Bridging Micro and Macro Production Functions: The Fiscal Multiplier of Infrastructure Investment*

Minsu Chang† Hanbaek Lee‡

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Abstract

This paper investigates the fiscal multiplier of infrastructure investment using an estimated heterogeneous-firm general equilibrium model. We show theoretically and quantitatively that the firm-level non-rivalry in infrastructure usage drives a significant discrepancy between the estimated input elasticities at the firm level and the state level. In addition, we link our firm-level production function to a canonical production function in a representative-agent framework (Baxter and King, 1993). The quantitative findings indicate a fiscal multiplier of approximately 1.09 over a 2-year horizon, suggesting a moderate net economic benefit from infrastructure investment, which is significantly higher than the representative-agent model prediction. This is due to the low sensitivity of the firm-level investment to the general equilibrium effect, leading to a dampened crowding out.

Keywords: Infrastructure investment, fiscal multiplier, heterogeneous-agent model, non-rivalry.

JEL codes: E23, E60, H54.

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†Georgetown University. Email: minsu.chang@georgetown.edu

‡University of Tokyo. Email: hanbaeklee1@gmail.com

1 Introduction

Infrastructure spending and its effect on output and welfare have become one of the central issues in recent policy discussions. Particularly noteworthy is the Infrastructure Investment and Jobs Act, which allocates over $1.2 trillion for transportation and various physical infrastructure projects over a decade. Fiscal multipliers serve as fundamental tools to gauge the economic advantages of government expenditure. Our research delves into the fiscal multiplier associated with infrastructure investments, incorporating an innovative analysis of investment decisions made by individual firms — a dimension unexplored in existing literature.

In contrast to the commonly employed representative firm model in the literature, our analysis of the infrastructure fiscal multiplier through a heterogeneous firm model yields two critical findings. Firstly, our heterogeneous agent model yields a higher output multiplier in response to an infrastructure spending shock compared to the representative agent model. This discrepancy arises from the fact that the average adjustment burden for heterogeneous firms, subject to convex adjustment costs, exceeds that of the representative firm. Consequently, our model captures a more realistic sensitivity in firm-level investment, mitigating negative investment responses induced by the crowding out and thereby increasing the overall output multiplier.

Secondly, we observe substantial variations in fiscal multipliers across various values of the elasticity of substitution between private and public capital in firms' production. In contrast, the representative agent model exhibits only modest differences in fiscal multipliers concerning this elasticity parameter. The heightened variability in fiscal multipliers within the heterogeneous agent model stems from the impact of the non-rivalry of public capital, which benefits each firm's production and is absent in the representative firm model. This highlights the importance of precisely estimating the elasticity parameter within a heterogeneous firm model, a contribution our paper seeks to make.

In this paper, we present a novel theoretical and quantitative analysis demonstrating that the inclusion of non-rivalry of public capital in the CES (Constant Elasticity of Substitution) firm-level production function leads to a notable disparity between the elasticities of substitution estimated at the firm-level and the state-level when inputs are aggreagted.[1] In addition, we demonstrate that the inclusion of non-rivalry within the firm-level CES production function, encompassing substitutable inputs, implies increasing returns to scale for the aggregate Cobb-Douglas production function. To the best of our knowledge, our paper is the first one to bridge the firm-level and state-level input elasticities and micro-found the increasing returns to scale in the aggregate production function of Baxter and King (1993).

The key ingredient of our baseline model is the firm-level CES production function that utilizes private capital, public capital, and labor input factors. Public capital enters firms' production function in a non-rivalrous manner as in Glomm and Ravikumar (1994).[2] Subject to idiosyncratic productivity shocks, firms make lumpy investment decisions with both fixed and convex adjustment costs (Cooper and Haltiwanger, 2006; Winberry, 2021). Using revenue financed from household income tax and corporate tax, the government spends through infrastructure investment, lump-sum subsidy, and public employment. The infrastructure evolves with an exogenous law of motion subject to convex adjustment costs similar to private investment.

To connect with the available state-level micro data, our model incorporates two regions distinguished by their infrastructure levels: one with poor infrastructure and another with good infrastructure.[3] Motivated by the cross-state variations

[1] We aggregate the firm-level equilibrium allocations up to the state level and compare the elasticities. However, the theoretical implications of this aggregation are not limited to a particular level of aggregation.

[2] In contrast to Glomm and Ravikumar (1994), we incorporate firm-level heterogeneity in the model.

[3] In the model, these two states are also assumed to feature different state-specific TFP levels. These heterogeneous TFP levels are also estimated.

in the infrastructure spending data that has stayed almost invariant over the sample period, we assume the allocation of expenditures between these regions is exogenously given. In our model, the elasticity of substitution between private and public capital in the firm's production function serves as a critical parameter. If private and public capital exhibit a stronger degree of complementarity, it is anticipated that the region with high infrastructure will possess a greater proportion of private capital stock. Unlike the commonly employed approach of utilizing time-series variations, our study offers a novel approach by utilizing cross-sectional variations at the state level to identify and estimate the input elasticity parameter at the firm level.

Estimating a general equilibrium model with heterogeneous agents, such as our model, is widely recognized as computationally challenging due to the need to solve for market clearing prices for every potential value of the model parameters. However, we introduce a novel extension to an existing estimation method that significantly reduces computational costs. Our method is closely related to estimation method to match the model-simulated moments to the data moments. To handle the general equilibrium, we extend this method by including market clearing conditions as additional moments. In this paper, we employ the multi-block Metropolis Hastings algorithm, which involves dividing the parameter space into two blocks: one for the price block and another for the model parameters. From running this algorithm, we generate draws that bring the market clearing conditions closer to zero, while ensuring a closer fit to the empirical moments.

Our estimates indicate that the elasticity of substitution is around 1.19, suggesting gross substitutability between private and public capital inputs. To validate our model, we compute the elasticity at the state level from our model and compare it to the empirical elasticity derived from U.S. state-level data. In the model, the state-level elasticity is computed by aggregating firms' behaviors in two regions and estimating the state-level production functions. Our estimation yields a state-level elasticity of 0.48. In comparison, by estimating the state-level production function following An et al. (2019), we obtain an empirical counterpart of 0.45. Our findings suggest that public and private capital inputs exhibit gross complementarity at the state level, while demonstrating substitutability at the firm level. This observation aligns with our theoretical result, where we establish that the nature of substitution can change as the micro-level (firm-level) input elasticity is aggregated to the state-level counterpart.

Given our estimated model, we conduct the quantitative analysis to compute fiscal multipliers with one-time unexpected infrastructure spending shock whose magnitude is 1% of steady-state GDP value. We assume that the fiscal policy shock is financed by a lump-sum tax on impact. The main focus is on the national-level fiscal output multiplier that is obtained after aggregating the micro-level allocations. An increase in the public capital leads to a boost in output. However, the fiscal policy shock causes an increase in the interest rate due to lump-sum financing which initially reduces consumption. Consequently, these general equilibrium effects result in crowding out of private investment on impact. Accounting for these opposing forces, the short-run aggregate fiscal multiplier over a two-year period is estimated to be 1.09, while the short-run multiplier in the partial equilibrium is estimated to be 1.86. This discrepancy is mainly due to the crowding out of the private investments induced by the general equilibrium effect. In the representative-agent model, the private investment is more responsive to the general equilibrium effect than the baseline, resulting in the fiscal multiplier of 0.99.

The fiscal spending shock leads to a substantially heterogeneous effect across the states. According to our baseline model, per \$1 spending shock, the Poor states receives only \$0.016 output gains, while the Good states receive \$1.072 out of total \$1.088 gains. Then, we consider a counterfactual scenario where the \$1 spending shock is equally spent on the Poor and Good states. In this scenario, the inequality is mitigated substantially (\$0.062 vs. \$0.810) at the cost of total output multiplier

(1.088 $\rightarrow$ 0.873). The more money is spent on the Poor state, the less efficiently the resources are utilized in the economy leading to a lowered output multiplier. This result sharply shows the trade off between the efficiency and the equality the government faces in their spending.

Furthermore, our analysis reveals that varying micro-level elasticities of substitution yields significantly different fiscal multipliers. A lower elasticity corresponds to a larger fiscal multiplier, as the private investment is crowded out less. This emphasizes the importance of sharply estimating the input elasticity to quantify fiscal multipliers while considering firms' investment. In the Appendix, we also report that the fiscal multipliers vary with the inclusion of time-to-build assumption.[4] Consistent with Ramey (2020), we find that the aggregate fiscal multiplier decreases when compared to scenarios without the extended time-to-build assumption.[5]

Our paper contributes to several strands of existing literature. First, it is closely connected to the literature on government spending multipliers (Ramey and Zubairy, 2018; Chodorow-Reich, 2019; Hagedorn et al., 2019; Auerbach et al., 2020; Ramey, 2020; Hasna, 2021). Our focus is specifically on quantifying the multipliers associated with infrastructure spending, which represents a distinct category of public investment. In empirical research, there have been attempts to estimate the output elasticity of the public investment (An et al., 2019; Espinoza et al., 2020; Ramey, 2020, An et al., 2022). Alongside empirical analysis using the data on the American Recovery and Reinvestment Act (ARRA), Ramey (2020) analyzes the impacts of government investment using a stylized neoclassical and New Keynesian mod-

[4] Suárez Serrato and Zidar (2016) identify local incidence of corporate taxation using a spatial model where firms are heterogeneous in productivity and imperfectly mobile. If we had firms' location choice in our model, time-to-build assumption would generate imperfectly mobile firms as firms cannot make decisions that perfectly insure themselves against idiosyncratic shocks.

[5] The time-to-build assumption impacts fiscal multipliers through two key channels. First, there is a news effect where individuals adjust their behaviors as they expect a future increase in the infrastructure. Second, there is a general equilibrium effect endogenously stemming from the news effect. More details are available in Appendix H.

els. Our contribution lies in quantifying the infrastructure spending multipliers based on a heterogeneous firm model, incorporating firm-level investment, which has not been addressed in the aforementioned papers. Also, our theoretical result bridging the firm-level and the macro-level production function provides a micro-foundation for the widely used macro-level production function in this literature (Baxter and King, 1993).

Second, we contribute to the literature that bridges the gap between aggregate estimates and micro-level estimates using a structural model. Similar to Nakamura and Steinsson (2018) and Oberfield and Raval (2021), we estimate the elasticity of substitution between private and public capital stocks at the firm level using cross-state variations. We obtain the firm-level elasticity of substitution of 1.19. Then we aggregate the capital stock within states to measure the state-level elasticity of substitution based on our model, which yields an estimate of 0.48. This estimate closely aligns with the empirical estimate of 0.45 obtained from the data. Our results suggest that private capital and public capital are gross substitutes at the firm level, while they act as gross complements at the state level. We provide theoretical support for this relations by demonstrating the non-rivalry effect of infrastructure in the firm-level production function.

Lastly, our paper is related to the literature that studies firm-level investments. This literature has empirically and theoretically investigated the firm-level lumpy investment patterns and their macroeconomic implications (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006; Abel and Eberly, 2002; Khan and Thomas, 2008; Winberry, 2021). Based on the literature, we incorporate the convex and fixed adjustment cost for the firm-level capital adjustment and estimate the cost parameters to capture the observed investment dynamics at the firm level. Our goal is to establish a micro-level foundation for analyzing the fiscal multiplier of infrastructure spending, specifically taking into account firm-level investment and incorporating the non-rivalrous nature of the public capital stock. According to the estimated model, the firm-level heterogeneity under the capital adjustment friction leads to a significantly different fiscal multiplier from the representative-firm model.

The rest of this paper proceeds as follows. Section 2 presents a theory showing that the nature of substitution between private and public capital flips with the aggregation from firm-level to state-level. Section 3 presents the model. Section 4 presents the estimation results and validates the model using the state-level data. Section 5 presents a comprehensive quantitative analysis on the infrastructure spending multipliers and compare them with the results in the existing literature. Section 6 concludes.

2 A simple theory on micro and macro production functions

In this section, we theoretically demonstrate that non-rivalry in public capital usage (such as infrastructure) at the firm level leads to a noteworthy disparity between the estimated input elasticities with micro and macro production functions. Specifically, we will use the terms "micro" to denote firm-level and "macro (or aggregate)" to refer to state-level unless otherwise indicated. It is essential to note that the theoretical implications presented in this section extend beyond a specific level of aggregation, transcending the state-level focus addressed in this paper.

Consider a CES production function $F(K, N, L; \lambda, z)$ with constant returns to scale (CRS):

$$F(K, N, L; \lambda, z) = z(\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}\alpha} L^{1-\alpha}$$

where λ is the elasticity of substitution between private and public capital; K is the private capital input; N is the public capital input, L is the labor input; z is the productivity level. $\theta \in (0,1)$ is the weight parameter between the private and public capital. Then, we consider a static labor demand problem:

$$\max_{L} \; F(K,N,L;\lambda,z) - wL$$

Using the solution of this problem $L^* = z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$, we can rewrite the production function with the implicit labor demand:

$$F(K,N,L^*;\lambda,z) = f(K,N;\lambda,z) := z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \quad (1)$$

Then, we consider estimation of the elasticity of substitution at the firm level and at the state level using the CES production function as in equation (1). Suppose we use a dataset that contains firm-level observations (k_1, k_2, y_1, y_2, N), where the subscript $i \in \{1,2\}$ represents two different firms in the same state.[6] It is important to note that the state-level capital stock N is shared among all firms in the same state. In the firm-level estimation, we estimate the firm-level elasticity and the productivity (z, λ) that satisfy

$$f(k_1,N;\lambda,z) = y_1$$
$$f(k_2,N;\lambda,1) = y_2$$

where the second firm's productivity is normalized to be unity.

In the state-level estimation, we estimate the state-level elasticity ξ that satisfies

$$f(k_1+k_2,N;\xi,1) = y_1 + y_2.$$

where the state-level productivity is normalized to be unity.

[6] Propositions in this section can be generalized to a continuum of firms indexed by $i \in [0,1]$. These results are available in Appendix L.

We show that, due to the non-rivalrous nature of public capital, firm-level estimate λ and state-level estimate ξ can be starkly different. Under mild conditions, to be formally outlined later, private and public capital are gross substitutes at the firm level, despite their gross complementary nature at the state level.

The intuition behind the logic is that when the elasticity is estimated with the aggregate production function, the non-rivalry of public capital stock is missing in the estimation. Therefore, in our paper's context, the state-level estimate supports a substantially stronger complementarity between private and public capital stocks than the firm-level estimate does. The following proposition formally states and proves this discrepancy in the firm-level and state-level estimates.

Proposition 1. *Suppose we are given the micro-level data set* (k_1, k_2, y_1, y_2, N) *s.t.*

$$\exists i \in \{1,2\} \text{ s.t. } k_i < N, \quad N \leq k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}.$$

Suppose the micro-level estimates (z, λ) *and the aggregate-level estimate* ξ *are exactly identified by fitting the data with the production functions as follows:*

$$\begin{aligned} f(k_1, N; \lambda, z) &= y_1 \\ f(k_2, N; \lambda, 1) &= y_2. \\ f(k_1 + k_2, N; \xi, 1) &= y_1 + y_2 \end{aligned}$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$*, the aggregate-level input elasticity satisfies* $\xi < 1$.

Proof. See Appendix J. ■

2.1 Link to the production function of Baxter and King (1993)

In this section, we connect the CES production function in equation (1) with the widely employed production function proposed by Baxter and King (1993). To examine the macroeconomic implications of changes in public capital, Baxter and King (1993) employs the following formulation of a production function:

$$H(K, N, L; \zeta, z) = zK^{\alpha}L^{1-\alpha}N^{\zeta}$$

where α is the capital share between the private input factors; ζ is the scale parameter for the public capital stock. By rewriting the production function with the implicit labor demand, we obtain

$$h(K, N; \zeta, z) = z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} KN^{\frac{\zeta}{\alpha}}$$

In Proposition 2, we show that the non-rivalry between the private and public capital stocks in the firm-level production function of (1) and these inputs being gross substitutes lead to the estimate of $\zeta > 0$ in the aggregate Cobb-Douglas production function under mild assumptions.

Proposition 2. *Suppose we are given the micro-level data set (k_1, k_2, y_1, y_2, N) s.t.*

$$\exists i \in \{1, 2\} \text{ s.t. } k_i < N, \quad 1 < N \le k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}.$$

Suppose the micro-level estimates (z, λ) and the aggregate-level estimate ζ are exactly identified by fitting the data with the production functions as follows:

$$\begin{aligned} f(k_1, N; \lambda, z) &= y_1 \\ f(k_2, N; \lambda, 1) &= y_2. \\ h(k_1 + k_2, N; \zeta, 1) &= y_1 + y_2 \end{aligned}$$

Then, if the micro-level input elasticity satisfies $\lambda \ge 1$, the public capital scale parameter satisfies $\zeta > 0$.

Proof. See Appendix K. ∎

Proposition 1 and Proposition 2 provide a theoretical connection between the firm-level and the aggregate-level production functions. In particular, the latter shows that the non-rivalrous characteristic of public capital, acting as a gross substitute for private capital within the firm-level CES production function, can potentially lead to the emergence of increasing returns to scale (IRS) Cobb-Douglas production functions, aligning with the framework of Baxter and King (1993). The following corollary summarizes the propositions' economic implications.

Corollary 1. *If the assumptions of Proposition 2 are satisfied, the separately identified* (ξ,ζ) *imply that*

(i) private and public capital are gross complement in aggregate CRS-CES production function, and

(ii) under the Cobb-Douglas production function, the aggregate production function is IRS as in Baxter and King (1993).

Proof. The proof of this corollary is immediate from the prior propositions. ■

In our subsequent analysis, we employ the firm-level production function described in equation (1). To calculate infrastructure spending multipliers while accounting for endogenous firm decisions, a reliable estimate of the elasticity of substitution between private and public capital is essential. However, directly obtaining this elasticity from data presents challenges due to the limited availability of firm-level data concerning public infrastructure usage. While it may be possible to estimate the production function using state-level data on relevant inputs, Proposition 1 emphasizes that directly applying state-level input elasticity estimates as firm-level elasticities is inappropriate. Consequently, in Section 3, we introduce a structural model as a framework for estimating firm-level input elasticity.

3 Model

3.1 Household

Time is discrete and lasts forever. We consider the standard representative household with temporal utility u_t, of which the arguments are consumption c_t and labor supply L_t:

$$u(c_t, L_t) = log(c_t) - \frac{\eta}{1+\frac{1}{\chi}} L_t^{1+\frac{1}{\chi}}$$

where χ is the Frisch labor supply elasticity parameter, and η is the labor disutility parameter. The temporal utility in the future periods is discounted by the discount factor β. The household is subject to the following budget constraint:

$$c_t + \frac{a_{t+1}}{1+r_t} + \frac{B_{t+1}}{1+r_t^B} = w_t(\mathcal{E}_t + L_t)(1-\tau^h) + D_t(1-\tau^h) + (a_t - D_t) + T_t + B_t$$

where a_{t+1} and a_t are the wealth based on equity holding. D_t is the dividend from the equity holding. r_t is the market interest rate to be determined in the competitive market, and r_t^B is the interest rate of the government bond. L_t is labor supply; $\mathcal{E}_t$ is exogenously determined public employment; B_t is savings in government bonds; T_t is the lump-sum subsidy.[7] τ^h is the income tax rate that symmetrically applies to both labor and capital income. The household maximizes the sum of the discounted expected temporal utilities through the choice of $\{c_t, L_t, a_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ based on the rational expectation.

3.2 Production technology

A measure one of ex-ante homogenous firms are considered. Each firm owns capital. It produces a unit of goods from the inputs of labor and capital. The production

[7] We assume the public sector's wage follows the competitive level at the private market.

technology of a firm i located at a region j follows a CES form as specified below:[8]

$$z_{i,t}x_{j,t}f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) = z_{i,t}x_{j,t}\left(\theta(k_{i,t})^{\frac{\lambda-1}{\lambda}} + (1-\theta)\mathcal{N}_{j,t}^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}\alpha} l_{i,t}^{\gamma}$$

where $k_{i,t}$ is capital input, $l_{i,t}$ is labor input, and $\mathcal{N}_{j,t}$ is a region-specific infrastructure stock. $z_{i,t}$ is idiosyncratic productivity and $x_{j,t}$ is a region-specific productivity. We do not separately consider the congestion effects as in Alder et al. (2023). However, the estimated region-specific productivity heterogeneity implicitly reflects the heterogeneous congestion effects. $\lambda > 0$ is the elasticity of substitution between private capital and the infrastructure. $\theta \in (0, 1)$ is the weight parameter between the private and public capital. α is capital share, and γ is labor share such that $\alpha + \gamma < 1$.[9]

Idiosyncratic productivity $z_{i,t}$ is specified as below:

$$ln(z_{i,t+1}) = \rho_z ln(z_{i,t}) + \epsilon_{z,i,t+1}, \quad \epsilon_{z,i,t+1} \sim_{iid} N(0, \sigma_z^2)$$

where ρ_z and σ_z are persistence and standard deviation of independent and identically distributed (*iid*) innovation in the process. The idiosyncratic shock process is discretized using the Tauchen method for computation.

In the economy, there are two regions $j \in \{P, G\}$ of which infrastructure levels and productivity levels are different from each other. We denote the poor infrastructure region as P and the good infrastructure region as G: $N_G > N_P$. Firms

[8]In the baseline specification, we normalize the aggregate productivity as unity, as our estimation and the fiscal multiplier analysis are based on the stationary recursive competitive equilibrium. The extension of including the stochastic aggregate productivity process would allow the state-dependent fiscal multiplier analysis, which we leave for future research.

[9]Proposition 1 is based on the production function with constant returns to scale. This assumption is intended for the theoretical clarity of the statement. For example, with decreasing returns to scale, additional boundary conditions of parameters are necessary for the proposition. For our quantitative analysis, we assume the decreasing returns to scale production function to capture the empirically-supported dividend stream level used in the literature. In Section 4.5, we show that the theoretical implications of Proposition 1 is unaffected by this assumption.

switch from one region to another following an exogenous Markov process:

$$\begin{bmatrix} p^P_{t+1} \\ p^G_{t+1} \end{bmatrix} = \begin{bmatrix} \pi_{PP} & \pi_{PG} \\ \pi_{GP} & \pi_{GG} \end{bmatrix}' \begin{bmatrix} p^P_t \\ p^G_t \end{bmatrix}$$

Using the production function, firms at a region j earn operating profit in each period by solving the following problem:

$$\pi(z_{i,t}, k_{i,t}, j; , x_{j,t}\mathcal{N}_{j,t}, w_t) = \max_{l_{i,t}} \; z_{i,t} x_{j,t} f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) - w_t l_{i,t}$$

where w_t is the real wage, that will be endogenously determined in the competitive market.

3.3 Firm-level investment

Firms make an investment decision as in Khan and Thomas (2008). A small-scale capital adjustment is specified as $\Omega(k_{i,t}) := [-\nu k_{i,t}, \nu k_{i,t}]$. When they make a large-scale capital adjustment, $I_{i,t} \notin \Omega(k_{i,t})$, they need to pay a fixed adjustment cost $\xi_{i,t}$, where $\xi_{i,t} \sim_{iid} Uniform[0, \overline{\xi}]$. This cost is regarded as a labor overhead cost, so the actual cost is $w_t \xi_{i,t}$. If a firm makes a small-scale capital adjustment, $I_{i,t} \in \Omega(k_{i,t})$, it does not need to pay a fixed adjustment cost.[10]

Following Cooper and Haltiwanger (2006) and Winberry (2021), we assume all investments are subject to a convex adjustment cost, $C(I_{i,t}, k_{i,t}) = \frac{\mu}{2}\left(\frac{I_{i,t}}{k_{i,t}}\right)^2 k_{i,t}$. The convex adjustment cost plays an essential role in this paper, as it helps to capture the realistic sensitivity of aggregate investment in response to the exogenous shocks such as fiscal policy shocks (Zwick and Mahon, 2017; Koby and Wolf, 2020; Lee, 2022).

[10] As in Khan and Thomas (2008), there exists a threshold rule for the fixed cost shock ξ realization regarding the large-scale investment. For the brevity, we omit the detailed description.

3.4 Government

The government collects income tax from households at the rate of τ^h and corporate tax at τ^c. Household income is the sum of labor income $w_t l_t$ and dividend income D_t. The tax rates are exogenously determined. Government issues a bond B_{t+1} which matures in one period and is discounted by the gross bond return, $1 + r_t^B$ and pays back the maturing bond, B_t. Using the revenue $\mathcal{G}_t$ financed from the taxation and the net debt issuance, the government spends through three channels: infrastructure investment $\mathcal{F}_t$, public employment $w_t\mathcal{E}_t$, and lump-sum subsidy T_t:

$$
\begin{aligned}
\mathcal{G}_t &= \tau^h(w_t l_t + D_t) + \int \tau^c \pi(z_{i,t}, k_{i,t}, j; \mathcal{N}_t, w_t, r_t) d\Phi_t + \frac{B_{t+1}}{1 + r_t^B} - B_t \quad &\text{[Revenue]} \\
&= \mathcal{F}_t + w_t\mathcal{E}_t + T_t &\text{[Spending]}
\end{aligned}
$$

We assume the overhead fixed cost of infrastructure investment is covered by public sector workers, $\mathcal{E}_t$, without an extra cost. The public employment $\mathcal{E}_t = \mathcal{E}$ is exogenously determined. The split between the lump-sum subsidy and the infrastructure investment is determined exogenously by φ. To be specific, for $\varphi > 0$, $\mathcal{F}_t = \varphi(\mathcal{G}_t - w_t\mathcal{E}_t)$, and $T_t = (1 - \varphi)(\mathcal{G}_t - w_t\mathcal{E}_t)$.

The country-level infrastructure $\mathcal{N}_{A,t}$ and state-level infrastructure $\mathcal{N}_{j,t}$ ($j \in \{P, G\}$) evolve according to the following law of motion:

$$
\begin{aligned}
\mathcal{N}_{A,t+s} &= \mathcal{N}_{A,t+s-1}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2}\left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t+s-1}}\right)^2 \mathcal{N}_{A,t+s-1} \\
\mathcal{N}_{j,t} &= \zeta_j \mathcal{N}_{A,t} \quad \text{for } j \in \{P, G\}
\end{aligned}
$$

where the aggregate infrastructure $\mathcal{N}_{A,t}$ satisfies $\mathcal{N}_{A,t} = \mathcal{N}_{P,t} + \mathcal{N}_{G,t}$. The split between the poor infrastructure region and the good infrastructure region is exogenously determined by ζ_j, which is calibrated to match the distribution of infrastructures described in Table 2. A positive integer s represents time to build for the infrastructure investment. Infrastructure investment is subject to the same convex capital adjustment cost as private investment.

To summarize the state variables, the individual state variables are idiosyncratic productivity shock, $z_{i,t}$, and individual capital stock, $k_{i,t}$. The aggregate state variables are the tuple of each region's infrastructure stocks, $\mathcal{N}_t = (\mathcal{N}_{P,t}, \mathcal{N}_{G,t})$, infrastructure spending history and plan, $\mathbb{F}_t = (\mathcal{F}_{t+\tilde{s}})_{\tilde{s}=-s}^{\infty}$, the government bond holdings, B_t, and the distribution of individual state variables, Φ_t.

3.5 Recursive formulation

For the brevity of notation, we drop the time subscripts for each allocation from this point on. A representative household consumes, saves, and supplies labor. We define the collection of aggregate state variables $\mathcal{S} := (B, \Phi, \mathcal{N}, \mathbb{F})$. The recursive formulation of the household problem is as follows:

$$V(a; \mathcal{S}) = \max_{c,a',L,B'} log(c) - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}} + \beta V(a'; \mathcal{S}')$$

s.t.

$$c + \frac{a'}{1+r(\mathcal{S})} + \frac{B'}{1+r^B(\mathcal{S})} = w(\mathcal{S})(\mathcal{E}+L)(1-\tau^h) + D(1-\tau^h) + (a-D) + T(\mathcal{S}) + B$$

$$\mathcal{S}' = G_H^{ALM}(\mathcal{S})$$

In the recursive formulation, a firm's problem is as follows:

$$\begin{aligned} J(z,k,j;\mathcal{S}) = \max_{I,I^c} \quad & \pi(z,k,j;\mathcal{S})(1-\tau^c)(1-\tau^h) + \delta \tau^c k (1-\tau^h) \\ & + \int_0^{\bar{\xi}} \max\{(-I - w(\mathcal{S})\xi - C(I,k))(1-\tau^h) + \frac{1}{1+r(\mathcal{S})} \mathbb{E} J(z',k',j';\mathcal{S}'), \\ & (-I^c - C(I^c,k))(1-\tau^h) + \frac{1}{1+r(\mathcal{S})} \mathbb{E} J(z',k^c,j';\mathcal{S}')\} dG(\xi) \end{aligned}$$

$$
\begin{aligned}
\text{s.t.} \quad & k' = (1-\delta)k + I, \quad I \notin \Omega(k_t) = [-\nu k_t, \nu k_t] \\
& k^c = (1-\delta)k + I^c, \quad I^c \in \Omega(k_t) \\
& \mathcal{S}' = G_F^{ALM}(\mathcal{S}) \\
& dG(\xi) = \frac{1}{\bar{\xi}} d\xi \\
& \pi(z,k,j;\mathcal{S}) = \max_n \; z x_j f(k,n,N_j) - w(\mathcal{S})n \\
& C(I,k) = \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k
\end{aligned}
$$

We assume the optimal dividend payout policy fully internalizes the income tax of households, τ^h. Without this assumption, there would be an inefficient allocation of dividends, which is beyond the scope of this paper.[11] Firms earn tax benefit from tax shield out of the depreciated capital δk. By allowing the fixed cost ξ to be uniformly distributed *iid* shock, the value function becomes smooth without a kink. G_H^{ALM} and G_F^{ALM} are the aggregate law of motion that reflects the rational expectation for the future aggregate state allocations. The full elaboration of the equilibrium is in Appendix D.

4 Estimation

We postulate how we estimate the parameters of our general-equilibrium model with heterogeneous firms. This has been a computationally demanding task than estimating a partial-equilibrium model since the market clearing prices have to be solved for each candidate value for the model parameters. We provide a novel way to bypass this bottleneck by estimating market clearing prices simultaneously with the model parameters.

[11] Without this assumption, the firm's profit maximization would not take into account the household's income tax. This contrasts with the household's saving decision, which is based on the future after-income-tax dividend, leading to a distortionary effect of the corporate tax. Analyzing this distortionary effect is beyond the scope of this paper.

We first illustrate how we choose the values of externally calibrated parameters. Then we explain our estimation procedure for the remaining parameters of the general-equilibrium model. We also provide identification arguments with our targeted moments and report the estimation results.

4.1 External calibration

We first fix a few parameters at the common level in the literature: $\beta = 0.96$, $\alpha = 0.28$, and $\gamma = 0.64$. Some parameters are externally calibrated outside of the model, and their values are reported in Table 1.

Parameter	Description	Value
τ^h	household income tax rate	0.15
τ^c	corporate tax rate	0.27
ζ_G	infrastructure portion for G	0.81
$\mathcal{E}$	public employment	0.05
φ	infrastructure spending	0.09
s	time to build	1
χ	Frisch elasticity	4
δ	depreciation rate of private capital	0.09
$\delta_{\mathcal{N}}$	depreciation rate of public capital	0.02
ρ_z	idiosyncratic shock persistence	0.75
σ_z	idiosyncratic shock volatility	0.13

Table 1: Externally calibrated parameters

Notes: Each period in the model corresponds to one year in the data.

For the average of household income tax rate, we use 0.15 as in Krueger and Wu (2021) where they compute the tax rate with the data from Blundell et al. (2016). For corporate tax rate, we use 0.27 from Gravelle (2014) that is the effective tax paid after deductions and credits. We use 0.05 for the fraction of public employment, using the FRED data on the government employees ($USGOVT$) and the private employees ($USPRIV$). We use 0.09 for the infrastructure spending out of tax revenue. This comes from the fact that the infrastructure spending as share of GDP is 2.4% and the tax revenue as share of GDP is 27.1%. We assume one year of time-to-build for the baseline analysis. We set Frisch elasticity to be 4 as in Ramey (2020). We use 0.09 for the private capital depreciation rate, and 0.02 for the public capital depreciation rate from the BEA depreciation data. We use the estimates of the persistence and volatility of the idiosyncratic productivity shocks in Lee (2022), which applies the methodology of Ackerberg et al. (2015) on Compustat data from Standard and Poor's.

Furthermore, our model captures state-level variations by including two regions P, G that differ in infrastructure levels. To map this to the data pooled across years after detrending, we divide states into two groups by the median infrastructure level. Table 2 shows some summary statistics between poor and good infrastructure groups. The data is from Bennett et al. (2020). The transition probabilities are set to be persistent ($\pi_{PP} = 0.90, \pi_{GG} = 0.98$).[12] The infrastructure portion for group G, ζ_G, is set at 0.81, and the Poor's portion ζ_P is 0.19.

	Poor infrastructure	Good infrastructure
Infrastructure portion	0.19	0.81
	(0.001)	(0.001)
Establishment (#) portion	0.17	0.83
	(0.005)	(0.005)
Firm (#) portion	0.173	0.827
	(0.006)	(0.006)
GDP ($) portion	0.151	0.849
	(0.005)	(0.005)

Table 2: Comparison of two states: regions with good vs. poor infrastructure

Notes: Standard errors are in parentheses. # stands for the number of observations.

[12]Transition probabilities are constructed using the state-level data in Table A.1 in the appendix.

4.2 Estimation method

We extend the Simulated Method of Moments (SMM) to mitigate the computational challenges associated with estimating a general equilibrium model. The fundamental objective of the SMM method is to minimize the discrepancy between the moments generated by the model and those observed empirically. Our innovative approach involves incorporating market clearing conditions as additional moments and finding prices that bring these moments close to zero.

4.2.1 Challenge of estimating a general equilibrium model with SMM

Let Θ denote the parameters of interest and $\hat{\mathbf{m}}$ denote the vector of M empirical moments from the data for estimation. Under SMM, the moment conditions to satisfy are

$$\mathbf{m}(\Theta) - \hat{\mathbf{m}} = \mathbf{0}$$

where $\mathbf{m}(\Theta)$ is the model's prediction for the moments under parameter Θ and $\mathbf{0}$ is a zero vector of length M.

Suppose we estimate parameters of the model in which market clearing conditions need to be satisfied as general equilibrium conditions. Given each candidate parameter vector, the model is solved with an additional loop that makes sure the market clearing conditions become zero with numerical precision. This additional layer regarding general equilibrium conditions is likely to result in prohibitively high computational costs.

4.2.2 Extension of SMM to incorporate general equilibrium conditions

In order to make the estimation procedure computationally feasible, we extend the SMM method by augmenting data moments with market clearing conditions. In other words, we treat market clearing prices as parameters to be estimated where the associated moments in estimation procedure are market clearing conditions.

With the standard estimation with general equilibrium models, the computational bottleneck lies in that we need to satisfy market clearing conditions for each candidate parameter vector. Instead, our suggested method treats market clearing conditions as additional moments. Given the lens of our model, we need to track both of the market clearing prices: wage w and marginal utility of consumption p given which we can back out the interest rate through the Euler equation. Thus, we treat (p, w) as additional parameters to estimate. From our model (given state $\mathcal{S}$), $p = 1/c(\mathcal{S})$ and $w = \eta L(\mathcal{S})^{\frac{1}{\chi}} c(S)/(1-\tau^h)$.

In addition to the moment conditions $\hat{\mathbf{m}} - \mathbf{m}(\Theta) = \mathbf{0}$, we include the following market clearing conditions:

$$\begin{bmatrix} p - 1/c(\mathcal{S}) \\ w - \eta L(\mathcal{S})^{\frac{1}{\chi}} c(S)/(1-\tau^h) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Given candidates of the market clearing prices (p, w), we compute the model-generated prices $(1/c(\mathcal{S}), \eta L(\mathcal{S})^{\frac{1}{\chi}} c(S)/(1-\tau^h))$ after solving the model. Then we can check whether the difference between the model-generated prices and (p, w) is zero.

We implement our estimation method in a Bayesian way as in Fernández-Villaverde et al. (2016) and use the multiple-block Metropolis-Hastings where we break the parameter space into two blocks, one for the price block and the other for the other model parameters. We include more details on the algorithm in Appendix B. As the RWMH chain runs, we obtain the posterior draws that render market clearing conditions closer to zero as well as fitting the target empirical moments closely.

4.3 Identification and target moments

The upper bound for fixed cost $\bar{\xi}$ is identified using the portion of the firms making lumpy investments. The convex adjustment cost parameter μ is identified from the average investment-to-capital ratio. Parameter ν associated with the constrained investment region is identified from the standard deviation of investment-to-capital ratio. Private capital share parameter θ is identified from the private-to-infrastructure capital ratio. Productivity level parameter x is identified from the high region's output y portion. Government spending level parameter G is identified from the government spending to output ratio. Labor disutility parameter η is identified from total working hours. The elasticity of substitution parameter λ is identified from the difference in private capital stocks between the state with a high level of infrastructure and the state with a low level of infrastructure. We assume that the firm-level production function is identical across firms within a state. As private and public capital are more complementary, the portion of private capital stock in the high infrastructure region is expected to be greater. To the best of our knowledge, our study is the first to use a structural model to identify and estimate the elasticity of substitution between private and public capital at the firm level.

4.4 Estimation results

Table 4 reports the posterior means and the 90% credible intervals of parameters from our estimation. The firm-level elasticity of substitution λ is estimated to be 1.185, which supports the Cobb-Douglas production function as a reasonable specification. The productivity of infrastructure abundant region is approximately double that of infrastructure poor region. It is worth noting that we do not consider endogenous evolution of productivity. Overall, the credible intervals are much narrower than the uniform priors, suggesting that the variations from the data is

Target moment	Data	Source
Lumpy investment portion	0.140	Zwick and Mahon (2017)
Average investment-to-capital ratio (i/k)	0.100	Zwick and Mahon (2017)
Standard deviation of (i/k)	0.160	Zwick and Mahon (2017)
Private-to-infrastructure capital ratio	0.750	Bureau of Economic Analysis
High region's private capital k portion	0.830	Census Business Dynamics Statistics
High region's output y portion	0.849	Bennett et al. (2020)
Government spending to output ratio	0.155	World Bank Database
Total working hours	0.330	Current Employment Statistics

Table 3: Target moments used in estimation

Notes: High region refers to the state with high infrastructure capital stock.

useful to infer the parameters of interest.

		Posterior		Uniform Prior
Parameter	Description	Mean	90% interval	[Min, Max]
$\bar{\xi}$	fixed cost upper bound	0.519	[0.511,0.523]	[0.001,1.900]
μ	convex adjustment cost	3.124	[3.118,3.135]	[0.200,3.500]
ν	constrained investment	0.0406	[0.0405,0.0407]	[0.001,0.080]
θ	private capital share	0.667	[0.666,0.668]	[0.500,0.999]
λ	elasticity of substitution	1.185	[1.180,1.190]	[0.300,2.500]
x	productivity of high region	2.064	[2.042,2.080]	[0.500,2.500]
G	government spending level	0.103	[0.101,0.107]	[0.010,0.400]
η	labor disutility	2.845	[2.831,2.860]	[2.100,3.500]

Table 4: Estimation results

Table 5 shows the model fit for the targeted moments. The model-generated moments fit the empirical moments from the data reasonably well.[13]

[13]In addition, the market clearing prices are tightly pinned down. When using the posterior mean value for market clearing prices, the market clearing conditions are satisfied with the numerical accuracy of e^{-4}.

Target moment	Data	Model
Lumpy investment portion	0.140	0.139
Average investment-to-capital ratio (i/k)	0.100	0.100
Standard deviation of (i/k)	0.160	0.160
Private-to-infrastructure capital ratio	0.750	0.798
High region's private capital k portion	0.830	0.870
High region's output y portion	0.849	0.984
Government spending to output ratio	0.155	0.154
Total working hours	0.330	0.344

Table 5: Model fit

4.5 External validation with empirical state-level elasticity

As external validation, we compute the state-level elasticity from our model and compare it to the empirical estimate using the state-level data. We find that the state-level input elasticity from our model indicates the complementarity between private and public capital and this is consistent with the empirical elasticity obtained from the state-level production function estimation.

4.5.1 State-level elasticity from the model

In our model, the infrastructure stock is shared among the firms in the same region. We conduct the state-level aggregation as follows: we fix the firm-level estimates except for the elasticity λ and spatial productivity heterogeneity x_1.[14] We estimate

[14]Since the production function in our model is decreasing returns to scale, there is no guarantee that the firm-level elasticity and productivity is aggregated to have the same value in the state-level.

these two parameters under the state-level production functions.[15]

$$\begin{bmatrix} x_1 \left(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_1^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}\alpha} l_1^{\gamma} \\ \left(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_2^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}\alpha} l_2^{\gamma} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where (x_1, λ) are unknown, while all the other allocations and parameters, $(y_1, y_2, k_1, k_2, N_1, N_2, l_1, l_2, \theta, \alpha, \gamma)$ are obtained from the estimated baseline model.[16] Using the same nonlinear least squares optimization used in An et al. (2019), we get the estimate of the state-level production function $(x_1, \lambda) = (1.766, 0.349)$.

If we assume a CRS state-level production function with $\gamma = 1 - \alpha$, the estimates are $(x_1, \lambda) = (1.923, 0.482)$. Therefore, our model suggests that public capital and private capital are gross complements at the state level.[17]

4.5.2 State-level elasticity from the data

Using the state-level data available, we estimate the elasticity of substitution between private and public capital given a CES production technology. We closely follow An et al. (2019) in which the elasticity is estimated using the nonlinear least squares using the following:

$$\ln\left(\frac{Y_{i,t}}{Y_{i,t-1}}\right) = c + (1-a)\ln\left(\frac{L_{i,t}}{L_{i,t-1}}\right) + \frac{a}{\psi}\ln\left[\frac{bK_{i,t}^{\psi} + (1-b)N_{i,t}^{\psi}}{bK_{i,t-1}^{\psi} + (1-b)N_{i,t-1}^{\psi}}\right] + (\epsilon_{i,t} - \epsilon_{i,t-1}).$$

i denotes the state, t denotes the time, and ϵ is the error term. Y is the output, K is the private capital stock, N is the infrastructure capital stock, and L is employ-

[15] We cannot identify the public capital stock share, θ separately from the elasticity, λ in the state-level model. This is the main reason why we introduce the micro-level heterogeneity in our structural model. Therefore, in the state-level model, we fix the public capital stock share at the firm-level estimate.

[16] The two parameters (x_1, λ) are obtained from the exact identification.

[17] Among the unreported results, we estimate the Cobb-Douglas production function as in Baxter and King (1993) using the simulated state-level data. The returns to scale parameter for the public capital is estimated to be greater than 0, consistent with the increasing returns to scale as shown in Proposition 2. The result is available upon request.

ment. ψ is the capital substitution parameter which implies a public-private capital elasticity of substitution (ES) of $1/(1-\psi)$.

Using the state-level data, we compute local estimates of input elasticity. We first compute the net investment on public and private capital stocks. The state-level net public investment is approximated by the portion of aggregate net infrastructure investment, where the weight is obtained by the state-level real public highway infrastructure investment from Bennett et al. (2020).[18] This is from the assumption that the infrastructure spending at the state level for each of the different items (e.g., highway, water supply, etc.) is identically distributed across the states. The state-level net private investment is approximated by the portion of aggregate net non-residential fixed investment from National Income and Product Accounts (NIPA) data of Bureau of Economic Analysis (table 5.2.6), where the weight is obtained by the number of establishments at the state level from the Business Dynamics Statistics (BDS) at the United States Census Bureau. This approximation is based on the assumption that the capital stock at each establishment does not vary significantly. After we obtain the net investment for public and private capital, we construct public and private capital stocks using the perpetual inventory method. For this approach, the initial capital stocks are needed for both public and private capital stocks. The state-level initial infrastructure stock is obtained by the portion of the infrastructure stock in 1977 from Bennett et al. (2020), where the weight is from the highway infrastructure spending in 1977. The state-level initial private capital stock is from the portion of the aggregate private capital stock in 1977 from NIPA of Bureau of Economic Analysis (table 4.1), where the weight is from the number of establishments in 1977. All the data is at the annual frequency. All real

[18]The aggregate net infrastructure investment is also from Bennett et al. (2020). In the state-level calculation, the weight is computed in the following way:

$$weight_{i,t} = \frac{\text{highway infrastructure investment}_{i,t}}{\sum_i \text{highway infrastructure investment}_{i,t}}$$

variables are chained in 2012 dollar value.

	Estimates	90% confidence interval
a	0.402	[0.351, 0.453]
b	0.070	[0.018, 0.123]
Elasticity of substitution	0.445	[-0.099, 0.989]

Table 6: Results from nonlinear least squares estimation

Notes: Elasticity of substitution is $\frac{1}{(1-\psi)}$. Its confidence interval is derived by the delta method.

Table 6 shows the estimation results from nonlinear least squares. The elasticity of substitution between public and private capital is estimated to be 0.445.[19] In other words, the state-level variations indicate the complementarity between private and public capital. However, this result does not imply the complementarity between private and public capital at the firm level. In fact, the private and public capitals can be gross substitutes at the firm level, whereas they are gross complements at the state level as shown in Section 2.

It is worth noting that our model bridges the gap between the firm-level estimates and the state-level estimates. According to our estimates, private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level. At the state level, the elasticity of substitution includes a good public nature of the infrastructure benefiting all firms in a state. Therefore, the non-rivalry of the infrastructure generates the complementarity between the state-level private capital and the public capital.

[19] As robustness check, we apply GMM estimation where $L_{it-2}, K_{it-2}, N_{it-2}$ are used in exogeneity conditions. The elasticity of substitution is estimated to be 0.44. This result is available upon request.

5 Analyses of fiscal multipliers

We analyze the fiscal multipliers of infrastructure investment based on our estimated structural model. We define the fiscal multiplier as follows:

$$\text{Fiscal Multiplier} = \frac{\sum_{t=1}^{T} \text{Present value of } \Delta x_t}{\sum_{t=1}^{T} \text{Present value of } \Delta G_t}$$

where Δx_t is the deviation at period t of the equilibrium allocation of interest from the steady-state level; ΔG_t is the fiscal spending shock at period t.[20] In the short run, we assume $T = 2$, and in the long run, we assume $T = 5$. In this section, we focus on the impact of a sudden shock in fiscal spending specifically through infrastructure investment. We assume the fiscal spending shock is a one-time unexpected shock (MIT shock) without any persistence.

5.1 Baseline analysis

The magnitude of the one-time shock is assumed at 1% of the steady-state output level as in Ramey (2020). We assume that all the fiscal policy shock is financed by a lump-sum tax.[21]

The following laws of motion determine the time path of the public capital stocks after the fiscal spending shock ΔG at $t = 1$:

$$\mathcal{N}_{A,t+1} = \mathcal{N}_{A,t}(1-\delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2}\left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t}}\right)^2 \mathcal{N}_{A,t}$$

$$\mathcal{N}_{j,t} = \zeta_j \mathcal{N}_{A,t} \quad \text{for } j \in \{P, G\}$$

$$F_t = \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}$$

[20]The shock is deviation from the steady-state level.

[21]In Appendix E, we also consider changes in corporate tax policy on top of the lump-sum taxation for fiscal financing.

where F^{ss} is the stationary equilibrium level of infrastructure spending.

Figure 1 plots the impulse responses of the fiscal policy shock. The impulse response is obtained from the perfect-foresight transition path after the one-time fiscal spending shock. The dashed line in each panel shows the government expenditure changes from the steady-state level in percent of the steady-state output. The solid line is the impulse response of the equilibrium allocations.[22]

Private investment contemporaneously decreases. The response of private investment is the outcome of two countervailing forces: 1) increase in the investment incentive with increased infrastructure stock and 2) adjustment in the interest rate that dampens investment (general equilibrium effect; GE effect hereafter). The increase in the investment incentive comes from the imperfect substitution between public and private capital stock. For a simple illustration, we consider a two-period model with the firm-level investment decision where the production functions are the same as in Proposition 1, and investment is subject to the convex adjustment cost. From the first-order condition of the investment, the following equation holds:

$$\underbrace{1+\mu\left(\frac{k'}{k}-(1-\delta)\right)}_{\text{marginal cost}} = \underbrace{\overbrace{\frac{1}{1+r}}^{\text{GE channel}} \; \overbrace{z^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}\left(\theta k'^{\frac{\lambda-1}{\lambda}}+(1-\theta)N^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}-1}k'^{-\frac{1}{\lambda}}\theta}^{\text{Future MPK}}}_{\text{marginal benefit = discounted future MPK}}$$

The left-hand side of the equation above is the marginal cost of the firm-level investment, and the right-hand side is the marginal benefit. To analyze how the increase in the public capital stock N affects the marginal benefit of firm-level in-

[22]The responses of output, consumption, public capital, wage, and government investment decay in slow rates due to the low infrastructure depreciation rate at $\delta_{\mathcal{N}} = 0.02$.

vestment, we a take a partial derivative with respect to N.

$$\frac{\partial}{\partial N}\text{Marginal benefit} = \left(\frac{1}{1+r}\right) \times \frac{\partial}{\partial N}\text{Future MPK} + \overbrace{\text{Future MPK} \times \frac{\partial}{\partial N}\left(\frac{1}{1+r}\right)}^{\text{GE effect}} \tag{2}$$

$$\begin{aligned}\frac{\partial}{\partial N}\text{Future MPK} &= \frac{\partial}{\partial N}\left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}}\right)^{\frac{1}{\lambda-1}} F(\Theta) \\ &= \left(\frac{1}{\lambda-1}\right)\left(\frac{\lambda-1}{\lambda}\right)(1-\theta)N^{-\frac{1}{\lambda}}\left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}}\right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) \\ &= \frac{1}{\lambda}(1-\theta)N^{-\frac{1}{\lambda}}\left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}}\right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) > 0 \end{aligned} \tag{3}$$

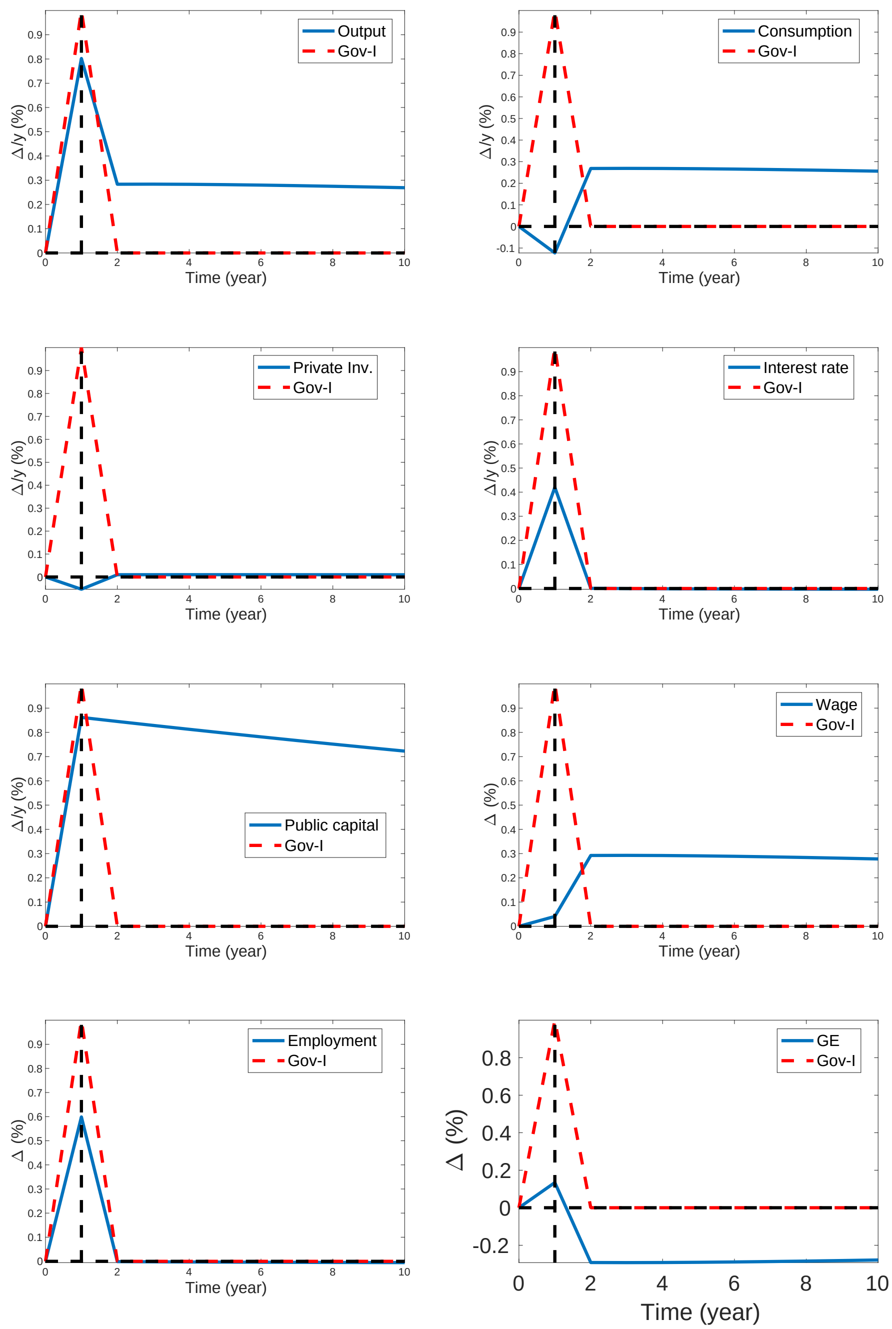
where F is a function of the parameters, Θ. If the elasticity of substitution λ is a finite positive number, the marginal benefit of firm-level investment increases in N through the increased future marginal product of capital, given the general equilibrium effect is fixed. However, if λ goes to infinity, the marginal benefit of investment does not depend on N. It is worth noting that the marginal benefit increases in N regardless of whether the public and private capital stocks are gross complements ($\lambda < 1$) or substitutes ($\lambda > 1$).

However, a fiscal policy affects the prices, which we denote as the general equilibrium effect. Regarding this, one of the most important channels is lump-sum taxation to finance infrastructure investment: the household reduces consumption to pay this lump-sum tax. Thus, the marginal utility of contemporaneous consumption increases, leading to an increase in the interest rate in the equilibrium. Then, the heightened interest rate strongly crowds out the firm-level investment despite the increased future MPK as in Equation (3).[23]

In the baseline impulse response, the interest rate increases by 0.39 percent after the infrastructure spending shock. This general equilibrium effect dominates the increase in the marginal benefit from the greater public capital stock, leading to a

[23] Households have no precautionary saving motivation against the aggregate risk, as the economy is abstract from the aggregate uncertainty.

Figure 1: The impulse responses to the infrastructure spending shock



net crowding out of the private investment.

In contrast, the employment response is not completely dampened by the general equilibrium (wage) effect. As the shock hits, employment increases by 0.52% despite the wage increase. Consumption and the GE effect are the mirror image of each other as the GE effect refers to the inverse of consumption under the log utility. In the shock period, consumption decreases strongly due to the lump-sum taxation, but it increases in the following period from the strong consumption smoothing motivation: all the expected future gains out of the infrastructure spending smoothly shift up the consumption level.

5.2 The role of elasticity of substitution between private and public capital stocks

The elasticity of substitution between private and public capital stock plays a key role in determining the marginal benefit of firm-level investment given a fiscal expenditure shock. Analytically, the change in the marginal benefit of firm-level investment over the elasticity given the fiscal spending shock can be captured by the cross derivative $\left[\frac{\partial^2}{\partial\lambda\partial N}\text{Marginal benefit}\right]$ in the simple two-period model. Using Equation (2), we have the following equation:

$$\frac{\partial^2}{\partial\lambda\partial N}\text{Marginal benefit} = \frac{\partial}{\partial\lambda}\left[\underbrace{\frac{1}{\lambda}}_{\text{Direct}}\underbrace{\frac{\left(\frac{1}{1+r}\right)MPK}{\left(\theta k'\left(\frac{N}{k'}\right)^{\frac{1}{\lambda}}+(1-\theta)N\right)}}_{\text{Indirect}}\right] + \frac{\partial}{\partial\lambda}\text{GE effect}$$

As displayed in the equation above, the elasticity of substitution affects the response of marginal benefit through two channels: 1) direct and 2) indirect channels. The direct channel refers to newly added capital being relatively less valuable when the public capital stocks are more substitutable with the private capital. The indirect channel refers to a change in marginal benefit of investment due to the change in the relative values of the existing public and private capital stocks.

The direct channel predicts the marginal benefit of firm-level investment decreases in the elasticity, while the sign of the indirect channel cannot be analytically determined.[24]

Table 7: Fiscal multipliers

Fiscal multipliers	High ($\lambda = 3$)	Baseline ($\lambda = 1.185$)	Low ($\lambda = 0.5$)
Heterogeneous-agent			
Output	0.6721	1.0878	1.3639
Investment	-0.1657	-0.0434	0.0950
Partial eq. - Output	0.9012	1.8582	2.2832
Partial eq. - Investment	0.0285	0.1892	0.2247
	$\zeta = 0.2436$	Baseline ($\zeta = 0.2708$)	$\zeta = 0.2832$
Representative-agent			
Output	0.9699	0.9914	0.9980
Investment	-0.1643	-0.1573	-0.1552

Table 7 provides a summary of our fiscal multipliers across various scenarios over a 2-year horizon. The benchmark values stem from a heterogeneous agent general equilibrium model, with the elasticity of substitution between public and private capital denoted as $\lambda = 1.185$. Specifically, the output multiplier is 1.088, while the investment multiplier is -0.043. In other words, the output increases by \$1.088 and investment decreases by \$0.043 in the short run for every \$1 spending in the infrastructure.

In contrast, the fiscal multipliers calculated using a heterogeneous agent partial equilibrium model yield significantly larger results: a 1.858 output multiplier and a 0.189 investment multiplier given $\lambda = 1.185$.[25] Notably, this is due to the miss-

[24]The sign of the effect also depends on the firm-level capital stock.

[25]The partial equilibrium is defined as the equilibrium path where the prices are fixed at the

ing crowding-out effect of private investment because there are no interest rate adjustments in the partial equilibrium framework.

We also compare the fiscal multipliers under our benchmark model with heterogeneous firms to those from the representative-agent model, where the production function is following Baxter and King (1993) and the other ingredients are the same as the baseline model except for absence of the fixed adjustment cost.[26] The detailed description of the representative-agent model is available in Appendix I.[27] The output multiplier under the representative agent model is observed to be less than 1 (0.991), in contrast to the baseline value of 1.088. This difference arises from a more subdued investment multiplier (-0.157 vs. -0.043). First, the lower sensitivity in the heterogeneous-firm model is due to Jensen's inequality effect on the convex adjustment cost: the heterogeneous firms' average adjustment burden is greater than that of the representative (average) firm due to the convexity. Specifically, at the steady state, both the representative-agent and heterogeneous-agent models feature the same i/k ratio, which is identical to δ. Then, the aggregated adjustment cost is greater for the heterogeneous-agent model than the representative-agent model due to the dispersion over the convex cost.

Second, our baseline model incorporates the fixed adjustment cost, which allows the model to capture realistic firm-level investment dynamics, while the representative-agent model does not. The fixed adjustment cost dampens the sensitivity of the firm-level investment due to the extensive margin decision to stay inactive. For these reasons, the dampened negative investment response in the heterogeneous-agent model leads to a greater output multiplier.

Finally, the comparison extends to fiscal multipliers across different values of the elasticity of substitution. In the benchmark heterogeneous agent model, the

steady-state level.

[26]If the fixed adjustment cost is considered at the macro level, the aggregate investment becomes too lumpy, which is inconsistent with the observed patterns in the data.

[27]The representative-agent model's parameter levels are assumed to be at the same level as the baseline model, except for the scale parameter in the production function.

fiscal multipliers exhibit notable variation with changes in λ values. Specifically, under $\lambda = 3$ (indicating high substitutability between public and private capital), the output multiplier is 0.672, significantly smaller than the benchmark value of 1.088. Conversely, under $\lambda = 0.5$, reflecting complementarity between private and public capital, the output multiplier is markedly higher at 1.364.

For a comparable analysis within the representative agent model framework, we calculate the implied returns to scale parameter (ζ) corresponding to each λ value. We find that the variations in fiscal multipliers across different λ values are relatively modest. The reason for observing more pronounced variations under the heterogeneous agent model is the influence of the non-rivalry of public capital in each heterogeneous firm's production function. This underscores the significance of accurately estimating the elasticity parameter when seeking to analyze the fiscal multiplier of non-rivalrous public investment incorporating heterogeneous firms' decisions.

Table 8: Short-run responses of other equilibrium allocations

	Baseline	Partial eq.
Employment (average annual % change from ss)	0.3044	0.8519
Wage (average annual % change from ss)	0.3888	
Earning (dollars per 1$ spending)	0.7134	1.2935
Comsumption (dollars per 1$ spending)	0.1479	0.6052

Table 8 presents changes in various equilibrium allocations resulting from the fiscal shock. Compared to the stationary equilibrium, there is an average annual increase of 0.304% in employment and 0.389% in wages. Consequently, this leads to a \$0.713 increase in earnings and \$0.148 increase in consumption for every \$1 of fiscal spending. In the partial equilibrium scenario without changes in factor prices, the increase in labor demand is significantly more pronounced. This results in a \$1.294 surge in earnings and a \$0.605 increase in consumption for every \$1 of infrastructure spending.

5.3 Firm-level responses to the fiscal shock: Extensive vs. intensive

In this section, we analyze how the investment responds to the fiscal shock at the firm level. The literature on firm-level frictional capital adjustment has highlighted the distinguished nature of investments between the extensive and intensive margins. Empirically observed firm-level investments are significantly lumpy. One of the most popular explanations for this observed pattern is the presence of the fixed adjustment cost, which separates the decision of whether to make an investment today or not (extensive) and the decision of the investment size (intensive). In this section, we investigate through which channel the investment crowding out happens in our baseline model.

Table 9: Decomposition of fiscal multipliers by the investment margins

	Baseline	Extensive-only	Intensive-only	No resp.
Output	1.0878	1.1034	1.1228	1.1386
Investment	-0.0434	-0.0321	-0.0108	

Table 9 reports the decomposition of fiscal multipliers along with the different firm-level investment margins. The column labeled "Extensive-only" corresponds to the scenario where only adjustments to the extensive margin of investment are allowed, while the intensive margin is fixed at the stationary equilibrium level. In contrast, the column labeled "Intensive-only" represents the case where only adjustments to the intensive margin of investment are permitted, while keeping the extensive margin at the stationary equilibrium level.

In this analysis, we note that both the extensive and intensive margins play a role in the crowding out of investment, with a more pronounced effect originating from the extensive margin. Specifically, the investment multiplier is -0.043 under the baseline, and approximately 74% of this baseline multiplier is obtained in the ”Extensive-only” scenario.

The column labeled “No resp.” corresponds to the scenario where investment responses are held constant at the stationary equilibrium level in response to the fiscal spending shock. Without alterations in investment behavior, the fiscal multipliers would have been higher at 1.139 compared to the baseline value of 1.088.

5.4 Efficiency vs. Equality: Cross-state analysis

In this section, we analyze the cross-state heterogeneous impact of the infrastructure investment. In our structural analysis, we divide all the states in the U.S. into two groups: Poor vs. Good. The Poor states are bottom half states in terms of the size of infrastructure and the rest is defined as the Good states.[28] For example, the Top side of the Good states include New York, California, Texas, Florida, and Illinois, and the bottom side of the Poor states include Vermont, New Hampshire, North Dakota, Maine, and Rhodes Island.

Table 10: Cross-state inequality in short-run fiscal multipliers

	Baseline			Partial eq.			Equal spending		
	Total	Poor	Good	Total	Poor	Good	Total	Poor	Good
Y	1.0878	0.0157	1.072	1.8582	0.0274	1.8309	0.8727	0.0623	0.8104
Inv.	-0.0434	-0.0007	-0.0426	0.1892	0.0048	0.1844	-0.0516	-0.0004	-0.0512
Earnings	0.7134	0.0167	0.6967	1.2935	0.0192	1.2743	0.5459	0.0420	0.5039
C	0.1479	-0.1272	0.2751	0.6052	-0.0993	0.7045	-0.0556	-0.1010	0.0454

Table 10 reports the heterogeneous state-specific fiscal multipliers. Per $1 spending, out of the total output increase of $1.088, $1.072 goes to the output increase in the Good states, while only the incremental of $0.016 belongs to the Poor states. In

[28] The assumptions necessary for the cross-state analysis are specified in Appendix F.

contrast, the crowding-out effect of the private investment is significantly greater in the Good states, featuring -0.0426 out of total crowding-out effect of -0.0434. In terms of the earnings, out of the total increase of \$0.713 per \$1 fiscal spending, \$0.017 belongs to the Poor states, while \$0.697 goes to the Good states. The aggravation of the cross-state inequality is more severe in the consumption side, as the infrastructure spending is financed by the lumpy-sum taxation impartially across the states, while the benefit of the spending is relatively more skewed to the Good states.[29]

In the baseline model, the general equilibrium effect weakly alleviates the unequal distribution of the benefit of the fiscal spending across the states, as can be seen from the earnings ratio: the earnings multiplier share of the Good states in the baseline is smaller than the counterpart in the partial equilibrium $(0.697/0.713 < 1.274/1.294)$. This is mainly due to the Good state's substantially more susceptible investment response to the GE crowding-out effect than the Poor state's investment (from 0.184 to -0.043 vs. from 0.005 to -0.001).

Finally, we consider a counterfactual policy experiment where the fiscal shock is equally spent on the Good and Poor states. The outcome of this experiment shows a drastic trade-off a government would face between aggregate-level efficiency and cross-state inequality. Compared to the status quo policy, equal spending leads to a substantially lower output multiplier (0.873) and a greater degree of crowding-out effect. Not surprisingly, the Good states' fiscal multiplier drops down significantly (0.810 vs. 1.072). However, the Poor states' output fiscal multiplier is around four times greater than the baseline (0.062 vs. 0.016); the earnings multiplier is more than twice the baseline level (0.042 vs. 0.017). This result is largely due to the estimated state-specific productivity differences across the states. Also, due to the non-rivalry, the cross-state heterogeneity in the number of firms (capital) utilizing the additional infrastructure strongly affects the policy outcome.

[29] The tax is imposed more lightly for the Poor states than the Good states, but the tax burden is significantly greater than the benefit in the poor states in the baseline model.

As this paper does not focus on the normative optimal policy, the implication is limited to a positive evaluation, but the evaluated trade-offs between efficiency and equality are highly policy-relevant takeaways of this model.

5.5 Comparison with the fiscal multipliers in the literature

Ramey (2011) constructs government spending shocks that control for the anticipation effects and finds the government spending multipliers ranging from 0.6 to 1.2. Nakamura and Steinsson (2018) use state-level variations in military buildups (increases in federal purchases associated with military buildups) and find a state GDP multiplier of 1.4. They show their multiplier is equal to the aggregate multiplier in a small open economy with a fixed exchange rate and is larger than the closed economy aggregate multiplier for normal monetary policy. Chodorow-Reich (2019) studies the fiscal multiplier using the cross-sectional variations of fiscal spending controlling for state-specific heterogeneity. The paper concludes that the cross-sectional multiplier is around 1.8, and the lower bound for the national multiplier without the monetary policy response is 1.7. Our quantitative result shows that the short-run national fiscal multiplier is around 1.86 without the general equilibrium effect, consistent with Chodorow-Reich (2019). However, once the general equilibrium effect (the real interest rate) is considered, the spending multiplier reduces to 1.09, which is within the range of Ramey (2011).

Our focus is on infrastructure spending multipliers, which may have a different nature compared to defense or overall government spending, as pointed out in Leduc and Wilson (2017). The paper empirically analyzes the fiscal multiplier using the state-level variation in highway spending and structurally analyzes the national-level multiplier based on the empirical analysis. They provide the impact multiplier of 1.4 and the cumulative multiplier of 6.6 over 10 years. Our paper distinguishes itself from Leduc and Wilson (2017) by including a firm-level frictional adjustment that accurately models micro-level investment responses after the implementation of fiscal policies (Cooper and Haltiwanger, 2006). We also estimate the micro-level parameters using the firm and state-level moments.

Fishback and Kachanovskaya (2015) discuss that the possible spillover effects across the states after fiscal spending hinder the translation of the cross-sectional multiplier analysis to the national-level multiplier. For this, recent literature has found a breakthrough by quantifying the spillover effects through the cross-state network (Peri et al., 2023). In our paper, we take a different route to handle this issue. We group the states into two groups: good and poor. The identification comes from the variations in the infrastructure spending and equilibrium allocations between these two groups. The spillover effect is crucially an issue within the good states: e.g., how does New York affect Massachusetts once the highway within New York is significantly improved? However, in our setup, such spillover does not bias our estimates as such influential states are grouped together, and the total productivity effect from the spillover is separately identified as the group-specific (good state-specific) productivity.

6 Conclusion

This paper theoretically bridges the micro-level and the macro-level production functions that take the infrastructure stock as an input factor. Then, it analyzes the infrastructure investment multipliers through the lens of an estimated heterogeneous-firm general equilibrium model. Our theory establishes that the non-rivalry of public capital at the firm-level combined with the gross substitutability between private and public capital leads to the aggregate-level gross complementarity in CES or the increasing returns to scale in Cobb-Douglas production function. We show the theory is consistent with the measured outcomes through a quantitative analysis. According to our estimated general equilibrium model, the output multiplier is around 1.09, which is significantly greater than the level (0.99) predicted by the representative-firm model. This is due to a lower sensitivity of the firm-level investment responses to the general equilibrium effect than the representative firm's response, resulting in a dampened crowding out.

References

Abel, A. B. and J. C. Eberly (2002). Investment and q with Fixed Costs: An Empirical Analysis. *Working Paper*.

Ackerberg, D. A., K. Caves, and G. Frazer (2015). Identification properties of recent production function estimators. *Econometrica 83*, 2411–2451.

Alder, S., Z. M. Song, and Z. Zhu (2023). A Quantitative Analysis of Infrastructure Investment Efficiency using Truck GPS Data. Working Paper.

An, Z., A. Kangur, and C. Papageorgiou (2019). On the substitution of private and public capital in production. *European Economic Review 118*, 296–311.

An, Z., F. Zhang, and H. Li (2022). Elasticity of substitution between public and private capital: Evidence from manufacturing firms in europe. *Economics Letters 219*, 1–3.

Auerbach, A., Y. Gorodnichenko, and D. Murphy (2020). Local fiscal multipliers and fiscal spillovers in the usa. *IMF Economic Review 68*, 195–229.

Baxter, M. and R. G. King (1993). Fiscal policy in general equilibrium. *The American Economic Review 83*(3), 315–334.

Bennett, J., R. Kornfeld, D. Sichel, and D. Wasshausen (2020). Measuring infrastructure in the bureau of economic analysis national economic accounts. BEA Working Paper.

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2016). Consumption inequality and family labor supply. *American Economic Review 106*, 387–435.

Bureau of Economic Analysis (1925-2020b). NIPA Fixed Asset Table 4.1, Annual. https://apps.bea.gov/iTable/index_nipa.cfm, used June 15th, 2022.

Bureau of Economic Analysis (1967-2020a). NIPA Domestic Product and Income Table 5.2.6, Annual. https://apps.bea.gov/iTable/index_nipa.cfm, used June 15th, 2022.

Caballero, R. J. and E. M. R. A. Engel (1999, July). Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S, s) Approach. *Econometrica 67*(4), 783–826.

Chodorow-Reich, G. (2019). Geographic cross-sectional fiscal spending multipliers: What have we learned? *American Economic Journal: Economic Policy 11*, 1–34.

Cooper, R. W. and J. C. Haltiwanger (2006). On the nature of capital adjustment costs. *Review of Economic Studies 73*, 611–633.

Espinoza, R., J. Gamboa-Arbelaez, and M. Sy (2020). The fiscal multiplier of public investment: The role of corporate balance sheet. IMF Working Paper.

Fernández-Villaverde, J., J. Rubio-Ramírez, and F. Schorfheide (2016). Solution and estimation methods for dsge models. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2, pp. 527–724. Elsevier.

Fishback, P. and V. Kachanovskaya (2015). The multiplier for federal spending in the states during the great depression. *The Journal of Economic History 75*(1), 125–162.

Glomm, G. and B. Ravikumar (1994). Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control 18*, 1173–1187.

Gravelle, J. G. (2014). International corporate tax rate comparisons and policy implications. Technical report, Congressional Research Service.

Hagedorn, M., I. Manovskii, and K. Mitman (2019). The fiscal multiplier. NBER Working Paper.

Hasna, Z. (2021). The grass is actually greener on the other side: Evidence on green multipliers from the united states. Working Paper.

Khan, A. and J. K. Thomas (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica 76*, 395–436.

Koby, Y. and C. K. Wolf (2020). Aggregation in heterogeneous-firm models: Theory and measurement. Working Paper.

Krueger, D. and C. Wu (2021). Consumption insurance against wage risk: Family labor supply and optimal progressive income taxation. *American Economic Journal: Macroeconomics 13*, 79–113.

Leduc, S. and D. Wilson (2017, May). Are state governments roadblocks to federal stimulus? evidence on the flypaper effect of highway grants in the 2009 recovery act. *American Economic Journal: Economic Policy 9*(2), 253–92.

Lee, H. (2022). Striking while the iron is cold: Fragility after a surge of lumpy investments. Working Paper.

Nakamura, E. and J. Steinsson (2018). Identification in macroeconomics. *Journal of Economic Perspectives 32*, 59–86.

Oberfield, E. and D. Raval (2021). Micro data and macro technology. *Econometrica 89*, 703–732.

Peri, A., O. Rachedi, and I. Varotto (2023). Public investment in a production network: Aggregate and sectoral implications. Working Paper.

Ramey, V. A. (2011, September). Can government purchases stimulate the economy? *Journal of Economic Literature 49*(3), 673–85.

Ramey, V. A. (2020). The macroeconomic consequences of infrastructure investment. NBER Working Paper.

Ramey, V. A. and S. Zubairy (2018). Government spending multipliers in good times and in bad: Evidence from us historical data. *Journal of Political Economy 126*, 850–901.

Suárez Serrato, J. C. and O. Zidar (2016). Who benefits from state corporate tax cuts? a local labor markets approach with heterogeneous firms. *American Economic Review 106*, 2582–2624.

Winberry, T. (2021). Lumpy investment, business cycles, and stimulus policy. *American Economic Review 111*, 364–396.

Zwick, E. and J. Mahon (2017). Tax policy and heterogeneous investment behavior. *American Economic Review 107*, 217–248.

Appendix: For online publication

Contents

A State-level data on infrastructure

Table A.1 summarizes the state-level data that is based on the highway infrastructure investment data from Bennett, Kornfeld, Sichel, and Wasshausen (2020).

State	Avg. Rank (Infra.)	# Good Group	Portion (Infra.)	Portion (GDP)	Avg. Rank (Estab.)	Portion (Estab.)
New York	1.708	24.000	0.072	0.081	2.439	0.072
California	1.833	24.000	0.071	0.133	1.000	0.114
Texas	2.458	24.000	0.071	0.079	2.659	0.069
Florida	4.000	24.000	0.064	0.050	4.000	0.060
Illinois	5.000	24.000	0.049	0.046	5.341	0.044
Ohio	6.542	24.000	0.035	0.036	7.000	0.039
New Jersey	7.125	24.000	0.034	0.034	8.415	0.033
Georgia	8.458	24.000	0.032	0.029	11.171	0.027
Pennsylvania	8.708	24.000	0.032	0.040	5.561	0.044
Massachusetts	9.708	24.000	0.030	0.027	12.341	0.024
Minnesota	10.458	24.000	0.029	0.018	18.561	0.019
North Carolina	12.208	24.000	0.025	0.027	9.976	0.028
Wisconsin	13.083	24.000	0.025	0.017	17.439	0.020
Washington	14.250	24.000	0.024	0.024	14.561	0.022
Virginia	14.458	24.000	0.024	0.027	12.585	0.025
Michigan	16.083	24.000	0.022	0.030	9.024	0.032
Tennessee	16.917	24.000	0.021	0.018	19.195	0.019
Missouri	18.167	24.000	0.019	0.018	15.171	0.021
Indiana	18.833	24.000	0.018	0.019	15.171	0.021
Kentucky	20.292	24.000	0.018	0.011	27.415	0.013
Louisiana	21.333	24.000	0.017	0.014	22.805	0.015
Iowa	21.625	24.000	0.017	0.010	28.951	0.012
Arizona	22.875	24.000	0.016	0.017	23.756	0.016
Colorado	25.625	15.000	0.015	0.017	19.439	0.018
Kansas	25.833	13.000	0.014	0.009	30.829	0.011
Alabama	26.000	23.000	0.015	0.012	24.951	0.014
Maryland	26.042	11.000	0.015	0.020	20.415	0.018
Connecticut	26.542	10.000	0.014	0.016	25.951	0.014
Oklahoma	29.458	0.000	0.012	0.010	27.634	0.013
Mississippi	30.208	0.000	0.011	0.006	33.317	0.009
Oregon	30.500	0.000	0.011	0.011	25.659	0.014
South Carolina	31.917	0.000	0.011	0.011	26.634	0.013
Nevada	33.083	0.000	0.010	0.008	38.000	0.006
Nebraska	34.417	0.000	0.010	0.006	34.927	0.007
Arkansas	34.708	0.000	0.010	0.007	32.439	0.009
New Mexico	35.542	0.000	0.010	0.006	37.000	0.006
West Virginia	37.000	0.000	0.009	0.004	37.244	0.006
Utah	38.375	0.000	0.008	0.007	34.122	0.008
Alaska	39.167	0.000	0.007	0.003	51.000	0.002
Hawaii	39.458	0.000	0.007	0.005	41.854	0.005
Idaho	41.667	0.000	0.006	0.004	40.512	0.005
Montana	41.958	0.000	0.006	0.002	42.512	0.004
Delaware	42.375	0.000	0.006	0.004	47.317	0.003
Wyoming	44.167	0.000	0.005	0.002	49.707	0.003
South Dakota	45.042	0.000	0.005	0.002	45.073	0.003
Rhode Island	46.083	0.000	0.004	0.003	42.963	0.004
Maine	47.208	0.000	0.004	0.004	39.098	0.005
North Dakota	47.500	0.000	0.004	0.002	47.146	0.003
New Hampshire	49.000	0.000	0.003	0.004	39.963	0.005
District of Columbia	50.000	0.000	0.002	0.007	47.927	0.003
Vermont	51.000	0.000	0.002	0.002	47.829	0.003

Table A.1: State-level summary

Notes: Avg. Rank (Infra.) is the average time-series ranking of infrastructure (this variable is the sorting variable). # Good Group is how many times the state belonged to the good infrastructure group (Max:24). Portion (Infra.) is the portion of infrastructure on average. Avg. Rank (Estab.) is the average time-series ranking of the number of establishments. Portion (Estab.) is the portion of establishments on average.

B Estimation details

B.1 Implementation of SMM in a Bayesian way

The limited-information Bayesian method, as described in Kim (2002) and later advocated by Christiano, Trabandt, and Walentin (2010) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016) among others, can be viewed as the Bayesian version of the simulated method of moments (SMM). The limited-information Bayesian method only uses a set of moments from the data for parameter inference.

Let Θ denote the parameters of interest and $\hat{\mathbf{m}}$ denote the vector of M empirical moments from the data for estimation. The likelihood of $\hat{\mathbf{m}}$ conditional on Θ is approximately

$$f(\hat{\mathbf{m}}|\Theta) = (2\pi)^{-\frac{M}{2}} |S|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\mathbf{m}} - \mathbf{m}(\Theta)\right)' S^{-1} \left(\hat{\mathbf{m}} - \mathbf{m}(\Theta)\right)\right],$$

where $\mathbf{m}(\Theta)$ is the model's prediction for the moments under parameter Θ, and S is the covariance matrix of $\hat{\mathbf{m}}$. The covariance matrix S is often unknown but can be replaced by a consistent estimator of it, which can be obtained through bootstrap. Bayes' theorem tells us that the posterior density $f(\Theta|\hat{\mathbf{m}})$ is proportional to the product of the likelihood $f(\hat{\mathbf{m}}|\Theta)$ and the prior density $p(\Theta)$:

$$f(\Theta|\hat{\mathbf{m}}) \propto f(\hat{\mathbf{m}}|\Theta)p(\Theta),$$

and we can then apply the standard Markov Chain Monte Carlo (MCMC) techniques to obtain a sequence of random samples from the posterior distribution.

B.2 Implementation of multiple-block Metropolis-Hastings

We use the multiple-block Metropolis Hastings algorithm to estimate the model parameters as well as finding market clearing prices. Let's denote the moments to match (including the market clearing conditions) as $y \equiv [\hat{\mathbf{m}}, \mathbf{0}]$. $\hat{\mathbf{m}}$ is for the moments constructed from the data and $\mathbf{0}$ is associated with solving for general equilibrium. We break the parameter space into two blocks as follows: $\Theta = (\Theta^1, \Theta^2)$ where Θ^1 is for the price block and Θ^2 is for the other model parameter block. Starting from an initial value $\Theta_0 = (\Theta_0^1, \Theta_0^2)$, the algorithm works as follows:

For iteration $j = 1, \ldots, M$, and for block $k = 1, 2$.

- Propose a value $\tilde{\Theta}^k$ for the kth block, conditional on Θ_{j-1}^k for the kth block and the current value of the other block (Θ^{-k}). Θ^{-k} stands for the remaining block except for the kth block.[1]

- Compute the acceptance probability $\alpha^k = \min\left\{1, \frac{f(\tilde{\Theta}^k|\Theta^{-k},y)}{f(\Theta_{j-1}^k|\Theta^{-k},y)}\right\}$.
  Update the kth block as

$$\Theta_j^k = \begin{cases} \tilde{\Theta}^k & \text{w.p. } \alpha^k \\ \Theta_{j-1}^k & \text{w.p. } (1-\alpha^k) \end{cases}$$

For each iteration, we first update the price block conditional on the previous iteration's value for the price block and the remaining model parameter block. Then we sequentially update the model parameter block conditional on the updated price block.

We apply the multiple-block RWMH algorithm to simulate draws from the posterior density $f(\Theta|\hat{\mathbf{m}})$ with uniform priors. The posterior distribution is character-

[1]In our application, $\Theta^{-1} = \Theta^2$ and $\Theta^{-2} = \Theta^1$.

ized by a sequence of 2000 draws after a burn-in of 2000 draws. We initialize the chain at the point estimate from particle swam optimization routine from MATLAB.

C Notes on the market clearing conditions

In the model, there are two centralized markets: the capital market and the labor market. Thus, there are two prices to be determined endogenously.

C.1 Interest rate and capital market

Define $p := U'(c(\mathcal{S}))$. Then, from the Euler equation of the representative household,

$$\beta \mathbb{E} \frac{U'(c(\mathcal{S}'))}{U'(c(\mathcal{S}))} = \frac{1}{1 + r^B(\mathcal{S})} \iff \beta \frac{p(\mathcal{S}')}{p(\mathcal{S})} = \frac{1}{1 + r^B(\mathcal{S})}$$

As there is no aggregate uncertainty, the expectation operator can be ignored.

Then, define a modified value function $\tilde{J}(z, k, j; \mathcal{S}) = p(\mathcal{S}) J(z, k, j; \mathcal{S})$.

In the following original recursive formulation,

$$\begin{aligned} J(z, k, j; \mathcal{S}) = &\max_{I, I^c} \; \pi(z, k, j; \mathcal{S})(1 - \tau^c)(1 - \tau^h) \\ &+ \int_0^{\bar{\xi}} \max\{-I - \xi w(\mathcal{S}) - C(I, k) \\ &+ \frac{1}{1 + r(\mathcal{S})} \mathbb{E} J(z', k', j'; \mathcal{S}'), \\ &- I^c - C(I^c, k) + \frac{1}{1 + r(\mathcal{S})} \mathbb{E} J(z', k^c; \mathcal{S}')\} dG(\xi) \end{aligned}$$

replace $\frac{1}{1+r(\mathcal{S})}$ with $\beta\frac{p(\mathcal{S}')}{p(\mathcal{S})}$. So we have,

$$
\begin{aligned}
J(z,k,j;\mathcal{S}) = \max_{I,I^c}\ & \pi(z,k,j;\mathcal{S})(1-\tau^c)(1-\tau^h) \\
& + \int_0^{\bar{\bar{\xi}}} \max\{-I - \xi w(\mathcal{S}) - C(I,k) \\
& + \beta\frac{p(S')}{p(\mathcal{S})}\mathbb{E}J(z',k',j';\mathcal{S}'), \\
& - I^c - C(I^c,k) + \beta\frac{p(S')}{p(\mathcal{S})}\mathbb{E}J(z',k^c;\mathcal{S}')\}dG(\xi)
\end{aligned}
$$

Then, multiply $p(\mathcal{S})$ to both sides. It leads to

$$
\begin{aligned}
p(\mathcal{S})J(z,k,j;\mathcal{S}) = \max_{I,I^c}\ & p(\mathcal{S})\pi(z,k,j;\mathcal{S})(1-\tau^c)(1-\tau^h) \\
+ \int_0^{\bar{\bar{\xi}}} \max\{ & -p(\mathcal{S})I - p(\mathcal{S})w(\mathcal{S})\xi \\
& - p(\mathcal{S})C(I,k) \\
& + \beta p(\mathcal{S}')J(z',k',j';\mathcal{S}'), \\
& - p(\mathcal{S})I^c - p(\mathcal{S})C(I^c,k) \\
& + \beta p(\mathcal{S}')J(z',k^c;\mathcal{S}')\}dG(\xi)
\end{aligned}
$$

Thus, we have

$$\begin{aligned}\tilde{J}(z,k,j;\mathcal{S}) = \max_{I,I^c}\ & p(\mathcal{S})\pi(z,k,j;\mathcal{S})(1-\tau^c)(1-\tau^h) \\ + \int_0^{\bar{\xi}} \max\{ & -p(\mathcal{S})I - p(\mathcal{S})w(\mathcal{S})\xi \\ & - p(\mathcal{S})C(I,k) \\ & + \beta\tilde{J}(z',k',j';\mathcal{S}'), \\ & - p(\mathcal{S})I^c - p(\mathcal{S})C(I^c,k) \\ & + \beta\tilde{J}(z',k^c;\mathcal{S}')\}dG(\xi)\end{aligned}$$

Therefore, a firm's problem is perfectly characterized by the price

$$p(\mathcal{S}) = U'(c(\mathcal{S})) = 1/c(\mathcal{S}).$$

C.2 Wage and labor market

From the representative household's intra-temporal optimality condition (with respect to the labor supply),

$$\eta L^{\frac{1}{\chi}} = U'(c(\mathcal{S}))w(\mathcal{S})(1-\tau^h)$$

Therefore,

$$\eta L^{\frac{1}{\chi}} = p(\mathcal{S})w(\mathcal{S})(1-\tau^h) \implies w(\mathcal{S}) = \frac{\eta}{p(\mathcal{S})(1-\tau^h)}L^{\frac{1}{\chi}}$$

The optimal labor supply L depends upon w, and w can be determined only when the labor supply L is known, leading to a fixed-point problem. Therefore, w needs to be tracked together with p for the computation.[2]

D Equilibrium

In the stationary recursive competitive equilibrium, the interest rate and the wage are determined in the competitive market. Specifically, the following market clearing conditions determine each price.[3]

$$\text{[Capital]} \quad \int \underbrace{\mathbb{E}J(z', k'(z,k); \mathcal{S})d\Phi}_{\text{Capital demand}} = \underbrace{a'(a; \mathcal{S})}_{\text{Capital supply}}$$

$$\text{[Labor]} \quad \underbrace{\int \left(n(z,k,j;\mathcal{S}) + \left(\frac{\min\{\xi^*, \bar{\xi}\}^2}{2\bar{\xi}} \right) \right) d\Phi}_{\text{Private labor demand}} = \underbrace{L(a; \mathcal{S})}_{\text{Labor Supply in the private market}}$$

The aggregate dividend is a sum of individual after-corporate-tax operating profits net of investment, and the ex-dividend portfolio value $P(\mathcal{S})$ is a sum of all the firms' values after the dividend payout:

$$\begin{aligned} \text{[Aggregate Dividend]} \quad D(\mathcal{S}) = \int \Big(& \pi(z,k,j;\mathcal{S})(1-\tau^c) \\ & - I^*(z,k,j;\mathcal{S}) - C(I^*(z,k,j;\mathcal{S}),k) - \mathbb{I}\{I^* \notin \Omega(k)\} w(\mathcal{S})\xi \Big) d\Phi \end{aligned}$$

$$\text{[Ex-dividend Portfolio Value]} \quad P(\mathcal{S}) = \int J(z,k,j;\mathcal{S})d\Phi - D(\mathcal{S})$$

And the government budget constraint and the spending constraint clear:

[2] If $\chi \to \infty$, p is the only price to be tracked as in Khan and Thomas (2008).

[3] On the private labor demand side, overhead labor demand is computed by multiplication of the probability of implementing lumpy investment $\frac{\min\{\xi^*, \bar{\xi}\}}{\bar{\xi}}$ and the conditional expectation $\frac{\min\{\xi^*, \bar{\xi}\}}{2}$, where $\xi^* = \xi^*(z,k,j;\mathcal{S})$ is the threshold rule for making lumpy investments, as in Khan and Thomas (2008).

$$[\text{Government Budget}] \quad \mathcal{G}(\mathcal{S}) = \tau^h(w(\mathcal{S})L(a;\mathcal{S}) + D(\mathcal{S})) + \int \tau^c(\pi(z,k,j;\mathcal{S}) - \delta k)d\Phi + \frac{B'}{1+r^B(\mathcal{S})} - B$$

$$[\text{Infrastructure Investment}] \quad \mathcal{F}(\mathcal{S}) = \varphi(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E})$$

$$[\text{Lump-sum Subsidy}] \quad \mathcal{T}(\mathcal{S}) = (1-\varphi)(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E})$$

From the law of motion of the infrastructure, the stationary infrastructure stock is obtained.[4]

$$[\text{Infrastructure}] \quad \mathcal{N}_A = \frac{1+\sqrt{1-2\mu\delta_{\mathcal{N}}}}{2\delta_N}\mathcal{F}(\mathcal{S}), \qquad \mathcal{N}_j = \zeta_j\mathcal{N}_A \quad \text{for } j \in \{P, G\}$$

Lastly, there is no arbitrage between the wealth return and the bond return.

$$[\text{No Arbitrage}] \quad r(\mathcal{S}) = r^B(\mathcal{S})$$

E Fiscal multipliers and corporate taxation

In this section, we compare the fiscal multipliers when the infrastructure spending is combined with different tax policies. Three different policies are considered. The first policy is decreasing the corporate tax rate by 33% from the baseline level (27%→18%). The second policy uses the baseline level (27%), and the last policy increases the corporate tax rate by 33% from the baseline level (27%→36%).[5] The remaining balance in the fiscal budget after the change in taxation is financed by the lump-sum tax. Thus, the third policy collects the least amount of lump-sum

[4]There are two fixed points for the stationary infrastructure stock. We focus only on the greater one, which is a stable fixed point.

[5]The third policy mimics the Biden administration's original plan to increase the corporate tax rate by 33%. As our baseline tax level is 27% while the corporate tax rate of 2022 is 21%, there is a level difference in the tax rate.

tax among the three policies.[6]

Table E.2: Fiscal multipliers

Fiscal multipliers	Low Tax	Baseline	High Tax
Output			
Short-run	1.2267	1.0878	0.9517
Long-run	2.1738	1.9206	1.6721
Short-run (2 years)			
Consumption	0.1014	0.1479	0.1933
Investment	0.0891	-0.0434	-0.1734
Public capital	1.6683	1.6695	1.6709
Labor income	0.8764	0.7134	0.5550
Long-run (5 years)			
Consumption	1.0035	0.9376	0.8727
Investment	0.1245	-0.0137	-0.1495
Public capital	4.0092	4.0102	4.0114
Labor income	1.6049	1.3636	1.1281

Table E.2 reports the fiscal multipliers across the three corporate tax policies. In the first policy with low corporate tax, the short-run multiplier is around 1.227, which is the greatest among the three. In the last policy with high corporate tax, the short-run multiplier is around 0.952, which is the lowest among the three. The same ranking is observed for the long-run multipliers.

One of the main channels that cause the differences in the fiscal multipliers is the firm-level investment. When the fiscal spending is combined with the low corporate tax policy, due to the increased incentive of cumulating the future capital stock, the private investment crowds in, as can be seen from the positive investment multiplier of 0.089. However, in other cases, the greater public capital stock

[6]Our fiscal multiplier analysis is based on the impulse response to the MIT fiscal spending shock under perfect foresight. Therefore, the representative household becomes indifferent between lump-sum tax financing and debt financing as long as the lifetime income is unaffected. If the model considers household heterogeneity under the borrowing limit and frictional financial market, this indifference collapses, leading to divergent fiscal multipliers between tax financing and debt financing as in Hagedorn, Manovskii, and Mitman (2019).

crowds out the private capital investment. A similar pattern is observed in the long-run fiscal multipliers of private investment.

The differences in the response of private capital investment to the fiscal policy lead to the differences in the labor income response. The greater the private investment, the greater the employment effect on the economy. In the low corporate tax policy, the labor income multiplier is 0.876; in the baseline corporate tax policy, the labor income multiplier is 0.713; in the high corporate tax policy, the labor income multiplier is 0.555.

However, the low corporate tax policy is not a free lunch. The low corporate tax policy leads to the lowest consumption multiplier of 0.101 in the short run. This is because this tax policy requires the greatest lump-sum tax to finance the spending shock. This clearly shows what is the trade-offs in corporate tax policies; the low tax policy sacrifices the short-run welfare to achieve long-run welfare. In the long run, due to the private investment and labor income channels, the fiscal multiplier is the greatest for the low corporate tax policy.

F Assumptions for the cross-state analysis

In this section, we specify the assumptions to implement the cross-state analysis using the baseline model. As there is only a representative household in this economy, the state-level consumption is defined under the following assumptions:

- All the incomes are state-specific, and there is no cross-state transfer.
- Each equity is exclusively owned by the state's household.

- Bond holding and lump-sum subsidies are attributed to each state proportionately to the exogenous fiscal spending ratio.

Given these assumptions, the state-level consumption can be properly defined due to the separate budget clearing across the states. One can introduce two households in the model to capture Poor and Good households separately, but this can be done only at a high computational cost and the model complication.

G The marginal product of private and infrastructure capital

In this section, we assess the equilibrium level of the marginal product of private and public capital stock. Table G.3 shows the marginal product of private and infrastructure capital stocks for the entire economy (column 1), the Good state (column 2), and the Poor state (column 3). In this economy, due to the presence of the capital adjustment cost at the firm level, the marginal product of capital varies across the firms.[7] We use the average marginal product of capital for the analysis.

Table G.3: The marginal product of private and public capital

Marginal product of capital (MPK)	Aggregate	Good state	Poor state
Private	0.2840	0.3345	0.0426
Infrastructure	0.4799	0.5449	0.1691

The marginal product of public capital stock is substantially higher than the private counterpart. This shows that the current stock of public capital is less than

[7]The convex adjustment cost depends on the capital stock of the firm, which makes the marginal cost of investment different across the firms. This leads to the heterogeneous marginal product of capital stock in equilibrium.

the socially desired level.[8] Moreover, the public-to-private MPK ratio is more than twice greater in the Poor state than in the Good state. This shows that the relative shortage of public capital provision is more severe in the Poor state than in the Good state in equilibrium. However, this relative shortage is an efficient outcome in the model, so the normative interpretation is limited.

H The role of time to build

In this section, we analyze the role of time to build on the fiscal multiplier. On top of the one-year time to build in the baseline, we assume there is an extra year of time to build for capital stock to be utilizable after the investment as in Ramey (2020) (two years, in total). Therefore, the law of motion of the public capital stock is as follows:[9]

$$\mathcal{N}_{A,t+2} = \mathcal{N}_{A,t+1}(1-\delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2}\left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t+1}}\right)^2 \mathcal{N}_{A,t+1}$$

$$\mathcal{N}_{j,t+1} = \zeta_j \mathcal{N}_{A,t+1} \quad \text{for } j \in \{P, G\}$$

$$F_t = \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}$$

where F^{ss} is the stationary equilibrium level of fiscal spending on infrastructure. Due to the time lag between the fiscal policy shock and the arrival of the public capital stock, there exists a news component in the policy, which will be analyzed further in this section.

For this analysis, the fiscal multiplier is measured by the sum of the present values over the first three years for the short run and over the six years for the long

[8] If there were a competitive market for public capital, the price of the public capital would adjust in the direction to equate the shadow value of private and public capital.

[9] For the consistency in the notation with the previous formulations, we leave the time index of the future public capital stock to be $t+1+s$ where $s = 1$.

Table H.4: Fiscal multipliers across the states under time to build of two years

Fiscal multipliers	T2B	T2B - PE	Baseline	Baseline - PE
Output				
Short-run	1.0280	2.0301	1.0878	1.8582
Long-run	1.8476	5.4015	1.9206	5.0314
Short-run (2 years)				
Consumption	0.1274	0.6414	0.1479	0.6052
Investment	-0.0726	0.2897	-0.0434	0.1892
Public capital	1.6327	1.6383	1.6695	1.6728
Labor income	0.6596	1.4504	0.7134	1.2935
Long-run (5 years)				
Consumption	0.9038	3.6506	0.9376	3.4216
Investment	-0.0435	0.5858	-0.0137	0.4769
Public capital	3.9734	3.9789	4.0102	4.0135
Labor income	1.3011	3.7605	1.3636	3.4781

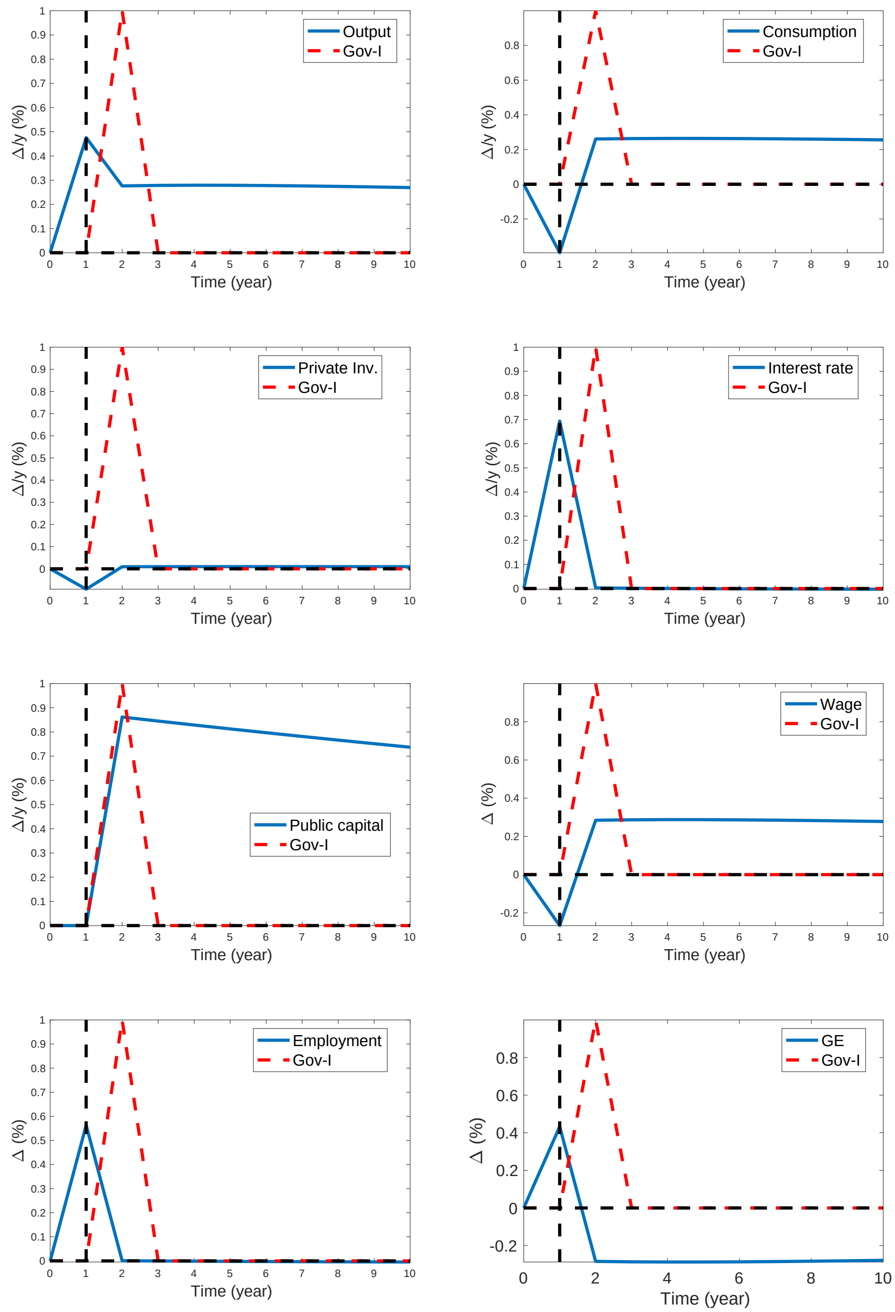
run after the initial fiscal spending shock.[10]

Table H.4 reports the fiscal multipliers when there is time to build of two years. The first column is the general equilibrium multipliers under the two years of the time to build; the second reports the same one in the partial equilibrium; the third is the baseline model; and the last is the baseline model in the partial equilibrium. The output fiscal multiplier decreases in general equilibrium, when the time-to-build is extended to two years (1.088→1.028), consistent with Ramey (2020). On the other hand, the output fiscal multiplier increases in the time to build in the partial equilibrium (1.858→2.030).

To illustrate the role of the extended time to build, Figure H.1 plots the impulse responses of equilibrium allocations. Due to the extended time to build, the capitalized government expenditure in the dashed line spikes one year after the beginning of the endogenous responses in the equilibrium allocations. As the fiscal

[10]Previously, it was 2 years for the short run and 5 years for the long run without the extended time to build.

Figure H.1: The impulse responses under the time to build of two years



spending shock hits, consumption immediately drops as the lump-sum tax immediately puts downward pressure on the household's consumption. This makes the household more willing to supply the labor. On the other hand, the production side does not face any change in the infrastructure until one year after the shock. Therefore, the increased labor supply at the period of shock ($t = 1$) leads to a lower wage and greater employment. Then, this feeds back into increased output at $t = 1$. The interest rate increases as the marginal utility of consumption at $t = 1$ increases, resulting in a decrease in private investment. After the infrastructure spending becomes capitalized, the demand for labor increases while the willingness for the labor supply decreases (income effect). This leads to an increase in the wage while the employment stays almost unchanged from the stationary equilibrium level.

The news effect impacts the fiscal multiplier in the partial equilibrium, as it allows the agents with the rational expectation to adjust their allocations optimally even before the spending shock is capitalized. However, this effect is dominated by changes in the price once we consider the general equilibrium effect. The agents' adjustment before the shock capitalization results in wage and interest rate adjustment, dampening the fiscal multiplier even in a greater magnitude than the one-year time-to-build. This is because the interest rate adjustment occurs at one time, and the increased cost of investment at the period before the spending shock leads to a lowered capital stock. Under the real friction such as the convex adjustment cost, the lowered capital stock leads to a greater adjustment cost in the following period when the fiscal spending shock is materialized, leading to a substantially dampened fiscal multiplier. Therefore, this is an outcome of the interaction between the news effect and the real friction.

I The representative-agent model: An extended version of Baxter and King (1993)

We consider the following representative-firm problem where the notations are the same as the baseline model except for ζ, which is the scale parameter for the infrastructure capital. It is worth noting that we use the same Φ level for the representative-agent model as in the baseline. This is to preserve the symmetry in the adjustment costs between the private sector and the public sector.[11] Also, the household and government sides are identical to the baseline model, so we abstract from the description for the sake of brevity.

$$
\begin{aligned}
J(k;S) = \max_{k'} \quad & (1-\gamma)\left(\frac{\gamma}{w(S)}\right)^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} N^{\frac{\zeta}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}} (1-\tau^c)(1-\tau^h) \\
& + (-k' + (1-\delta)k)(1-\tau^h) + \tau\delta k(1-\tau^h) \\
& - \frac{\mu}{2}\left(\frac{k'}{k} - (1-\delta)\right)^2 k(1-\tau^h) + \frac{1}{1+r(S)}\mathbb{E}J(k';S')
\end{aligned}
$$

where J is the value of the representative firm; S is the aggregate state that include the same components as the baseline model's aggregate state, except for the distribution of capital Φ replaced by the aggregate capital stock K. The first-order optimality conditions are as follows:

$$
[k'] : \quad \left(1 + \mu\left(\frac{k'}{k} - (1-\delta)\right)\right)(1-\tau^h) = \frac{1}{1+r(S)}\mathbb{E}J_1(k';S')
$$

[11]If the representative-agent economy's private capital adjustment cost is differently calibrated, it necessarily implies less or more efficient adjustment than the infrastructure capital adjustment. Also, it is not desirable to change the public capital adjustment cost parameter for the sake of a fair comparison of the fiscal multipliers across the models.

Also, from the envelope theorem, we have

$$[k]: \quad J_1(k;S) = \frac{\alpha}{1-\gamma}(1-\gamma)\left(\frac{\gamma}{w(S)}\right)^{\frac{\gamma}{1-\gamma}} A^{\frac{1}{1-\gamma}} N^{\frac{\zeta}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}-1}(1-\tau^c)(1-\tau^h)$$
$$+ (1-\delta+\tau^c\delta)(1-\tau^h) + \left(\frac{\mu}{2}\left(\frac{k'}{k}\right)^2 - \frac{\mu}{2}(1-\delta)^2\right)(1-\tau^h)$$

J Proof of Proposition 1

Proposition 1. *Suppose we are given the micro-level data set (k_1, k_2, y_1, y_2, N) s.t.*

$$\exists i \in \{1,2\} \text{ s.t. } k_i < N, \quad N \leq k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}.$$

Suppose the micro-level estimates (z, λ) and the aggregate-level estimate ξ are exactly identified by fitting the data with the production functions as follows:

$$f(k_1, N; \lambda, z) = y_1$$
$$f(k_2, N; \lambda, 1) = y_2.$$
$$f(k_1 + k_2, N; \xi, 1) = y_1 + y_2$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$, the aggregate-level input elasticity satisfies $\xi < 1$.

Proof. Without loss of generality suppose $k_1 > k_2$, $z > 1$, and let $k_2 < N$. From the production functions, we have

$$
\begin{aligned}
y_1 &= z^{\frac{1}{\alpha}} B(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\
y_2 &= B(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\
y_1 + y_2 &= B(\theta (k_1+k_2)^{\frac{\xi-1}{\xi}} + (1-\theta) N^{\frac{\xi-1}{\xi}})^{\frac{\xi}{\xi-1}}
\end{aligned}
$$

where $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Therefore, the following relationships hold (from the second and the third equations above):

$$
\begin{aligned}
\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} &= \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} \\
\left(\frac{y_1+y_2}{B(k_1+k_2)}\right)^{\frac{\xi-1}{\xi}} &= \theta + (1-\theta)\left(\frac{N}{k_1+k_2}\right)^{\frac{\xi-1}{\xi}}.
\end{aligned}
$$

Suppose we are given $\lambda \geq 1$. We will prove the proposition by contradiction, so we assume $\xi \geq 1$. As $N > k_2$, $\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1$. Thus,

$$
\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1.
$$

Hence, $\frac{y_2}{Bk_2} > 1$. From the condition $\frac{y_1}{k_1} = \frac{y_2}{k_2}$,

$$
1 < \frac{y_2}{Bk_2} = \frac{y_1+y_2}{B(k_1+k_2)}.
$$

As $\xi \geq 1$, we have

$$
1 < \left(\frac{y_1+y_2}{B(k_1+k_2)}\right)^{\frac{\xi-1}{\xi}} = \theta + (1-\theta)\left(\frac{N}{k_1+k_2}\right)^{\frac{\xi-1}{\xi}}.
$$

However, $N \leq k_1 + k_2$. Thus, $\left(\frac{N}{k_1+k_2}\right)^{\frac{\xi-1}{\xi}} \leq 1$. This leads to

$$\theta + (1-\theta)\left(\frac{N}{k_1+k_2}\right)^{\frac{\xi-1}{\xi}} \leq 1,$$

which is a contradiction. Therefore, if the micro-level input elasticity satisfies $\lambda \geq 1$, then the aggregate-level input elasticity satisfies $\xi < 1$. ■

K Proof of Proposition 2

Proposition 2. *Suppose we are given the micro-level data set (k_1, k_2, y_1, y_2, N) s.t.*

$$\exists i \in \{1,2\} \text{ } s.t. \text{ } k_i < N, \quad 1 < N \leq k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}.$$

Suppose the micro-level estimates (z, λ) and the aggregate-level estimate ζ are exactly identified by fitting the data with the production functions as follows:

$$f(k_1, N; \lambda, z) = y_1$$
$$f(k_2, N; \lambda, 1) = y_2.$$
$$h(k_1 + k_2, N; \zeta, 1) = y_1 + y_2$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$, the public capital scale parameter satisfies $\zeta > 0$.

Proof. Without loss of generality suppose $k_1 > k_2$, $z > 1$, and let $k_2 < N$. From the production functions, we have

$$
\begin{aligned}
y_1 &= z^{\frac{1}{\alpha}} B(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\
y_2 &= B(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\
y_1 + y_2 &= B(k_1 + k_2) N^{\frac{\zeta}{\alpha}}
\end{aligned}
$$

where $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Therefore, the following relationships hold (from the second and the third equations above):

$$
\begin{aligned}
\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} &= \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} \\
\frac{y_1 + y_2}{B(k_1 + k_2)} &= N^{\frac{\zeta}{\alpha}}.
\end{aligned}
$$

Suppose we are given $\lambda \geq 1$. We will prove the proposition by contradiction, so we assume $\zeta < 0$. As $N > k_2$, $\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1$. Thus,

$$
\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1.
$$

Hence, $\frac{y_2}{Bk_2} > 1$. From the condition $\frac{y_1}{k_1} = \frac{y_2}{k_2}$,

$$
1 < \frac{y_2}{Bk_2} = \frac{y_1 + y_2}{B(k_1 + k_2)}.
$$

Thus, we have

$$
1 < \frac{y_1 + y_2}{B(k_1 + k_2)} = N^{\frac{\zeta}{\alpha}},
$$

which is a contradiction, as $\zeta < 0$ and $N > 1$. Therefore, if the micro-level input elasticity satisfies $\lambda \geq 1$ under the non-rivalry, then the public capital scale parameter satisfies $\zeta > 0$ (Baxter and King, 1993). ■

L Simple theory with the continuum of firms

Proposition 3. *Suppose we are given the micro-level data set* (k_j, y_j, N)*,* $j \in [0,1]$ *s.t.*

$$\exists i \in [0,1] \text{ s.t. } k_i < N, \quad N \leq \int_0^1 k_j dj, \quad \frac{y_j}{k_j} = C \in \mathbb{R}.$$

where C *is a constant. Suppose the micro-level estimates* (z_j, λ) *and the aggregate-level estimate* ξ *are exactly identified by fitting the data with the production functions as follows:*

$$(Normalizer) \quad z_0 = 1$$
$$f(k_j, N; \lambda, z_j) = y_j$$
$$f\left(\int k_j dj, N; \xi, 1\right) = \int_0^1 y_j dj$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$*, the aggregate-level input elasticity satisfies* $\xi < 1$*.*

Proof. Without loss of generality suppose $k_0 < N$. From the production functions, we have

$$y_0 = B(\theta k_0^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$$
$$\int_0^1 y_j dj = B\left(\theta\left(\int k_j dj\right)^{\frac{\xi-1}{\xi}} + (1-\theta)N^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi}{\xi-1}}$$

where $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Therefore, the following relationships hold (from the second and the third equations above):

$$\left(\frac{y_0}{Bk_0}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_0}\right)^{\frac{\lambda-1}{\lambda}}$$
$$\left(\frac{\int_0^1 y_j dj}{B\left(\int_0^1 k_j dj\right)}\right)^{\frac{\xi-1}{\xi}} = \theta + (1-\theta)\left(\frac{N}{\int_0^1 k_j dj}\right)^{\frac{\xi-1}{\xi}}.$$

Suppose we are given $\lambda \geq 1$. We will prove the proposition by contradiction, so we assume $\xi \geq 1$. As $N > k_0$, $\left(\frac{N}{k_0}\right)^{\frac{\lambda-1}{\lambda}} > 1$. Thus,

$$\left(\frac{y_0}{Bk_0}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_0}\right)^{\frac{\lambda-1}{\lambda}} > 1.$$

Hence, $\frac{y_0}{Bk_0} > 1$. From the condition $\frac{y_j}{k_j} = C$,

$$1 < \frac{y_2}{Bk_2} = \frac{\int_0^1 y_j dj}{B\int_0^1 k_j dj}.$$

As $\xi \geq 1$, we have

$$1 < \left(\frac{\int_0^1 y_j dj}{B\left(\int_0^1 k_j dj\right)}\right)^{\frac{\xi-1}{\xi}} = \theta + (1-\theta)\left(\frac{N}{\int k_j dj}\right)^{\frac{\xi-1}{\xi}}.$$

However, $N \leq \int k_j dj$. Thus, $\left(\frac{N}{\int k_j dj}\right)^{\frac{\xi-1}{\xi}} \leq 1$. This leads to

$$\theta + (1-\theta)\left(\frac{N}{\int k_j dj}\right)^{\frac{\xi-1}{\xi}} \leq 1,$$

which is a contradiction. Therefore, if the micro-level input elasticity satisfies $\lambda \geq$

1, then the aggregate-level input elasticity satisfies $\xi < 1$. ■

Proposition 4. *Suppose we are given the micro-level data set (k_j, y_j, N), $j \in [0,1]$ s.t.*

$$\exists i \in [0,1] \; s.t. \; k_i < N, \quad 1 < N \leq \int_0^1 k_j dj, \quad \frac{y_j}{k_j} = C \in \mathbb{R}.$$

where C is a constant. Suppose the micro-level estimates (z_j, λ) and the aggregate-level estimate η are exactly identified by fitting the data with the production functions as follows:

$$(Normalizer) \quad z_0 = 1$$
$$f(k_j, N; \lambda, z_j) = y_j$$
$$h\left(\int k_j dj, N; \eta, 1\right) = \int_0^1 y_j dj$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$, the public capital scale parameter satisfies $\eta > 0$.

Proof. Without loss of generality suppose $k_0 < N$. From the production functions, we have

$$y_0 = B(\theta k_0^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$$
$$\int y_j dj = B\left(\int k_j dj\right) N^{\frac{\eta}{\alpha}}$$

where $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Therefore, the following relationships hold (from the second and the third equations above):

$$\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}}$$
$$\frac{\int y_j dj}{B \int k_j dj} = N^{\frac{\eta}{\alpha}}.$$

Suppose we are given $\lambda \geq 1$. We will prove the proposition by contradiction, so we assume $\eta < 0$. As $N > k_2$, $\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1$. Thus,

$$\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta)\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1.$$

Hence, $\frac{y_2}{Bk_2} > 1$. From the condition $\frac{y_1}{k_1} = \frac{y_2}{k_2}$,

$$1 < \frac{y_2}{Bk_2} = \frac{\int y_j dj}{B \int k_j dj}.$$

Thus, we have

$$1 < \frac{\int y_j dj}{B \int k_j dj} = N^{\frac{\eta}{\alpha}},$$

which is a contradiction, as $\eta < 0$ and $N > 1$. Therefore, if the micro-level input elasticity satisfies $\lambda \geq 1$ under the non-rivalry, then the public capital scale parameter satisfies $\eta > 0$ (Baxter and King, 1993). ■

References

BENNETT, J., R. KORNFELD, D. SICHEL, AND D. WASSHAUSEN (2020): "Measuring Infrastructure in the Bureau of Economic Analysis National Economic Accounts," BEA Working Paper.

CHRISTIANO, L. J., M. TRABANDT, AND K. WALENTIN (2010): "DSGE Models for Monetary Policy Analysis," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3, pp. 285–367. Elsevier.

FERNÁNDEZ-VILLAVERDE, J., J. RUBIO-RAMÍREZ, AND F. SCHORFHEIDE (2016): "Solution and Estimation Methods for DSGE Models," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2, pp. 527–724. Elsevier.

HAGEDORN, M., I. MANOVSKII, AND K. MITMAN (2019): "The Fiscal Multiplier," NBER Working Paper.

KHAN, A., AND J. K. THOMAS (2008): “Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics,” *Econometrica*, 76, 395–436.

KIM, J.-Y. (2002): “Limited Information Likelihood and Bayesian Analysis,” *Journal of Econometrics*, 107, 175–193.

RAMEY, V. A. (2020): “The Macroeconomic Consequences of Infrastructure Investment,” NBER Working Paper.