Task Sheet 8

Rate equations

Deadline 15.00, 27.06.2024

Lecture: *Collective Robotics and Scalability*, Summer Term 2024

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Objectives:

- ▷ understanding rate equations and delay equations

Task 8.1 Rate equations

We use the rate equations model of searching and avoiding from the lecture:

$$\frac{dn_s(t)}{dt} = -\alpha_r n_s(t)(n_s(t)+1) + \alpha_r n_s(t-\tau_a)(n_s(t-\tau_a)+1))$$
$$\frac{dm(t)}{dt} = -\alpha_p n_s(t) m(t)$$

a) Use a tool of your choice to calculate the temporal course of this system of ordinary differential equations (a simple forward integration in time can also be implemented from scratch for this system; make sure you have a sufficient small step size, e.g., $\delta_t = 0.0001$, otherwise the system may diverge or shows weird behavior). Notice that we have delay equations. How should the delays be treated especially early in the simulation ($t < \tau_a$)? Use the following setting for the parameters: $\alpha_r = 0.6$, $\alpha_p = 0.2$, $\tau_a = 2$, $n_s(0) = 1$, $m(0) = 1$. Calculate the values of n_s and m for $t \in (0, 50]$ and plot them. Interpret your result.

b) Now we want to extend the model. In addition to *searching* and *avoiding* we introduce a third state: *homing* (n_h). Robots that have found a puck do a transition to the state *homing* in which they stay for a time $\tau_h = 15$. We assume that for unspecified reasons robots in state *homing* do not interfere with each other or with robots of any other state (assumption: no avoidance behavior for robots in state *homing* necessary). After the time of τ_h they have reached the home base and do a transition back to *searching*. Add an equation for n_h and edit the equation of n_s accordingly. Calculate the values of n_h, n_s and m for $t \in (0, 160]$ and plot them. In a second calculation, reset the ratio of pucks at time $t = 80$ to $m(80) = 0.5$ and plot the results. Interpret your result.