PC3231

Tutorial 4: Radiation

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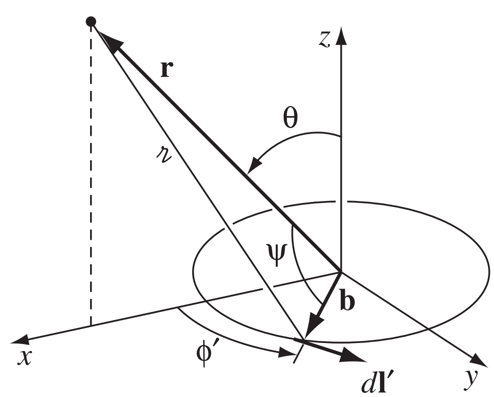
1. (a) Find the radiation resistance of the wire joining the two ends of the dipole. (This is the resistance that would give the same average power loss–to heat–as the oscillating dipole in fact puts out in the form of radiation.)

   Show that $R = 790\,(d/\lambda)^2\ \Omega$, where λ is the wavelength of the radiation.

   (b) Similarly, find the radiation resistance for the oscillating magnetic dipole in the figure below.



   Express your answer in terms of λ and b, and compare the radiation resistance of the *electric* dipole.

2. A parallel-plate capacitor C, with plate separation d, is given an initial charge $(\pm)Q_0$. It is then connected to a resistor R, and discharges, $Q(t) = Q_0 e^{-t/RC}$.

   (a) What fraction of its energy $(Q_0^2/2C)$ does it radiate away?

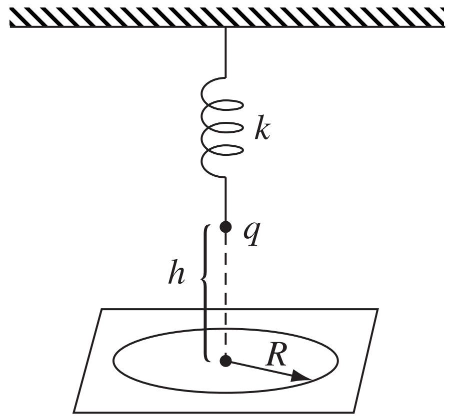
   (b) If $C = 1$ pF, $R = 1000\ \Omega$, and $d = 0.1$ mm, what is the actual *number*?

3. An insulating circular ring (radius b) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\phi$, where λ_0 is constant and ϕ is the usual azimuthal angle. The ring is now set spinning at a constant angular velocity ω about the z axis. Calculate the power radiated.

4. A positive charge q is fired head-on at a distant positive charge Q (which is held stationary), with an initial velocity v_0. It comes in, decelerates to $v = 0$, and returns out to infinity. What fraction of its initial energy $(\frac{1}{2}mv_0^2)$ is radiated away? Assume $v_0 \ll c$, and that you can safely ignore the effect of radiative losses on the motion of the particle.

   Note: $\displaystyle\int_\infty^a \frac{1}{x^4\sqrt{1-\frac{a}{x}}}\,\mathrm{d}x = -\frac{16}{15a^3}, \quad a \neq 0$

5. A particle of mass m and charge q is attached to a spring with force constant k, hanging from the ceiling.



Its equilibrium position is a distance h above the floor. It is pulled down a distance d below equilibrium and released, at time $t = 0$.

(a) Under the usual assumptions ($d \ll \lambda \ll h$), calculate the intensity of the radiation hitting the floor, as a function of the distance R from the point directly below q. [Note: The intensity here is the average power per unit area of *floor*.] At what R is the radiation most intense?

(b) As a check on your formula, assume the floor is of infinite extent, and calculate the average energy per unit time striking the entire floor. Is it what you'd expect?

(c) Because it is losing energy in the form of radiation, the amplitude of the oscillation will gradually decrease. After what time τ has the amplitude been reduced to d/e? (Assume the fraction of the total energy lost in one cycle is very small.)