Scaling Decision—Theoretic Probabilistic Programming Through Factorization

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Probabilistic programs are designed to specify probabilistic models.

Probabilistic models can do more than inference, like decision making. Why can't probabilistic programs?

Goal: Let's design a probabilistic language that can reason about rational decision making under uncertainty!

What are our language specifications? Our semantics?

We construct a decision choosing between n choices by declaring `[a1,...,an]`, all fresh variables

We destruct a decision by declaring `choose d | a1 => e1 |...| an => en`

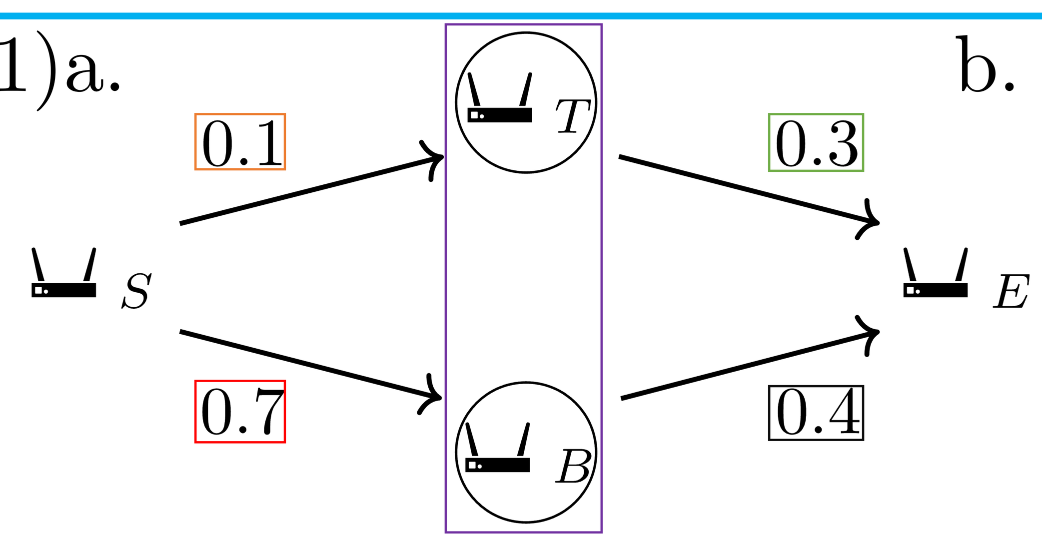
We add in reward primitives `reward k`, where `k` is a real number, to denote reward/cost

Two mutually recursive denotational semantics $[\![\cdot]\!]_{\mathrm{Pr}}, [\![\cdot]\!]_{\mathbb{E}U}$:

- $[\![\cdot]\!]_{\mathrm{Pr}}$ specifies the underlying probability distribution given the choices made
- $[\![\cdot]\!]_{\mathbb{E}U}$ specifies the accumulated reward/cost given the choices made, scaled by probability

1) A sample decision making scenario where investigate whether router T or B is faulty after a network failure. If we find the faulty router, we get a reward of 10. a. A graphical depiction, b. corresponding decision—theoretic probabilistic program, c. the compiled Boolean formula.
2) Selected denotational semantics and their respective types.
3) Selected compilation rules. Programs compile into a quadruple of the Boolean formula, an "accepting" function witnessing observations, a weight function on literals, and a set of accumulated reward variables.
4) A graph demonstrating promising initial results of our compilation and inference algorithm over ProbLog in a generalization of the example in 1). n denotes the number of vertices along the top or bottom route. D-constrained is best in this example but fails to support fast nested decision making.

1) a.



b.

```
let st, te = flip 0.1, flip 0.3 in
let sb, be = flip 0.7, flip 0.4 in
let toproute = if st then te else false in
let botroute = if sb then be else false in
let _ = observe(!toproute and !botroute) in
choose [t,b]
| t => if st and !te then reward 10 else reward 0
| b => if sb and !be then reward 10 else reward 0
```

c.

$$\begin{aligned}\varphi = &(\overline{toproute} \wedge \overline{botroute})\\ &\wedge [(t \wedge ((st \wedge \overline{te} \wedge r_{10}) \vee (\overline{st \wedge \overline{te}} \wedge r_0)))\\ &\vee (b \wedge ((sb \wedge \overline{be} \wedge r_{10}) \vee (\overline{sb \wedge \overline{be}} \wedge r_0)))]\\ &\wedge \mathrm{ExactlyOne}(t,b) \wedge \mathrm{ExactlyOne}(r_{10}, r_0)\end{aligned}$$

2)

$[\![\cdot]\!]_{\mathrm{Pr}} : \texttt{Programs} \to \texttt{Choices} \to (\{T,F\} \to [0,1])$

$[\![\texttt{reward}\ k]\!]_{\mathrm{Pr}}(\pi)(v) = \delta_T(v)$

$$[\![[a_1,\cdots,a_n]]\!]_{\mathrm{Pr}}(\pi)(v) = \begin{cases}\delta_T(v) & \exists i.\ a_i \in \pi\\ 0 & else\end{cases}$$

$[\![\texttt{choose}\ [a_1,\cdots,a_n]\ \{a_i \implies e_i\}]\!]_{\mathrm{Pr}}(\pi)(v) = [\![e_i]\!]_{\mathrm{Pr}}(\pi)(v)$ for i s.t. $a_i \in \pi$

$[\![\cdot]\!]_{\mathbb{E}U} : \texttt{Programs} \to \texttt{Choices} \to \mathbb{R}$

$[\![\texttt{reward}\ k]\!]_{\mathbb{E}U}(\pi) = k$

$[\![[a_1,\cdots,a_n]]\!]_{\mathbb{E}U}(\pi) = 0$

$[\![\texttt{choose}\ [a_1,\cdots,a_n]\ \{a_i \implies e_i\}]\!]_{\mathbb{E}U}(\pi) = [\![e_i]\!]_{\mathbb{E}U}(\pi)$ for i s.t. $a_i \in \pi$

$$[\![\texttt{let}\ x := e\ \texttt{in}\ e']\!]_{\mathbb{E}U} = \sum_{v' \in V} [\![e]\!]_{\mathrm{Pr}}(v')([\![e]\!]_{\mathbb{E}U}[\![e'[x \mapsto v']]\!]_{\mathrm{Pr}}(T) + [\![e'[x \mapsto v']]\!]_{\mathbb{E}U})$$

How do we make our language fast?

We compile to a Boolean formula where literals are weighted by the expectation semiring $\mathbb{R}^{\geq 0} \times \mathbb{R}$

This Boolean formula is then represented in a **factorized** manner in binary decision diagrams

We developed a **new, highly general branch-and-bound style algorithm** that computes the correct maximum expected utility/reward

Proven correct, implemented in Rust, outperforms ProbLog's default solver by >=10x on average

We prove an adequacy result: **maximizing the accumulated reward denotational semantics equals the maximum expected utility on the compiled Boolean formula. Moreover, the corresponding optimal decisions taken will also match.**

Proof is an induction on the Boolean compilation rules, such as the ones below!

3)

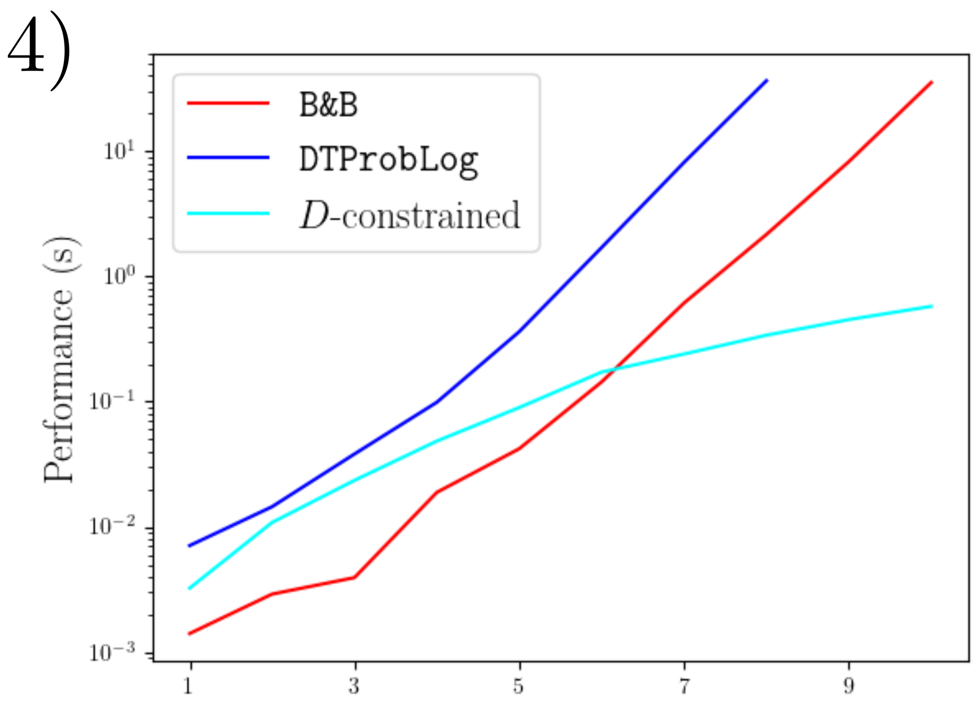
$$\frac{k \in \mathbb{R} \quad r_k \text{ fresh}}{\texttt{reward}\ k \rightsquigarrow (r_k, \top, (r_k \mapsto (1,k), \overline{r} \mapsto (1,0)), \{r_k\})}\ \texttt{bc/reward}$$

$$\frac{a_1, \cdots a_n \text{ fresh}}{[a_1,\cdots,a_n] \rightsquigarrow (\mathrm{ExactlyOne}(a_1,\cdots,a_n), \top, (a_i \mapsto (1,0), \overline{a_i} \mapsto (1,0)), \varnothing)}\ \texttt{bc/[]}$$

$$\frac{e \rightsquigarrow (\varphi, \gamma_1, w_1, R_1) \quad e' \rightsquigarrow (\psi, \gamma_2, w_2, R_2)}{\texttt{let}\ x := e\ \texttt{in}\ e' \rightsquigarrow (\psi[x \mapsto \varphi], \gamma_1 \wedge \gamma_2[x \mapsto \varphi], w_1 \cup w_2, R_1 \cup R_2)}\ \texttt{bc/let}$$

$$\frac{d \rightsquigarrow (\varphi, \top, w_d, \varnothing) \quad \forall i.\ e_i \rightsquigarrow (\psi_i, \gamma_i, w_i, R_i) \quad \forall i.\ a_i \in d}{\texttt{choose}\ d\ \{a_i \implies e_i\} \rightsquigarrow (\varphi \wedge \mathrm{ExactlyOne}(R_i) \wedge \bigvee_i (a_i \wedge \psi_i), \bigwedge_i(\gamma_i), w_d \cup \bigcup_i w_i, \bigcup_i R_i)}\ \texttt{bc/choose}$$

4)



What is future work?

Implementing the language with additional features to run more sophisticated benchmarks

Generalizing our branch-and-bound to work on a broader class of optimization problems in PPLs

Learn more at my DRAGSTERS talk:

If you have any questions or comments, contact me at minsung@ccs.neu.edu!