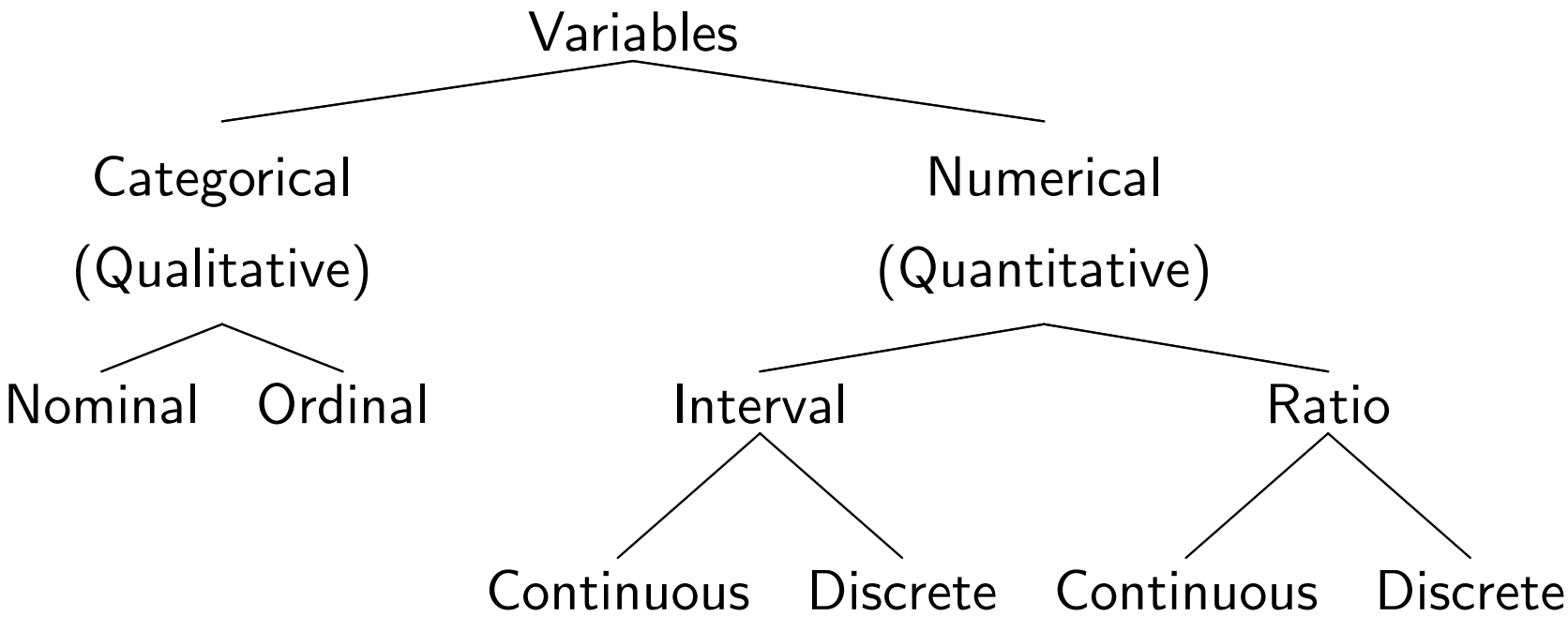
Economics 103 – Statistics for Economists

Minsu Chang

University of Pennsylvania

Lecture # 2

A Taxonomy of Variables



From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

- **Nominal** no order to the categories

From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

Nominal no order to the categories

Ordinal categories with natural order

From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

Nominal no order to the categories

Ordinal categories with natural order

Numerical

Quantitative, number meaningful

From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

Nominal no order to the categories

Ordinal categories with natural order

Numerical

Quantitative, number meaningful

Interval only differences meaningful, no natural zero

From Weakest to Strongest

Categorical

Qualitative, assigns each unit to category, number either meaningless or indicates order only

Nominal no order to the categories

Ordinal categories with natural order

Numerical

Quantitative, number meaningful

Interval only differences meaningful, no natural zero

Ratio differences and ratios meaningful, natural zero

And For Numerical Variables (interval or ratio)...

Discrete

Takes value from discrete set of numbers, typically count data

And For Numerical Variables (interval or ratio)...

Discrete

Takes value from discrete set of numbers, typically count data

Continuous

Value could be any real number within some range (even though *measurements* are made with finite precision)

What kind of variable is...

...Handspan?

(a) Nominal

(b) Ordinal

(c) Interval

(d) Ratio

What kind of variable is...

...Temperature?

(a) Nominal

(b) Ordinal

(c) Interval

(d) Ratio

What kind of variable is...

...Eye Color?

(a) Nominal

(b) Ordinal

(c) Interval

(d) Ratio

What kind of variable?

On course evaluations you can rate your professor as follows:

0 = Poor, 1 = Fair, 2 = Good, 3 = Very Good, 4 = Excellent.

What kind of data is your rating?

(a) Nominal

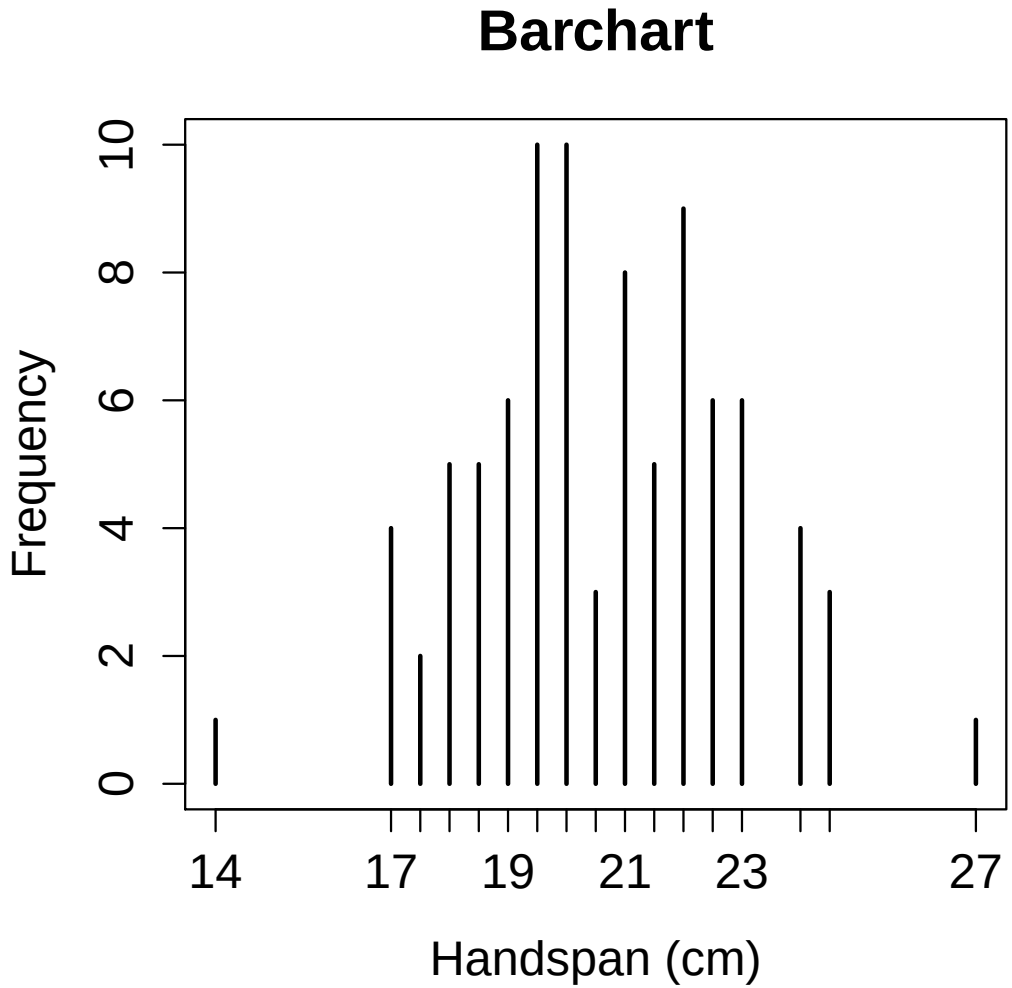
(b) Ordinal

(c) Interval

(d) Ratio

Handspan - Frequency and Relative Frequency

cm	Freq.	Rel. Freq.
14.0	1	0.01
17.0	4	0.05
17.5	2	0.02
18.0	5	0.06
18.5	5	0.06
19.0	6	0.07
19.5	10	0.11
20.0	10	0.11
20.5	3	0.03
21.0	8	0.09
21.5	5	0.06
22.0	9	0.10
22.5	6	0.07
23.0	6	0.07
24.0	4	0.05
24.5	3	0.03
27.0	1	0.01
	$n = 89$	1.00



Handspan - Summarize Barchart by "Smoothing"

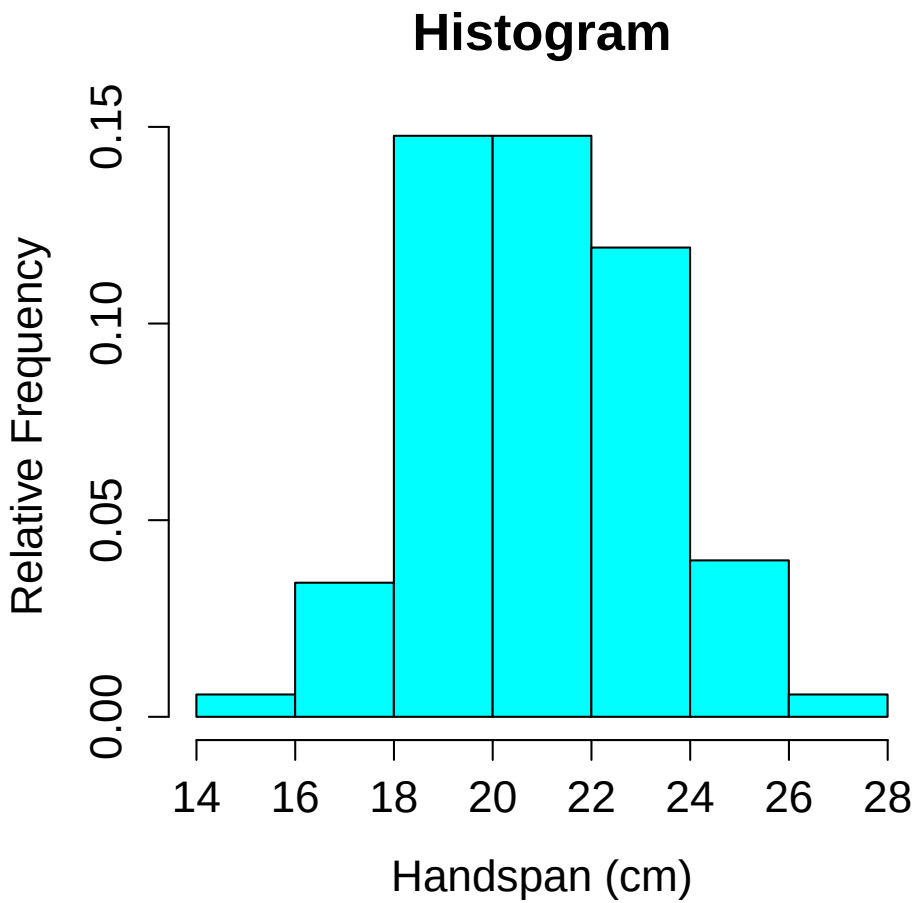
cm	Freq.	Rel. Freq.
14.0	1	0.01
17.0	4	0.05
17.5	2	0.02
18.0	5	0.06
18.5	5	0.06
19.0	6	0.07
19.5	10	0.11
20.0	10	0.11
20.5	3	0.03
21.0	8	0.09
21.5	5	0.06
22.0	9	0.10
22.5	6	0.07
23.0	6	0.07
24.0	4	0.05
24.5	3	0.03
27.0	1	0.01
	$n = 88$	1.00

Group data into non-overlapping bins of equal width:

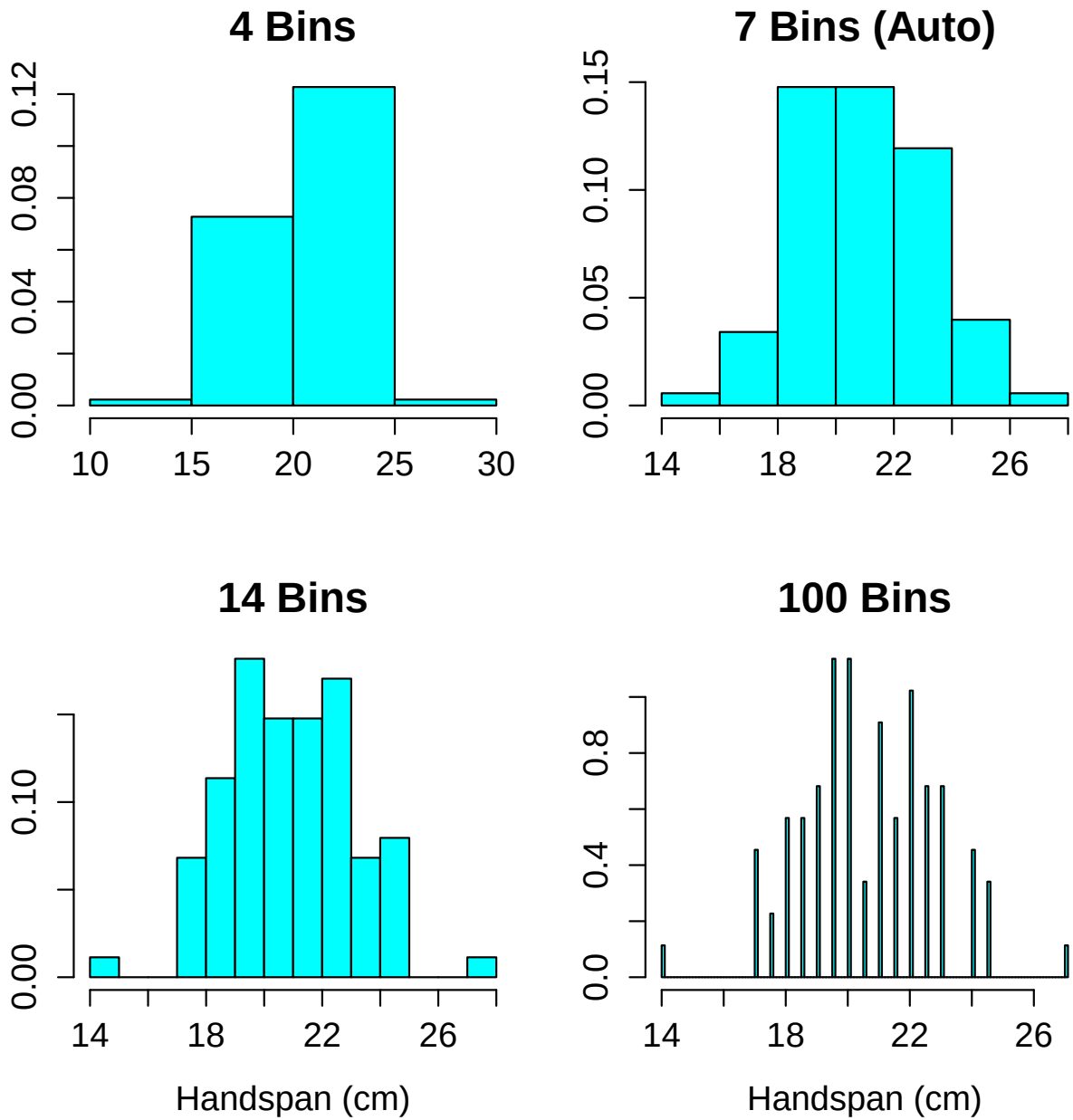
Bins	Freq.	Rel. Freq.
$[14, 16)$	1	0.01
$[16, 18)$	6	0.07
$[18, 20)$	26	0.30
$[20, 22)$	26	0.30
$[22, 24)$	21	0.24
$[24, 26)$	7	0.08
$[26, 28)$	1	0.01
	$n = 88$	1.00

Histogram – Density Estimate by Smoothing Barchart

Bins	Freq.	Rel. Freq.
$[14, 16)$	1	0.01
$[16, 18)$	6	0.07
$[18, 20)$	26	0.30
$[20, 22)$	26	0.30
$[22, 24)$	21	0.24
$[24, 26)$	7	0.08
$[26, 28)$	1	0.01
	$n = 88$	1.00



Number of Bins Controls Degree of Smoothing



Histograms are *Really* Important

Why Histogram?

Summarize numerical data, especially continuous (few repeats)

Histograms are *Really* Important

Why Histogram?

Summarize numerical data, especially continuous (few repeats)

Too Many Bins – Undersmoothing

No longer a summary (lose the shape of distribution)

Histograms are *Really* Important

Why Histogram?

Summarize numerical data, especially continuous (few repeats)

Too Many Bins – Undersmoothing

No longer a summary (lose the shape of distribution)

Too Few Bins – Oversmoothing

Miss important detail

Histograms are *Really* Important

Why Histogram?

Summarize numerical data, especially continuous (few repeats)

Too Many Bins – Undersmoothing

No longer a summary (lose the shape of distribution)

Too Few Bins – Oversmoothing

Miss important detail

Don't confuse with barchart!

Summary Statistic: Numerical Summary of Sample

1. Measures of Central Tendency
   - Mean
   - Median

Summary Statistic: Numerical Summary of Sample

1. Measures of Central Tendency
   - Mean
   - Median
2. Measures of Spread
   - Variance
   - Standard Deviation
   - Range
   - Interquartile Range (IQR)

Summary Statistic: Numerical Summary of Sample

1. Measures of Central Tendency
   - Mean
   - Median
2. Measures of Spread
   - Variance
   - Standard Deviation
   - Range
   - Interquartile Range (IQR)
3. Measures of Symmetry
   - Skewness

Summary Statistic: Numerical Summary of Sample

1. Measures of Central Tendency
   - Mean
   - Median
2. Measures of Spread
   - Variance
   - Standard Deviation
   - Range
   - Interquartile Range (IQR)
3. Measures of Symmetry
   - Skewness
4. Measures of relationship between variables
   - Covariance
   - Correlation
   - Regression

Questions to Ask Yourself about Each Summary Statistic

1. What does it measure?
2. What are its units compared to those of the data?
3. (How) do its units change if those of the data change?
4. What are the benefits and drawbacks of this statistic?

Some of the information regarding items 2 and 3 is on the homework rather than in the slides because working it out for yourself is a good way to check your understanding.

Measures of Central Tendency

Suppose we have a dataset with observations $x_1, x_2, \ldots, x_n$

Sample Mean

- $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
- Only for numeric data
- Works best when data are symmetric and there are no outliers

Measures of Central Tendency

Suppose we have a dataset with observations $x_1, x_2, \ldots, x_n$

Sample Mean

- $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
- Only for numeric data
- Works best when data are symmetric and there are no outliers

Sample Median

- Middle observation if n is odd, otherwise the mean of the two observations closest to the middle.
- Applicable to numerical or ordinal data
- Robust to outliers and skewness

What is an Outlier?

Outlier

A very unusual observation relative to the other observations in the dataset (i.e. very small or very big).

Mean is Sensitive to Outliers, Median Isn't

First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

Mean is Sensitive to Outliers, Median Isn't

First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

Second Dataset: 1 2 3 4 4990

Mean = 1000, Median = 3

Mean is Sensitive to Outliers, Median Isn't

First Dataset: 1 2 3 4 5

Mean = 3, Median = 3

Second Dataset: 1 2 3 4 4990

Mean = 1000, Median = 3

When Does the Median Change?

Ranks would have to change so that 3 is no longer in the middle.

Percentiles (aka Quantiles) – Generalization of Median

Approx. $P\%$ of the data are at or below the P^{th} percentile.

Percentiles (aka Quantiles)

P^{th} Percentile = Value in $(P/100) \cdot (n+1)^{th}$ Ordered Position

Percentiles (aka Quantiles) – Generalization of Median

Approx. $P\%$ of the data are at or below the P^{th} percentile.

Percentiles (aka Quantiles)

P^{th} Percentile = Value in $(P/100) \cdot (n+1)^{th}$ Ordered Position

Quartiles

Q1 = 25th Percentile

Q2 = Median (i.e. 50th Percentile)

Q3 = 75th Percentile

An Example: n = 12

60 63 65 67 70 72 75 75 80 82 84 85

Q_1 = value in the $0.25(n+1)^{th}$ ordered position

An Example: n = 12

60 63 65 67 70 72 75 75 80 82 84 85

$$
\begin{aligned}
Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\
&= \text{value in the } 3.25^{th} \text{ ordered position}
\end{aligned}
$$

An Example: n = 12

60 63 65 67 70 72 75 75 80 82 84 85

$$
\begin{aligned}
Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\
&= \text{value in the } 3.25^{th} \text{ ordered position} \\
&= 65 + 0.25 * (67 - 65)
\end{aligned}
$$

An Example: n = 12

60 63 65 67 70 72 75 75 80 82 84 85

$$
\begin{aligned}
Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\
&= \text{value in the } 3.25^{th} \text{ ordered position} \\
&= 65 + 0.25 * (67 - 65) \\
&= 65.5
\end{aligned}
$$

Student Debt

Guess the 90th percentile of student loan debt in the U.S. That is, guess the amount of money such that 10% college students graduate with *more* than this amount of debt and 90% graduate with less than or equal to this amount of debt.

Student Debt

Would you guess that the median amount of student loan debt in the U.S. is above, below, or equal to the mean amount?

(a) Median $>$ Mean

(b) Median $=$ Mean

(c) Median $<$ Mean

Source: Avery & Turner (2012)

Table 4

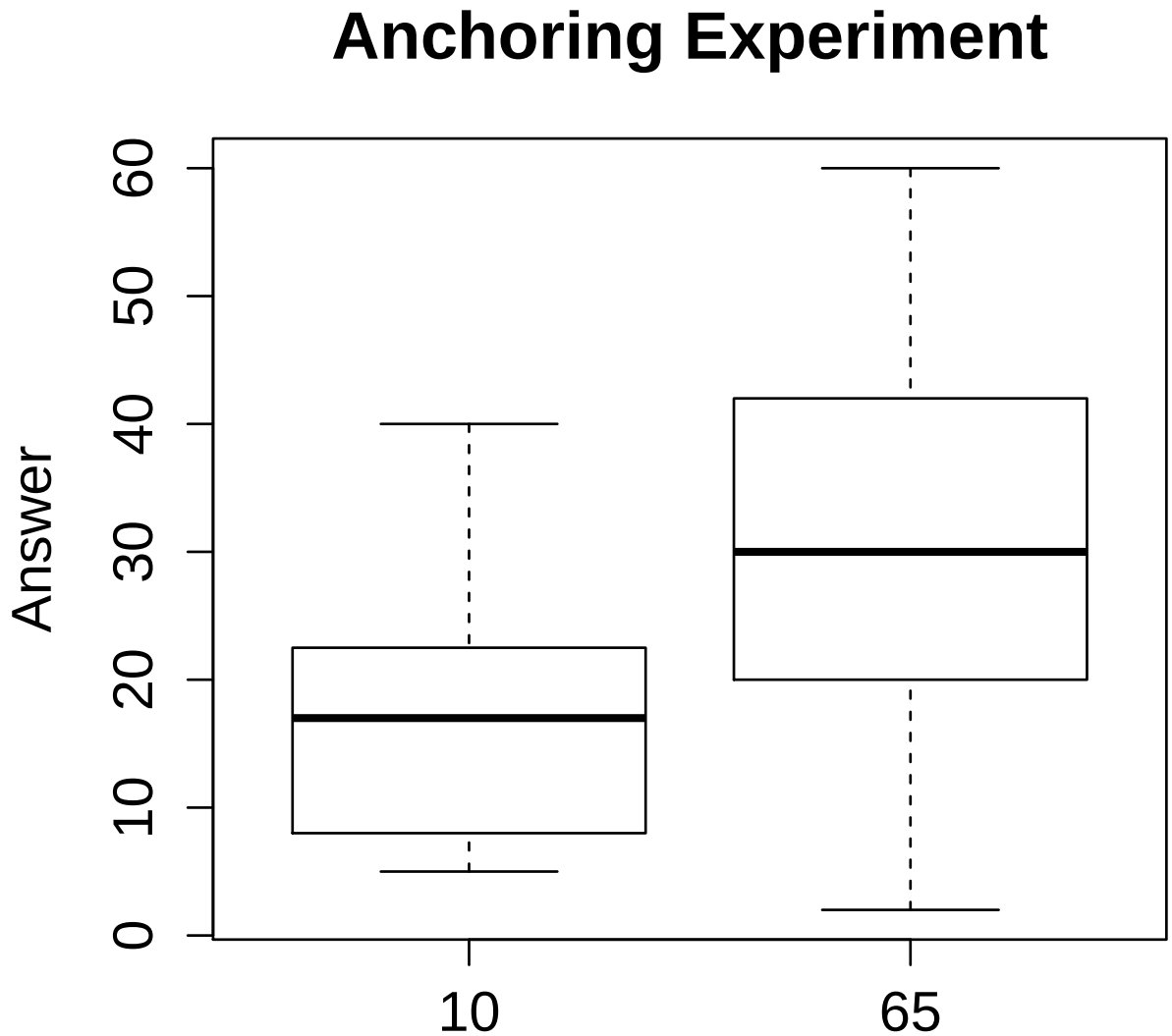
Borrowing Distribution after Six Years, by Degree Type and First Institution

	Type of institution of first enrollment			
	Public 4-year	*Private nonprofit 4-year*	*Private for-profit 4-year*	*Public 2-year*
All students beginning in 2004				
% Borrowing	61%	68%	89%	41%
Percentile of borrowers				
10th	$0	$0	$0	$0
25th	$0	$0	$6,376	$0
50th	$6,000	$11,500	$13,961	$0
75th	$19,000	$24,750	$28,863	$6,625
90th	$30,000	$40,000	$45,000	$18,000
Mean	**$11,706**	**$16,606**	**$19,726**	**$5,586**
BA recipients				
BA completion	61.5%	70.7%	14.8%	13%
% Borrowing	59%	66%	92%	69%
Percentile of borrowers				
10th	$0	$0	$12,000	$0
25th	$0	$0	$30,000	$0
50th	$7,500	$15,500	$45,000	$11,971
75th	$20,000	$27,000	$50,000	$23,265
90th	$32,405	$45,000	$100,000	$40,000
Mean	**$12,922**	**$18,700**	**$45,042**	**$15,960**

Source: Authors' tabulations based on the Beginning Postsecondary Survey 2004:2009.

Boxplots and the Five-Number Summary

Minimum $<$ Q1 $<$ Median $<$ Q3 $<$ Maximum



Measures of Variability/Spread

Range

Maximum Observation - Minimum Observation

Measures of Variability/Spread

Range

Maximum Observation - Minimum Observation

Interquartile Range (IQR)

IQR$= Q_3 - Q_1$

Measures of Variability/Spread

Range

Maximum Observation - Minimum Observation

Interquartile Range (IQR)

IQR$= Q_3 - Q_1$

Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Measures of Variability/Spread

Range

Maximum Observation - Minimum Observation

Interquartile Range (IQR)

IQR$= Q_3 - Q_1$

Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Standard Deviation

$$s = \sqrt{s^2}$$

Variance

Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

Variance

Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

Standard Deviation

$\sqrt{\text{Variance}}$, but more convenient since same units as data

Variance

Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

Standard Deviation

$\sqrt{\text{Variance}}$, but more convenient since same units as data

Range

Difference between largest and smallest observations. *Very* sensitive to outliers. Displayed in boxplot.

Variance

Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

Standard Deviation

$\sqrt{\text{Variance}}$, but more convenient since same units as data

Range

Difference between largest and smallest observations. *Very* sensitive to outliers. Displayed in boxplot.

Interquartile Range

Range of middle 50% of the data. Insensitive to outliers, skewness. Displayed in boxplot.