# PC2135: Thermodynamics and Statistical Physics

Semester 1, AY2526

# Tutorial 1: Chapter 2

**Instructions:**
Please submit the questions with the asterisk by 5pm, Friday of the tutorial week.

### Question 1: Probability Distribution I – Two Outcomes

In this question, we derive the general probability distribution of a two-outcome scenario. Consider a system of $N$ identical coins with probability $p$ of landing on heads and $1-p$ of landing on tails.

Note: Throughout this question, we are using the terms macrostates and microstates rather loosely.

(a) When $N = 4$:
  i. List down all the possible macrostates and and their corresponding microstates for this system[a].
  ii. For a fair coin ($p = 1/2$), compute the probability of each of the macrostates on your list?

(b) When $N = 100$:
  i. How many distinct macrostates are possible for this system?
  ii. How many distinct microstates are possible for this system?
  iii. How many distinct microstates correspond to the each of the first four macrostates of this system[b]?
  iv. Determine the general formula for calculating the number of microstates associated with a specific macrostate.
  v. Hence determine the general formula for computing the probability of obtaining a specific macrostate.

### Question 2: Probability Distribution II – Two Outcomes

Utilizing a computation software (or otherwise), reproduce Table 1.2 in Lecture 1, and plot out the full probability distribution.

| No of particles on left, $n$ | $0 \leq n \leq 10$ | $10 < n \leq 20$ | $20 < n < 30$ | $30 \leq n < 40$ | $40 \leq n \leq 50$ |
|---|---|---|---|---|---|
| Probability, $P_n$ | $1.19 \times 10^{-5}$ | 0.101 | 0.797 | 0.101 | $1.19 \times 10^{-5}$ |

Table 1: Probabilities for 4 different macroscopic scenarios when $N = 50$

Note that in this scenario, a particle has an equal chance of being found in the left or right partition of the container. Take further note of when the symbol above includes the equality and when it does not.

### Question 3: Beyond Two Outcomes*

What is the probability of throwing a total of 6 points or less with 3 dice?

### Question 4: Dispersions*

Use the general properties of mean values to show that the dispersion of $u$ can be calculated by the general relation

$$\overline{(\Delta u)^2} \equiv \overline{(u - \bar{u})^2} = \overline{u^2} - \bar{u}^2. \tag{1}$$

Explain clearly the reasoning behind each step. If you utilized the properties of mean as derived in Section 2.3.1, do clearly denote it.

[a] Because we're using the terms macrostate and microstate rather loosely here, this is not the most precise question. But consider how you would describe this system. What is the most precise way *you* can describe the outcomes of these coin flips? What is a less precise way of describing these outcomes?

[b] You may define your own order of listing the macrostates.

### Question 5: Mean Values for A Single Spins

The magnetic moment of a spin$-\frac{1}{2}$ particle is such its component in the up direction has probability $p$ of being equal to $\mu_0$ and probability $q = 1 - p$ of being equal to $-\mu_0$.

(a) Calculate $\bar{\mu}$ and $\overline{\mu^2}$

(b) Use Eq. (1) to calculate $\overline{\Delta\mu^2}$

(c) We reached a similar expression in our derivations in Section 2.4.3 in Lecture 2 for a single spin (from which we proceeded to obtain Eq. 2.17 for a system of $N$ spins). Reproduce that derivation here and explain how the derivation of (b) differed from this and simplified matters.

### Question 6: The Inequality $\overline{u^2} \geq \bar{u}^2$

Suppose that the variable $u$ can assume the possible values $u_r$ with respective probabilities $P_r$

(a) By using the definitions of $\bar{u}$ and $\overline{u}^2$, and remembering the normalization requirement $\sum_r P_r = 1$, show that

$$\overline{u^2} - \bar{u}^2 = \frac{1}{2} \sum_r \sum_s P_r P_s \left(u_r - u_s\right)^2$$

where each sum extends over all possible values of the variable $u$

(b) Since no term in the sum can ever be negative, show that

$$\overline{u^2} \geq \bar{u}^2$$

where the equals sign applies only in the case where only one value of $u$ occurs with non-zero probability.

### Question 7: System of Nuclei with Spin-1*

Consider a system where the component of its magnetic moment along a given direction can have *three* possible values, $+\mu_0$, 0 or $-\mu_0$. Suppose further that there is a probability $p$ that $\mu = \mu_0$ and a probability $p$ that $\mu = -\mu_0$; the probability that $\mu = 0$ is then equal to $1 - 2p$.

(a) Calculate $\bar{\mu}$ and $\overline{\mu^2}$

(b) Calculate $\overline{(\Delta\mu)^2}$

(c) Suppose that we now have $N$ such magnetic moments which interact with each other to a negligible extent. Denoting $M$ as the total component of magnetic moment along the specified direction, calculate $\overline{M}$ and the standard deviation $\underset{\sim}{\Delta} M$ in terms of $N$, $p$ and $\mu_0$.

Explain clearly the reasoning behind each step. If you utilized the properties of mean as derived in Section 2.3.1, do clearly denote it. Clarify further when the summation is over the spins in the system, and when the summation is over the ensemble average.