

Name: \_\_\_\_\_

Score: \_\_\_\_\_ /50

**Instructions:** You will have 25 minutes to complete this test. As this is merely a diagnostic test, your score will not be factored into your course grade.

1. Evaluate each of the following expressions.

(a) 1 point  $5!$

(a) 120

(b) 2 points  $\frac{100!}{98!}$

(b) 9900

(c) 2 points  $\binom{5}{3}$

(c) 10

2. True or False?

(a) 2 points  $\sum_{i=1}^n (x_i/n) = (\sum_{i=1}^n x_i)/n$

(a) TRUE

(b) 2 points  $\sum_{k=1}^n x_k z_k = z_k \sum_{k=1}^n x_k$

(b) FALSE

(c) 2 points  $\sum_{k=1}^m x_k y_k = (\sum_{k=1}^m x_k)(\sum_{k=1}^m y_k)$

(c) FALSE

(d) 2 points  $(\sum_{i=1}^n x_i)(\sum_{j=1}^m y_j) = \sum_{i=1}^n \sum_{j=1}^m x_i y_j$

(d) TRUE

(e) 2 points  $(\sum_{i=1}^n x_i)/(\sum_{i=1}^n z_i) = \sum_{i=1}^n (x_i/z_i)$

(e) FALSE

3. 5 points Suppose that  $-c < (a - x)/b < c$  where  $b > 0$ . Find a lower bound  $L$  and an upper bound  $U$  such that  $L < x < U$ .

**Solution:** Rearranging,

$$-bc - a < -x < bc - a$$

and multiplying through by  $-1$ ,

$$a - bc < x < a + bc$$

4. Let  $f(a) = (x - a)^2 + (y - a)^2$  where  $x$  and  $y$  are constants.

- (a) 5 points What value of  $a$  minimizes  $f(a)$ ?

**Solution:** The first order condition is  $-2(x - a) - 2(y - a) = 0$ . Solving for  $a$ , we find  $a^* = (x + y)/2$ .

- (b) 5 points How can you be sure this is a minimum rather than a maximum?

**Solution:** The second derivative is 4, which is positive.

5. Imagine a rectangle whose base is the  $x$ -axis and whose bottom left corner is the origin  $(0, 0)$ . Suppose its area is 1 and the length of its base is 3. The area of this rectangle can be expressed as the definite integral of a function  $f(x)$  from  $a$  to  $b$ .

- (a) 3 points What is the function  $f$ ?

**Solution:**  $f(x) = 1/3$

- (b) 2 points What are the limits of integration  $(a, b)$ ?

**Solution:**  $a = 0, b = 3$

6. 5 points Solve  $\int_0^2 cx^2 dx = 1$  for  $c$ . Your answer should not involve  $x$ .

**Solution:** The integral evaluates to  $\frac{cx^3}{3} \Big|_0^2 = 8c/3$ . Setting this equal to 1 and solving,  $c = 3/8$ .

7. 5 points Let  $z = \sum_{i=1}^n (y_i - a - bx_i)^2$ . Evaluate  $\partial z / \partial b$ .

**Solution:**  $-2 \sum_{i=1}^n (y_i - a - bx_i)x_i$

8. 5 points Let  $F(x) = \int_a^x f(t) dt$ . Express the derivative  $F'(x)$  in terms of  $f$ .

**Solution:** By the Fundamental Theorem of Calculus,  $F'(x) = f(x)$ .