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## RATIONAL HOUSEHOLD LABOR SUPPLY

BY PIERRE-ANDRÉ CHIAPPORI<sup>1</sup>

Traditionally, household behavior is derived from the maximization of a unique utility function. In this paper, we propose an alternative approach, in which the household is modeled as a two-member collectivity taking Pareto-efficient decisions. The consequences of this assumption are analyzed in a three-good model, in which only total consumption and each member's labor supply are observable. If the agents are assumed egoistic (i.e., they are only concerned with their own leisure and consumption), it is possible to derive falsifiable conditions upon household labor supplies from both a parametric and nonparametric viewpoint. If, alternatively, agents are altruistic, restrictions obtain in the nonparametric context; useful interpretation stems from the comparison with the characterization of aggregate demand for a private-good economy.

**KEYWORDS:** Micro economy, labor supply, household behavior, nonparametric analysis.

### 1. INTRODUCTION

THE USUAL APPROACH of household decision making derives household behavior from the maximization of a unique utility function. This “neoclassical” formalization, however, can be criticized on the ground that it somehow contradicts the basic requirements of methodological individualism. Indeed, since the household consists of several members, its behavior should be analyzed as the result of several individually rational decisions. That is to say, each member should be characterized by his (her) own utility function; and the “collective” household decisions should be analyzed within a formal framework which would model the interactions between members. Of course, it is not clear—and, in fact, it seems rather doubtful—that such a “collective” decision making model will lead to the same predictions about household behavior as the “neoclassical” setting. For instance, Slutsky relations may not be satisfied by the “collective” household demand functions. This means, of course, that forcing household behavior into an aggregate “neoclassical” framework by using ad hoc assumptions will generally be unacceptable.

The literature upon “collective” household decision processes, so far, has borrowed its tools from cooperative game theory.<sup>2</sup> Manser and Brown (1980) consider the implications of various bargaining rules, including Nash and Kalay-Smorodinsky bargaining equilibrium concepts. A paper by McElroy and Horney (1981) provides a careful analysis of the Nash-bargained case; in particular, the authors derive a “Nash generalization” of the Slutsky equations for household demand functions. It turns out, however, that these conditions are not restrictive, unless the agents’ premarital preferences are known.<sup>3</sup>

<sup>1</sup> I am indebted to the editor and an anonymous referee for numerous valuable comments. I also benefited from discussions with F. Bourguignon, J. P. Benassy, A. Boyer, R. Guesnerie, F. Laisney, and H. Sonnenschein. This paper was presented at the European Meeting of the Econometric Society, Budapest, September, 1986, and at the Séminaire d’Economie Théorique, Paris, March, 1986.

<sup>2</sup> With the exception of Bourguignon (1984) who considers noncooperative Nash equilibria. See also Trognon (1981).

<sup>3</sup> For a careful analysis of this point, see Chiappori (1986).

The aim of the present paper is to generalize previous works upon collective household decision in two directions. First, it tries to derive falsifiable conditions upon household behavior from a “collective rationality” hypothesis; however, instead of referring to some definite bargaining concept, it only makes a very weak and general assumption—namely, that the household always reaches Pareto-efficient agreements. The question which is investigated through the paper is thus the following: does Pareto efficiency alone imply restrictions upon observable household behavior?

A second generalization is that intra-household distribution of consumption is not assumed observable. That is to say, though the household’s total consumption is known, each member’s is not. The reason behind this assumption is essentially empirical. Most, if not all, available data upon household behavior describe each member’s labor supply, but only aggregate consumption; each member’s share remains unknown. This means that a set of restrictions which would necessitate the observation of each member’s private consumption could simply not be used to test empirically the relevance of the collective approach.

We thus consider a very simple model of a two-member household in a three-good economy (namely, each member’s labor plus consumption), the aim being to derive falsifiable restrictions upon labor supplies from the sole Pareto-efficiency hypothesis. Conclusions can be summarized as follows:

(i) A crucial hypothesis is whether household members are egoistic (they are only concerned with their own leisure and consumption) or altruistic.

(ii) In case of egoistic agents, one can derive conditions from a parametric (i.e., partial differential equations upon labor supply) as well as nonparametric point of view. Moreover, these conditions are totally independent from the “neoclassical” ones, so that empirical tests may be performed in order to compare the two settings.

(iii) In the “general” case (“altruistic” agents), though the class of collectively rational labor supply functions is very broad (it includes, in particular, all the neoclassical ones), it is still possible to derive necessary and sufficient conditions in a nonparametric context. These conditions basically reflect the fact that the number of agents is smaller than the number of goods; a simple counterexample shows that there exist data which do *not* satisfy the restrictions.

The model is set forth and discussed in Section 2. Section 3 concentrates upon the case of egoistic agents, first from a parametric, then from a nonparametric viewpoint. The case of altruistic agents is analyzed in Section 4.

## 2. RATIONAL COLLECTIVE DECISION

### A. *The Model*

We consider a two-member household, in which both members supply labor;  $\ell^i$ ,  $L^i$ , and  $T$  respectively denote member  $i$ ’s labor supply, leisure, and total time, so that  $\ell^i = T - L^i$ . There is a unique consumption good, the price of which is set to one; total household consumption is denoted by  $C$ . The consumption good

is purely private;  $C$  is thus divided between both members. Let  $Z^i$  denote member  $i$ 's private consumption. We shall assume in what follows that  $Z^1$  and  $Z^2$  are *not observable*; that is to say, though household consumption is known as a whole, we do not possess any data about its repartition between the members. Lastly, the agents make their decisions conditionally on given values of wages,  $w_1$  and  $w_2$ , and nonlabor income  $y$ , so that the budget constraint is

$$Z^1 + Z^2 \leq C = y + w_1 \ell^1 + w_2 \ell^2.$$

In what follows, we assume that the couple  $(w_1, w_2)$  belongs to a compact subset  $S$  of  $]0, +\infty[^2$ ; wages are thus bounded away from zero.

The next step, in the neoclassical setting, would be to assume the existence of a (unique) household utility function, depending on  $L^1$ ,  $L^2$ ,  $Z^1$ , and  $Z^2$ , and which is maximized subject to a budget constraint. Well known conditions can then be derived upon household behavior. We only emphasize the fact that restrictions upon labor supplies obtain in this context even though  $Z^1$  and  $Z^2$  are not observable. Indeed, a pair of  $C^1$  labor supply functions  $\ell^i(w_1, w_2, y)$ ,  $i = 1, 2$ , will not possibly derive from the maximization of a unique utility function unless it satisfies homogeneity and the Slutsky condition:

$$\frac{\partial \ell^1}{\partial w_2} - \ell^2 \frac{\partial \ell^1}{\partial y} = \frac{\partial \ell^2}{\partial w_1} - \ell^1 \frac{\partial \ell^2}{\partial y}.$$

In what follows, we shall derive similar conditions upon labor supplies from an alternative, "collective" decision framework. In the "collective" setting, the household is characterized by a pair of utility functions, which we shall assume strictly monotonic, strongly quasi-concave, and continuously twice differentiable. In general, member  $i$ 's utility depends on both  $i$ 's and  $j$ 's ( $j \neq i$ ) leisure and consumption:

$$U^i = U^i(L^1, L^2, Z^1, Z^2).$$

However, in the first part of the paper, we shall consider the special case of *egoistic* agents. An agent is said to be egoistic if his (her) utility depends only on his (her) own leisure and consumption, i.e., if

$$\frac{\partial U^i}{\partial L^j} \equiv \frac{\partial U^i}{\partial Z^j} \equiv 0 \quad \text{for } j \neq i.$$

### B. Hypothesis upon the Household's Decision Process

The restrictions upon labor supplies will of course crucially depend upon the assumptions made about the decision process. As we mentioned in the introduction, earlier works on the subject have referred to particular concepts of cooperative game theory (in fact, essentially to Nash bargaining, though the Manser-Brown (1980) paper also considers Kalai-Smorodinsky). We shall not follow this path; instead, we impose a much less restrictive condition, namely, Pareto efficiency of outcomes. That is to say, any pair of labor supply functions  $(\ell^1, \ell^2)$  will be considered as compatible with the collective setting if there exist two

individual consumption functions,  $Z^1$  and  $Z^2$ , which sum up to  $C$ , such that  $(L^1, L^2, Z^1, Z^2)$  is Pareto efficient within the household, among all possible choices of leisure and private consumption which satisfy the budget constraint.

Two reasons can be invoked in order to advocate this choice. First, Nash bargaining is neither a realistic, nor a very convenient tool to manipulate (see Chiappori (1986)). A second, and probably more important, reason is the impossibility of testing separately the various hypotheses which are implicit in the cooperative concept which has been chosen. In the absence of sociological data about the decision process within the household, the latter has to be considered as a “black box.” But this situation is likely to generate a typical example of the so-called “Duhem problem.” Suppose, for instance, that empirical evidence disconfirms the predictions of the model. Then it will be impossible to decide whether the failure must be attributed to the “collective” setting in general (as opposed, for instance, to the neoclassical one) or to the casual bargaining concept which has been used.<sup>4</sup>

Vice versa, since the paper is aimed at providing analytic conditions which allow testing of the collective setting versus the neoclassical one, it seems necessary to limit the assumption on the decision process to some kind of minimum minimorum. The obvious candidate is of course Pareto efficiency. All cooperative concepts lead to Pareto-efficient outcomes; and it is very doubtful that a cooperative decision could be considered as “rational” in any meaningful sense, if it results in nonefficient outcomes.<sup>5</sup>

This leads to the following definitions (which, for simplicity, relate to demands for leisure rather than to labor supplies):

DEFINITION 1: Household demand for leisure  $(\bar{L}^1(w_1, w_2, y), \bar{L}^2(w_1, w_2, y))$  is said to be *collectively rational (CR)*, if there exist two functions  $\bar{Z}^1$  and  $\bar{Z}^2$ , mapping  $S \times \mathbb{R}$  to  $\mathbb{R}^+$ , and two utility functions  $U^i(L^1, L^2, Z^1, Z^2)$  ( $i = 1, 2$ ), such that, for every  $(w_1, w_2, y)$  in  $S \times \mathbb{R}$ ,

$$(\alpha) \quad \bar{Z}^1(w_1, w_2, y) + \bar{Z}^2(w_1, w_2, y) \leq y + w_1(T - \bar{L}^1(w_1, w_2, y)) \\ + w_2(T - \bar{L}^2(w_1, w_2, y))$$

$$(\beta) \quad (\bar{L}^1(w_1, w_2, y), \bar{L}^2(w_1, w_2, y), \bar{Z}^1(w_1, w_2, y), \bar{Z}^2(w_1, w_2, y))$$

is Pareto optimal (for  $U^1$  and  $U^2$ ) in the set

$$\{(L^1, L^2, Z^1, Z^2) \in [0, T]^2 \times (\mathbb{R}^+)^2 / Z^1 + Z^2 \leq y + w_1(T - L^1) \\ + w_2(T - L^2)\}.$$

<sup>4</sup> See, for instance, Popper (1967). This kind of problem is commonly encountered in empirical economics (for instance, Varian (1983) invokes, as a major drawback of parametric testing, the fact that failure can always be attributed to inadequate functional forms).

<sup>5</sup> This solution—i.e., to use a very weak (hence uncontestable) version of the rationality principle, so that any empirical falsification can be attributed to the collective setting itself—follows exactly Popper's requirements (op. cit.).

DEFINITION 2: Household demand for leisure ( $\bar{L}^1(w_1, w_2, y)$ ,  $\bar{L}^2(w_1, w_2, y)$ ) is said to be *collectively rational for egoistic agents* (CREA) if there exist two functions  $\bar{Z}^1$  and  $\bar{Z}^2$  and two egoistic utility functions  $U^1(L^1, Z^1)$  and  $U^2(L^2, Z^2)$  such that conditions  $(\alpha)$  and  $(\beta)$  above are satisfied.

Definition 2 restricts Definition 1 to the case of egoistic agents. Here,  $(\alpha)$  is the admissibility condition, and  $(\beta)$  is the Pareto efficiency requirement. An alternative formulation is that there exists, for every vector of wages and nonlabor income, some household welfare function  $W$  which is maximized. Of course,  $W$  will in general depend not only on  $U^1$  and  $U^2$ , but also on  $w_1$ ,  $w_2$  and  $y$ ; that is to say, it is not necessarily the *same* welfare function which is maximized in each situation. Considering, for instance, a linear welfare function, this means that the weight of each member is allowed to depend on the wage he receives, or on the share of his labor income in total household resources. The particular case in which  $W$  depends only on  $U^1$  and  $U^2$  will be met further.

Two remarks are needed to help understanding these definitions.

(i) For any fixed  $(w_1, w_2, y)$ , there exists a continuum  $L(w_1, w_2, y)$  of couples  $(L^1, L^2)$  which are CR or CREA (in the sense defined above). This means that it is of course *not* possible to derive a *unique* couple of labor supply functions from these definitions. The derivation of labor supplies would require an additional element, namely a rule defining *which* of the (infinitely many) Pareto-optimal couples is chosen. (This is, for instance, what the Nash-bargaining argument concerned.<sup>6</sup>)

(ii) The question which will be investigated throughout the paper is the following: does, however, the Pareto efficiency requirement *alone* generate restrictions upon observable behavior? In other terms: take any couple of functions  $(L^1(w_1, w_2, y), L^2(w_1, w_2, y))$ ; under which conditions is it possible to find functions  $Z^1, Z^2, U^1$ , and  $U^2$  such that, for every  $(w_1, w_2, y)$ ,

$$(L^1(w_1, w_2, y), L^2(w_1, w_2, y)) \in L(w_1, w_2, y),$$

i.e.,

$$\{L^1(w_1, w_2, y), L^2(w_1, w_2, y), Z^1(w_1, w_2, y), Z^2(w_1, w_2, y)\}$$

is *one* of the (infinitely many) Pareto optimal allocations? Or is it the case that *any* couple of labor supplies can be collectively rationalized?

### C. Income-Sharing Rules

Before looking for answers to this question, let us briefly indicate a different, but equivalent, formulation of the *case of egoistic agents*. Suppose that the

<sup>6</sup> A consequence is that Definitions 2 and 3 do not imply, in general homogeneity of  $L^1$  and  $L^2$ . The reason is that if all prices, wages, and incomes are multiplied by the same nonnegative constant, the *set* of Pareto optimal outcomes does not change; but the particular outcome chosen may still vary. However, we shall, in what follows, only consider homogeneous solutions (an assumption we implicitly made when setting to one the price of the consumption good).

household is characterized by a pair of utility functions,  $U^1(L^1, Z^1)$  and  $U^2(L^2, Z^2)$  plus an income-sharing rule, which defines how nonlabor income is divided between members. Formally we have the following definition.

DEFINITION 3: An *income-sharing rule* is a mapping

$$G: (\mathbb{R}^+)^2 \times \mathbb{R} \rightarrow \mathbb{R}^2$$

$$(w_1, w_2, y) \mapsto G(w_1, w_2, y) = (y_1, y_2)$$

such that  $y_1 + y_2 = y$ .

Now, each member freely chooses its leisure and consumption, subject to its own budget constraint; that is, member  $i$ 's program is

$$(P_i) \quad \begin{aligned} &\text{Max } U^i(L^i, Z^i) \\ &\text{subject to } Z^i \leq y_i(w, w, y) + w_i(T - L^i). \end{aligned}$$

Note that, for a *given* income-sharing rule, it is possible to derive a unique pair of labor supply functions; but, of course, we have no way to observe the household income-sharing rule. Also, for any  $G$ ,  $(P_1)$  and  $(P_2)$  will generate Pareto-optimal outcome; conversely, any Pareto optimum can be considered as a solution of a program of this type for a well chosen income-sharing rule. Thus, our basic question can now be reformulated as follows. Take a pair of labor supply (or demand for leisure) functions; under which conditions is it possible to find two utility functions  $U^1, U^2$  and a sharing rule  $G$ , such that  $L^i$  is derived as a solution of  $(P^i)$  ( $i = 1, 2$ )?<sup>7</sup>

### 3. THE CASE OF EGOISTIC AGENTS

We first assume that both agents are egoistic. The conditions which arise, in this context, from the Pareto efficiency hypothesis, can be analyzed from either of two points of view. We may, first, consider labor supplies as everywhere differentiable functions of wages and nonlabor income; in that case, the conditions will generally take the form of partial differential equations or inequalities, similar to Slutsky relations. This approach is called “parametric”, because, since derivatives are usually considered as nondirectly observable, empirical testing will resort to specific functional forms for direct utility (or indirect utility, or demand, or expenditure, ...) functions. In that case, the restrictions will be translated into relations between the parameters that have to be estimated.

The second, “nonparametric” approach, as pioneered by Samuelson and by Afriat, Diewert, and Varian, uses a direct, revealed-preference type analysis. It derives algebraic relations about an assumed *finite* number of observations; no knowledge of the functional forms observed is required. In what follows, we exhibit conditions for the “collective” household decision-making model with egoistic agents successively from parametric and nonparametric viewpoints.

<sup>7</sup> This formulation cannot be easily extended to the (general) case of nonegoistic agents. Indeed,  $(P_i)$  depends on  $L^j$  and  $Z^j$ ; thus the resolution of  $(P_1)$  and  $(P_2)$  will give a kind of Nash equilibrium, which will generally *not* lead to a Pareto optimal allocation.

## A. The Parametric Approach

### A.1. The Main Result

We consider twice differentiable functions  $L^1, L^2$  from  $S \times \mathbb{R}$  to  $[0, T]^2$ . We use the following notation:

$X_K$  stands for  $\partial X / \partial K$ , where  $X = L^1, L^2, Z^1, Z^2$ , etc., and  $K = w_1, w_2, y$ .

$$A = L_{w_2}^1 / L_y^1, \quad B = L_{w_1}^2 / L_y^2, \quad \text{and}$$

$$\alpha = \left[ 1 - \frac{BA_y - A_{w_1}}{AB_y - B_{w_2}} \right]^{-1} \quad \text{if } AB_y - B_{w_2} \neq 0, \quad 0 \text{ if not, and } \beta = 1 - \alpha.$$

Then we have the following result:

PROPOSITION 1: (i) For  $L^1, L^2$  to be CREA in the sense of Definition 4, it is necessary that, for each  $(w_1, w_2, y)$  in  $S \times \mathbb{R}$  such that  $L_y^1 \cdot L_y^2 \neq 0$ :

(CREA a)

$$AB_y - B_{w_2} \neq BA_y - A_{w_1} \quad \text{or} \quad AB_y - B_{w_2} = BA_y - A_{w_1} = 0.$$

(ii) If  $AB_y - B_{w_2} \neq BA_y - A_{w_1}$  ("general case"), the following conditions are moreover necessary:

(CREA b)

$$\alpha_y A + \alpha A_y - \alpha_{w_2} = 0,$$

(CREA c)

$$\beta_y B + \beta B_y - \beta_{w_1} = 0,$$

(CREA d)

$$\alpha A [L_{w_1}^1 \alpha B + L_{w_2}^1 (\beta B - T + L^1)]^{-1} \leq 0,$$

(CREA e)

$$\beta B [L_{w_2}^2 \beta A + L_{w_1}^2 (\alpha A - T + L^2)]^{-1} \leq 0.$$

If these conditions are fulfilled,  $Z^1$  and  $Z^2$  are unique up to an additive constant, and  $Z^i$  depends only on  $L^i(w_1, w_2, y)$  and  $w_i$  ( $i = 1, 2$ ).

(iii) If  $AB_y - B_{w_2} = BA_y - A_{w_1} = 0$  ("special case"), there exist functions  $X^1, X^2, Y^1, Y^2$  such that

$$L_{w_1}^1 - L_y^1 B = X^1(L^1, w_1), \quad L_{w_2}^2 - L_y^2 A = X^2(L^2, w_2),$$

$$B(w_1, w_2, y) = Y^1(L^1, w_1), \quad A(w_1, w_2, y) = Y^2(L^2, w_2),$$

and it is necessary that the couple of partial differential equations:

(CREA f)

$$\varphi_w^i(L^i, w_i) = -\varphi_L^i(L^i, w) X_i(L^i, w_i) + Y_i(L^i, w_i), \quad i = 1, 2,$$

have solutions  $\varphi^1, \varphi^2$  satisfying

(CREA g)

$$\varphi_L^i(T - L^i - \varphi_w^i) \leq 0.$$

The complete proof is in Appendix 1. Basically, the result can be interpreted as follows. The Pareto efficiency requirement can be written:

$$(1) \quad (\bar{L}^1, \bar{L}^2, \bar{Z}^1, \bar{Z}^2) \in \text{Arg Max}_{L^1, L^2, Z^1, Z^2} \left| \begin{array}{l} U^1(L^1, Z^1) + k(w_1, w_2, y) U^2(L^2, Z^2) \\ Z^1 + Z^2 \leq y + w_1(T - L^1) + w_2(T - L^2) \end{array} \right.$$

(where  $L^i$  and  $Z^i$  stand respectively for  $L^i(w_1, w_2, y)$  and  $Z^i(w_1, w_2, y)$ ) for some strictly positive function  $k$ .

In this problem, the maximand possesses obviously a separability property, since it is a linear combination of two functions of, respectively,  $(L^1, Z^1)$  and  $(L^2, Z^2)$  alone. This separability property implies the restrictions of Proposition 1.<sup>8</sup> To be more precise, the first order conditions give:

$$\forall (w_1, w_2, y) \in S \times \mathbb{R}, \\ U_L^i[L^i(w_1, w_2, y), Z^i(w_1, w_2, y)] = w_i U_Z^i[L^i(w_1, w_2, y), Z^i(w_1, w_2, y)]$$

for  $i = 1, 2$ .

Thus  $Z^i$  can be expressed as a function on  $L^i$  and  $w_i$  alone. This fact and the budget constraint result in (CREA a). In the general case, it is also possible to derive (CREA b), (CREA c), and the uniqueness of  $Z^1$  (hence of  $Z^2$ ) up to an additive constant. Lastly, (CREA d) and (CREA e) are necessary for indifference curves of  $U^1$  and  $U^2$  to be convex.

In the *special case* ( $AB_R - B_{w_2} = BA_R - A_{w_1} = 0$ ), it is no longer possible to characterize  $Z^1$  and  $Z^2$  up to an additive constant. Here,  $\varphi^i = w_i(T - L^i) - Z^i$  is only characterized by (CREA f); (CREA g) writes the convexity of indifference curves. If, however,  $\varphi^1$ , for instance, satisfies (CREA f), then  $\varphi^2 = y - \varphi^1$  automatically does.

The conditions of Proposition 1 are not sufficient. In particular, nonnegativity restrictions ( $Z^i \geq 0$ ,  $i = 1, 2$ ) are ignored; they have to be tested directly upon the solutions of the equations which characterize  $Z^i$ . Also, it is generally possible to recover the utility functions from the labor supplies; an example is given in Appendix 4.

Lastly, the results above can also be formulated in terms of an income-sharing rule:

**COROLLARY:** *The conditions (CREA) of Proposition 1 are necessary for the existence of a sharing rule (in the sense of Definition 3). Moreover, in the general case (i.e., if  $L^1$  and  $L^2$  are such that  $AB_y - B_{w_2} \neq BA_y - A_{w_1}$ ), the income-sharing rule is defined up to an additive constant; that is, if  $G = (y_1, y_2)$  and  $G' = (y'_1, y'_1)$  are two such rules, there exists a constant  $k$  such that*

$$\forall (w_1, w_2, y) \quad \left| \begin{array}{l} y_1(w_1, w_2, y) = y'_1(w_1, w_2, y) + k \\ y_2(w_1, w_2, y) = y'_2(w_1, w_2, y) - k. \end{array} \right.$$

<sup>8</sup> Note, however, that (1) is quite different from the (well known) maximization of a separable utility. First the maximand is not a utility function in the usual sense, since it depends directly on  $w_1$ ,  $w_2$ , and  $y$  ("price dependent preferences"). Second,  $Z^1$  and  $Z^2$  are not observable; so we are looking only for conditions on  $L^1$  and  $L^2$ , the demands for leisure, which arise from the existence of  $Z^1$  and  $Z^2$ .

PROOF: Immediate from Proposition 1 since  $y_i \stackrel{\text{def}}{=} Z^i - w_i(T - L^i)$ .

### A.2. Comparison with Neoclassical Conditions

The Pareto efficiency hypothesis, in the case of egoistic agents, thus results in a set of partial differential equations upon  $L^1$  and  $L^2$ . These conditions are similar to, though probably more complex than, Slutsky relations, which characterize the neoclassical setting. An important point is that both sets of conditions (Slutsky on the one hand, CREA on the other hand) are totally independent. There exists neither any inclusion, nor any exclusion relationship between the classes of functions which satisfy Slutsky or CREA. Or, in other words, the fact that a given function satisfies one set of conditions does not tell anything about the other set.

This point is easily shown on the following examples. Consider, for instance, log-linear labor supply functions:

$$\ell^i = k_i w_1^{a_i} w_2^{b_i} y^{c_i} \quad (i = 1, 2)$$

where  $k_i$ ,  $a_i$ ,  $b_i$ ,  $c_i$  are reals.

It can easily be seen that two forms are compatible with the Slutsky equations:

$$(i) \begin{cases} \ell^1 = k_1 w_1^{a_1} y^{c_1} \\ \ell^2 = k_2 w_2^{b_2} y^{c_2} \end{cases} \quad \text{or} \quad (ii) \begin{cases} \ell^1 = k b w_1^{a-1} w_2^a y^c \\ \ell^2 = k a w_1^b w_2^{b-1} y^c \end{cases}$$

In the same way, conditions CREA imply  $a_2 b_1 = 0$ , i.e., any of the two following forms (the proof is left to the reader):

$$(iii) \begin{cases} \ell^1 = k_1 w_1^{a_1} y^{c_1} \\ \ell^2 = k_2 w_1^{a_2} w_2^{b_2} y^{c_2} \end{cases} \quad \text{or} \quad (iv) \begin{cases} \ell^1 = k_1 w_1^{a_1} w_2^{b_1} y^{c_1} \\ \ell^2 = k_2 w_2^{b_2} y^{c_2} \end{cases}$$

Also, concavity and positivity restrictions are fulfilled if  $c_i \leq 0$  ( $i = 1, 2$ ); it can even be shown that, in that case, conditions CREA are sufficient.

Now, choose any functions satisfying (ii) for  $a > 1$ ,  $b > 1$ . They are compatible with the neoclassical setting, but not with the collective setting with egoistic agents. Conversely, a solution of (iii) with  $a_2 \neq 0$  and  $a_1 \neq -1$  is not compatible with the neoclassical setting, though it is with the collective one. Lastly, a solution of (i) is compatible with both settings.

This example proves, first, the absence of any inclusion or exclusion relationship between both sets of solutions. Second, it shows that, depending on the functional form adopted, CREA may lead to conditions which are more realistic than Slutsky. In the log-linear case studied here, Slutsky would imply, either (i) that each member's labor supply does not depend on the other member's wage, or (ii) that nonlabor income elasticities of labor supply are the same for both members, and the elasticity of member  $i$ 's labor supply, with respect to member  $j$ 's wage, is simply member  $j$ 's own wage elasticity of labor supply plus one (a conclusion which is specially surprising when  $i$  is the husband and  $j$  is the wife).

Almost all empirically estimated elasticities would reject (i) as well as (ii). On the other hand, CREA holds true, for instance, if the husband's labor supply does not depend, in a significative way, on his wife's wage; this condition sounds much less unrealistic, and does not contradict too heavily the existing estimations.

TABLE I  
A COMPARISON BETWEEN NEOCLASSICAL AND COLLECTIVE CONDITIONS

Functional Form for Labor Supply (1) or Demand for Leisure ( $L$ )	"Neoclassical" (Slutsky) Conditions	"Collective" Conditions
<i>Log Linear</i> $\ell^i = k_i w_1^a w_2^b y^c$  or $L^i = T - k_i w_1^a w_2^b y^c$	$c_1 = c_2 \text{ and } \begin{cases} a_2 = b_1 = 0 \\ \text{or} \\ k_2 a_2 = k_1 b_1 \\ 1 + a_1 = a_2 \\ 1 + b_2 = b_1 \end{cases}$  Plus concavity restrictions	$a_2 = 0 \quad \text{or} \quad b_1 = 0$  Then the special case ( $AB_y - B_{w_2} = BA - A_{w_1} = 0$ ) is satisfied.  Concavity: $c_i \leq 0$ (sufficient)
<i>Semi Log</i> $\ell^i = a_i \text{ Log } w_1 + b_i \text{ Log } w_2 + c_i y$  or $L^i = T - \ell^i$	$\begin{cases} a_2 = b_1 = 0 \\ a_1 c_2 = 0 \\ b_2 c_1 = 0 \end{cases}$  Plus concavity restrictions	Special case always satisfied Plus concavity restrictions
<i>Linear Expenditures</i> Derived, in the neoclassical setting, from the utility: $U(L^1, L^2, C) = a \text{ Log } (C - \bar{C}) + b_1 \text{ Log } L^1 + b_2 \text{ Log } L^2$ Hence, $w_i L^i = b_i (w_1 + w_2) T - \bar{C} + y$	Always satisfied	Special case always satisfied

<p><i>Shares Linear in Logarithms</i></p> $\frac{w_i \ell^i}{y} = a_i + b_i \text{Log } y + c_i^1 \text{Log } w_1 + c_i^2 \text{Log } w_2$ <p>or</p> $\frac{w_i L_i}{y} = \frac{w_i T}{y} - \frac{w_i \ell^i}{y}$	$c_1^2 + a_2(a_1 + b_1) = c_2^1 + a_1(a_2 + b_2)$ $c_1^1(a_2 + b_2) + a_1 c_2^1 = c_1^2(a_1 + b_1) + a_2 c_1^1$ $c_2^2(a_1 + b_1) + a_2 c_2^2 = c_2^1(a_2 + b_2) + a_1 c_2^2$ $b_2 c_1^1 + b_1 c_2^1 = b_1 c_1^2 + b_2 c_2^2$ $b_2 c_2^1 + b_1 c_1^1 = b_1 c_2^2 + b_2 c_1^2$ <p>Plus concavity restrictions</p>	$c_2^1 = c_1^2 = 0$ <p>and the special case is satisfied</p> <p>or</p> $c_1^2 = c_2^1 = c_1^1 = c_2^2 = b_1 = b_2$ <p>and the general case is satisfied</p> <p>Plus concavity restrictions</p>
$L_i = a_i y + b_i y \text{Log } y + c_i \text{Log } w_1 + d_i \text{Log } w_2$	$\begin{cases} a_1 = a_2 \\ b_1 = b_2 \end{cases} \text{ and } c_1 = c_2 = d_1 = d_2 = 0$ <p>or</p> $a_1 = b_1 = a_2 = b_2 = d_1 = c_2 = 0$ <p>Plus concavity restrictions</p>	<p>General case always satisfied</p> <p>Concavity (sufficient conditions):</p> $a_1 b_2 - a_2 b_1 \geq 0, \quad b_2 \geq 0,$ $c_1 b_2 - c_2 b_1 \leq 0, \quad b_1 \leq 0,$ $d_2 b_1 - d_1 b_2 \geq 0$

A comparison for a few other functional forms is given in Table I. Note that some forms, which are more or less incompatible with the neoclassical setting, turn out to be very convenient for the collective model (e.g. semi-log and generalized semi-log).

An interesting interpretation of conditions (CREA) obtains when we consider labor supply functions which satisfy both (CREA) and Slutsky. First (CREA) imply the existence of a couple  $(Z^1, Z^2)$  such that (see Appendix 1):

$$(2) \quad Z_{w_2}^1 L_y^1 - Z_y^1 L_{w_2}^1 = Z_{w_1}^2 L_y^2 - Z_y^2 L_{w_1}^2 = 0.$$

For any  $(w_1, w_2, y)$  in  $S \times \mathbb{R}$  such that  $L_y^1 \cdot L_y^2 \neq 0$ , define

$$\lambda = (L_{w_2}^1 - (T - L^2)L_y^1) / L_y^1 L_y^2;$$

then

$$L_{w_2}^1 - (T - L^2)L_y^1 = \lambda L_y^1 L_y^2;$$

from Slutsky, it follows that

$$L_{w_1}^2 - (T - L^1)L_y^2 = \lambda L_y^1 L_y^2;$$

now, (2) gives

$$Z_{w_2}^1 - (T - L^2)Z_y^1 = \lambda Z_y^1 L_y^2, \quad \text{and}$$

$$Z_{w_1}^2 - (T - L^1)Z_y^2 = \lambda Z_y^2 L_y^1.$$

These equations are nothing else than Gorman's separability conditions. The similarity is not unexpected; to see this, consider the following program:

$$(P') \quad \begin{aligned} & \text{Max}_{L^1, Z^1, L^2, Z^2} W[U^1(L^1, Z^1), U^2(L^2, Z^2)] \\ & Z^1 + Z^2 = y + w_1(T - L^1) + w_2(T - L^2) \end{aligned}$$

where  $W$  is strictly increasing and quasi-concave.

First,  $(P')$  can be viewed as the maximization of a (utility) function of  $L^1, Z^1, L^2, Z^2$  under a budget constraint; hence, the solutions will satisfy Slutsky. Second, the separable form of the maximand can be interpreted in either of two ways: (i) From the neoclassical viewpoint, the household utility function is separable in  $L^1$  and  $Z^1$  on the one hand,  $L^2$  and  $Z^2$  on the other hand; thus Gorman's condition must be satisfied. (ii) From the collective viewpoint, the household is characterized by a *fixed welfare function*  $W$ .<sup>9</sup> This implies, in particular, that the solutions of  $(P)$  are Pareto optimal; hence they must verify (CREA).

Thus CREA can be viewed, in a quite informal way, as "generalized separability" conditions, which restriction to functions satisfying Slutsky coincide with Gorman's condition.

<sup>9</sup>  $W$  is fixed in the sense that it does *not* depend on  $w_1, w_2$ , and  $y$ , but only on  $U^1$  and  $U^2$ : the *same* welfare function is maximized for all  $(w_1, w_2, y)$ .

## B. The Nonparametric Approach

### B.1. The Main Result

We now turn to the conditions that must be fulfilled by a *finite* set of data,

$$D = \{(L_j^1, L_j^2, C_j; w_1^j, w_2^j), j = 1, \dots, T\},$$

to be compatible with the collective setting when agents are egoistic. In what follows, we shall assume that  $i \neq j \Rightarrow (L_i^1, L_i^2, C_i) \neq (L_j^1, L_j^2, C_j)$ ; this technical condition is necessary for the differentiability of the utility functions.

**DEFINITION 4:** A pair of egoistic utility functions  $(U^1, U^2)$  provides a *collective rationalization with egoistic agents (CREA)* of the data  $D$  if there exists  $T$  pairs of positive reals  $(Z_j, \theta_j)$ ,  $j = 1, \dots, T$ , such that:

$$Z_j \leq C_j,$$

and

$$U^1(L_j^1, Z_j) + \theta_j U^2(L_j^2, C_j - Z_j) > U^1(L^1, Z^1) + \theta_j U^2(L^2, Z^2)$$

for all  $(L^1, Z^1, L^2, Z^2) \neq (L_j^1, Z_j, L_j^2, C_j - Z_j)$ , such that

$$Z^1 + Z^2 + w_1^j L^1 + w_2^j L^2 \leq C_j + w_1^j L_j^1 + w_2^j L_j^2.$$

This definition is no more than a transposition of the usual revealed preferences conditions: among all the bundles which are financially available when wages are  $w_1^j$  and  $w_2^j$  and nonlabor income is  $y^j$ , the observed one is the “best.” The two main differences are, first, that “best” is intended here in a collective, Pareto optimality sense; hence, for each  $j$ , there must exist a  $\theta_j$  such that  $U^1 + \theta_j U^2$  is maximized. Second, private consumptions are assumed not observable; only their sum is given by the budget constraint. Thus it is sufficient that the data are CR for some  $(Z_j^1, Z_j^2)$  satisfying this constraint.

**PROPOSITION 2:** *There exists a pair of strongly concave, strictly monotonic, infinitely differentiable utility functions which provide a CREA of the data if and only if there exists  $T$  nonnegative reals  $(Z_j, j = 1, \dots, T)$ , with  $Z_j \leq C_j$  for each  $j$ , such that one of the following equivalent conditions is fulfilled:*

(CREA'a) *The data  $\{(L_j^1, Z_j; w_1^j), j = 1, \dots, T\}$  on the one hand, and  $\{(L_j^2, C_j - Z_j; w_2^j), j = 1, \dots, T\}$  on the other hand, both satisfy the Strong Axiom of Revealed Preferences (SARP).*

(CREA'b) *There exists numbers  $U_j^1, U_j^2$  and  $\lambda_j, \mu_j > 0, j = 1, \dots, T$ , such that for each  $i, j \in \{1, \dots, T\}, i \neq j$ :*

$$U_i^1 - U_j^1 \leq \lambda_j w_1^j (L_i^1 - L_j^1) + \lambda_j (Z_i - Z_j),$$

$$U_i^2 - U_j^2 \leq \mu_j w_2^j (L_i^2 - L_j^2) + \mu_j (C_i - Z_i - C_j + Z_j),$$

*the equality holding in the first (resp. the second) inequality only if  $L_i^1 = L_j^1$  and  $Z_i = Z_j$  (resp.  $L_i^2 = L_j^2$  and  $C_i - Z_i = C_j - Z_j$ ).*

PROOF: See Appendix 2.

Again, these results can be understood as stemming from a separability property of the problem. In particular, the income-sharing rule interpretation is illuminating. If  $y_1$  is member 1's share, the latter will then solve

$$\begin{cases} \text{Max } U^1(L^1, Z) \\ w_1 L^1 + Z = y_1 + w_1 T. \end{cases}$$

This explains why the data  $(L^1, Z_1)$  must satisfy SARP. The same holds true for member 2.<sup>10</sup>

### B.2. Comparison with Neoclassical Conditions

Again, conditions CREA' are independent of neoclassical restrictions. Indeed, the neoclassical approach would require the data  $D$  to satisfy SARP; we now proceed to show, with the help of a few examples, that there is neither any inclusion, nor any exclusion relation between the set of data satisfying SARP and the set of data satisfying CREA'.

We use the following notations:

$$\begin{aligned} P_i &= (L_i^1, L_i^2, Z_i, C_i - Z_i; w_1^i, w_2^i), \\ P_i \Re P_j &\Leftrightarrow w_1^i(L_j^1 - L_i^1) + w_2^i(L_j^2 - L_i^2) + C_j - C_i \leq 0, \\ P_i \Re_1 P_j &\Leftrightarrow w_1^i(L_j^1 - L_i^1) + Z_j - Z_i \leq 0, \\ P_i \Re_2 P_j &\Leftrightarrow w_2^i(L_j^2 - L_i^2) + C_j - Z_j - C_i + Z_i \leq 0. \end{aligned}$$

Here,  $\Re$  denotes the usual direct revealed preferences relation; and  $\Re_1$  (resp.  $\Re_2$ ) the same, restricted to the  $(L_i^1, Z_i)$  (resp. to the  $(L_i^2, C_i - Z_i)$ ).

EXAMPLE 1:

$$\begin{cases} P_1 = (5, 5, 80, 20; 1, 1), \\ P_2 = (10, 10, 20, 30; 10, 10). \end{cases}$$

Then  $P_1 \Re P_2$ , since  $w_1^1(L_2^1 - L_1^1) + w_2^1(L_2^2 - L_1^2) + C_2 - C_1 = -40 < 0$ , and  $P_2 \Re P_1$ , since  $w_1^2(L_1^1 - L_2^1) + w_2^2(L_1^2 - L_2^2) + C_1 - C_2 = -50 < 0$ . Hence this couple violates SARP; but

$$\begin{aligned} w_1^2(L_1^1 - L_2^1) + Z_1 - Z_2 &= 10 > 0, \quad \text{hence } P_2 \not\Re P_1, \quad \text{and} \\ w_2^1(L_2^2 - L_1^2) + C_2 - C_1 - Z_2 + Z_1 &= 15 > 0, \quad \text{hence } P_1 \not\Re_2 P_2. \end{aligned}$$

Thus  $(P_1, P_2)$  satisfy (CREA').

This example shows that the set of functions satisfying CREA' is not included in the set of functions satisfying SARP. The converse is less obvious, but nevertheless true, as can be seen in Example 2.

<sup>10</sup> This argument can be used directly to prove that the (CREA') is necessary. The interest of (CREA'b) is to allow an explicit construction of the utility functions.

EXAMPLE 2:

$$\begin{cases} P_1 = (60, 20, Z_1, 10 - Z_1; 2, 32), \\ P_2 = (10, 10, Z_2, 90 - Z_2; 1, 3.4), \\ P_3 = (50, 25, Z_3, 5 - Z_3; 10, 5). \end{cases}$$

This set of data satisfies SARP; indeed,

$$P_2 \nprec P_1, \quad \text{since} \quad w_1^2(L_1^1 - L_2^1) + w_2^2(L_1^2 - L_2^2) + C_1 - C_2 = 4 > 0,$$

$$P_1 \nprec P_3, \quad \text{since} \quad w_1^1(L_3^1 - L_1^1) + w_2^1(L_3^2 - L_1^2) + C_3 - C_1 = 135 > 0,$$

$$P_2 \nprec P_3, \quad \text{since} \quad w_1^2(L_3^1 - L_2^1) + w_2^2(L_3^2 - L_2^2) + C_3 - C_2 = 6 > 0,$$

which is sufficient to prove SARP.

But, for any  $(Z_1, Z_2, Z_3)$  such that  $0 \leq Z_1 \leq 10$ ,  $0 \leq Z_2 \leq 90$ ,  $0 \leq Z_3 \leq 5$ , CREA' is violated; indeed we have the following:

If  $Z_2 < 46$ , then

$$P_1 \nprec_2 P_2, \quad \text{since} \quad w_2^1(L_2^2 - L_1^2) + C_2 - C_1 - Z_2 + Z_1 = -250 - Z_2 + Z_1 < 0,$$

$$\text{since } Z_1 \leq 10, \quad \text{and}$$

$$P_2 \nprec_2 P_1, \quad \text{since} \quad w_2^2(L_1^2 - L_2^2) + C_1 - C_2 - Z_1 + Z_2 = -46 - Z_1 + Z_2 < 0,$$

and CREA' is violated.

If  $Z_2 \geq 46$ , then

$$P_3 \nprec_1 P_2, \quad \text{since} \quad w_1^3(L_2^1 - L_3^1) + Z_2 - Z_3 = -400 + Z_2 - Z_3 < 0, \quad \text{and}$$

$$P_2 \nprec_1 P_3, \quad \text{since} \quad w_1^2(L_3^1 - L_2^1) + Z_3 - Z_2 = 40 + Z_3 - Z_2 < 0,$$

$$\text{since } Z_3 \leq 5,$$

so that CREA' is also violated.

EXAMPLE 3: Lastly, SARP and CREA' are not incompatible. For instance, the data:

$$\begin{cases} P = (20, 20, 20, 20; 1, 1), \\ P = (10, 10, 10, 10; 2, 2), \end{cases}$$

obviously satisfy both.

What can we say about data satisfying both neoclassical (SARP) and collective (CREA') conditions? In particular, do they stem from the maximization of a separable utility function? More generally, consider the following question (which is a straightforward generalization).

QUESTION: Consider data  $D = \{(x_j^1, \dots, x_j^k, y_j^1, \dots, y_j^n; p_j^1, \dots, p_j^k, q_j^1, \dots, q_j^n), j = 1, \dots, T\}$  such that:

- (i)  $D$  satisfy SARP.
- (ii) The data  $\{(x_j^1, \dots, x_j^k; p_j^1, \dots, p_j^k), j = 1, \dots, T\}$  satisfy SARP.
- (iii) The data  $\{(y_j^1, \dots, y_j^n; q_j^1, \dots, q_j^n), j = 1, \dots, T\}$  satisfy SARP.

Can the data in  $D$  be derived from the maximization of a separable utility function?

It is well known that (i), (ii), and (iii) are necessary for the existence of a separable utility. Also, we do know sufficient conditions which are stronger than these (see Varian (1983)). But it is not clear (at least to me) whether (i), (ii), and (iii) are sufficient, though my conjecture is that they are *not*.

#### 4. THE GENERAL CASE

We now release the assumption of egoistic agents, and investigate the properties of collectively rational (CR) behavior, in the sense of Definition 2. The class of CR labor supplies will clearly be very broad. It will include CREA labor supplies, as characterized in Section 3, as well as neoclassical labor supplies (take an household with two identical agents; then collective rationality amounts to the maximization of a single utility function). The question is whether it will include any labor supply at all; i.e., is collective rationality, in the more general sense, falsifiable from empirical data? We now proceed to show that the answer is positive. Indeed, it is possible to derive a set of necessary and sufficient conditions which characterize CR labor supplies, *at least from a nonparametric viewpoint*; also, it can be shown from a simple counterexample that there exist data which do *not* satisfy those conditions.

##### A. The Main Result

Assume, again, that  $(L_i^1, L_i^2, C_i) \neq (L_j^1, L_j^2, C_j)$  unless  $i = j$ .

DEFINITION 5: A pair of utility functions  $U^1, U^2$  provide a *collective rationalization (CR) of the data*

$$D = \{(L_j^1, L_j^2, C_j; w_1^j, w_2^j), j = 1, \dots, T\}$$

if there exist  $T$  couples of positive numbers  $(Z_j, \theta_j), j = 1, \dots, T$ , such that:

- (i)  $Z_j \leq C_j$ ,
- (ii)  $U^1(L_j^1, L_j^2, Z_j, C_j - Z_j) + \theta_j U^2(L_j^1, L_j^2, Z_j, C_j - Z_j) > U^1(L^1, L^2, Z^1, Z^2) + \theta_j U^2(L^1, L^2, Z^1, Z^2)$ ,

for all  $(L^1, L^2, Z^1, Z^2) \neq (L_j^1, L_j^2, Z_j^1, Z_j^2)$  such that

$$Z^1 + Z^2 + w_1^j L^1 + w_2^j L^2 \leq C_j + w_1^j L_j^1 + w_2^j L_j^2.$$

Note that Definition 5 only extends Definition 4 to nonegoistic utility functions. We have the following result:

**PROPOSITION 3:** *There exists a pair of strongly concave, strictly monotonic, infinitely differentiable utility functions which provide a CR of the data if and only if there exist nonnegative numbers  $(Z_j, \alpha_1^j, \alpha_2^j, \beta_1^j, \beta_2^j)$ ,  $j = 1, \dots, T$ , with  $Z_j \leq C_j$ ,  $\alpha_1^j < w_1^j$ ,  $\alpha_2^j < w_2^j$ ,  $\beta_1^j < 1$ ,  $\beta_2^j < 1$ , such that one of the following equivalent conditions is fulfilled:*

(CR'a) *The data  $(L_j^1, L_j^2, Z_j, C_j - Z_j; \alpha_1^j, \alpha_2^j, \beta_1^j, \beta_2^j)$  on the one hand, and  $(L_j^1, L_j^2, Z_j, C_j - Z_j; w_1^j - \alpha_1^j, w_2^j - \alpha_2^j, 1 - \beta_1^j, 1 - \beta_2^j)$  on the other hand, both satisfy SARP.*

(CR'b) *There exist numbers  $U_j^1, U_j^2$ , and  $\lambda^j, \mu^j > 0$ , such that for each  $i, j$  in  $\{1, \dots, T\}$ ,  $i \neq j$ :*

$$\begin{cases} U_1^1 - U_j^1 < \lambda^j \alpha_1^j (L_1^1 - L_j^1) + \lambda^j \alpha_2^j (L_2^1 - L_j^2) + \lambda^j \beta_1^j (z_i - z_j) \\ \quad + \lambda^j \beta_2^j (C_i - Z_i - C_j + Z_j), \\ U_i^2 - U_j^2 < \mu^j (w_1^j - \alpha_1^j) (L_i^1 - L_j^1) + \mu^j (w_2^j - \alpha_2^j) (L_i^2 - L_j^2) \\ \quad + \mu^j (1 - \beta_1^j) (Z_i - Z_j) + \mu^j (1 - \beta_2^j) (C_i - Z_i - C_j + Z_j). \end{cases}$$

**PROOF:** See Appendix 3.

A natural interpretation of Proposition 3 is in terms of public goods. Here, both members' leisure and consumption are public goods for the household; there must exist a set of personal prices for each member, which add up to market prices. This shows that (CR'a) is necessary; the proof of the converse uses the traditional nonparametric arguments. We shall come back to this interpretation later.

Lastly, note that for  $\alpha_1^j = w_1^j$ ,  $\alpha_2^j = 0$ ,  $\beta_1^j = 1$ ,  $\beta_2^j = 0$ , (CR') collapses to (CREA'); and that for  $\alpha_1^j = w_1^j/2$ ,  $\alpha_2^j = w_2^j/2$ ,  $\beta_1^j = \beta_2^j = 1/2$ , (CR') collapses to Afriat conditions. This allows verification that both neoclassical and CREA are special cases of CR.

## B. A Counterexample

Consider the following set of three observations:

$$P_1 = (L_1^1, L_1^2, C_1; w_1^1, w_2^1) = (10, 1, 1; 4, .3),$$

$$P_2 = (L_2^1, L_2^2, C_2; w_1^2, w_2^2) = (1, 10, 1; .3, 4),$$

$$P_3 = (L_3^1, L_3^2, C_3; w_1^3, w_2^3) = (1, 1, 10; .3, .3).$$

We now show that, for any set  $E = \{(\alpha_1^j, \alpha_2^j, \beta_1^j, \beta_2^j, Z_j), j = 1, 3\}$  such that  $Z_j \leq C_j$ ,  $\alpha_1^j \leq w_1^j$ ,  $\alpha_2^j \leq w_2^j$ ,  $\beta_1^j \leq 1$ ,  $\beta_2^j \leq 1$ , either the data  $Q = (Q_1, Q_2, Q_3)$  or the data  $S = (S_1, S_2, S_3)$  (where  $Q_j = (L_j^1, L_j^2, Z_j, C_j - Z_j; \alpha_1^j, \alpha_2^j, \beta_1^j, \beta_2^j)$  and  $S_j = (L_j^1, L_j^2, Z_j, C_j - Z_j; w_1^j - \alpha_1^j, w_2^j - \alpha_2^j, 1 - \beta_1^j, 1 - \beta_2^j)$ ) violate SARP. In fact, either  $Q$  or  $S$  must even violate the Weak Axiom of Revealed Preference, WARP.

Note  $\mathfrak{R}$  for “is directly preferred to”,  $\mathfrak{N}$  for “is not directly preferred to”, and suppose there exists some  $E$  such that the  $Q_j$  and the  $S_j$  satisfy WARP. Then contradiction is reached in four steps:

STEP 1: From WARP, we have either  $S_1 \mathfrak{N} S_2$  or  $S_2 \mathfrak{N} S_1$ . Note that the initial set  $\{P_1, P_2, P_3\}$  is not affected if goods 1 and 2 are permuted. Since this permutation permutes also  $S_1$  and  $S_2$ , we can, without loss of generality, assume that  $S_1 \mathfrak{N} S_2$ :

$$(4 - \alpha_1^1)(-9) + (.3 - \alpha_2^1)9 + (1 - \beta_1^1)(Z_2 - Z_1) - (1 - \beta_2^1)(Z_2 - Z_1) \geq 0.$$

Since  $Z_1 \leq 1$ ,  $Z_2 \leq 1$ ,  $\beta_1^1 \leq 1$ ,  $\beta_2^1 \leq 1$ , this gives

$$\alpha_1^1 \geq 4 - .3 - \frac{1}{9} \geq 3.58.$$

STEP 2: Then  $Q_1 \mathfrak{N} Q_2$ , for

$$\alpha_1^1(-9) + 9\alpha_2^1 + \beta_1^1(Z_2 - Z_1) - \beta_2^1(Z_2 - Z_1) \leq -32.22 + 2.7 + 1 < 0.$$

Hence  $Q_2 \mathfrak{N} Q_1$ :

$$9\alpha_1^2 + \alpha_2^2(-9) + \beta_1^2(Z_1 - Z_2) - \beta_2^2(Z_1 - Z_2) \geq 0$$

which gives

$$\alpha_2^2 \leq \alpha_1^2 - \frac{1}{9}(\beta_2^1 - \beta_2^2)(Z_2 - Z_1) \leq .42 \quad \text{by} \quad \alpha_1^2 \leq .3$$

$$\text{and} \quad \beta_2^1, \beta_2^2, Z_1, Z_2 \leq 1.$$

STEP 3: This, in turn, implies  $S_2 \mathfrak{N} S_3$ ; indeed:

$$\begin{aligned} & (4 - \alpha_2^2)(-9) + (1 - \beta_1^2)(Z_3 - Z_2) + (1 - \beta_2^2)(9 - Z_3 + Z_2) \\ & \leq -36 + 3.78 + 10 + 9 < -14 < 0. \end{aligned}$$

Thus  $S_3 \mathfrak{N} S_2$ :

$$(.3 - \alpha_2^3)9 + (1 - \beta_1^3)(Z_2 - Z_3) + (1 - \beta_2^3)(-9 - Z_2 + Z_3) \geq 0$$

or

$$(\beta_2^3 - \beta_1^3)(Z_2 - Z_3) + 9\beta_2^3 \geq 9(1 - .3) = 6.3$$

or

$$-(\beta_2^3 - \beta_1^3)Z_3 + 9\beta_2^3 \geq 5.3.$$

STEP 4: Now  $Q_3 \mathfrak{N} Q_1$ , for

$$9\alpha_1^3 + \beta_1^3(Z_1 - Z_3) + \beta_2^3(-9 - Z_1 + Z_3) \leq -5.3 + 9 \times .3 + 1 = -1.6 < 0$$

and  $Q_1 \Re Q_3$ , for (remember  $\alpha_1^1 \geq 3.58$  from Step 1)

$$\alpha_1^1(-9) + \beta_1^1(Z_3 - Z_1) + \beta_2^1(9 - Z_3 + Z_1) \leq -32 + 10 + 9 < 0.$$

Thus WARP is violated by  $(Q_1, Q_2, Q_3)$ , a contradiction.

### C. Interpretation

A few remarks may help in understanding the results above. First, that the efficiency hypothesis by itself generates falsifiable restrictions upon household behavior is not really surprising. Basically, restrictions appear because the number of goods is greater than the number of agents in the model. This fact is known to generate restrictions in a different context, namely the characterization of aggregate demand for private goods (see, for instance, Shafer-Sonnenschein, 1982). It turns out, however, that the latter problem is closely related to the one at stake here.

The best way to understand the links between them is the following. In our model, we analyze the demand of a two agents economy for three *public* goods (since each good enters *both* utility functions), when the decisions taken are Pareto optimal. The public goods interpretation suggests consideration of the set of personal prices (marginal willingness to pay) which correspond to any Pareto-efficient allocation. The only condition upon these personal prices is that, for each good, *they must add up to the market price*. Now, by using the traditional duality transform, we may interpret the personal prices (resp. market prices, quantities) as individual consumptions (resp. aggregate consumption, prices) of three private goods. The adding-up constraint, in this context, only states that aggregate consumption must be the sum of individual consumptions for each good. That is to say, the initial problem has been transformed into the characterization of aggregate demand in a three-private-good, two-agent economy.<sup>11</sup>

It must however be stressed that the two problems are not totally equivalent. Within the literature on aggregate demand for private goods, it is generally assumed that the distribution of income is fixed. But such an assumption is clearly irrelevant in our context. A consequence is that the restrictions upon aggregate demand which have been established earlier (Sonnenschein (1973); Diewert (1977); Mantel (1977)) cannot be transposed to the present problem.<sup>12</sup> They are not sufficient, since demand functions which satisfy them only “behave locally like” (i.e., have the same value and first-order derivative as) the aggregate demands of some economy; also, they ignore nonnegativity restrictions. On the other hand, these conditions are not necessary in our model, since we do not assume a fixed distribution of income. It is thus of interest to note that Proposition 3 can be viewed as providing a set of *necessary and sufficient* conditions for this kind of problem, under the general assumptions of variable income distribution. This

<sup>11</sup> The duality transform between private and public goods is well known since Samuelson (1956). A general presentation of the properties of this transform is in Milleron (1972). For a detailed application to the aggregate demand problem, see Chiappori (1986).

<sup>12</sup> Cf. the survey by Shafer and Sonnenschein (1982).

might be of some interest, since no set of necessary and sufficient conditions are known so far in the aggregate demand for private goods problems.

What can we say about parametric restrictions for the CR case? Almost nothing but this: such conditions (if any) would probably appear as partial differential equations. However, this would mean, informally, that the set of labor supply functions which satisfy these conditions is of measure zero—i.e., that “almost all” labor supplies would fail to be CR. Whether such a conclusion could be expected is not clear; in any case, note that it is *not* necessarily implied in the nonparametric case.

In conclusion, two points can be mentioned. First, an interesting test would be to compare empirically Slutsky and CREA restrictions from data upon labor supply of a set of two-member households. The two members requirement, however, prohibits the use of aggregate data; only individual data could be used for this purpose. Second, nonparametric tests could be done upon data of this kind; they would allow us to check: (i) whether the “collective” rationality requirements (in the broadest sense) are fulfilled; (ii) whether the latter collapse, either to the usual neoclassical conditions, or to conditions (CREA') (i.e., CR with egoistic agents).

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## APPENDIX I

### PROOF OF PROPOSITION 1

1. Pareto optimality requires  $(L^1, L^2, Z_1, Z_2)$  to be a solution of the following problem:

$$\begin{aligned} \text{Max } & U^1(L^1, Z_1) + \lambda(w_1, w_2, y)U^2(L^2, Z_2) \\ & Z_1 + Z_2 \leq y + w_1(T_1 - L^1) + w_2(T_2 - L^2), \end{aligned}$$

for some *positive* mapping  $\lambda$  (note that  $\lambda$  must be taken as a *function* of  $(w_1, w_2, y)$ ). The first order conditions give

$$\begin{aligned} \text{(a)} \quad & U_L^1(L^1, Z_1) = w_1 U_C^1(L^1, Z_1), \\ \text{(b)} \quad & U_L^2(L^2, Z_2) = w_2 U_C^2(L^2, Z_2), \\ \text{(c)} \quad & Z_1 + Z_2 = y + w_1(T_1 - L^1) + w_2(T_2 - L^2). \end{aligned}$$

The equality in (c) is required because  $U^1$  and  $U^2$  are assumed strictly increasing. Also, note that  $\lambda$  has been eliminated from these equations.

Consider any two functions  $L^1$  and  $L^2$  from  $S \times \mathbb{R}$  to  $[0, T_1]$  and  $[0, T_2]$ . For these functions to define a CR demand for leisure, it is necessary that there exist two functions  $Z_1$  and  $Z_2$ , from  $S \times \mathbb{R}$  to  $\mathbb{R}^+$  such that (a), (b), (c) are satisfied.

Let  $(w_1, w_2, y)$  be a point in  $S \times \mathbb{R}$  such that  $L_y^i(w_1, w_2, y) \neq 0$  ( $i = 1, 2$ ). Relation (a) can be written:

$$(A1) \quad (U_{LL}^1 - w_1 U_{LC}^1)_{(L^1, Z_1)} \cdot \overline{\text{grad}} L^1 + (U_{LC}^1 - w_1 U_{CC}^1)_{(L^1, Z_1)} \cdot \overline{\text{grad}} Z_1 - U_C^1(L^1, Z_1) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0.$$

We now use the following lemma:

**LEMMA:** Let  $\varphi, X, Y$  be any three  $C^\infty$  functions from some open, non-empty subset  $B$  of  $\mathbb{R}^n$  to  $\mathbb{R}$ , such that  $\overline{\text{grad } X}$  and  $\overline{\text{grad } Y}$  are noncollinear. For a function  $\theta$ , from  $\mathbb{R}^2$  to  $\mathbb{R}$ , such that

$$\forall (x_1, \dots, x_n) \in B, \quad \varphi(x_1, \dots, x_n) = \theta[X(x_1, \dots, x_n), Y(x_1, \dots, x_n)]$$

to exist in a neighborhood of any point of  $B$ , it is necessary and sufficient that the vectors  $\overline{\text{grad } \varphi}, \overline{\text{grad } X}, \overline{\text{grad } Y}$  are always colinear.

**PROOF OF THE LEMMA:** The condition is obviously necessary. To prove sufficiency, note that, since  $\overline{\text{grad } X}$  and  $\overline{\text{grad } Y}$  are noncollinear, for each point in  $B$ , there exists, in the  $(n \times 2)$  matrix  $M = (\text{grad } X, \text{grad } Y)$  a  $2 \times 2$  determinant different from 0.

Let  $x^0 = (x_1^0, \dots, x_n^0)$  be some point in  $B$ , and assume for instance that the upper determinant of matrix  $M$  is different from zero. There exists an open neighborhood  $U$  of  $x^0$  upon which this determinant is different from zero. Now, consider the mapping  $\tau: U \rightarrow \mathbb{R}^n$

$$(x_1, \dots, x_n) \rightarrow (X(x_1, \dots, x_n), Y(x_1, \dots, x_n), x_3, \dots, x_n).$$

This mapping is invertible at every point of  $U$ ; thus there exists, in some neighborhood  $V$  of  $x^0$ , a mapping  $\tau^{-1}$  such that:

$$\tau^{-1}(X(x_1, \dots, x_n), Y(x_1, \dots, x_n), x_3, \dots, x_n) = (x_1, \dots, x_n).$$

In particular,  $\varphi$  can be expressed as a function of  $X, Y, x_3, \dots, x_n$ :

$$\begin{aligned} \varphi(x_1, \dots, x_n) &= \varphi_0 \tau^{-1}[X(x_1, \dots, x_n), Y(x_1, \dots, x_n), x_3, \dots, x_n] \\ &= \theta[X(x_1, \dots, x_n), Y(x_1, \dots, x_n), x_3, \dots, x_n]. \end{aligned}$$

Now, it is easy to check that if  $\overline{\text{grad } \varphi}, \overline{\text{grad } X}, \overline{\text{grad } Y}$  are colinear, then:

$$\frac{\partial \theta}{\partial x_3} = \dots = \frac{\partial \theta}{\partial x_n} = 0 \quad \text{at each } x \text{ of } V.$$

The lemma directly applies here, with  $n=3$  and  $X = L^1(w_1, w_2, y)$ ,  $Y = w_1$ , since  $L_y^1 \neq 0$ . Thus, locally,  $Z_1$  can be written as a function of  $L^1$  and  $w_1$ ; in the same way,  $Z_2$  is a function of  $L^2$  and  $w_2$ .

From (c), one gets:

$$(A2) \quad Z_2 - w_2(T_2 - L^2) = y + w_1(T_1 - L^1) - Z_1.$$

Define  $\varphi(L^1, w_1) = w_1(T_1 - L^1) - Z_1$ . From (A2),  $y + \varphi(L^1, w_1)$  only depends on  $L^2$  and  $w_2$ . Applying the lemma, one gets:

$$\begin{vmatrix} \varphi_L \cdot L_{w_1}^1 + \varphi_w & L_{w_1}^2 & 0 \\ \varphi_L \cdot L_{w_2}^1 & L_{w_2}^2 & 1 \\ 1 + \varphi_L \cdot L_y^1 & L_y^2 & 0 \end{vmatrix} = 0, \quad \text{hence} \\ \varphi_L(L_{w_1}^1 L_y^2 - L_y^2 L_{w_1}^1) + \varphi_w L_y^2 = L_{w_1}^2.$$

Pose  $B = L_{w_1}^2 / L_y^2$ , then

$$(A3) \quad \varphi_w = -\varphi_L(L_{w_1}^1 - L_y^1 \cdot B) + B.$$

Thus  $B + \varphi_L(L_y^1 \cdot B - L_{w_1}^1)$  depends only on  $L^1$  and  $w_1$ ; again:

$$0 = \begin{vmatrix} B_{w_2} - \varphi_{LL} \cdot L_{w_2}^1 (L_{w_1}^1 - L_y^1 B) + \varphi_L(L_y^1 B_{w_2} + L_y^1 B_{w_2} - L_{w_1}^1 w_2) & L_{w_2}^1 \\ B_y - \varphi_{LL} \cdot L_y^1 (L_{w_1}^1 - L_y^1 B) + \varphi_L(L_y^1 B + L_y^1 B_y - L_{w_1}^1 y) & L_y^1 \end{vmatrix},$$

hence, if  $A = L_{w_2}^1 / L_y^1$ :

$$(A4) \quad B_{w_2} - AB_y = \varphi_L \cdot L_y [A_{w_1} - BA_y - (B_{w_2} - AB_y)].$$

If  $A_{w_1} - BA_y = B_{w_2} - AB_y$ , this relation implies that both members are equal to zero. In that case, the right-hand side of (A4) is a function of  $L^1$  and  $w_1$ , and we get a partial differential equation which characterizes a class of possible functions  $\varphi$ .

On the other hand, if  $A_{w_1} - BA_y \neq B_{w_2} - AB_y$ , pose

$$\alpha = \left(1 - \frac{A_{w_1} - BA_y}{B_{w_2} - AB_y}\right)^{-1} \quad \text{if } B_{w_2} - AB_y \neq 0,$$

$$\alpha = 0 \quad \text{otherwise.}$$

Then (A4) becomes

$$\varphi_L = -\frac{\alpha}{L_y^1}.$$

The right-hand side depends only on  $L^1$  and  $w_1$ , which gives, again from the Lemma:

$$(A5) \quad \underline{\alpha A_y + \alpha_y A = \alpha_{w_2}}.$$

If this relation is satisfied, then, from (A3):

$$(a) \quad \begin{cases} \varphi_L = -\frac{\alpha}{L_y^1}, \\ \varphi_w = \alpha \left( \frac{L_{w_1}^1}{L_y^1} - B \right) + B. \end{cases}$$

For these two relations to be compatible, it is necessary that  $\varphi_{Lw} = \varphi_{wL}$ . To compute this, consider the mapping

$$\begin{aligned} \pi: A &\rightarrow R^2[0, T_1], \\ (w_1, w_2, y) &\rightarrow (w_1, w_2, L^1). \end{aligned}$$

$\pi$  is locally invertible, since  $L_y^1 \neq 0$ ; the Jacobian matrix of  $\pi^{-1}$  is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_{w_1}^1/L_y^1 & -L_{w_2}^1/L_y^1 & 1/L_y^1 \end{pmatrix}.$$

Then we get from (a)

$$\varphi_{Lw} = -\frac{\partial}{\partial w_1} \left( \frac{\alpha}{L_y^1} \right) - \frac{\partial}{\partial y} \left( \frac{\alpha}{L_y^1} \right) \cdot \left( -\frac{L_w}{L_y^1} \right)$$

and from (b)

$$\varphi_{wL} = \frac{\partial}{\partial y} \left[ \alpha \left( \frac{L_{w_1}^1}{L_y^1} - B \right) + B \right] \cdot \frac{1}{L_y^1}$$

which gives

$$(A6) \quad \begin{aligned} \alpha B_y - \alpha_{w_1} + \alpha_y B - B_y &= 0, \quad \text{or} \\ \beta B_y + \beta_y B &= \beta_{w_1}, \quad \text{where } \beta = 1 - \alpha. \end{aligned}$$

Note that (A6) can be deduced from (A5) by permuting 1 and 2, hence  $A$  and  $B$  and  $\alpha$  and  $\beta$ : the conditions imposed upon  $L^1$  and  $L^2$  are symmetrical. Also, (a) and (b), once the compatibility condition is fulfilled, define  $\varphi$ , hence  $Z_1$  and  $Z_2$ , up to an additive constant.

2. We now analyze the conditions upon  $L^1$  and  $L^2$  for the existence of two *quasi-concave* solutions to the problem.

First, note that  $\overline{\text{grad } L^1}$  and  $\overline{\text{grad } Z_1}$  are not colinear; for, in that case, (A1) would imply that

$$\overline{\text{grad } L^1} \text{ is colinear with } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ which is impossible since } L_R^1 \neq 0.$$

But the determinant

$$\begin{vmatrix} L_{w_2}^1 & Z_{w_2}^1 \\ L_y^1 & Z_y^1 \end{vmatrix}$$

is zero, again by (A1). Thus the determinant

$$\begin{vmatrix} L_{w_1}^1 & Z_{w_1}^1 \\ L_{w_2}^1 & Z_{w_2}^1 \end{vmatrix}$$

must be different from zero.

Consider now the mapping

$$\theta: \begin{pmatrix} w_1 \\ w_2 \\ y \end{pmatrix} \rightarrow \begin{pmatrix} L \\ Z \\ y \end{pmatrix}.$$

Its Jacobian matrix is always of full rank; thus  $\theta$  is locally invertible. The first line of its Jacobian matrix is, by straightforward calculations:

$$(L_{w_1}^1 Z_{w_2}^1 - L_{w_2}^1 Z_{w_1}^1)^{-1} (Z_{w_2}^1, -L_{w_2}^1, L_{w_2}^1 Z_y^1 - L_y^1 Z_{w_2}^1).$$

Condition (a) can be written:

$$\Phi^1(L^1, Z_1) = P_1 \circ \theta^{-1}(L^1, Z_1),$$

where

$$\Phi^1 = \frac{U_L^1}{U_C^1}$$

and  $P_1$  is the projection on the first component:

$$P_1(x_1, x_2, x_3) = x_1.$$

$\Phi^1$ , being minus the MRS between consumption and leisure, must satisfy, for  $U^1$  to be quasi-concave:

$$\begin{cases} \Phi^1(L^1, Z_1) > 0, \\ \Phi_L^1(L^1, Z_1) - \Phi^1 \Phi_C^1(L^1, Z_1) < 0. \end{cases}$$

Using  $\theta^{-1}$ , and recalling that  $\Phi(L^1, Z_1) \equiv w_1$ , these relations become

$$w_1 > 0,$$

which is always true, and

$$(A7) \quad \frac{Z_{w_2}^1 + w_1 L_{w_2}^1}{L_{w_1}^1 Z_{w_2}^1 - L_{w_2}^1 Z_{w_1}^1} < 0.$$

We now consider two cases. Suppose, first, that  $AB_y - B_{w_2} \neq BA_y - A_{w_1}$ . Then (a) and (b) apply. Since  $Z_1 = w_1(T - L^1) - \varphi(L^1, w_1)$ , one gets:

$$\begin{aligned} Z_{w_1}^1 &= (T - L^1) - w_1 L_{w_1}^1 - \varphi_L L_{w_1}^1 - \varphi_{w_1} \\ &= (T - L^1) - w_1 L_{w_1}^1 + \alpha B - B \end{aligned}$$

and

$$Z_{w_2}^1 = -w_1 L_{w_2}^1 + \alpha A.$$

Thus (A7) can be written:

$$(A8) \quad \alpha A [L_{w_1}^1 \alpha A + L_{w_2}^1 (\beta B - T + L^1)]^{-1} \leq 0.$$

By symmetry, we would have also:

$$(A9) \quad \beta B [L_{w_2}^2 \beta B + L_{w_1}^2 (\alpha A - T + L^2)]^{-1} \leq 0.$$

If, on the other hand,  $AB_y - B_{w_2} = BA_y - A_{w_1} = 0$ , then  $\varphi_w$  and  $\varphi_L$  must only satisfy the partial differential equation (A3):

$$\varphi_w = -\varphi_L(L_{w_1}^1 - L_y^1 B) + B$$

since, in that case, both  $(L_{w_1}^1 - L_y^1 B)$  and  $B$  are functions of  $L^1$  and  $w_1$  only. Pose:

$$L_{w_1}^1 - L_y^1 B = X_{(L^1, w_1)},$$

$$B = Y_{(L^1, w_1)},$$

and let  $\Phi^1$  be defined hereafter. Since  $Z_1 = w_1(T - L^1) - \varphi$ , one gets from:

$$\Phi^1(L^1, w_1(T - L^1) - \varphi_{(L^1, w_1)}) \equiv w_1$$

the relations

$$\begin{cases} \Phi_L^1 - (w_1 + \varphi_L)\Phi_C^1 = 0, \\ \Phi_C^1(T - L^1 - \varphi_w) = 1, \end{cases}$$

and the condition  $\Phi_L - \Phi \cdot \Phi_C < 0$  becomes:

$$(A10) \quad \varphi_L[T - L^1 - \varphi_w]^{-1} \leq 0.$$

So the problem is whether the equation

$$(A11) \quad \varphi_{w(L, w)} = -\varphi_{L(L, w)}X_{(L, w)} + Y_{(L, w)}$$

has at least one solution  $\varphi$  which satisfies (A10). Moreover, the same must be true for the second agent. Pose  $\varphi_{(L^1, w_1)}^1 = \varphi_{(L^1, w_1)}$ , and

$$\varphi^2 = y - \varphi_{(L^1, w_1)}^1.$$

Then  $AB - B_{w_2} = BA - A_{w_1} = 0$  implies that  $\varphi^2$  is a function of  $L^2$  and  $w_2$  only; it is easy to check, moreover, that if  $\varphi^1$  satisfies the equation (A11), then  $\varphi^2$  also satisfies the corresponding equation:

$$\varphi_{w(L^2, w_2)}^2 = -X_{2(L^2, w_2)} \cdot \varphi_{L(L^2, w_2)}^2 + Y_{2(L^2, w_2)}.$$

It is then necessary that  $\varphi^2$  satisfy the convexity requirement:

$$\varphi_L^2[T - L^2 - \varphi_w^2]^{-1} \leq 0.$$

## APPENDIX II

### PROOF OF PROPOSITION 2

PROOF: The equivalence between (CREA'a) and (CREA'b) is well known; we only show the property for (CREA'b).

We first show that (CREA'b) is necessary. Suppose that there exists  $U^1$ ,  $U^2$  and  $(Z_j, \theta_j)$ ,  $j = 1, \dots, T$ , as in Definition 4. For each  $j$ ,  $(L_j^1, L_j^2, Z_j, C_j - Z_j)$  is a solution of

$$\begin{cases} \text{Max } U^1(L^1, Z) + \theta_j U^2(L^2, C - Z), \\ C + w_1^1 L^1 + w_2^1 L^2 \leq C_j + w_1^1 L_j^1 + w_2^1 L_j^2. \end{cases}$$

First order conditions are (if  $\lambda_j$  is the Lagrange multiplier of the budget constraint):

$$(4) \quad \begin{cases} U_L^1 = \lambda_j w_1^1, \\ \theta_j U_L^2 = \lambda_j w_2^1, \\ U_Z^1 = \lambda_j, \\ \theta_j U_Z^2 = \lambda_j, \end{cases}$$

where the derivatives of  $U^1$  and  $U^2$  are taken respectively in  $(L_j^1, Z_j)$  and  $(L_j^2, C_j - Z_j)$ . Now, the strong concavity of  $U^1$  and  $U^2$  gives

$$(5) \quad \begin{cases} U^1(L_i^1, Z_i) - U^1(L_j^1, Z_j) \\ \quad \leq U_L^1(L_j^1, Z_j)(L_i^1 - L_j^1) + U_Z^1(L_j^1, Z_j)(Z_i - Z_j) \\ \text{and} \\ U^2(L_i^2, C_i - Z_i) - U^2(L_j^2, C_j - Z_j) \\ \quad \leq U_L^2(L_j^2, C_j - Z_j)(L_i^2 - L_j^2) + U_Z^2(L_j^2, C_j - Z_j)(C_i - C_j - Z_i + Z_j) \end{cases}$$

with equality only if  $L_i^1 = L_j^1$  and  $Z_i = Z_j$  (resp.  $L_i^2 = L_j^2$  and  $C_i - Z_i = C_j - Z_j$ ). Pose  $U_j^1 = U^1(L_j^1, Z_j)$ ,  $U_j^2 = U^2(L_j^2, C_j - Z_j)$ , and  $\mu_j = \lambda_j / \theta_j$ ; then reporting (4) into (5) gives (CREA'b).

Conversely, suppose that (CREA'b) is fulfilled. First, note that it is always possible, by using "small" changes of the  $Z_j$ , to guarantee that the  $Z_j$  are all different (if  $Z_i = Z_j$  and  $L_i^1 = L_j^1$ , a small change of  $U_j^2$  is needed too); and the same is true for the  $C_j - Z_j$ . We can thus assume that the inequalities in (CREA'b) are strict (a detailed proof of these points is left to the reader). Now, define

$$U^1(L^1, Z) = \text{Min} (U_j^1 + \lambda_j w_1^j (L^1 - L_j^1) + \lambda_j (Z - Z_j)),$$

$$U^2(L^2, Z) = \text{Min} (U_j^2 + \mu_j w_2^j (L^2 - L_j^2) + \mu_j (Z - C_j + Z_j)).$$

First,  $U^1(L_j^1, Z_j^1) = U_j^1$ , for we have, for some  $m$ :

$$U^1(L_j^1, Z_j^1) = U_m^1 + \lambda_m w_1^m (L_j^1 - L_m^1) + \lambda_m (Z_j^1 - Z_m^1) \\ \leq U_j^1 + \lambda_j w_1^j (L_j^1 - L_j^1) + \lambda_j (Z_j^1 - Z_j^1) = U_j^1$$

which violates (CREA'b) unless  $m = j$ .

In the same way,  $U^2(L_j^2, C_j - Z_j) = U_j^2$ .

Lastly, suppose that, for some  $(L^1, L^2, Z^1, Z^2)$ , we have

$$(6) \quad Z^1 + Z^2 + w_1^1 L^1 + w_2^2 L^2 \leq C_i + w_1^1 L_i^1 + w_2^2 L_i^2.$$

Then, for  $\theta_i = \lambda_i / \mu_i$

$$U^1(L^1, Z^1) + \theta_i U^2(L^2, Z^2) \leq U_i^1 + \lambda_i w_1^i (L^1 - L_i^1) + \lambda_i (Z^1 - Z_i^1) \\ + \theta_i (U_i^2 + \mu_i w_2^i (L^2 - L_i^2) + \mu_i (Z^2 - C_i + Z_i^1)) \\ \leq U_i^1 + \theta_i U_i^2 + \lambda_i (w_1^i (L^1 - L_i^1) + w_2^i (L^2 - L_i^2) \\ + Z^1 + Z^2 - C_i) \\ \leq U_i^1 + \theta_i U_i^2 = U^1(L_i^1, Z_i^1) + \theta_i U^2(L_i^2, C_i - Z_i^1)$$

which proves that  $(L_i^1, L_i^2, Z_i, C_i - Z_i)$  maximizes  $U^1(L^1, Z^1) + \theta_i U^2(L^2, Z^2)$  under (6).

Lastly,  $U^1$  and  $U^2$  are concave, continuous, and strictly monotonic; since, moreover, the  $Z_j$  are different and the inequalities in (CREA'b) are strict, we can "smooth" them in order to obtain a strongly concave, infinitely differentiable utility function (for a proof of this last result, see Chiappori-Rochet (1987)).

### APPENDIX III

#### PROOF OF PROPOSITION 3

Again, the equivalence between (CR'a) and (CR'b) is clear.<sup>13</sup>

We first show that (CR'b) is necessary. If  $U^1$ ,  $U^2$ ,  $Z_j$ , and  $\theta_j$  exist (as in Definition 5), then  $(L_j^1, L_j^2, Z_j, C_j - Z_j)$  is always a solution of:

$$\left| \begin{array}{l} \text{Max } U^1(L^1, L^2, Z^2) + \theta_j U^2(L^1, L^2, Z^1, Z^2), \\ Z^1 + Z^2 + w_1^1 L^1 + w_2^2 L^2 \leq C_j + w_1^1 L_j^1 + w_2^2 L_j^2. \end{array} \right|$$

First order conditions are:

$$(7) \quad \begin{aligned} U_{L^1}^1 + \theta_j U_{L^1}^2 &= \lambda^j w_1^j, \\ U_{L^2}^1 + \theta_j U_{L^2}^2 &= \lambda^j w_2^j, \\ U_{Z^1}^1 + \theta_j U_{Z^1}^2 &= \lambda^j, \\ U_{Z^2}^1 + \theta_j U_{Z^2}^2 &= \lambda^j, \end{aligned}$$

where  $\lambda^j$  is the Lagrange multiplier, and where partial derivatives are taken at  $(L_j^1, L_j^2, Z_j, C_j - Z_j)$ .

Pose  $\alpha_1^j = U_{L^1}^1 / \lambda^j$ ,  $\alpha_2^j = U_{L^2}^1 / \lambda^j$ ,  $\beta_1^j = U_{Z^1}^1 / \lambda^j$ ,  $\beta_2^j = U_{Z^2}^1 / \lambda^j$ , and  $\mu^j = \lambda^j / \theta_j$ . Now, from (7),

$$U_{L^1}^1 = \mu^j (w_1^j - \alpha_1^j), \quad U_{L^2}^1 = \mu^j (w_2^j - \alpha_2^j), \\ U_{Z^1}^1 = \mu^j (1 - \beta_1^j), \quad U_{Z^2}^1 = \mu^j (1 - \beta_2^j).$$

Now, the strong concavity of  $U^1$  and  $U^2$  implies (CR'b).

<sup>13</sup> (CR'b)  $\Rightarrow$  (CR'a) is straightforward. To see that (CR'a)  $\Rightarrow$  (CR'b) note that the  $Z_i$  can be "slightly" modified without violating SARP (the set of data satisfying SARP is open). The detailed proof is left to the reader.

Conversely, assume that (CR'b) is fulfilled; define:

$$(8) \quad \begin{cases} U^1(L^1, L^2, Z^1, Z^2) = \text{Min}_j [U_j^1 + \lambda^j \alpha_1^j (L^1 - L_j^1) + \lambda^j \alpha_2^j (L^2 - L_j^2) \\ \quad + \lambda^j \beta_1^j (Z^1 - Z_j) + \lambda^j \beta_2^j (Z^2 - C_j + Z_j)] \quad \text{and} \\ U^2(L^1, L^2, Z^1, Z^2) = \text{Min}_j [U_j^2 + \mu^j (w_1^j - \alpha_1^j) (L^1 - L_j^1) \\ \quad + \mu^j (w_2^j - \alpha_2^j) (L^2 - L_j^2) + \mu^j (1 - \beta_1^j) (Z^1 - Z_j) \\ \quad + \mu^j (1 - \beta_2^j) (Z^2 - C_j + Z_j)]. \end{cases}$$

The proof that  $U^a(L_j^1, L_j^2, Z_j, C_j - Z_j) = U_j^a (a = 1, 2)$  is as in Proposition 2; it then results from (8) that whenever

$$Z^1 + Z^2 + w_1^j L^1 + w_2^j L^2 \leq C_j + w_1^j L_j^1 + w_2^j L_j^2,$$

then

$$\begin{aligned} U^1(L^1, L^2, Z^1, Z^2) &+ \frac{\lambda^j}{\mu^j} U^2(L^1, L^2, Z^1, Z^2) \\ &\leq U^1(L_j^1, L_j^2, Z_j, C_j - Z_j) + \frac{\lambda^j}{\mu^j} U^2(L_j^1, L_j^2, Z_j, C_j - Z_j). \end{aligned}$$

Lastly, the inequality in (CR'b) being strict, it is always possible to "smooth"  $U^1$  and  $U^2$  into strongly concave, infinitely differentiable functions (Chiappori-Rochet (1987)).

#### APPENDIX IV

##### RECOVERING UTILITY FUNCTIONS

Given a couple of labor supply (or demand for leisure) functions which satisfy conditions (CREA), it is possible to recover the utility functions from which they stem. For simplicity, we indicate the modulus operandi on a convenient example. Consider the following functional forms (defined for  $y > 0$ ):

$$\begin{cases} L^1 = a_1 y + b_1 y \text{ Log } y + c_1 \text{ Log } w_1 + d_1 \text{ Log } w_2, \\ L^2 = a_2 y + b_2 y \text{ Log } y + c_2 \text{ Log } w_1 + d_2 \text{ Log } w_2, \end{cases}$$

where  $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$  are real parameters. Then

$$A = \frac{d_1}{w_2[a_1 + b_1(1 + \text{Log } y)]} \quad \text{and} \quad B = \frac{c_2}{w_1[a_2 + b_2(1 + \text{Log } y)]}.$$

(i) The first task is to determine  $\alpha$  and  $\beta$ . By straightforward calculation, we get here:

$$\alpha = \lambda(a_1 + b_1 + b_1 \text{ Log } y), \quad \text{where} \quad \lambda = \frac{b_2}{a_1 b_2 - a_2 b_1}$$

and

$$\beta = \mu(a_2 + b_2 + b_2 \text{ Log } y), \quad \text{where} \quad \mu = \frac{b_1}{a_2 b_1 - a_1 b_2}.$$

(ii) Since we are in the general case, conditions (CREA b) and (CREA c) must be satisfied; the proof that this is the case here is left to the reader. Now, we consider the equations (A5) of the Appendix:

$$\begin{cases} \varphi_L = -\frac{\alpha}{L_y^1}, \\ \varphi_w = \alpha \left( \frac{L_{w_1}^1}{L^1 y} - B \right) + B, \end{cases}$$

where  $\varphi = w_1(T - L^1) - Z^1$ . Here:

$$\varphi_L = -\lambda,$$

$$\varphi_w = (\lambda c_1 + \mu c_2)/w_1,$$

thus  $\varphi = -\lambda L^1 + (\lambda c_1 + \mu c_2) \text{Log } w_1 + k$  where  $k$  is a constant. This gives  $Z^1$  as a function of  $L^1$  and  $w_1$ , say  $Z^1 = \Psi(L^1, w_1)$ : here

$$(3) \quad Z^1 = w_1(T - L^1) + \lambda L^1 - (\lambda c_1 + \mu c_2) \text{Log } w_1 - k.$$

Note that  $Z^1$  is, as announced, defined up to an additive constant. A similar relation for  $Z^2$ , as a function of  $L^2$  and  $w_2$ , can be obtained either in the same way or through the budget constraint.

(iii) The last step is to recover  $U^1$  and  $U^2$ . The easiest way is probably to define a function  $Z^1 = f(L^1)$  by

$$U^1(L^1, f(L^1)) \equiv K$$

where  $K$  is any constant. In that case, indeed, one gets

$$\frac{df}{dL^1} = -\frac{U_L^1}{U_c^1} = -w_1$$

from the first order conditions. Replacing in (3) gives the following partial differential equation in  $f$

$$f(L^1) = \Psi \left[ L^1, -\frac{df}{dL^1} \right];$$

here, for instance

$$f(L^1) = \frac{df}{dL^1} (L^1 - T) + \lambda L^1 - (\lambda c_1 + \mu c_2) \text{Log} \left[ -\frac{df}{dL^1} \right] - k.$$

This differential equation has a family of solutions which corresponds exactly to the family of indifference curves of  $U^1$ ; and the same path can be used for  $U^2$ .

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