

The Answer of Assignment 2

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Problem 1 Solution

- (1) 根据线性空间的同构理论，任何维数相同的线性空间都是同构的。因此可知 $\sum_{i,j=0}^1 c_{ij} |i, j\rangle$ 与 $\{(x_1, x_2, x_3, x_4)\}, x \in C$ 同构。即 $v_1 = (c_{00}, c_{01}, c_{02}, c_{03})^*, v_2 = (d_{00}, d_{01}, d_{10}, d_{11})^*$
- (2) $\langle \psi_1 | \psi_2 \rangle = v_1^\dagger v_2$ 成立, 这是因为
- (3)
- (4)

Problem 2 Solution

- (1)
- (2)
- (3)
- (4)

Problem 3 Solution

- (1)
- (2)
- (3)

(4) how should the Pauli operators σ_i^+ and σ_i^z can be written in terms of the f operators? we have the definition that:

$$f_i^\dagger = \left(\prod_{j < i} \sigma_j^z \right) \sigma_i^+$$

so from $\langle \overline{n_1 n_2 \dots n_i} | f_i^\dagger | \overline{n_1 n_2 n_3 \dots n_i} \rangle = \langle \overline{n_1 n_2 \dots n_i} | f_i | \overline{n_1 n_2 \dots n_i} \rangle^*$ we can get the relation between f_i and σ_i^+, σ_i^z as follows:

$$f_i = \left(\prod_{j < i} \sigma_j^z \right) \sigma_i^-$$

then try to represent σ_i^z and σ_i^+ in terms of f_i :

$$\sigma_i^z = \left(\prod_{j < i} f_j^\dagger \right) f_i^\dagger f_i \left(\prod_{j < i} f_j \right)$$