rationality Extension: Research Question

Individual Rationality  $\uparrow \Rightarrow$  Collective Rationality  $\uparrow$ ?

# Rationality Extension: First-Order Stochastic Dominance



# Rationality Extension: First-Order Stochastic Dominance

- We do a series of Kolmogorov-Smirnov tests:

$$H_0 : F_{\text{group } i}(X) = F_{\text{group } j}(X) \quad \text{for all values of } X.$$

- Test statistic:  $D_{ij} = \sup_{x \in X} ||F_{\text{group } i}(x) - F_{\text{group } j}(x)||$ .
- (Low, Low) v.s. (High, High): 0.17
  - The corresponding p-value is 0.01.
- (Low, High) v.s. (High, High): 0.21
  - The corresponding p-value is 0.00.

# Rationality Extension: Econometric Analysis

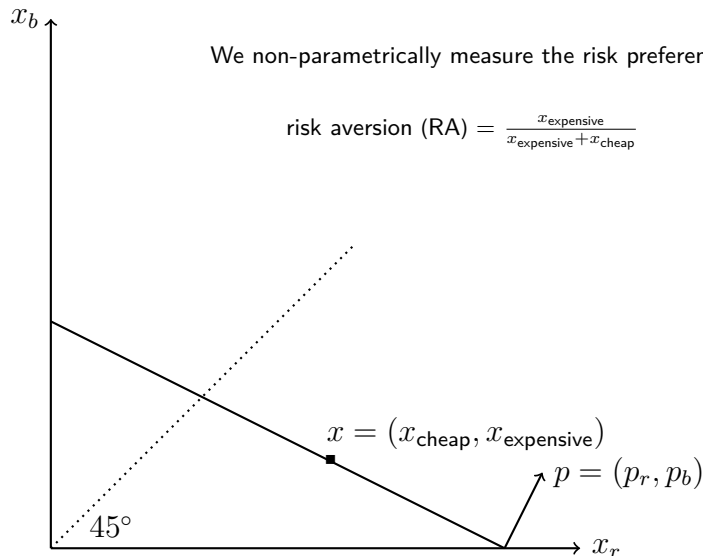
Collective CCEI	Coefficient	
	Model 1	Model 2
CCEI_Max	0.368*** (0.083)	0.302*** (0.089)
CCEI_Distance	-0.277*** (0.056)	-0.233*** (0.058)
Math_Score_Max		0.012** (0.005)
Math_Distance		-0.010** (0.005)
Constant	0.582*** (0.077)	0.664*** (0.084)
Class Fixed Effect	Yes	Yes
Individual Characteristics	No	Yes
School Characteristics	No	Yes
Friendship	No	Yes
Observations	786	786
R-squared	0.200	0.235
*Throughout the paper, we clustered standard error in class level.		

## **Result 2: Preference Aggregation**

# Measurement: Risk Preferences

We non-parametrically measure the risk preference by a ratio:

$$\text{risk aversion (RA)} = \frac{x_{\text{expensive}}}{x_{\text{expensive}} + x_{\text{cheap}}}$$



# Measurement: Risk Preferences

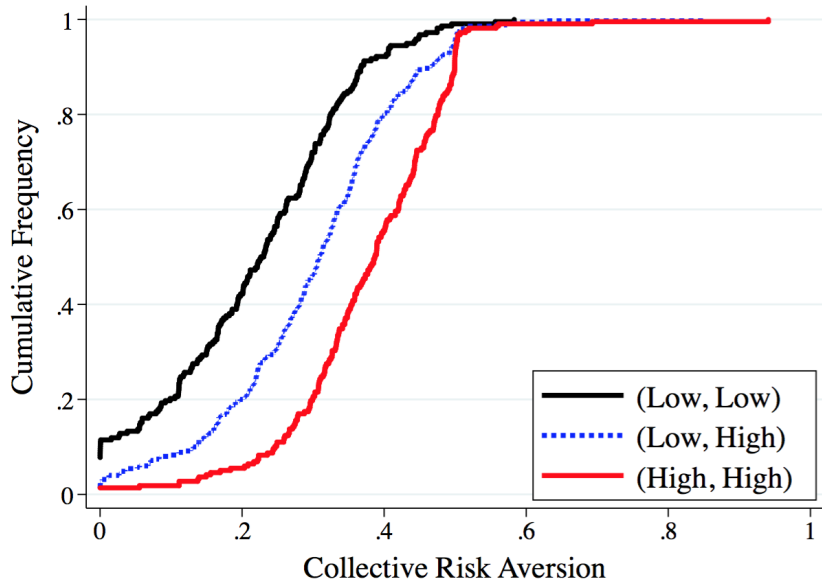
- By definition,  $RA \in [0, 1]$ .
- The **bigger** ratio is, the **higher** risk aversion.
- Basic statistics of individual and collective RAs:
  - Average: 0.324 (0.132)
  - Average: 0.298 (0.139).

# Risk Preference Aggregation: Research Question

Individual risk aversion  $\uparrow \Rightarrow$  Collective risk aversion  $\uparrow$ ?



## Preference Aggregation: FOSD by Relative Ratio



# Preference Aggregation: Econometric Analysis

Collective Risk Aversion	Coefficient	
	Model 1	Model 2
Risk_Aversion_Max	0.792*** (0.066)	0.759*** (0.073)
Risk_Aversion_Distance	-0.421*** (0.053)	-0.434*** (0.062)
Math_Score_Max		0.000 (0.005)
Math_Distance		0.005 (0.004)
Constant	0.004 (0.027)	-0.175** (0.077)
Class Fixed Effect	Yes	Yes
Individual Characteristics	No	Yes
School Characteristics	No	Yes
Friendship	No	Yes
Observations	786	786
R-squared	0.372	0.382

### **Result 3: Efficiency and Welfare**

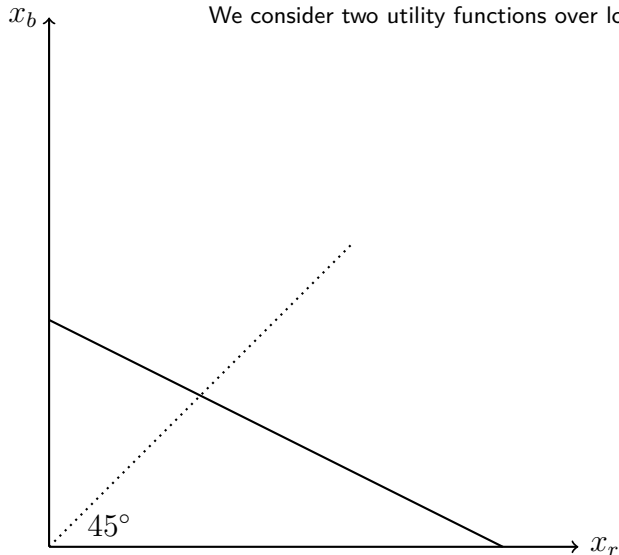
# Measurement: Idea

- We analyze the **quality** of collective decisions as a function of the degrees of rationality and preference alignment.
- Idea:
  - We consider a class of utility functions over lotteries.
  - For each subject, we estimate the utility function parametrically.
  - We characterize a set of Pareto efficient choices.
  - For collective choices which are **not** Pareto efficient, we measure the degree of welfare loss.

# Measurement: Utility Estimation

We assume a CARA utility function:  $u(x) = -e^{-\rho x}/\rho$ .

We consider two utility functions over lotteries.



# Measurement: Utility Estimation

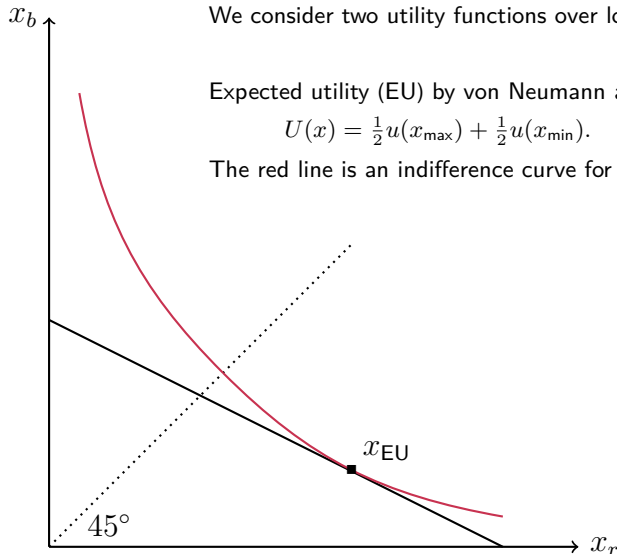
We assume a CARA utility function:  $u(x) = -e^{-\rho x}/\rho$ .

We consider two utility functions over lotteries.

Expected utility (EU) by von Neumann and Morgenstern (1953):

$$U(x) = \frac{1}{2}u(x_{\max}) + \frac{1}{2}u(x_{\min}).$$

The red line is an indifference curve for the EU.



# Measurement: Utility Estimation

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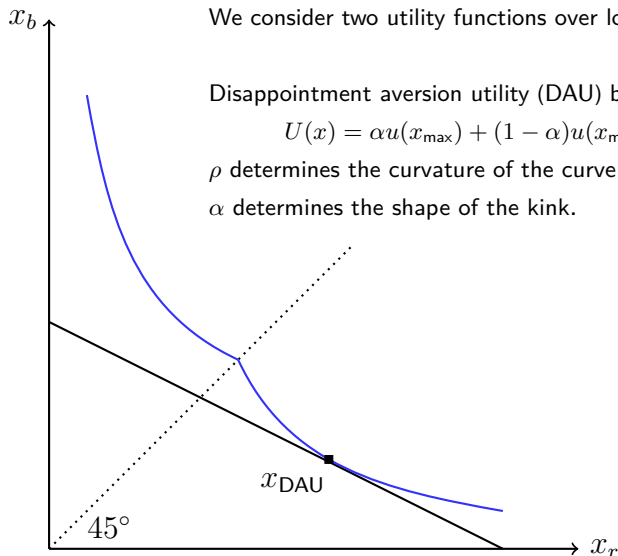
We consider two utility functions over lotteries.

Disappointment aversion utility (DAU) by Gul (1991):

$$U(x) = \alpha u(x_{\max}) + (1 - \alpha)u(x_{\min}).$$

$\rho$  determines the curvature of the curve.

$\alpha$  determines the shape of the kink.

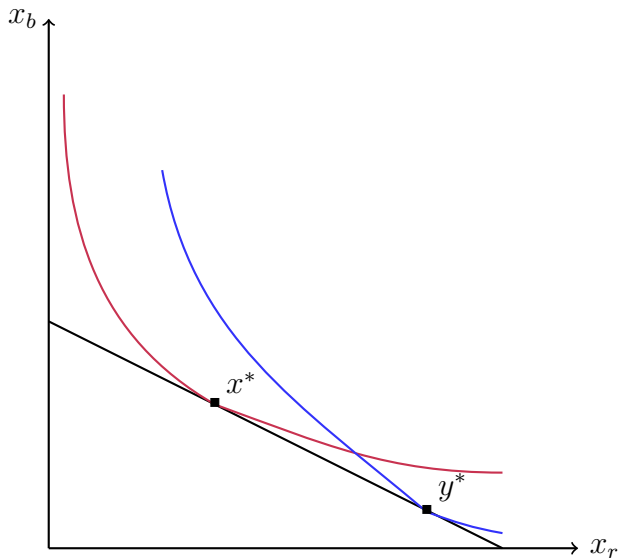


# Measurement: Utility Estimation

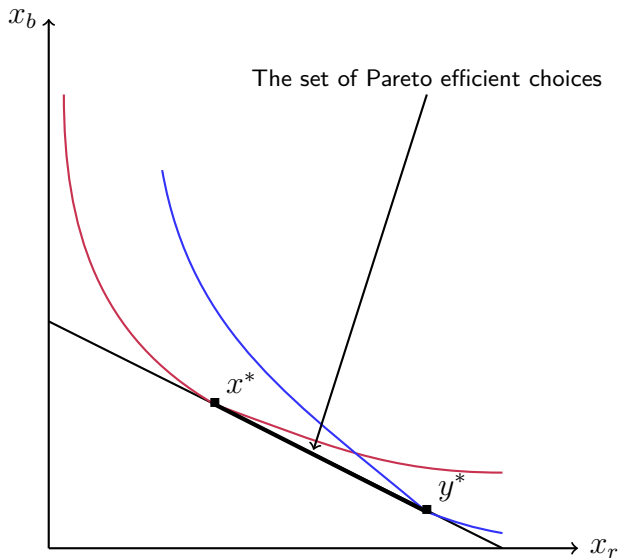
- We restrict our attention to a CARA utility function over outcomes.
- We consider two different types of utility function over lotteries:
  - Expected utility (EU)
  - Disappointment aversion utility (DAU).
- We estimate  $\rho$  and  $\beta$  simultaneously by using a combination of a bootstrapping and the non-linear least square (NLLS) methods:
  - 1 Find subsample of size 18 with replacement.
  - 2 For given subsample, estimate  $\alpha$  and  $\rho$  by NLLS.
  - 3 Repeat the above for 250 times.
  - 4 If  $0.5 \in [\alpha_{2.5}, \alpha_{97.5}]$ , then set  $\alpha = 0.5$  as an EU.
  - 4' Otherwise, set  $\alpha = \bar{\alpha}$  as a DAU.



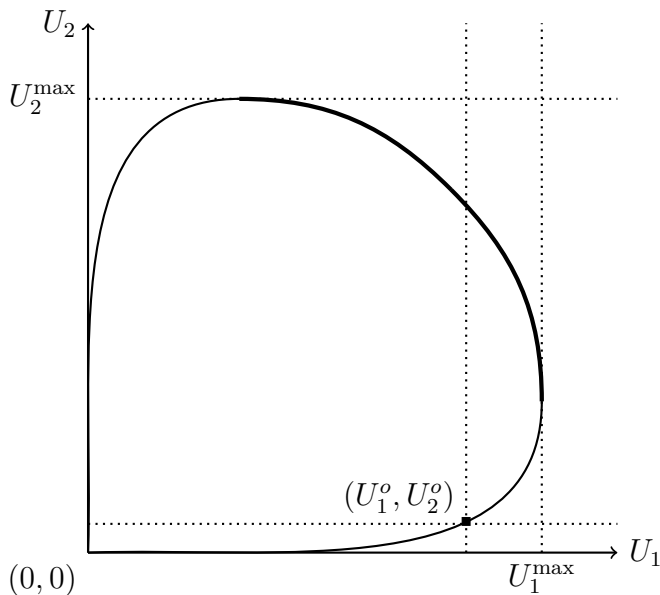
# Measurement: Efficiency and Welfare Loss



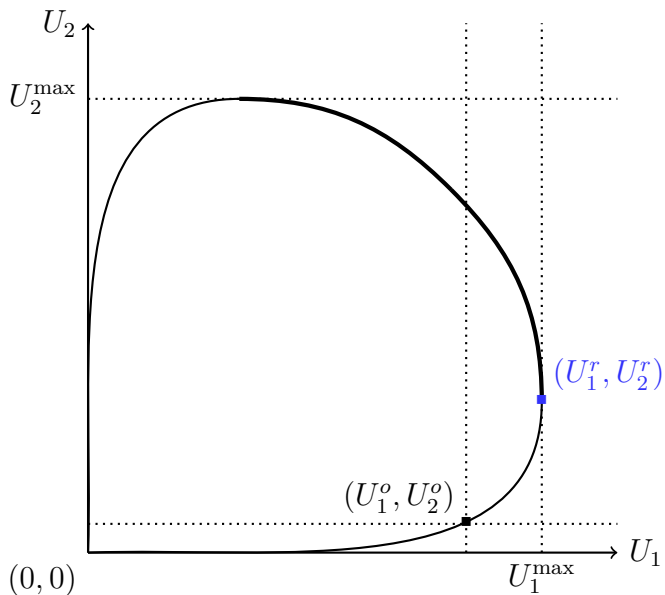
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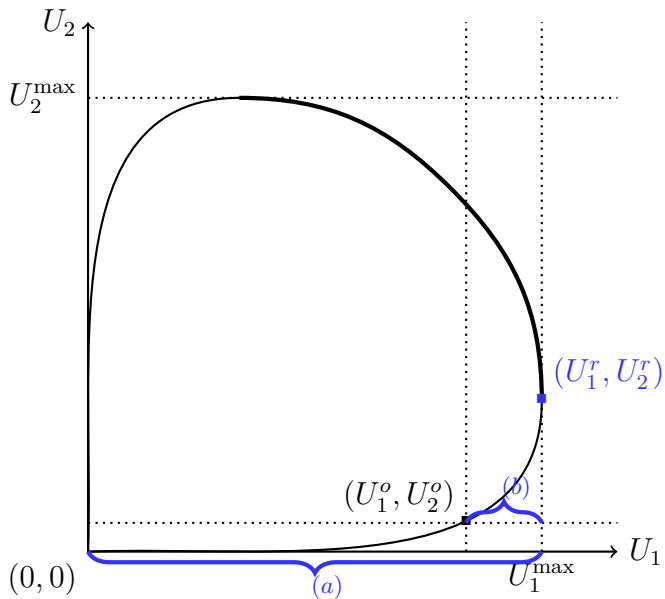
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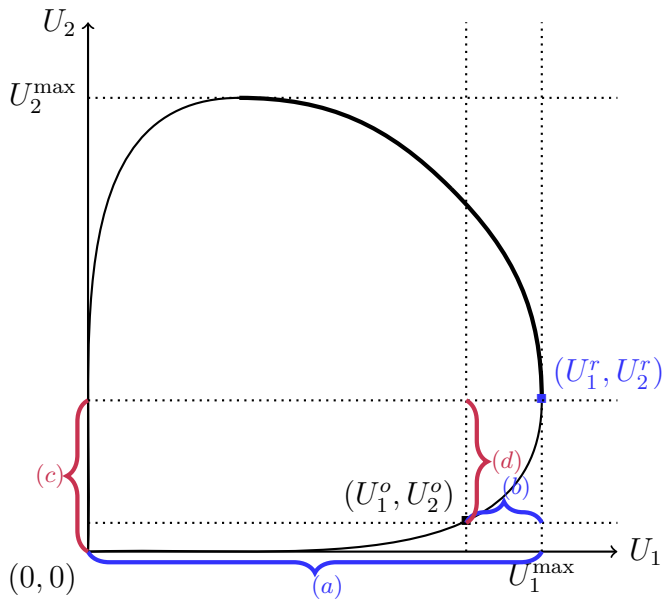
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# Efficiency and Welfare: Measurement

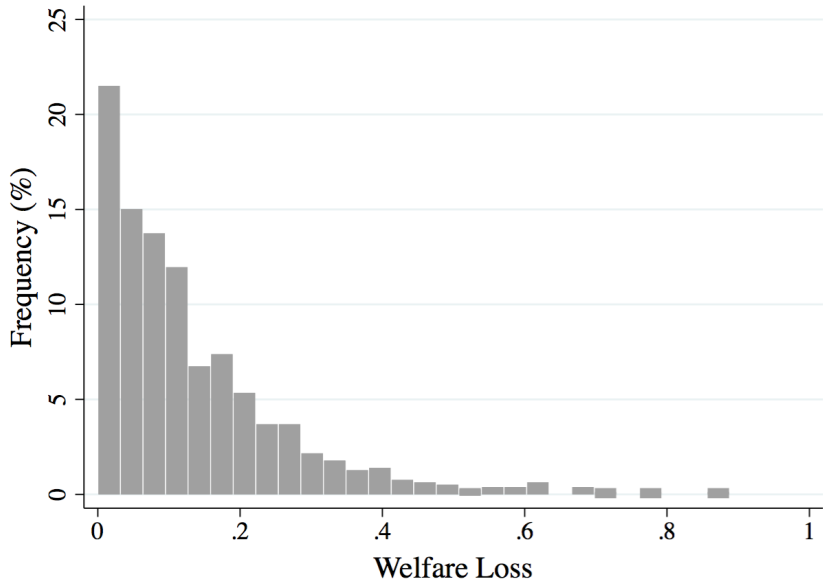
- We focus on the group choices which are not Pareto efficient (60%).
- For those choices, we measure **welfare loss** of a group as

$$\text{Welfare Loss} = \frac{1}{18} \sum_{k=1}^{18} \frac{1}{2} \sum_{i=1}^2 \frac{U_i(x_{kr}) - U_i(x_{kc})}{U_i(x_{kr}) - U_i(x_{kw})},$$

where

- $x_{kc}$ : group's choice in  $k$ -th round
  - $x_{kr}$ : group's reference choice in  $k$ -th round
  - $x_{kw}$ : group's worst choice in  $k$ -th round.
- By definition, Welfare Loss  $\in [0, 1]$ .

## Efficiency and Welfare: Distribution of Welfare Loss

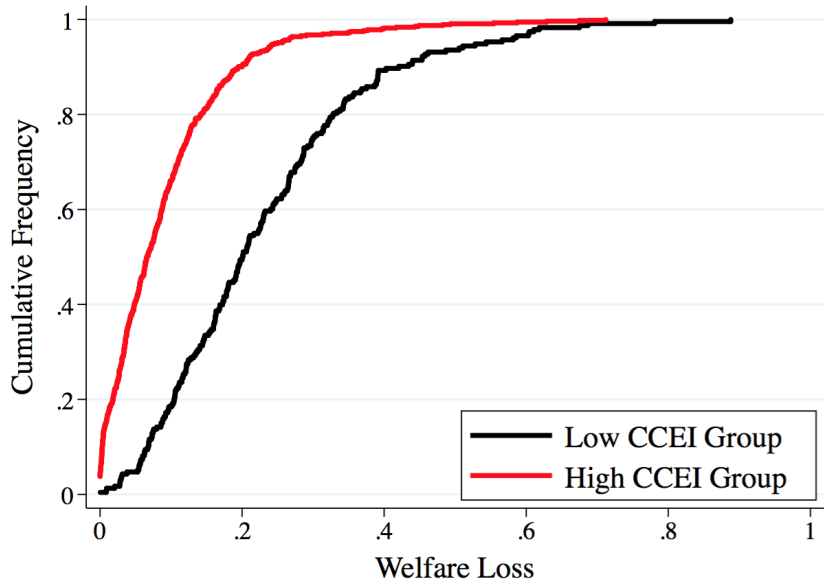




# Efficiency and Welfare: Research Question

How is the welfare loss related to individual rationality and risk preference?

## Welfare: FOSD by Group Rationality



# Welfare: Econometric Analysis

Group Inefficiency	Coefficient			
	Model 1	Model 2	Model 3	Model 4
CCEI_Group	-0.571*** (0.042)	-0.503*** (0.039)	-0.527*** (0.043)	-0.414*** (0.078)
CCEI_Max		-0.296*** (0.045)	-0.242*** (0.051)	-0.692 (0.671)
CCEI_Distance		0.165*** (0.042)	0.178*** (0.052)	0.289 (0.257)
Risk_Aversion_Max		-0.009 (0.056)	0.023 (0.063)	-0.024 (0.089)
Risk_Aversion_Distance		-0.057* (0.031)	-0.073* (0.040)	-0.123* (0.066)
Math_Score_Max			0.002 (0.005)	0.008 (0.007)
Math_Distance			-0.004 (0.003)	-0.010 (0.006)
Constant	0.651*** (0.038)	0.866*** (0.048)	0.807*** (0.061)	1.154* (0.653)
Class Fixed Effect	Yes	Yes	Yes	Yes
Individual Characteristics	No	No	Yes	Yes
School Characteristics	No	No	Yes	Yes
Friendship	No	No	Yes	Yes
Observations	786	786	786	274
R-squared	0.442	0.487	0.497	0.436

# Conclusion

- We measure rationality and risk preference in individual and group levels.
- We observe rationality extension and preference aggregation.
- We develop a measure of efficiency and utility loss of group decisions.
- We find that
  - Rational groups are more likely to make efficient decisions.
  - Preference-aligned individuals need not make efficient decisions.

# Conclusion

- Our main findings are robust with respect to
  - another rationality measure (Varian's efficiency index)
  - other cutoff values of CCEI (0.99 or 0.95)
  - another measure of risk preferences (risk premium).

## **Robustness**

# Varian's Efficiency Index

- Varian modifies CCEI by allowing  $e_k$  to vary across the different price vectors.
- Consider a vector  $\theta = (e^k)_{k=1}^K$  of numbers in  $[0, 1]$ , one for each observation.
- Define the binary relation  $R_\theta$  as  $x^k R_\theta x^l$  if  $e^k p^k \cdot x^k \geq p^k \cdot x^l$ . Let  $P_\theta$  be the corresponding strict relation.
- There is a set  $\Theta$  of vectors  $\theta$  such that the corresponding  $\langle R_\theta, P_\theta \rangle$  satisfies GARP.
- Varian's efficiency index (VEI) is the closest distance of a vector  $\theta$  to the unit vector ( $e_k = 1$  for all  $k$ ), among those  $\theta$  for which the preference pair is acyclic:

$$\text{VEI} = \inf \left\{ \|1 - \theta\| \mid \langle R_\theta, P_\theta \rangle \text{ is acyclic} \right\}.$$

# Disappointment Aversion Utility

- A functional form:

$$u(x_1, x_2) = u(\max\{x_1, x_2\}) + (1 - \gamma)u(\min\{x_1, x_2\}),$$

where  $\gamma = 1/(2 + \beta)$ .

- $\beta > 0$  represents the **disappointment aversion**: the better outcome is under-weighted relative to the objective probability.
- $\beta \in (-1, 0)$  represents **elation seeking**: the better outcome is over-weighted relative to the objective probability.
- Of course, this utility function is aligned with the first-order stochastic dominance relationships between lotteries.