The Answer of Assignment 2

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31/8/2025

Problem 1 Solution

(1) 根据线性空间的同构理论，任何维数相同的线性空间都是同构的。因此可知 $\sum_{i,j=0}^{1} c_{ij}\,|i,j\rangle$ 与 $\{(x_1,x_2,x_3,x_4)\}, x \in C$ 同构。即 $v_1 = (c_{00}, c_{01}, c_{02}, c_{03})^*$,$v_2 = (d_{00}, d_{01}, d_{10}, d_{11})^*$

(2) $\langle\psi_1|\psi_2\rangle = v_1^\dagger v_2$ 成立, 这是因为 $\langle\psi_1|\psi_2\rangle = (\sum_{i,j=0}^{1} c_{i,j}^* \langle ij|)(\sum i,, j = 0^1 c_{i,j}\,|ij\rangle) = v_1^\dagger v_2$

(3) $O\,|pq\rangle = O_{ij,kl}\,|ij\rangle\langle kl|\,|pq\rangle = O_{ij,kl}\,|ij\rangle\,\delta_{kp}\delta_{lq} = O_{ij,pq}\,|ij\rangle$ if we define $|00\rangle$ as e_1, $|01\rangle$ as e_2, $|10\rangle$ as e_3, $|11\rangle$ as e_4, then we can get the matrix representation of O: $Oe_i = \sum_{j=1}^{4} O_{ji}e_j$ Therefore, the matrix representation of O is:

$$M = \begin{pmatrix} O_{11} & O_{12} & O_{13} & O_{14} \\ O_{21} & O_{22} & O_{23} & O_{24} \\ O_{31} & O_{32} & O_{33} & O_{34} \\ O_{41} & O_{42} & O_{43} & O_{44} \end{pmatrix}$$

$O\,|\psi_1\rangle$ is the same as Mv_1 **(4)**

Problem 2 Solution

(1)

(2)

(3)

(4)

Problem 3 Solution

(1)

(2)

(3)

(4) how should the Pauli operators σ_i^+ and σ_i^z can be written in terms of the f operators? we have the definition that:

$$f_i^\dagger = \left(\prod_{j<i} \sigma_j^z\right) \sigma_i^+$$

so from $\langle \overline{n_{1'} n_{2'} \cdot n_{i'}} | f_i^\dagger | \overline{n_1 n_2 n_3 \cdot n_i} \rangle = \langle \overline{n_1 n_2 \cdot n_i} | f_i | \overline{n_{1'} n_{2'} \cdot n_{i'}} \rangle^*$ we can get the relation between f_i and σ_i^+, σ_i^z as follows:

$$f_i = \left(\prod_{j<i} \sigma_j^z\right) \sigma_i^-$$

then try to represent σ_i^z and σ_i^+ in terms of f_i: we use the matrix representation of the Pauli operators:

$$\sigma_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

σ_i^zcan be expressed as:$2\sigma_i^+\sigma_i^- - I = 2f_i^\dagger f_i - I$

like wise:

$$\sigma_i^+ = f_i^\dagger \left(\prod_{j<i} \sigma_j^z\right)^{-1}$$

$$= f_i^\dagger \left(\prod_{j<i} (2f_j^\dagger f_j - I)\right)$$