Collective choices which can not be Pareto improved

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December 19, 2016

Denote (x_c, y_c)= collective choice of x and y, (x_i^*, y_i^*)=i's optimal choice estimated from estimated utility function. Also, assume that one's utility function has the form of $U(x_{min}, y_{min}) = \alpha u(x_{min}) + (1-\alpha)u(x_{max}), \alpha >= \frac{1}{2}$

Claim 1. If $\alpha >= \frac{1}{2}$, any $(x_c, y_c) = t(x_1^*, y_1^*) + (1-t)(x_2^*, y_2^*)$ can not be Pareto improved.

Claim 2. This is also same when $\alpha < \frac{1}{2}$. From now one, assume $(p_x < p_y)$ w.l.o.g.

step 1. Any combination in $\{(x, y)|p_x < p_y, x < y\}$ can't not be an optimal solution.

Suppose there exist $(x^*, y^*) \in \{(x, y)|p_x < p_y, x < y\}$. Then, there exists $(\tilde{x}, \tilde{y}) = (y^*, x^*)$ such that $U(x^*, y^*) = U(\tilde{x}, \tilde{y})$. However, $(p_x\tilde{x} + p_y\tilde{y}) - (p_x x^* + p_y y^*) = (p_x y^* + p_y x^*) - (p_x x^* + p_y y^*) = (p_x - p_y)(y^* - x^*) < 0$. Therefore, by non-satiation property of utility function, there exists $(\hat{x}, \hat{y})$ s.t $(\hat{x}, \hat{y}) > (\tilde{x}, \tilde{y})$ elementwise and $U(\hat{x}, \hat{y}) > U(\tilde{x}, \tilde{y}) = U(x^*, y^*)$.

Therefore, $(x_1^*, y_1^*), (x_2^*, y_2^*) \in \{(x, y)|p_x < p_y, x > y\}$.

step 2. Note that in the subset of budget line $\{(x, y)|p_x x + p_y y = M, p_x < p_y, x > y\}$, indifference curve follows standard assumptions. It means both individual's indifferent curve moves same as in single peaked preference. Therefore, any $(x_c, y_c) \in [(x_1^*, y_1^*), (x_2^*, y_2^*)]$ can't be Pareto improved and $(x_c, y_c) \notin [(x_1^*, y_1^*), (x_2^*, y_2^*)]$ can be Pareto improved when $(x_c, y_c) \in \{(x, y)|p_x x + p_y y = M, p_x < p_y, x > y\}$.

If $(x_c, y_c) \in \{(x, y)|p_x x + p_y y = M, p_x < p_y, x < y\}$, as in step 1, there exists $(\tilde{x_c}, \tilde{y_c}) = (y_c, x_c)$ s.t. $U_1(\tilde{x_c}, \tilde{y_c}) = U_1(x_c, y_c)$, $U_2(\tilde{x_c}, \tilde{y_c}) = U_2(x_c, y_c)$ and $(p_x\tilde{x_c} + p_y\tilde{y_c}) < (p_x x_c + p_y y_c)$. Therefore, we

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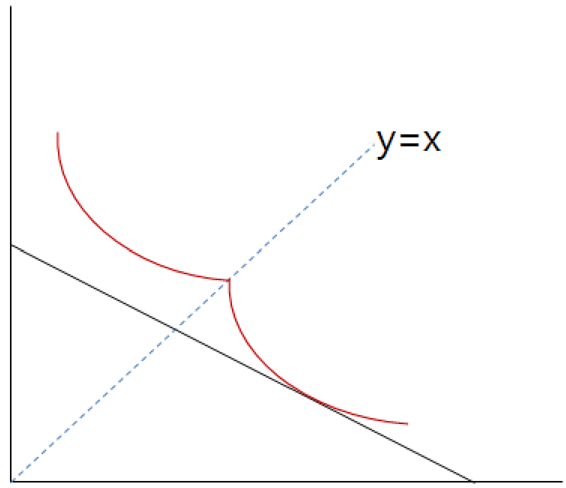
Figure 1: Indifference curve when $\alpha < \frac{1}{2}$.



can find $(\hat{x_c}, \hat{y_c})$ as in step 1. However, this contradicts that any $(x_c, y_c) \in [(x_1^*, y_1^*), (x_2^*, y_2^*)]$ can't be Pareto improved within $\{(x, y)|p_x x + p_y y = M, p_x < p_y, x > y\}$

1 A simple proof

Without loss of generality, suppose that $p_2 \geq p_1$. Then, both x^A and x^B are located in a region where $x_2 \leq x_1$. Let I be the budget line defined as

$$B(p_1, p_2, I) = \{(x_1, x_2) \in \mathbb{R}^2 | p_1 x_1 + p_2 y_2 = I, x_1 \geq 0, x_2 \geq 0\}.$$

Let x^C be a point on the budget line, which is also a convex combination of x^A and x^B. That is, there exists $\alpha \in [0, 1]$ such that $x^C = \alpha x^A + (1 - \alpha) x^B$. Now we want to show that x^C is not Pareto dominated by any point on the budget line.

Suppose, by the way of contradiction, that there exists x^D that Pareto dominates x^C. That is,

$$u_A(x^D) \geq u_A(x^C) \quad \text{and} \quad u_B(x^D) \geq u_B(x^C),$$

where at least one inequality is strict. Then, $x^D = (x_1^D, x_2^D)$ cannot be located in the region where $x_2 \leq x_1$. This follows from the fact that independent of the risk preference parameter, the **restricted** indifference curve of x^C is concex for both agent A and agent B. This enables us to assume that $x_2^D > x_1^D$.

Suppose that there exists $x^D \in B(p_1, p_2, I)$ such that $x_2^D > x_1^D$ and x^D Pareto dominates x^C. Suppose, without loss of generality, that $u_A(x^D) > u_A(x^C)$. Let $x^E = (x_2^D, x_1^D)$. Then, since the indifference is symmetric about the line $x_2 = x_1$, it follows that

$$u_A(x^E) = u_A(x^D) > u_A(x^C).$$

Hence, x^E is located in the upper-contour set of the indifference curve passing through x^C. In addition, since $p_2 \geq p_1$, it follows that

$$p \cdot x^E = p_1 x_2^D + p_2 x_1^D < I.$$

Now, since the indifference curves of agent A and agent B are convex in the region where $x_2 \leq x_1$

and $p \cdot x^E < I$, it follows that

$$u_B(x^E) = u_B(x^D) < u_B(x^C),$$

which contradicts the assumption that x^D Pareto dominates x^C. Therefore, x^C is not Pareto dominated by any point on the budget line.