PC3231

Tutorial 2: Electromagnetic Waves

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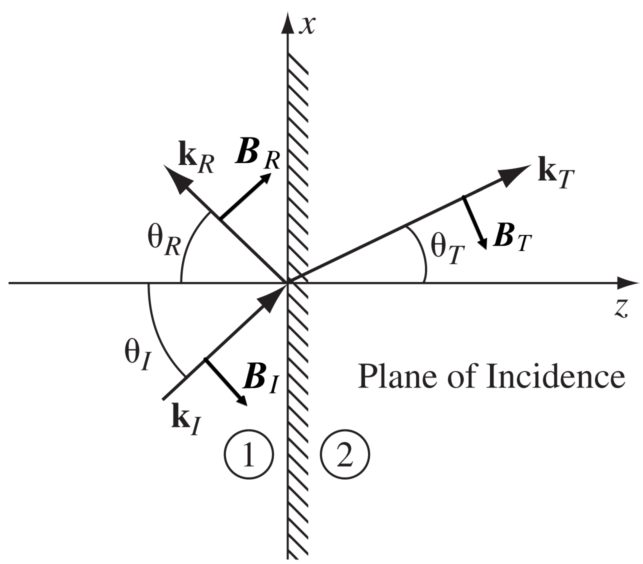
1. Analyze the case of polarization *perpendicular* to the plane of incidence (i.e. electric fields in the y direction and magnetic fields parallel to the plane of incidence).



Impose the boundary conditions

$$
\begin{cases}
\text{(i)} \;\; \epsilon_1(\tilde{\boldsymbol{E}}_{0_I} + \tilde{\boldsymbol{E}}_{0_R})_z = \epsilon_2(\tilde{\boldsymbol{E}}_{0_T})_z \\[2ex]
\text{(ii)} \;\; (\tilde{\boldsymbol{B}}_{0_I} + \tilde{\boldsymbol{B}}_{0_R})_z = (\tilde{\boldsymbol{B}}_{0_T})_z \\[2ex]
\text{(iii)} \;\; (\tilde{\boldsymbol{E}}_{0_I} + \tilde{\boldsymbol{E}}_{0_R})_{x,y} = (\tilde{\boldsymbol{E}}_{0_T})_{x,y} \\[2ex]
\text{(iv)} \;\; \frac{1}{\mu_1}(\tilde{\boldsymbol{B}}_{0_I} + \tilde{\boldsymbol{B}}_{0_R})_{x,y} = \frac{1}{\mu_2}(\tilde{\boldsymbol{B}}_{0_T})_{x,y}
\end{cases}
$$

and obtain the Fresnel equations for $\tilde{E}_{0_R}$ and $\tilde{E}_{0_T}$. Confirm that the Fresnel equations reduce to the proper forms at normal incidence. Show that there is no Brewster's angle for typical materials with $\mu \approx \mu_0$. Compute the reflection and transmission coefficients, and check that they add up to 1.

2. (a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $(2/\sigma)\sqrt{\epsilon/\mu}$ (independent of frequency).

   (b) Show that the skin depth in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$ (where λ is the wavelength *in the conductor*). Find the skin depth (in nanometers) for a typical metal ($\sigma \approx 10^7$ $(\Omega\mathrm{m})^{-1}$) in the visible range ($\omega \approx 10^{15}$/s), assuming $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. Why are metals opaque?

   (c) Show that in a good conductor the magnetic field lags the electric field by 45°, and find the ratio of their amplitudes. For a numerical example, use the "typical metal" in part (b).

3. Suppose

$$\boldsymbol{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}[\cos(kr-\omega t) - (1/kr)\sin(kr-\omega t)]\,\hat{\boldsymbol{\phi}}$$

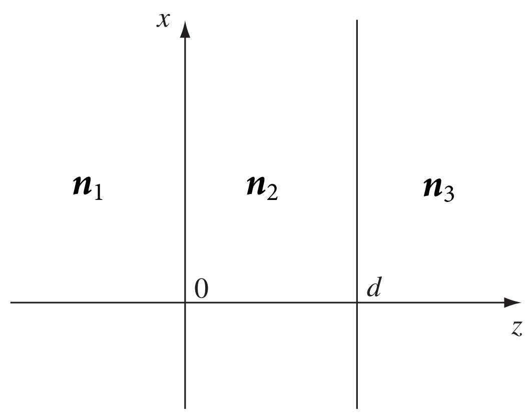
with $\omega/k = c$.

   (a) Show that $\boldsymbol{E}$ obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.

   (b) Calculate the Poynting vector. Average $\boldsymbol{S}$ over a full cycle to get the intensity vector $\boldsymbol{I}$. (Does it point in the expected direction? Does it fall off like r^{-2}, as it should?)

   (c) Integrate $\boldsymbol{I} \cdot d\boldsymbol{a}$ over a spherical surface to determine the total power radiated.

4. Light of angular frequency ω passes from medium 1, through a slab (thickness d) of medium 2, and into medium 3 as in the figure below.



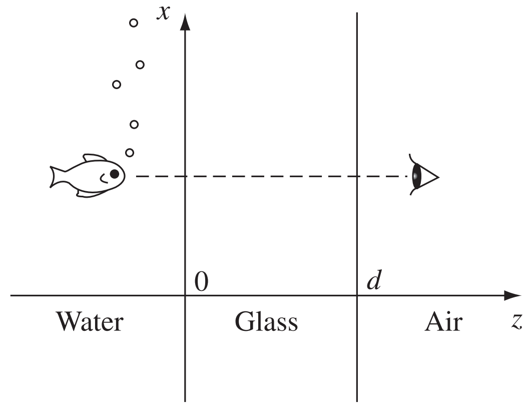
Show that the transmission coefficient for normal incidence is given by

$$T^{-1} = \frac{1}{4n_1n_3}\left[(n_1+n_3)^2 + \frac{(n_1^2-n_2^2)(n_3^2-n_2^2)}{n_2^2}\sin^2\left(\frac{n_2\omega d}{c}\right)\right] \quad (1)$$

All three media are linear and homogeneous. Assume that $\mu_1 = \mu_2 = \mu_3 = \mu_0$.

[Hint: To the *left*, there is an incident wave and a reflected wave; to the *right*, there is a transmitted wave; inside the slab, there is a wave going to the right and a wave going to the left. Express each of these in terms of its complex amplitude, and relate the amplitudes by imposing suitable boundary conditions at the two interfaces.]

5. Light from an aquarium goes from water ($n = \frac{4}{3}$) through a plane of glass ($n = \frac{3}{2}$) into air ($n = 1$).



Assuming it's a monochromatic plane wave and that it strikes the glass at normal incidence, find the minimum and maximum transmission coefficients (See Problem 4, Eq. (1) above).

You can see the fish clearly; how well can it see you?