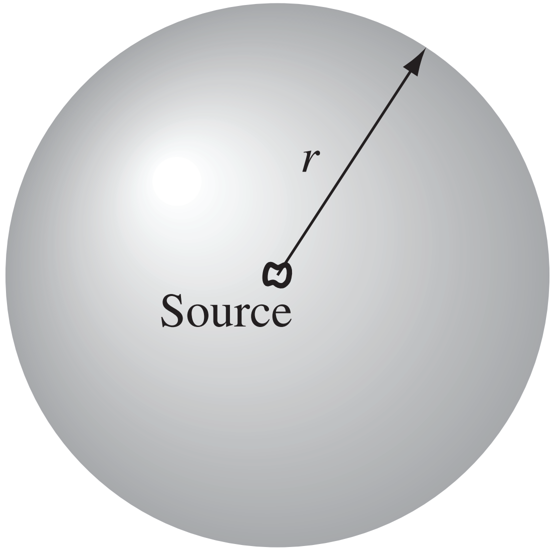
Radiation
❖ Dipole Radiation
❖ Dipole ... ii
❖ Dipole ... iii
Electric Dipole Radiation
Magnetic Dipole Radiation
Radiation from an Arbitrary Source

Radiation

- When charges *accelerate*, their fields can transport energy irreversibly out to infinity–a process called **radiation**
- Assume source is localized near the origin, and we shall calculate the energy it is radiating at time t_0
- Imagine a gigantic spherical shell, out at radius r



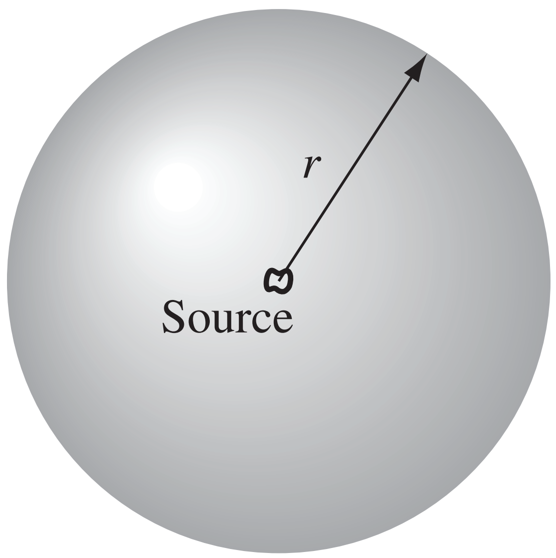
Power passing through its surface is the integral of the Poynting vector:

$$P(r,t) = \oint \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a} = \frac{1}{\mu_0} \oint (\boldsymbol{E} \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{a} \tag{11.1}$$

Because electromagnetic “news” travels at the speed of light, this energy actually left the source at the earlier time $t_0 = t - r/c$, so the power radiated:

$$P_{\mathrm{rad}}(t_0) = \lim_{r\to\infty} P\left(r, t_0 + \frac{r}{c}\right) \qquad (11.2)$$

(with t_0 held constant).



This is the energy (per unit time) that is transported out to infinity, and never comes back.

Dipole Radiation (cont'd)

- Area of the sphere is $4\pi r^2$, so for radiation to occur the Poynting vector must decrease (at large r) no faster than $1/r^2$

  E.g. If it went like $1/r^3$, then P would go like $1/r$, and P_{rad} would be zero.

- According to Coulomb's law, electrostatic fields fall off like $1/r^2$, and the Biot-Savart law says that magnetostatic fields go like $1/r^2$, which means that $S \sim 1/r^4$, for static configurations.

  So *static* sources do not radiate.

- Jefimenko's equations indicate that *time-dependent* fields include terms (involving $\dot{\rho}$ and $\dot{\boldsymbol{J}}$) that go like $1/r$; it is these terms that are responsible for electromagnetic radiation.

- Study of radiation involves picking the parts of $\boldsymbol{E}$ and $\boldsymbol{B}$ that go like $1/r$ at large distances from the source, constructing from them the $1/r^2$ term in $\boldsymbol{S}$, integrating over a large spherical surface, and taking limit $r \to \infty$.

Radiation

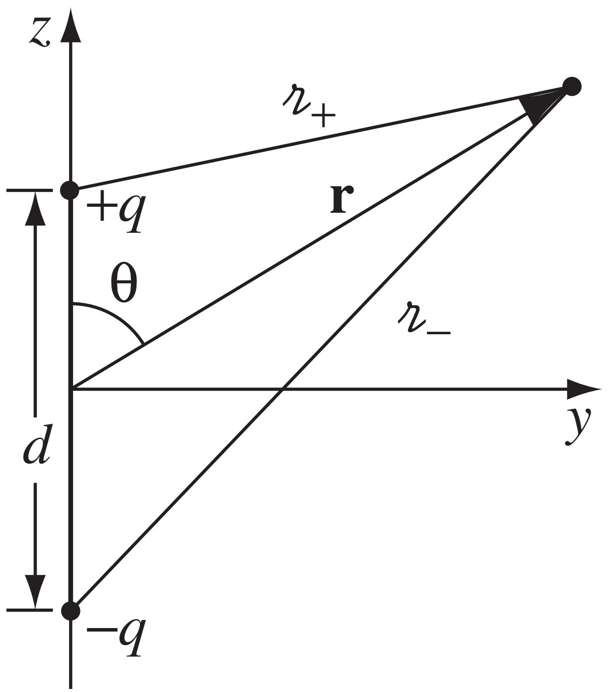
Electric Dipole Radiation
❖Electric ... i
❖Electric ... ii
❖Electric ... iii
❖Electric ... iv
❖Electric ... v
❖Electric ... vi
❖Electric ... vii
❖Electric ... viii
❖Electric ... ix
❖Electric ... x

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

Electric Dipole Radiation

Electric Dipole Radiation



Picture two tiny metal spheres separated by a distance d and connected by a fine wire. At time t the charge on the upper sphere is $q(t)$, and the charge on the lower sphere is $-q(t)$.

Suppose the charge is driven back and forth through the wire, from one end to the other, at an angular frequency ω:

$$q(t) = q_0 \cos(\omega t) \tag{11.3}$$

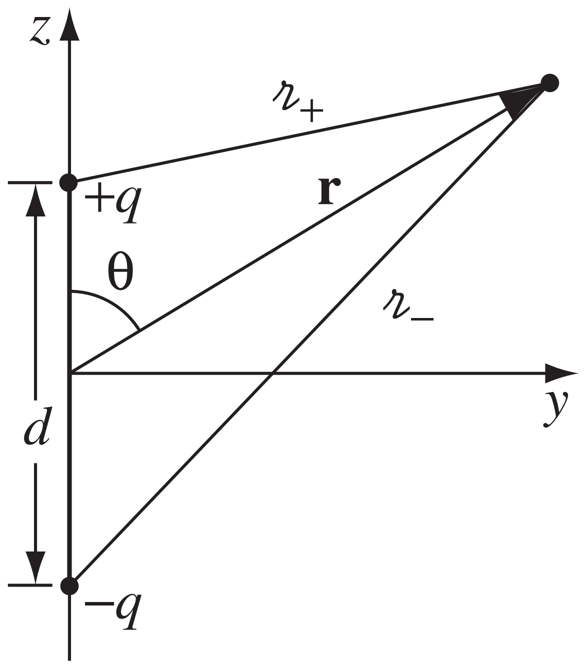
The result is an oscillating electric dipole:

$$\boldsymbol{p}(t) = p_0 \cos(\omega t)\,\hat{\boldsymbol{z}} \tag{11.4}$$

where

$$p_0 \equiv q_0 d$$

is the the maximum value of the dipole moment.



The retarded potential Eq. (10.18) is

$$V(\boldsymbol{r},t) = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_0\cos[\omega(t-\mathscr{r}_+/c)]}{\mathscr{r}_+} - \frac{q_0\cos[\omega(t-\mathscr{r}_-/c)]}{\mathscr{r}_-}\right\} \qquad (11.5)$$

where, by the law of cosines,

$$\mathscr{r}_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2} \qquad (11.6)$$

Now, to make this physical dipole into a perfect dipole, we want the separation distance to be extremely small:

$$\textbf{approximation 1} : d \ll r \qquad (11.7)$$

Expanding to first order in d

$$\mathscr{r}_\pm \cong r\left(1 \mp \frac{d}{2r}\cos\theta\right) \qquad (11.8)$$

Electric Dipole Radiation (cont'd)

It follows that

$$\frac{1}{\mathscr{r}_\pm} \cong \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right) \tag{11.9}$$

and

$$\cos[\omega(t - \mathscr{r}_\pm/c)] \cong \cos\left[\omega(t - r/c) \pm \frac{\omega d}{2c}\cos\theta\right]$$
$$= \cos[\omega(t - r/c)]\cos\left(\frac{\omega d}{2c}\cos\theta\right) \mp \sin[\omega(t - r/c)]\sin\left(\frac{\omega d}{2c}\cos\theta\right)$$

In the perfect dipole limit we have, further,

$$\textbf{approximation 2} : d \ll \frac{c}{\omega} \text{ or } d \ll \lambda \tag{11.10}$$

Under these conditions

$$\cos[\omega(t - \mathscr{r}_\pm/c)] \cong \cos[\omega(t - r/c)] \mp \frac{\omega d}{2c}\cos\theta\,\sin[\omega(t - r/c)] \tag{11.11}$$

Electric Dipole Radiation (cont'd)

Radiation

Electric Dipole Radiation

❖Electric ... i
❖Electric ... ii
❖Electric ... iii
❖Electric ... iv
❖Electric ... v
❖Electric ... vi
❖Electric ... vii
❖Electric ... viii
❖Electric ... ix
❖Electric ... x

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

Putting Eqs. (11.9) and (11.11) into Eq. (11.5), we obtain the potential of an oscillating perfect dipole:

$$V(r,\theta,t) = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r}\left\{-\frac{\omega}{c}\sin[\omega(t-r/c)] + \frac{1}{r}\cos[\omega(t-r/c)]\right\} \tag{11.12}$$

In the static limit $(\omega \to 0)$ the second term reproduces the old formula for the potential of a stationary dipole:

$$V = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r^2}$$

We are only interested in the fields that survive at large distances from the source, in the so-called **radiation zone**.

Radiation

Electric Dipole Radiation
❖Electric ... i
❖Electric ... ii
❖Electric ... iii
❖Electric ... iv
❖Electric ... v
❖Electric ... vi
❖Electric ... vii
❖Electric ... viii
❖Electric ... ix
❖Electric ... x

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

Electric Dipole Radiation (cont'd)

$$\textbf{approximation 3} : r \gg \frac{c}{\omega} \text{ or } r \gg \lambda \tag{11.13}$$

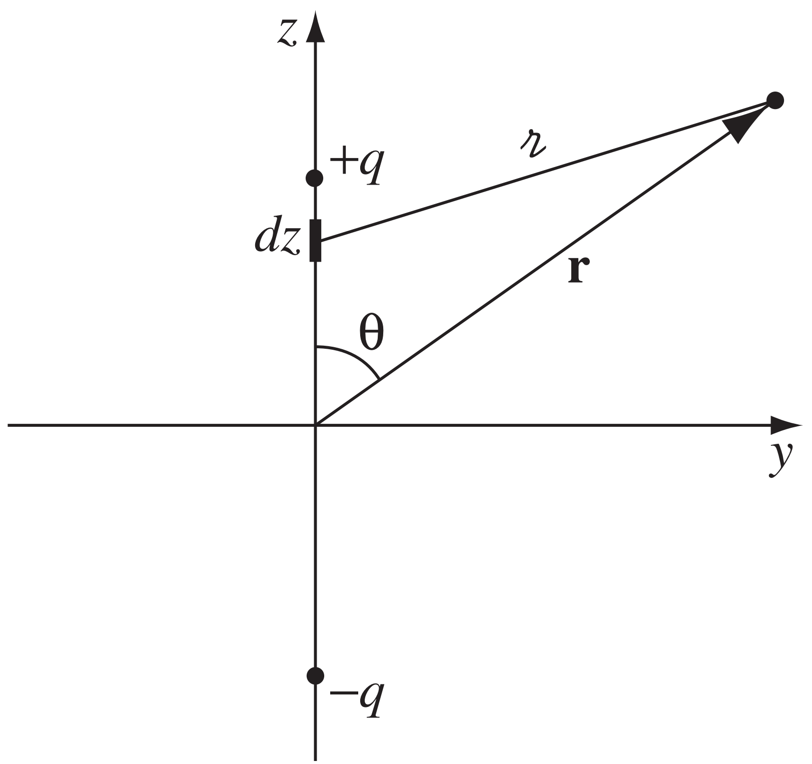
In this region the potential reduces to

$$V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t - r/c)] \tag{11.14}$$

Meanwhile, the vector potential is determined by the current flowing in the wire:

$$\boldsymbol{I}(t) = \frac{\mathrm{d}q}{\mathrm{d}t}\,\hat{\boldsymbol{z}} = -q_0\omega\sin(\omega t)\,\hat{\boldsymbol{z}} \tag{11.15}$$

Electric Dipole Radiation (cont'd)



The integral can be integrated to give

$$\boldsymbol{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin[\omega(t-r/c)]\,\hat{\boldsymbol{z}} \quad (11.17)$$

Now

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2}\frac{-q_0\omega\sin[\omega(t-\mathscr{r}/c)]\,\hat{\boldsymbol{z}}}{\mathscr{r}}\,\mathrm{d}z$$

(11.16)

Electric Dipole Radiation (cont'd)

From the potentials, we have

$$
\begin{aligned}
\nabla V &= \frac{\partial V}{\partial r}\,\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\hat{\boldsymbol{\theta}} \\
&= -\frac{p_0\omega}{4\pi\epsilon_0 c}\left\{\cos\theta\left(-\frac{1}{r^2}\sin[\omega(t-r/c)] - \frac{\omega}{rc}\cos[\omega(t-r/c)]\right)\hat{\boldsymbol{r}}\right. \\
&\quad \left. -\frac{\sin\theta}{r^2}\sin[\omega(t-r/c)]\,\hat{\boldsymbol{\theta}}\right\} \\
&\cong \frac{p_0\omega^2}{4\pi\epsilon_0 c^2}\left(\frac{\cos\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\boldsymbol{r}}
\end{aligned}
$$

(First and last terms are dropped in accordance with approximation 3)

Likewise,

$$
\frac{\partial \boldsymbol{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi r}\cos[\omega(t-r/c)](\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})
$$

and therefore

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\theta}} \qquad (11.18)$$

Meanwhile

$$\begin{aligned}\nabla \times \mathbf{A} &= \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}} \\ &= -\frac{\mu_0 p_0 \omega}{4\pi r}\left\{\frac{\omega}{c}\sin\theta\cos[\omega(t - r/c)] + \frac{\sin\theta}{r}\sin[\omega(t - r/c)]\right\}\hat{\boldsymbol{\phi}}\end{aligned}$$

The second term is again eliminated by approximation 3, so

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\phi}} \qquad (11.19)$$

Electric Dipole Radiation (cont'd)

Radiation

Electric Dipole Radiation

❖ Electric ... i

❖ Electric ... ii

❖ Electric ... iii

❖ Electric ... iv

❖ Electric ... v

❖ Electric ... vi

❖ Electric ... vii

❖ Electric ... viii

❖ Electric ... ix

❖ Electric ... x

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

Eqs. (11.18) and (11.19) represent monochromatic waves of frequency ω traveling in the radial direction at the speed of light. $\boldsymbol{E}$ and $\boldsymbol{B}$ are in phase, mutually perpendicular, and transverse; the ratio of their amplitudes is $E_0/B_0 = c$.

The energy radiated by an oscillating electric dipole is determined by the Poynting vector:

$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\right\}^2 \hat{\boldsymbol{r}} \qquad (11.20)$$

The intensity is obtained by averaging (in time) over a complete cycle:

$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\,\hat{\boldsymbol{r}} \qquad (11.21)$$

Electric Dipole Radiation (cont'd)

Radiation

Electric Dipole Radiation

❖ Electric ... i

❖ Electric ... ii

❖ Electric ... iii

❖ Electric ... iv

❖ Electric ... v

❖ Electric ... vi

❖ Electric ... vii

❖ Electric ... viii

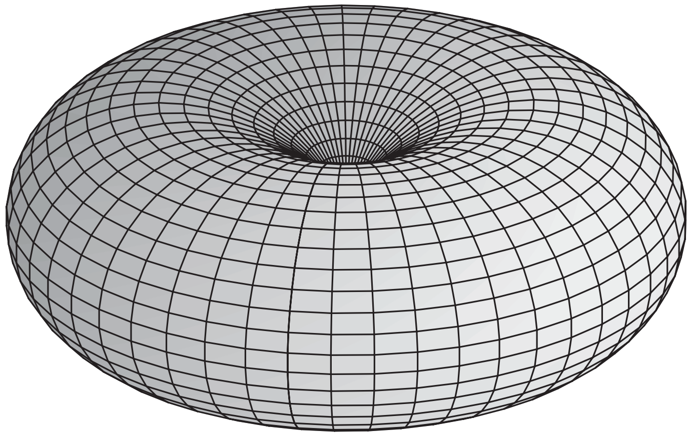
❖ Electric ... ix

❖ Electric ... x

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

Along the axis of the dipole ($\sin\theta = 0$), there is no radiation. The intensity profile takes the form of a donut, with its maximum in the equatorial plane.



The total power radiated is found by integrating $\langle \boldsymbol{S} \rangle$ over a sphere of radius r:

$$\langle P \rangle = \int \langle \boldsymbol{S} \rangle \cdot \mathrm{d}\boldsymbol{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \qquad (11.22)$$

Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation

❖ Magnetic ... i
❖ Magnetic ... ii
❖ Magnetic ... iii
❖ Magnetic ... iv
❖ Magnetic ... v
❖ Magnetic ... vi
❖ Magnetic ... vii
❖ Magnetic ... viii
❖ Magnetic ... ix
❖ Magnetic ... x

Radiation from an Arbitrary Source

Magnetic Dipole Radiation

Magnetic Dipole Radiation

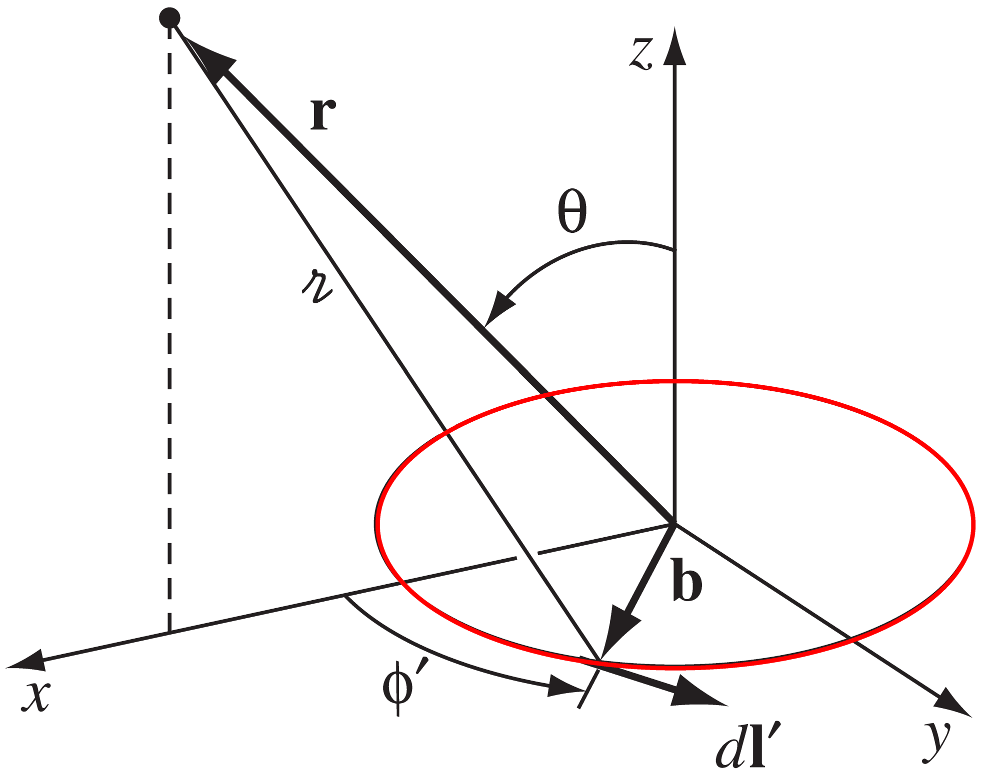
Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation

❖ Magnetic ... i
❖ Magnetic ... ii
❖ Magnetic ... iii
❖ Magnetic ... iv
❖ Magnetic ... v
❖ Magnetic ... vi
❖ Magnetic ... vii
❖ Magnetic ... viii
❖ Magnetic ... ix
❖ Magnetic ... x

Radiation from an Arbitrary Source



We have a wire loop of radius b, around which we drive an alternating current:

$$I(t) = I_0 \cos(\omega t) \tag{11.23}$$

This is a model for an oscillating magnetic dipole,

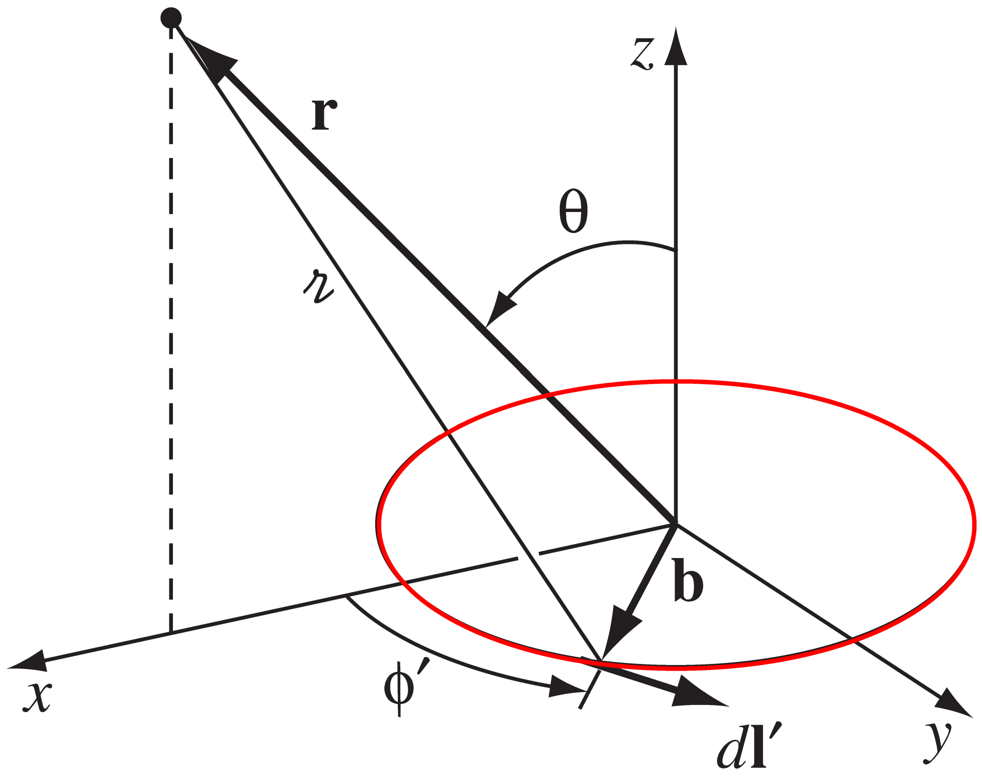
$$\boldsymbol{m}(t) = \pi b^2 I(t)\,\hat{\mathbf{z}} = m_0 \cos(\omega t)\,\hat{\mathbf{z}} \tag{11.24}$$

where

$$m_0 = \pi b^2 I_0 \tag{11.25}$$

is the maximum value of the magnetic dipole moment.

The loop is uncharged, so the scalar potential is zero.

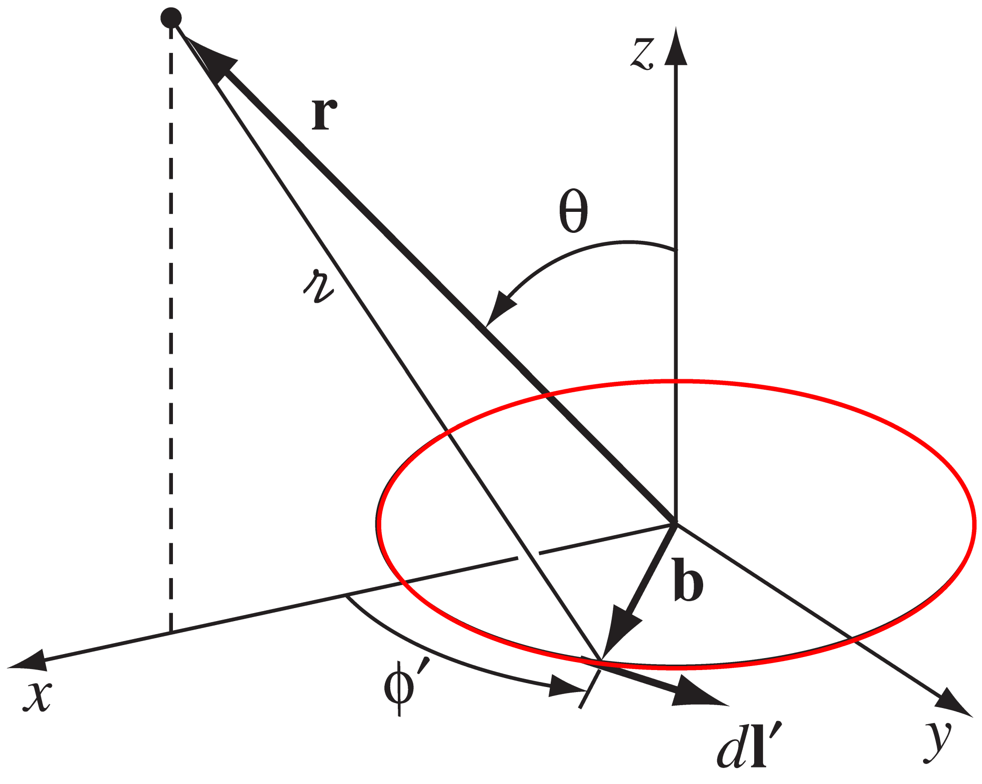


The retarded vector potential is

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int \frac{I_0\cos[\omega(t-\mathscr{r}/c)]}{\mathscr{r}}\,\mathrm{d}\boldsymbol{l}' \quad (11.26)$$

For a point $\boldsymbol{r}$ directly above the x axis, $\boldsymbol{A}$ must aim in the y direction, since the x components from symmetrically placed points on either side of the x axis will cancel. Thus

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0 I_0 b}{4\pi}\,\hat{\boldsymbol{y}}\int_0^{2\pi}\frac{\cos[\omega(t-\mathscr{r}/c)]}{\mathscr{r}}\cos\phi'\,\mathrm{d}\phi' \quad (11.27)$$



By the law of cosines,

$$\mathscr{r} = \sqrt{r^2 + b^2 - 2\boldsymbol{r}\cdot\boldsymbol{b}},$$

where vectors $\boldsymbol{r}$ and $\boldsymbol{b}$ are given by

$$\begin{aligned}\boldsymbol{r} &= r\sin\theta\,\hat{\boldsymbol{x}} + r\cos\theta\,\hat{\boldsymbol{z}}\\ \boldsymbol{b} &= b\cos\phi'\,\hat{\boldsymbol{x}} + b\sin\phi'\,\hat{\boldsymbol{y}}\end{aligned}$$

So, $\boldsymbol{r}\cdot\boldsymbol{b} = rb\sin\theta\cos\phi'$, and therefore

$$\mathscr{r} = \sqrt{r^2 + b^2 - 2rb\sin\theta\cos\phi'} \qquad (11.28)$$

For a "perfect" dipole, we want the loop to be extremely small:

$$\textbf{approximation 1}: \quad b \ll r \qquad (11.29)$$

Magnetic Dipole Radiation (cont'd)

To first order in b, then,

$$\mathscr{r} \cong r\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right)$$

so

$$\frac{1}{\mathscr{r}} \cong \frac{1}{r}\left(1 + \frac{b}{r}\sin\theta\cos\phi'\right) \tag{11.30}$$

and

$$\begin{aligned}\cos[\omega(t - \mathscr{r}/c)] &\cong \cos\left[\omega(t - r/c) + \frac{\omega b}{c}\sin\theta\cos\phi'\right] \\ &= \cos[\omega(t - r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right) - \sin[\omega(t - r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)\end{aligned}$$

Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation
❖ Magnetic ... i
❖ Magnetic ... ii
❖ Magnetic ... iii
❖ Magnetic ... iv
❖ Magnetic ... v
❖ Magnetic ... vi
❖ Magnetic ... vii
❖ Magnetic ... viii
❖ Magnetic ... ix
❖ Magnetic ... x

Radiation from an Arbitrary Source

Magnetic Dipole Radiation (cont'd)

We assume the size of the dipole is small compared to the wavelength radiated:

$$\textbf{approximation 2}: b \ll \frac{c}{\omega} \text{ or } b \ll \lambda \tag{11.31}$$

In that case,

$$\cos[\omega(t - \mathscr{r}/c)] \cong \cos[\omega(t - r/c)] - \frac{\omega b}{c} \sin\theta \cos\phi' \sin[\omega(t - r/c)] \tag{11.32}$$

Inserting Eqs. (11.30) and (11.32) into Eq. (11.27), and dropping the second-order term:

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{r}, t) \cong \frac{\mu_0 I_0 b}{4\pi r} \hat{\boldsymbol{y}} \int_0^{2\pi} \bigg\{ &\cos[\omega(t - r/c)] \\ &+ b \sin\theta \cos\phi' \left(\frac{1}{r} \cos[\omega(t - r/c)] - \frac{\omega}{c} \sin[\omega(t - r/c)] \right) \bigg\} \cos\phi' \, d\phi' \end{aligned}$$

Magnetic Dipole Radiation (cont'd)

$$\boldsymbol{A}(\boldsymbol{r},t) \cong \frac{\mu_0 I_0 b}{4\pi r}\hat{\boldsymbol{y}} \int_0^{2\pi} \left\{ \cos[\omega(t - r/c)] \quad + \right.$$
$$\left. b \sin\theta \cos\phi' \left(\frac{1}{r} \cos[\omega(t - r/c)] - \frac{\omega}{c} \sin[\omega(t - r/c)] \right) \right\} \cos\phi' \, \mathrm{d}\phi'$$

First term integrates to zero: $\int_0^{2\pi} \cos\phi' \, \mathrm{d}\phi' = 0$

The second term involves the integral of cosine squared: $\int_0^{2\pi} \cos^2\phi' \, \mathrm{d}\phi' = \pi$

In general $\boldsymbol{A}$ points along $\hat{\boldsymbol{\phi}}$-direction. $\boldsymbol{A}$ of an oscillating perfect magnetic dipole

$$\boldsymbol{A}(r,\theta,t) = \frac{\mu_0 m_0}{4\pi} \left(\frac{\sin\theta}{r} \right) \left\{ \frac{1}{r} \cos[\omega(t - r/c)] - \frac{\omega}{c} \sin[\omega(t - r/c)] \right\} \hat{\boldsymbol{\phi}} \qquad (11.33)$$

Magnetic Dipole Radiation (cont'd)

Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation

- Magnetic ... i
- Magnetic ... ii
- Magnetic ... iii
- Magnetic ... iv
- Magnetic ... v
- Magnetic ... vi
- Magnetic ... vii
- Magnetic ... viii
- Magnetic ... ix
- Magnetic ... x

Radiation from an Arbitrary Source

In the static limit $(\omega = 0)$ we recover the formula for the potential of a magnetic dipole

$$\boldsymbol{A}(r, \theta) = \frac{\mu_0}{4\pi} \frac{m_0 \sin\theta}{r^2} \hat{\boldsymbol{\phi}}$$

In the radiation zone,

$$\textbf{approximation 3} : r \gg \frac{c}{\omega} \tag{11.34}$$

the first term in $\boldsymbol{A}$ is negligible, so

$$\boldsymbol{A}(r, \theta, t) = -\frac{\mu_0 m_0 \omega}{4\pi c} \left(\frac{\sin\theta}{r}\right) \sin[\omega(t - r/c)] \hat{\boldsymbol{\phi}} \tag{11.35}$$

Magnetic Dipole Radiation (cont'd)

Radiation
Electric Dipole Radiation
Magnetic Dipole Radiation
❖ Magnetic ... i
❖ Magnetic ... ii
❖ Magnetic ... iii
❖ Magnetic ... iv
❖ Magnetic ... v
❖ Magnetic ... vi
❖ Magnetic ... vii
❖ Magnetic ... viii
❖ Magnetic ... ix
❖ Magnetic ... x
Radiation from an Arbitrary Source

From $\boldsymbol{A}$ we obtain the fields at large r:

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\phi}} \tag{11.36}$$

and

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\theta}} \tag{11.37}$$

(approximation 3 is used in calculating $\boldsymbol{B}$)

These fields are in phase, mutually perpendicular, and transverse to the direction of propagation ($\hat{\boldsymbol{r}}$), and the ratio of their amplitudes is $E_0/B_0 = c$.

Magnetic Dipole Radiation (cont'd)

Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation

- ❖ Magnetic ... i
- ❖ Magnetic ... ii
- ❖ Magnetic ... iii
- ❖ Magnetic ... iv
- ❖ Magnetic ... v
- ❖ Magnetic ... vi
- ❖ Magnetic ... vii
- ❖ Magnetic ... viii
- ❖ Magnetic ... ix
- ❖ Magnetic ... x

Radiation from an Arbitrary Source

The energy flux for magnetic dipole radiation is

$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{\mu_0}{c}\left\{\frac{m_0\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\right\}^2 \hat{\boldsymbol{r}} \tag{11.38}$$

the intensity is

$$\langle \boldsymbol{S} \rangle = \left(\frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3}\right)\frac{\sin^2\theta}{r^2}\hat{\boldsymbol{r}} \tag{11.39}$$

and the total radiated power is

$$\langle P \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} \tag{11.40}$$

The intensity profile has the shape of a donut, and the power radiated goes like ω^4.

Magnetic Dipole Radiation (cont'd)

Radiation
Electric Dipole Radiation
Magnetic Dipole Radiation
❖Magnetic ... i
❖Magnetic ... ii
❖Magnetic ... iii
❖Magnetic ... iv
❖Magnetic ... v
❖Magnetic ... vi
❖Magnetic ... vii
❖Magnetic ... viii
❖Magnetic ... ix
❖Magnetic ... x
Radiation from an Arbitrary Source

There is, however, one important difference between electric and magnetic dipole radiation: For configurations with comparable dimensions, the power radiated electrically is enormously greater. Comparing Eqs. (11.22) and (11.40),

$$\frac{P_{\text{magnetic}}}{P_{\text{electric}}} = \left(\frac{m_0}{p_0 c}\right)^2 \tag{11.41}$$

where $m_0 = \pi b^2 I_0$, and $p_0 = q_0 d$. The amplitude of the current in the electrical case was $I_0 = q_0 \omega$. Setting $d = \pi b$, we have

$$\frac{P_{\text{magnetic}}}{P_{\text{electric}}} = \left(\frac{\omega b}{c}\right)^2 \tag{11.42}$$

But $\omega b/c$ is very small (approximation 2), and here it appears squared. Ordinarily, then, one should expect electric dipole radiation to dominate.

Radiation

Electric Dipole Radiation

Magnetic Dipole Radiation

Radiation from an Arbitrary Source

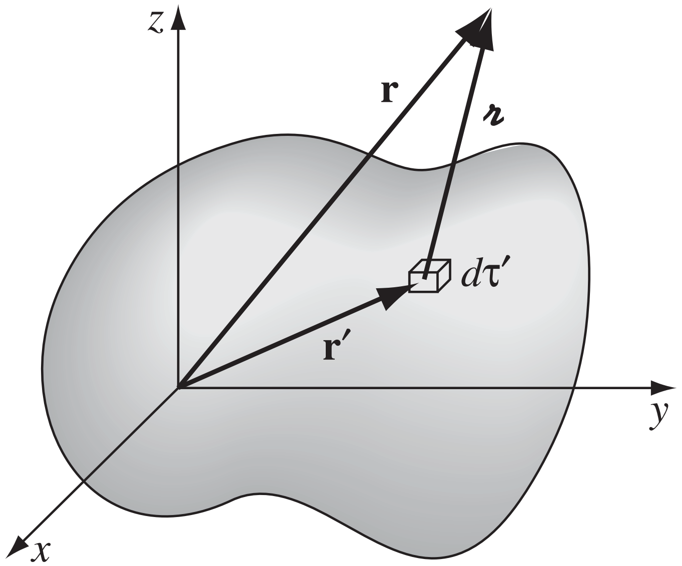
❖ Radiation from Arbitrary Source

❖ Radiation ii

❖ Example

Radiation from an Arbitrary Source

Radiation from an Arbitrary Source



Consider a configuration of charge and current that is entirely arbitrary, except that it is localized within some finite volume near the origin.

It can be shown that the fields are given by

$$\boldsymbol{E}(\boldsymbol{r},t) \cong \frac{\mu_0}{4\pi r}[\hat{\boldsymbol{r}} \times (\hat{\boldsymbol{r}} \times \ddot{\boldsymbol{p}})] \qquad (11.43)$$

$$\boldsymbol{B}(\boldsymbol{r},t) \cong -\frac{\mu_0}{4\pi r c}[\hat{\boldsymbol{r}} \times \ddot{\boldsymbol{p}}] \qquad (11.44)$$

where $\ddot{\boldsymbol{p}}$ is evaluated at time $t_0 = t - r/c$. $\boldsymbol{E}$ and $\boldsymbol{B}$ are mutually perpendicular, transverse to the direction of propagation $(\hat{\boldsymbol{r}})$, and in the ratio $E/B = c$.

Radiation from an Arbitrary Source (cont'd)

Radiation
Electric Dipole Radiation
Magnetic Dipole Radiation
Radiation from an Arbitrary Source
❖ Radiation from Arbitrary Source
❖ Radiation ii
❖ Example

The Poynting vector is

$$\boldsymbol{S}(\boldsymbol{r}, t) = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{\mu_0}{16\pi^2 c}[\ddot{p}(t_0)]^2 \left(\frac{\sin^2\theta}{r^2}\right) \hat{\boldsymbol{r}} \tag{11.45}$$

The power passing through a giant spherical surface at radius r is

$$P(r, t) = \oint \boldsymbol{S}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{a} = \frac{\mu_0}{6\pi c}\left[\ddot{p}\left(t - \frac{r}{c}\right)\right]^2$$

and the total radiated power (Eq. (11.2)) is

$$P_{\text{rad}}(t_0) \cong \frac{\mu_0}{6\pi c}[\ddot{p}(t_0)]^2 \tag{11.46}$$

Example

Radiation
Electric Dipole Radiation
Magnetic Dipole Radiation
Radiation from an Arbitrary Source
❖ Radiation from Arbitrary Source
❖ Radiation ii
❖ Example

For a single point charge q, the dipole moment is

$$\boldsymbol{p}(t) = q\boldsymbol{d}(t)$$

where d is the position of q with respect to the origin.

Therefore,

$$\ddot{\boldsymbol{p}}(t) = q\boldsymbol{a}(t)$$

where $\boldsymbol{a}$ is the acceleration of the charge.

From (11.46), the power radiated by a point charge is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \tag{11.47}$$

The Larmor formula, good for non-relativistic speeds.