

PRACTICE MIDTERM II

ECON 103, STATISTICS FOR ECONOMISTS

Graphing calculators, notes, and textbooks are not permitted.

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

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Signature: _____

Student ID #: _____ Recitation #: _____

Instructions: Answer all questions in the space provided, continuing on the back of the page if you run out of space. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement above and to write your name and student ID number on *each page* in the space provided. Make sure that you have all pages of the exam before starting.

Warning: If you continue writing after we call time, even if this is only to fill in your name, twenty-five points will be deducted from your final score. In addition, two points will be deducted for each page on which you do not write your name and student ID.

1. Indicate whether each of the following statements is True or False. If False, provide a one sentence explanation. If True, no explanation is needed.

(a) The sample covariance between x and *itself* equals the sample variance of x .

(b) The sample variance of the z-scores always equals zero.

(c) If a variable has a positive skewness, then we would generally expect its mean to be *less than* its median.

(d) In a large population that is approximately bell-shaped, roughly 68% of the z-scores will be between -2 and 2.

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(e) For any events A and B , $P(A \cap B) = P(B \cap A)$.

(f) For any events A and B , $P(B) = P(A \cap B)/P(B|A)$

(g) A random variable is neither random, nor a variable.

(h) The expected value of a Bernoulli(p) random variable equals $1/2$.

(i) If $X \sim \text{Bernoulli}(p)$ then $E[X^2] = p$.

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(j) If X is a discrete RV with pmf $p(x)$, then $\sum_{\text{all } x} xp(x) = 1$.

2. Five friends want to lose weight so they sign up for a five-week program of fitness classes. At the end of the program, they note down how many weeks of classes they each attended, x , and the change in their weight in kilograms, y . The data appear in the table below. Since y denotes a *change* in weight, $y_i = -1$ means that person i lost one kilogram.

Note: *If you get an early part to this question incorrect and have to use the results of this part later on, we will take account of this in grading and assign partial credit accordingly. We have left space in the table for your calculations.*

x	y	
3	0	
3	-1	
4	0	
5	-2	
5	-2	

(a) Calculate \bar{x} and \bar{y} .

(b) Calculate the sample variances, s_x^2 and s_y^2 .

- (c) Calculate the sample covariance between x and y .

3. Use what you know about summation notation to prove the following equality:

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \left(\sum_{i=1}^n x_i y_i \right) - n\bar{x}\bar{y}$$

4. Suppose I flip a fair coin and roll a single fair die at the same time. Define the events

A = the coin comes up tails

B = the die shows a 3 *or* 5

C = the die shows an *odd* number

- (a) Calculate $P(B|C)$.

- (b) Calculate $P(A \cap B)$.

(c) Calculate $P(A \cup B)$.

5. This question asks you to prove some results about probability that we did not cover in class. Let A , B and C be three arbitrary events.

(a) Prove *Boole's Inequality*: $P(A \cup B) \leq P(A) + P(B)$.

(b) Prove *Bonferroni's Inequality*: $P(A \cap B) \geq P(A) + P(B) - 1$.

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6. New England Patriots quarterback Tom Brady has a lucky coin, which he flips five times before every game. The coin is fair, so it has a probability $1/2$ of coming up heads on any given flip.
- (a) What is the probability that Tom gets the same outcome for all 5 flips of his lucky coin? Express your answer as a fraction.
- (b) Tom is a superstitious person. If (and only if) he gets 5 heads or 5 tails he will cheat by deflating the balls used in the game. If Tom cheats, the Patriots win with probability $\frac{3}{4}$. But if Tom does not cheat, they win with probability $\frac{2}{5}$. Given that the Patriots won the game, what is the probability that Tom cheated?

7. This question asks you for the R commands needed to carry out tasks similar to those in the tutorials. The first two parts refer to the dataframe `jobsearch` that we created in R Tutorial 1. The entire dataframe is printed below.

##	location	salary	title	hours
## 1	New York	70000	Office Manager	50
## 2	Chicago	80000	Research Assistant	56
## 3	Boston	60000	Analyst	65
## 4	Boston	50000	Office Manager	40
## 5	New York	45000	Analyst	50

- (a) Create a new dataframe called 'highpay' by taking a subset of the 'jobsearch' dataframe to include only jobs which pay more than \$55,000.
- (b) Write code to calculate the number of observations in your new dataframe, `highpay`, and call this number `n`.

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The next four parts of this question concerns the dataframe `survey` from R Tutorial 2. This dataframe has observations of six variables. The variable names, and the first six observations are printed below:

##	sex	credits	eye.color	handedness	height	handspan
## 1	Male	5	Brown	1.0	67	20.0
## 2	Female	5	Brown	0.4	63	19.5
## 3	Female	NA	Brown	0.6	62	19.0
## 4	Female	5	Brown	0.6	65	19.5
## 5	Female	4	Brown	1.0	62	18.5
## 6	Female	NA	Brown	1.0	68	18.5

- (c) Write the command that prints the first six rows of `survey`.
- (d) Write the command to draw a scatter plot with handedness on the horizontal axis and height on the vertical axis.
- (e) Create a dataframe called `numerical` that contains only the numerical variables in our dataframe above.