

PBL Effect  $\Rightarrow$  composition effect + effect of interest

<Treatment>						<Control>					
			Post						Post		
		N	R	G	M			N	R	G	M
pre	N	$P_{NN}^T$	$P_{NR}^T$	$P_{NG}^T$	$P_{NM}^T$	pre	N	$P_{NN}^C$	$P_{NR}^C$	$P_{NG}^C$	$P_{NM}^C$
	R						R				
	G						G				
	M						M				

where  $P_{ij} = (\text{Dyadic Relation } t=1 = j \mid \text{Dyadic Relation } t=0 = i)$

Treatment effect for Pre-Null relationship

$\tau_i^{(*)}$   
 $= \rho_N$

$$= \sum_j P_{Nj}^T (E(Y_{11} \mid X_{11}=j, T, X_{10}=N) - E(Y_{10} \mid X_{11}=j, T, X_{10}=N)) \\ - \sum_j P_{Nj}^C (E(Y_{11} \mid X_{11}=j, C, X_{10}=N) - E(Y_{10} \mid X_{11}=j, C, X_{10}=N))$$

$$= \sum_j \underbrace{P_{Nj}^T - P_{Nj}^C}_{\text{diff in transition}} (E(Y_{11} \mid X_{11}=j, T, X_{10}=N) - E(Y_{10} \mid X_{11}=j, T, X_{10}=N) \\ - (E(Y_{11} \mid X_{11}=j, C, X_{10}=N) - E(Y_{10} \mid X_{11}=j, C, X_{10}=N))) \quad \left. \vphantom{\sum_j} \right\} \text{effect of interest}$$

$$+ \sum_j (P_{Nj}^T - P_{Nj}^C) (E(Y_{11} \mid X_{11}=j, C, X_{10}=N) - E(Y_{10} \mid X_{11}=j, C, X_{10}=N))$$

diff in transition

learning by each group in control

$$(*) Y_{it} = \beta_0 + \beta_1 t_i + \beta_2 t_i PBL_i + \sum_{j \neq N} \delta_j X_{10j} + \sum_j \gamma_j X_{10j} t_i + \sum_j \alpha_j X_{10j} \cdot PBL_i \\ + \sum_j \rho_j X_{10j} \cdot PBL_i \cdot t_i$$

$$\rho_N = E(Y_{11} \mid PBL=1, X_{10j}=N) - E(Y_{10} \mid PBL=1, X_{10j}=N) \\ - (E(Y_{11} \mid PBL=0, X_{10j}=N) - E(Y_{10} \mid PBL=0, X_{10j}=N))$$

$\hookrightarrow$  whole sample에 대한 transition matrix 계산하는 것이 아님