

Problem Set (Week 4)

Econ 103

Lecture 12

Note: In the following five questions $X_1, X_2 \sim \text{iid } N(\mu, \sigma^2)$, $Y = (X_1 - \mu)/\sigma$, $Z = (X_2 - \mu)/\sigma$.

1. (a) What is the distribution of $X_1 + X_2$?
(b) Use R to calculate $P(X_1 + X_2 > 5)$ if $\mu = 5$ and $\sigma^2 = 50$.
(c) Use R to calculate the 10th percentile of the distribution of $X_1 + X_2$.
2. (a) What is the distribution of $Z/\sqrt{Y^2}$?
(b) What value of c satisfies $P(-c \leq Z/\sqrt{Y^2} \leq c) = 0.95$?
(c) How does the interval in part (b) compare to the corresponding interval for Z ?
3. (a) What is the distribution of Y^2/Z^2 ?
(b) Use R to calculate the 95th percentile of the distribution of Y^2/Z^2 .

Lecture 13

4. In this question, you will use R to study the sampling distribution of the sample mean where we take as our population the heights of all students in Econ 103.
 - (a) Load the class survey data used in R Tutorial # 2, extract the height column and assign it to a variable called `height`. Use `!is.na` to remove all missing values from `height`.
 - (b) Make a histogram of height and calculate the mean height for students in the class. For the purposes of this exercise, these correspond to the *population*.
 - (c) Write a function that takes n as its only input and returns the sample mean of an iid random sample of size n drawn from the vector `height`. Call this function `x.bar.draw`. [Hint: use `sample` with `replace = TRUE`.]

- (d) Test the function you wrote for part (d) by running it with $n = 10000$. What value do you get? Your answer should be approximately equal the population mean you calculated above. If it isn't, something is wrong with your code.

Lecture 14

5. Textbook question: Chapter 7-18.

Lecture 15-16

6. For this question assume that we have a random sample from a normal distribution with unknown mean but *known* variance.
- (a) Suppose that we have 36 observations, the sample mean is 5, and the population variance is 9. Construct a 95% confidence interval for the population mean.
 - (b) Repeat the preceding with a population variance of 25 rather than 9.
 - (c) Repeat the preceding with a sample size of 25 rather than 36.
 - (d) Repeat the preceding but construct a 99% rather than a 95% confidence interval.
 - (e) How would you construct the confidence interval if the variance is *unknown*? Explain what estimator you will use to approximate the variance and what distribution you will use to construct the interval.