# Practice Final II

Econ 103, Statistics for Economists

**Graphing calculators, notes, and textbooks are not permitted.**

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

Name: ______________________________

Student ID #: ______________________________

Signature: ______________________________

**Instructions:** Answer all questions in the space provided, continuing on the back of the page if you run out of space. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement above and to write your name and student ID number on *each page* in the space provided. Make sure that you have all pages of the exam before starting.

**Warning:** If you continue writing after we call time, even if this is only to fill in your name, twenty-five points will be deducted from your final score. In addition, two points will be deducted for each page on which you do not write your name and student ID.

1. Mark each statement as TRUE or FALSE. If FALSE provide a one sentence explanation.

(a) If $(2, 6)$ is a 95% CI for $\mu$, we do not reject $H_0\colon \mu = 1$ vs. $H_A\colon \mu \neq 1$ with $\alpha = 0.05$.

**Solution:** FALSE: we would reject $\mu = 1$ since 1 lies outside the CI.

(b) A Type I error is rejecting a false null hypothesis.

**Solution:** FALSE: it is rejecting a *true* null hypothesis.

(c) The smaller the p-value the stronger the evidence against $H_0$.

**Solution:** TRUE

(d) The power of a hypothesis test equals the probability of making a Type II error.

**Solution:** FALSE: it is one *minus* the probability of making a type II error.

(e) If $A$ and $B$ are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.

**Solution:** TRUE

(f) If $A$ and $B$ events such that $A$ implies $B$ then $P(A) \leq P(B)$.

**Solution:** TRUE

(g) If $X$ is a continuous RV with pdf $f(x)$ then $f(0)$ gives $P(X = 0)$.

**Solution:** FALSE: if $X$ is continuous $P(X = x) = 0$ for any $x$.

(h) If $X$ and $Y$ are two RVs, then $E[XY] = Cov(X, Y) + E[X]E[Y]$.

**Solution:** TRUE

(i) If $X$ and $Y$ are discrete RVs then $p_Y(y) = \sum_{\text{all } y} p_{XY}(x, y)$.

**Solution:** FALSE: the sum should be taken over all $x$.

2. For each of the following, provide R code to generate the specified result.

   (a) Suppose I have a dataframe called `gradebook` with a column called `midterm1`. Write down R code to display only those rows of `gradebook` for which the entry in `midterm1` is at least 80.

   **Solution:**

   ```
   subset(gradebook, midterm1 >= 80)
   ```

   (b) Write code to plot the CDF of a $\chi^2(5)$ RV from 0 to 10 over a grid of 1001 values.

   **Solution:**

   ```
   x <- seq(from = 0, to = 10, by = 0.01)
   y <- pchisq(x, df = 5)
   plot(x, y, type = ‘l’)
   ```

   (c) Write code to create a vector containing 20 simulated rolls of a fair, six-sided die.

   **Solution:** Various possibilities, the simplest of which is

   ```
   sample(1:6, size = 20, replace = TRUE)
   ```

   (d) Write an R function called `my.rt` to make one random draw from the $t(\nu)$ distribution. Your function should take one input argument, the degrees of freedom `nu`, and return the random draw. In your answer you may use any R functions you like *except* for `rt`.

   **Solution:**

   ```
   my.rt <- function(nu){
     normal.draw <- rnorm(1)
     chisq.draw <- rchisq(1, nu)
     return(normal.draw / sqrt(chisq.draw / nu))
   }
   ```

3. This question concerns a game played by rolling a fair, six-sided die with sides numbered 1–6. To play the game you roll the die once. Let $x$ denote the number on the side that shows face-up. If $x$ is even you win $x$ dollars but if $x$ is odd you win $2x$ dollars.

(a) Suppose you were to play this game an extremely large number of times. On average, how much would you win per play?

**Solution:** The expected winnings in this game are

$$\frac{1}{6}\left[(2+4+6)+2\times(1+3+5)\right]=30/6=5$$

so you will win, on average, 5 dollars per play in a long sequence of plays.

(b) Your winnings in one play of this game can be viewed as the realization of a discrete random variable $Z$. Calculate $Var(Z)$.

**Solution:** By the Shortcut Formula $Var(Z)=E[Z^2]-E[Z]^2$. We have

$$\begin{aligned} E[Z^2] &= \sum_{\text{all } z} z^2p(z)=\frac{1}{6}\left[(2^2+4^2+6^2)+(2\times 1)^2+(2\times 3)^2+(2\times 5)^2\right] \\ &= \frac{1}{6}\left[(4+16+36)+\left(2^2+6^2+10^2\right)\right]=\frac{1}{6}\left[56+(4+36+100)\right] \\ &= 196/6 \end{aligned}$$

Therefore $Var(Z)=196/6-25=23/3\approx 7.67$

(c) Suppose you play this game 69 times consecutively. Based on the approximation provided by the CLT, roughly what is the probability that your average winnings will be less than \$4.34 per play?

**Solution:** Since individual throws of a fair die are independent and identically distributed we have $Z_1,\dots Z_{69}\sim$ iid with mean 5 and variance 23/3 by parts (a) and (b). The question asks us to approximate the value of $P(\bar{Z}<4.34)$ where $\bar{Z}=(Z_1+\cdots+Z_{69})/69$. By the CLT $\bar{Z}$ is approximately normally distributed with mean 5 and standard deviation $\sqrt{(23/3)/69}=1/3$. We have

$$P(\bar{Z}<4.34)=P\left(\frac{\bar{Z}-5}{1/3}<-1.98\right)=\texttt{pnorm}(-1.98)\approx 0.025$$

4. Professor Quack has developed a diet plan where you are allowed to eat anything you want as long you wash down every meal with a spoonful of pickle juice. He claims that if you follow this diet you will lose, on average, 3kg over 4 weeks. Matt decides to carry out an experiment to test this claim. He recruits a random sample of 25 subjects and puts all of them on the "pickle juice diet." Let $X_i$ be person $i$'s weight (in kg) before beginning the diet and $Y_i$ be person $i$'s weight (in kg) after four weeks on the diet. The summary statistics from Matt's experiment are as follows:

| | $X$ | $Y$ |
|---|---|---|
| Sample Mean | 83 | 82 |
| Sample S.D. | 6 | 10 |
| Correlation | 0.3 | |

Throughout this question please work with the approximation provided by the CLT. For simplicity you may treat this approximation as though it were exact.

(a) Is this an independent samples or matched pairs problem? Explain in one sentence.

**Solution:** Matched pairs: we're interested in how much weight people lose over the course of the diet which involves a before-and-after comparison for each subject.

(b) Let $L_i$ denote the (positive) amount of weight that subject $i$ lost over the course Matt's experiment: $L_i = X_i - Y_i$. Calculate $\bar{L}$, the sample mean of the $L_i$.

**Solution:** $\bar{L} = \bar{X} - \bar{Y} = 83 - 82 = 1$

(c) Continuing from the preceding part calculate $S_L^2$, the sample variance of the $L_i$.

**Solution:** $S_L^2 = S_X^2 + S_Y^2 - 2r_{XY}S_XS_Y = 6^2 + 10^2 - 2 \times 0.3 \times 6 \times 10 = 100$

Suppose Matt decides to test the null hypothesis that population mean weight loss for people on the "pickle juice" diet is zero against the one-sided alternative of positive weight loss at the 2.5% significance level.

(d) What is the critical value for Matt's test?

**Solution:** `qnorm(0.975)`$\approx 2$

(e) What is the value of Matt's test statistic?

**Solution:**

$$\frac{\bar{L}}{S_L/\sqrt{n}} = \frac{1}{10/\sqrt{25}} = 0.5$$

(f) Does Matt reject the null hypothesis at his specified significance level?

**Solution:** No: his test statistic is smaller than his critical value.

(g) Write down the R command Matt would use to calculate the p-value for his test.

**Solution:** `1 - pnorm(0.5)`

5. An R dataframe called `houses` contains the sale price and characteristics of a random sample of 128 houses sold in Kansas City in a single year. The first few rows of the dataframe are as follows:

```
> head(houses)
  neighborhood offers sqft brick bedrooms bathrooms  price
1            B      3 1990    No        2         2 105600
2            A      3 1900    No        3         3 102500
3            A      3 1860    No        2         2  91100
4            A      2 1780    No        3         2 114600
5            C      3 2150   Yes        4         3 160600
6            C      2 2110    No        3         2 142600
```

In this question we will only work with the columns `sqft`, `brick` and `price`: `brick` is a categorical variable that indicates whether or not the house in question is made of brick, `sqft` gives the size of the house in square feet, and `price` is the sale price of the house in US dollars. The final two pages of this exam contain regression results and plots that relate to this question. You may want to tear them out for easy reference when answering the following. You may assume throughout this question that there are no missing values.

Parts (a) and (b) refer to the *first* of the two plots on the final page of this exam.

(a) Give R code to create the plot, including axis labels and title.

**Solution:**

```
boxplot(price ˜ brick, data = houses, xlab = ‘‘Brick House?’’,
                       ylab = ‘‘House Price ($)’’,
                       main = ‘‘House Prices in Kansas City’’)
```

(b) Explain what this plot shows using bullet points with no more than three bullets.

**Solution:** This is a boxplot. It compares the minimum, 25th percentile, median, 75th percentile and maximum of the sale prices of a sample of brick houses in Kansas city to those of non-brick houses. Brick houses cost more.

Suppose I wanted to test the null hypothesis that the average price for brick and non-brick houses in Kansas City are the same against the two-sided alternative.

(c) Which set of regression results should I consult?

**Solution:** Regression #2

(d) On average, how much more does a brick house cost in Kansas City?

**Solution:** About 26,000 more.

(e) Approximately what is the p-value of my test?

**Solution:** The test statistic is roughly $25.8/4.5 \approx 5.7$ so the p-value is less than 0.001.

(f) Is there convincing evidence that brick houses cost more? Explain in one sentence.

**Solution:** Yes: we would resoundingly reject the null hypothesis here even if we chose a tiny value for $\alpha$.

Suppose I wanted to use square-footage *alone* to predict house prices in Kansas City based on a simple linear regression model.

(g) Which set of regression results should I consult?

**Solution:** Regression #4

(h) What is the sample correlation between house prices and square-footage?

**Solution:** $\sqrt{0.31} \approx 0.56$

(i) Based on the regression results, how much more would we predict that a house would cost if it were 100 square feet larger?

**Solution:** The regression slope is about 70 dollars per square foot, so we would predict that a house that is 100 square feet larger would cost 7000 dollars more.

(j) Construct an approximate 95% confidence interval for the regression slope, including the appropriate units.

**Solution:** Approximately $70 \pm 19$ or $(51, 89)$ dollars per square foot.

Now suppose I wanted to use both brick and sqft to predict house prices. There are two ways I could do this: by allowing *only* a different intercept for brick houses or by allowing *both* a different intercept and and a different slope.

(k) Suppose I only allow a different intercept, *not* a different slope. Based on the appropriate set of regression results, how much larger would a non-brick house have to be for us to predict it to have the same sale price as a brick house?

**Solution:** From Regression #1 a brick house commands a premium of around 23,450 dollars while the predicted increase in price per additional square foot is about 66 dollars. Dividing, the non-brick house would have to be about 355 square feet larger to command the same price premium.

(l) Do the regression results provide convincing evidence that brick houses command a higher premium *per square foot* than non-brick houses? Explain briefly in bullet points using no more than two bullets.

**Solution:** From Regression #3, the slope for brick houses is estimated to be about 25 dollars/sqft greater than for non-brick houses. The approximate 95% CI, however, is about $25 \pm 37$ dollars per square foot which comfortably includes zero: the evidence is suggestive but not particularly convincing.

### Regression #1

```
lm(formula = price ~ brick + sqft, data = houses)
            coef.est coef.se
(Intercept) -9444.29 16577.13
brickYes    23445.10  3709.81
sqft           66.06     8.27
---
n = 128, k = 3
residual sd = 19644.14, R-Squared = 0.47
```

### Regression #2

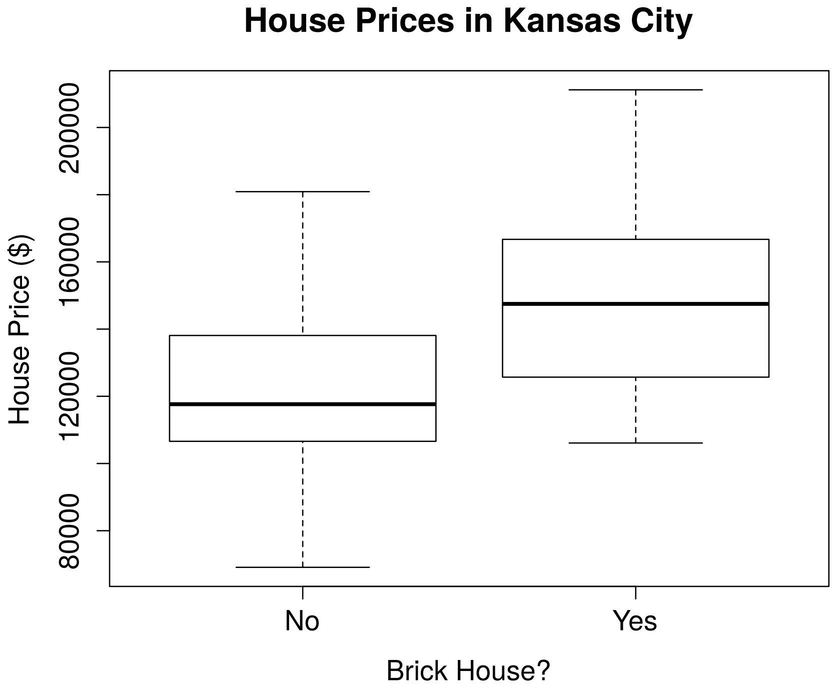
```
lm(formula = price ~ brick, data = houses)
            coef.est  coef.se
(Intercept) 121958.14   2593.50
brickYes     25810.91   4527.59
---
n = 128, k = 2
residual sd = 24051.17, R-Squared = 0.21
```

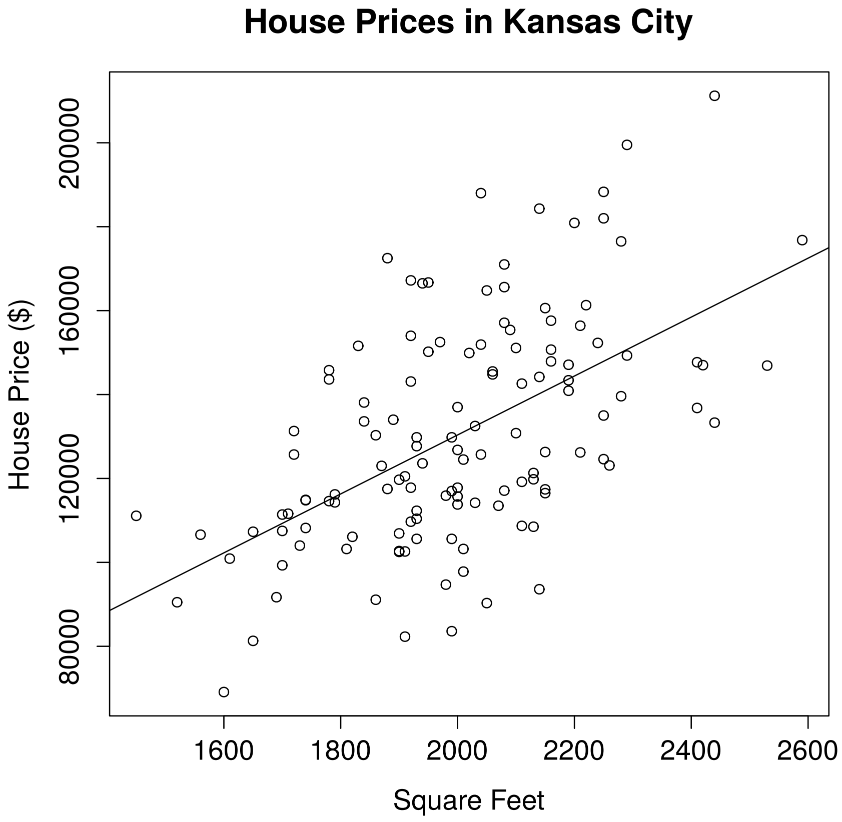
### Regression #3

```
lm(formula = price ~ brick + sqft + brick:sqft, data = houses)
              coef.est  coef.se
(Intercept)     4448.23  19396.56
brickYes      -27193.38  37234.31
sqft              59.07      9.69
brickYes:sqft     25.13     18.39
---
n = 128, k = 4
residual sd = 19576.29, R-Squared = 0.48
```

### Regression #4

```
lm(formula = price ~ sqft, data = houses)
            coef.est  coef.se
(Intercept) -10091.13  18966.10
sqft            70.23      9.43
---
n = 128, k = 2
residual sd = 22475.53, R-Squared = 0.31
```





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