PC3231

Tutorial 3: Potentials & Fields

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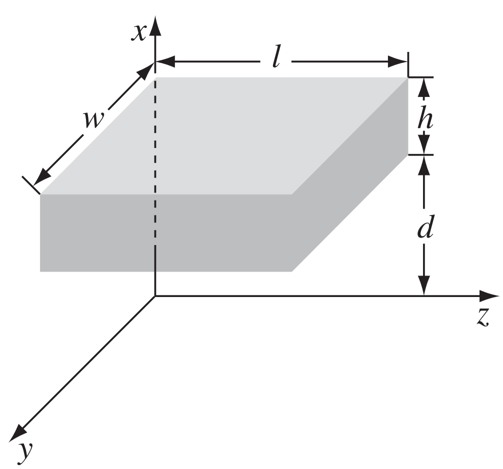
1. Given the field configuration,

$$\boldsymbol{E} = -\frac{\mu_0 k}{2}(ct - |x|)\,\hat{\boldsymbol{z}} \quad \text{for } |x| < ct,$$
$$\boldsymbol{B} = \pm\frac{\mu_0 k}{2c}(ct - |x|)\,\hat{\boldsymbol{y}} \quad \text{for } |x| < ct,$$

   where plus for $x > 0$, minus for $x < 0$, consider a rectangular box of length l, width w, and height h, situated a distance d above the yz plane.

   (Note that, for $|x| > ct$, $\boldsymbol{E} = \boldsymbol{B} = 0$.)



   (a) Find the energy in the box at time $t_1 = d/c$, and at $t_2 = (d + h)/c$.

   (b) Find the Poynting vector, and determine the energy per unit time flowing into the box during the interval $t_1 < t < t_2$.

(c) Integrate the result in (b) from t_1 to t_2 and confirm that the increase in energy (part (a)) equals the net influx.

2. (a) Find the fields, and the charge and current distributions, corresponding to

$$V(\boldsymbol{r},t) = 0, \quad \boldsymbol{A}(\boldsymbol{r},t) = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\,\hat{\boldsymbol{r}}$$

(b) Use the gauge function $\lambda = -\dfrac{1}{4\pi\epsilon_0}\dfrac{qt}{r}$ to transform the potentials, and comment on the result.

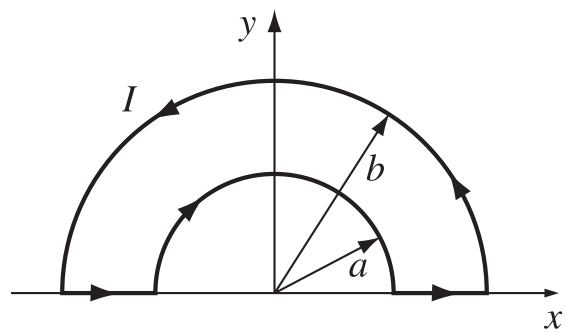
3. A time-dependent point charge $q(t)$ at the origin, $\rho(\boldsymbol{r},t) = q(t)\,\delta^3(\boldsymbol{r})$, is fed by a current $\boldsymbol{J}(\boldsymbol{r},t) = -\dfrac{1}{4\pi}\dfrac{\dot{q}}{r^2}\,\hat{\boldsymbol{r}}$, where $\dot{q} \equiv \dfrac{\mathrm{d}q}{\mathrm{d}t}$.

(a) Check that charge is conserved, by confirming that the continuity equation is obeyed.

(b) Find the scalar and vector potentials in the Coulomb gauge. If you get stuck, try working on (c) first.

(c) Find the fields, and check that they satisfy all of Maxwell's equations.

4. A piece of wire bent into a loop



carries a current that increases linearly with time:

$$I(t) = kt \quad (-\infty < t < \infty)$$

Calculate the retarded vector potential $\boldsymbol{A}$ at the center. Find the electric field at the center. Why does this (neutral) wire produce an *electric* field? (Why can't you determine the *magnetic* field from this expression for $\boldsymbol{A}$?)

5. Suppose you take a plastic ring of radius a and glue charge on it, so that the line charge density is $\lambda_0|\sin(\theta/2)|$. Then you spin the loop about its axis at an angular velocity ω. Find the (exact) scalar and vector potentials at the center of the ring.