Econ 103 – Quiz 5

Name: _____

Instructions: This is closed-book, closed-notes quiz. Please write your answers in the blanks provided. Non-programmable calculators are permitted.

1. (2 points) Suppose you have $W_1, ...W_{12} \sim iid\, N(3,1)$ independently of $V_1, ...V_6 \sim iid\, N(2,1)$.

   The random variable $\frac{(\bar{W}_{12} - \bar{V}_6 - (3-2))}{\sqrt{\frac{1}{12}+\frac{1}{6}}}$ follows a:

   a. Standard Normal distribution

   b. Normal distribution $N(\mu = 1, \sigma^2 = 1)$

   c. χ^2 distribution

   d. F-distribution

   1. _____

2. (2 points) The results of a recent survey suggests that 25% of Americans are able to hold a conversation in a second language. For this survey 1,000 people were polled. Use the Central Limit Theorem to construct an approximate 95% confidence interval for the actual proportion of Americans that are able to hold a conversation in a second language.

   2. _____

3. (1 point) True or false: a Type I error is when you fail to reject a null that is false.

   3. _____

4. (2 points) Steve wants to test the null hypothesis that freshman students weigh on average 170 pounds against the two-sided alternative. Write down the null hypothesis and alternative hypothesis, where μ is the true average weight in the population.

   4. _____

5. (2 points) Following on from the previous question, to test this hypothesis Steve gathered data on 9 students. We assume that the weights collected come from a normal distribution: $X_1 ... X_9 \sim iid N(\mu, \sigma^2)$. He calculates the sample mean weight, $\bar{X}$. We assume that he knows the variance σ^2. Write down an expression for an appropriate test statistic if he wants to test the null hypothesis $\mu = \mu_0$.

   5. _____

6. (2 points) Suppose you are testing $\mu = \mu_0$ when $X_1, ...X_n \sim N(\mu, \sigma^2)$. Which of the following will be our rejection criterion if we are testing against the two sided alternative hypothesis $\mu \neq \mu_0$ with a significance level of α?

   1. $\left| \frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} \right| > \text{qnorm}(1 - \alpha/2)$.
   2. $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} > \text{qnorm}(\alpha)$.
   3. $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} < \text{qnorm}(1 - \alpha)$.
   4. $\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} > \text{qnorm}(\alpha/2)$.

   6. _____

7. (2 points) Which of the following is true about p values?

   a. p value gives the probability that the null hypothesis is true.

   b. p value is the probability under the null hypothesis of observing a test statistic at least as extreme as the one actually obtained.

   c. p value gives us reliable results in the sense that, if we could repeat the experiment a great number of times, we would obtain a significant results on $100p\%$ of those occasions.

   d. p value greater than the significance level leads to rejection of the null hypothesis.

   7. _____

8. (3 points) From a polling data, 45% of 130 Republicans correctly knew that John Roberts is the current Chief Justice whereas 40% of 120 Democrats knew it. We can model each sample as draws from i.i.d. Bernoulli distribution. Suppose we want to test whether the population parameters from the two samples are equal. Calculate the value of the test statistic fully imposing the null hypothesis.

   8. _____

9. (2 points) Suppose we want to conduct hypothesis testing against two-sided alternative with 5% significance level. Given the test statistic you computed above, express the p-value using R command.

   9. _____

10. (2 points) Following on from the previous question, should we reject the null hypothesis? Why?

    10. _____