The Non-Existence of Representative Agents

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Abstract

We characterize environments in which there exists a representative agent: an agent who inherits the structure of preferences of the population that she represents. The existence of such a representative agent imposes strong restrictions on individual utility functions—requiring them to be *linear* in the allocation and additively separable in any parameter that characterizes agents' preferences (e.g., a risk aversion parameter, a discount factor, etc.). Commonly used classes of utility functions (exponentially discounted utility functions, CRRA or CARA utility functions, logarithmic functions, etc.) do not admit a representative agent.

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1 Introduction

1.1 Overview

Groups of people, in aggregate, can behave very differently from individuals. For example, the classic Sonnenschein-Mantel-Debreu Theorem (Sonnenschein, 1973; Mantel, 1974; Debreu, 1974) illustrated that even if individuals each satisfy standard conditions on their demand functions, some of the most vital of those conditions—e.g., the weak axiom of revealed preference—are lost when demands are aggregated.

This can be problematic, a model that admits arbitrary aggregate behavior is hard to work with. Thus, a usual approach is to assume aggregate behavior that is well-behaved,

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implicitly presuming that agents in the underlying economy satisfy certain necessary restrictions for the model to be consistent. In particular, the literature has often assumed the existence of a well-behaved *representative agent*, one whose choices or preferences reflect the aggregate choices of society. The notion itself can be traced back to Edgeworth (1881) and Marshall (1890).[1] Since the publication of the Lucas Critique (1976), micro-founding economic models has become pervasive. Given the challenges of analyzing heterogeneous societies, the use of a representative agent as a modeling tool has become standard practice.

The existence of one sort of representative agent was theoretically founded in the mid-twentieth century by Gorman (1953, 1961). Gorman showed that in order to have a representative Marshallian demand function for an economy, such that the representative demand at the aggregate income level is equal to the sum of individual demands, agents' indirect utility functions have to take a particular restrictive form, termed the "Gorman Form," and have identical dependence on income.

Specifically, let $D(p, y)$ denote a Marshallian demand as a function of a vector of prices p and an income level y. Gorman's (1953) results imply that in order for there to exist a representative D such that

$$D(p, \sum_i y_i) = \sum_i D_i(p, y_i)$$

for all vectors of individual income levels y_i, it must be that the agents have linear and identical Engel curves, up to a parallel shift. As Gorman (1961) showed later, this imposes strong restrictions on the preferences in society—essentially requiring that they either be quasi-linear in income, or identical (up to a normalization) and homothetic.

Although Gorman's results are discouraging, most of the settings that researchers have analyzed with representative agents are not modeled through Marshallian demand functions nor do they require that a representation hold for all distributions of income. Most models involve decisions that are far more restricted. For instance, representative agents have been used to analyze how agents make consumption and savings decisions in the face of returns to savings that are impacted by various policies (e.g., Lucas, 1978), how agents choose their labor supply in the face of a tax schedule (e.g., Chamley, 1986), and how agents select public goods (e.g., Rogoff, 1990). Even though these decision problems involve maximizing a utility function with respect to some resource constraints, none of them fit into the Gorman setting.

Instead of presuming a common demand function, researchers assume that there is a single agent in the economy and specify that agent's preferences rather than their demand function. This allows a derivation of the agent's behavior in reaction to various influences and policies, as well as the analysis of inefficiencies and welfare. Ultimately, models often specify an agent with some characteristics a, who has a utility function of the form $V(x, a)$, where x can correspond to one dimension of consumption, a stream of consumption, etc. The agent's choice of x can then be subject to various feasibility constraints. If, for example, a captures the agent's income and x stands for a bundle of goods, then this formalization

[1]Edgeworth (1881) referred to a "representative particular," while Marshall (1890) referred to a "representative firm".

can be translated back into the Gorman form. But if, instead, a captures the level of risk aversion, or a discount factor, or a political ideology, etc., then this specification no longer fits the Gorman framework. In particular, the conditions under which this specification can represent a heterogeneous society do not follow as a corollary from Gorman's results.

Although researchers are generally careful not to claim that such a representative-agent formulation is a valid substitute for the analysis of a heterogeneous population, that hope is implicit. Such results are clearly of much more limited interest if there does not exist *any* population in which individual agents' preferences also take the form $V(x, a_i)$, with heterogeneous characteristics a_i, for which the analysis of one agent with preferences described by $V(x, a)$ for some characteristics a, could ever be consistent with.

Thus, we ask whether there exists *at least one* possible set of weights λ_i—e.g., representing the relative fractions of different groups in the population—such that if the population is comprised of λ_i agents in group i, each with a preference parameter a_i, then there exists some representative agent with preference parameter a for whom the utility of the average outcome is a proxy for the average utility. For private goods, this restriction takes the form:

$$V(\sum \lambda_i x_i, a) = \sum_i \lambda_i V(x_i, a_i), \tag{1}$$

while for common consumption, or a public good, this restriction takes the form:

$$V(x, a) = \sum_i \lambda_i V(x, a_i).$$

One requires this sort of formulation, for instance, when one looks for a policy that maximizes society's utilitarian welfare. For example, in Chamley (1986), a representative agent chooses labor supply, consumption, and savings decisions, responding to a taxation schedule given by government. The taxation schedule affects the relative returns to labor and savings, and thus the resulting consumption streams. Denote an agent i's labor, savings, and consumption choices, be it dynamic or static, by x_i. Denote that agent's risk-aversion parameter by a_i. The agent's utility can then be represented by some $V(x_i, a_i)$. The planner is presumed to choose a taxation schedule that maximizes the overall welfare subject to some revenue constraints. If the society is heterogeneous, then a planner who has a welfare function that is represented as a weighted sum of utilities of the agents would be evaluating the welfare of profiles of the agents' choices, $(x_i)_i$, by evaluating $\sum_i \lambda_i V(x_i, a_i)$ for some weights $(\lambda_i)_i$. In Chamley's formulation, which is quite typical, the planner maximizes the utility of a single "representative agent," and takes the form $V(x, a)$. In order for that agent to actually "represent" a heterogeneous society beyond a single agent, it must be that the planner's choice of x is somehow related to the actual choices $(x_i)_i$, either in per-capita value or some transformation thereof. Thus, the representative agent is evaluating some $V(\sum_i \lambda_i x_i, a)$, where the parameter a allows for any sort of transformation of $\sum_i \lambda_i x_i$ necessary to make things work. Having this evaluation of $V(\sum_i \lambda_i x_i, a)$ "represent" $\sum_i \lambda_i V(x_i, a_i)$, so that it yields the same welfare evaluations, is then the requirement that (1) hold.

The question that we pose is simply whether there exists *at least one* possible heterogeneous society that could be represented in this way. We show that the classes of utility functions that admit a representative agent are quite restricted.

Before providing our main results, and discussing more of the literature, we offer a couple of simple examples to illustrate the issues arising with the representative-agent assumption. One could imagine that if the source of heterogeneity in preferences is differences in discount factors or risk-aversion coefficients, then finding a representative agent would be easy, and there would exist some aggregate discount factor or risk-aversion coefficient that would capture the overall evaluation of utilitarian welfare. The following examples illustrate why this is not the case. We illustrate both of these with common consumption. The representation problem is even more restrictive when consumption is heterogeneous.

Example 1 (CRRA Utility Functions): Consider a population of n agents with CRRA (isoelastic) utility functions. Each agent i is identified by a CRRA parameter $a_i \in (0,1)$ and gets a utility from reward x given by

$$V(x; a_i) = \frac{x^{1-a_i} - 1}{1 - a_i}.$$

A representative agent would have utility proportional to some convex combination of the population. Namely, for a profile of coefficients of relative risk aversion $\mathbf{a} \equiv (a_1, ..., a_n)$, up to an affine transformation, her utility of the common reward x would be given by:

$$U(x, \mathbf{a}) = \sum_i \lambda_i V(x; a_i) = \sum_i \lambda_i \frac{x^{1-a_i} - 1}{1 - a_i},$$

for some positive weights λ_i.

Straightforward calculations show that, whenever the a_i's are not all identical, the resulting coefficient of relative risk aversion,

$$-\frac{xU''(x, \mathbf{a})}{U'(x, \mathbf{a})} = \frac{\sum_i \lambda_i a_i x^{-a_i}}{\sum_i \lambda_i x^{-a_i}},$$

changes with x.

This means that the representative agent cannot be characterized by a utility function $V(x, a)$ that satisfies the same property (constant relative risk aversion) satisfied by the utility functions of all members of the population she represents.

If we move to a setting with private allocations, the problem becomes even starker. In this case, the weighted sum of agents' utilities is

$$\sum_i \lambda_i V(x_i; a_i) = \sum_i \lambda_i \frac{x_i^{1-a_i} - 1}{1 - a_i}.$$

This cannot be represented by any function of $\sum_i \lambda_i x_i$ when the x_i's differ, unless all the a_i's are 0—so all agents have to be risk neutral and evaluating a linear function.

Example 2 (Exponential Discounting): Consider a population of n agents who assess consumption streams with a horizon of T. Assume consumption at time t is $x(t) \in [0,1]$. Suppose each individual is characterized by a discount factor $a_i \in [0,1]$. The resulting utility of individual i is generated by exponentially discounting the consumption stream. That is,

$$V(\mathbf{x}; a_i) = \sum_{t=1}^{T} a_i^{t-1} x(t).$$

For a profile of discount factors, $\mathbf{a} \equiv (a_1, ..., a_n)$, the representative agent would be characterized, up to an affine transformation, by the utility function

$$U(\mathbf{x}; \mathbf{a}) = \sum_{i=1}^{n} \lambda_i V(\mathbf{x}; a_i) = \sum_{t=1}^{T} \sum_{i=1}^{n} \lambda_i a_i^{t-1} x(t),$$

for some positive weights λ_i, as before.

The effective discount factor corresponding to the representative agent at any time $t \geq 1$ is then

$$\frac{\sum_{i=1}^{n} \lambda_i a_i^t}{\sum_{i=1}^{n} \lambda_i a_i^{t-1}}.$$

Given any heterogeneity in the a_i's, this effective discount factor increases in t.[2] Again, the representative agent has a utility function that has fundamentally different properties than those corresponding to the underlying population. In this setting, the representative agent is time inconsistent and exhibits a present bias, even though all members of the population are time consistent.

Our main results below prove that these examples are not special. We fully characterize the classes of preferences for which representative agents exist. That is, we identify the conditions under which the population's preferences can be represented by an agent who has preferences in the same class. As we show, such a representative agent exists only if there are extreme restrictions on individual utility functions. When consumption is common, we show that only parametrized classes of utility functions that are *separable in agents' utility parameters* admit representative agents. The assumption that consumption is common applies to environments corresponding to members of a household sharing consumption and savings, or a community—a neighborhood, a state, or a country—benefiting from a common public good, etc. When consumption is private, corresponding to settings of consumer behavior, and encompassing the original examples offered by Lucas (1978), the existence of a representative agent turns out to be even more demanding. In this case, we show that *only utility functions that are linear in consumption and additively separable in agents' utility parameters* admit representative agents.

[2] For a more general analysis of the issue of finding a common discount factor, as well as related references, see Jackson and Yariv (2015).

The literature using representative agents almost never assumes linear utility functions, nor utility functions that are additively separable in agents' individual preference parameters. It then follows from our results that the commonly used classes of utility functions (logarithmic, weighting mean and variance, prospect theoretic, etc., in addition to exponentially-discounted utilities and CRRA or CARA utilities as described above) cannot be aggregated to generate a representative agent who is characterized by preferences from the same class. Thus, our results imply the only possible underlying societies that could rationalize most representative-agent models are ones in which some agents must have preferences outside of any standard class.

1.2 Related Literature

We are certainly not the first to point out issues with the use of representative agents. Beyond Gorman's contributions discussed above, the notion has endured scrutiny practically since its inception, and actively since the beginning of the twentieth century. For instance, one of its most vehement early criticisms appeared in Robbins (1928) (e.g., see Kirman, 1992 and Hartley, 1996 for surveys).

Nonetheless, as mentioned above, the publication of the Lucas Critique (1976) brought new life to micro-founding economic models using the representative-agent construct. Examples of ensuing models relying on representative agents abound. For instance, classical business cycle theories posit that observed aggregate fluctuations of an economy are partly driven by decisions of a representative household (e.g., Kydland and Prescott, 1982; King, Plosser, and Rebelo, 1988). In these models, the cyclical variation of aggregate consumption and employment is a consequence of the continuous optimization by a household that trades goods and leisure intertemporally in response to exogenous factors and movements in prices. Representative-agent models have also been used in the design of tax systems (e.g., see Chamley, 1986; Judd, 1985; and literature that followed), to estimate tax rates on factor incomes and consumption (e.g., Mendoza, Razin, and Tesar, 1994), moral hazard and adverse selection constraints in insurance markets (see Prescott and Townsend, 1984 and literature that followed), and so forth.

Although much of this literature mentions a background assumption of population homogeneity, the fragility of some of its conclusions to heterogeneity have been inspected only fairly recently and in particular contexts. For instance, An, Chang, and Kim (2009) consider an economy with some heterogeneity, incomplete capital markets, and indivisible labor supply. They illustrate that, in their setting, a "representative household" would correspond to an agent with non-concave utility. Constantinides (1982) studies the challenges of heterogeneity in an asset pricing model. Gollier (2001) shows that a mean-preserving spread of endowments among agents that are otherwise identical can affect the level of the equity premium and the risk-free rate in an Arrow-Debreu exchange economy. More recently, Kaplan, Moll, and Violante (2018) highlight the potential importance of accounting for household heterogeneity in monetary policy. Mazzoco (2004) studies households' saving decisions when members have heterogenous risk preferences and make efficient joint choices. Among other results, he shows that an increase in risk aversion and prudence of one household member can reduce the household's risk aversion and prudence, absent harsh restrictions on preferences. In a completely different realm, Mongin (1998) considers a group of individuals satisfying the axioms of subjective expected utility. He shows that when either individuals' beliefs or utility functions are sufficiently heterogeneous, it is impossible to aggregate their preferences while respecting Pareto efficiency and the axioms of subjective expected utility. Golman (2011) examines the existence of representative agents in quantal-response equilibria. There are several other papers that illustrate the sensitivity of the representative-agent framework to heterogeneity of various sorts in particular environments. Our contribution is in highlighting a basic and general principle that drives all such observations.

As mentioned above, there is also a literature characterizing conditions under which aggregate demand, or aggregate behavior, features similar properties to underlying demands (for more recent references, see Chiappori and Ekelend, 1999). In contrast, our focus is on whether any modeler who assumes some properties of a representative agent's preferences must be making errors when presuming that these preferences are consistent with the preferences of an underlying heterogeneous population satisfying similar properties. As discussed above, this question is not answered by the demand-based literature, and yet it covers many, if not most, of the settings in which representative agents are used.

The insights of this paper are in the spirit of Jackson and Yariv (2015) and several of the papers cited there, which showed that there is no utilitarian aggregation of exponentially-discounted preferences that satisfies time consistency.[3] Here we show that such impossibilities are a much more pervasive phenomenon—applying to many different preference formulations and for quite general sources of heterogeneity—and can be argued quite directly.

Our results also provide insight into the observed differences between individual and group decision making. It is well-documented in the experimental and empirical literature that groups exhibit different behavioral patterns than individuals in various environments, ranging from choices between uncertain and thereby risky alternatives—chronicled in a large literature starting from Wallach, Kogan, and Bem (1962)—to choices of timing of events (see Ibanez, Czermak, and Sutter, 2009; Schaner, 2015; and references therein), allocation decisions (Cason and Mui, 1997; Ambrus, Greiner, and Pathak, 2015), etc. Our conclusions are in line with these observations when groups behave in line with some convex combination of their members' preferences. In fact, experimental evidence suggests that group members place substantial weight on utilitarian motives (e.g., Charness and Rabin, 2002; Jackson and Yariv, 2014). The characterization that we provide then suggests that if individual behavior is inconsistent with linear utilities, there is no reason to expect groups composed of such individuals to echo choices of any well-behaved individual.

[3]See also Apesteguia and Ballester (2016) for a similar approach considering several stochastic models of choice.

2 Representative Agents with Private Allocations

We first consider the case in which individuals each have their own allocation and the representative agent evaluates the aggregate/average allocation. For example, the allocation could stand for consumption, investment, and/or savings levels. Agents may exhibit heterogeneity in their discount factors, risk aversion parameters, or other preference parameters, as well as their endowments of human capital, wealth, and so on.

Formally, $n \geq 2$ agents evaluate allocations, generically denoted by x that come from some set D_x, which is a closed and convex subset of $\mathbb{R}_+^\ell$ for some ℓ. Also, we assume that there exists some $x \in D_x$ for which x is positive in all dimensions and $0 \leq y \leq x$ implies that $y \in D_x$.[4]

The heterogeneity of agents' preferences is captured by an index $a \in D_a$, where D_a is some index set. Depending on the application, the parameter a would represent an agent's risk aversion parameter, discount factor, endowment of human capital or wealth, etc.

Utility functions are functions $V : D_x \times D_a \to \mathbb{R}$ that are continuous in the allocation (the first variable).[5]

We say that there exists *a representative agent* with private allocations if there exists *some* $(\lambda_1, ..., \lambda_n) \in [0,1)^n$, where $\sum_{i=1}^{n} \lambda_i = 1$,[6] such that for *some* $(a_1, \ldots, a_n) \in D_a^n$, there exists $\bar{a} \in D_a$ for which for all $(x_1, \ldots, x_n) \in D_x^n$:[7]

$$\sum_{i=1}^{n} \lambda_i V(x_i; a_i) = V(\sum_{i=1}^{n} \lambda_i x_i; \bar{a}).$$

When utility functions are concave, a Pareto optimal allocation is a solution to the maximization $\sum_{i=1}^{n} \lambda_i V(x_i; a_i)$ for some weights. The utilitarian social-welfare function corresponds

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[4]We could also assume that D_x is an open set. Given that our utility functions are continuous, they extend to points of closure, and the results generalize. One can admit allocations with negative dimensions (so D_x is a closed convex subset of $\mathbb{R}^\ell$), simply by presuming there is an open ball in D_x containing 0 and the proof extends. In addition, by simply translating the utility functions (which preserves our definition of representative agents), it becomes without loss of generality to presume that $0 \in D_x$. So, technically, the necessary assumption for our results amounts to presuming that D_x contains an open ball.

[5]Using techniques from Corollary 3 of Rado and Baker (1987), one can extend the key lemmas in our proof to hold for Lebesgue measurable functions, but the proof will be more transparent with continuous functions on D_x and preferences are generally assumed to be continuous in representative-agent models.

[6]The proofs of Theorems 1 and 2 below can be extended to the case in which the $\sum_{i=1}^{n} \lambda_i$ is not required to be one, provided that D_x is unbounded above. It is then then still required that $\lambda_i > 0$ for at least two agents (which is implied above since $\lambda_i < 1$ for all i and the sum is 1). Otherwise, the setting boils down to one with a single agent and representation is trivial.

[7]Note that we have not placed any restrictions on how V depends on a, and so this allows $V(\cdot : \bar{a})$ to be any arbitrary function $W : D_x \to \mathbb{R}$ that is continuous in x.

to the special case in which each coefficient λ_i, $i = 1, ..., n$, is the fraction of the population characterized by preference parameter a_i and allocation x_i per person.

One could also contemplate situations in which welfare is assessed with a set of weights that does not necessarily coincide with the respective fractions of "types" in the population. In that case, the weights on the right-hand-side of the definition of a representative agent may differ from those on the left-hand-side. In Section 6, we show that such an assumption leads to even harsher restrictions on utility functions and requires them to be independent of the allocation altogether. We maintain the definition above since it corresponds to most applications of representative agents in the literature, and since it provides for more "conservative" insights in that it places weaker restrictions on preferences.

The existence of a representative agent is a type of convexity requirement on the space of utility functions. Indeed, consider the special case in which the x_i's are all equal. The existence of a representative agent requires that a convex combination of individuals' utility functions is in the same class to which those individual utility functions belong.

In our formulation, $\bar{a}$ is the representative agent's preference parameter. The representative agent's utility function is often assumed to take the form $be^{c-\bar{a}x}$, $(c + bx)^{\bar{a}}/\bar{a}$, $\bar{a}\ log(x)$, etc.[8]

Note that this setting fits a classic example in Lucas (1978). Lucas considers choices of consumption levels. He assumes individuals make a consumption versus savings decision each period, and the sequence of consumption decisions determines the remaining savings decisions. In that example, a_i would be the agent's initial holdings of the asset, and x_i would be the agent's consumption choice in a given period.

Lucas assumes a single consumer represents the entire population, which, as he notes, would be valid if all agents in the economy were identical. Clearly, the presumption that all agents in the economy are identical is made purely for tractability, as agents in most economies have very different wealth levels and thus face different consumption versus savings trade-offs (as was certainly noted before, see the above literature review). The hope is that the single "representative" consumer analysis provides insights into more general settings than those pertaining to a homogeneous society. If so, the modeler could consider a single representative agent, with utility $V(x, \bar{a})$ instead of considering a heterogeneous society. As in Lucas (1978), the ultimate goal is to analyze welfare-maximizing policies. In particular, we want to be able to compare two different policies—inducing, say, $(x_1, \ldots, x_n)$ and $(x'_1, \ldots, x'_n)$—according to the social welfare function $\sum_i \lambda_i V(x_i; a_i)$. In order for the researcher to simplify her analysis and consider a representative agent with some preference parameter $\bar{a}$, whose preferences over aggregate bundles correspond to the welfare evaluation of the heterogeneous bundles, evaluations of the form $\sum_i \lambda_i V(x_i; a_i)$ should be equivalent to those of $V(\sum_i \lambda_i x_i; \bar{a})$ for some $\bar{a}$.

[8]In these formulations, b and c are taken as constants. For instance, the form $be^{c-\bar{a}x}$ with $b = 1$ and $c = 0$ would correspond to a representative agent with a CARA utility function and the form $(c + bx)^{\bar{a}}/\bar{a}$ with $c = 0$ and $b = 1$ would correspond to a representative agent with a CRRA utility function.

The following is the characterization of utility functions that admit the existence of a representative agent when allocations are private.

THEOREM 1 *There exists a representative agent $\bar{a}$ in the case of private allocations, relative to some $\lambda \in [0,1)^n$ and some $(a_1, \ldots, a_n) \in D_a^n$, if and only if $V(x;a) = c \cdot x + h(a)$ for all $x \in D_x$ and $a \in \{a_i \mid \lambda_i > 0\} \cup \{\bar{a}\}$, where $c \in \mathbb{R}^\ell$ and $h : D_a \to \mathbb{R}$ satisfies $h(\bar{a}) = \sum_i \lambda_i h(a_i)$.*

The structure characterized by Theorem 1 requires linearity in the allocation x and additive separability in the type parameter a. It is clearly not satisfied by utility functions that are commonly used in economic modeling. For example, strictly concave utility functions do not satisfy the restriction, nor do CRRA or CARA utility functions, nor do exponentially-discounted utilities. In such cases, the theorem implies that assuming a representative agent whose utility is taken from the same class of heterogeneous individuals' preferences would generate inaccurate estimates of aggregate behavior and welfare.

If we additionally require the representative-agent restriction hold for *all* preference profiles, then the structural implications of Theorem 1 then apply to all preference parameters. In particular, if we assume that $D_a = [0,1]$ and that $V(x;a)$ is continuous in a, the existence of a representative agent is tantamount to $V(x;a) = c \cdot x + h(a)$ for all $(x,a) \in D_x \times D_a$, where $c \in \mathbb{R}^\ell$ and $h : D_a \to \mathbb{R}$ is any continuous function.

The proofs of our results appear in Section 6. Intuitively, if a representative agent exists, a marginal change in the private allocation x_i of any agent i has a proportional effect on the allocation the representative agent considers (where the proportional factor corresponds to the individual's weight in society). The only way to get marginal utility calculations line up for all agents is to have linearity in x.

3 Representative Agents with Common Alternatives

The case of private allocations applies to most of the work built upon representative agents in macroeconomics and finance. We now expand the analysis to admit alternatives that are jointly evaluated. For example, in household decision making, expenditures and savings are often common across household members. Furthermore, common consumption is central to many models of political economy and public finance. In these models, agents make decisions over the level of some public good. As we now show, the existence of a representative agent in such environments is still very restrictive, but entails a different sort of separability.

We maintain the same basic structure of preference heterogeneity as above. For the rest of the paper, we add the conditions that D_a is $[0,1]$, that utility functions $V(x;a)$ are continuous in a, and that there exists at least one $x^* \in D_x$ for which $V(x^*;a)$ is strictly monotone (increasing or decreasing) in a.[9]

[9] The assumption that $V(x;a)$ is continuous in a simplifies our proof presentation, but is, in fact, not necessary. The continuity of $V(x^*;a)$ in a is implied by montonicity combined with the existence of a representative agent defined below, and is all that is required for our main result.

The restriction that $V(x^*; a)$ is monotone in a for some $x^* \in D_x$ is weak and satisfied for many classes of commonly used utility functions. For instance, exponential discounting, CRRA, and CARA satisfy the condition. Although we maintain this condition for presentation simplicity and since it allows for most cases covered in the literature, we note that the proofs imply that this condition can, in fact, be weakened to a requirement that $V(x^*; a)$ be piece-wise monotonic for some $x^* \in D_x$, which is satisfied for practically all preference specifications appearing in the literature.[10]

We say that there exists *a representative agent* with common alternatives if there exists some $(\lambda_1, ..., \lambda_n) \in [0, 1)^n$, where $\sum_{i=1}^{n} \lambda_i = 1$, such that for any $(a_1, ..., a_n) \in D_a^n$, there exists $\bar{a} \in [0, 1]$ for which:

$$\sum_{i=1}^{n} \lambda_i V(x; a_i) = V(x; \bar{a})$$

for all x.

In the case of common alternatives, an alternative definition that would require our restriction to hold for only one profile of preference parameters $(a_1, ..., a_n)$ could be trivially satisfied in a mechanical fashion. For instance, for any two continuous functions $f(x), g(x)$ such that $f(x) > g(x)$ for all x, defining $V(x; a) = f(x)$ for low values of a, $V(x; a) = g(x)$ for high values of a, and $V(x; a) = \frac{1}{2} f(x) + \frac{1}{2} g(x)$ for intermediate values of a would suffice for the existence of a representative agent with respect to particular weights and particular preference parameters. This is why we require the restriction to hold for *all* preference parameters.

Theorem 2 characterizes the class of utility functions that admit a representative agent.

THEOREM **2** *There exists a representative agent with common alternatives if and only if* $V(x; a) = h(a)f(x) + g(x)$ *for all* $(x, a) \in D_x \times D_a$, *for some continuous functions* $h(a)$, $f(x)$, *and* $g(x)$ *such that* $h(\cdot)$ *is monotone, and* $f(x^*) \neq 0$.

Although the restrictions implied by the existence of a representative agent for common alternatives are weaker than those for private allocations, they are still sufficiently strong as to rule out nearly all commonly assumed utility functions. From the examples mentioned so far, exponential discounting, CARA and CRRA utility functions with risk-aversion parameters do not satisfy the restrictions of Theorem 2, nor do concave loss functions with bliss points serving as parameters—e.g., single-peaked preferences.

[10]Without some such assumption, one admits the possibility that all preferences are completely independent of types, in which case there is no meaningful heterogeneity in the population and everyone has the same preferences. In such cases, a representative agent exists trivially. This requirement is not needed in the case of private allocations since there agents can differ in their consumption. That potential variation imposes a stronger requirement on a representative agent, even with identical preferences.

One contrast between the common-alternative and private-allocation cases pertains to the class of concave utility functions. Certainly, a mixture of concave functions is concave. Thus, when considering the full class of concave functions, with common consumption, a representative agent does exist, and is characterized by the convex combination of agents' utility functions. Of course, that function must look quite different from the functions that are being aggregated (as required by the theorem above). This does not violate the theorem since there is no representation of the class of all concave functions that satisfies the monotonicity requirement.

4 Strongly Representative Agents

We now consider a more demanding notion of a representative agent. Under this variant, the representative agent's preference parameter $\bar{a}$ is the weighted average of individual agents' preference parameters. For instance, suppose an empiricist observes individual preference parameters with noise and erroneously assumes the population is homogenous. A natural estimate for the preference parameter corresponding to that population, as well as its legitimate representative agent under the assumption of homogeneity, would be the average of observed parameters. In the context of discount factor estimations, see the survey by Frederick, Loewenstein, and O'Donoghue (2002) for examples. With a large population of individuals, the estimated average parameter may not be biased. However, as we now show, welfare assessments based on the estimated utility function may be inaccurate. In fact, the classes of utility functions admitting such strongly representative agents are even more restrictive than those identified above.

As before, we start with the case of private allocations.[11]

We say that there exists a *strongly representative agent* with private allocations if there exists some $(\lambda_1, ..., \lambda_n) \in [0,1)^n$, where $\sum_{i=1}^{n} \lambda_i = 1$, such that for all $(a_1, \ldots, a_n) \in D_a^n$ and $(x_1, \ldots, x_n) \in D_x^n$:

$$\sum_{i=1}^{n} \lambda_i V(x_i; a_i) = V\left(\sum_{i=1}^{n} \lambda_i x_i; \sum_{i=1}^{n} \lambda_i a_i\right). \tag{2}$$

Proposition 1 *There exists a strongly representative agent when allocations are private if and only if there exist constants b_1, b_2, and $c \in \mathbb{R}^\ell$ such that $V(x;a) = c \cdot x + b_1 a + b_2$ for all $(x,a) \in D_x \times D_a$.*

Proposition 1 states that a strongly representative agent exists only when utility functions are additively separable in the preference parameter and the allocation, and *linear* in both.

[11] As mentioned, we maintain our assumptions from Section 3 that D_a is $[0,1]$, that $V(x;a)$ is continuous in a, and that there exists at least one $x^* \in D_x$ for which $V(x^*;a)$ is strictly monotone (increasing or decreasing) in a.

The intuition is similar to that described for Theorem 1. If a strongly representative agent exists, a marginal change in either the allocation or the parameter of any individual has a proportional effect on the representative agent's allocation and utility parameter (where the proportional factor corresponds to the individual's weight in society). This implies the linear structure of utility functions.

With common alternatives, we analogously say that there exists *a strongly representative agent* with common alternatives if there exists some $(\lambda_1, ..., \lambda_n) \in [0,1)^n$, where $\sum_{i=1}^{n} \lambda_i = 1$, such that for any $a_1, ..., a_n \in D_a^n$ and $x \in D_x$:

$$\sum_{i=1}^{n} \lambda_i V(x; a_i) = V\left(x; \sum_{i=1}^{n} \lambda_i a_i\right). \tag{3}$$

PROPOSITION **2** *There exists a strongly representative agent when alternatives are common if and only if there exist continuous functions $f(x), g(x)$ such that $V(x;a) = af(x) + g(x)$ for all $(x,a) \in D_x \times D_a$.*

The intuition behind Proposition 2 is again similar to the intuition provided for previous results. When an average representative agent exists, a marginal change in one individual's utility parameter has a proportional impact on the marginal change of the representative agent's utility parameter. This maps into a linearity requirement with respect to the utility parameter a.

5 Discussion

The assumption of a representative agent, who has preferences representing the aggregate of the population, is commonplace in modern economics. We have shown that for the representative agent to inherit the structure of preferences in the population that she represents, extreme restrictions need to be satisfied. In particular, utility functions need to be additively separable in allocations and parameters characterizing preferences; and, in the case including private allocations, *linear* in the allocation. Unfortunately, these restrictions are not satisfied by any commonly used classes of utility functions. For instance, a society in which each agent is characterized by a CRRA or CARA utility does not admit a representative agent with a similar utility function.

While others have pointed out the challenges of using a representative-agent model when individuals in society are aggregated in particular ways or interact strategically, the results in this paper are more fundamental. They illustrate an impossibility result for wide classes of utility functions, including practically all those studied in the literature. Our results make it imperative that researchers give more careful consideration to the use and formulation of representative agents, and model and account for heterogeneity directly.

6 Proofs

We begin with some lemmas that provide the key structure behind the proofs. The lemmas provide a variation on the analysis of Pexider's equation.[12] The proofs use techniques developed in Azcél (1966, 1969), Eichhorn (1978), and Diewert (2011).

We begin with a proof about a representation on one dimension and then use that to prove results for more dimensions.

LEMMA 1 *Let $f(x)$ be a continuous function on $[0,t]$. Suppose that, for some $\lambda \in (0,1)$,*

$$f(\lambda x + (1-\lambda)y) = f(\lambda x) + f((1-\lambda)y) \text{ for all } x, y \in [0,t]$$

then $f(x) = cx$ for all $x \in [0,t]$, where c is a scalar.

Proof of Lemma 1: Let $t' = \min\{\lambda, 1-\lambda\}t$.

We first show that for any positive integer k and any $z \in [0,t']$, $f(z) = kf(z/k)$.

Let $x = \frac{z}{\lambda k}$ and $y = \frac{(k-1)z}{(1-\lambda)k}$. Note that, since $z \leq t'$ then by construction, $x, y \in [0,t]$. Then,

$$f(z) = f\left(\frac{z}{k}\right) + f\left(\frac{(k-1)z}{k}\right).$$

For $k = 1$ this establishes the claim. For $k \geq 2$, writing $\frac{(k-1)z}{k} = \lambda \frac{z}{\lambda k} + (1-\lambda)\frac{(k-2)z}{(1-\lambda)k}$, it follows that

$$f(z) = f\left(\frac{z}{k}\right) + f\left(\frac{z}{k}\right) + f\left(\frac{(k-1)z}{k}\right) = 2f\left(\frac{z}{k}\right) + f\left(\frac{(k-2)z}{k}\right).$$

Continuing recursively, establishes that $f(z) = kf(z/k)$ for all $z \in [0,t']$ and for all positive integers k.

Next, we show that this implies that $f(x) = cx$ for all $x \in [0,t']$. Let $c = \frac{f(t')}{t'}$. For any $x = \frac{m}{n}t'$, where m and n are integers such that $m < n$, we have $\frac{x}{m} = \frac{t'}{n}$ and therefore from above $f(x) = \frac{m}{n}f(t') = cx$. From continuity, it follows that $f(x) = cx$ for all $x \in [0,t']$.[13]

Now, suppose $\min\{\lambda, 1-\lambda\} = \lambda$ so that $\lambda z \in [0,t']$ for all $z \in [0,t]$. Then:

$$\begin{aligned}
f(z) &= f(\lambda z) + f((1-\lambda)z) = c\lambda z + f((1-\lambda)z) = \\
&= c\lambda z + f(\lambda(1-\lambda)z) + f((1-\lambda)^2 z) = c(\lambda z + \lambda(1-\lambda))z + f((1-\lambda)^2 z) = \\
&= cz\lambda \sum_{i=0}^{\infty}(1-\lambda)^i + \lim_{n\to\infty} f((1-\lambda)^n z) = cz + \lim_{n\to\infty} f((1-\lambda)^n z).
\end{aligned}$$

Since $f(0) = 0$ (which follows from $f(0) = kf(0)$) and f is continuous, it follows that $f(z) = cz$ for all $z \in [0,t]$. A similar argument follows for $\min\{\lambda, 1-\lambda\} = 1-\lambda$. ∎

[12]Our results in an earlier version of this paper presumed analytic functions. We thank an anonymous reviewer for suggesting that some variation of Pexider's equation might be used to strengthen our results.

[13]Note that were D_x unbounded, e.g. $D_x = [0,\infty)$, the proof would be completed here.

LEMMA **2** *Let $f(x)$ be a continuous function on D_x such that there exists some $\lambda \in (0,1)$ for which*

$$f(\lambda x + (1-\lambda)y) = f(\lambda x) + f((1-\lambda)y) \text{ for all } x, y \in D_x.$$

Then there exists $c \in \Re^\ell$ such that $f(x) = c \cdot x$ for all $x \in D_x$.

Proof of Lemma 2: First, note that $f(0) = 0$, since $\lambda \times 0 + (1-\lambda) \times 0 = 0$ and so $f(0) = f(0) + f(0) = 2f(0)$.

Next, note that, by assumption, there exists $x \in D_x$ that is positive in all dimensions such that $0 \leq y \leq x$ implies $y \in D_x$. Let $D' = \{y : 0 \leq y \leq x\}$. Let D'_j be the subset of D' such that $y \in D'_j$ implies $y_k = 0$ for all $k \neq j$.

Applying Lemma 1 to each D'_j implies that for each dimension j, there exists c_j for which $f(y) = c_j y_j$ whenever $y \in D'_j$.

Next, let $d = \min\{\lambda, 1-\lambda\}$ and $D'' = \{y : 0 \leq \frac{y}{d} \leq x\}$. For $y \in D''$, abuse notation and write y_j to denote the vector that is the projection of y onto its j-th dimension, and y_{-j} to be $y - y_j$. Then, for any $y \in D''$, by the definition of D'', it follows that $(1-\lambda)y_j + (1-\lambda)\frac{y_{-j}}{1-\lambda} \in D'$. Therefore,

$$f(y) = f\left(\lambda y_j + (1-\lambda)y_j + (1-\lambda)\frac{y_{-j}}{1-\lambda}\right) = \lambda c_j y_j + f\left((1-\lambda)y_j + (1-\lambda)\frac{y_{-j}}{1-\lambda}\right).$$

Repeating the argument for the second term, we get $f(y) = c_j y_j + f(y_{-j})$. Then, iterating on the remaining dimensions,

$$f(y) = c \cdot y.$$

for any $y \in D''$.

Next, let $d' = \max\{\lambda, 1-\lambda\}$ and consider any $z \in D_x$ such that $d'z \in D''$. Then,

$$f(z) = f(\lambda z) + f((1-\lambda)z) = \lambda c \cdot z + (1-\lambda)c \cdot z = c \cdot z.$$

For any $z \in D_x$, there exists some t for which $d'^t z \in D''$, and so by iterating on the above argument, the result follows. ∎

LEMMA **3** *Let $f_1(x)$, $f_2(x)$, and $f_3(x)$ be continuous functions on D_x. If, for some $\lambda \in (0,1)$,*

$$f_1(\lambda x + (1-\lambda)y) = \lambda f_2(x) + (1-\lambda) f_3(y) \text{ for all } x, y \in D_x,$$

then there exist constants $a, b \in \mathbb{R}$ and $c \in \mathbb{R}^\ell$ such that

$$\begin{aligned} f_1(x) &= c \cdot x + \lambda a + (1-\lambda)b \\ f_2(x) &= c \cdot x + a, \\ f_3(x) &= c \cdot x + b. \end{aligned}$$

Proof of Lemma 3: Let $x = 0$. Then

$$f_1((1-\lambda)y) = \lambda f_2(0) + (1-\lambda)f_3(y) \text{ for all } y \in D_x.$$

Define $a \equiv f_2(0)$. Then,

$$f_3(y) = \frac{1}{1-\lambda}\left[f_1((1-\lambda)y) - \lambda a\right].$$

Similarly, if we define $b \equiv f_3(0)$, we get:

$$f_2(x) = \frac{1}{\lambda}\left[f_1(\lambda x) - (1-\lambda)b\right].$$

Plugging into the assumed equality, we have:

$$\begin{aligned} f_1(\lambda x + (1-\lambda)y) &= \lambda f_2(x) + (1-\lambda)f_3(y) = \\ &= f_1(\lambda x) + f_1((1-\lambda)y) - \lambda a - (1-\lambda)b. \end{aligned}$$

Define $f(x) \equiv f_1(x) - \lambda a - (1-\lambda)b$. Then, from the last equality we have

$$f(\lambda x + (1-\lambda)y) = f(\lambda x) + f((1-\lambda)y).$$

Lemma 2 then implies that $f(x) = c \cdot x$ and the result follows. ∎

6.1 Proofs Pertaining to Private Allocations

Proof of Theorem 1: Suppose that for some $\lambda_1, ..., \lambda_n \in [0,1)$ for which $\sum_{i=1}^{n} \lambda_i = 1$, and some $(a_1, \ldots, a_n) \in D_a^n$, there exists $\bar{a} \in D_a$ such that for all $(x_1, \ldots, x_n) \in D_x^n$:

$$\sum_{i=1}^{n} \lambda_i V(x_i; a_i) = V(\sum_{i=1}^{n} \lambda_i x_i; \bar{a}).$$

There must exist some i for which $0 < \lambda_i < 1$. Let $x_i = x$, $x_j = y$ for all $j \neq i$. It follows that

$$V(\lambda_i x + (1-\lambda_i)y; \bar{a}) = \lambda_i V(x; a_i) + (1-\lambda_i)\sum_{j\neq i} V(y; a_j),$$

for any $x \in D_x$ and $y \in D_x$ and the characterization of V follows from Lemma 3. In particular, the first application of the lemma uses $f_1(x) = V(x; \bar{a})$, $f_2(x) = V(x; a_i)$, and $f_3(x) = \sum_{j\neq i} V(\cdot; a_j)$ and so gives

$$\begin{aligned} &V(x; \bar{a}) = c \cdot x + h(\bar{a}), \\ &V(x; a_i) = c \cdot x + h(a_i), \\ &\textstyle\sum_{j\neq i} V(\cdot; a_j) = c \cdot x + b_i, \end{aligned}$$

where $h(\bar{a}) = \lambda_i h(a_i) + (1 - \lambda_1)b_i$. Iterating to apply the lemma to any j for which $\lambda_j > 0$ (and necessarily $\lambda_j < 1$ by definition), one similarly gets that $h(\bar{a}) = \lambda_j h(a_j) + (1 - \lambda_j)b_j$. These iterative applications of the lemma imply that for any j for which $\lambda_j > 0$,

$$V(x; a_j) = c \cdot x + h(a_j)$$

and

$$\sum_{i \neq j} V(x; a_i) = c \cdot x + b_j.$$

Thus, putting all of these together, it follows that $b_j = \sum_{i \neq j : \lambda_i > 0} \lambda_i h(a_i)$. Therefore, it follows that

$$h(\bar{a}) = \sum_{i : \lambda_i > 0} \lambda_i h(a_i) = \sum_i \lambda_i h(a_i),$$

as claimed and any extension of $h(\cdot)$ to D_a would do. The converse follows directly. ∎

A weaker definition of the existence of a representative agent would impose that for some $\lambda_i, \lambda_i' \in [0, 1)$ for $i = 1, .., n$ with $\sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \lambda_i' = 1$, and some $(a_1, \ldots, a_n) \in D_a^n$, there exists $\bar{a} \in D_a$ such that for all $(x_1, \ldots, x_n) \in D_x^n$:

$$\sum_{i=1}^{n} \lambda_i V(x_i; a_i) = V(\sum_{i=1}^{n} \lambda_i' x_i; \bar{a}).$$

Without loss of generality, assume $\lambda_1' \in (0, 1)$. As in the proof of Theorem 1, Let $x_1 = x$, $x_2 = x_3 = ... = x_n = y$. It follows that

$$\begin{aligned} V(\lambda_1' x + (1 - \lambda_1')y; \bar{a}) &= \lambda_1 V(x; a_1) + (1 - \lambda_1)V(y; a_2) \\ &= \lambda_1' V_1(x; a_1) + (1 - \lambda_1')V_2(y; a_2), \end{aligned}$$

where

$$V_1(x; a_1) = \frac{\lambda_1}{\lambda_1'} V(x; a_1) \quad \text{and} \quad V_2(y; a_2) = \frac{1 - \lambda_1}{1 - \lambda_1'} V(y; a_2).$$

Lemma 3 then implies that whenever $\lambda_1 \neq \lambda_1'$, $V(x; a)$ is independent of x (noting that all three functions must have the same c, which is not possible if $\lambda_1 \neq \lambda_1'$ and $c \neq 0$) .

Proof of Proposition 1: The proof follows combining the implications of the functional forms from Theorem 1 together with Proposition 2, as both representations hold by either simply fixing any profile of a_is, or working with all agents having the same allocation. ∎

6.2 Proofs Pertaining to Common Alternatives

We start with the proof of Proposition 2, which is useful for proving Theorem 2.

Proof of Proposition 2: We show that (3) implies that there exist continuous functions $f(x), g(x)$ such that $V(x;a) = af(x) + g(x)$ for all x, a, as the converse is straightforward. Let $a_1 = r$, $a_2 = a_3 = ... = a_n = s$, and $x_1 = x_2 = ... = x_n = x$. Then, the existence of a strongly representative agent, for some $\lambda_i \in (0,1)$, implies that:

$$V(x; \lambda_i r + (1-\lambda_i)s) = \lambda_i V(x;r) + (1-\lambda_i)V(x;s)$$

and Lemma 3 (now applied on the a dimension), together with the continuity of V in a, imply the result. ∎

Proof of Theorem 2: Let

$$h(a) \equiv V(x^*; a).$$

Notice that $h(\cdot)$ is monotone in a and, therefore, from continuity, $\mathrm{Im}_h\, D_a = \tilde{D}_{h,a}$ is a compact set and $h^{-1} : \tilde{D}_{h,a} \to D_a$ is continuous and monotone as well. Now let

$$G(x;a) = V(x; h^{-1}(a)).$$

By our assumption on V, for some $\lambda_1, ..., \lambda_n \geq 0, \sum_{i=1}^{n} \lambda_i = 1$, and for any $a_1, ..., a_n \in D_a^n$, there exists $\bar{a}$ such that for all x,

$$\sum_{i=1}^{n} \lambda_i G(x; a_i) = G(x; \bar{a}),$$

In particular:

$$\sum_{i=1}^{n} \lambda_i G(x^*; a_i) = \sum_{i=1}^{n} \lambda_i a_i = G(x^*; \bar{a}) = \bar{a}.$$

Therefore, G satisfies the assumptions of Proposition 2, so that there exist continuous functions $f(x), g(x)$ such that

$$G(x;a) = af(x) + g(x),$$

which, in turn, implies that

$$V(x;a) = h(a)f(x) + g(x).$$

This establishes the theorem, as the converse is immediate. ∎

7 References

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