

## Econ 103 – Quiz 5

Name: \_\_\_\_\_

**Instructions:** This is closed-book, closed-notes quiz. Please write your answers in the blanks provided. Each question is worth one point but no partial credit will be awarded. Non-programmable calculators are permitted.

1. True or false: a Type I error is when you fail to reject a null that is false

1. \_\_\_\_\_ **false** \_\_\_\_\_

2. Steve wants to test the null hypothesis that freshman students weigh on average 170 pounds against the two-sided alternative. Write down the null hypothesis and alternative hypothesis, where  $\mu$  is the true average weight in the population.

2. \_\_\_\_\_  $H_0 : \mu = 170, H_A : \mu \neq 170$  \_\_\_\_\_

3. Following on from the previous question, to test this hypothesis Steve gathered data on 9 students. We assume that the weights collected come from a normal distribution:  $X_1 \dots X_9 \sim iid N(\mu, \sigma^2)$ . He calculates the sample mean weight,  $\bar{X}$ . We assume that he knows the variance  $\sigma^2$ . Write down an expression for an appropriate test statistic if he wants to test the null hypothesis  $\mu = \mu_0$ .

3. \_\_\_\_\_  $T_n = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{9}}$  \_\_\_\_\_

4. Suppose you are testing  $\mu = \mu_0$  when  $X_1, \dots, X_n \sim N(\mu, \sigma^2)$ . Which of the following will be our rejection criterion if we are testing against the two sided alternative hypothesis  $\mu \neq \mu_0$  with a significance level of  $\alpha$ ?

1.  $\left| \frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} \right| > \text{qnorm}(1 - \alpha/2).$

2.  $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} > \text{qnorm}(\alpha).$

3.  $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} < \text{qnorm}(1 - \alpha).$

4.  $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} > \text{qnorm}(\alpha/2).$

4. \_\_\_\_\_ **1** \_\_\_\_\_

5. Which of the following is true about  $p$  values?

a. The  $p$  value gives the probability that the null hypothesis is true.

b. The  $p$  value is the probability under the null hypothesis of observing a test statistic at least as aberrant as the one actually obtained.

c. The  $p$  value gives us reliable results in the sense that, if we could repeat the experiment a great number of times, we would obtain a significant results on  $100p\%$  of those occasions.

d. The  $p$  value can be used to disprove the null hypothesis.

5. \_\_\_\_\_ **b.** \_\_\_\_\_

6. Suppose that we observe a sample proportion of  $\hat{p} = 0.6$  with a sample size of 100 and want to test  $H_0 : p = 0.5$  against the one-sided alternative that  $p < 0.5$  at the 5% significance level. Calculate the value of the test statistic, fully imposing the null (i.e. using the value of  $p$  from our null hypothesis wherever possible)?

6. \_\_\_\_\_  $SE(p) = \sqrt{\frac{p(1-p)}{n}}, T_n = (0.6 - 0.5)/(5/100) = 2$  \_\_\_\_\_

7. Following on from the previous question, should we reject the null hypothesis?

7. \_\_\_\_\_ **no** \_\_\_\_\_

8. Kevin and Sara poll a random sample of 100 Penn Undergraduates to find out the proportion who prefer Coke to Pepsi. Unbeknownst to them, the true proportion is exactly 65%. Using the *exact same* dataset, both Kevin and Sara carry out two-sided hypothesis tests of the null hypothesis  $H_0: p = 0.5$ . Whereas Sara uses a 5% significance level for her test, Kevin uses a 1% significance level. Whose test has more power?

A. Kevin's Test   B. Sara's Test   C. Both have Equal Power   D. Not Enough Information to Determine

8. \_\_\_\_\_ **Sara's Test** \_\_\_\_\_