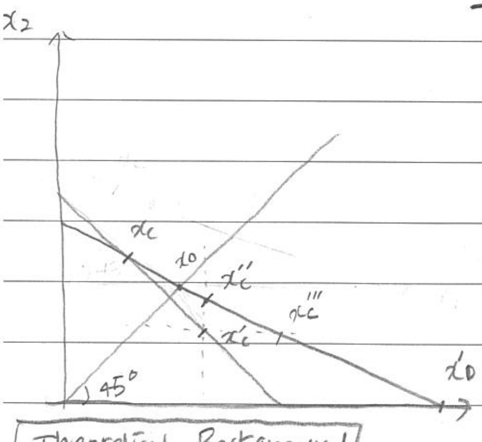
How to find a Pareto improvement of x_c in a numerical way?

Motivation



- All idiosyncratic values are from when subjects put more resource on more expensive goods.
- But as alpha < 0.5 ($\forall$ such cases), it's better for them to choose such x_c rather than partners' optimal point which is very close to $x_1 = x_2$. ($U_{i}pec < U_{i}c$)

Theoretical Background

By the symmetry of our utility functional form,

$U_i(x_c) = U_i(x_c')$ if $x_c' = (x_{c2}, x_{c1})$ when $x_c = (x_{c1}, x_{c2})$

In addition, by the monotonicity $U_i(x) > U_i(x_c')$ if $x \in \{x \mid x_1 \geq x_{c1}$ & $x_2 \geq x_{c2}\}$ (strict inequality always holds because $p \cdot x_c' < w$)

Denote $x_c'' = (x_{c2}, w - p_1 \cdot x_{c2})$, $x_0 = \left(\frac{w}{(p_1+p_2)}, \frac{w}{(p_1+p_2)}\right)$, $x_0' = (x_1, 0)$,

$x_c''' = (w - p_2 x_{c1}, x_{c1})$.

case 1) $U_i(x_0) \leq U_i(x_c')$ $(\leq U_i(x_c''))$

$U_i(x)$ is continous in x and quasi concave.

$\therefore \exists\, x$ s.t $U_i(x_0) < U(x) = U(x_c') \leq U_i(x_c'')$

and $x = t x_0 + (1-t) x_c''$

(by middle point theorem)

case 2) $U_i(x_0) > U_i(x_c')$

i) no point $x \in [x_0, x_0']$ s.t $U(x_c) = U(x)$

$\Leftrightarrow \forall\, x,\ U(x) > U(x_c)$

$\Rightarrow \Leftarrow$ $U_i(x_{PE}) < U_i(x_c)$ ($\because x_{PE} \in [x_0, x_0']$)

which was our motivation for all these works

ii) then by continuity and quasi concavity

$\forall\, x \in [x_0, x_c')$ $U(x) > U_i(x_c')$

∴ Find $x \in [x_c''', x_0']$ s.t $U_i(x) = U_i(x_c)$

Algorithm

$U_i(x_0) > U_i(x_c)$

Yes | No (0 case!!)

- **Yes** (Repeat)
  - $x_1 = \frac{x_0 + x_c''}{2}$
  - $U_i(x_1) > U_i(x_c)$
    - Yes: $x_2 = \frac{x_0 + x_1}{2}$
    - No: If $U_i(x_1) = U_i(x_c)$ stop. O.W $x_2 = \frac{x_1 + x_c''}{2}$
- **No** (Repeat)
  - $x_1 = \frac{x_c''' + x_0'}{2}$
  - $U_i(x_1) > U_i(x_c)$
    - Yes: $x_2 = \frac{x_0' + x_1}{2}$
    - No: If $U_i(x_1) = U_i(x_c)$ stop. O.W $x_2 = \frac{x_1 + x_c'''}{2}$

Repeat until $|U_i(x_k) - U_i(x_c)| \le$ tolerance.

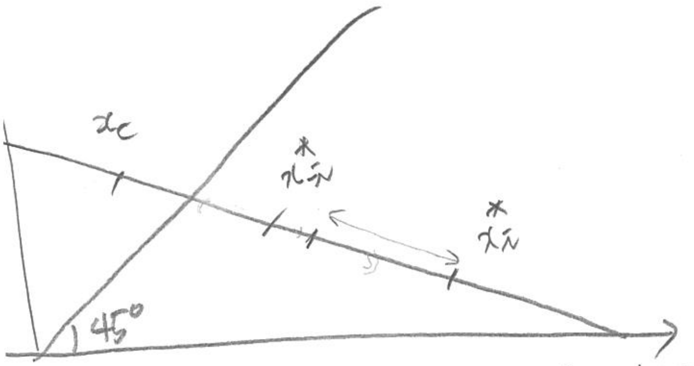
(*) For convenience, change all data to ($x_{cheaper}$, $x_{expensive}$) and mirror estimated demand for cases where the original data has the form of ($x_{expensive}$, $x_{cheaper}$)

partner's point of view

By the algorithm, we find an improvement for i.

Is it also better to $-i$? ("pareto" improvemet?)

⇒ the answer is, yes.



(대소관계는 x_1을 기준으로 함)

개 our case, $x_{-i}^* < x_{-i}^*$

$x_{pI} > x_{-i}^*$ where $U(x_{pI}) = U(x_c)$.

(If not, $U_i(x_{pe}) < U_i(x_c)$ doesn't hold, which is the starting point of all these process)

and we found $x_{pI} < x_i^*$ ($\because U_i(x_0) > U_i(x_c)$ 인 cases)

$-i$의 입장에서 improvement set $= \{x \mid [x_{-i}^* - p,\ x_{-i}^* + q]\}$

i의 〃 $= \{x \mid [x_{pI},\ x_{-i}^* + r]\}$

$\therefore$ If $x_{pI} \notin \{x \mid [x_{-i}^* - p,\ x_{-i}^* + q]\}$

then no improvement which contradicts our finding on pareto efficiency.