MOOC Learnersourcing Experiment

July 2018

Proofs for Learning

In the learning phase, the student will be assigned to one of these four conditions:

1. **Expert Example**: Student watches a video of an expert (Keith) going through the proof, like they do in the MOOC.
2. **Passive**: Student passively reads three peer proofs along with a rubric score for each proof.
3. **Active**: Student evaluates three peer proofs using the rubric.
4. **Constructive**: Student reads three peer proofs and uses ideas from the three examples to construct their own proof.

The following question (along with the peer or expert proofs) will be used in all of the four conditions:

Sum of Odd Fibonacci Numbers

The *Fibonacci sequence* is defined iteratively by setting $F_1 = F_2 = 1$ and thereafter letting $F_{n+2} = F_n + F_{n+1}$.

Theorem: For any natural number n,

$$\sum_{k=1}^{n} F_{2k-1} = F_{2k}$$

Test Questions

Constructing a Proof for the Sum of Fibonacci Sequence

Question: Write a proof for the following theorem:
The *Fibonacci sequence* is defined iteratively by setting $F_1 = F_2 = 1$ and thereafter letting $F_{n+2} = F_n + F_{n+1}$.

Theorem: For any natural number n,

$$\sum_{k=1}^{n} F_k = F_{k+2} - 1$$

Sum of Even Fibonacci Numbers

Question: What is the sum of the even Fibonacci numbers?
Question: How did you derive your answer?

Spotting the Error

Question: Is the following proof correct? If not, what is wrong with it?
Claim: For any natural number n,

$$\sum_{k=1}^{n} 2k = n^2 + n + 1$$

Proof: We will proceed by induction. Suppose the statement is true for n. We have that

$$\begin{aligned}\sum_{k=1}^{n+1} 2k &= \sum_{k=1}^{n} 2k + 2(n+1) \text{ by removing the last term}\\ &= n^2 + n + 1 + 2(n+1) \text{ by the induction hypothesis}\\ &= n^2 + 3n + 3 = (n^2 + 2n + 1) + (n+1) + 1 \text{ by rearranging terms}\\ &= (n+1)^2 + (n+1) + 1\end{aligned}$$

Thus the induction hypothesis holds for $n+1$. Hence we have proved the claim by induction.

Evaluating Proofs

Evaluate two proofs of varying quality.