

Rationality and Preference Aggregation of Group Decision under Risk

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Introduction

- In various contexts, many important decisions are made by groups.
- Individual heterogeneity exists in various dimensions:
 - Risk preference: risk assessment in environmental policy
 - Time preference: household savings and consumption decisions
 - Rationality
- It is important to understand how individual heterogeneity in a collective influences final outcomes.

Introduction: Research Questions

1. Rationality extension:

- Do rational members make more collectively rational decisions?

2 Risk preference aggregation:

- Are individual's risk preferences reflected into that of a group?

3. Efficiency and welfare:

- How is the efficiency of group decisions related to individual's rationality and preferences?
- How is social welfare related to individual' rationality and preferences?

Introduction: Examples of Individual Heterogeneity

- Rationality
 - High-income, high-education, men, and young subjects tend more toward utility maximization (Choi et al., 2014).
- Risk preference
 - White males are more likely to perceive risks as being smaller (Bickerstaff, 2004; Flynn et al., 1994).
 - There is no substantial difference between men and women (Kagel and Roth, 2016).
 - Subjects' risk preferences are closer to neutrality when they make decisions on behalf of other participants (Batteux et al., 2017).
 - High-power groups adopt a more positive attitude toward potential risks (Anderson and Galinsky, 2006; Magee et al., 2007; Geng et al, 2018).

Experimental Design

Experimental Design (Choi et al., 2007; Choi et al., 2014)

x_b

Two **equally likely** states: R and B .

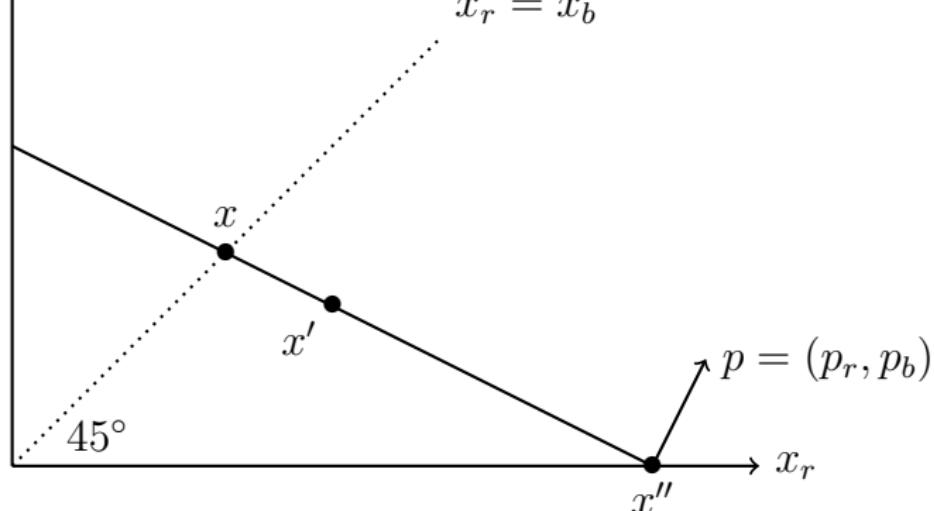
There are two associated Arrow securities.

x_r is the demand for the security that pays off in state R .

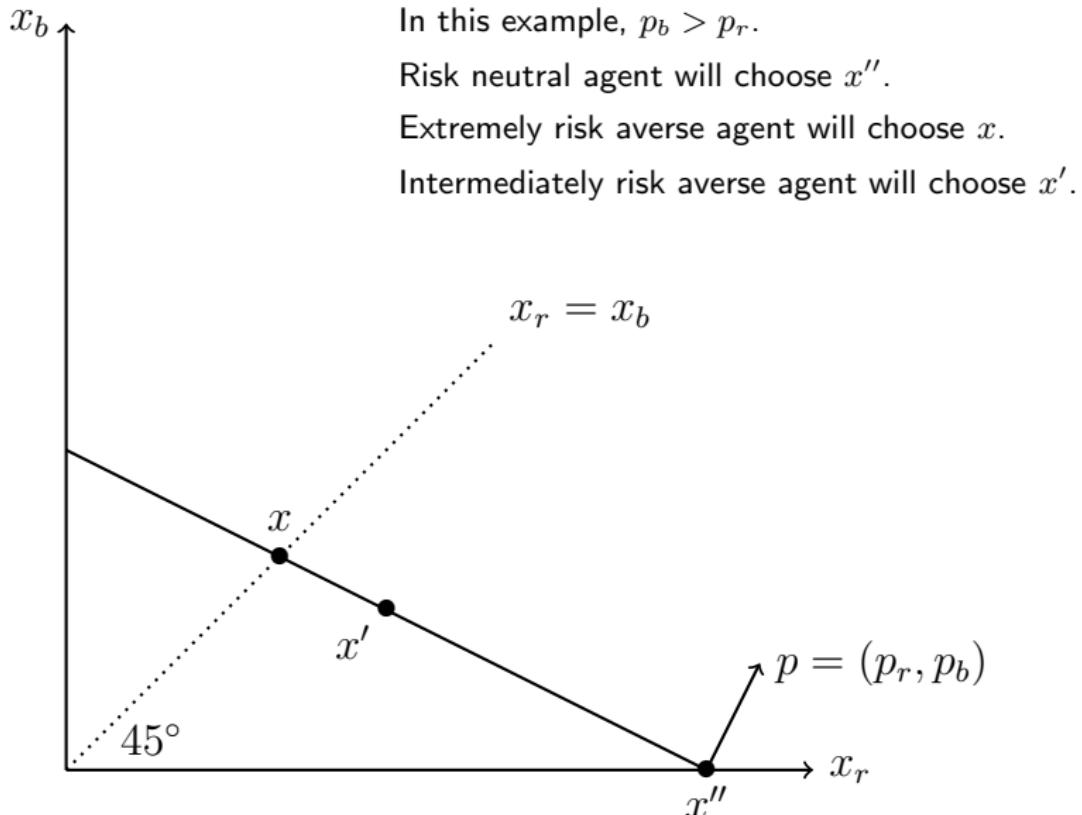
x_b is the demand for the security that pays off in state B .

Budget constraint: $p_r x_r + p_b x_b = 1$.

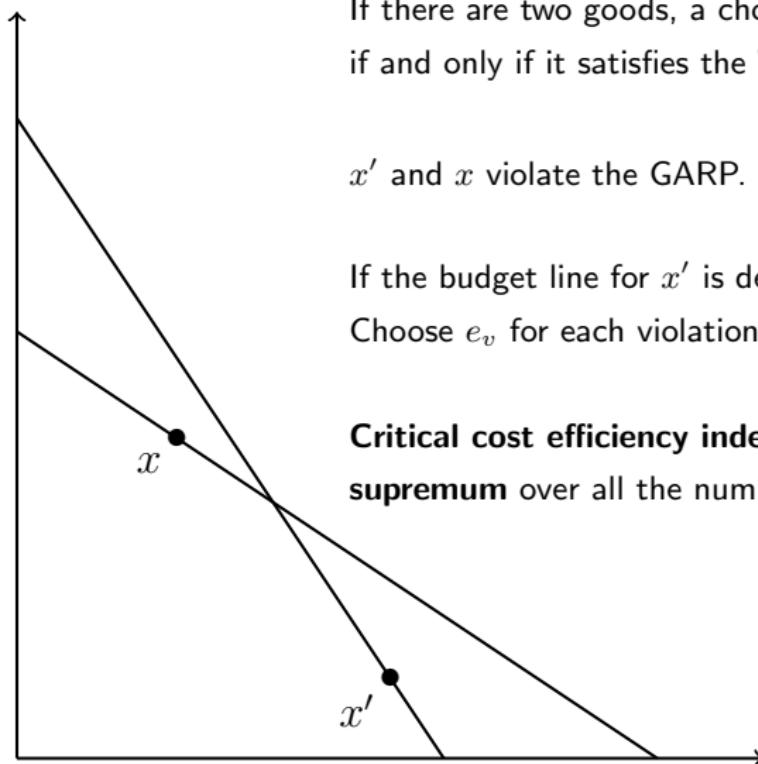
$$x_r = x_b$$



Experimental Design (Choi et al., 2007; Choi et al., 2014)



Measurement: Afriat's Efficiency Index (a.k.a. CCEI)



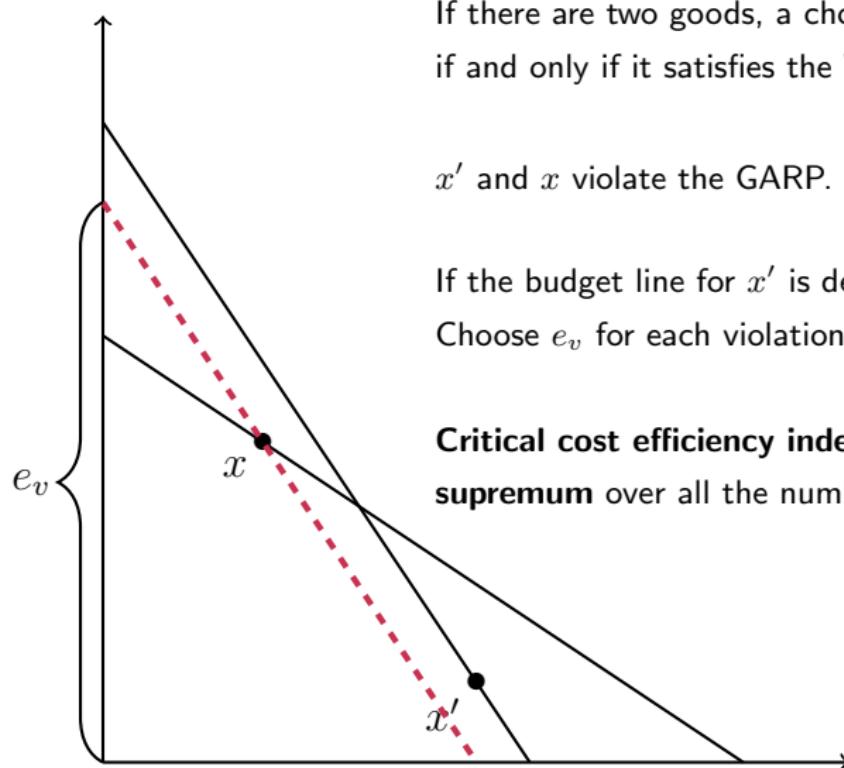
If there are two goods, a choice dataset satisfies the GARP if and only if it satisfies the WARP.

x' and x violate the GARP.

If the budget line for x' is deflated, the GARP is satisfied.
Choose e_v for each violation v .

Critical cost efficiency index (CCEI) is defined as the **supremum** over all the numbers e_v 's.

Measurement: Afriat's Efficiency Index (a.k.a. CCEI)



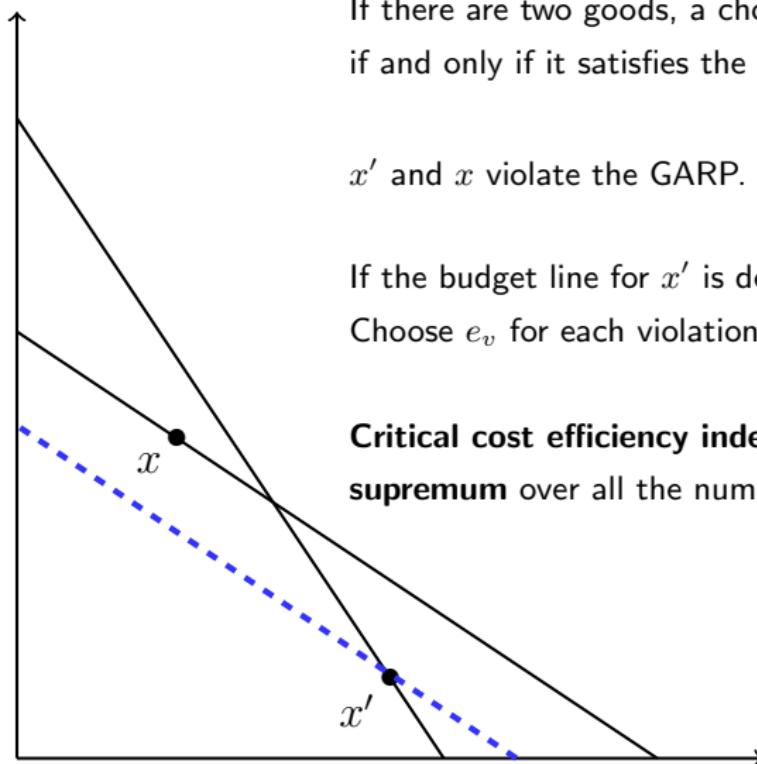
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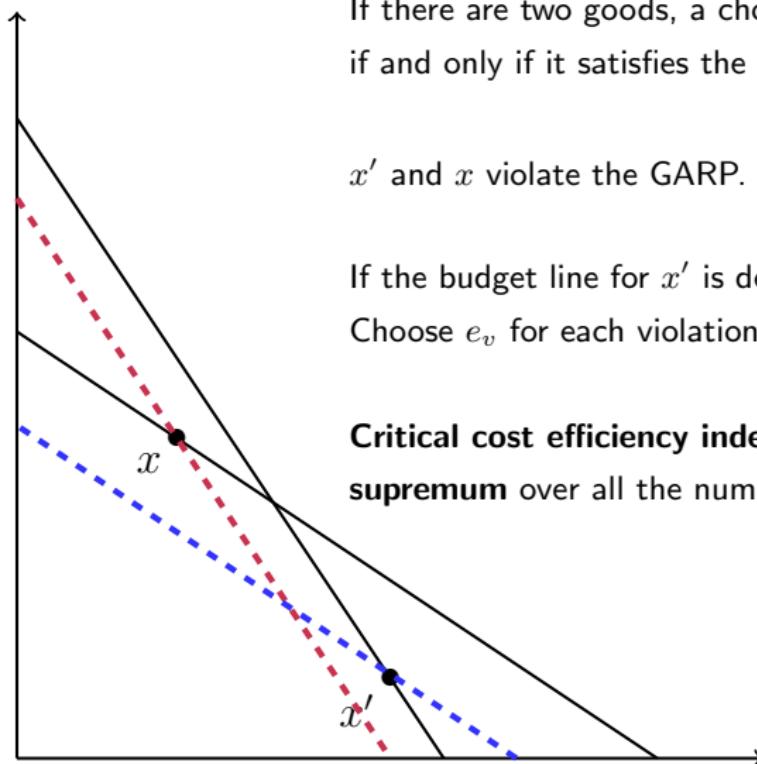


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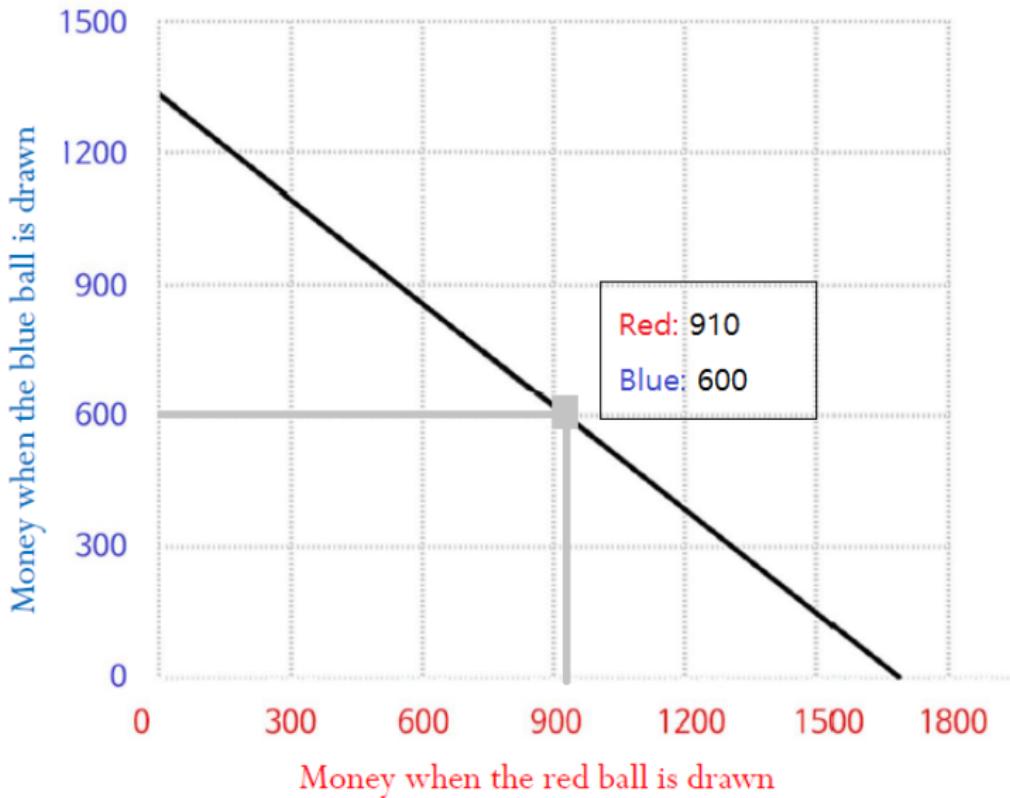
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Screenshot



Field



Procedure and Subjects

- We conducted the experiment in 12 middle schools in Daegu.
- The number of students: 1572.
- The number of groups: 786.
- The instructions were read by an experimenter in each classroom.
- Each subject participated in two sessions: individual and group decisions.

Procedure and Subjects

- Each round started by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above 300 KRW or below 1500 KRW.
- Each session consisted of 18 independent decision rounds.
- At the end of each round, the computer randomly selected one of the two states (*R* and *B*).
- Subjects were not informed of the state that was selected at the end of each round.

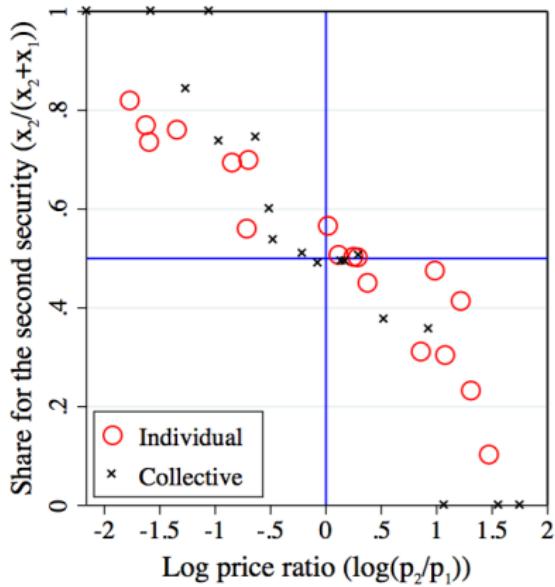
Procedure and Subjects

- Two students in the same classroom were randomly matched.
- One of the two students was randomly chosen to move to the other partner's desk.
- They made a series of collective decisions by sharing one computer.
- We allowed students to discuss how to make decisions for 1 min before starting the second session.
- Each subject was paid for he/she earned in a randomly selected round.

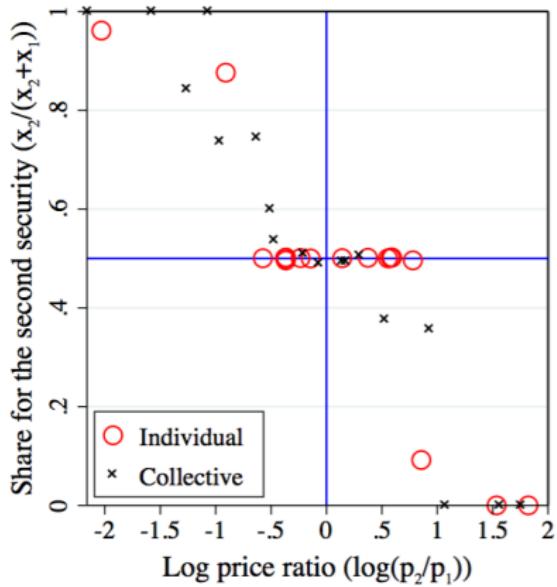
Example of Choice Data

Group ID: 284

Collective CCEI: 1.00
Risk Preference: 0.27, DAU



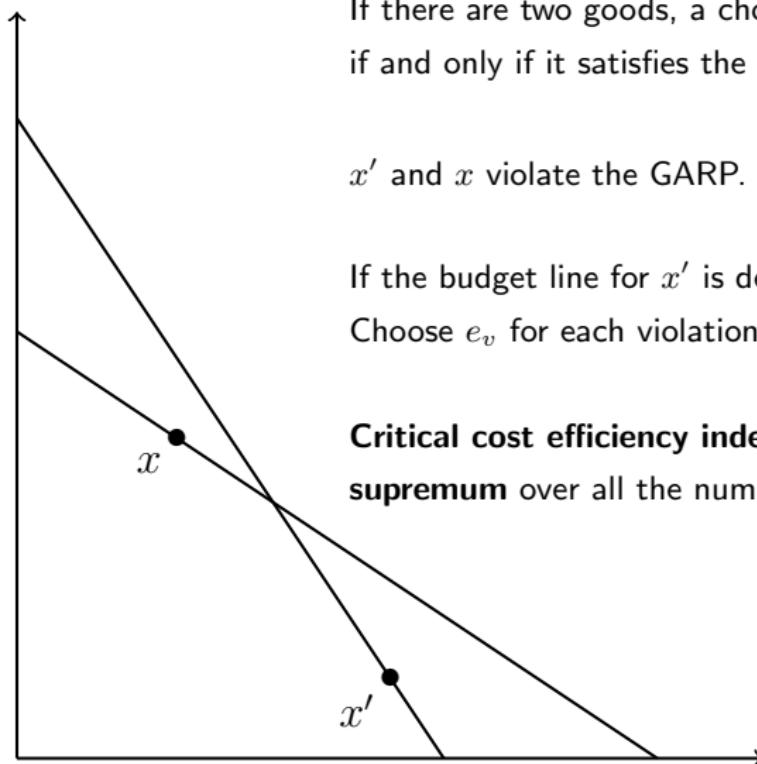
Individual CCEI: 1.00
Risk Preference: 0.35, EU
Id: 1410707



Individual CCEI: 1.00
Risk Preference: 0.38, DAU
Id: 1410721

Result 1: Rationality Extension

Measurement: Afriat's Efficiency Index (a.k.a. CCEI)



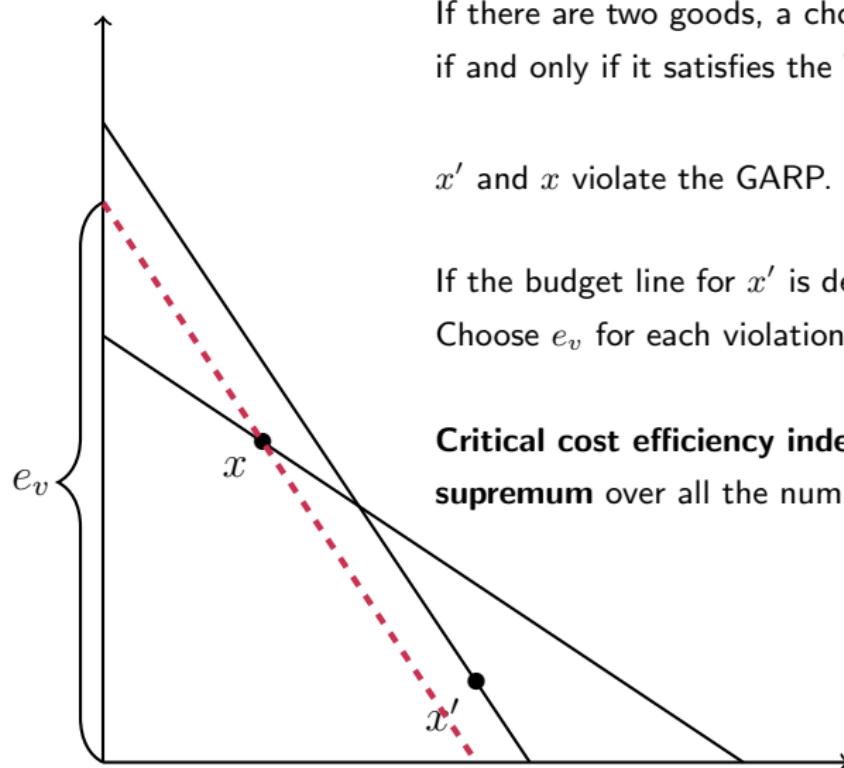
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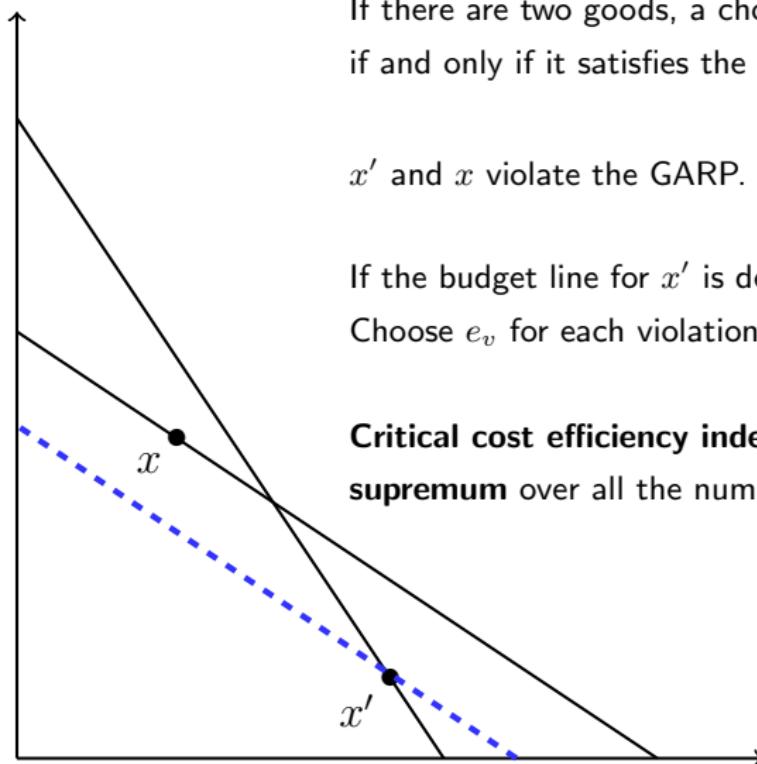


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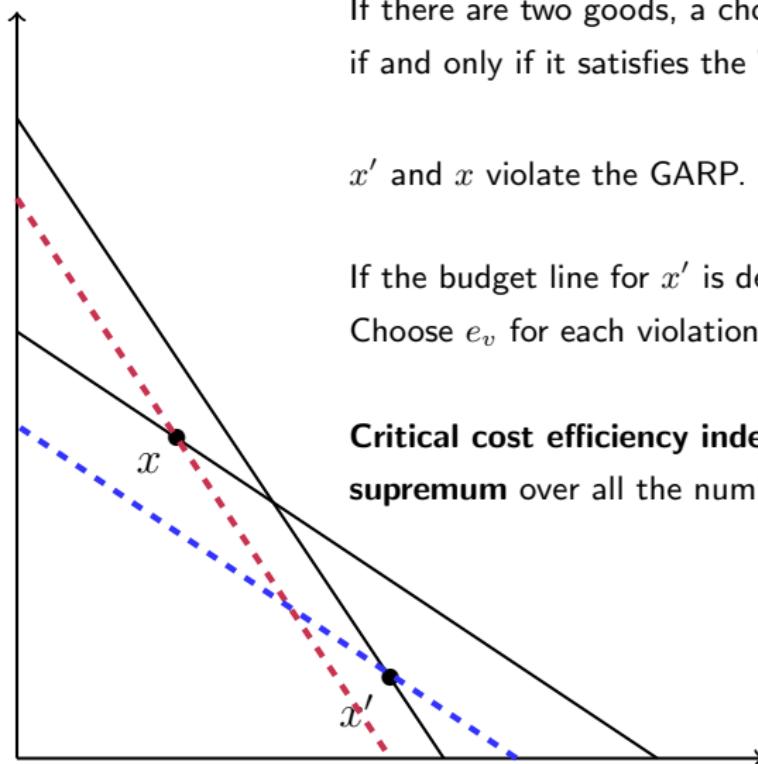
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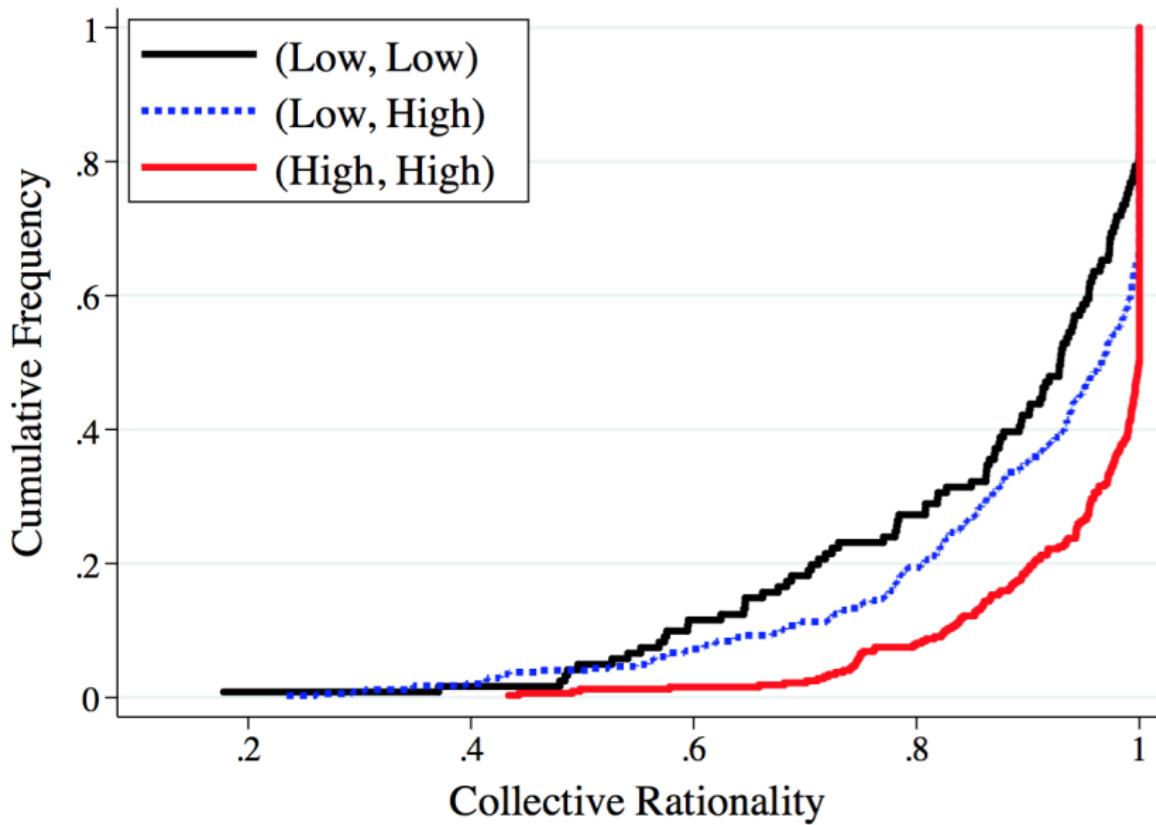
Measurement: Afriat's Efficiency Index (a.k.a. CCEI)

- By definition, $\text{CCEI} \in [0, 1]$.
- The **bigger** CCEI is, the **less** severe violation of GARP.
- Basic statistics of individual CCEI:
 - Average: 0.897 (0.136)
 - Quantiles: 0.838 (25%), 0.953 (50%), 1 (75%).
- Basic statistics of collective CCEI:
 - Average: 0.910 (0.141)
 - Quantiles: 0.868 (25%), 0.981 (50%), 1 (75%).

Rationality Extension: Research Question

Individual Rationality $\uparrow \Rightarrow$ Collective Rationality $\uparrow?$

Rationality Extension: First-Order Stochastic Dominance



Rationality Extension: First-Order Stochastic Dominance

- We do a series of Kolmogorov-Smirnov tests:

$$H_0 : F_{\text{group } i}(X) = F_{\text{group } j}(X) \quad \text{for all values of } X.$$

- Test statistic: $D_{ij} = \sup_{x \in X} ||F_{\text{group } i}(x) - F_{\text{group } j}(x)||$.

- (Low, Low) v.s. (High, High): 0.17

- The corresponding p-value is 0.01.

- (Low, High) v.s. (High, High): 0.21

- The corresponding p-value is 0.00.

Rationality Extension: Econometric Analysis

Collective CCEI	Coefficient		
	Model 1	Model 2	Model 3
CCEI_Max	0.368*** (0.083)	0.327*** (0.074)	0.302*** (0.089)
CCEI_Distance	-0.277*** (0.056)	-0.250*** (0.053)	-0.233*** (0.058)
Risk_Aversion_Max		-0.189*** (0.056)	-0.172** (0.070)
Risk_Aversion_Distance		0.087* (0.048)	0.093* (0.055)
Math_Score_Max			0.012** (0.005)
Math_Distance			-0.010** (0.005)
Constant	0.582*** (0.077)	0.679*** (0.070)	0.664*** (0.084)
Class Fixed Effect	Yes	Yes	Yes
Individual Characteristics	No	No	Yes
School Characteristics	No	No	Yes
Friendship	No	No	Yes
Observations	786	786	786
R-squared	0.200	0.212	0.235

*Throughout the paper, we clustered standard error in class level.

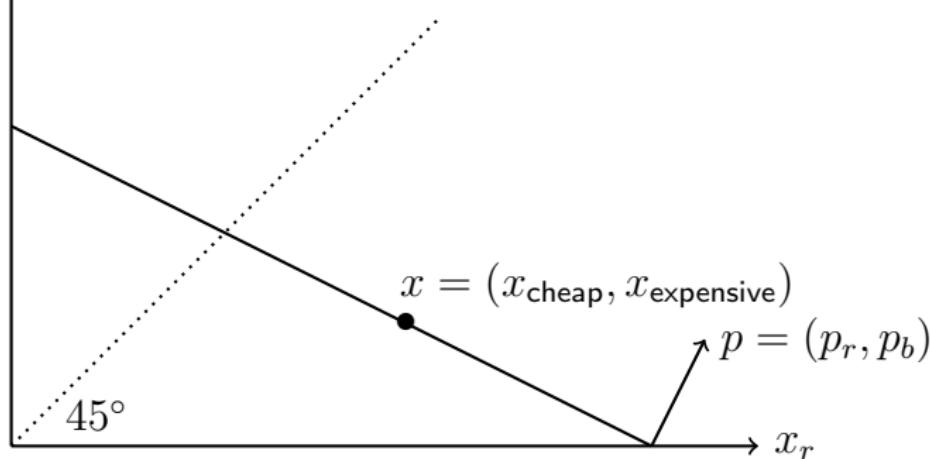
Result 2: Preference Aggregation

Measurement: Indifference Curves

x_b

We non-parametrically measure the risk preference by a ratio:

$$\text{risk aversion (RA)} = \frac{x_{\text{expensive}}}{x_{\text{expensive}} + x_{\text{cheap}}}$$



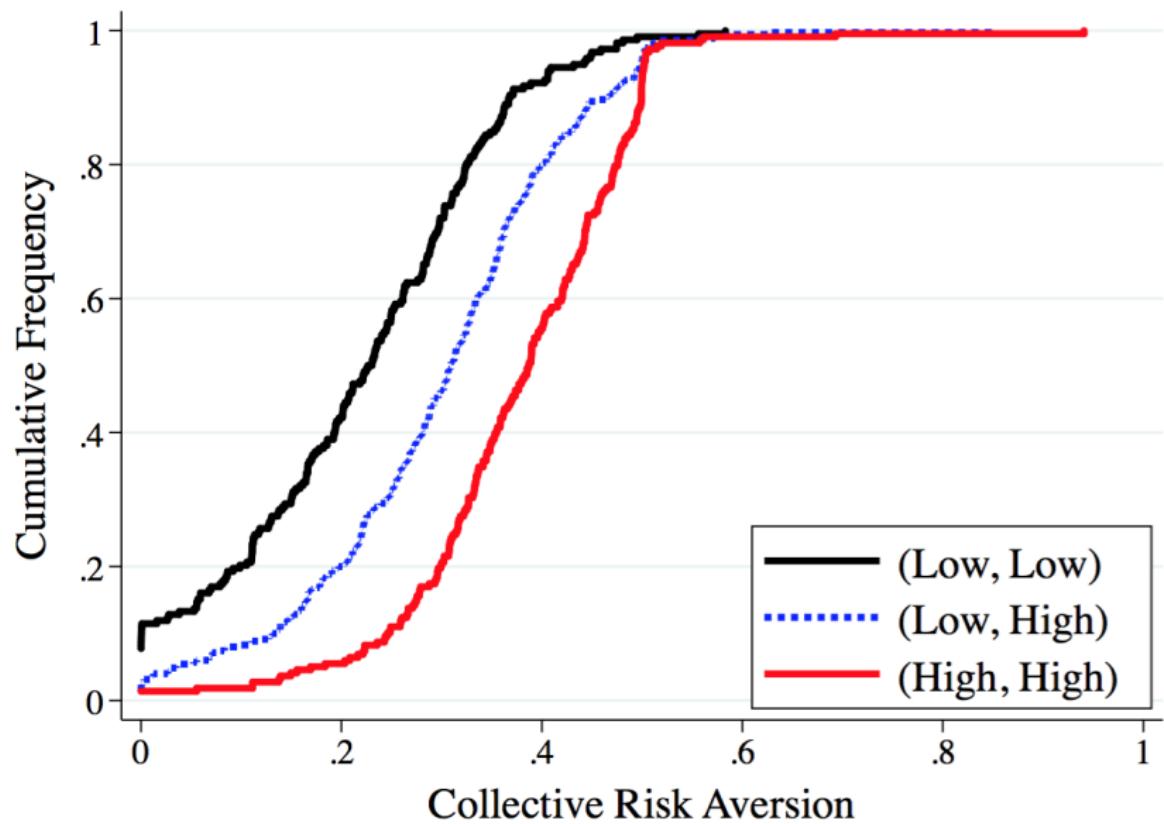
Measurement: Risk Preferences

- By definition, $RA \in [0, 1]$.
- The **bigger** ratio is, the **higher** risk aversion.
- Basic statistics of individual RA:
 - Average: 0.324 (0.132).
 - Quantiles: 0.213 (25%) , 0.310 (50%), 0.390 (75%), 0.499 (99%).
- Basic statistics of collective RA:
 - Average: 0.298 (0.139).
 - Quantiles: 0.255 (25%), 0.348 (50%), 0.413 (75%), 0.497 (99%).

Risk Preference Aggregation: Research Question

Individual risk aversion $\uparrow \Rightarrow$ Collective risk aversion \uparrow ?

Preference Aggregation: FOSD by Relative Ratio



Preference Aggregation: Econometric Analysis

Collective Risk Aversion	Coefficient		
	Model 1	Model 2	Model 3
Risk_Aversion_Max	0.792*** (0.066)	0.808*** (0.063)	0.759*** (0.073)
Risk_Aversion_Distance	-0.421*** (0.053)	-0.432*** (0.053)	-0.434*** (0.062)
CCEI_Max		0.165** (0.064)	0.196*** (0.071)
CCEI_Distance		0.014 (0.033)	0.040 (0.045)
Math_Score_Max			0.000 (0.005)
Math_Distance			0.005 (0.004)
Constant	0.004 (0.027)	-0.156** (0.063)	-0.175** (0.077)
Class Fixed Effect	Yes	Yes	Yes
Individual Characteristics	No	No	Yes
School Characteristics	No	No	Yes
Friendship	No	No	Yes
Observations	786	786	786
R-squared	0.372	0.378	0.382

Result 3: Efficiency and Welfare

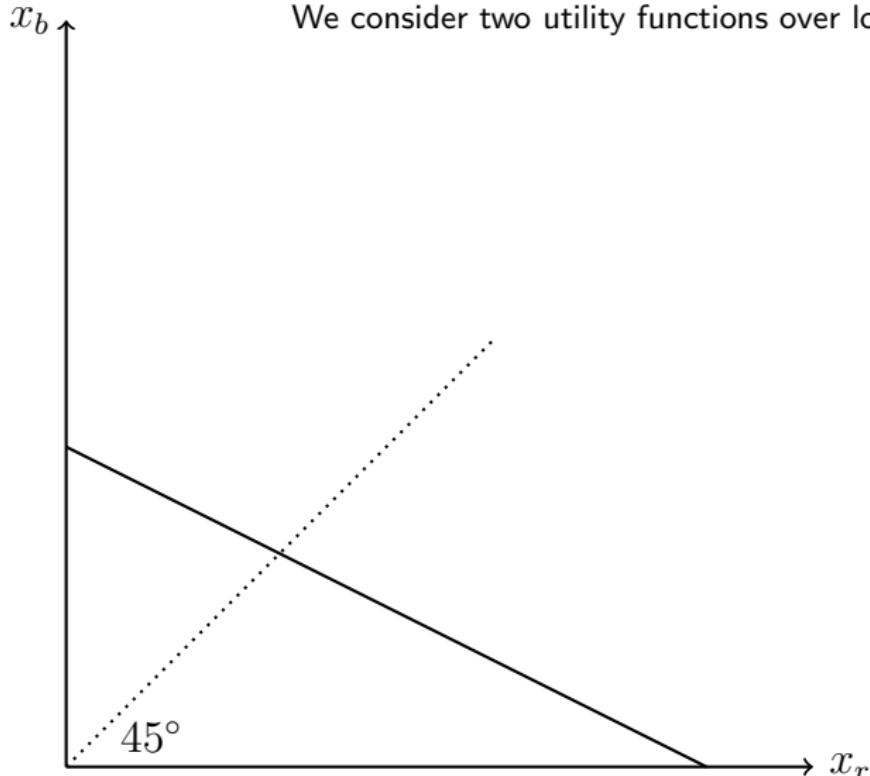
Measurement: Idea

- We analyze the **quality** of collective decisions as a function of the degrees of rationality and preference alignment.
- Idea:
 - We consider a class of utility functions over lotteries.
 - For each subject, we estimate the utility function parametrically.
 - We characterize a set of Pareto efficient choices.
 - For collective choices which are **not** Pareto efficient, we measure the degree of welfare loss.

Measurement: Utility Estimation

We assume a CARA utility function: $u(x) = -e^{-\rho x} / \rho$.

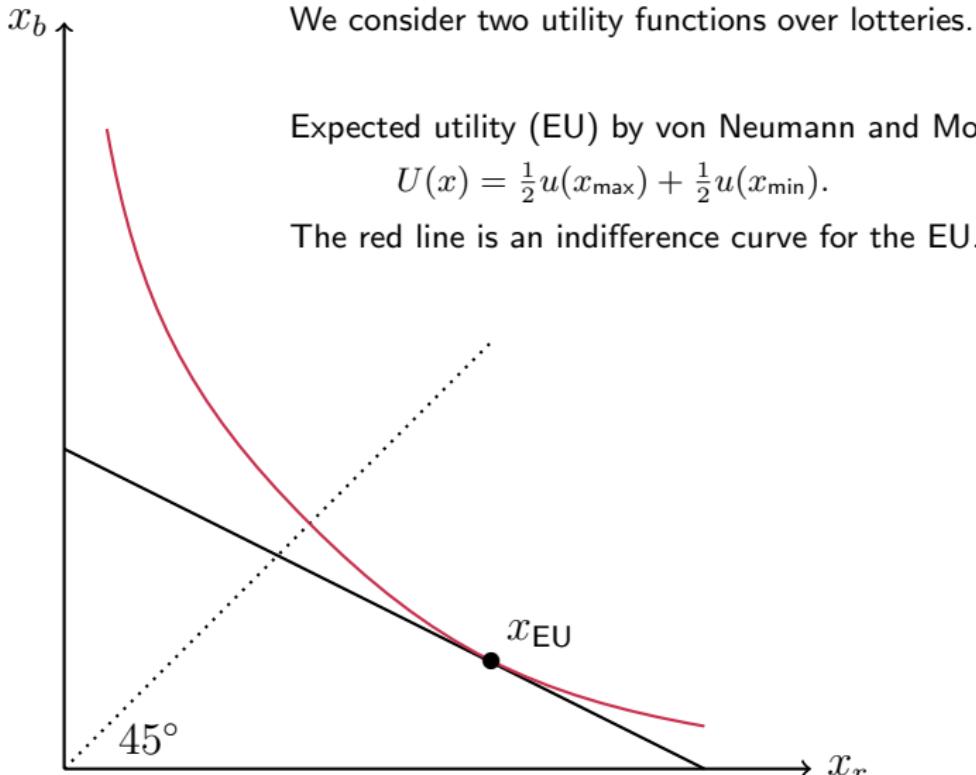
We consider two utility functions over lotteries.



Measurement: Utility Estimation

We assume a CARA utility function: $u(x) = -e^{-\rho x} / \rho$.

We consider two utility functions over lotteries.



Expected utility (EU) by von Neumann and Morgenstern (1953):

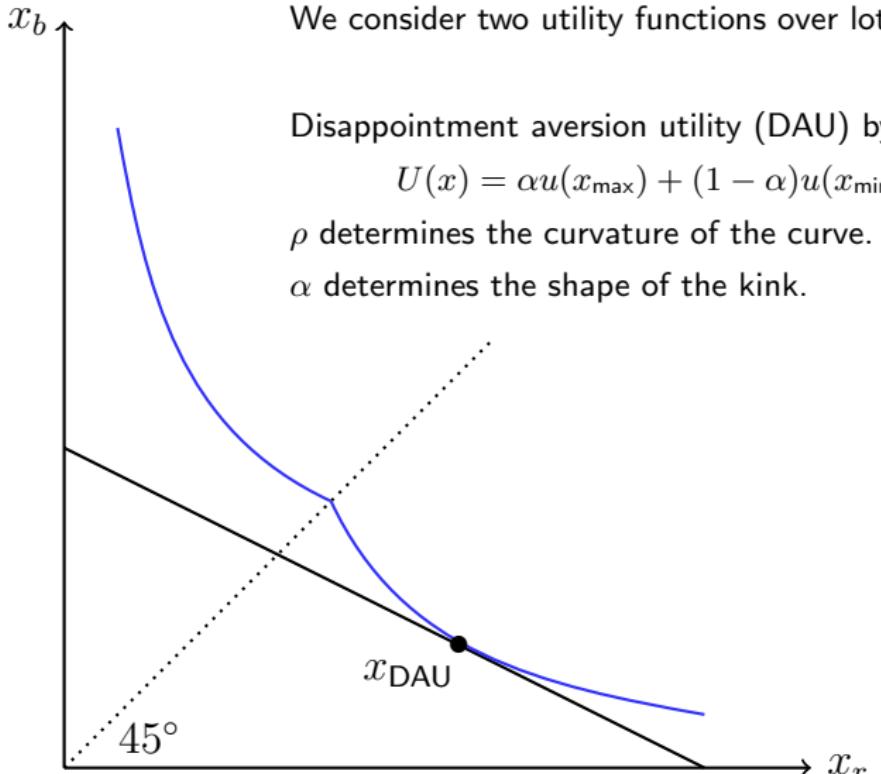
$$U(x) = \frac{1}{2}u(x_{\max}) + \frac{1}{2}u(x_{\min}).$$

The red line is an indifference curve for the EU.

Measurement: Utility Estimation

We assume a CARA utility function: $u(x) = -e^{-\rho x} / \rho$.

We consider two utility functions over lotteries.



Disappointment aversion utility (DAU) by Gul (1991):

$$U(x) = \alpha u(x_{\max}) + (1 - \alpha)u(x_{\min}).$$

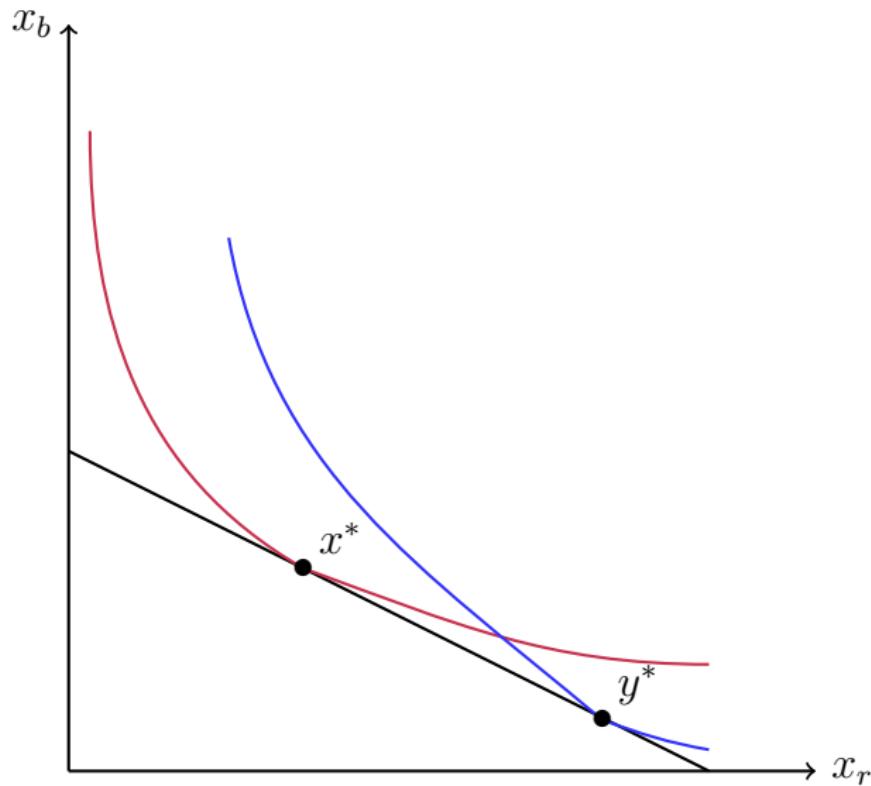
ρ determines the curvature of the curve.

α determines the shape of the kink.

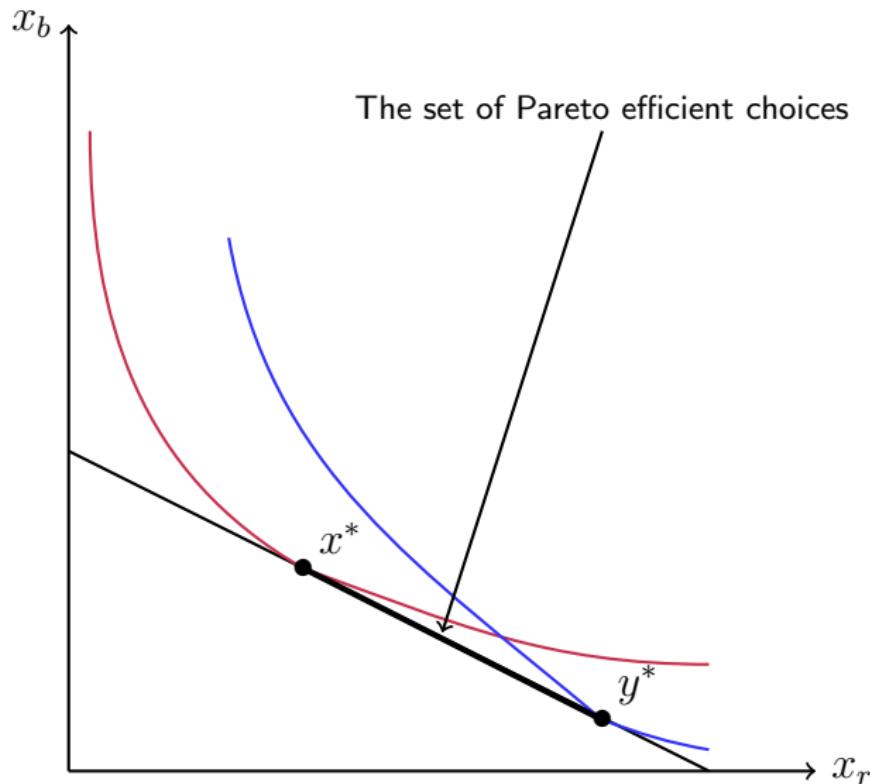
Measurement: Utility Estimation

- We restrict our attention to a CARA utility function over outcomes.
- We consider two different types of utility function over lotteries:
 - Expected utility (EU)
 - Disappointment aversion utility (DAU).
- We estimate ρ and β simultaneously by using a combination of a bootstrapping and the non-linear least square (NLLS) methods:
 - 1 Find subsample of size 18 with replacement.
 - 2 For given subsample, estimate α and ρ by NLLS.
 - 3 Repeat the above for 250 times.
 - 4 If $0.5 \in [\alpha_{2.5}, \alpha_{97.5}]$, then set $\alpha = 0.5$ as an EU.
 - 4' Otherwise, set $\alpha = \bar{\alpha}$ as a DAU.

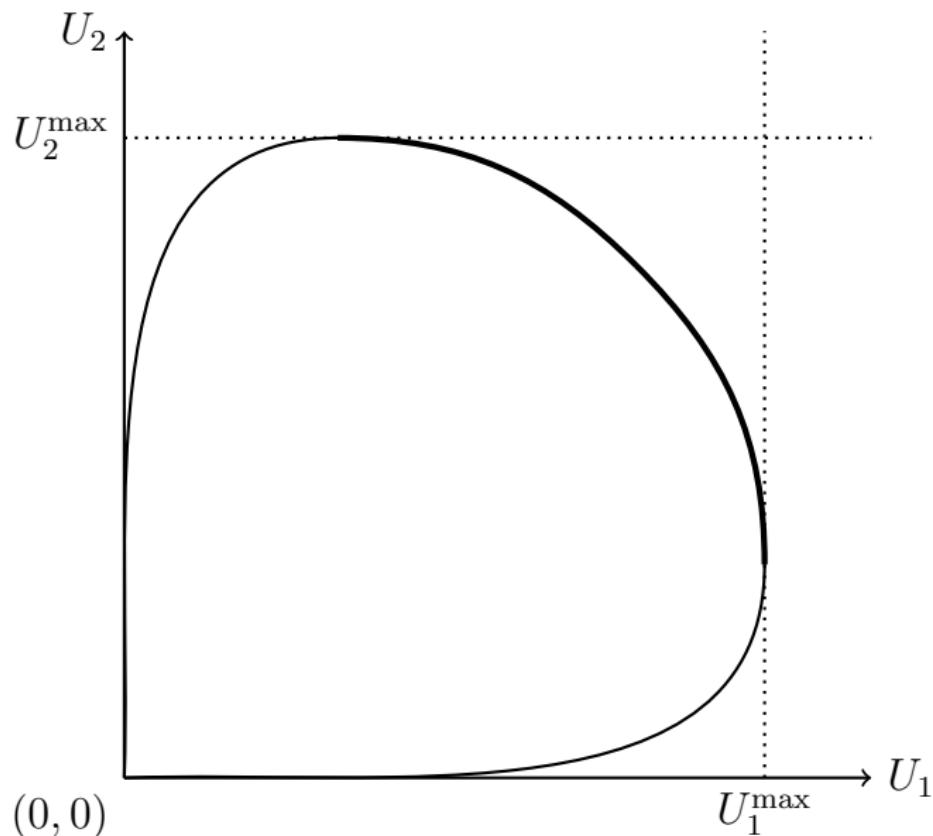
Measurement: Efficiency and Welfare Loss



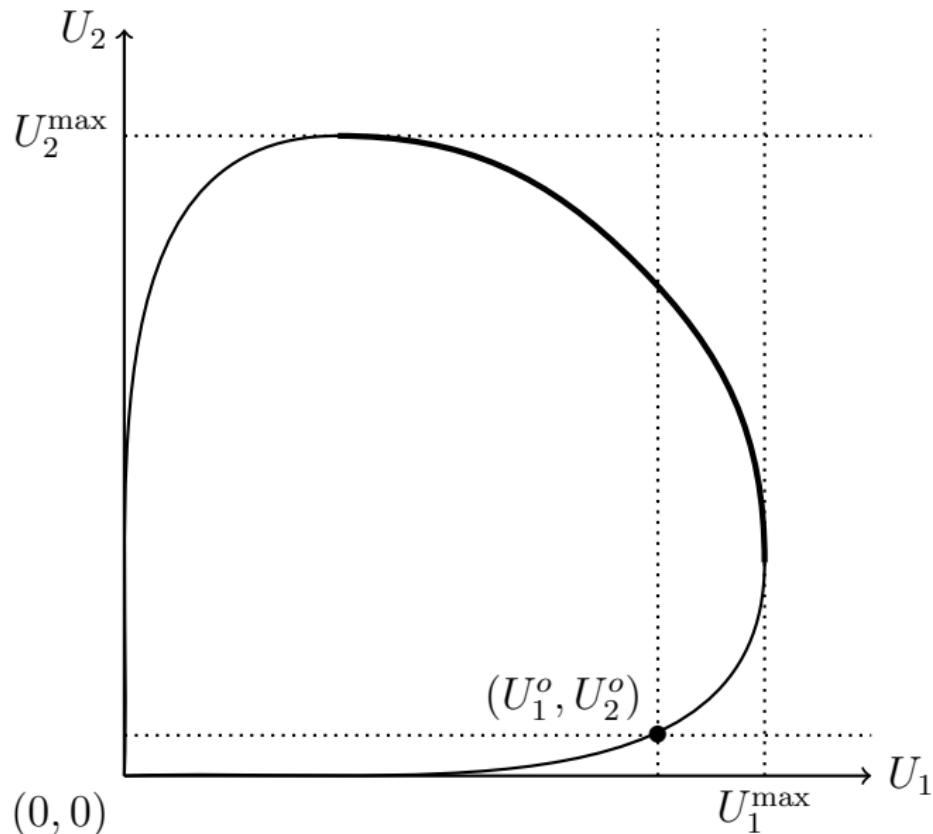
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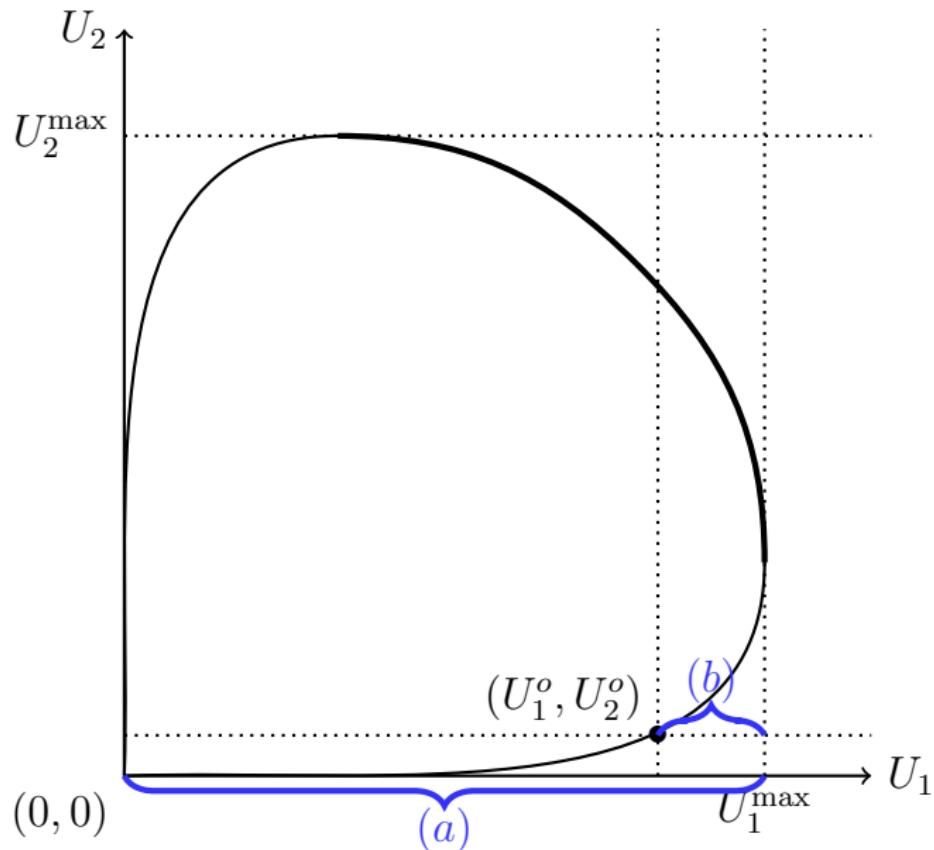
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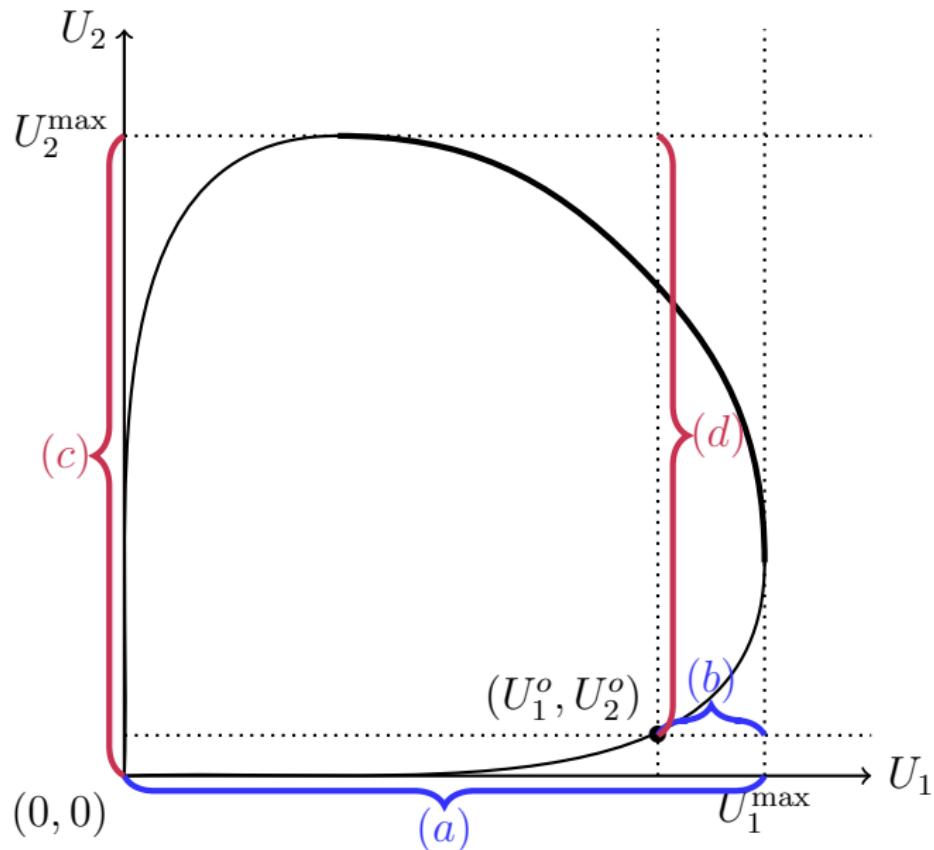
Measurement: Efficiency and Welfare Loss



Measurement: Efficiency and Welfare Loss



Measurement: Efficiency and Welfare Loss



Efficiency and Welfare: Measurement

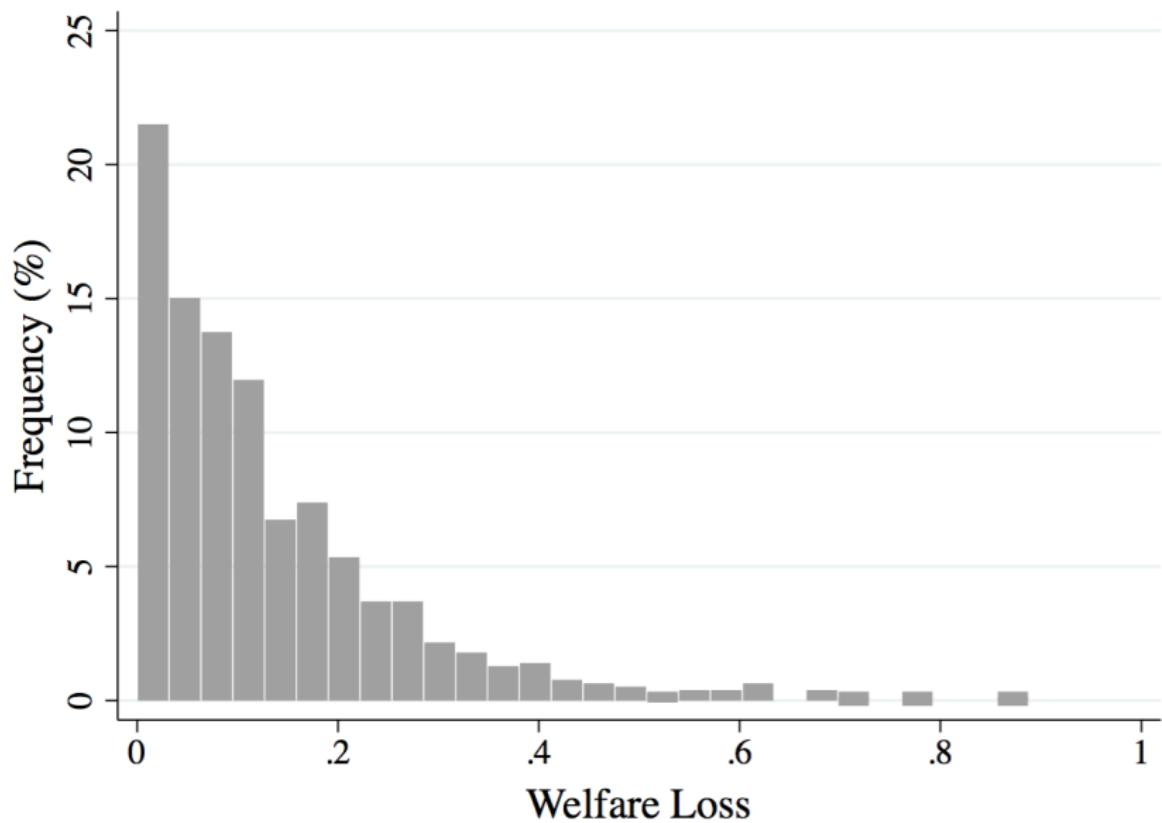
- We focus on the group choices which are not Pareto efficient (60%).
- For those choices, we measure **welfare loss** of a group as

$$\text{Welfare Loss} = \frac{1}{18} \sum_{k=1}^{18} \frac{1}{2} \sum_{i=1}^2 \frac{U_i(x_{ikb}) - U_i(x_{ikc})}{U_i(x_{ikb}) - U_i(x_{ikw})},$$

where

- x_{ikc} : group choice in k -th round
 - x_{ikb} : member i 's best choice in k -th round
 - x_{ikw} : member i 's worst choice in k -th round.
-
- By definition, Welfare Loss $\in [0, 1]$.

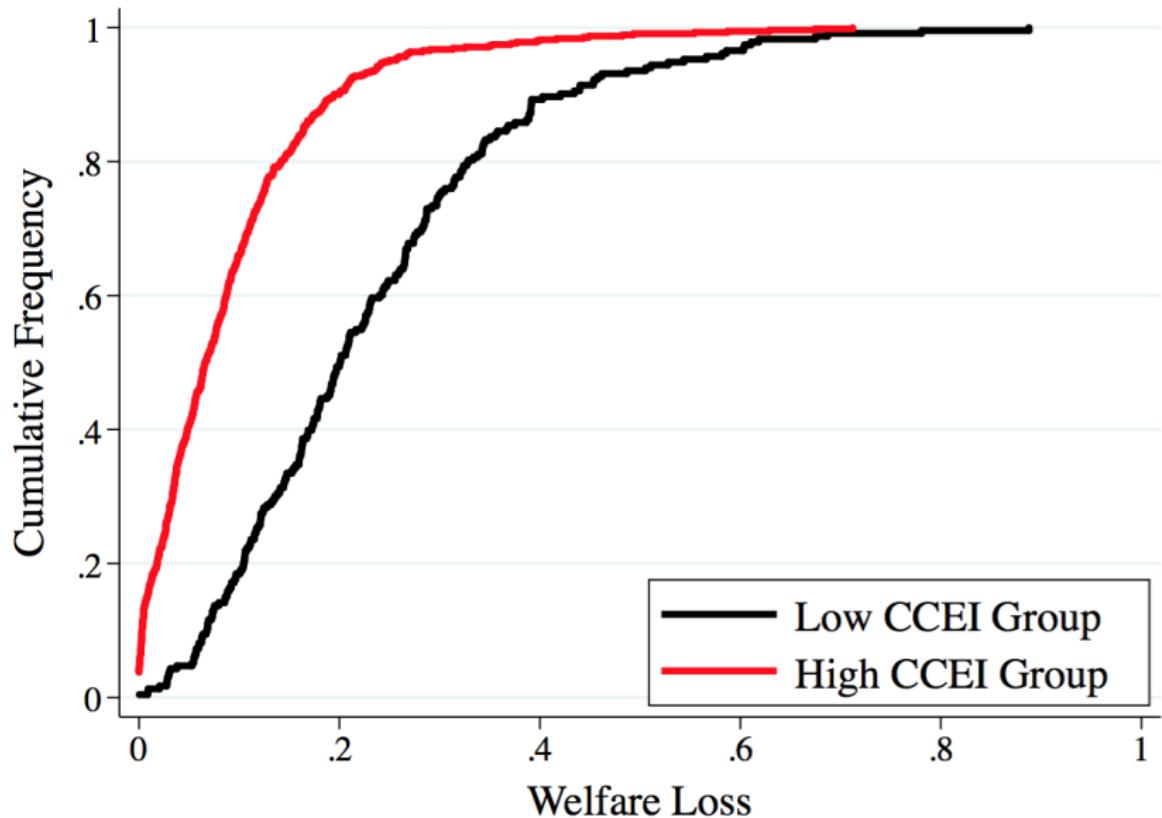
Efficiency and Welfare: Distribution of Welfare Loss



Efficiency and Welfare: Research Question

How is the welfare loss related to individual rationality and risk preference?

Welfare: FOSD by Group Rationality



Welfare: Econometric Analysis

Group Inefficiency	Coefficient			
	Model 1	Model 2	Model 3	Model 4
CCEI_Group	-0.571*** (0.042)	-0.503*** (0.039)	-0.527*** (0.043)	-0.414*** (0.078)
CCEI_Max		-0.296*** (0.045)	-0.242*** (0.051)	-0.692 (0.671)
CCEI_Distance		0.165*** (0.042)	0.178*** (0.052)	0.289 (0.257)
Risk_Aversion_Max		-0.009 (0.056)	0.023 (0.063)	-0.024 (0.089)
Risk_Aversion_Distance		-0.057* (0.031)	-0.073* (0.040)	-0.123* (0.066)
Math_Score_Max			0.002 (0.005)	0.008 (0.007)
Math_Distance			-0.004 (0.003)	-0.010 (0.006)
Constant	0.651*** (0.038)	0.866*** (0.048)	0.807*** (0.061)	1.154* (0.653)
Class Fixed Effect	Yes	Yes	Yes	Yes
Individual Characteristics	No	No	Yes	Yes
School Characteristics	No	No	Yes	Yes
Friendship	No	No	Yes	Yes
Observations	786	786	786	274
R-squared	0.442	0.487	0.497	0.436

Conclusion

- We measure rationality and risk preference in individual and group levels.
- We observe rationality extension and preference aggregation.
- We develop a measure of efficiency and utility loss of group decisions.
- We find that
 - Rational groups are more likely to make efficient decisions.
 - Preference-aligned individuals need not make efficient decisions.

Conclusion

- Our main findings are robust with respect to
 - another rationality measure (Varian's efficiency index)
 - other cutoff values of CCEI (0.99 or 0.95)
 - another measure of risk preferences (risk premium).

Robustness

Varian's Efficiency Index

- Varian modifies CCEI by allowing e_k to vary across the different price vectors.
- Consider a vector $\theta = (e^k)_{k=1}^K$ of numbers in $[0, 1]$, one for each observation.
- Define the binary relation R_θ as $x^k R_\theta x^l$ if $e^k p^k \cdot x^k \geq p^k \cdot x^l$. Let P_θ be the corresponding strict relation.
- There is a set Θ of vectors θ such that the corresponding $\langle R_\theta, P_\theta \rangle$ satisfies GARP.
- Varian's efficiency index (VEI) is the closest distance of a vector θ to the unit vector ($e_k = 1$ for all k), among those θ for which the preference pair is acyclic:

$$\text{VEI} = \inf \left\{ \|1 - \theta\| \mid \langle R_\theta, P_\theta \rangle \text{ is acyclic} \right\}.$$

Disappointment Aversion Utility

- A functional form:

$$u(x_1, x_2) = u(\max\{x_1, x_2\}) + (1 - \gamma)u(\min\{x_1, x_2\}),$$

where $\gamma = 1/(2 + \beta)$.

- $\beta > 0$ represents the **disappointment aversion**: the better outcome is under-weighted relative to the objective probability.
- $\beta \in (-1, 0)$ represents **elation seeking**: the better outcome is over-weighted relative to the objective probability.
- Of course, this utility function is aligned with the first-order stochastic dominance relationships between lotteries.