

# The Answer of Assignment 2

WEI SHUANG

31/8/2025

## Problem 1 Solution

- (1) 根据线性空间的同构理论，任何维数相同的线性空间都是同构的。因此可知  $\sum_{i,j=0}^1 c_{ij} |i, j\rangle$  与  $\{(x_1, x_2, x_3, x_4)\}, x \in C$  同构。即  $v_1 = (c_{00}, c_{01}, c_{02}, c_{03})^*, v_2 = (d_{00}, d_{01}, d_{10}, d_{11})^*$
- (2)  $\langle \psi_1 | \psi_2 \rangle = v_1^\dagger v_2$  成立, 这是因为
- (3)
- (4)

## Problem 2 Solution

- (1)
- (2)
- (3)
- (4)

## Problem 3 Solution

- (1)
- (2)
- (3)

(4) how should the Pauli operators  $\sigma_i^+$  and  $\sigma_i^z$  can be written in terms of the  $f$  operators? we have the definition that:

$$f_i^\dagger = \left( \prod_{j < i} \sigma_j^z \right) \sigma_i^+$$

so from  $\langle \overline{n_1 n_2 \dots n_i} | f_i^\dagger | \overline{n_1 n_2 n_3 \dots n_i} \rangle = \langle \overline{n_1 n_2 \dots n_i} | f_i | \overline{n_1 n_2 \dots n_i} \rangle^*$  we can get the relation between  $f_i$  and  $\sigma_i^+, \sigma_i^z$  as follows:

$$f_i = \left( \prod_{j < i} \sigma_j^z \right) \sigma_i^-$$

then try to represent  $\sigma_i^z$  and  $\sigma_i^+$  in terms of  $f_i$ : we use the matrix representation of the Pauli operators:

$$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$\sigma_i^z$  can be expressed as:  $2\sigma_i^+ \sigma_i^- - I = 2f_i^\dagger f_i - I$

like wise:

$$\begin{aligned} \sigma_i^+ &= f_i^\dagger \left( \prod_{j<i} \sigma_j^z \right)^{-1} \\ &= f_i^\dagger \left( \prod_{j<i} (2f_j^\dagger f_j - I) \right) \end{aligned}$$