COE321K-HW1

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Question 1

[Equilibrium]

① $\Sigma F_x = A_x + AB \cdot \cos\theta = 0$
$\Sigma F_y = A_y + AB \cdot \sin\theta = 0$

② $\Sigma F_x = C_x - \frac{4}{5}CD = 0$
$\Sigma F_y = C_y - CD - \frac{3}{5}CD = 0$

③ $\Sigma F_x = AD - \frac{4}{5}BD = 0$
$\Sigma F_y = CD - F + \frac{3}{5}BD = 0$

$$\underbrace{F^{ext} = \begin{bmatrix} A_x \\ A_y \\ 0_x \\ 0_y \\ C_x \\ C_y \\ D_x \\ D_y \end{bmatrix}}_{8\times 1} \quad \underbrace{F_{int} = \begin{bmatrix} AB \\ BD \\ CD \end{bmatrix}}_{3\times 1} \quad \underbrace{f = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{4}{5} & 1 \\ 0 & -1 & -\frac{3}{5} \end{bmatrix}}_{8\times 3}$$

[Force-Deflection]

$$\underbrace{\begin{pmatrix} AB \\ BD \\ CD \end{pmatrix}}_{3\times 1} = \underbrace{\begin{pmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{EA}{L} & 0 \\ 0 & 0 & \frac{EA}{L} \end{pmatrix}}_{3\times 3} \underbrace{\begin{pmatrix} S_{bar} \\ S_{cable} \\ S_{rod} \end{pmatrix}}_{3\times 1}$$

$$[C_b]$$

[Kinematics]

$$\underbrace{\begin{pmatrix} S_{bar} \\ S_{cable} \\ S_{rod} \end{pmatrix}}_{3\times 1} = \underbrace{\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & -1 \end{bmatrix}}_{3\times 8} \underbrace{\begin{pmatrix} u_1^x = 0 \\ u_1^y = 0 \\ u_2^x = 0 \\ u_2^y = 0 \\ u_3^x = 0 \\ u_3^y = 0 \\ u_4^x \\ u_4^y \end{pmatrix}}_{8\times 1}$$

Note: $M_b = M^T$

$$\underbrace{[M]}_{3\times 8}\underbrace{[C_b]}_{3\times 3}\underbrace{[M_b]}_{3\times 8}\underbrace{(U_{nodes})}_{8\times 1} = \underbrace{(F^{ext}_{nodes})}_{8\times 1}$$

K-unstable

⇒ they are row all zeroes or ZERO. rank deficient.

⇒ Here, K_{nodes} is a 2x2 matrix

$$K = \begin{bmatrix} \frac{192\,EA}{7\,L} & 0 & -\frac{12\,EA}{7\,L} & 0 & -\frac{12\,EA}{7\,L} & 0 & 0 & 0 \\ 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{12\,EA}{7\,L} & 0 & \frac{48\,EA}{7\,L} & -\frac{EA}{L} & \frac{12\,EA}{7\,L} & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & -\frac{EA}{L} & \frac{EA}{L} & 0 & 0 & 0 & 0 \\ -\frac{12\,EA}{7\,L} & 0 & \frac{12\,EA}{7\,L} & 0 & \frac{192\,EA}{7\,L} & 0 & -\frac{12\,EA}{7\,L} & 0 \\ 0 & 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{12\,EA}{7\,L} & 0 & \frac{48\,EA}{7\,L} & -\frac{EA}{L} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$

$$K_{nodes} = \begin{bmatrix} \frac{192\,EA}{7\,L} & -\frac{12\,EA}{7\,L} \\ -\frac{12\,EA}{7\,L} & \frac{48\,EA}{7\,L} \end{bmatrix}$$

2x2

⇒ $u_{nodes} = \begin{pmatrix} u_1^x \\ u_1^y \end{pmatrix}$; $F^{ext}_{nodes} = \begin{pmatrix} 0 \\ -P \end{pmatrix}$

2x1 2x1

$$\begin{bmatrix} \frac{192\,EA}{7\,L} & -\frac{12\,EA}{7\,L} \\ -\frac{12\,EA}{7\,L} & \frac{48\,EA}{7\,L} \end{bmatrix} \begin{pmatrix} u_1^x \\ u_1^y \end{pmatrix} = \begin{pmatrix} 0 \\ -P \end{pmatrix}$$

$$\begin{pmatrix} u_1^x \\ u_1^y \end{pmatrix} = \left[K_{nodes}\right]^{-1} \begin{pmatrix} 0 \\ -P \end{pmatrix} = \frac{1}{EA}\begin{bmatrix} \frac{7}{3} & \frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} \end{bmatrix} \begin{pmatrix} 0 \\ -P \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} u_1^x \\ u_1^y \end{pmatrix} = \begin{pmatrix} -\frac{7}{4}\frac{P}{EA} \\ -\frac{7}{4}\frac{P}{EA} \end{pmatrix}$$

⇒ Now, from U^{ext}_{nodes} (U_{nodes})

$$(U_{nodes}) = \frac{P}{EA}\left(0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -\frac{7}{4} \;\; -\frac{7}{4}\right)^T$$

⇒ find internal forces:

$$[C_b][M_b](U_{nodes}) = (N_{bars})$$

$$\begin{bmatrix} 0.707107037037037 \cdot P \\ -0.222222222222222 \cdot P \\ 0.707107037037037 \cdot P \\ 0 \\ -0.222222222222222 \cdot P \\ 0.296296296296296 \cdot P \\ 0 \\ 0.707107037037037 \cdot P \\ 2.77555756156289 \cdot 10^{-17} \cdot P \\ -1.0 \cdot P \end{bmatrix}$$

⇒ find $(F_{int}) = [M](N_{bars}) - (F^{ext}_{nodes})$

$$\begin{bmatrix} 0.707107037037037 \cdot P \\ 0 \\ -0.222222222222222 \cdot P \\ 0.296296296296296 \cdot P \\ 0 \\ 0.707107037037037 \cdot P \\ 2.77555756156289 \cdot 10^{-17} \cdot P \\ -1.0 \cdot P \end{bmatrix}$$

→ compute Angles

$\sqrt{4+\frac{9}{4}}$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2}$

[Equilibrium]

Node A: $\Sigma F_x = A_x + AB \cdot \frac{2}{\sqrt{13}} = 0$
$\Sigma F_y = -AB - \frac{3}{2}AC = 0$

Node B: $\Sigma F_x = B_x + AB + \frac{2}{\sqrt{13}}BC = 0$
$\Sigma F_y = B_y + AB + \frac{3}{2}BC = 0$

Node C: $\Sigma F_x = \frac{4}{5}CE - \frac{2}{\sqrt{13}}BC - \frac{4}{5}BC = 0$
$\Sigma F_y = -CD + \frac{3}{5}BC - \frac{3}{5}BC - \frac{3}{5}CE = 0$

Node D: $\Sigma F_x = DE - AD = 0$
$\Sigma F_y = CD - P = 0$

Node E: $\Sigma F_x = -DE - \frac{4}{5}CE = 0$
$\Sigma F_y = E_y + \frac{3}{5}CE = 0$

⇒ Now, form: $[M](N_{bars}) - [F_{ext}]$

$$\begin{bmatrix} -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 \end{bmatrix} \underbrace{\begin{pmatrix} AB \\ AC \\ BC \\ BD \\ CD \\ CE \\ DE \end{pmatrix}}_{(N_{bars})_{9\times 1}} = \underbrace{\begin{pmatrix} A_x \\ A_y = 0 \\ B_x \\ B_y \\ C_x = 0 \\ C_y = 0 \\ D_x = 0 \\ D_y = -P \\ E_x = 0 \\ E_y \\ (F^{ext})_{8\times 1} \end{pmatrix}}_{8\times 1}$$

$[M]_{10\times 9}$

[Force-Bar-Deflection]

$$\underbrace{\begin{pmatrix} AB \\ AC \\ BC \\ BD \\ CD \\ CE \\ DE \end{pmatrix}}_{(N_{bars})_{9\times 1}} = \frac{EA}{L} \underbrace{\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}}_{[C_b]_{9\times 7}} \underbrace{\begin{pmatrix} S_{AB} \\ S_{AC} \\ S_{BC} \\ S_{BD} \\ S_{CD} \\ S_{CE} \\ S_{DE} \end{pmatrix}}_{(S_{bars})_{7\times 1}}$$

[Kinematics]

$$\underbrace{\begin{pmatrix} S_{AB} \\ S_{AC} \\ S_{BC} \\ S_{BD} \\ S_{CD} \\ S_{CE} \\ S_{DE} \end{pmatrix}}_{(S_{bars})_{7\times 1}} = \underbrace{[N_b]}_{7\times 10} \underbrace{\begin{pmatrix} u_1^x = 0 \\ u_1^y \\ u_2^x = 0 \\ u_2^y = 0 \\ u_3^x \\ u_3^y \\ u_4^x \\ u_4^y \\ u_5^x \\ u_5^y \end{pmatrix}}_{(U_{nodes})_{10\times 1}}$$

$\downarrow$

M^T

[Cable]

$$\underbrace{[M]}_{10\times 7}\underbrace{[C_b]}_{7\times 7}\underbrace{[N_b]}_{7\times 10}\underbrace{(U_{nodes})}_{10\times 1} = \underbrace{(F^{ext})}_{10\times 1} \quad \underbrace{\begin{pmatrix} A_x \\ A_y = 0 \\ B_x \\ B_y \\ C_x = 0 \\ C_y = 0 \\ D_x = 0 \\ D_y = -P \\ E_x = 0 \\ E_y \\ (F^{ext})_{8\times 1} \end{pmatrix}}_{10\times 1}$$

$[K]_{10\times 10}$

$$\begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

③ Compute rows 1, 3, 4, 10 as we don't know these external forces.

↳ this is because we want to do (U_{nodes}) and (F^{ext}_{int}) have to known values.

④ use a smaller step corresponding columns with these rows.

$$\begin{bmatrix} 0.500000000000000 & 0.500000 & 0 & 0 & 0 \\ 0.500000 & 0.500000 & 0 & 0 & 0 \\ 0 & 0 & 0.500000 & 0.500000 & 0 \\ 0 & 0 & 0 & 0.500000 & 0.500000 \\ 0 & 0 & 0 & 0 & 0.500000 \end{bmatrix}$$

$$\hookrightarrow [K_{nodes}]_{5\times 5} \quad \hookrightarrow \begin{pmatrix} u_1^x \\ u_1^y \\ u_2^x \\ u_2^y \\ u_3^x \\ u_3^y \\ u_4^x \\ u_4^y \end{pmatrix} = (U_{nodes}) \quad \hookrightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -P \\ -P \end{pmatrix} = (F^{ext}_{nodes}) \Rightarrow$$

now $U_1^x, U_1^y, U_3^x, U_3^y$ and U_5^x are zero, leaving only $U_2^x, U_2^y, U_4^x, U_4^y$ to be solved to F^{ext}_{nodes}

⇒ solve for (U_{nodes}):

$$\begin{bmatrix} 0.000000000000000 \cdot P & 0.000000000000000 \cdot P & 0.000000000000000 \cdot P & 0.000000000000000 \cdot P & 0.000000000000000 \cdot P \end{bmatrix}$$

$$(U_{nodes})^T = \begin{bmatrix} -0.000000000000000 \cdot P & 0 & 0 & 0 & 0 \end{bmatrix}$$

⇒ find internal forces:

$$[C_b][N_b](U_{nodes}) = (N_{bars})$$

$$(N_{bars})^T = \begin{bmatrix} 0.707107037037037 \cdot P & -0.333333333333333 \cdot P & 0.296296296296296 \cdot P & 1.0 \cdot P & -0.333333333333333 \cdot P & 0.296296296296296 \cdot P & 0 & -1.0 \cdot P & -0.333333333333333 \cdot P & 0.296296296296296 \cdot P \end{bmatrix}$$

⇒ find $(F^{ext}) = [M](N_{bars})$

$$(F^{ext})^T = \begin{bmatrix} -0.398412698765432 \cdot P \\ -0.555111512312578 \cdot 10^{-17} \cdot P \\ 0.398412698765432 \cdot P \\ 0.792857497857857 \cdot P \\ -0.555111512312578 \cdot 10^{-17} \cdot P \\ -0.588274839857857 \cdot P \\ 0 \\ -1.0 \cdot P \\ 2.22044604925031 \cdot 10^{-16} \cdot P \\ 0.297175925925926 \cdot P \end{bmatrix} \quad \begin{pmatrix} -0.398P \\ 0 \\ 0.398P \\ 0.792P \\ 0 \\ -0.588P \\ 0 \\ -1P \\ 0 \\ 0.297P \end{pmatrix}$$

3. Determine the result of the operation $\mathbf{A}^{-1}\mathbf{B}$, where,

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 1 & -2 \\ -1 & 5 & -2 & 1 \\ 2 & -3 & 2 & -1 \\ 1 & 2 & -1 & 3 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 6 \end{bmatrix}$$

↳ we are not a invertible if A is row-equivalent to I_n, and in that case any row operations that reduce A to I_n also transform I_n to A^{-1}

$$A^{-1}A = A^{-1}I = A^{-1}$$

1) construct augmented matrix $[A \mid I]$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ -1 & 5 & -2 & 1 & 0 & 1 & 0 & 0 \\ 2 & -3 & 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -1 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

2) turn A into I + apply same row-ops to I.

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & -2 & 0 & 1 & 0 \\ 0 & 4 & -2 & 5 & -1 & 0 & 0 & 1 \end{array}\right]$$

⇒ $R_2 + 2R_1$

⇒ $R_3 - 2R_1$

$$\xrightarrow{R_4 - R_1} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & -2 & 0 & 1 & 0 \\ 0 & 4 & -2 & 5 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 - 3R_2} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & 8 & -5 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -4 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_4 - 4R_2} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & 8 & -5 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -4 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_4 + \frac{2}{5}R_3} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & 8 & -5 & -1 & 1 & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \div -5} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{cccc|cccc} 1 & -2 & 0 & \frac{6}{5} & 0 & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 3 & -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{cccc|cccc} 1 & -2 & 0 & \frac{6}{5} & 0 & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 3 & 0 & -\frac{3}{5} & 2 & \frac{6}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_1 + 2R_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{12}{5} & 4 & \frac{7}{5} & \frac{2}{5} & 0 \\ 0 & 3 & 0 & -\frac{3}{5} & 2 & \frac{6}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \div 3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{12}{5} & 4 & \frac{7}{5} & \frac{2}{5} & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{2}{3} & \frac{2}{5} & -\frac{1}{15} & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{21}{5} & -\frac{18}{5} & -\frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_4 \div \frac{21}{5}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{12}{5} & 4 & \frac{7}{5} & \frac{2}{5} & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & \frac{2}{3} & \frac{2}{5} & -\frac{1}{15} & 0 \\ 0 & 0 & 1 & -\frac{8}{5} & 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & -\frac{6}{7} & -\frac{2}{7} & \frac{2}{21} & \frac{5}{21} \end{array}\right]$$

$$\xrightarrow{\substack{R_1 - \frac{12}{5}R_4 \\ R_2 + \frac{1}{5}R_4 \\ R_3 + \frac{8}{5}R_4}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{16}{7} & \frac{11}{7} & \frac{4}{7} & -\frac{12}{7} \\ 0 & 1 & 0 & 0 & \frac{4}{7} & \frac{4}{7} & -\frac{1}{7} & \frac{1}{7} \\ 0 & 0 & 1 & 0 & \frac{1}{7} & \frac{3}{7} & -\frac{1}{7} & \frac{8}{7} \\ 0 & 0 & 0 & 1 & -\frac{6}{7} & -\frac{2}{7} & \frac{2}{21} & \frac{5}{21} \end{array}\right]$$

$$\Rightarrow A^{-1} = \frac{1}{21}\begin{bmatrix} 48 & 33 & 12 & -36 \\ 12 & 12 & -3 & 3 \\ 3 & 9 & -3 & 24 \\ -18 & -6 & 2 & 15 \end{bmatrix}$$

⇒ compute: $A^{-1}B$:

$$A^{-1}B = \frac{1}{21}\begin{bmatrix} 48 & 33 & 12 & -36 \\ 12 & 12 & -3 & 3 \\ 3 & 9 & -3 & 24 \\ -18 & -6 & 2 & 15 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 3 \\ 6 \end{bmatrix} = \frac{1}{21}\begin{bmatrix} 48 + 0 + 36 - 216 \\ 48 + 0 + 9 + 18 \\ 12 + 0 - 9 + 144 \\ -72 + 0 + 6 + 90 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -\frac{132}{21} \\ \frac{75}{21} \\ \frac{147}{21} \\ -\frac{16}{21} \end{bmatrix}$$