Corporate Finance

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Introduction

These notes are based on the lectures delivered by Professor Giancarlo Giudici during the Corporate Finance course at Politecnico di Milano during Semester 2024-2. Official course information can be found on the official program (Giudici, 2024).
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1.1 Basics of Financial Markets

Companies create **value** through their **investments**. Actually, efficient investments may generate cash flows in the future.
Financers provide cash to companies, that is paid back and remunerated thanks to future cash flows of the borrower.
Investors may be individuals, banks, and investment funds with a capital surplus, and are available to offer cash on the market in exchange for remuneration in the future.
The expected profitability is balanced against the **risk** of the business.
The cash sources are from:

- Issuance of new liabilities
- Divestment of existing assets

The financial market allows the match between companies demanding capital and investors willing to lend it. Anyway, financial markets are not always efficient. There are information asymmetries and market imperfections that create problems in this system.
These transactions are regulated by **contracts**, which are called **securities**. Examples of them are bonds, loans, shares, and derivatives.

Companies typically finance their assets through a combination of:

- **Equity (E):** Capital provided by shareholders. It is a title of ownership of the firm, and the remuneration is residual
- **Debt (D):** Capital raised on the market by third parties. The remuneration is contractually declared at the time of signing the contract. When is financed with debts, it is said that is **levered**.

1.2 Taxonomy of Financial Markets

There are different types of markets where the companies borrow capital.

- **Primary Market:** Direct financing channel through securities issuance.
- **Secondary Market:** Financial Market in which financial instruments issued on the primary market are bought and sold. In other words, in this market, the securities change through different portfolios.
- **Financial Intermediaries:** Collect deposits on the market and convey it to businesses that need to be financed. For instance, banks.

1.3 Basic concepts

The EBIT is the Earnings Before Interest and Taxes, and measures a company's profitability from its core operations.

$$EBIT = Revenue - Operating\ Expenses$$

It is also called operating profit The earnings are calculated as:

$$Earnings = (EBIT - r \cdot D) \cdot (1 - t_C)$$

where t_C are the corporate taxes.
Therefore, if we invest in the t% of the company value, our returns on investment will be

$$\pi = t\% \cdot Earnings = t\%(EBIT - r \cdot D)$$

Now, we introduce the concept of the expected profitability on equity capital k_E, and the expected profitability of the company k_A:

$$k_E = \frac{Earnings}{E}$$

$$k_A = \frac{EBIT_t}{V}$$

where $EBIT_t$ is the EBIT in the year t.
Define leverage as the use of borrowed funds to increase the potential return on investment:

$$Leverage = \frac{D}{E}$$

On the other hand, the cost of capital k is the required return that a company must earn to justify the cost of funding its operations. It represents the opportunity cost for investors.

The present value is the current worth of a future sum of money (Future value) or cashflows, discounted at the cost of capital. It reflects the time value of money, reflecting that money today is worth more than the same amount in the future.

$$PV = \frac{C}{(1+r)^t}$$

where t is the time period of the money return.
In particular, we have specific cases:

- **Perpetuity:** If we have a constant cash flow up to infinity, the present value is:

$$PV = \sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = \frac{C}{r}$$

- **Annuity:** If a constant cash flow is paid a fixed number of periods:

$$PV = \sum_{i=1}^{T} \frac{C}{(1+r)^i} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

- **Growing Perpetuity:** Perpetuity with C increasing each period at the rate $g < r$:

$$PV = \sum_{i=1}^{\infty} \frac{C(1+g)^{i-1}}{(1+r)^i} = \frac{C}{r-g}$$

- **Growing Annuity:** Annuity with C increasing each period at a rate $g < r$:

$$PV = \sum_{i=1}^{T} \frac{C(1+g)^{i-1}}{(1+r)^i} = \frac{C}{r-g} \cdot \left[1 - \frac{(1+g)^T}{(1+r)^T}\right]$$

There exist two different types of interest rate:

- **Simple:** The interest is determined by multiplying the interest rate r by the principal K:

$$I = r \cdot K$$

- **Compounded:** The interest is determined by the principal K but it accumulates over the periods, so if the interest are paid m times in each period:

$$I = \frac{r \cdot K}{m}$$

The relation between the simple r_S and compounded r_C interest rate is.

$$(1 + r_S) = (1 + r_C)^C$$

Then, r_S would be the real cost of capital in the whole period
There is another distinction between interest rates. A nominal interest rate is an interest rate that is computed in terms of the money numbers. Instead, a real interest rate is the one that takes into account the inflation g. The relation between those is called Fisher Relation:

$$(1 + r_{NOM})(1 + r_{REAL}) \cdot (1 + g)$$

1.4 Capital Structure Irrelevance

Modigliani and Miller developed famous prepositions about capital structure capital in perfect markets under the following assumptions:

1. No corporate taxes. (Actually, a less strict condition is needed: “Taxation does not create a difference between equity and debt”)
2. There is a unique interest rate (r%) for companies and investors
3. There is no asymmetric information or transaction costs
4. Firms can choose how to finance their operations with no endogenous limits

it does not matter which capital structure a firm decides to adopt.

Proof. If we invest in the 5% of each company (In the case of the leveraged company we invest 5% as a shareholder and 5% as a debtholder):

$$I_L = 5\%(V_L) = 5\% \cdot E_L + 5\% \cdot D_L$$

$$I_U = 5\%(V_U) = 5\% \cdot E_U$$

Then, since there are no corporate taxes, our return on each portfolio is

$$\pi_L = 5\%[Earnings_L] + 5\%(r \cdot D_L) = 5\%(EBIT - r \cdot D_L) + 5\%(r \cdot D_L) = 5\% \cdot EBIT$$

$$\pi_U = 5\%[Earnings_U] = 5\%(EBIT - r \cdot D_U) = 5\% \cdot EBIT \qquad \text{(As } D_U \text{ is 0)}$$

□

Note that it occurs despite the earnings of the leverage company are smaller in absolute terms
Anyway, these assumptions are not fulfilled in the real world.

1.5 Tradeoff theory of leverage

Modigliani & Miller also developed the tradeoff theory of leverage. It says: Let be the return on investments larger than the interest rate on debt, and under the same assumptions as before. If the proportion of debt in the firm's capital structure increases, the profitability expected by shareholders increases in a linear function:

$$k_E = k_A + Leverage \cdot (k_A - r) = k_A + \frac{D_L}{E_L} \cdot (k_A - r)$$

Proof. Let's assume that a company every year exhibits the same constant operating margin (EBIT).

$$\begin{aligned} k_E &= \frac{Earnings}{E_L} = \frac{EBIT - r \cdot D_L}{E_L} \cdot \frac{D_L + E_L}{D_L + E_L} \\ &= \frac{EBIT}{E_L} \cdot \frac{D_L + E_L}{D_L + E_L} - \frac{r \cdot D_L}{E_L} = \frac{EBIT}{E_L + D_L} \cdot \left(\frac{D_L}{E_L} + 1\right) - r \cdot \frac{D_L}{E_L} \\ &= \frac{EBIT}{V_L} \cdot \left(\frac{D_L}{E_L} + 1\right) - r \cdot \frac{D_L}{E_L} = k_A \cdot \left(\frac{D_L}{E_L} + 1\right) - r \cdot \frac{D_L}{E_L} = k_A + \frac{D_L}{E_L} \cdot (k_A - r) \end{aligned}$$

□

Note that there is a similarity with the leverage formula:

$$ROE = ROI + \left(\frac{D}{E}\right) \cdot (ROI - r)$$

where returns are computed on accounting numbers, here we are using market values.

Note that if $k_A \leq r$, the leverage produces a reduction (or it remains equal) on k_E

1.6 The effect of Risk

Although the share value is equal between leveraged and unleveraged companies, there is a risk associated with leverage.
The expected profitability of leverage companies increases when we raise the debt, but the variance of the profitability does it as well.

If r is known, and therefore does not have variance:

$$\sigma^2[k_E] = \sigma^2\left[k_A + \frac{D}{E}(k_A - r)\right] = \sigma^2\left[k_A\left(1+\frac{D}{E}\right) - \frac{D}{E}r\right]$$
$$= \left(1+\frac{D}{E}\right)^2 \sigma[k_A]$$

1.7 Effect of Corporate Taxes on M&M propositions

If there are corporate taxes, the Earnings are

$$Earnings = EBIT \cdot (1 - t_C)$$

Then, the return on investments are

$$\pi_U = EBIT \cdot (1 - t_C)$$

$$\pi_L = (1 - t_C) \cdot (EBIT - r \cdot D) + r \cdot D = EBIT \cdot (1 - t_C) + t_C \cdot r \cdot D$$

Then, the difference in the return on investments is:

$$\Delta\pi = t_C \cdot r \cdot D$$

Further, the cash flow distributed to debtholders is tax-deductible, whereas the one distributed to equity holders is not.
π is the annual differential cash flow, but as present value at today:

$$\text{Present Value (tax savings)} = \frac{t_C \cdot r \cdot D}{(1+k)} + \frac{t_C \cdot r \cdot D}{(1+k)^2} + \dots$$

where k is the cost of capital for the company. If tax savings do not depend on the business risk (i.e. the level of debt each year is pre-determined), then k=r. Assuming the debt constant:

$$PV = \sum_{t=1}^{\infty}\left(t_C \cdot r \cdot D \cdot \frac{1}{(1+r)^t}\right) = \frac{t_C \cdot r \cdot D}{r} \qquad \text{(geometric series)}$$

$$\implies PV = t_C \cdot D$$

Thus, the assets of the levered company are more valuable than the unlevered company:

$$V_L = V_U + t_C \cdot D$$

Then, k_A will depend on D:

$$k_A^* = \frac{EBIT}{V_L}$$

So k_E^*:

$$
\begin{aligned}
k_E^* &= \frac{Earnings}{E_L} = \frac{(EBIT - r \cdot D) \cdot (1 - t_C)}{E_L} \cdot \frac{V_L}{V_L} = \left[\frac{(EBIT - r \cdot D)}{V_L} \cdot \frac{V_L}{E_L} \right] (1 - t_C) \\
&= \left[\frac{EBIT}{V_L} \cdot \frac{V_L}{E_L} - \frac{r \cdot D}{E_L} \right] (1 - t_C) = \left[k_A^* \cdot \frac{E_L + D}{E_L} - \frac{r \cdot D}{E_L} \right] (1 - t_C) \\
&= \left[k_A^* \cdot \left(1 + \frac{D}{E_L} \right) - \frac{r \cdot D}{E_L} \right] (1 - t_C) = \left[k_A^* + \frac{D}{E_L} (k_A^* - r) \right] (1 - t_C)
\end{aligned}
$$

This is the second proposition by M&M. Summarising, if we introduce corporate taxes, the expected return on equity capital will be affected by this tax, but also for the company leverage, since V_L and therefore k_A^* depends on this.

1.8 Problems upon high debt

Regardless, it is not convenient to increase D as much as we can.
A firm will not increase debt indefinitely because:

- Tax savings are limited up to when gross income is equal to 0.
- Some countries impose restrictions on the deduction of interests.
- When debt is too high, some number of problems arise.

1. The first of the problems that arise when debt is too high is the costs of financial distress. As the debt increases, the probability of a default increase, since the company may not be able to pay back its contracts.

   When a company goes bankrupt, the assets are sold and creditors receive cash according to a hierarchy: Suppliers, employees, and debtholders. If the company runs out of money, the lower positions in the hierarchy are not paid.

   Plus some associated costs like legal fees, liquidation costs, image damage, advanced market discount, among others can appear.

2. The second of the problems is the agency costs, the principal-agent problem. The agent, are the ones that can take actions (equity investor). They are delegated by the principal, the ones that invest through lending money (debt-holders). The latter have a information asymmetry with respect to the agent.

   There is a dissonance between the agent and principal utility function. In particular, equity-holders usually take riskier decisions that does not maximize the equity-holders interests. This is because if the company goes bankrupt, the equity-holders have 0 money left. On the other hand, debt holders-loses what the company owes them, so they are losing more money than share-holders.

   This problem causes that debt-holders raise the cost of borrowing.

Therefore, there is a point where D is optimum, maximizing the firm value

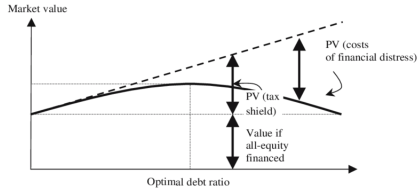

Figure 1: Optimum D considering Financial Distress Costs

1.9 Pecking Order Theory

The pecking order theory is a competing theory that states that the cost of financing increases with asymmetric information. Then, companies prioritize their sources of financing, first preferring internal financing, and then debt, lastly raising equity as a last choice, avoiding the market.
Equity is the less proferred source since when managers issue new equity, investors believe that managers know that the firm is overvalued and managers are taking advantage of this over-valuation.
Akerlof showed it with a game theory example: A lemon is a car found defective only after it has been bought. In the market, buyers can't distiguish a good quality car (G) from a lemon (B). B value is much lower than G value $V_B \ll V_G$, so sellers have an incentive to say that they are type-G cars
The information given by sellers is not credible so buyers attribute a medium expected value (e.g. if the probability is 50/50, $V = \frac{V_G + V_B}{2}$).
Therefore, as buyers only submit offer under the V_G, seller will sell only lemon cars, then, all good cars will be outside of the market. Finally, buyers will offer only V_B when buying a car, being this the nash equilibrium.

2 Bonds

2.1 Introduction

The bonds are certificates that show that a borrower owes a specific sum. To repay the money, the latter has agreed to pay interests and a 'principal' payment in designated dates. Most of the time, the principal is the 'par value' or 'face value', that is the fixed book accounting value of the single bond.
There are different kinds of bonds in capital markets:

- **Corporate Bonds:** The borrowers are private firms or corporations
- **Sovereign Bonds ('GOVIES'):** The borrowers are local or national goverments
- **International Organization Bonds:** as World Bank, or European Investment Bank.

2.2 Rating

Bond commitments may be disregarded, actually, the main discrimination among issuers is the risk of insolvency or default.

Bonds have parameters in case of insolvency:

- **Probability of Default (PD):** The issuer probability to fall into default. The larger is the PD, the larger is the return on capital required
- **Recovery Rate (RR):** The percentage of the investment recovered in case of default

Rating is a synthetic measure of solvency capacity. The measure is calculated by independent agencies with different methodologies. The calculation is done with accounting data and a qualitative assessment. There are different scales, but usually starts from AAA, when they are very secure, and finishes in D or C when they are almost a sure default.
We can use the Credit Default Swaps (CDS) to hedge against the risk of default of an issuer. It can be useful to measure the probability of default as well, since while higher is its price, higher is the default probability.

2.3 Currency

Issuers can raise debt in different currencies. In the case of buying debt in different currencies, there's also a currency risk, related to the fluctuation of the exchange rate.
Raising capital in foreign currencies may be useful to diversify the risk related to interest rates. Plus, the currency risk can be offset from the assets and liabilities

2.4 Maturity

The maturity is the moment in which the bond expires. It can be as short as some months or as long as a century, but typically bonds mature in some years. There also exist perpetual bonds, but they are very rare.
Typical bonds pay back the principal at the final maturity, this is called 'bullet'. There also exist bonds that pay back the principal during their life, this is called 'amortizing'.
Maturity is also a proxy of the bond risk, since while larger the maturity is, larger is the risk of default, and larger is the information asymmetry.

2.5 Coupons

Periodically, bonds pay certain amount of money, those are called coupons. Therefore, the coupons are divided in two types:

- **Coupon Bond:** Periodically, in correspondence with the payment date, a percentage of the par value of the bond is paid to creditors. The payment frequency is normally annual, biannual, quarterly, and so on.
- **Zero Coupond Bond (ZC):** In correspondence with the expiration date, the principal is paid.

The coupon rates can be fixed or variable. In the latter case, it is often determined by a spread + a benchmark, like interbank rates, Euribor, Libor, or other measures (anything observable in the market).
If the coupon rate exceeds r, the bound price will be over the par value. The difference between the bound price when it is larger and the par value is called premium

2.6 Seniority

The seniority refers to the priority of repayment in the event of issuer default. Each security has a specific seniority. For instance, by default, debt has a major security than equity. Further, different classes of debt bonds can have different seniority. While higher is the seniority, lower is the return and risk.
It is said that a bond is subordinated when it has less priority than another one. For example, bonds are generally subordinated with respect to bank debt.

Bonds can also have guarantees, that are compensations in case of default, supported by assets. They are often not the whole value, they are usually near to the 80%:

- **Secured Bond**
- **Unsecured Bond**

2.7 Particular Bonds

There are also specific types of bonds:

- **Callable Bond:** It is a bond type that allows the issuer to retain the privilege, but not the obligation of redeeming the bond at some time (call dates) before the bond reaches its date of maturity.
- **Putable Bond:** It is like a callable bond, but the option is granted to the investor.
- **Convertible Bond:** It is a bond type that the holder can convert into a specified number of shares of a common stock of the issuing company. The conversion is exercised if the market price of the equity securities is sufficient large.
- **Green Bond:** It is a bond issued to finance projects with a positive environment impact.
- **Social Bond:** It is issued to have a positive social impact.
- **Sustainable Bond:** It will finance projects with a generic sustainable impact
- **Sustainable-linked bond:** It is liked to the outcome of a sustainable project, measured by a certain metric, and certified by a third part.

2.8 Evaluate Bonds

Some notation:

- t=0 is the moment when the bond is issued on the primary market (or traded on the secondary market)
- F is the principal
- C_t is the coupon paid at time t
- P_0 is the equilibrium price

- r'_t is the return rate of risk-free bonds at t
- Δr is the spread, in the case of risky bonds

The calculation of P_0 is done by calculating the present value of the coupons plus the principal:

$$P_0 = \sum_{t=0}^{T} \frac{C_t}{(1+r_t)^t} + \frac{F}{(1+r_T)^T}$$

Where $r_t = r'_t + \Delta r$

- Example for risk free zero coupon bond:
  The P_0 is calculated as the present value of the coupon payment:

  $$P_0 = \frac{F}{(1+r_T)^T}$$

  $F = 1000$, T=5 months, $r_{5/12} = 1.2\%$

  $$\implies P_0 = \frac{1000}{(1+1.2\%)^{5/12}} = 995.04$$

  Therefore, the adimensional price (the proportion over F=100) is 99.504.
  If suddenly $r_{5/12} = 2.2\%$

  $$\implies P_{0,\ adimensional} = \frac{100}{(1+2.2\%)^{5/12}} = 99.097$$

- Example for risk free coupon paying bonds:
  $F = 100$, T = 2 years, yearly coupons=3%, r=2%

  $$\implies P_0 = \frac{3}{(1+2\%)^1} + \frac{103}{(1+2\%)^2} = 101.94$$

Bonds are also evaluated differently when they are between the defined coupon periods:

$$P_i = P_{i+1-\varepsilon} \cdot (1+r)$$

$$P_i = P_{i+\varepsilon} = P_{(i+1)-\varepsilon} \cdot (1+r)$$

Actually, the bond prices go up within the period, until just before the coupon payment, due to the accrual interest, the interest accumulated in the period from the last coupon. This is added to the clean price, and the result is called dirty price. When the next coupon is paid, the price falls abruptly, and the dirty price become equal to the clean price. (see figure 2.8) The accrual can be calculated with:

$$accrual = C \cdot \frac{\Delta t(\text{from the payment of last coupon})}{\Delta t(\text{between two coupons})}$$

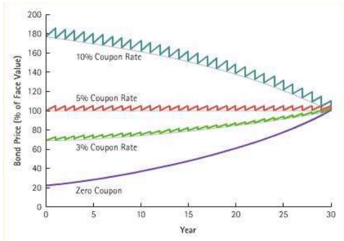

Figure 2: Dirty price with different coupon rates

2.9 Yield to Maturity (YTM)

The Yield to Maturity (YTM) of a bond is the rate r_{eff} that satisfies:

$$P_0 = \sum_{t=0}^{T} \frac{C_t}{(1+r_{Eff})^t} + \frac{F}{(1+r_{Eff})^T}$$

The Interest Rate Term Structure (IRTS) shows how the interest rates change along the maturity of the bonds

- If IRTS is flat, $r_{Eff} = r$
- If IRTS is not flat, r_{Eff} is the average of r_t
- If IRTS is growing, long term bonds have r_{Eff} higher than long term ones

2.10 Duration

The bond duration is the weighted average of the maturities of the coupons and principal payment:

$$D = \frac{1}{P_0}\left[\sum_{t=t_1}^{T} \frac{C_t \cdot t}{(1+r_t)^t} + \frac{F \cdot T}{(1+r_T)^T}\right]$$

For a Zero Coupon Bond, $D = T$, but in general $D \leq T$
If we don't know the IRTS, we can estimate the duration using the YTM
Duration fulfills linearity. The duration of a portfolio made from p of bond 1 and $1-p$ of bond 2, can be calculated as:

$$D_3 = \frac{p \cdot P_1 \cdot D_1 + (1-p) \cdot P_2 \cdot D_2}{p \cdot P_1 + (1-p) \cdot P_2}$$

where

- D_i : Duration of i
- P_i : Value or price of i

2.11 Volatility

Volatility σ is the percentage change in the bond market price when interest rates change.

$$\sigma = -\frac{1}{P_0}\frac{\partial P_0}{\partial r}$$

σ often has a negative value. Actually, when market interest rates increase, the price decreases, and vice versa.
Volatility is proportional to duration. Duration computed with r_{Eff} is a good estimator for volatility:

$$\hat{\sigma} \cong \frac{-D}{(1 + r_{Eff})}$$

Precisely, if we change the interest rates in a $\Delta r\%$, the percentage change in the bond price will be:

$$\frac{-D}{(1 + r_{Eff})} \Delta r$$

Therefore, the bond market price with a D duration will change about by $\frac{-D}{(1+r_{Eff})} \Delta r\%$ if the interest rates go up by $\Delta r\%$.

2.12 Taxation

Bonds are taxed in 3 ways:

- **Coupons:** The coupon payments are taxed with withholding tax
- **Capital Gain/Loss:** The difference between the purchase and sale, or alternatively the principal payment, is taxed as a capital gain/loss. Usually, losses can be offset with future capital gains.
- **Issue Discount:** This is applied when a bond is sold with a discount, i.e. the principal is higher than the P_0

For example, in Italy the tax rates are 26% for corporate and 12.5% for government bonds. If the individual has a capital loss, it can be offset with the next 3-years capital gains.

2.13 Issuing and trading

Bonds can be issued on primary markets in three ways:

- **Auction:** The price is offered by various investors that compete with each other. In this way, issuers can learn about the demand function and decide the bond quantity ($\bar{Q}$) and the issue price ($\bar{P}$).
- **Private Placement:** Are offered to a restricted number of investors. Usually, a survey between institutional investors is carried out to prove the willingness to invest and the average price. It is the cheapest way to place a bond in the market, so it is used by small companies.
- **Public Offering:** The market authority (In Italy CONSOB) requires the publication of the prospectus. PO is the most expensive and risky way to put a bond in the market, since the public may not appreciate the offer

Bonds may be traded on secondary markets as well. In Italy, MOT is the regulated market for retail and professional investors.

3 Shares

3.1 Introduction

When a firm needs to raise capital for new investments, one option is to issue equity. Equity issuing consists of issuing shares. Shares are instruments like bonds, but dividends are not contractually driven, and the profit is residual. The cash flows related to equity are:

- **Dividends:** It is a regular payment received by those with shares in a firm that pays dividends. It can be taken out as money, or it can be re-invested in company shares
- **Trading:** The buying and selling of shares can generate gains or losses

3.2 Types

The equity capital of a firm can be composed of different share classes:

- **Common Shares:** They have standard features provided by the law and the company statutes
- **Preference Shares:** They have limited voting rights, only in extraordinary shareholders meetings. On the other hand, there is a benefit consisting of a priority in the dividend distribution, compared to shareholders owning common shares.
- **Saving Shares:** They do not have voting rights, but they do have a priority in distributing dividends.

Anyway, there are several degrees of freedom in the shares classes: Restriction in selling voting power, priority in case of liquidation of assets, and convertibility into other classes of shares.

3.3 Tax Impact

The taxation is different in each country, but there are some patterns that are repeated across a lot of countries Dividends received by domestic investors are taxed differently depending on the investor typology:

- Individuals: There is a withholding tax rate
- Unlimited partnerships: Partners' income tax rate
- Corporate income tax rate

In the case of capital gains and losses, it is done by a fixed tax rate

3.4 Evaluate Shares

To evaluate shares, we use the Discounted Cash Flows method (DCF). This method still uses the Discounted Cash Flows method (DCF). This time, we cannot use r_f as a discount rate, and we do not have a benchmark spread for the risk according to a rating. We must use an appropriate rate, the cost of capital, which includes a risk premium:

$$k = r_f + \Delta r$$

Δr is not observable, but there are models to estimate it, using different present value calculations (see section 1.3), just at the moment next to the payment of DIV_0, if exists, so the latter is not considered in P_0 calculation.

- **The Dividend Discount Model (DDM)**
  It uses the price of a share equal to the perpetuity present value of the future cash flows (dividends and expected future price).

$$P_0 = \sum_{t=1}^{\infty} \frac{DIV_t}{(1+k)^t}$$

  If the dividends are a constant $\overline{DIV}$:

$$P_0 = \frac{\overline{DIV}}{k}$$

- **The Gordon & Shapiro Model**
  It uses the growing perpetuity formula and assumes:
  - The dividend growth is constant for the future ($g_t = g$)
  - The dividend growth is strictly lower than the discount rate ($k > g$)

$$P_0 = \sum_{i=1}^{\infty} \frac{DIV_i}{(1+k)^i} = \sum_{i=1}^{\infty} \frac{DIV_1 \cdot (1+g)^{i-1}}{(1+k)^i}$$

  Therefore:

$$P_0 = \frac{DIV_1}{k-g} = \frac{DIV_0 \cdot (1+g)}{k-g}$$

3.5 Useful Concepts

- n = Number of shares of a company
- p = Price of a unitary share of a company
- **Earnings per share**

$$EPS_t = \frac{Earnings_t}{n}$$

- **Book value:** Accounting value of equity capital
- **Book value per share**

$$B_t = \frac{Book\ value_t}{n}$$

- **Payout Ratio**

$$PR_t = \frac{DIV_t}{EPS_t}$$

- **Ploughed-back ratio**

$$h_t = 1 - PR_t$$

- **Return on equity**

$$ROE_t = \frac{Earnings_t}{Book\_value_{t-1}} = \frac{EPS_t}{B_{t-1}}$$

- **Growth rate of dividends**

$$g(DIV_t) = g_t = \frac{DIV_t - DIV_{t-1}}{DIV_{t-1}}$$

- **Growth rate of book value**

$$g(B_t) = \frac{B_t - B_{t-1}}{B_{t-1}}$$

- **Equity value:** If a company is public, the Equity value is calculated as:

$$E = n \cdot p$$

3.6 Explaining value with profitability

There are some relations between these variables:

1.
$$EPS_t = ROE_t \cdot B_{t-1}$$

2.
$$DIV_t = EPS_t \cdot (1 - h_t) = ROE_t \cdot B_{t-1} \cdot (1 - h_t)$$

3. Growth is financed by ploughed-back:
$$B_t - B_{t-1} = EPS_t - DIV_t = h_t \cdot EPS_t$$

4. Growth increases with h and ROE
$$g(B_t) = \frac{h \cdot EPS_t}{B_{t-1}} = h_t \cdot ROE_t$$

Then, if we assume that h and ROE are constant in the future:

$$g(EPS) = \frac{EPS_t - EPS_{t-1}}{EPS_{t-1}} = \frac{ROE \cdot B_t - ROE \cdot B_{t-1}}{ROE \cdot B_{t-1}} = g(B) = h \cdot ROE$$
$$g(DIV) = \frac{DIV_t - DIV_{t-1}}{DIV_{t-1}} = \frac{(1-h) \cdot EPS_t - (1-h) \cdot EPS_{t-1}}{(1-h) \cdot EPS_{t-1}} = g(EPS) = h \cdot ROE$$

But the growth is not enough condition to create value. Using the Gordon and Shapiro model:

$$\begin{aligned} P_0 &= \frac{DIV_1}{k - g} = \frac{(1-h) \cdot EPS_1}{k - h \cdot ROE} = \frac{(1-h) \cdot B_0 \cdot ROE}{k - h \cdot ROE} \\ &= B_0 \cdot \left(\frac{ROE}{k}\right) \cdot \frac{1-h}{1 - h\left(\frac{ROE}{k}\right)} \qquad (k > h \cdot ROE) \end{aligned}$$

Where k is the cost of capital.
Then, we have 3 different situations (see figure 3.6):

- If ROE=k, then $P_0 = B_0$, no matter if the company pays or not dividends, since the cost of capital is the same as return the offered on the equity capital.

- The situation is different when $ROE > k$. In this case, it is better to retain profits in order to seize investment opportunities. If h is higher, P_0 is higher. Therefore, this increases the share value

- If $ROE < k$, it is worse to retain profits, since we are destroying part of the value firm. Therefore, this decreases the share value. When the company has liquidity and no opportunities to invest it, it can give extraordinary dividends.

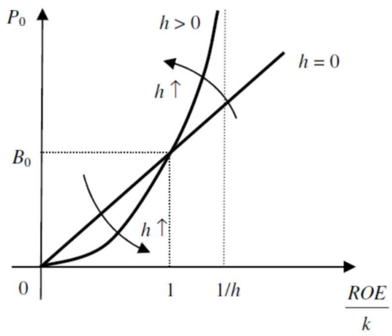

Figure 3: P_0 vs $\frac{ROE}{k}$

Under the same assumptions, ROE and h constants, we can reformulate P_0:

$$P_0 = \frac{(1-h) \cdot EPS_1}{k - h \cdot ROE} = \frac{(1-h) \cdot B_0 \cdot ROE}{k - h \cdot ROE} = \frac{EPS_1}{k} + \frac{h \cdot EPS_1 \cdot (ROE - k)}{k \cdot (k - h \cdot ROE)}$$

The first part is the perpetuity, which provides the present value of future profits in absence of growth. On the other hand, the second component is the Present Value of Growth Opportunity (PVGO).

$$P_0 = \frac{EPS_1}{k} + PVGO$$

This is true for every ROE and h, but the PVGO is different as above

The conditions that must be fulfilled so the firm value is greater than no-growth are:

- $h > 0$: The dividends we pay are less than the earnings

- $ROE > k$: The return on investment is greater than the cost of capital

Anyway ROE and h are not constant in real life, but companies try to cover the variation making DIVs stable, this is called dividend smoothing. If a company raises the dividends, probably it hopes convincingly that its earnings will be enough better in the future

3.7 Market Efficiency

The Efficient-market hypothesis (EMH) states that markets are efficient when security prices reflect instantly all available information. Under this assumption, equilibrium prices should be rationally determined on the markets discounting future expectations, which are homogeneous.

In consequence, past prices and past information are not useful for pricing securities on the

market.
Anyway, there are information asymmetries, and investors are not always rational.

3.8 Risk diversification

In finance, the risk is measured in statistical terms, adopting the investing volatility as a measure.

Due to the sum of the two normal distributions, whose variance is:

$$Var(Ax + By) = A^2Var(x) + B^2Var(y) + 2 \cdot A \cdot B \cdot Cov(X, Y)$$

Variables with low correlation reduce the variance, and therefore the risk of a portfolio. Indeed, if the correlation were -1, we could emulate the free risk case. Obviously, reducing the risk also has an effect on the maximum possible return on the investment.

Given all risky shares, there is a curve of possible portfolios that can be taken at a certain money level. However, different investors have a different Relative Risk Aversion (RAR), so there is not a market optimum (see example in figure 3.8).

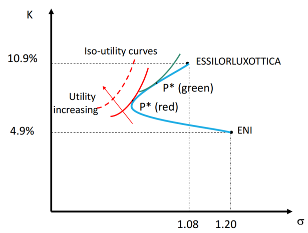

Figure 4: Possible Portfolios vs iso-utilities curves: Example with 2 risky securities

We can also mix the securities with risk-free securities, and even increase the k leveraging on debt. This new model is called Tobin Model

Furthermore, The best new portfolios between the risk and risk-free securities is given by the 'Capital Market Line'. A line that mix the portfolio M in the risky securities and the free-risk securities. M is given by the point where the gradient of risky securities is equal to the gradient between M and the whole risk-free point. (see figure 3.8)

Remarkably, there are zones in the risky security curve that should never be met (actually most of the curve), since there are points in the Capital Market Line that can achieve the same expected return at a lower risk. The risk we avoid choosing the better combination of risky securities is called diversifiable risk, it is the risk that can be surpassed by diversifying the portfolio risk. On the contrary, we have the idiosyncratic risk, that is given by the investor aversion to the risk.

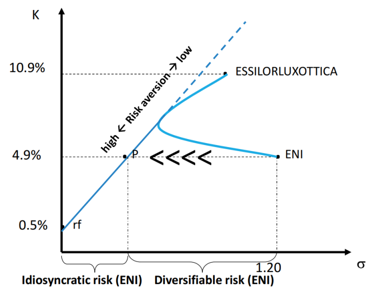

Figure 5: Possible Portfolios, including risk-free and risky securities. Also including debt leverage: Example with 2 risky securities pointing risk types of ENI

Note that this model assumes we can borrow and lend money at risk-free rate.

3.9 Cost of Capital

We introduced the idea of the cost of capital in section 3.4. More specifically, it is the opportunity cost of capital offered by alternative investment opportunities on the market, given the level of risk.
At first glance:

$$k = \underbrace{r_f}_{\text{return of risk-free securities}} + \underbrace{Risk - premium}_{\text{Specific firm premium}}$$

The first term is observable, while the second one is not.

There are different alternative models to estimate the risk premium. The assumptions that they take is that investors are risk-averse to the same expected return.
The most-known models are:

- Capital Asset Pricing Model (CAMP) and its evolutions
- The Arbitrage Pricing Theory (APT)

3.10 The Capital Asset Pricing Model

Starting from the Tobin model, this model proves that the expected return K requested by the market on the risky asset i is equal to:

$$k_i = r_f + \frac{k_M - r_f}{\sigma_M} \cdot \frac{\partial \sigma_p}{\partial x_i}$$

where:

- k_M is the expected return of the portfolio M (mix of risky ones)
- σ_M is the risk of the portfolio M (mix of risky ones)
- σ_p is the idiosyncratic risk
- x_i is the amount of asset i

It can be rewritten as follows:

$$k_i = r_f + \beta_i \cdot (k_M - r_f)$$

The β_i is the ratio between the derivative over x_i and σ_M
It can be proved that idiosyncratic risk has the additive property (linearity of derivative plus uncorrelated assets in the point):

$$\beta_P = \sum_{i=1}^{n} x_i \cdot \beta_i$$

Therefore:

$$\beta_{\text{risk free bonds}} = 0$$
$$\beta_{\text{market portfolio}} = 1$$

In particular:

$$\beta > 1 \implies \text{Agressive portfolio, it can beat the market, but with more risk}$$
$$\beta < 1 \implies \text{Conservative portfolio, expect lower return than market, but less risk}$$

For listed companies, the β can be estimated with a linear regression, having k_i, r_f, and k_M. (k_M is the return on the market index)
For unlisted companies, there are sectoral averages of betas of listed firms (e.g. Professor Aswath Damodaran's web site).
Note that this uses the past β to estimate the future ones, that is not necessarily true, but it is the best approximation that can be done.

3.11 Arbitrage Pricing Theory (APT)

The Arbitrage Pricing Theory is a multinomial model that posits that stock returns are explained also by both firm-specific factors and macro-economic factors:

- GPD
- Inflation
- Trade Surplus
- Interest Rates on Markets

It is closer to reality but harder to estimate

3.12 Fama & French three-factor model

In the 1990s, Eugene Fama and Kenneth French introduced a three-variable model. They observe that two classes of stocks generally perform better than the market as a whole:

- Small Caps: Small minus big portfolio. Empirically, small companies are on average undervalued concerning the big companies.

- Stocks with a low price-to-book ratio (P/B) or income stocks: HML (High minus low portfolio)

Then, they added two factors to the CAPM to reflect a portfolio's exposure to these two additional factors:

$$k = \alpha + \beta_1 \cdot k_M + \beta_2 \cdot SMB + \beta_3 \cdot HML$$

Carhart adds a fourth variable, the liquidity.

3.13 Technical Analysis

According to the Efficient Market Hypothesis (EMH, see section 3.7), stock prices only reflect expectations about the future, so the more efficient the markets are, the faster they discount the information. Criticism of EMH is moved from the fact that markets are not rational
Despite this, technical analysis is a moved from the opposite idea: Past prices and trades can explain future prices. Recurrent trends and figures can predict stock returns.
It relies on that some patterns (known) are repeated in the markets.
Criticism of technical analysis says that it works due to the self-realising expectation. The market reflects what everyone thinks because of the alleged patterns.

4 Interactions Value vs Finance

4.1 Value and Financing

We know now that financing decisions do affect corporate assets. Therefore, also the value of a single project will depend on how he initial investment will be financed (whether equity, debt, leasing, cash available, raising new capital, etc).
In order to take into account this effect, there are two methodologies generally used:

- The Adjusted Present Value (APV)
- The Weighted Average Cost of Capital (WACC)

4.2 APV

The Net Present Value (NPV) is the sum of the present value of all the cash flows in a determined context, often a project. The Adjusted Present Value of a project is defined as:

$$\text{APV} = \text{NPV (base case)} + \text{NPV (financing)}$$

The NPV (base case) in the base case is computed in the situation in which both the firm and the project are unlevered. Cash flows are discounted with the unlevered cost of capital k^*.
The NPV (financing) is related to the differential effects of financing choice only. E.g.: Tax savings, costs of raising capital, contributions, grants, etc. These cash flows are discounted with the cost of capital k^* if they depend on the operating cash flows of the project. otherwise, they will be discounted with the debt cost of capital r_d.

4.3 WACC

The second approach to compute an adjusted measure of the cost of capital is the weighted average cost of capital (WACC).
This is a less powerful approach, because it is taken into account only tax savings on debt and not other possible differential effects of the financing.

$$WAC = \frac{E}{D+E}k_E + \frac{D}{D+E}r_D(1 - t_C)$$

E is the market value of equity, and D is the value of debt.
WACC is used to calculate a new 'unlevered' present value, using WACC as k^*. Then, we can just add the tax shield to the value.
The problem is that, as seen in section 1.7, k_E depends on D. In other words, we know that k^* is the unlevered cost of capital, but the definition of the WACC contains the levered cost of capital k_E, that depends on D.
Moreover, we can use WACC to discount cash flows only when the leverage is constant (Because, indeed, WACC changes according to the leverage).
To compute WACC, we can use two formulas:

- **The Miles & Ezzell Formula:** This is used when debt D is adjusted in each period to keep the leverage L constant

$$WAC = k^* - L \cdot r_D \cdot t_C \cdot \frac{1 + k^*}{1 + r_D}$$

- **The Modigliani & Miller Formula:** This is used when cash flows CF and debt D are constant and perpetual, and when D is not linked to the project, and thus the leverage is constant, since the value of the project is always $CF/WACC$ (the sum can be shifted and it would continue giving the same result, multiplied by the period discount rate):

$$WACC = k^*(1 - L \cdot t_C)$$

This time, leverage is characterized as $L = D/V$

4.4 Hints

- The cost of capital for shareholders k_E depends on:
  - The risk of the project
  - The leverage
- Knowing the debt's value makes it easier to proceed with APV. If L is not constant, WACC cannot be used.
- Knowing the target leverage of the project, if it is constant, make easier to proceed with WACC
- In the real world neither k_E nor $WACC$ are constant when the financing policy is modified, and this impacts the project and firm value, and equity risk.

5 Derivatives

5.1 Introduction

A derivative is a financial contract whose payoffs and values are derived from, or depending on, another asset. This asset, named underlying asset, can be real, financial, or any other asset with a visible market value. Currencies, due to their different free-risk rates, are used as underlying assets.
Derivatives are mainly used to hedge against the risk of the underlying market price. Derivatives are also used to build portfolios with aggressive returns. Actually, it is possible to enhance the risk and return of the underlying asset, and even lose more money than the one we invested in.
Derivatives are also known as 'term contracts', because they typically regulate a future trade between two parties on the underlying asset.
There are two types of derivatives:

- Forward
- Options

5.2 Forwards Contracts

A forward contract is a mutually binding term agreement for the sale of an asset at a certain future date T at a pre-determined price (called the 'delivery price' X). The buyer is said to be in the 'long position', while the seller is in a 'short position'.
At the end of the forward maturity, the short position delivers the physical underlying or an equivalent cash payment, while the long position gives the agreed money to another one.

5.2.1 Evaluation

Let be:

- T**:** Time to maturity of the forward contract
- S_0**:** Spot price of the underlying at time 0
- X**:** Delivery price determined in the forward contract
- f**:** Value of the forward contract
- S_T**:** Price at maturity T of the underlying (a priori unknown)

The payoffs at time T are:

- For long position: $f = f^{long} = S_T - X$
- For short position: $f^{short} = X - S_T = -f$

5.2.2 Value at time 0

The long value of a forward agreement is given by:

$$f = f^{long} = S_0 - X \cdot e^{-r \cdot T} \text{ or in general } f^{long} = S_0 - PV(X)$$

where r is the continuously compounded annual interest rate, which can be discounted with $(1 + r)^T$ as well.
The short value is the contrary:

$$f^{short} = -f^{long} = PV(X) - S_0$$

The values can be positive or negative.

5.2.3 Forward Price

The 'forward price' F is the price that, being discounted at maturity T, is equal to S_0. This value is commonly used as the delivery price in forward contracts since neither counterpart has to pay or receive money to enter into the agreement (sometimes, if there is a difference with F, it is paid in t=0).

5.2.4 Cash flows

The underlying assets in a forward contract might generate positive or negative cash flows before the expiry of the contract (e.g.: dividends, interests, coupons, costs of carry, etc). In this case, the non-arbitrage equation must be adjusted:

$$f = f^{long} = [S_0 + PV(costs) - PV(revenues)] - X \cdot e^{-r \cdot T}$$

5.3 Futures

Future Contracts are similar to forward contracts, with some differences:

- They are generally traded on exchanges and standardized, while forward contracts are typically private (OTC: Over the Counter).
- They may be bought and sold frequently, while OTC forward contracts are generally kept to maturity (even a fee can be paid in case of early termination)
- The payoff is subject to the mark-to-market rule
- They may use a multiplier that multiplies the cash flows when there is a change in the asset value, this is called leverage effect

5.3.1 Marking-to-Market

In future contracts, the mechanism to deal with counterparty risk is based on two pillars:

- **Mark-to-market:** Investors are charged only with a % margin deposit (that depends on volatility) of the notional value (total value of the asset) of the contract when selling or buying the future, and each day money is credited or debited according to the change in the underlying market price.

- **Clearing House:** Third parties, responsible for all futures and option contracts, guarantee that the payoffs associated with a future contract will be paid.

5.4 Swap Contracts

It is a specific type of forward security. It consists of an agreement between two parties to swap a stream of cash flows for a defined period of time.
The most common swap is the interest rate swap (IRS), against a floating interest rate (typically euribor).
In practice, the difference between the interest rates is paid for just one of the parties.

5.5 Hedging using derivatives

As seen, future contracts may generate positive or negative payoffs according to the underlying market price. Therefore, we may use them to 'offset' losses or gains of the underlying itself.
Anyway, perfect hedging is not always possible using the available derivatives:

- The time-to-delivery may not coincide as we want
- The underlying asset may not be the same as we want

Then, we can:

- Use derivatives with significantly correlated underlying assets
- Choose expiry dates closer to the hedging objective
- Buy more forward contracts, to take down β_P

5.6 Options

An option gives the right, but not the obligation, to buy or sell an underlying asset, at time T, at a given predetermined price X, that is called 'strike price'.
Specifically:

- **Call:** Option gives the decision to the buyer (long position)
- **Put:** Option gives the decision to the seller (short position)

To acquire the option, a premium upfront is needed (value). Obviously, the option will be exercised only when convenient, so it will never generate a negative cash flow for the buyer at T (in the worst case will be 0).

5.6.1 Payoff

Let be:

- S_0**:** Current price of the underlying asset
- X**:** Strike price
- T**:** Time to maturity of the option

- S_T**:** Price of the underlying asset at maturity (unknown at t=0)
- **c (or C):** Value of the European (or American) option call
- **p (or P):** Value of the European (or American) option put

At maturity, i.e. just when the contract is finishing, the value of the call and put option for the buyer is:

$$c = C = \max(0, S_T - X)$$
$$p = P = \max(0, X - S_T)$$

This is exactly the payoff. Note that the call option payoff could be unlimited, while the put option payoff is at maximum T. The seller has just the contrary values.

5.6.2 American vs European

There are two types of options. American and European options:

- European options can be exercised only at time T
- American options can be exercised at any moment between 0 and T

The distinction is not related to the markets in which the options are written, it is just the name

5.6.3 Put-Call Parity Theorem

The values of European put (p) and call (c) options are tightly correlated, by the put-call parity theorem:

$$p + S_0 = c + PV(X)$$

Proof. At time t=0, the call side invests on the call and invests at risk free rate:

$$c + PV(X)$$

The put side instead, invests on the put and on the underlying:

$$S_0 + p$$

At time T, we have 2 cases:

- $S > X$**:**
  Call: $(S_T - X) + X = S_T$
  Put: $S_T + 0 = S_T$
- $S < X$**:**
  Call: $0 + X = X$
  Put: $S_T + (X - S_T) = X$

□

If the underlying asset generates cash flows, we have to adjust:

$$p + S_0 + PV(costs) - PV(revenues) = c + PV(X)$$

5.6.4 Assumptions over Evaluation

Contrary to forward contracts, there are no powerful formulas to determine the value of an option. Indeed, we have to make some assumptions on how the price of the underlying asset S will change in the future.
There are also some limits that option values must be in:

1. All Options: Their value is always positive, never 0 or negative ('no free lottery)

2. Call Options: Worth more than $S_0 - PV(X)$, i.e. the value of the long forward contract, but less than S_0

3. Put Options: Worth less than X (P(V) for European options), but more than $P(V) - S_0$, i.e. the value of the short forward contract.

It is never optimal to strike an American call option in advance. Since we saw on point 2, the call option is always worth more than $S_0 - PV(X)$, this is also true at time $t^* < T$, thinking about it as t_0. Since the pay we will receive is $S_{t^*} - X$, and this is lower than $S_{t^*} - P(X)$, we own an option worth more than what we will receive if we call the option. Therefore, it is not optimum to call an option in advance.
On the contrary, it might be optimal to exercise earlier an American put option, when it is close to 0. This happens since there is a limit on the price, it can't cost less than 0.
There is only one situation where it could be better to execute an American call option in advance. It is when we want to have the underlying asset in advance to take advantage of the cashflows that it pays, if these cashflows are at least higher than the difference between $S_{t^*} - PV(X)$ and $S_{t^*} - X$.

5.6.5 Evaluation

The two most well-known evaluation models for options are:

- **Binomial Model (or 'Martingale' model):** It simply assumes that the value of the underlying asset at any moment can take only two values. Therefore the distribution of $S(t)$ is discrete and Binomial

- **Black & Scholes Model:** It hypothesizes that the function $S(t)$ is continuous and partly stochastic with a random walk. Therefore, the underlying return is distributed as a Gaussian Distribution.

Binomial Model This model assumes that the return of the underlying asset at time T may have two values only:

$$u \cdot S_0 \quad \text{or} \quad d \cdot S_0$$

To each event is assigned a risk-neutral probability p_u and p_d
The solution is obtained by:

- Determine the option payoff both outcomes Π_1 and Π_2 with respect to S_0

- Compute the risk-neutral probabilities:

$$p_u = \frac{(1+r)^T - d}{u-d}$$

$$p_d = \frac{u - (1+r)^T}{u-d}$$

- Finally, compute the value V:

$$\begin{aligned} V &= \frac{p_u}{(1+r)^T} \cdot \Pi_1 + \frac{p_d}{(1+r)^T} \cdot \Pi_2 \\ &= \frac{\Pi_1 - \Pi_2}{u-d} - \frac{1}{(1+r)^T} \cdot \left(\frac{d}{u-d}\Pi_1 - \frac{u}{u-d}\Pi_2 \right) \end{aligned}$$

We can compute the hedge ratio δ

$$\delta = \frac{\frac{\Pi_1 - \Pi_2}{u-d}}{S_0}$$

that is an important parameter, because it represents the ratio between the option value and the underlying price.
We say that the option can be 'replicated' through:

- A fraction of the underlying equal to δ
- Debt

If we want to hedge 1 underlying unit, we need $1/\delta$ options. In the forward and future contracts, the hedge ratio is equal to 1
Black & Scholes This model assumes that the underlying return is normally distributed, and that the price follows a stochastic process ('brownian motion').
It introduces a formula to compute the value of an European call option, with only 5 variables:

- **S:** The spot price of the underlying
- **r:** The risk free rate
- **T:** Expiration
- σ**:** Volatility of the underlying return

It calculates the call option value as follows:

$$c = S_0 \cdot N(d_1) - X \cdot e^{-r \cdot T} \cdot N(d_2)$$

where:

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S_0}{X}) + (r + \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{S_0}{X \cdot e^{-rT}}\right)}{\sigma \cdot \sqrt{T}} + \frac{\sigma \cdot \sqrt{T}}{2} \\ d_2 &= \frac{\ln(\frac{S_0}{X}) + (r - \frac{\sigma^2}{2}) \cdot T}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{S_0}{X \cdot e^{-rT}}\right)}{\sigma \cdot \sqrt{T}} - \frac{\sigma \cdot \sqrt{T}}{2} = d_1 - \sigma \cdot \sqrt{T} \\ N(d) &= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{d} e^{-\frac{1}{2}x^2} dx = \text{Cumulative function of the normal distribution} \end{aligned}$$

In practice, there are tables for calculating $N(d_1)$ and $N(d_2)$.
Let's look at how the values change with respect to S_0

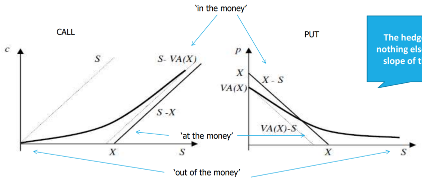

Figure 6: Example call and put graphics against S_0

The derivative of the curve is the hedge ratio δ. When S_0 is low, it is said the option is 'out of the money', $\delta \approx 0$. When S_0 is close to PV(X), it is said that the option is 'in the money', $\delta \approx 1$

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