Introducing Household Production in Collective Models of Labor Supply

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I. Introduction

In a recent paper (Chiappori 1992), I proposed a model of labor supply based on a "collective" representation of household behavior. In this framework, each household member is characterized by his or her own utility function, and decisions are only assumed to result in Pareto-efficient outcomes. I showed that these simple assumptions were sufficient to (i) generate testable restrictions on labor supply functions and (ii) recover individual preferences and the outcome of the decision process from observed behavior. For the sake of simplicity, the results were derived in the simplest possible case, namely, a static labor supply model with private consumptions. Such a framework was appropriate only for the purpose of the paper, that is, a preliminary exploration of the properties of collective models of labor supply. The next step is to evaluate the robustness of the results to various extensions of the basic model. This opens a general research program and generates a wide range of particular problems that it is hoped will be considered in the future.

Among the most serious shortcomings of the 1992 model is the absence of household production: as in most labor supply models, agents were assumed to divide their time between market activities and leisure. As Apps and Rees (1997, in this issue) very rightly point out in their comment, the absence of domestic production is far from innocuous as soon as welfare issues are considered. For instance, a low level of market labor supply will automatically be interpreted as a large consumption of leisure, whereas it may in fact reflect the specialization of one of the members in domestic

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production. This remark clearly suggests that the estimation of individual utilities in a collective labor supply model is likely to be very sensitive to the presence of household production. Also, Apps and Rees raise a major issue, namely, To what extent is it possible to recover the intrahousehold decision process in the presence of household production? They argue that the answer is negative in general, which suggests that the introduction of household production within the collective framework may lead to serious identification problems.

In the present note, I make four basic points. (1) A key issue is whether the commodity that is domestically produced is marketable (the "complete market" case) or not (the "incomplete market" framework). In their paper, Apps and Rees consider only the latter situation. I briefly discuss under which circumstances each setting can be seen as appropriate. (2) In the complete market case, I show that the results of the initial framework can be extended to household production. That is, testable restrictions on domestic and market labor supply functions can be derived, and the intrahousehold distribution of resources (the "sharing rule") can be recovered up to an additive constant. (3) In the alternative situation (the one Apps and Rees implicitly consider), the price of the household commodity is endogenous to household decisions and has to be estimated as well (as a function of wages and incomes). Then their result on the impossibility of exact identification is correct. It can actually be made more precise; specifically, I show that if the household production exhibits constant returns to scale, the sharing rule can be recovered only *up to an additive function of wages*. In particular, I show, in a counterexample, why parametric identification can then be revealed as seriously misleading. (4) However, complete identification (up to an additive constant) may still be possible, but it requires additional assumptions on the decision process, examples of which are provided.

The model is similar to Apps and Rees's, though, for the sake of consistency, I adopt the same notation as in my 1992 paper. The household consists of two members, 1 and 2. Respective demands for leisure are denoted by L_i, market labor supplies by l_i, and wages by w_i; let m_1 and m_2 be the members' respective nonlabor incomes.[1]

[1] What is actually needed for identification is in fact the existence of at least one "distribution factor" (see Bourguignon, Browning, and Chiappori 1994), i.e., factors that influence the decision process but neither the preferences nor the budget constraint. If m_1 and m_2 can be assumed exogenous, they will play this role: while their sum has an income effect, the distribution should matter only for the decision process. But other variables could be used, e.g., extra environmental parameters à la McElroy (1990).

There are two consumption goods: a market good x, whose price is set to one, and a domestic good y, which can be produced within the household. Let $h(t_1, t_2)$ be the production function of the domestic good, where t_i is member i's household work.[2] Following a remark made some years ago by Pollak and Wachter (1975), I assume, as in Apps and Rees, that h exhibits constant returns to scale.

All goods are privately consumed. Also, following my initial model, I consider the case of cross-sectional data. Then wages and incomes vary across households, but other prices are assumed to be constant over the sample. Accordingly, let us assume that labor supplies l_i and t_i are observed as functions of w_1, w_2 and m_1, m_2. Finally, I forget about the tax system and assume that budget sets are linear.

The household consists of two members. In what follows, I assume that preferences are either "egoistic," as in Apps and Rees, in which case U^i has the form

$$U^i(x_i, y_i, L_i), \tag{1}$$

or "caring," in which case each agent maximizes an index of the form

$$W^i[U^1(x_1, y_1, L_1), U^2(x_2, y_2, L_2)]. \tag{2}$$

Finally, decisions are made according to some intrahousehold process. Following the basic idea of the collective approach, I simply assume that the decision process, whatever its true nature, always generates Pareto-efficient outcomes; this framework was used in my initial models (Chiappori 1988, 1992) as well as in Apps and Rees (1988) and various other contributions (e.g., Bourguignon and Chiappori [1992] and Browning et al. [1994]; see, however, Udry [1996]). In the present framework, any efficient decision process can be interpreted as follows: members agree on some efficient production plan and some intrahousehold distribution of resources; then each member freely chooses his or her own leisure–domestic production–consumption bundle subject to the specific budget constraint he or she faces. Formally, production is determined by

$$\max_{t_1, t_2} p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2, \tag{3}$$

and member i maximizes

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \tag{4}$$

$$\text{subject to} \quad x_i + p \cdot y_i + w_i \cdot L_i = s_i.$$

[2] Note that production is not assumed to depend only on total time $t_1 + t_2$; i.e., I allow for differences in marginal productivity of labor. One advantage of this generalization is that it can account for partial specialization (time input being nonzero for each member).

Here, s_i stands for member i's "potential income," with

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2 + p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2. \tag{5}$$

In other words, s_i is a given function of (w_1, w_2, m_1, m_2) that "summarizes" the decision process; it can be seen as the natural generalization of the sharing rule, as introduced in my 1992 paper, to the case of household production.

As will be clear in what follows, a crucial issue is whether good y can be bought and sold on the market or is household specific. In the former case, let p denote the market price of the domestic good (p is then *exogenous* for the household) and z denote the quantity sold (if positive) or bought (if negative) on the market, so that the household's budget constraint is

$$x_1 + x_2 \leq w_1 l_1 + w_2 l_2 + m_1 + m_2 + p \cdot z.$$

In the second case, $z = 0$ for all households. As mentioned by Apps and Rees, we can still define a price p for the domestic good. The difference, however, is that p is now *endogenous* to the household preferences and decision process; technically, it has to be estimated as a function of the various parameters.

II. Is the Domestic Good Marketable?

Since the two cases above lead to totally opposite results, a quick discussion of their empirical relevance may be useful.

A. *The Case for a Marketable Domestic Good*

Farm Household

Whether the domestic good can be assumed marketable depends on the interpretation that is given to the notion of domestic production. In most cases, this assumption is in fact quite natural. The best example is agricultural households. In all developing countries (and in most developed ones as well), rural households have an agricultural production activity. Models of household production based on the maximization of a unique household utility, subject to a budget set and a production constraint, have become basic tools for the analysis of farm households.[3]

In the standard framework, as studied by many authors (including

[3] See Singh, Squire, and Strauss (1986) for a comprehensive survey of this literature.

Lau, Lin, and Yotopoulos [1978], Barnum and Squire [1979], and others), markets are complete, and households are price takers for every commodity (including labor) they buy, sell, or consume. This means, in particular, that household production (consisting of commodities such as rice and coffee) can always be sold on sufficiently well-organized markets. As Singh et al. (1986, p. 3) put it, "most households in agricultural areas produce partly for sale and partly for their own consumption." A well-known consequence is the existence of a "separation" property between production and consumption. Production can be analyzed as stemming from the maximization of profit at market prices. In the same way, consumption is represented by utility maximization under a traditional budget constraint, again at market prices. The key point is that the household can make its production decision independently of its consumption and labor supply decisions.

As will be clear later on, the results of the next section rely on exactly the same separability property as in the standard "unitary" model of a farm household. The present paper can in fact be viewed as an extension of this literature to a collective framework, in which each household member is represented by specific preferences instead of being aggregated within the somewhat ad hoc fiction of a single household utility. The good news is that the nice properties of the standard, complete market framework (in terms of estimation in particular) do extend to the collective setting.

Domestic Production in Nonrural Households

While agricultural households do constitute a standard area of application of household production, it is by no means the only one. The general relevance of the concept of domestic production has been emphasized by many authors, starting with Becker (1991). In many cases, the same goods or services can also be bought on outside markets, at a given price. For example, meals can be taken at home or at a restaurant, children can be kept at home or in kindergartens, and so on. In that case, the corresponding price will play exactly the role described in the formal model: it will determine the trade-off between internal production and outside trade. Specifically, household members will devote their time to domestic production up to the point at which marginal productivity equals the ratio of the wage to the price of the good.[4]

[4] In some cases, the goods traded on outside markets are not perfect substitutes for domestic goods. Albeit this distinction may seem excessively subtle in the elementary framework of a standard labor supply model (where, for simplicity, all outside commodities are aggregated within a single good x, no fixed costs are considered, etc.), we may note that the argument above can be seen as an approximation: marginal

As another illustration, take the case of house cleaning. The trade-off faced by household members here is either doing the job themselves or hiring someone to do the work. Then a natural unit of measure for household productive output is the number of outside hours, and p can be interpreted as the market price of one hour of maid services. One could argue that a slight distinction should be introduced here, because hiring a maid should be interpreted as buying a productive input (rather than directly buying the output). But this fact can easily be reconciled with the framework at stake. Assume, for instance, that the output is actually produced from the work of the maid (who receives a wage s) plus some inputs (bought on the markets at a price q) according to some constant returns to scale technology. Then the production side defines the output price p as a *fixed* function of s and q; the crucial point is that p is again exogenous for the household.[5]

B. *The Case for a Nonmarketable Domestic Good*

While the complete market can be considered as the benchmark case, there may be various situations in which this assumption fails to hold true. Becker (1991, p. 24), for instance, considers such domestic goods as "prestige and esteem, health, altruism, envy, and pleasures of the senses," most of which (except perhaps the last one) can hardly be bought on outside markets. Some works on rural households have introduced the concept of *Z*-goods,[6] for which no market may exist (though this point has been disputed; see, e.g., Anderson and Leiserson [1980]). Note that, in both cases, not only is the output nonmarketable, but it is not observable in general (at least in a measurable way), which generates difficult identification problems.

Somewhat similar is the case in which, because of market imperfections of some kind, corner solutions obtain. Assume, for instance, that a given commodity is bought and sold at different prices, because of transaction costs, taxes, information asymmetries, or what-

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productivity has to be "close to" the wage/price ratio, the proximity level depending on the degree of substitutability between the commodities. It seems clear, however, that the dispersion between wages in a cross-sectional analysis will exceed, by and large, the gap between market and intrahousehold prices of domestic goods so that the latter distinction can probably be forgotten for empirical purposes.

[5] This assumes that the same technology is available to all households or, in a more realistic way, that differences in technologies can be neglected with regard to the huge dispersion between wages. If not—say, because differences in capital stocks across households are important enough—then an empirical estimation will have to control for the stock of durables in the usual way.

[6] See Hymer and Resnick (1969) and Singh et al. (1986) for a general discussion.

ever. A fraction of households will then exhibit a zero net demand. The shadow price of the domestic good, in that case, will lie somewhere between the buying and the selling price, but the exact value will be endogenous to household behavior as a whole.

As is well known, a consequence of missing markets is that the separability property no longer holds; household decisions cannot be modeled as recursive, as in the previous case. Estimation of nonseparable models is known to be a much more difficult task.

III. The Complete Market Case

The first question now is whether, from the observation of t_i and l_i, we can recover s_i as a function of the relevant variables. In addition, we would like to derive testable restrictions on observed behavior. Specifically, we are looking for conditions that have to be fulfilled by arbitrary functions t_i and l_i (or L_i) of (w_1, w_2, m_1, m_2) to be a solution of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2.

Both purposes appear to be feasible. Define, first,

$$Z_i = \frac{\partial L_i/\partial m_1}{\partial L_i/\partial m_2}. \tag{6}$$

Note that $Z_i = 1$ if income is pooled. In what follows, we assume that this is not the case, so that $Z_i \neq 1$. The main result can be stated as follows.

THEOREM 1. Let z, t_i, and L_i be given functions of (w_1, w_2, m_1, m_2), and assume that $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then the following results hold true: (1) One can find a list of necessary and sufficient conditions for the existence of U^1, U^2, and h such that (L_1, L_2, t_1, t_2, z) are solutions of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2 (they are given by conditions 1–7 in App. A). (2) For any functions satisfying these conditions, the sharing rule is identified up to an additive constant. In addition, individual consumption can be recovered up to an additive constant.

The complete proof is in Appendix A. I shall briefly sketch the basic steps. First, the observation of input demands $t_i(w_1, w_2)$ allows one to identify the production function h up to a multiplicative constant. Now, from (4), L_i can be written as some (Marshallian demand) function of w_i and s_i. From the observation of Z_i, one can identify the partials with respect to s_i. Finally, differentiating (5) in w_1 and w_2 leads to the identification of wage effects.

The properties of the complete market case are thus quite interesting. If one has a data base in which the time spent in household production by each member is recorded and distinct, exogenous income sources are observed, the conclusions of the initial model can readily be extended. Under the same assumptions (private consumptions and "caring" preferences), one can identify the sharing rule up to an additive constant, and individual, Marshallian demands for leisure can then be recovered. Furthermore, testable restrictions are generated on both market and household labor supply functions.

IV. The Incomplete Market Case

A. *Partial Identification of the Sharing Rule*

Let us now turn to the incomplete market case. In the maximization problems, p should no longer be interpreted as the actual price of the domestic good on some outside market, but rather as its shadow price. This is important because shadow prices are *endogenous* to household behavior. As such, p depends on wages and nonlabor income as well as on preferences and the decision process. How will this fact modify the results of the previous section?

I first consider the productive side. As before, the assumption of constant returns to scale allows the production function to be recovered up to a multiplicative constant. In addition, the output price is a given function of wages that can be estimated up to the same constant.[7] Note that p is homogeneous of degree one and depends only on wages.

I can now state the main result of this section.

THEOREM 2. Let t_i and L_i be given functions of (w_1, w_2, m_1, m_2). Assume that h exhibits constant returns to scale and that $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then the following results hold true: (1) If (L_1, L_2, t_1, t_2, z) are solutions of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2, then necessarily the ratio $\tau = t_1/t_2$ is a function of w_1/w_2 alone. (2) For any functions satisfying these conditions, the sharing rule is identified up to an additive function of (w_1, w_2). That is, if $s_i(w_1, w_2, m_1, m_2)$ is a sharing rule compatible with observed behavior, then any alternative solution must have the form

$$s_i'(w_1, w_2, m_1, m_2) = s_i(w_1, w_2, m_1, m_2) + f(w_1, w_2) \tag{7}$$

for some arbitrary function f.

[7] To see why, note that if $h(t_1, t_2) = t_1 H(\tau)$, where $\tau = t_2/t_1$, then H is identified up to a multiplicative constant, which is obviously not identifiable. Then p is given by

$$p = \frac{w_i}{\partial h/\partial t_i} = \frac{w_2}{H'[\tau(w_1/w_2)]} = p(w_1, w_2).$$

The proof is given in Appendix B.

The previous result suggests two remarks. First, endogeneity of the domestic price has a cost in terms of identification; indeed, the sharing rule can be estimated only up to an additive function of wages. This finding generalizes proposition 3 in Apps and Rees. Second, while complete identification is out of reach, there is still much to be learned from the observation of labor supplies. For one thing, the effect of nonlabor incomes on the sharing rule is exactly identified. The reason is that a change in m_1 and m_2 that leaves wages constant will not modify the price of the domestic good; the second-round effects alluded to above are thus ruled out. Note that this generalizes proposition 4 in Apps and Rees since we do not need to assume that the production function is linear. This is important information, especially in view of policies based on cash subsidies whose effect is exactly to alter the members' respective nonlabor incomes. For instance, we may be able to assess the consequences of a lump-sum distribution between husband and wife on the intrahousehold distribution of resources, even though the distribution is only partially identified. This has important implications, in particular for the "targeted" programs discussed in Haddad and Kanbur (1992); for an empirical investigation along these lines, see Lundberg, Pollak, and Wales (1995).

B. *Parametric versus Nonparametric Identification*

An important consequence of this result is that one has to be extremely cautious when identifying a collective model with household production. In particular, a parametric approach can be dangerously misleading. Assume that a given, parametric structural model generates a reduced form such that all the parameters of the model are identified. It may still be the case that other, *functionally different* structural models lead to the *same* reduced form. Then there is no reason to single out the initial model; as a consequence, all welfare recommendations based on the parametric identification are invalid.

This can be readily seen in the following example. Assume that utilities have the linear expenditure system form:

$$U^i(x_i, y_i, L_i) = \alpha_i \log(x_i - \bar{x}_i) + \beta_i \log(y_i - \bar{y}_i) + \gamma_i \log(L_i - \bar{L}_i),$$

with $\alpha_i + \beta_i + \gamma_i = 1$; the corresponding Marshallian demands are $\Lambda_i(p, w_i, s_i)$, with

$$w_i \Lambda_i = w_i \bar{L}_i + \gamma_i (s_i - \bar{x}_i - p\bar{y}_i - w_i \bar{L}_i).$$

Assume that the production function is Cobb-Douglas:

$$h(t_1, t_2) = (t_1)^{\delta}(t_2)^{1-\delta},$$

and take for the sharing rule the following form:

$$s_1(w_1, w_2, m_1, m_2) = m_1 + w_1 T + S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2},$$

$$s_2(w_1, w_2, m_1, m_2) = m_2 + w_2 T - S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}.$$

Then one can compute individual demand and labor supply functions. On the production side,

$$\frac{t_2}{t_1} = \frac{1-\delta}{\delta}\frac{w_1}{w_2}$$

identifies δ; then

$$p = K(w_1)^{\delta}(w_2)^{1-\delta},$$

where K is a known function of δ. Consider, now, demands for leisure:

$$\begin{aligned} w_1 L_1 = {} & -\gamma_1 \bar{x}_1 + [(1-\gamma_1)\bar{L}_1 + \gamma_1 T] w_1 - \gamma_1 \bar{y}_1 K(w_1)^{\delta}(w_2)^{1-\delta} \\ & + \gamma_1(m_1 + S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}), \\ w_2 L_2 = {} & -\gamma_2 \bar{x}_2 + [(1-\gamma_2)\bar{L}_2 + \gamma_2 T] w_2 - \gamma_2 \bar{y}_2 K(w_2)^{\delta}(w_2)^{1-\delta} \\ & + \gamma_2(m_2 - S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}). \end{aligned}$$

The main remark is that *all parameters are identifiable from the demand for leisure equation* (as can be readily checked). A naive econometrician might conclude that he has recovered the "true" utility functions and the "true" sharing rule. Even worse, he may try to deduce normative recommendations about welfare policies. Now, all this is a pure illusion. For assume that the true sharing rule is in fact

$$\tilde{s}_1(w_1, w_2, m_1, m_2) = m_1 + w_1 T + S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2} + \nu_1 w_1 - \nu_2 w_2,$$

$$\tilde{s}_2(w_1, w_2, m_1, m_2) = m_2 + w_2 T - S_o \cdot w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2} - \nu_1 w_1 + \nu_2 w_2,$$

where the ν_i are arbitrary, nonnegative scalars. Define the functions λ_i, $i = 1, 2$, by

$$\lambda_i(w_i, p, s_i) = \Lambda_i[w_i, p, s_i - \nu_i w_i + \nu_j K^{-1/(1-\delta)} p^{1/(1-\delta)} w_i^{-\delta/(1-\delta)}].$$

From standard consumer theory, we know that there exists some utility function for which λ_i is the Marshallian demand for leisure[8]

[8] The only condition is negativity of the diagonal Slutsky term, which is fulfilled provided that $\partial\Lambda_i/\partial s > 0$.

(of course, it will not have the linear expenditure system form). And it can be readily checked that the λ_i, together with the $\tilde{s}_i$, generate exactly the same reduced forms as above. Of course, the interpretation of the coefficients is totally different, and no normative conclusion will be robust to the new specification, though it is not empirically distinguishable from the initial one.

C. Particular Assumptions on the Decision Process

Finally, it should be noted that the model considered above is extremely general: it only assumes efficiency of household decisions and imposes no restriction on the location of the final outcome on the Pareto frontier. A natural conjecture is that stronger results (and possibly full identification) would obtain, should some additional structure be introduced. A possible solution is to impose some structural restriction on the decision process. As an illustration, consider the following assumption.

ASSUMPTION H. The sharing rules depend only on total income brought by each member; that is, s_i must be a function of $w_1 T + m_1$ and $w_2 T + m_2$ alone.

Then, again, the sharing rule can be fully identified. To see why, note that, from (7), we know that, for any solution s_i',

$$\frac{\partial s_i'}{\partial w_i} = \frac{\partial s_i}{\partial w_i} + \frac{\partial f}{\partial w_i} \tag{8}$$

and

$$\frac{\partial s_i'}{\partial m_i} = \frac{\partial s_i}{\partial m_i}. \tag{9}$$

Also, assumption H requires that

$$\frac{\partial s_i'}{\partial w_i} = T\frac{\partial s_i'}{\partial m_i}.$$

It follows that

$$\frac{\partial s_i}{\partial w_i} + \frac{\partial f}{\partial w_i} = T\frac{\partial s_i}{\partial m_i}.$$

Hence,

$$\frac{\partial f}{\partial w_i} = T\frac{\partial s_i}{\partial m_i} - \frac{\partial s_i}{\partial w_i}, \tag{10}$$

and f is identified up to an additive constant. In words, among all solutions of the form (7), one at most is compatible with assumption

H. In addition, assumption H generates new restrictions (second cross derivatives, as computed from [10], must be equal). So the identifying assumption is independently testable, as requested by standard methodological considerations.

Of course, assumption H is only an example. Basically, any restriction stating that s_i depends on (w_1, w_2, m_1, m_2) only through given functions $A_1(w_1, w_2, m_1, m_2)$ and $A_2(w_1, w_2, m_1, m_2)$ will lead to similar conclusions.[9] Finally, a more general question is the following: Can we think of particular, axiomatic restrictions on the decision process itself that will lead to complete identification of the sharing rule even in the nonmarketable case? Assume, for instance, that the process can adequately be described with the help of some specific cooperative bargaining concept (Nash bargaining, Kalai-Smorodinsky, Rubinstein's "shrinking cake," or whatever). Would this help identification in some way? This is an open question to which further work will be devoted. In addition, empirical applications are obviously needed, following the initial attempts by Blundell et al. (1996) and Fortin and Lacroix (1996).

Appendix A

Proof of Theorem 1

A. The Production Side

I start by considering production decisions. As stated above, the household maximizes profit:

$$\max_{t_1, t_2} p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2, \tag{A1}$$

where $h(t_1, t_2) = t_1 H(\tau)$, with $\tau = t_2/t_1$; note that τ, like t_1 and t_2, can be observed as a function of (w_1, w_2, m_1, m_2). Now efficiency requires that

$$\frac{\partial h/\partial t_1}{\partial h/\partial t_2} = \frac{H(\tau)}{H'(\tau)} - \tau = \frac{w_1}{w_2}.$$

[9] To see why, simply note that if this is the case, then the gradient of the sharing rule must belong to the subspace spanned by the gradients of A_1 and A_2. Now, among the solutions of the form (7), again at most one is compatible with this property. Indeed, assume that $Ds' = \alpha \cdot DA_1 + \beta \cdot DA_2$, where Dg denotes the gradient of g. Derivatives with respect to m_1 and m_2 give

$$\frac{\partial s'}{\partial m_i} = \alpha \cdot \frac{\partial A_1}{\partial m_i} + \beta \cdot \frac{\partial A_2}{\partial m_i}.$$

If the A_i are regular, in the sense that

$$\frac{\partial A_1/\partial m_1}{\partial A_1/\partial m_2} \neq \frac{\partial A_2/\partial m_1}{\partial A_2/\partial m_2},$$

these equations identify α and β. Then derivatives with respect to w_1 and w_2 identify the partials of f and, hence, the function itself up to an additive constant.

This differential equation defines H up to a multiplicative constant, which is obviously not identifiable unless total output is observed. Here, τ is defined as a function of w_1/w_2 alone, a fact that provides a first testable restriction.

CONDITION 1. t_2/t_1 must be a function of w_1/w_2 alone.

B. Demand for Leisure: Income Effects

We now come to labor supply, as determined by the program

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \tag{A2}$$

$$\text{subject to} \quad x_i + p \cdot y_i + w_i \cdot L_i = s_i.$$

A first remark is that demand for leisure must have the form

$$L_i(w_1, w_2, m_1, m_2) = \lambda_i(w_i, s_i), \tag{A3}$$

where λ_i is the Marshallian demand derived from U^i. It follows that

$$\frac{\partial L_i}{\partial m_j} = \frac{\partial \lambda_i}{\partial s_i}\frac{\partial s_i}{\partial m_j}; \tag{A4}$$

hence,

$$Z_i = \frac{\partial L_i/\partial m_1}{\partial L_i/\partial m_2} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2}, \tag{A5}$$

where Z_i is observable. Also, we know that

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2;$$

hence,

$$\frac{\partial s_1}{\partial m_j} + \frac{\partial s_2}{\partial m_j} = 1, \tag{A6}$$

for $j = 1, 2$. From (A4) and (A6), the partials of s with respect to m can be exactly identified.

LEMMA 1. Assume that $Z_1 \neq Z_2$. Then

$$\frac{\partial s_1}{\partial m_1} = Z_1 \frac{Z_1 - 1}{Z_2 - Z_1},$$

$$\frac{\partial s_1}{\partial m_2} = \frac{Z_2 - 1}{Z_2 - Z_1},$$

$$\frac{\partial s_2}{\partial m_1} = Z_2 \frac{Z_1 - 1}{Z_1 - Z_2},$$

$$\frac{\partial s_2}{\partial m_2} = \frac{Z_1 - 1}{Z_1 - Z_2}.$$

Note that if income is pooled, $Z_1 = Z_2 = 1$ and the partials are indeterminate.

Of course, the derivation of derivatives immediately generates cross-derivative restrictions.

CONDITION 2. The partials given by lemma 1 must satisfy

$$\frac{\partial^2 s_i}{\partial m_1 \partial m_2} = \frac{\partial^2 s_i}{\partial m_2 \partial m_1}.$$

In addition, (A4) gives the partials $\partial\lambda/\partial m$:

$$\frac{\partial\lambda_i}{\partial s_i} = \frac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j},$$

which does not depend on j by construction. This function must depend only on (w_i, s_i); in particular, the following condition must hold.

CONDITION 3. The functions L and s must satisfy

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2}.$$

C. Demand for Leisure: Wage Effects

From (A3), it follows that

$$\frac{\partial L_i}{\partial w_j} = \frac{\partial\lambda_i}{\partial s_i} \times \frac{\partial s_i}{\partial w_j}$$

for $i \neq j$. Hence,

$$\frac{\partial s_1}{\partial w_2} = \frac{\partial L_1/\partial w_2}{\partial\lambda_1/\partial s_1} = \frac{\partial s_1}{\partial m_1} \times \frac{\partial L_1/\partial w_2}{\partial L_1/\partial m_1}, \tag{A7}$$

where $\partial s_1/\partial m_1$ is given by lemma 1. Similarly,

$$\frac{\partial s_2}{\partial w_1} = \frac{\partial s_2}{\partial m_1} \times \frac{\partial L_2/\partial w_1}{\partial L_2/\partial m_1}. \tag{A8}$$

Finally, the own-wage effect can be recovered from (A5):

$$\frac{\partial s_1}{\partial w_1} = \frac{\partial s}{\partial w_1} - \frac{\partial s_2}{\partial w_1} = T - \frac{\partial s_2}{\partial w_1} \tag{A9}$$

and

$$\frac{\partial s_2}{\partial w_2} = T - \frac{\partial s_1}{\partial w_2}. \tag{A10}$$

It follows that all partials of s are identified: s can be recovered up to an additive constant. Of course, new testable restrictions arise.

Condition 4. Cross-derivative restrictions on s are

$$\frac{\partial^2 s_i}{\partial w_1 \partial w_2} = \frac{\partial^2 s_i}{\partial w_2 \partial w_1},$$
$$\frac{\partial^2 s_i}{\partial m_k \partial w_j} = \frac{\partial^2 s_i}{\partial w_j \partial m_k}.$$

Also, $\partial \lambda_i / \partial s_i$ depends on w_j only through s_i, which implies the following condition.

Condition 5.

$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k}.$$

Finally, we can now recover $\partial \lambda_i / \partial w_i$; indeed, note that

$$\frac{\partial L_i}{\partial w_i} = \frac{\partial \lambda_i}{\partial w_i} + \frac{\partial \lambda_i}{\partial s_i} \times \frac{\partial s_i}{\partial w_i},$$

which implies that

$$\frac{\partial \lambda_i}{\partial w_i} = \frac{\partial L_i}{\partial w_i} - \frac{\partial s_i}{\partial w_i} \times \frac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}. \tag{A11}$$

This partial must be a function of w_i and s_i alone, which generates the usual conditions; furthermore, it must be compatible with the partials $\partial \lambda / \partial s$ previously computed. Hence, the following condition must hold.

Condition 6.

$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k},$$

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2},$$

$$\frac{\partial^2 \lambda_i}{\partial w_i \partial s_i} = \frac{\partial^2 \lambda_i}{\partial s_i \partial w_i}.$$

However, Slutsky conditions do not impose new restrictions on λ because we do not observe changes in p.

D. *Recovering Private Consumptions*

Once the sharing rule has been identified, it is easy to recover the individual consumptions of both market and domestic goods. To see how, note that

$$y(w_1, w_2, m_1, m_2) = Y_1(w_1, s_1) + Y_2(w_2, s_2),$$

where y is observed but the Marshallian demands Y_1 and Y_2 are not. Now, compute the derivatives with respect to m_1 and m_2:

$$\frac{\partial y}{\partial m_i} = \frac{\partial Y_1}{\partial s_1} \times \frac{\partial s_1}{\partial m_i} + \frac{\partial Y_2}{\partial s_2} \times \frac{\partial s_2}{\partial m_i}. \tag{A12}$$

Then we have the following lemma.

LEMMA 2. Assume $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then $\partial Y_1/\partial s_1$ and $\partial Y_2/\partial s_2$ can be identified from (A12).

Proof. Equation (A12) provides a linear system with the two partials as unknowns. The determinant of the system is equal to $(Z_1 - 1) \cdot (Z_2 - 1) \cdot (Z_1 - Z_2)$ and hence is nonzero under the assumption.

Finally, once the partials with respect to s have been recovered, the two remaining partials, $\partial Y_1/\partial w_1$ and $\partial Y_2/\partial w_2$, follow immediately from

$$\frac{\partial y}{\partial w_i} = \frac{\partial Y_i}{\partial w_i} + \frac{\partial Y_1}{\partial s_1} \times \frac{\partial s_1}{\partial w_i} + \frac{\partial Y_2}{\partial s_2} \times \frac{\partial s_2}{\partial w_i}. \tag{A13}$$

Hence the Marshallian demands are known up to an additive constant.

As before, this implies two kinds of restrictions; namely, those derivatives must be functions of (w_i, s_i) alone and satisfy cross-derivative restrictions. This leads to the final bunch of testable restrictions.

CONDITION 7.

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial Y_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial Y_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k},$$

$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial Y_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial Y_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k} \quad \text{for } j \neq i,$$

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial Y_i}{\partial s_i}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial Y_i}{\partial s_i}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2},$$

$$\frac{\frac{\partial}{\partial w_j}\left(\frac{\partial Y_i}{\partial s_i}\right)}{\frac{\partial}{\partial m_k}\left(\frac{\partial Y_i}{\partial s_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k} \quad \text{for } j \neq i,$$

$$\frac{\partial_i^2 Y}{\partial w_i \partial s_i} = \frac{\partial_i^2 Y}{\partial s_i \partial w_i}.$$

Finally, the x_i can be recovered either through the same technique or, more directly, from the budget constraints, since the s_i are known. Note, in particular, that no additional restrictions appear at this stage: the x_i will automatically fulfill the conditions of the previous type from the budget constraint. This terminates the proof.

Appendix B

Proof of Theorem 2

Let us consider consumption behavior. The program becomes

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \tag{B1}$$

$$\text{subject to } x_i + p(w_1, w_2) \cdot y_i + w_i \cdot L_i = s_i.$$

As before, demand for leisure must have the form

$$L_i(w_1, w_2, m_1, m_2) = \lambda_i(w_i, p(w_1, w_2), s_i), \tag{B2}$$

where λ_i is the Marshallian demand derived from U^i and

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2. \tag{B3}$$

Assuming that $\partial L_i/\partial m_j \neq 0$ for some j, we can locally invert (B2) as

$$s_i(w_1, w_2, m_1, m_2) = F_i[w_i, p(w_1, w_2), L_i(w_1, w_2, m_1, m_2)]$$

for some function F_i that has to be estimated.

Equation (B3) becomes

$$\begin{aligned} &F_1[w_1, p(w_1, w_2), L_1(w_1, w_2, m_1, m_2)] \\ &+ F_2[w_2, p(w_1, w_2), L_2(w_1, w_2, m_1, m_2)] = (w_1 + w_2) \cdot T + m_1 + m_2. \end{aligned} \tag{B4}$$

Now, let (F_1, F_2) and (F_1', F_2') be two solutions of this equation, and define $G_i = F_i - F_i'$. I show that G_i can be written as a function of wages alone. Since (B4) is linear, we know that the G_i must satisfy the homogeneous equation

$$G_1[w_1, p(w_1, w_2), L_1(w_1, w_2, m_1, m_2)]$$

$$+ G_2[w_2, p(w_1, w_2), L_2(w_1, w_2, m_1, m_2)] = 0.$$

Differentiating in m_i gives

$$\frac{\partial G_1}{\partial L_1}\frac{\partial L_1}{\partial m_i} + \frac{\partial G_2}{\partial L_2}\frac{\partial L_2}{\partial m_i} = 0.$$

Since this system has full rank, its only solution is

$$\frac{\partial G_1}{\partial L_1} = \frac{\partial G_2}{\partial L_2} = 0;$$

hence $G_i = G_i[w_i, p(w_1, w_2)] = f_i(w_1, w_2)$.

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