PC3231

Tutorial 5: Electrodynamics & Relativity

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1. Consider a particle in hyperbolic motion,

$$x(t) = \sqrt{b^2 + (ct)^2}, \qquad y = z = 0$$

   (a) Find the proper time τ as a function of t, assuming the clocks are set so that $\tau = 0$ when $t = 0$. [*Hint*: Integrate $\mathrm{d}\tau = \sqrt{1 - u^2/c^2}\ \mathrm{d}t$]
   Note: $\int \frac{1}{\sqrt{a^2+x^2}}\ \mathrm{d}x = \ln[x + \sqrt{a^2 + x^2}]$

   (b) Find x and u (ordinary velocity) as functions of τ.

   (c) Find η^μ (proper velocity) as a function of τ.

2. Define proper acceleration in the obvious way:

$$\alpha^\mu \equiv \frac{\mathrm{d}\eta^\mu}{\mathrm{d}\tau} = \frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2}$$

   (a) Find α^0 and $\boldsymbol{\alpha}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{a}$ (the ordinary acceleration).

   (b) Express $\alpha_\mu \alpha^\mu$ in terms of $\boldsymbol{u}$ and $\boldsymbol{a}$.

   (c) Show that $\eta^\mu \alpha_\mu = 0$.

   (d) Write the Minkowski version of Newton's second law, $K^\mu \equiv \dfrac{\mathrm{d}p^\mu}{\mathrm{d}\tau}$, in terms of α^μ. Evaluate the invariant product $K^\mu \eta_\mu$.

3. In system $\mathcal{S}$, a static uniform line charge λ coincides with the z axis.

   (a) Write the electric field $\boldsymbol{E}$ in *Cartesian* coordinates, for the point (x, y, z).

   (b) Use the transformation rules

   $$\bar{E}_x = E_x, \quad \bar{E}_y = \gamma(E_y - vB_z), \qquad \bar{E}_z = \gamma(E_z + vB_y)$$

   $$\bar{B}_x = B_x, \quad \bar{B}_y = \gamma\left(B_y + \tfrac{v}{c^2}E_z\right), \quad \bar{B}_z = \gamma\left(B_z - \tfrac{v}{c^2}E_y\right)$$

   to find the electric in $\bar{\mathcal{S}}$, which moves with speed v in the x direction with respect to $\mathcal{S}$. The field is still in terms of (x, y, z); express it instead in terms of the coordinates $(\bar{x}, \bar{y}, \bar{z})$ in $\bar{\mathcal{S}}$. Finally, write $\bar{\boldsymbol{E}}$ in terms of the vector $\bar{\boldsymbol{R}}$ from the *present* location of the wire and the angle θ between $\bar{\boldsymbol{R}}$ and $\hat{\boldsymbol{x}}$. Does the field point away from the instantaneous location of the wire, like the field of a uniformly moving point charge?

4. Inertial system $\mathcal{S}'$ moves at constant velocity $\boldsymbol{v} = \beta c(\cos\phi\,\hat{\boldsymbol{x}} + \sin\phi\,\hat{\boldsymbol{y}})$ with respect to $\mathcal{S}$. Their axes are parallel to one another, and their origins coincide at $t = t' = 0$, as usual. Find the Lorentz transformation matrix Λ.

5. Calculate the threshold (minimum) momentum the pion must have in order for the process $\pi + p \to K + \sum$ to occur. The proton p is initially at rest. Use $m_\pi c^2 = 150$, $m_K c^2 = 500$, $m_p c^2 = 900$, $m_{\sum} c^2 = 1200$ (all in MeV).

   Hint: To formulate the threshold condition, examine the collision in the center-of-momentum frame.