# The Answer of Assignment 2

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### Problem 1 Solution

**(1)** 根据线性空间的同构理论，任何维数相同的线性空间都是同构的。因此可知 $\sum_{i,j=0}^{1} c_{ij} |i,j\rangle$ 与 $\{(x_1, x_2, x_3, x_4)\}, x \in C$ 同构。即 $v_1 = (c_{00}, c_{01}, c_{02}, c_{03})^*$,$v_2 = (d_{00}, d_{01}, d_{10}, d_{11})^*$

**(2)** $\langle\psi_1|\psi_2\rangle = v_1^\dagger v_2$ 成立, 这是因为

**(3)**

**(4)**

### Problem 2 Solution

**(1)**

**(2)**

**(3)**

**(4)**

### Problem 3 Solution

**(1)**

**(2)**

**(3)**

**(4)** how should the Pauli operators $\sigma_i^+$ and $\sigma_i^z$ can be written in terms of the $f$ operators? we have the definition that:

$$f_i^\dagger = \left(\prod_{j<i} \sigma_j^z\right) \sigma_i^+$$

so from $\langle\overline{n_{1'} n_{2'} \cdot n_{i'}}|f_i^\dagger|\overline{n_1 n_2 n_3 \cdot n_i}\rangle$=$\langle\overline{n_1 n_2 \cdot n_i}|f_i|\overline{n_{1'} n_{2'} \cdot n_{i'}}\rangle^*$ we can get the relation between $f_i$ and $\sigma_i^+$, $\sigma_i^z$ as follows:

$$f_i = \left(\prod_{j<i} \sigma_j^z\right) \sigma_i^-$$

then try to represent $\sigma_i^z$ and $\sigma_i^+$ in terms of $f_i$: we use the matrix representation of the Pauli operators:

$$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$\sigma_i^z$can be expressed as:$2\sigma_i^+\sigma_i^- - I$