

Econ 103 – Quiz 4

Name: _____

Instructions: This is closed-book, closed-notes quiz. Please write your answers in the blanks provided. Non-programmable calculators are permitted.

1. (2 points) What is the bias of the estimator $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$? Note that it is an estimator for population variance σ^2 . (Hint: $E[\sum_{i=1}^n (X_i - \bar{X})^2] = (n-1)\sigma^2$.)
2. (1 points) True or false? Consistency of an estimator is related to mean-squared error when the sample size is fixed.
3. (4 points) Let $X_1, X_2, \dots, X_n \sim i.i.d.$ with mean μ and variance σ^2 . You have two estimators of population mean μ . The first estimator is $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$. The other estimator is X_{max} , which is the maximum value of the sample. Which estimator do you prefer according to mean-squared error (MSE)? (Hint: The maximum value is one data point in the sample.)

4. (13 points) Assume that we have a random i.i.d. sample from a normal distribution with unknown mean but *known* variance. Suppose that we have 100 observations, the sample mean is 5, and the population variance is 16. (If necessary, use R command in your answer.)
- a. (2 points) Construct a 95% confidence interval.
 - b. (2 points) Compute the margin of error (ME) of the above CI.
 - c. (2 points) Compute the length of the above CI.
 - d. (2 points) Construct a 80% confidence interval.
 - e. (2 points) What happens to the confidence interval if the sample size increases? (Does CI become wider or tighter?)
 - f. (3 points) Consider the same setup as above except that we *do not know* the population variance. Instead, the sample standard deviation is 5. Construct a 95% confidence interval in this case.