

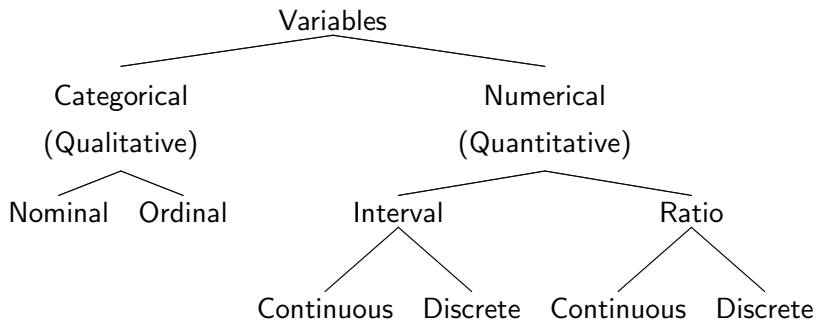
# Economics 103 – Statistics for Economists

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Lecture # 2

# A Taxonomy of Variables



# From Weakest to Strongest

## Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

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## Numerical

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**Interval** only differences meaningful, no natural zero

**Ratio** differences and ratios meaningful, natural zero



## And For Numerical Variables (interval or ratio)...

### Discrete

Takes value from discrete set of numbers, typically count data

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### Discrete

Takes value from discrete set of numbers, typically count data

### Continuous

Value could be any real number within some range (even though *measurements* are made with finite precision)



## What kind of variable is...

...Handspan?

- (a) Nominal
- (b) Ordinal
- (c) Interval
- (d) Ratio



## What kind of variable is...

...Temperature?

- (a) Nominal
- (b) Ordinal
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## What kind of variable is...

...Eye Color?

- (a) Nominal
- (b) Ordinal
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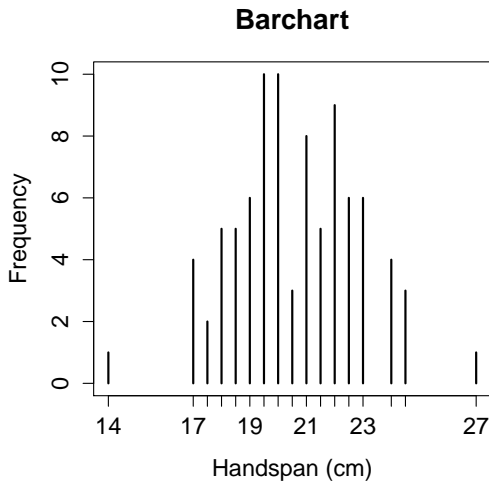
## What kind of variable?

On course evaluations you can rate your professor as follows:  
0 = Poor, 1 = Fair, 2 = Good, 3 = Very Good, 4 = Excellent.  
What kind of data is your rating?

- (a) Nominal
- (b) Ordinal
- (c) Interval
- (d) Ratio

## Handspan - Frequency and Relative Frequency

cm	Freq.	Rel. Freq.
14.0	1	0.01
17.0	4	0.05
17.5	2	0.02
18.0	5	0.06
18.5	5	0.06
19.0	6	0.07
19.5	10	0.11
20.0	10	0.11
20.5	3	0.03
21.0	8	0.09
21.5	5	0.06
22.0	9	0.10
22.5	6	0.07
23.0	6	0.07
24.0	4	0.05
24.5	3	0.03
27.0	1	0.01
<hr/> $n = 89$		1.00



## Handspan - Summarize Barchart by "Smoothing"

cm	Freq.	Rel. Freq.
14.0	1	0.01
17.0	4	0.05
17.5	2	0.02
18.0	5	0.06
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19.5	10	0.11
20.0	10	0.11
20.5	3	0.03
21.0	8	0.09
21.5	5	0.06
22.0	9	0.10
22.5	6	0.07
23.0	6	0.07
24.0	4	0.05
24.5	3	0.03
27.0	1	0.01
$n = 88$		1.00

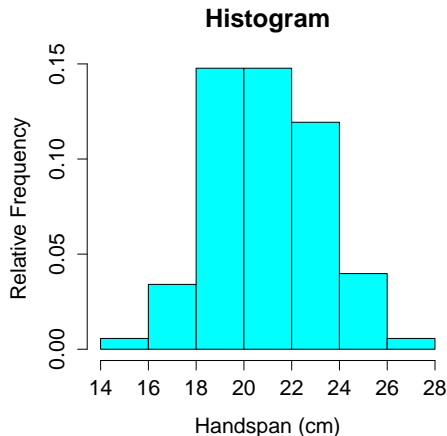
Group data into non-overlapping bins of equal width:

Bins	Freq.	Rel. Freq.
[14, 16)	1	0.01
[16, 18)	6	0.07
[18, 20)	26	0.30
[20, 22)	26	0.30
[22, 24)	21	0.24
[24, 26)	7	0.08
[26, 28)	1	0.01
$n = 88$		1.00

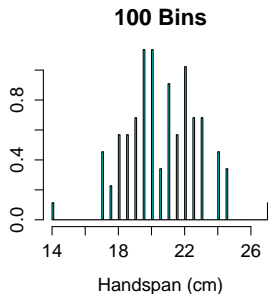
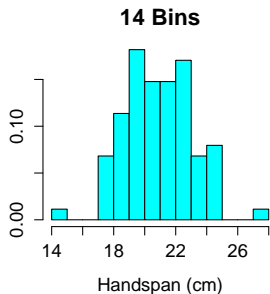
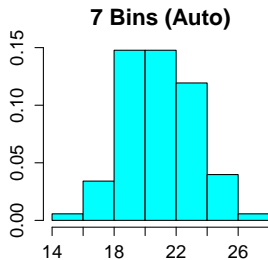
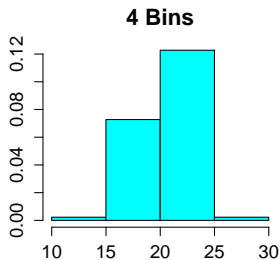


# Histogram – Density Estimate by Smoothing Barchart

Bins	Freq.	Rel. Freq.
[14, 16)	1	0.01
[16, 18)	6	0.07
[18, 20)	26	0.30
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[22, 24)	21	0.24
[24, 26)	7	0.08
[26, 28)	1	0.01
$n = 88$		1.00



# Number of Bins Controls Degree of Smoothing



# Histograms are *Really* Important

## Why Histogram?

Summarize numerical data, especially continuous (few repeats)

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Don't confuse with barchart!

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- ▶ Mean
- ▶ Median

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## 3. Measures of Symmetry

- ▶ Skewness

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  - ▶ Median
2. Measures of Spread
  - ▶ Variance
  - ▶ Standard Deviation
  - ▶ Range
  - ▶ Interquartile Range (IQR)
3. Measures of Symmetry
  - ▶ Skewness
4. Measures of relationship between variables
  - ▶ Covariance
  - ▶ Correlation
  - ▶ Regression

# Questions to Ask Yourself about Each Summary Statistic

1. What does it measure?
2. What are its units compared to those of the data?
3. (How) do its units change if those of the data change?
4. What are the benefits and drawbacks of this statistic?

Some of the information regarding items 2 and 3 is on the homework rather than in the slides because working it out for yourself is a good way to check your understanding.

# Measures of Central Tendency

Suppose we have a dataset with observations  $x_1, x_2, \dots, x_n$

## Sample Mean

- ▶  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
- ▶ Only for numeric data
- ▶ Works best when data are symmetric and there are no outliers

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## Sample Median

- ▶ Middle observation if  $n$  is odd, otherwise the mean of the two observations closest to the middle.
- ▶ Applicable to numerical or ordinal data
- ▶ Robust to outliers and skewness

# What is an Outlier?

## Outlier

A very unusual observation relative to the other observations in the dataset (i.e. very small or very big).

## Mean is Sensitive to Outliers, Median Isn't

First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

## Mean is Sensitive to Outliers, Median Isn't

First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

Second Dataset: 1 2 3 4 4990

Mean = 1000, Median = 3



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First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

Second Dataset: 1 2 3 4 4990

Mean = 1000, Median = 3

When Does the Median Change?

Ranks would have to change so that 3 is no longer in the middle.

## Percentiles (aka Quantiles) – Generalization of Median

Approx.  $P\%$  of the data are at or below the  $P^{th}$  percentile.

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## Quartiles

Q1 = 25th Percentile

Q2 = Median (i.e. 50th Percentile)

Q3 = 75th Percentile

## An Example: $n = 12$

60 63 65 67 70 72 75 75 80 82 84 85

$Q_1$  = value in the  $0.25(n + 1)^{th}$  ordered position

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$$\begin{aligned} Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\ &= \text{value in the } 3.25^{th} \text{ ordered position} \end{aligned}$$

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$$\begin{aligned} Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\ &= \text{value in the } 3.25^{th} \text{ ordered position} \\ &= 65 + 0.25 * (67 - 65) \end{aligned}$$

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## Student Debt

Guess the **90th percentile** of student loan debt in the U.S. That is, guess the amount of money such that 10% college students graduate with *more* than this amount of debt and 90% graduate with less than or equal to this amount of debt.





## Student Debt

Would you guess that the median amount of student loan debt in the U.S. is above, below, or equal to the mean amount?

- (a) Median  $>$  Mean
- (b) Median  $=$  Mean
- (c) Median  $<$  Mean

Source: Avery & Turner (2012)

*Table 4*

**Borrowing Distribution after Six Years, by Degree Type and First Institution**

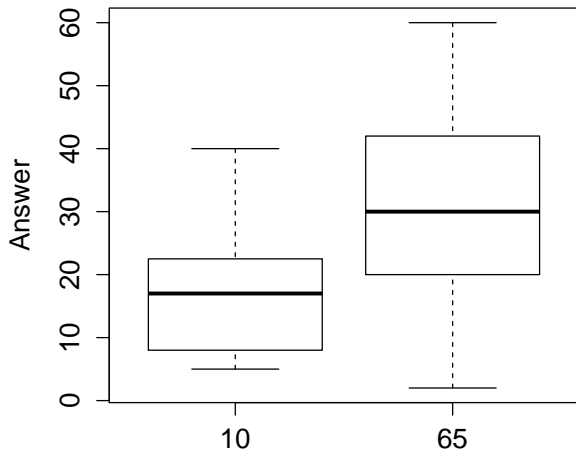
	<i>Type of institution of first enrollment</i>			
	<i>Public 4-year</i>	<i>Private nonprofit 4-year</i>	<i>Private for-profit 4-year</i>	<i>Public 2-year</i>
<i>All students beginning in 2004</i>				
% Borrowing	61%	68%	89%	41%
Percentile of borrowers				
10 <sup>th</sup>	\$0	\$0	\$0	\$0
25 <sup>th</sup>	\$0	\$0	\$6,376	\$0
50 <sup>th</sup>	\$6,000	\$11,500	\$13,961	\$0
75 <sup>th</sup>	\$19,000	\$24,750	\$28,863	\$6,625
90 <sup>th</sup>	\$30,000	\$40,000	\$45,000	\$18,000
<b>Mean</b>	<b>\$11,706</b>	<b>\$16,606</b>	<b>\$19,726</b>	<b>\$5,586</b>
<i>BA recipients</i>				
BA completion	61.5%	70.7%	14.8%	13%
% Borrowing	59%	66%	92%	69%
Percentile of borrowers				
10 <sup>th</sup>	\$0	\$0	\$12,000	\$0
25 <sup>th</sup>	\$0	\$0	\$30,000	\$0
50 <sup>th</sup>	\$7,500	\$15,500	\$45,000	\$11,971
75 <sup>th</sup>	\$20,000	\$27,000	\$50,000	\$23,265
90 <sup>th</sup>	\$32,405	\$45,000	\$100,000	\$40,000
<b>Mean</b>	<b>\$12,922</b>	<b>\$18,700</b>	<b>\$45,042</b>	<b>\$15,960</b>

*Source:* Authors' tabulations based on the Beginning Postsecondary Survey 2004:2009.

# Boxplots and the Five-Number Summary

Minimum < Q1 < Median < Q3 < Maximum

## Anchoring Experiment



# Measures of Variability/Spread

## Range

Maximum Observation - Minimum Observation

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## Standard Deviation

$$s = \sqrt{s^2}$$

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Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.



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Difference between largest and smallest observations. *Very* sensitive to outliers. Displayed in boxplot.

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Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

## Standard Deviation

$\sqrt{\text{Variance}}$ , but more convenient since **same units as data**

## Range

Difference between largest and smallest observations. *Very* sensitive to outliers. Displayed in boxplot.

## Interquartile Range

Range of middle 50% of the data. Insensitive to outliers, skewness. Displayed in boxplot.