The Answer of Assignment 2

WEI SHUANG

31/8/2025

Problem 1 Solution

(1) 根据线性空间的同构理论，任何维数相同的线性空间都是同构的。因此可知 $\sum\limits_{i,j=0}^{1} c_{ij}\,|i,j\rangle$ 与 $\{(x_1,x_2,x_3,x_4)\}, x \in C$ 同构。即 $v_1 = (c_{00}, c_{01}, c_{02}, c_{03})^*$,$v_2 = (d_{00}, d_{01}, d_{10}, d_{11})^*$

(2) $\langle\psi_1|\psi_2\rangle = v_1^\dagger v_2$ 成立, 这是因为 $\langle\psi_1|\psi_2\rangle = (\sum_{i,j=0}^{1} c_{i,j}^* \langle ij|)(\sum i,,j=0^1 c_{i,j}^* |ij\rangle) = v_1^\dagger v_2$ **(3)**

(4)

Problem 2 Solution

(1)

(2)

(3)

(4)

Problem 3 Solution

(1)

(2)

(3)

(4) how should the Pauli operators σ_i^+ and σ_i^z can be written in terms of the f operators? we have the definition that:

$$f_i^\dagger = \left(\prod_{j<i} \sigma_j^z\right)\sigma_i^+$$

so from $\langle\overline{n_{1'}n_{2'}\cdot n_{i'}}|f_i^\dagger|\overline{n_1n_2n_3\cdot n_i}\rangle$=$\langle\overline{n_1n_2\cdot n_i}|f_i|\overline{n_{1'}n_{2'}\cdot n_{i'}}\rangle^*$ we can get the relation between f_i and σ_i^+, σ_i^z as follows:

$$f_i = \left(\prod_{j<i} \sigma_j^z\right)\sigma_i^-$$

then try to represent σ_i^z and σ_i^+ in terms of f_i: we use the matrix representation of the Pauli operators:

$$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

σ_i^zcan be expressed as:$2\sigma_i^+\sigma_i^- - I = 2f_i^\dagger f_i - I$

like wise:

$$\begin{aligned} \sigma_i^+ &= f_i^\dagger \left(\prod_{j<i} \sigma_j^z \right)^{-1} \\ &= f_i^\dagger \left(\prod_{j<i} (2f_j^\dagger f_j - I) \right) \end{aligned}$$