I. Model

In an earlier paper, Choi et al. (2007), we adopted a model of disappointment aversion, proposed by Gul (1991), to measure the degree of disappointment aversion as well as standard risk aversion. We extend their approach to uncover not only disappointment aversion but also elation loving.

Let x_i denote the amount of assets associated at state $i = 1, 2$. Two states are equally probable. An individual is assumed to chooses a portfolio (x_1, x_2) to maximize the following utility function, for $\alpha \in (0, 1)$,

$$\alpha u(x_{\min}) + (1 - \alpha) u(x_{\max})$$

subject to the budget constraint, $p_1 x_1 + p_2 x_2 = 1$, where $x_{\min}$ denotes $\min\{x_1, x_2\}$ and $x_{\max}$ denotes $\max\{x_1, x_2\}$. If $\alpha = 1/2$, the model becomes the standard expected utility model. If $\alpha > (<) 1/2$, the individual shows *disappointment aversion* (*elation loving*). Note that the Gul(1991)'s model in the two-asset case is a version of the Rank-Depenent utility (RDU) model in which α is a probability weighting of 1/2. In terms used in the RDU model, the individual shows *pessimism* if $\alpha > 1/2$, whereas the individual shows *optimism* if $\alpha < 1/2$.

Our estimation proceeds with two specifications of utility function. First, we use the exponential utility function over monetary outcomes,

$$u(x) = -\frac{\exp(-\rho x)}{\rho},$$

where ρ represents the constant absolute risk aversion (CARA). Second, we use the power utility function over monetary outcomes,

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}.$$

In the latter case, the model is not able to generate the corner solutions that are often present in the data. In order to generate the corner solutions, the model requires an extra parameter to adjust a demand level. We will discuss this later.

Before we proceed, it is important to understand the implications of the RDU specification for individual behavior. Figure 1 below illustrates the indifference curves between the two securities, and Figure 2 illustrates the relationship between the log-price ratio $\ln(p_1/p_2)$

and the optimal relative demand $x_1^*/(x_1^*+x_2^*)$. The indifference curves have a *kink* at the 45-degree line, corresponding to portfolios with a certain outcome $x_1 = x_2$. The shape of the indifference curve on either side of the 45-degree line is determined by the individual's attitude toward risk and the nature of the kink is determined by the individual's pessimism/optimism measured by the parameter α. A pessimistic individual (black) $\alpha > \frac{1}{2}$ will choose safe portfolios satisfying $x_1 = x_2$ when the security prices, p_1 and p_2, are sufficiently similar. The effect of increasing the level of pessimism α is to make this intermediate range of prices larger. An optimistic individual (gray) $\alpha < \frac{1}{2}$ will not choose such portfolios, not even when the security prices are equal. In contrast, for an individual consistent with EUT $\alpha = \frac{1}{2}$ the indifference curves are smooth everywhere. Note that price vectors are randomly generated and give rise to portfolios satisfying $x_1 = x_2$ with probability zero if preferences are smooth.

[Figure 1 here]

[Figure 2 here]

II. CARA specification

A. Optimal demand

We can use the Lagrangian method to compute the optimal demand from the constrained maximization problem:

$$\max_{(x_1,x_2)} \alpha u\left(x_{\min}\right) + (1-\alpha)\, u\left(x_{\max}\right)$$
$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = 1; 0 \leq x_{\min} \leq x_{\max}.$$

Let $p_{\min}$ denote the minimum of two prices, p_1 and p_2. Similarly, $p_{\max}$ denotes the maximum of the two prices. The optimal portfolio allocation is characterized as below.

Case 1: $\ln\left(\frac{p_{\max}}{p_{\min}}\right) < \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$x_1^* = x_2^* = \frac{1}{p_1 + p_2}.$$

Case 2: $\ln\left(\frac{\alpha}{1-\alpha}\right) < \ln\left(\frac{p_{\max}}{p_{\min}}\right) < \frac{\rho}{p_{\min}} + \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$x^*_{\min} = \frac{1}{p_1+p_2} - \frac{p_{\min}}{\rho\left(p_1+p_2\right)}\left[\ln\left(\frac{p_{\max}}{p_{\min}}\right) - \ln\left(\frac{\alpha}{1-\alpha}\right)\right]$$
$$x^*_{\max} = \frac{1}{p_1+p_2} + \frac{p_{\max}}{\rho\left(p_1+p_2\right)}\left[\ln\left(\frac{p_{\max}}{p_{\min}}\right) - \ln\left(\frac{\alpha}{1-\alpha}\right)\right]$$

Case 3: $\ln\left(\frac{p_{\max}}{p_{\min}}\right) > \frac{\rho}{p_{\min}} + \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$x^*_{\min} = 0 \text{ and } x^*_{\max} = 1/p_{\min}.$$

Several remarks are in order.

- Case 1 occurs only when $\alpha > 1/2$. That is, if the individual chooses the equal amount of two assets in prices other than $p_1 = p_2$, the individual shows some degree of disappoint aversion unless he is extremely risk averse.
- When $\alpha < 1/2$ and $p_1 = p_2$, the optimal portfolio is not unique due to the nonconvexity of preferences. Depending on whether the conditions of Case 2 or 3 are met, $(x^*_1, x^*_2) = (x^*_{\min}, x^*_{\max})$ or $(x^*_{\max}, x^*_{\min})$.

III. CRRA specification

A. Optimal demand

We assume a parameter $\omega > 0$ inside the CRRA utility function which is positive and can be interpreted as a demand shifter with some reason such as a baseline level of consumption. This leads us to the following problem of utility maximization.

$$\max_{(x_1,x_2)} \alpha u\left(x_{\min}+\omega\right) + (1-\alpha)\, u\left(x_{\max}+\omega\right)$$
$$\text{s.t.} \quad p_1x_1 + p_2x_2 = 1; 0 \le x_{\min} \le x_{\max},$$

where $u(x) = x^{1-\gamma}/\left(1-\gamma\right)$. We assume that ω is so small that most curvature is attributed to the relative risk aversion coefficient. The optimal portfolio allocation is characterized as below.

Case 1: $\ln\left(\frac{p_{\max}}{p_{\min}}\right) < \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$x_1^* = x_2^* = \frac{1}{p_1 + p_2}.$$

Case 2: $\ln\left(\frac{\alpha}{1-\alpha}\right) < \ln\left(\frac{p_{\max}}{p_{\min}}\right) < \gamma \ln\left(\frac{1+\omega p_{\min}}{\omega p_{\min}}\right) + \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$\begin{aligned} x_{\min}^* &= \frac{K\left[1 + (1-K)\,\omega p_{\max}\right]}{K p_{\max} + p_{\min}} - (1-K)\,\omega \\ x_{\max}^* &= \frac{1 + (1-K)\,\omega p_{\max}}{K p_{\max} + p_{\min}}, \end{aligned}$$

where

$$K \equiv \left[\left(\frac{p_{\min}}{p_{\max}}\right)\left(\frac{\alpha}{1-\alpha}\right)\right]^{1/\gamma}.$$

Case 3: $\ln\left(\frac{p_{\max}}{p_{\min}}\right) > \gamma \ln\left(\frac{1+\omega p_{\min}}{\omega p_{\min}}\right) + \ln\left(\frac{\alpha}{1-\alpha}\right)$

$$x_{\min}^* = 0 \text{ and } x_{\max}^* = 1/p_{\min}.$$

IV. Estimation

Let's use the CARA specification. We can then estimate $\hat{\alpha}_n$ and $\hat{\rho}_n$ for each individual n by minimizing the Euclidean distance between observed data and optimal choices.