

GROUP 12

# CHE212: HEAT TRANSFER

CONDUCTION IN 2D SHEET

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# INTRODUCTION

The problem we have at hand is conduction through a thin metal sheet. We have a thin square metal sheet which we have divided in grid squares.

In one of the corner-most grid square, we have installed a small heater which we supply DC power to. We firstly let the heater reach a steady state temperature and then insert the corner into it.

We have attached temperature sensors at some specific nodes to read the temperatures at different times. We have also created a simulation which models the situation on Matlab.

# APPARATUS

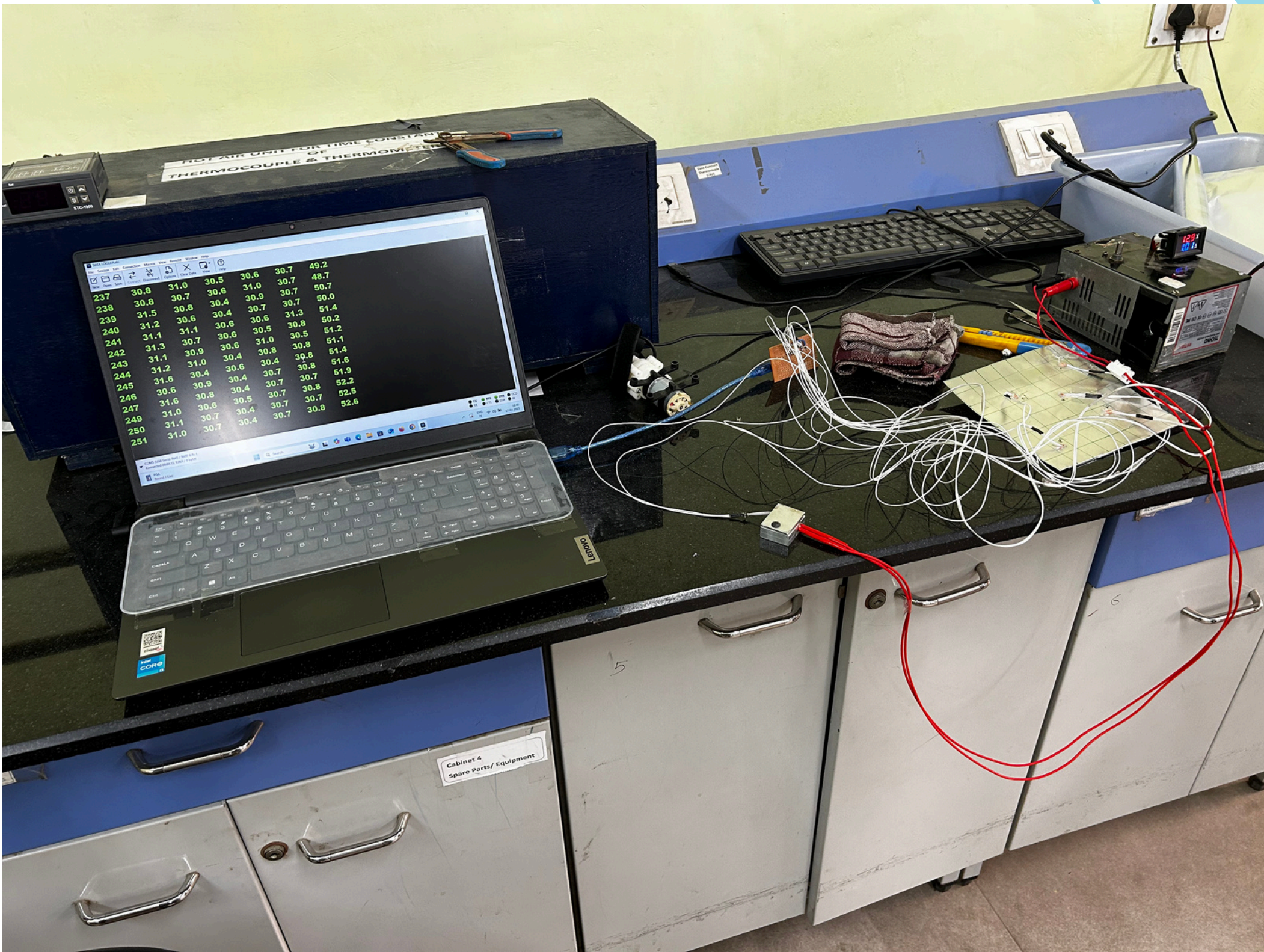
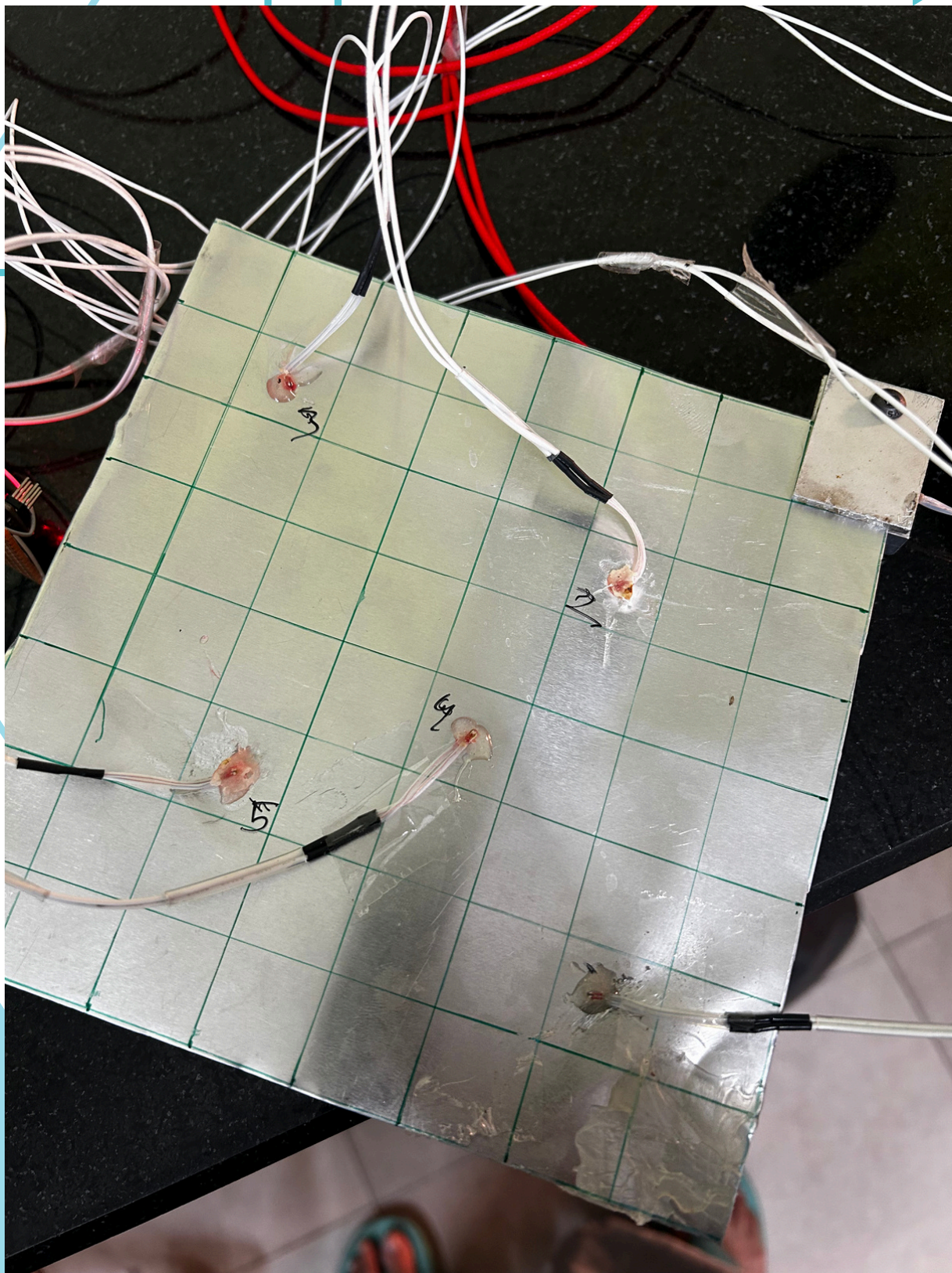
- GI Sheet with
  - $K = 55 \text{ W/mK}$
  - $C_p = 0.42 \text{ J/Kg.K}$ , Density =  $7850 \text{ kg/m}^3$
  - $16 \times 16 \text{ cm}$  dimensions and thickness  $0.2 \text{ mm}$
- Metal block (Iron or Aluminum)
- Cylindrical heater
- Power supply
- Temperature sensors
- Arduino board
- Laptop with CoolTerm Software installed
- Metal sheet of thickness  $3 \text{ mm}$ .
- Metal screw
- Workshop equipment

## INNOVATIVE ASPECT: HEATING ELEMENT

- First method we tried was inserting the relevant grid square into a plastic box filled with hot water through a slit. Issues like leakage of water, cooling down of water and non-square boundary were faced. Hence we made a small square heater.
- For the main heating element we used the cylindrical heater provided by the UOP lab.
- A 6mm hole was drilled into a 30\*30\*8mm metal block on a 30\*8 face. The heater was inserted in the hole.
- A 30\*30 metal flap was fixed on the top of this heater using a nail such that the flap could move sideways.
- The heater can be connected to a DC power source and on changing the power output we can reach different steady state temperatures.
- The sheet can be inserted between the flap and the block after reaching the steady state temperature. A sensor is attached to the heater surface to observe temperature



# EXPERIMENTAL SETUP





# PROCEDURE

- The metal sheet was of dimensions 16\*16 cm which was divided into an 8\*8 grid. If the grid square at the higher temperature is (1,1) then sensors were fit at (3,3),(7,2),(2,7),(6,6) and (5,4).
- The sensors were connected to the CoolTerm software.
- There were in total 6 temperature sensors connected to our system, five in the metal sheet and one on the heater block.
- First we connected the heater to a DC power source and observed the rise in temperature of the heater block.
- We waited till the heater reached a near about steady temperature after which the (1,1) grid square was inserted into the heater.
- The system was left undisturbed till no further changes could be observed in the sensor readings.



# THEORY

## Theory: 2D Transient Heat Conduction in a Thin Metal Sheet with Local Heating and Convective Losses

We consider a thin square metal sheet with a thickness  $\delta$ , placed in the  $x$ - $y$  plane. The sheet is divided into grid squares for temperature monitoring. A small heater is embedded at a corner node and supplies constant power. The rest of the sheet is initially at ambient temperature. Temperature sensors are embedded at certain points to record how heat spreads over time.

The top and bottom faces (i.e., surfaces normal to  $z$ -direction) are exposed to ambient air and lose heat by convection.

### Governing Equation:

The heat conduction in the  $x$ - $y$  plane with convective losses from the top and bottom surfaces is governed by:

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{2h}{\rho c_p \delta} (T - T_\infty)$$

- $T(x, y, t)$ : temperature field
- $\alpha = \frac{k}{\rho c_p}$ : thermal diffusivity
- $h$ : convective heat transfer coefficient (W/m<sup>2</sup>K)
- $\delta$ : sheet thickness (m)
- $T_\infty$ : ambient air temperature
- $\rho$ : density of the metal (kg/m<sup>3</sup>)
- $c_p$ : specific heat capacity (J/kg·K)
- $k$ : thermal conductivity (W/m·K)

## Initial Condition

$$T(x, y, 0) = T_{\infty} \quad (\text{uniform ambient temperature})$$

## Boundary Condition

- Heater contact at corner (Dirichlet condition):

$$T(0, 0, t) = T_H \quad (\text{constant heater temperature})$$

## Heat Transfer Coefficient Estimation (Flat Plate)

If forced convection is present on z-surfaces (say, due to a fan), the local convective heat transfer coefficient can be estimated using the Blasius solution:

$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3} \quad \Rightarrow \quad h_x = \frac{Nu_x \cdot k_{air}}{x}$$

- $Re_x = \frac{\rho V x}{\mu}$ : Reynolds number,
- $Pr$ : Prandtl number,
- $k_{air}$ : thermal conductivity of air.



## Numerical Implementation (FDM)

Each interior node satisfies:

$$\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \alpha \left( \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{\Delta x^2} + \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{\Delta y^2} \right) - \frac{2h}{\rho c_p \delta} (T_{i,j}^n - T_\infty)$$

## Heat Balance and Energy Loss Calculation due to Convection

At any time  $t$ , the total heat loss from the sheet is:

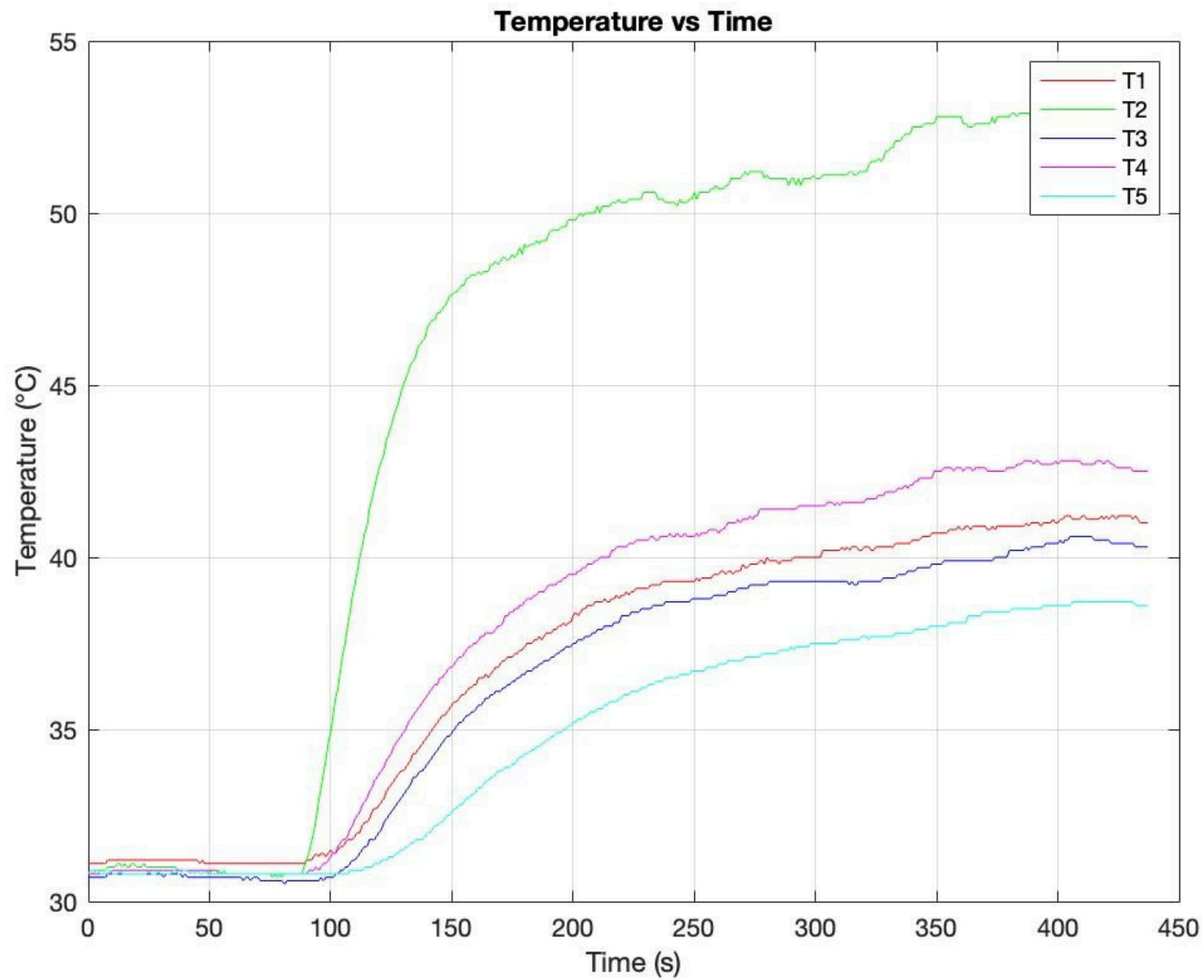
$$Q(t) = \iint_A 2h(T(x, y, t) - T_\infty) dx dy$$

## Assumptions

- Material is homogeneous and isotropic.
- Heat conduction is only in the x-y plane (thin sheet assumption).
- No radiation losses.
- Constant thermal properties.
- Convective losses from z-surfaces only.
- $h$  stands for local heat transfer coefficient.



# GRAPHS





# GRAPHS

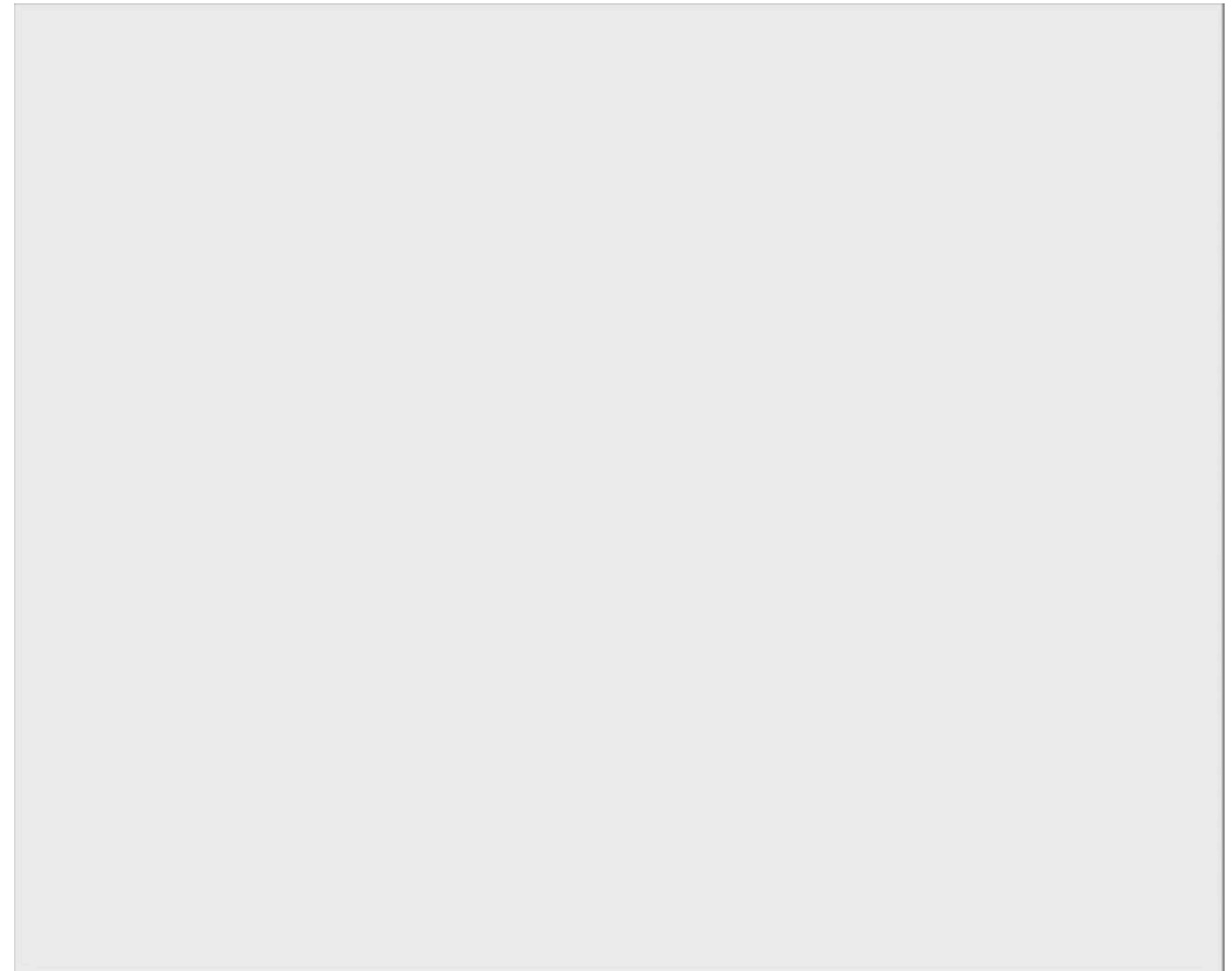
**$N_x=8, N_y=8$**

```
9-48ea-b47f-5471a4ff1fa3
ktop/HEAT_2D.m
ATEXP.m x HEATEXP2.m x HEAT_2D.m x +
- 1);
- 1);
d] = meshgrid(linspace(0,Lx,Nx), linspace(0,Ly,Ny));

erties

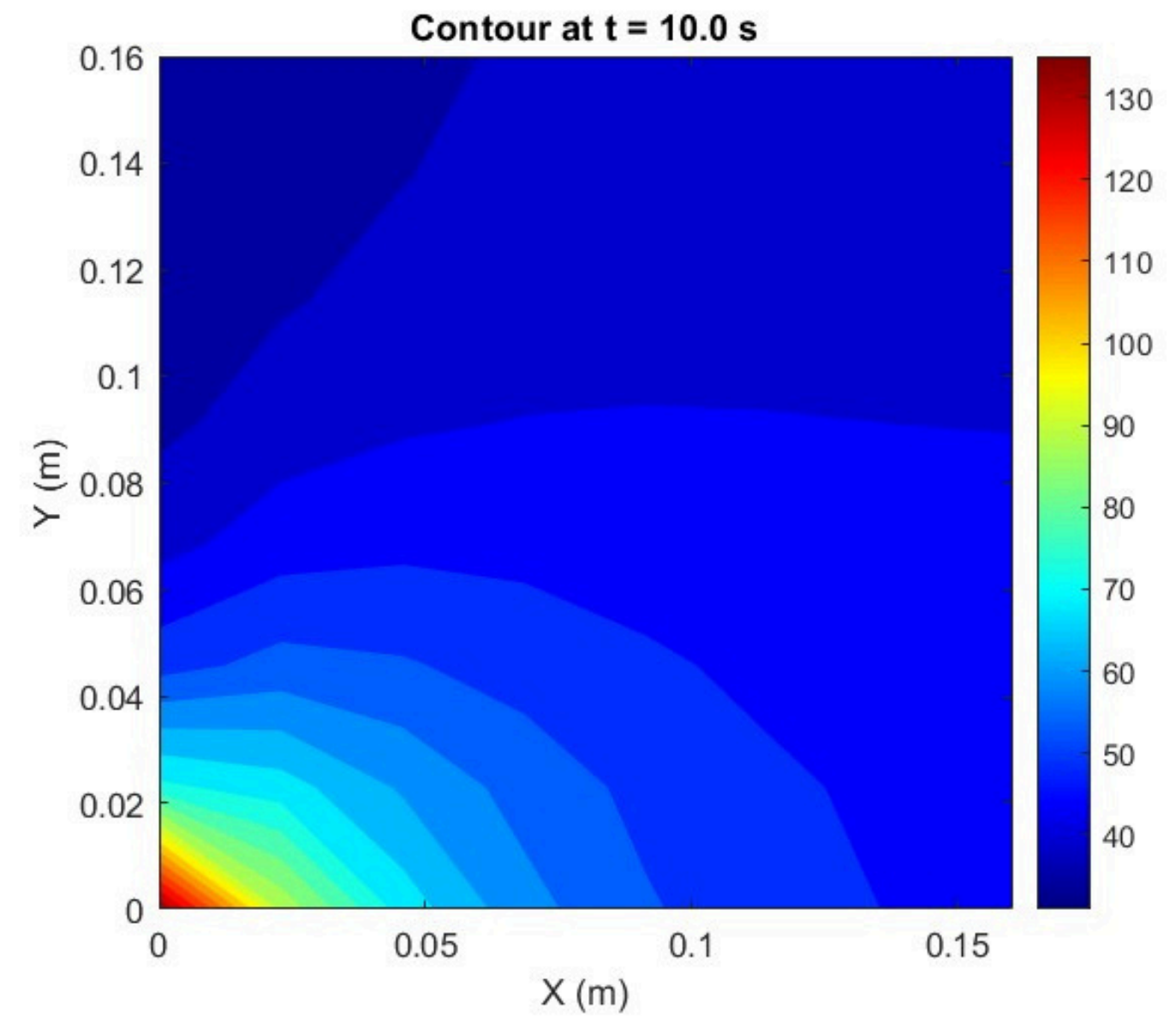
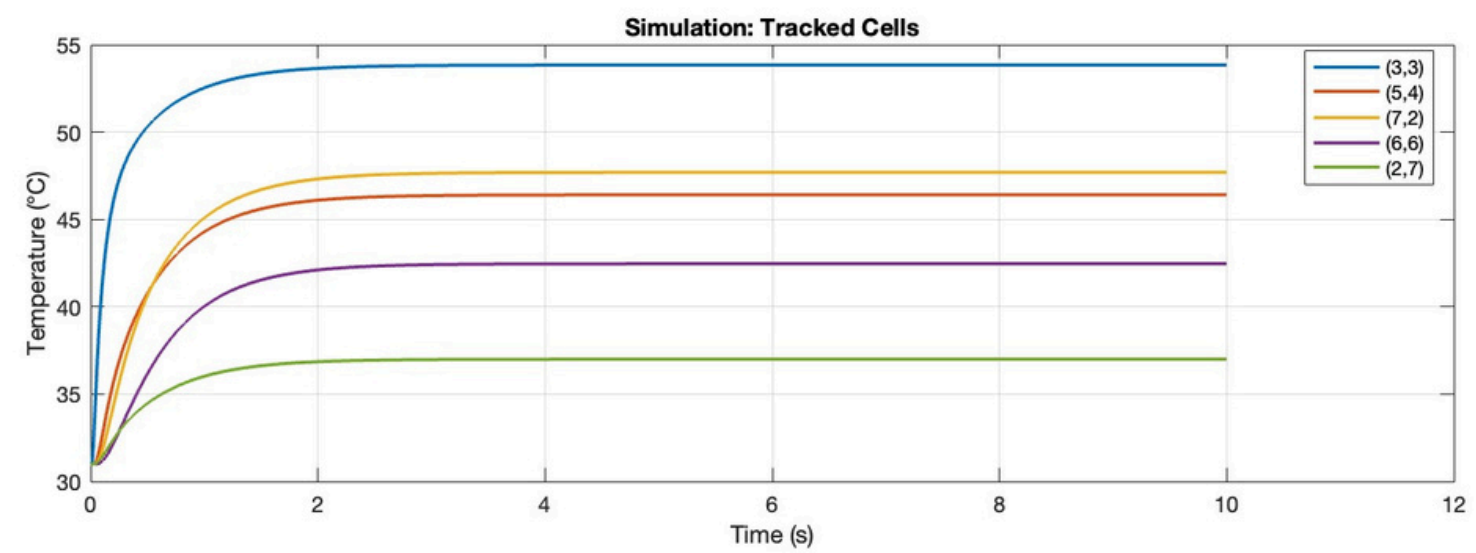
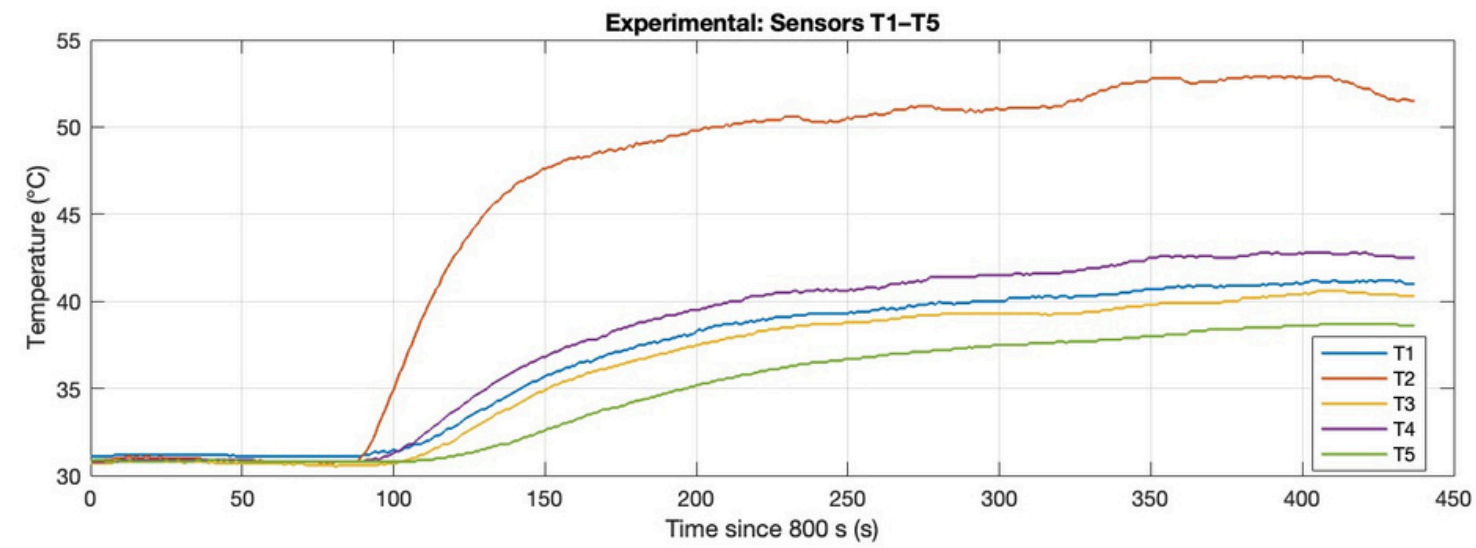
cp corrected to J/kg·K
ho * cp);
```

**$N_x=15, N_y=15$**



# GRAPHS

Temperature vs. Time: Experimental vs. Simulation





# DATA

T(s)	T1	T2	T3	T4	T5
0	31.1	30.8	30.7	30.9	30.8
1	31.1	30.9	30.7	30.9	30.8
2	31.1	30.9	30.7	30.9	30.8
3	31.1	30.9	30.7	30.9	30.8
4	31.1	30.9	30.7	30.8	30.8
5	31.1	30.8	30.7	30.8	30.8
6	31.1	30.9	30.7	30.8	30.8
7	31.1	30.8	30.7	30.8	30.8
8	31.1	30.8	30.7	30.8	30.8
9	31.1	30.8	30.7	30.8	30.8
10	31.1	30.8	30.7	30.8	30.8
11	31.1	30.8	30.7	30.8	30.8
12	31.1	30.8	30.7	30.8	30.8
13	31.1	30.8	30.7	30.8	30.8
14	31.1	30.8	30.6	30.8	30.8
15	31.1	30.8	30.7	30.8	30.8
16	31.1	30.8	30.7	30.8	30.8
17	31.1	30.8	30.6	30.8	30.8
18	31.1	30.8	30.7	30.8	30.8
19	31.1	30.8	30.7	30.8	30.8
20	31.1	30.8	30.7	30.8	30.8
21	31.1	30.8	30.6	30.8	30.8
22	31.1	30.8	30.6	30.8	30.8
23	31.1	30.8	30.6	30.8	30.8
24	31.1	30.8	30.6	30.8	30.8

25	31.1	30.8	30.6	30.8	30.8
26	31.1	30.7	30.6	30.8	30.8
27	31.1	30.8	30.6	30.8	30.8
28	31.1	30.7	30.6	30.8	30.8
29	31.1	30.8	30.6	30.8	30.8
30	31.1	30.7	30.6	30.8	30.8
31	31.1	30.8	30.5	30.8	30.8
32	31.1	30.8	30.6	30.8	30.8
33	31.1	30.8	30.6	30.8	30.8
34	31.1	30.8	30.6	30.8	30.8
35	31.1	30.8	30.6	30.8	30.8
36	31.1	30.8	30.6	30.8	30.8
37	31.1	30.8	30.6	30.8	30.8
38	31.1	30.8	30.6	30.8	30.8
39	31.1	31	30.6	30.8	30.8
40	31.2	31.2	30.6	30.8	30.8
41	31.2	31.4	30.6	30.9	30.8
42	31.2	31.7	30.6	30.9	30.8
43	31.3	32	30.6	30.9	30.8
44	31.4	32.4	30.6	31	30.8
45	31.3	32.9	30.6	30.9	30.8
46	31.4	33.3	30.7	31	30.8
47	31.4	33.7	30.6	31.1	30.8
48	31.3	34.1	30.7	31.1	30.8
49	31.4	34.6	30.7	31.2	30.8
50	31.5	34.9	30.7	31.3	30.8

[Check out the complete data by clicking on this link](#)

# RESULTS

- From our readings and simulation, we have observed several differences between the two. One major similarity between the two is that the graph of temperature increase of each sensor is qualitatively similar.
- The differences we have noticed are that steady state is reached very quickly in our simulation (~4-5 seconds) compared to the experiment. This discrepancy is due to incapacities in building an efficient environment for conducting the experiment.
- Our simulation does not account for heat loss from the boundaries while there is no way to avoid this in the experiment.
- Also we could not measure the velocity of air in the room, the value used in the simulation is based on assumption.
- The material properties used in the simulation were provided by the UOP lab technicians.
- Ultimately, the heat spread was qualitatively verified. Both simulation and experiments provide the knowledge that 2D heat conduction is somewhat radial in nature and that it reaches steady state exponentially.



# FUTURE WORK

- There is a necessity to make a better experimental setup. We can try to do the same by implementing the following things:
  - Ensure that the sheet is not deformed. Make very detailed grid lines.
  - Fit the temperature sensors using a different material such that it does not melt at high temperatures.
  - Make a proper stand with thermally insulating material to hold the sheet in air properly such that it does not touch any surfaces.
  - Use better sensors (glass bead sensors are unreliable at high temperatures)
  - More intricate sensor mapping.
  - Better simulation with accounting for heat loss through boundaries.

# FUTURE WORK

The glue we used currently melted when the heater reached 135 degree celsius. So to cure this issue, we can use the following sealant to fix the sensors in place

## DOWSIL 736 RTV Silicone Sealant

DOWSIL™ 736 Heat Resistant Sealant is a one-part, non-slumping paste that cures to a rubbery solid when exposed to moisture in the air. Formulated for use at high temperatures, it performs well at continuous temperatures up to 260°C (500°F) and intermittent exposure to 315°C (600°F). This silicone sealant is ideal for sealing, bonding, potting, and encapsulating high-temperature components. Non-slumping sealant designed for sealing and bonding applications exposed to temperatures as high as 315°C (600°F)





# CONCLUSION

We were able to implement our study to a limited qualitative estimation. We concluded that heat conduction equation which we used in our simulation is valid enough at least qualitatively in real life scenarios as well.

With more time and better setup, and also a good amount of trials and readings, we can extend this to a more accurate experiment. We also can use a more conducting sheet to observe better gradients.

The background features a light gray field with abstract teal geometric elements. In the top-left, there are nested rectangular outlines and a diagonal line. The top-right corner contains a 4x5 grid of small teal circles. The bottom-left has a 5x4 grid of similar circles. The bottom-right features more nested rectangular outlines. A diagonal teal line runs from the top-right towards the bottom-left, intersecting the other elements.

# THANK YOU