PC2135: Thermodynamics and Statistical Physics

Semester 1, AY2526

Tutorial 2: Chapter 2 & 3

Instructions:
Please submit the questions with the asterisk by 5pm, Friday of the tutorial week.

Extra Note:
In this tutorial, you might find the formula of a $3N$ dimensional sphere useful. The formula given below for when N is even.

$$V_{3N} = \frac{\pi^{\frac{3N}{2}}}{\left(\frac{3N}{2}\right)!} R^{3N}$$

Question 1: Probability Calculations I

Consider the system of four spins known to be in equilibrium, as discussed in Section 3.2.1. Suppose that the total energy of this system is known to be $-2\mu_0 B$.

(a) What is the mean magnetic moment $\bar{M}$ of the system?

(b) Consider the first spin in this system.

  (i) What is the probability this spin is pointing up?

  (ii) Given that this first spin has two possible states (spin up and spin down), we expect from the fundamental postulate of equilibrium statistical mechanics that the probability of it pointing up should be 1/2. Comment on your previous result in (b) in relation to this.

Question 2: Probability Calculations II

Consider a system with N ideal spin$-\frac{1}{2}$ systems, known to be in equilibrium.

(a) Let n denote the number of spins pointing up. Express the energy of the system E in terms of N, n and E_0, where $E_0 = \mu_0 B$.

(b) When $N = 10$ and $E = 8E_0$,

  i. how many microstates obey the constraint imposed by the initial preparation of the system?

  ii. what is the probability of the system being in any one of its possible microstates?

  iii. how many of these microstates (i.e. amongst the set of microstates of i.) have the first spin pointing up?

(c) Redo part (b) for $N = 100$.

(d) For each of the sub-questions answered in (c), which part of the generic four-ingredient framework of Lecture 3 did you utilize?

Question 3: Probability Calculations III

Consider a system of 2 ideal gas particles constrained within a 2D box of dimensions $L \times L$, having energy $30E_0$, where

$$E_0 = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}.$$

Utilizing a computational software (or otherwise), determine

(a) the highest energy (in terms of E_0) possible for the first particle.

(b) the probability of that particle having said energy.

Was the fundamental postulate of equilibrium statistical mechanics was applied in your solutions?

Question 4: Typical number of states accessible to a gas molecule

It is possible to determine the mean energy $\bar{E}$ of a diatomic gas molecule (such as N_2) using the density and pressure. Consider a scenario where this value is found to be 6×10^{-21}J for one diatomic molecule. It is known further that $m = 4.65 \times 10^{-26}$kg.

(a) Calculate $\Phi(\bar{E})$ for such a molecule enclosed in a box having a volume of one liter (10^3cm^3), where $\Phi(\bar{E})$ denotes the number of states with energy less than $\bar{E}$

(b) Consider a small energy interval $\delta E = 10^{-31}$J, which is much smaller than $\bar{E}$ itself. Calculate the number of states $\Omega(\bar{E})$ accessible to the molecule in the range between $\bar{E} + \delta E$

(c) Given the incredible small value of δE, what strikes you about the results in (b)?

Question 5: Number of states of an ideal gas*

Consider an ideal gas consisting of N particles confined within a box with edge length $L_x = L_y = L_z = L$. In this question, N is of the order of Avogadro's number.

Using approximate arguments similar to those used in lectures, show that the number of states $\Omega(E)$ in a given energy interval between E and $E + \delta E$ is given by

$$\Omega(\bar{E}) = C L^{3N} E^{3N/2}\, \delta E$$

where C is a constant of proportionality.

Question 6: Relaxing of Constraints

Consider a system of $N = 1000$ identical (assumed distinguishable) particles of mass m confined within a 3-dimensional box of length $L_x = L_y = L_z = L = 1$m,. Determine the ratio of Ω_f to Ω_i for an increment in volume of 0.1m^3.

Question 7 (Ch 2): Stirling's Approximation*

This course involves frequent use of combinations and therefore makes extensive use of factorial expressions. Given the large numbers involved, we will need to employ approximations.

(a) When n is large, show that

$$\ln n! \approx n \ln n - n$$

> A better approximation is given by *Stirling's formula*, which is
>
> $$\ln n! = n \ln n - n + \frac{1}{2}\ln(2\pi n)$$

(b) Determine the the probability of obtaining 500 heads when flipping 1000 coins via the two approximations given in (a). Compare both values to the actual value from the binomial distribution (compute with a computing software if necessary). Note that as n increases, Stirling's approximation will improve.

Question 8 (Ch 2): Beyond Two Outcomes II

Our binomial distribution works only when there are 2 possible outcomes.

Consider now a system of N identical (assumed distinguishable) spin-1 atoms. The component of its magnetic moment along a given direction can have three possible values, $+\mu_0$, 0 and $-\mu_0$. Assuming all three states are equally likely, determine the probability of of there being 20 spins with magnetic moment $+\mu_0$, 30 spins with magnetic moment 0 and 50 spins with magnetic moment $-\mu_0$.