Econ 103 – Quiz 4

Name: _____

Instructions: This is closed-book, closed-notes quiz. Please write your answers in the blanks provided. Each question is worth one point but no partial credit will be awarded. Non-programmable calculators are permitted.

1. Suppose you have $X_1, ... X_n \sim iid\, N(\mu_x, \sigma_x^2)$ independently of $Y_1, ... Y_m \sim iid\, N(\mu_y, \sigma_y^2)$.

   What is the sampling distribution of the difference of sample means?

   a. Normal distribution

   b. t-Distribution

   c. χ^2 distribution

   d. F-distribution

   1. _____ **Normal** _____

2. True or false? The central limit theorem says that sample means are approximately normally distributed if the sample size is large enough, even if the population is distributed chi-squared.

   2. _____ **True** _____

3. True or false? The central limit theorem says that sample means are approximately normally distributed only when the sample size is very small.

   3. _____ **False** _____

4. Suppose you have $W_1, ... W_{12} \sim iid\, N(2, \sigma_w^2)$ independently of $V_1, ... V_6 \sim iid\, N(1, \sigma_v^2)$.

   You don't know σ_w^2 and σ_v^2 . You can be sure that the random variable $\dfrac{\left(\bar{W}_{12} - \bar{V}_6 - (2-1)\right)}{\sqrt{\frac{S_w^2}{12} + \frac{S_v^2}{6}}}$ follows a:

   a. Normal distribution

   b. t-Distribution

   c. χ^2 distribution

   d. F-distribution

   4. _____ **t-distribution** _____

5. Suppose you have $W_1, ... W_{12} \sim iid\, N(3, 1)$ independently of $V_1, ... V_6 \sim iid\, N(2, 1)$.

   The random variable $\dfrac{\left(\bar{W}_{12} - \bar{V}_6 - (3-2)\right)}{\sqrt{\frac{1}{12} + \frac{1}{6}}}$ follows a:

   a. Standard Normal distribution

   b. Normal distribution $N(\mu = 1, \sigma^2 = 1)$

   c. χ^2 distribution

   d. F-distribution

   5. _____ **Standard Normal Distribution** _____

6. The results of a recent survey suggests that 25% of Americans are able to hold a conversation in a second language. For this survey 1,000 people were polled. Use the Central Limit Theorem to construct an approximate 95% confidence interval for the actual proportion of Americans that are able to hold a conversation in a second language.

6. _____ **(0.22;0.28)** _____

7. True or false: Suppose we have two individual samples $X_1, ..., X_n \sim iid$ with mean μ_X and variance σ_X^2 and $Y_1, ..., Y_m \sim iid$ with mean μ_Y and variance σ_Y^2, where $corr(X, Y) = 0.5$. Then a 68% confidence interval of the difference of population means is given by $(\bar{X}_n - \bar{Y}_m) \pm \sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}$

7. _____ **False** _____

8. True or false: Suppose we take the mean of a random sample. The 95% confidence interval for the population mean is roughly $\frac{1}{2}$ as wide as the 68% confidence interval

8. _____ **False** _____

Table 1: Fictional summary statistics for midterms this semester

	Midterm 1	Midterm 2	Difference
Sample size	100	100	100
Sample mean	110	100	10
Sample standard deviation	20	25	
Sample variance	400	625	
Sample correlation	0.5		

9. Calculate the variance of differences between midterm 1 and midterm 2 scores

9. $var(d) = var(m1) + var(m2) - 2sd(m1)sd(m2)corr(m1, m2) = 400 + 625 - 2 * 0.5 * 20 * 25 = 525$

10. Suppose we want to know whether midterm 2 was 'harder' than midterm 1. Using your answer to the previous question, and the numbers in the table, write down an expression for the 95% confidence interval for the difference between scores on the midterm. (Note - you can leave your answer in terms of a square root)

10. _____ $10 \pm 2\sqrt{525/100} \approx 10 \pm 2\sqrt{5.25} \Rightarrow (5.41, 14.58)$ _____