Exploiting Factorization For Scaling Decision-Theoretic Probabilistic Programs

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Abstract TBW

Definition 1. *A **branch-and-bound algebra** is a semiring $\mathcal{S} = (S, +, \times, 0, 1)$ equipped with orders $(\leq, \sqsubseteq)$ such that:*

(1) $(S, \leq)$ is a total order,
(2) $(S, \sqsubseteq)$ is a join–semilattice with join $\sqcup$ where $\sqsubseteq$ respects $+, \times$,
(3) $\leq, \sqsubseteq$ are compatible in the sense that for all $a \sqsubseteq b$, we also have $a \leq b$.

Example 1. The nonnegative real numbers $\mathbb{R}^{\geq 0}$ forms a branch-and-bound algebra with the usual semiring structure of $\mathbb{R}^{\geq 0}$ and the usual order serving as both $\leq$ and $\sqsubseteq$, with join being the max function.

Example 2. The Boolean semiring $\mathbb{B} = \{\top, \bot\}$ with $+ = \vee, \times = \wedge, 0 = \bot, 1 = \top$ forms a branch-and-bound algebra with the order $\bot \leq \top$ with join $\wedge$.

Example 3. The expected utility semiring $\mathbb{R}^{\geq 0} \times \mathbb{R}$ with the usual semiring operations forms a branch-and-bound algebra with:

(1) $(p, u) \leq (q, v)$ iff $u \leq v$ or $u = v$ and $p \leq q$
(2) $(p, u) \sqsubseteq (q, v)$ iff $p \leq q$ and $u \leq v$, with join being a coordinatewise max.

It is straightforward to see that these are compatible.

Example 4. For any branch-and-bound algebra $\mathcal{B}$ consider the collection of finite sets with elements in $\mathcal{B}$, $\mathcal{P}_{<\omega}(\mathcal{B})$.

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