1 Decision problem

An individual subject in the time preference experiment chooses a consumption profile over two periods, (c_t, c_{t+s}), to maximize his intertemporal utilities subject to a budget constraint:

$$u\left(c_t|\omega_t\right) + \beta_{t,s} u\left(c_{t+s}|\omega_{t+s}\right)$$
$$\text{s.t. } c_t + \frac{1}{1+r_{t,s}} c_{t+s} = m,$$

where $u\left(\cdot|\omega\right)$ is the per-period utility function based on the baseline consumption level, ω. $\beta_{t,s}$ denotes a discount factor. If we assume the exponential discounting function, the discount factor only depends on the time horizon between two periods, s. $r_{t,s}$ represents an experimental interest rate of payments between two periods.

2 Optimal demands

We derive the optimal demands in each of exponential and power utility functions.

2.1 Exponential utility function

Assume that the utility function u is represented by the exponential function:

$$u\left(c|\omega\right) = -\frac{1}{A}\exp\left(-A\left(c-\omega\right)\right),$$

where A determines the curvature of utility function, which is an absolute risk aversion coefficient in the decision making under risk. Note that in this case the optimal demand does not depend on the baseline consumption level, ω. From the constrained maximization problem, we have the following results.

Case 1: $-m \leq \frac{1}{A}\ln\left(\beta_{t,s}\left(1+r_{t,s}\right)\right) \leq \left(1+r_{t,s}\right)m$

$$c_t^* = \frac{1}{2+r_{t,s}}\left[\left(1+r_{t,s}\right)m - \frac{1}{A}\ln\left(\beta_{t,s}\left(1+r_{t,s}\right)\right)\right],$$
$$c_{t+s}^* = \frac{1+r_{t,s}}{2+r_{t,s}}\left[m + \frac{1}{A}\ln\left(\beta_{t,s}\left(1+r_{t,s}\right)\right)\right].$$

Case 2: $\frac{1}{A}\ln\left(\beta_{t,s}\left(1+r_{t,s}\right)\right) < -m$

$$c_t^* = m \text{ and } c_{t+s}^* = 0.$$

Case 3: $\frac{1}{A}\ln\left(\beta_{t,s}\left(1+r_{t,s}\right)\right) > \left(1+r_{t,s}\right)m$

$$c_t^* = 0 \text{ and } c_{t+s}^* = \left(1+r_{t,s}\right)m.$$

2.2 Power utility function

The utility function u is assumed to be represented by the power utility function:

$$u\left(c|\omega\right)=\frac{\left(c-\omega\right)^{1-\rho}}{1-\rho},$$

where ρ represents a relative risk aversion coefficient. Note that the optimal demand depends on the basline consumption level in this case and also that, for $\omega \leq 0$, the optimality of demands can not generate corner solutions, widely observed in the experimental data. Thus, we assume that $\omega > 0$.

Case 1: $0 < c_t^* < m$ **and** $0 < c_{t+s}^* < \left(1+r_{t,s}\right)m$

$$\begin{aligned} c_t^* &= \frac{1}{\left(1+r_{t,s}\right)+K}\left[\left(1+r_{t,s}\right)m+\left(K-1\right)\omega\right], \\ c_{t+s}^* &= \frac{K}{\left(1+r_{t,s}\right)+K}\left[\left(1+r_{t,s}\right)m+\left(K-1\right)\omega\right]-\left(K-1\right)\omega, \end{aligned}$$

where

$$K=\left[\beta_{t,s}\left(1+r_{t,s}\right)\right]^{1/\rho}.$$

Case 2 & 3:

$$\begin{aligned} c_t^* = m \quad &\text{and} \quad c_{t+s}^* = 0, \\ &\text{or} \\ c_t^* = 0 \quad &\text{and} \quad c_{t+s}^* = \left(1+r_{t,s}\right)m. \end{aligned}$$

3 Estimation

We estimate $\theta = \left(A, \beta_{t,s}\right)$ or $\left(\rho, \beta_{t,s}\right)$ using the individual-level experimental data. Each treatment consists of 50 decision problems for each individual, denoted by $\left\{\left(c_t^p, c_{t+s}^p, m^p, r_{t,s}^p\right)\right\}_{p=1}^{50}$. Our econometric method is to minmize the distance between observed demands and optimal demands:

$$D\left(\theta\right)=\sum_{p=1}^{50}\left[\left(c_t^p-c_t^{p*}\left(\theta\right)\right)^2+\left(c_{t+s}^p-c_{t+s}^{p*}\left(\theta\right)\right)^2\right].$$