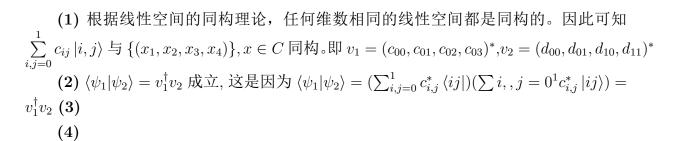
The Answer of Assignment 2

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Problem 1 Solution



Problem 2 Solution

- (1)
- **(2)**
- (3)
- **(4)**

Problem 3 Solution

- (1)
- **(2)**
- (3)
- (4) how should the Pauli operators σ_i^+ and σ_i^z can be written in terms of the f operators? we have the definition that:

$$f_i^{\dagger} = \left(\prod_{j < i} \sigma_j^z\right) \sigma_i^+$$

so from $\langle \overline{n_{1'}n_{2'} \cdot n_{i'}} | f_i^{\dagger} | \overline{n_1n_2n_3 \cdot n_i} \rangle = \langle \overline{n_1n_2 \cdot n_i} | f_i | \overline{n_{1'}n_{2'} \cdot n_{i'}} \rangle^*$ we can get the relation between f_i and σ_i^+ , σ_i^z as follows:

$$f_i = \left(\prod_{j < i} \sigma_j^z\right) \sigma_i^-$$

then try to represent σ_i^z and σ_i^+ in terms of f_i : we use the matrix representation of the Pauli operators:

$$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

 $\sigma_i^z {\rm can}$ be expressed as: 2 $\sigma_i^+ \sigma_i^- - I = 2 f_i^\dagger f_i - I$

like wise:

$$\sigma_i^+ = f_i^\dagger \left(\prod_{j < i} \sigma_j^z \right)^{-1}$$
$$= f_i^\dagger \left(\prod_{j < i} (2f_j^\dagger f_j - I) \right)$$