

PBL Effect \Rightarrow composition effect + effect of interest

<Treatment>

		Post			
Pre	N	R	G	M	
	P_{NN}^T	P_{NR}^T	P_{NG}^T	P_{NM}^T	
R	:				
G	:				
M	:				

<Control>

		Post			
Pre	N	R	G	M	
	P_{NN}^C	P_{NR}^C	P_{NG}^C	P_{NM}^C	
R	:				
G	:				
M	:				

$$\text{where } P_{ij} = (\text{Dyadic Relation}_{t=1} = j \mid \text{Dyadic Relation}_{t=0} = i)$$

Treatment effect for Pre-Null relationship

$$\stackrel{(*)}{=} \rho_N$$

$$\begin{aligned}
 &= \sum_j P_{Nj}^T (E(Y_{ti} \mid X_{ti}=j, T, X_{t0}=N) - E(Y_{t0} \mid X_{ti}=j, T, X_{t0}=N)) \\
 &\quad - \sum_j P_{Nj}^C (E(Y_{ti} \mid X_{ti}=j, C, X_{t0}=N) - E(Y_{t0} \mid X_{ti}=j, C, X_{t0}=N)) \\
 &= \sum_j P_{Nj}^T (E(Y_{ti} \mid X_{ti}=j, T, X_{t0}=N) - E(Y_{t0} \mid X_{ti}=j, T, X_{t0}=N)) \\
 &\quad - (E(Y_{ti} \mid X_{ti}=j, C, X_{t0}=N) - E(Y_{t0} \mid X_{ti}=j, C, X_{t0}=N)) \\
 &\quad + \underbrace{\sum_j (P_{Nj}^T - P_{Nj}^C)}_{\text{diff in transition}} \underbrace{(E(Y_{ti} \mid X_{ti}=j, C, X_{t0}=N) - E(Y_{t0} \mid X_{ti}=j, C, X_{t0}=N))}_{\text{learning by each group in control}}
 \end{aligned}$$

} effect of interest

$$(*) Y_{it} = \beta_0 + \beta_1 t_i + \beta_2 t_i \cdot PBL_i + \sum_{j \neq N} \delta_j X_{t0j} + \sum_j \gamma_j X_{t0j} \cdot t_i + \sum_j \alpha_j X_{t0j} \cdot PBL_i + \sum_j \rho_j X_{t0j} \cdot PBL_i \cdot t_i$$

$$\begin{aligned}
 \rho_N &= E(Y_{ti} \mid PBL=1, X_{t0j}=N) - E(Y_{t0} \mid PBL=1, X_{t0j}=N) \\
 &\quad - (E(Y_{ti} \mid PBL=0, X_{t0j}=N) - E(Y_{t0} \mid PBL=0, X_{t0j}=N))
 \end{aligned}$$

\hookrightarrow whole sample에 대한 transition matrix 계산하는 것이 아님