Rationality and Preference Aggregation of Group Decision under Risk

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Introduction

- In various contexts, many important decisions are made by groups.
- Individual heterogeneity exists in various dimensions:
  - Risk preference: risk assessment in environmental policy
  - Time preference: household savings and consumption decisions
  - Rationality
- It is important to understand how individual heterogeneity in a collective influences final outcomes.

Introduction: Research Questions

1. Rationality extension:
   - Do rational members make more collectively rational decisions?

2 Risk preference aggregation:
   - Are individual's risk preferences reflected into that of a group?

3. Efficiency and welfare:
   - How is the efficiency of group decisions related to individual's rationality and preferences?
   - How is social welfare related to individual' rationality and preferences?

Introduction: Examples of Individual Heterogeneity

- Rationality
  - High-income, high-education, men, and young subjects tend more toward utility maximization (Choi et al., 2014).
- Risk preference
  - White males are more likely to perceive risks as being smaller (Bickerstaff, 2004; Flynn et al., 1994).
  - There is no substantial difference between men and women (Kagel and Roth, 2016).
  - Subjects' risk preferences are closer to neutrality when they make decisions on behalf of other participants (Batteux et al., 2017).
  - High-power groups adopt a more positive attitude toward potential risks (Anderson and Galinsky, 2006; Magee et al., 2007; Geng et al, 2018).

Experimental Design and Subjects

Experimental Design (Choi et al.,2007; Choi et al., 2014)

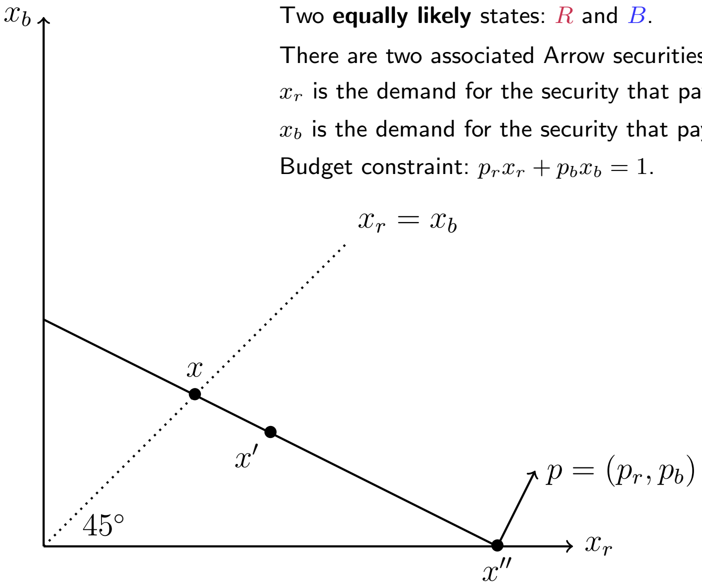
Two **equally likely** states: R and B.

There are two associated Arrow securities.

x_r is the demand for the security that pays off in state R.

x_b is the demand for the security that pays off in state B.

Budget constraint: $p_r x_r + p_b x_b = 1$.



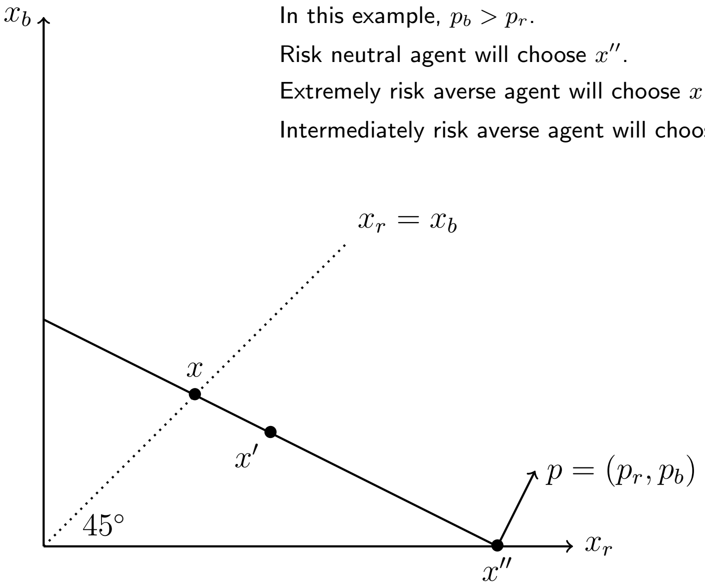
Experimental Design (Choi et al.,2007; Choi et al., 2014)

In this example, $p_b > p_r$.

Risk neutral agent will choose x''.

Extremely risk averse agent will choose x.

Intermediately risk averse agent will choose x'.

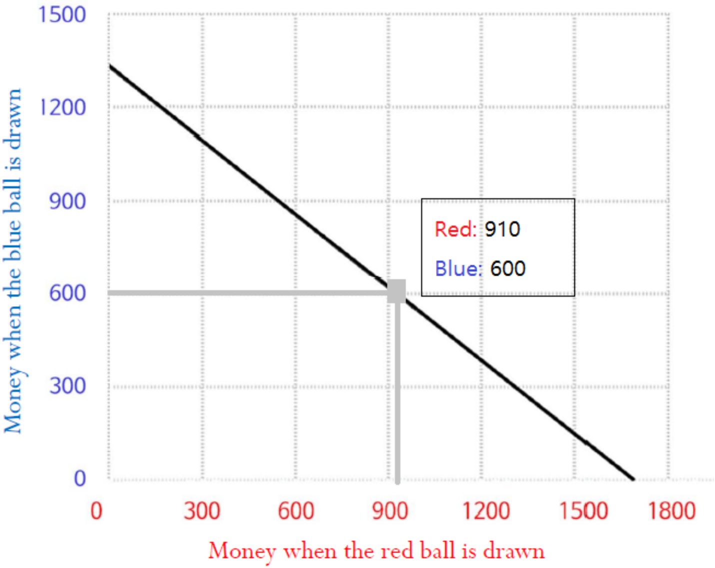


Procedure and Subjects

- We conducted the experiment in 12 middle schools in Daegu.
- The number of students: 1572.
- The number of groups: 786.
- The instructions were read by an experimenter in each classroom.
- Each subject participated in two sessions: individual and group decisions.

Field

Screenshot


1500
1200
900
600
300
0
Money when the blue ball is drawn
Red: 910
Blue: 600
0
300
600
900
1200
1500
1800
Money when the red ball is drawn

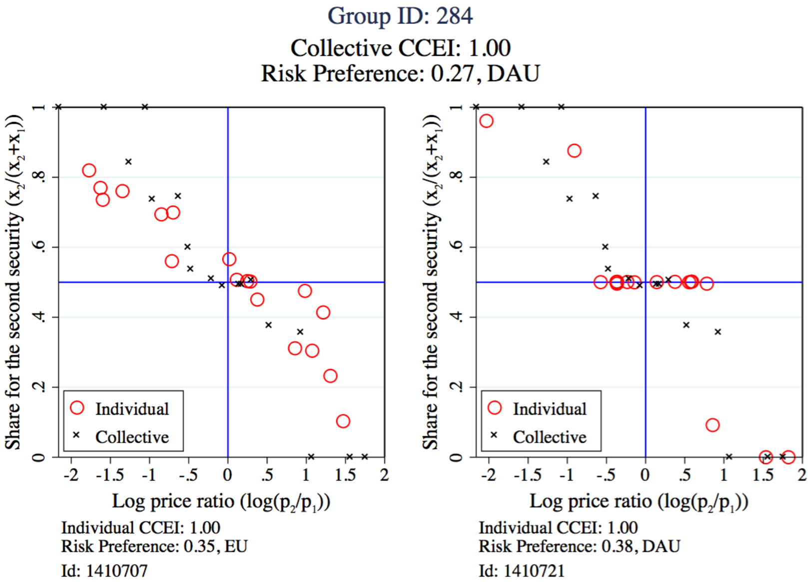
Procedure and Subjects

- Each round started by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above 300 KRW or below 1500 KRW.
- Each session consisted of 18 independent decision rounds.
- At the end of each round, the computer randomly selected one of the two states (R and B).
- Subjects were not informed of the state that was selected at the end of each round.

Procedure and Subjects

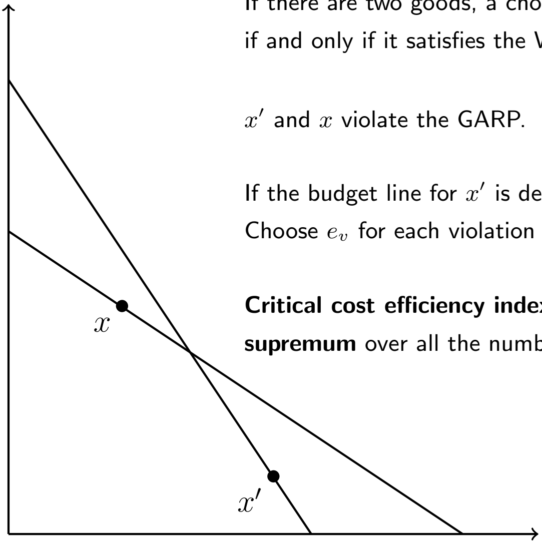
- Two students in the same classroom were randomly matched.
- One of the two students was randomly chosen to move to the other partner's desk.
- They made a series of collective decisions by sharing one computer.
- We allowed students to discuss how to make decisions for 1 min before starting the second session.
- Each subject was paid for he/she earned in a randomly selected round.

Example of Choice Data



Result 1: Rationality Extension

Measurement: Afriat's Efficiency Index (a.k.a. CCEI)



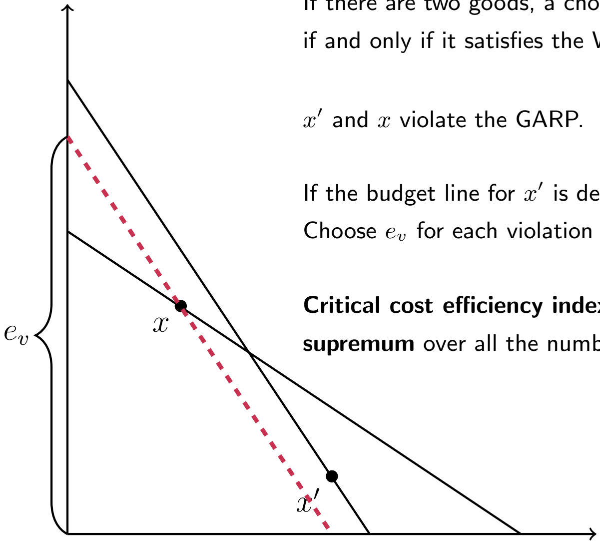
If there are two goods, a choice dataset satisfies the GARP if and only if it satisfies the WARP.

x' and x violate the GARP.

If the budget line for x' is deflated, the GARP is satisfied. Choose e_v for each violation v.

Critical cost efficiency index (CCEI) is defined as the **supremum** over all the numbers e_v's.

Measurement: Afriat's Efficiency Index (a.k.a. CCEI)



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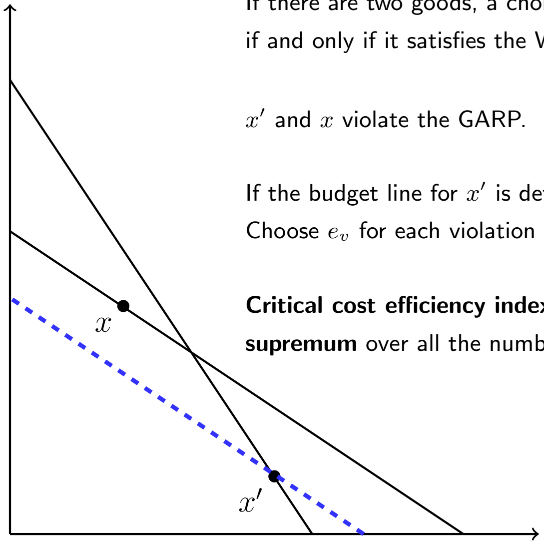

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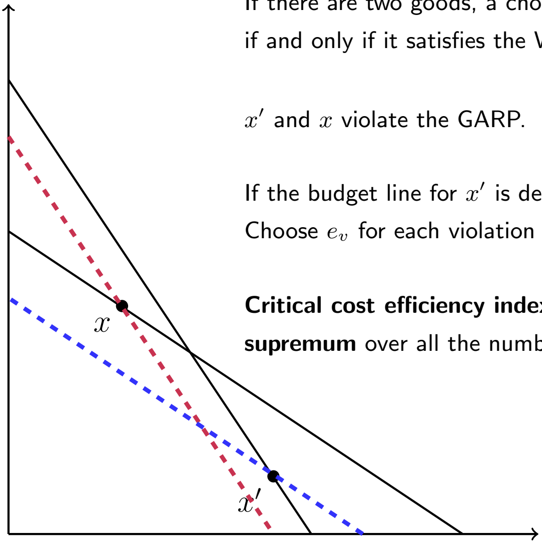

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If the budget line for x' is deflated, the GARP is satisfied. Choose e_v for each violation v.

Critical cost efficiency index (CCEI) is defined as the **supremum** over all the numbers e_v's.

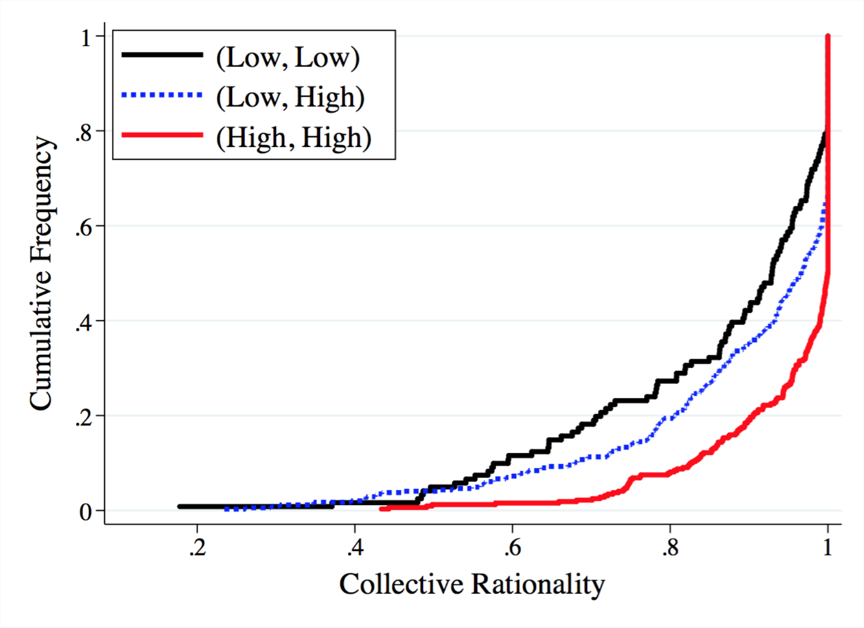
Measurement: Afriat's Efficiency Index (a.k.a. CCEI)

- By definition, CCEI $\in [0, 1]$.
- The bigger CCEI is, the less severe violation of GARP.
- Basic statistics of individual CCEI:
  - Average: 0.897 (0.136)
  - Quantiles: 0.838 (25%), 0.953 (50%), 1 (75%).
- Basic statistics of collective CCEI:
  - Average: 0.910 (0.141)
  - Quantiles: 0.868 (25%), 0.981 (50%), 1 (75%).

Rationality Extension: Research Question

Individual Rationality $\uparrow \Rightarrow$ Collective Rationality $\uparrow$?

Rationality Extension: First-Order Stochastic Dominance



Rationality Extension: First-Order Stochastic Dominance

- We do a series of Kolmogorov-Smirnov tests:

$$H_0 : F_{\text{group } i}(X) = F_{\text{group } j}(X) \quad \text{for all values of } X.$$

- Test statistic: $D_{ij} = \sup_{x \in X} ||F_{\text{group } i}(x) - F_{\text{group } j}(x)||$.
- (Low, Low) v.s. (High, High): 0.17
  - The corresponding p-value is 0.01.
- (Low, High) v.s. (High, High): 0.21
  - The corresponding p-value is 0.00.

Rationality Extension: Econometric Analysis

Collective CCEI	Coefficient		
	Model 1	Model 2	Model 3
CCEI_Max	0.368^{***}	0.327^{***}	0.302^{***}
	(0.083)	(0.074)	(0.089)
CCEI_Distance	-0.277^{***}	-0.250^{***}	-0.233^{***}
	(0.056)	(0.053)	(0.058)
Risk_Aversion_Max		-0.189^{***}	-0.172^{**}
		(0.056)	(0.070)
Risk_Aversion_Distance		0.087^{*}	0.093^{*}
		(0.048)	(0.055)
Math_Score_Max			0.012^{**}
			(0.005)
Math_Distance			-0.010^{**}
			(0.005)
Constant	0.582^{***}	0.679^{***}	0.664^{***}
	(0.077)	(0.070)	(0.084)
Class Fixed Effect	Yes	Yes	Yes
Individual Characteristics	No	No	Yes
School Characteristics	No	No	Yes
Friendship	No	No	Yes
Observations	786	786	786
R-squared	0.200	0.212	0.235

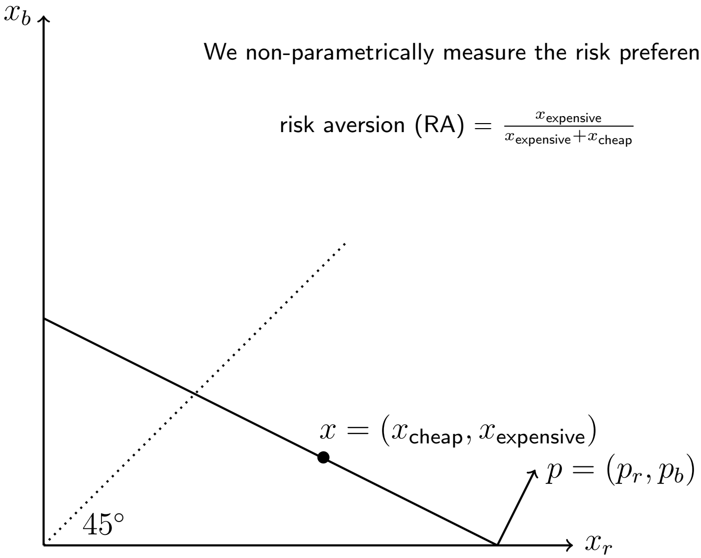
$*$Throughout the paper, we clustered standard error in class level.

Result 2: Preference Aggregation

Measurement: Indifference Curves

We non-parametrically measure the risk preference by a ratio:

$$\text{risk aversion (RA)} = \frac{x_{\text{expensive}}}{x_{\text{expensive}} + x_{\text{cheap}}}$$



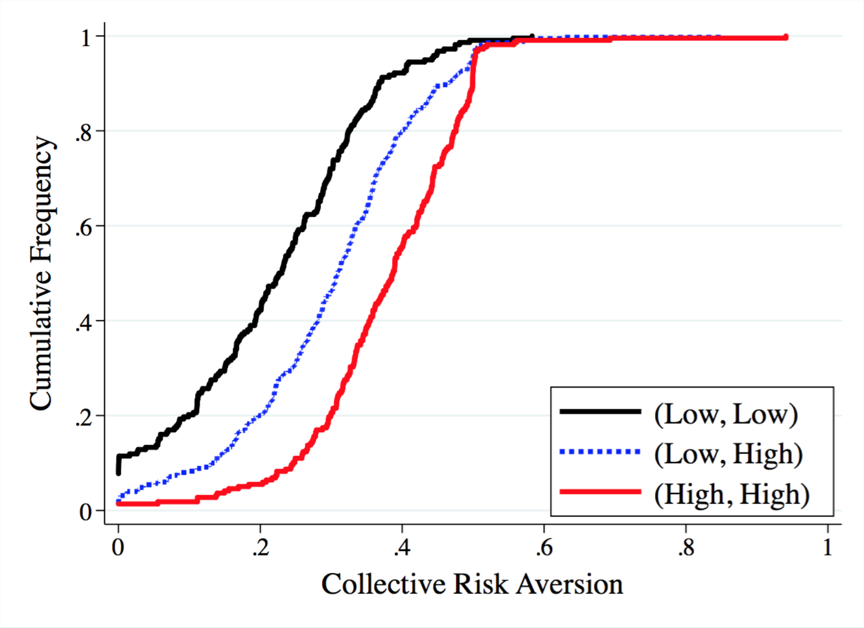
Measurement: Risk Preferences

- By definition, RA $\in [0, 1]$.
- The bigger ratio is, the higher risk aversion.
- Basic statistics of individual RA:
  - Average: 0.324 (0.132).
  - Quantiles: 0.213 (25%) , 0.310 (50%), 0.390 (75%), 0.499 (99%).
- Basic statistics of collective RA:
  - Average: 0.298 (0.139).
  - Quantiles: 0.255 (25%), 0.348 (50%), 0.413 (75%), 0.497 (99%).

Risk Preference Aggregation: Research Question

Individual risk aversion $\uparrow \Rightarrow$ Collective risk aversion $\uparrow$?

Preference Aggregation: FOSD by Relative Ratio



Preference Aggregation: Econometric Analysis

Collective Risk Aversion	**Coefficient**		
	Model 1	Model 2	Model 3
Risk_Aversion_Max	0.792***	0.808***	0.759***
	(0.066)	(0.063)	(0.073)
Risk_Aversion_Distance	−0.421***	−0.432***	−0.434***
	(0.053)	(0.053)	(0.062)
CCEI_Max		0.165**	0.196***
		(0.064)	(0.071)
CCEI_Distance		0.014	0.040
		(0.033)	(0.045)
Math_Score_Max			0.000
			(0.005)
Math_Distance			0.005
			(0.004)
Constant	0.004	−0.156**	−0.175**
	(0.027)	(0.063)	(0.077)
Class Fixed Effect	Yes	Yes	Yes
Individual Characteristics	No	No	Yes
School Characteristics	No	No	Yes
Friendship	No	No	Yes
Observations	786	786	786
R-squared	0.372	0.378	0.382

Result 3: Efficiency and Welfare

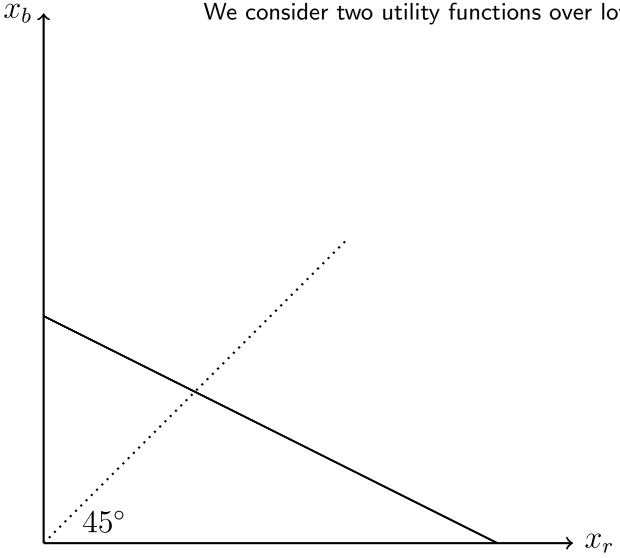
Measurement: Idea

- We analyze the **quality** of collective decisions as a function of the degrees of rationality and preference alignment.
- Idea:
  - We consider a class of utility functions over lotteries.
  - For each subject, we estimate the utility function parametrically.
  - We characterize a set of Pareto efficient choices.
  - For collective choices which are **not** Pareto efficient, we measure the degree of welfare loss.

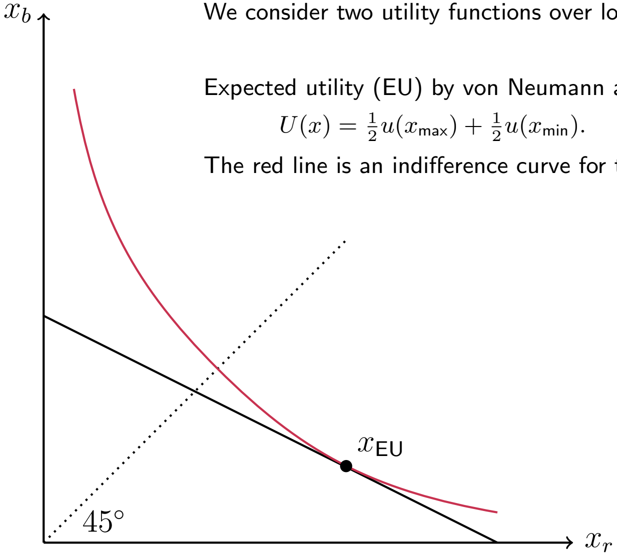
Measurement: Utility Estimation

We assume a CARA utility function: $u(x) = -e^{-\rho x}/\rho$.

We consider two utility functions over lotteries.



Measurement: Utility Estimation



We assume a CARA utility function: $u(x) = -e^{-\rho x}/\rho$.

We consider two utility functions over lotteries.

Expected utility (EU) by von Neumann and Morgenstern (1953):

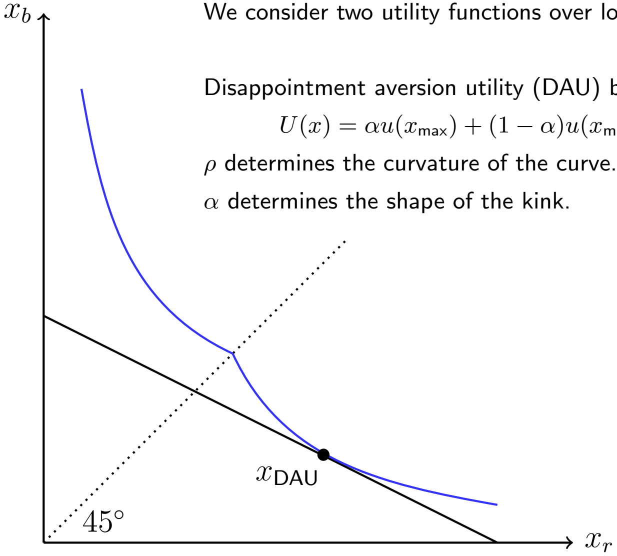
$$U(x) = \frac{1}{2}u(x_{\mathsf{max}}) + \frac{1}{2}u(x_{\mathsf{min}}).$$

The red line is an indifference curve for the EU.

Measurement: Utility Estimation

We assume a CARA utility function: $u(x) = -e^{-\rho x}/\rho$.

We consider two utility functions over lotteries.



Disappointment aversion utility (DAU) by Gul (1991):

$$U(x) = \alpha u(x_{\max}) + (1-\alpha)u(x_{\min}).$$

ρ determines the curvature of the curve.

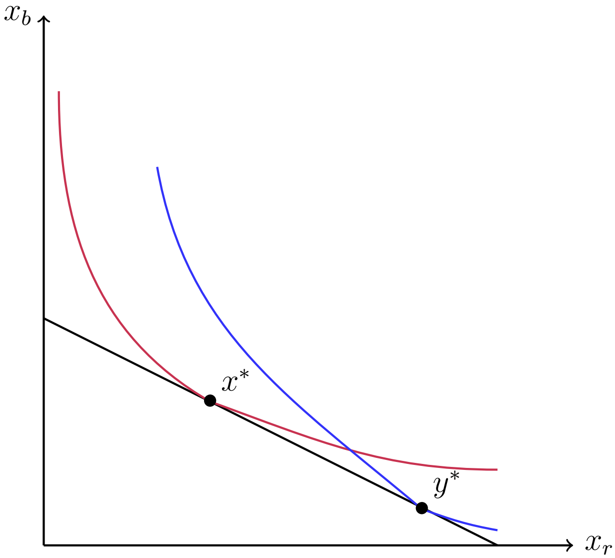
α determines the shape of the kink.

Measurement: Utility Estimation

- We restrict our attention to a CARA utility function over outcomes.
- We consider two different types of utility function over lotteries:
  - Expected utility (EU)
  - Disappointment aversion utility (DAU).
- We estimate ρ and β simultaneously by using a combination of a bootstrapping and the non-linear least square (NLLS) methods:
  1. Find subsample of size 18 with replacement.
  2. For given subsample, estimate α and ρ by NLLS.
  3. Repeat the above for 250 times.
  4. If $0.5 \in [\alpha_{2.5}, \alpha_{97.5}]$, then set $\alpha = 0.5$ as an EU.

  4' Otherwise, set $\alpha = \overline{\alpha}$ as a DAU.

Measurement: Efficiency and Welfare Loss



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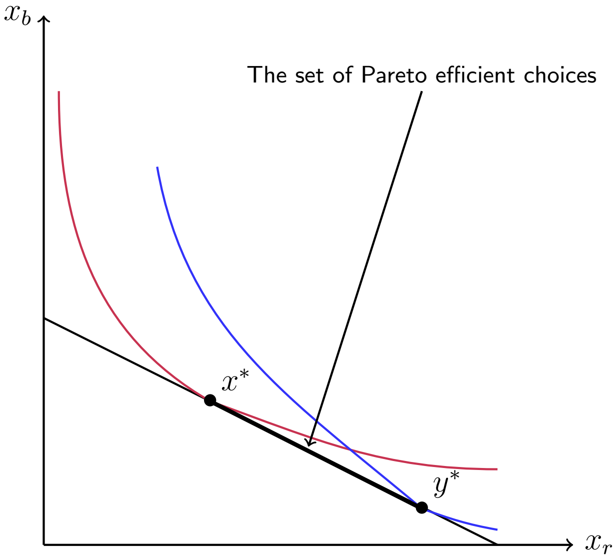

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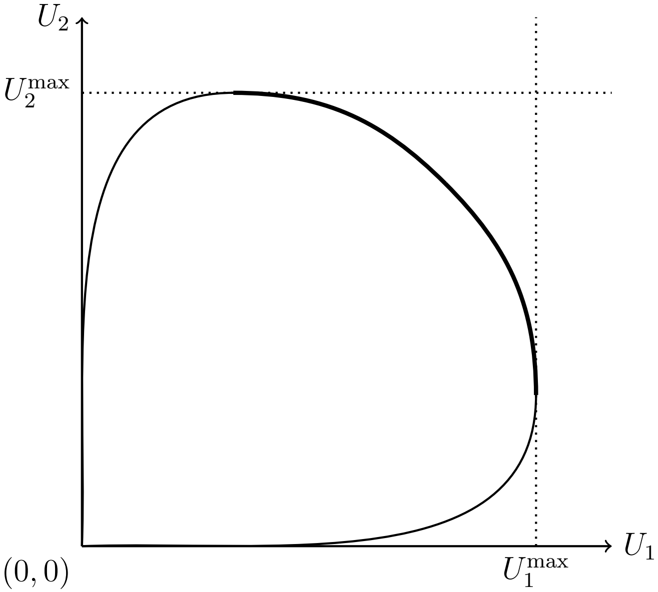

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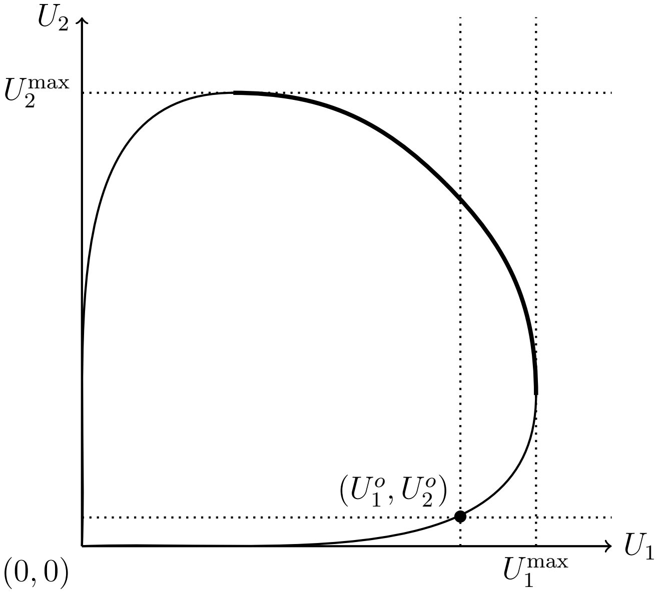
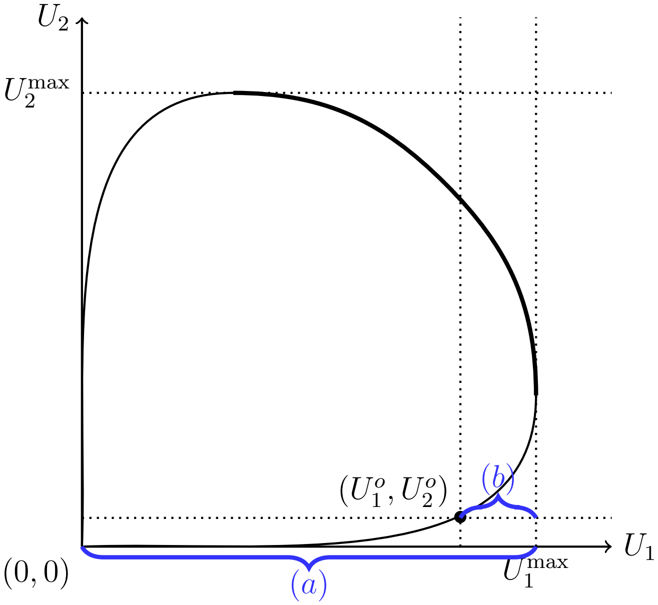
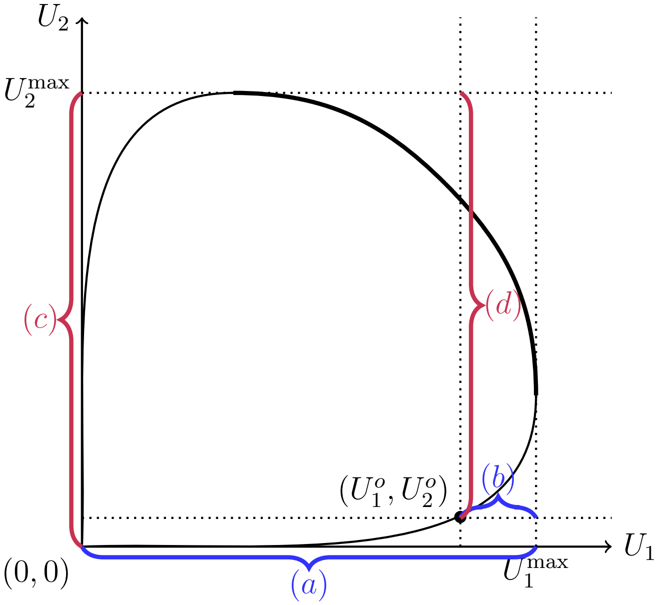


Measurement: Efficiency and Welfare Loss



Measurement: Efficiency and Welfare Loss



Efficiency and Welfare: Measurement

- We focus on the group choices which are not Pareto efficient (60%).
- For those choices, we measure **welfare loss** of a group as

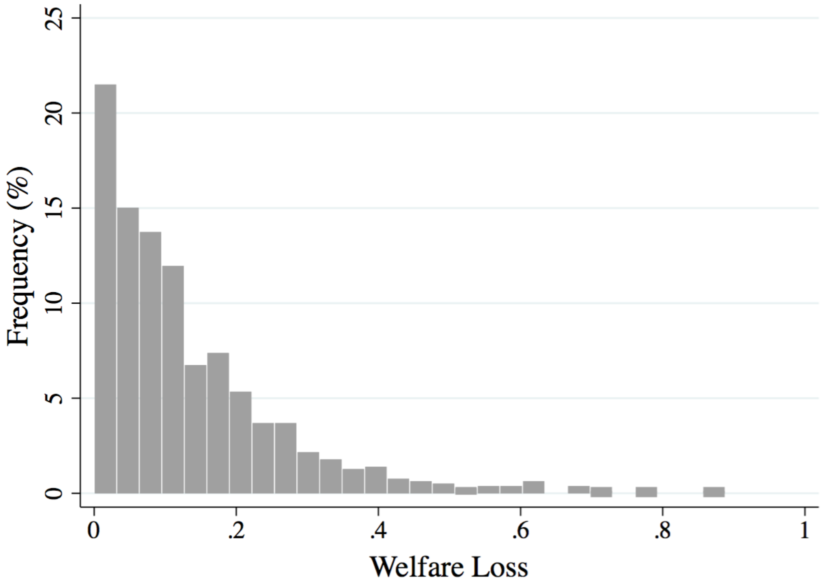
$$\text{Welfare Loss} = \frac{1}{18}\sum_{k=1}^{18}\frac{1}{2}\sum_{i=1}^{2}\frac{U_i(x_{ikb}) - U_i(x_{ick})}{U_i(x_{ikb}) - U_i(x_{ikw})},$$

  where

  - x_{ick}: group choice in k-th round
  - x_{ikb}: member i's best choice in k-th round
  - x_{ikw}: member i's worst choice in k-th round.

- By definition, Welfare Loss $\in [0, 1]$.

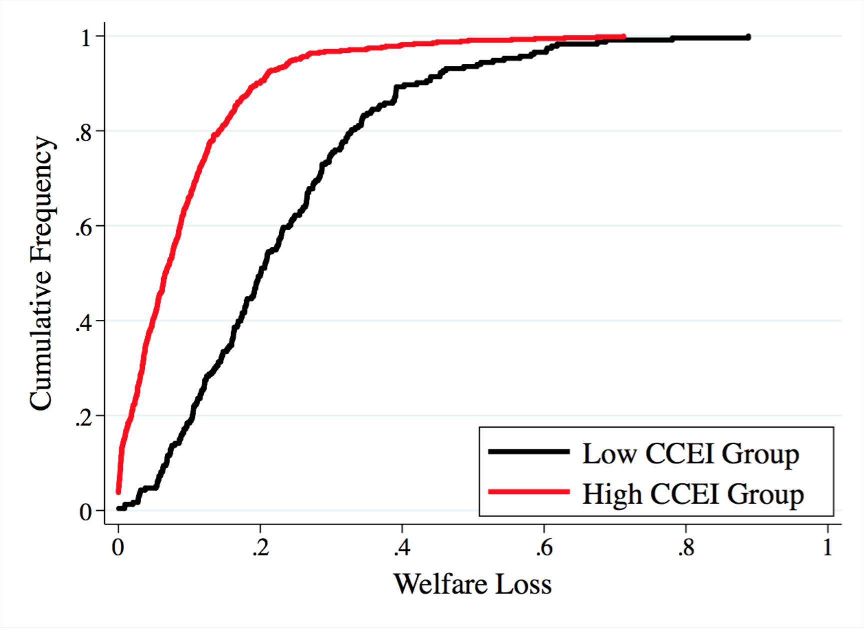
Efficiency and Welfare: Distribution of Welfare Loss



Efficiency and Welfare: Research Question

How is the welfare loss related to individual rationality and risk preference?

Welfare: FOSD by Group Rationality



Welfare: Econometric Analysis

Group Inefficiency	Coefficient			
	Model 1	Model 2	Model 3	Model 4
CCEI_Group	-0.571^{***}	-0.503^{***}	-0.527^{***}	-0.414^{***}
	(0.042)	(0.039)	(0.043)	(0.078)
CCEI_Max		-0.296^{***}	-0.242^{***}	-0.692
		(0.045)	(0.051)	(0.671)
CCEI_Distance		0.165^{***}	0.178^{***}	0.289
		(0.042)	(0.052)	(0.257)
Risk_Aversion_Max		-0.009	0.023	-0.024
		(0.056)	(0.063)	(0.089)
Risk_Aversion_Distance		-0.057^{*}	-0.073^{*}	-0.123^{*}
		(0.031)	(0.040)	(0.066)
Math_Score_Max			0.002	0.008
			(0.005)	(0.007)
Math_Distance			-0.004	-0.010
			(0.003)	(0.006)
Constant	0.651^{***}	0.866^{***}	0.807^{***}	1.154^{*}
	(0.038)	(0.048)	(0.061)	(0.653)
Class Fixed Effect	Yes	Yes	Yes	Yes
Individual Characteristics	No	No	Yes	Yes
School Characteristics	No	No	Yes	Yes
Friendship	No	No	Yes	Yes
Observations	786	786	786	274
R-squared	0.442	0.487	0.497	0.436

Conclusion

- We measure rationality and risk preference in individual and group levels.
- We observe rationality extension and preference aggregation.
- We develop a measure of efficiency and utility loss of group decisions.
- We find that
  - Rational groups are more likely to make efficient decisions.
  - Preference-aligned individuals need not make efficient decisions.

Conclusion

- Our main findings are robust with respect to
  - another rationality measure (Varian's efficiency index)
  - other cutoff values of CCEI (0.99 or 0.95)
  - another measure of risk preferences (risk premium).

Robustness

Varian's Efficiency Index

- Varian modifies CCEI by allowing e_k to vary across the different price vectors.

- Consider a vector $\theta = (e^k)_{k=1}^K$ of numbers in $[0, 1]$, one for each observation.

- Define the binary relation R_θ as $x^k R_\theta x^l$ if $e^k p^k \cdot x^k \geq p^k \cdot x^l$. Let P_θ be the corresponding strict relation.

- There is a set Θ of vectors θ such that the corresponding $\langle R_\theta, P_\theta \rangle$ satisfies GARP.

- Varian's efficiency index (VEI) is the closest distance of a vector θ to the unit vector ($e_k = 1$ for all k), among those θ for which the preference pair is acyclic:

$$\text{VEI} = \inf \left\{ ||1 - \theta|| \middle| \langle R_\theta, P_\theta \rangle \text{ is acyclic} \right\}.$$

Disappointment Aversion Utility

- A functional form:

$$u(x_1, x_2) = u(\max\{x_1, x_2\}) + (1-\gamma)u(\min\{x_1, x_2\}),$$

  where $\gamma = 1/(2+\beta)$.

- $\beta > 0$ represents the **disappointment aversion**: the better outcome is under-weighted relative to the objective probability.

- $\beta \in (-1, 0)$ represents **elation seeking**: the better outcome is over-weighted relative to the objective probability.

- Of course, this utility function is aligned with the first-order stochastic dominance relationships between lotteries.